Gyrokinetic Plasma-Sheath Transition

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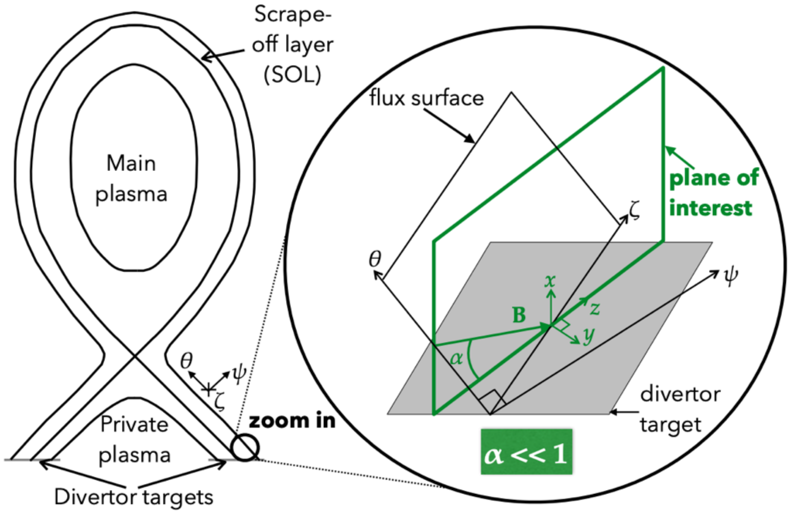
Tokamak Divertors

- Divertors handle the heat and particle exhaust for the tokamak.
- Crucial for protecting walls of fusion devices, pumping helium ash, impurities, etc.
- Can affect edge and core confinement.
- Sheath forms near divertor.



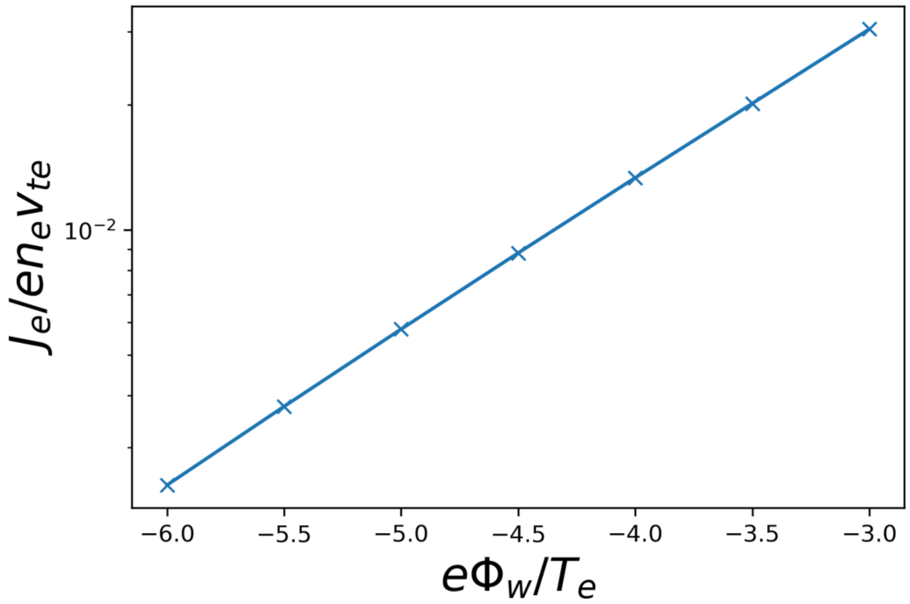
Importance for Electromagnetic Edge Simulations

- In the core, large parallel current perturbations J_e~ $en_e v_{te}$ can form from extended modes like MTMs [1] θ ~ $(m_i/m_e)^{1/2}$.

$$v_{\|}\hat{b}\cdot\nabla\theta\frac{\partial h_e}{\partial\theta}=\frac{eF_{0e}}{T_e}\omega_* J_0(k_\perp\rho_e)\phi$$

- But in SOL, electron current instead limited by wall potential.

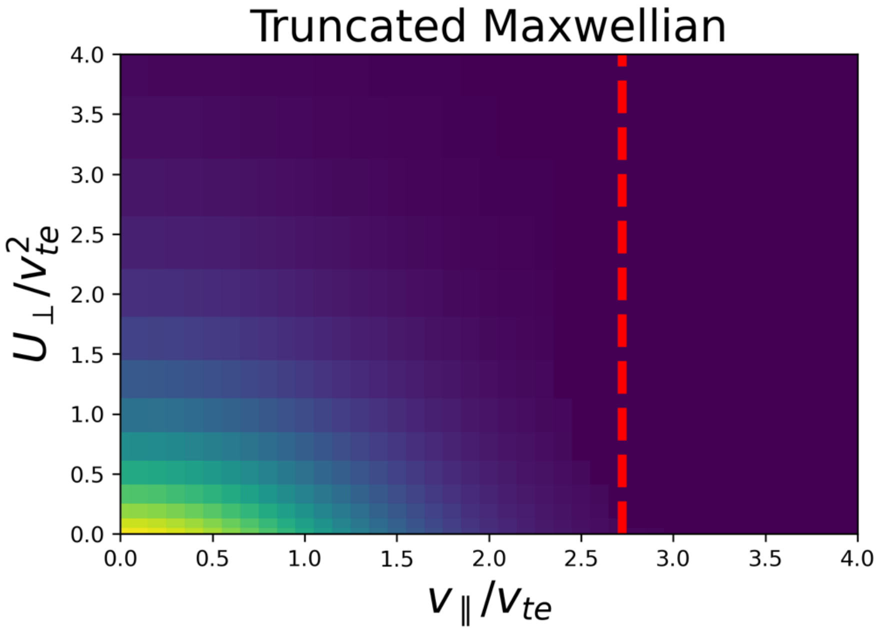
$$J_e \sim en_e v_{te}\exp\left(\frac{e\phi_w}{T_e}\right)$$



[1] Hardman et al, PPCF, 2023

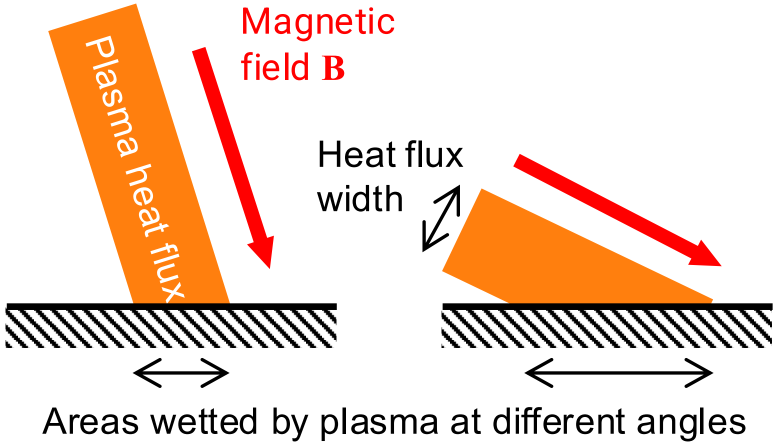
Importance for Heat Fluxes

- Sheaths control the electron heat flux by repelling most electrons.
- In conducting boundary conditions, electron cutoff determined purely by parallel energy $\frac{1}{2}\mathrm{m_e v_\parallel}^2 = \mathrm{e}\phi_w$.



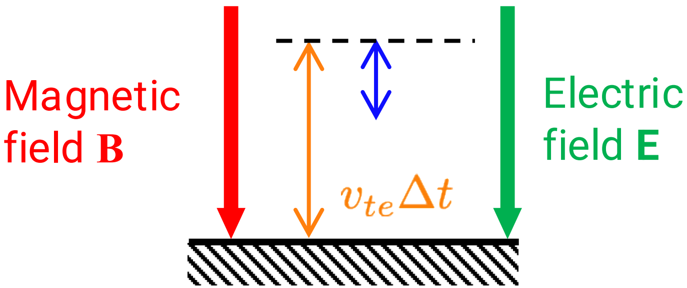
Shallow Angle Incidence

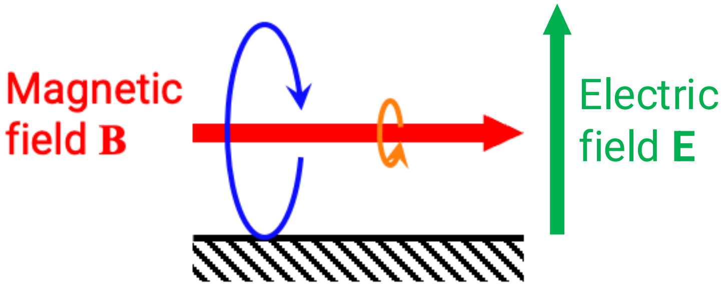
- B inclined at small angle α with respect to divertor.
- α ~ 2-5 degrees.
- Spreads out heat flux over a wider area, reducing damage to divertors.



Walls and Magnetized Plasmas

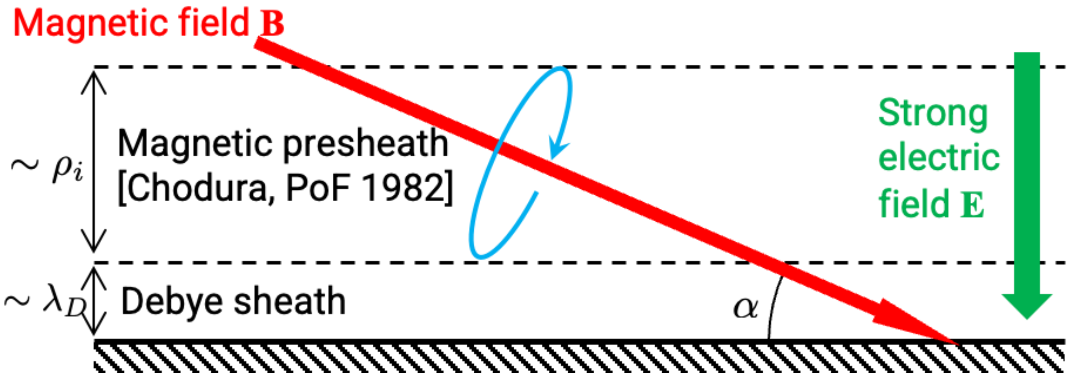
- Electric field in a magnetized sheath depends on the angle α that **B** makes with the wall
- If **B** is perpendicular to the wall, then wall is negatively-charged and **E** is electron-repelling since $v_{te} >> v_{ti}$.
- If **B** is parallel to the wall, then wall is positively-charged and **E** is ion-repelling since $\rho_i >> \rho_e$.
- If $\alpha >> (m_e/m_i)^{1/2}$, then wall is still negatively-charged.





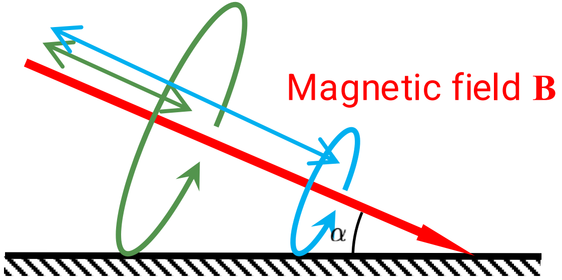
Boundary Layers in the Sheath

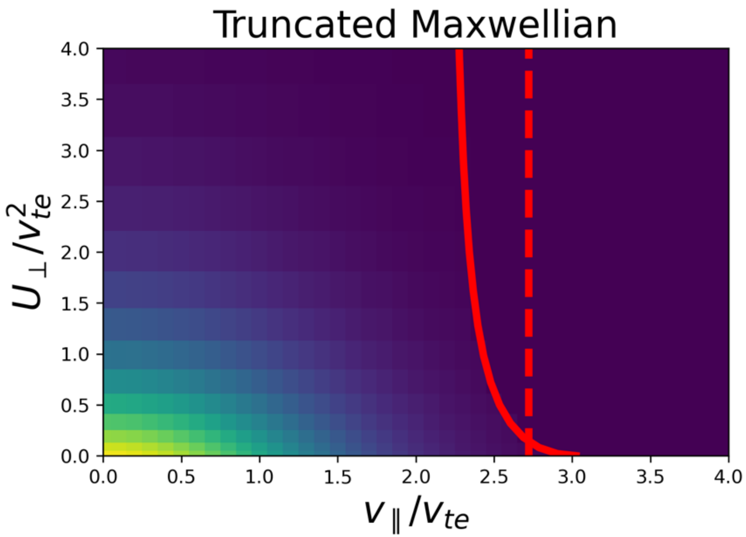
- Sheath has 2 main layers.
- Positively-charged Debye sheath ~ λ_D to shield plasma from wall potential.
- Quasi-neutral magnetic presheath ~ ρ_i to accelerate ions.
- Depending on collisionality, can also have a collisional presheath ~ $\alpha\lambda_{mfp}$



Electron FLR Effects

- ρ_e effects typically neglected.
- $\gamma = \rho_e / \lambda_D \sim 1$, so electron FLR effects important in the DS.
- Exact cutoff determined by perpendicular energy – electron with larger orbits can more easily reach wall.
- Can lead to substantial (30-40%) changes to heat fluxes [2].
- Will be important for non-monotonic potential profiles.

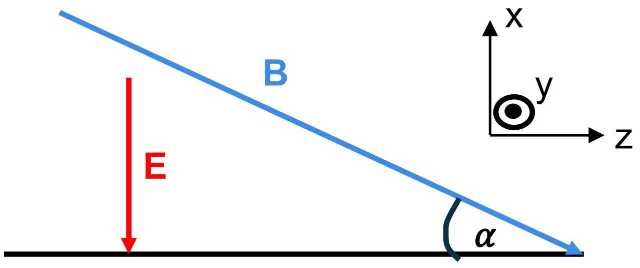




[2] Geraldini et al., 2025

Grazing Angle Gyrokinetic Model of the Sheath

Grazing-Angle Gyrokinetic Model of the Sheath



- Assume slab geometry, with constant |**B**|, and 1D **E**(x).
- Assume $\rho_e \sim \lambda_D << \rho_i << \alpha\lambda_{mfp}$.
- Dynamics are parallel streaming, gyromotion, and ExB drifts.

$$\dot{v_x} = -\frac{\Omega}{B}\frac{\partial\phi}{\partial x} + \Omega v_y \cos\alpha$$

$$\dot{v_y} = -\Omega v_x \cos\alpha - \Omega v_z \sin\alpha$$

$$\dot{v_x} = \Omega v_y \sin\alpha$$

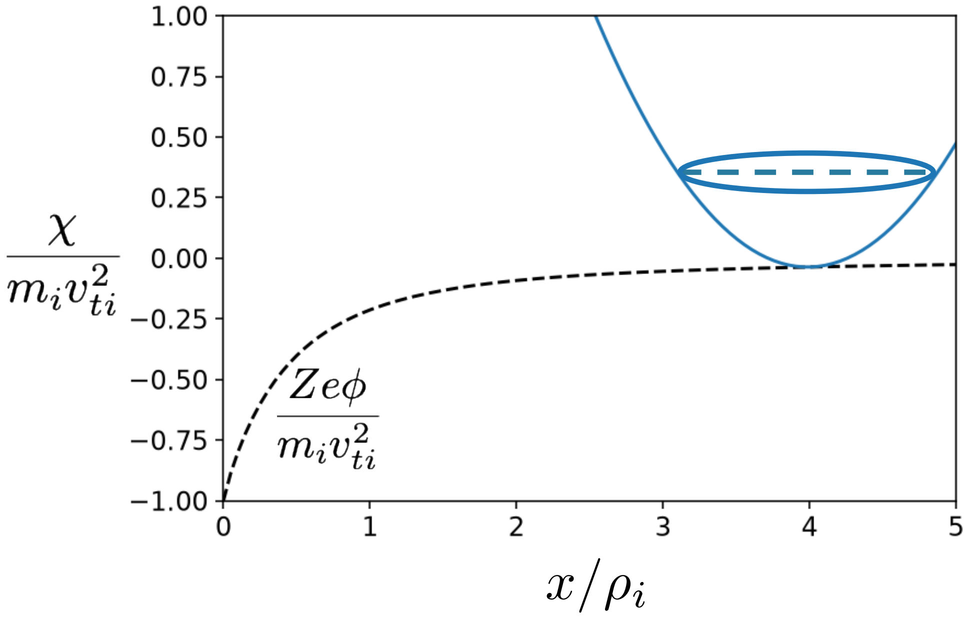
$$X = \frac{v_y}{\Omega_i} + x$$

- v_x can be written in terms of effective potential

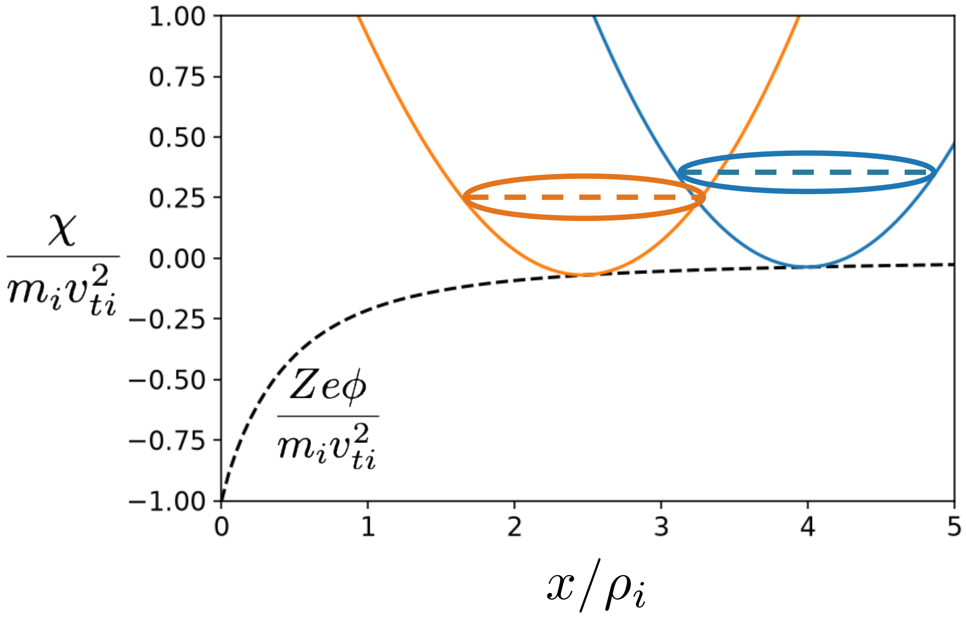
$$v_x = \sigma_x V_x = \sigma_x \sqrt{U_\perp - \chi(x, X)}$$

$$\chi(x, X) = \frac{1}{2}\Omega_i^2 (x - X)^2 + \frac{\Omega_i}{B}\phi(x)$$

- Far from the wall, corresponds to normal gyromotion

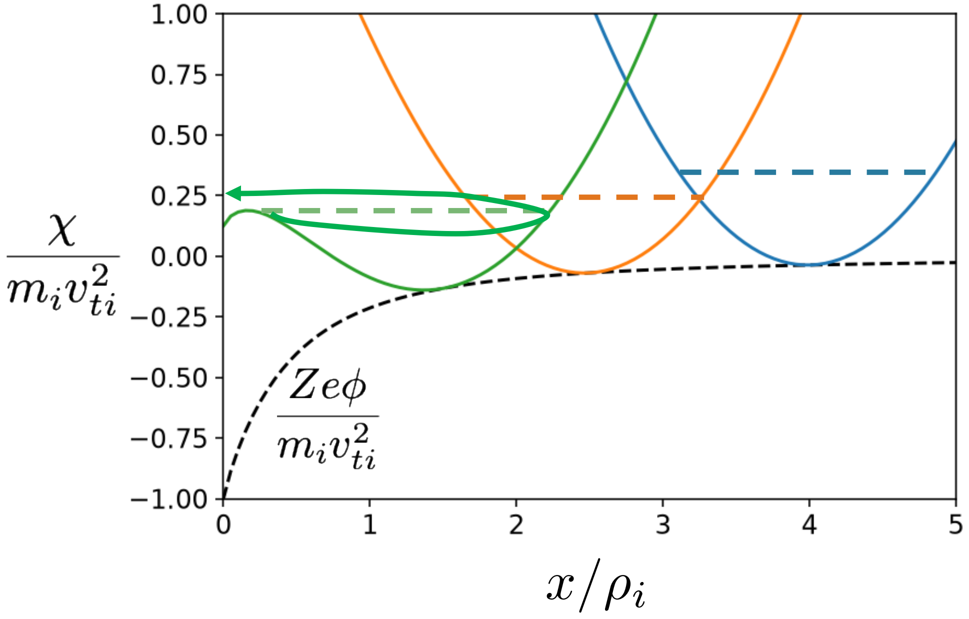


- As ions accelerate towards wall, $U_\perp$ and χ decrease.



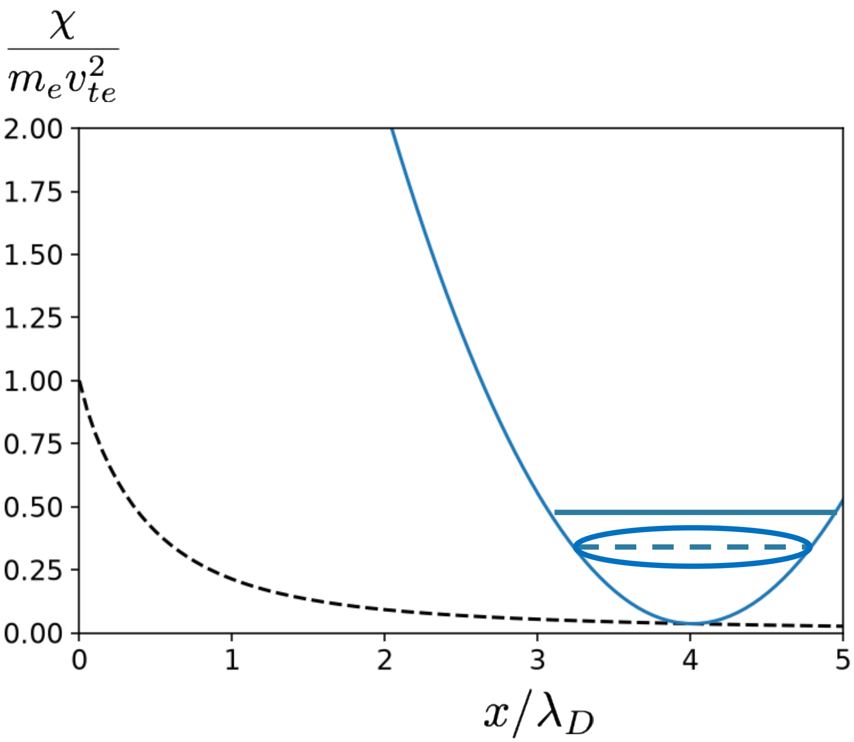
Ion Orbits in the Magnetic Presheath

- Eventually, $U_\perp = \chi_M$, and ion is lost to the DS.
- Corresponds to $F_E > F_B$, and gyro-orbits being "opened" up.
- Can treat ions in the DS with 1x1v model.



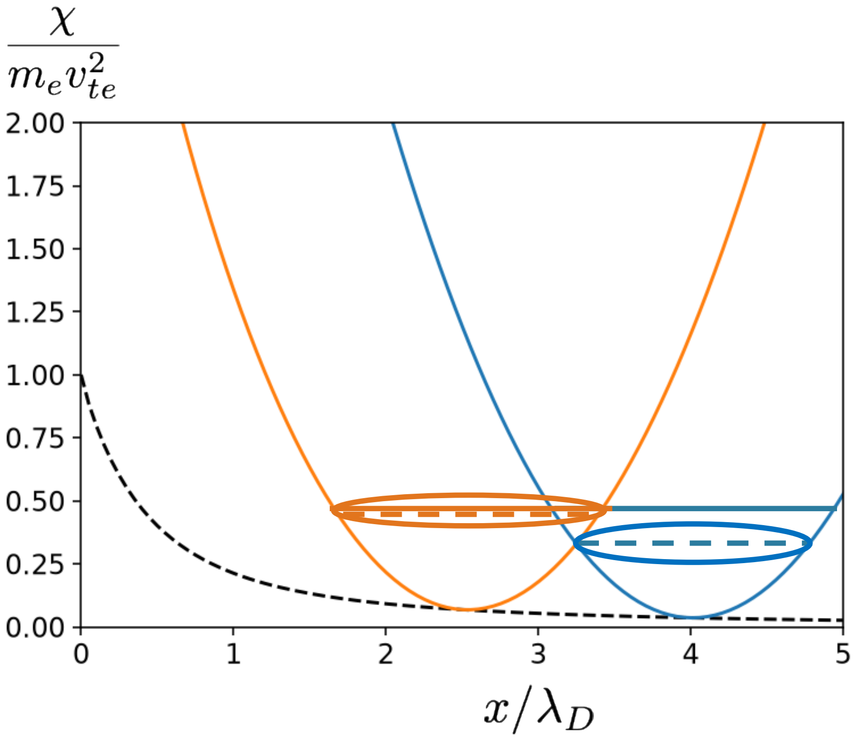
Electron orbits in the Debye sheath

- Normal gyromotion far from the wall.

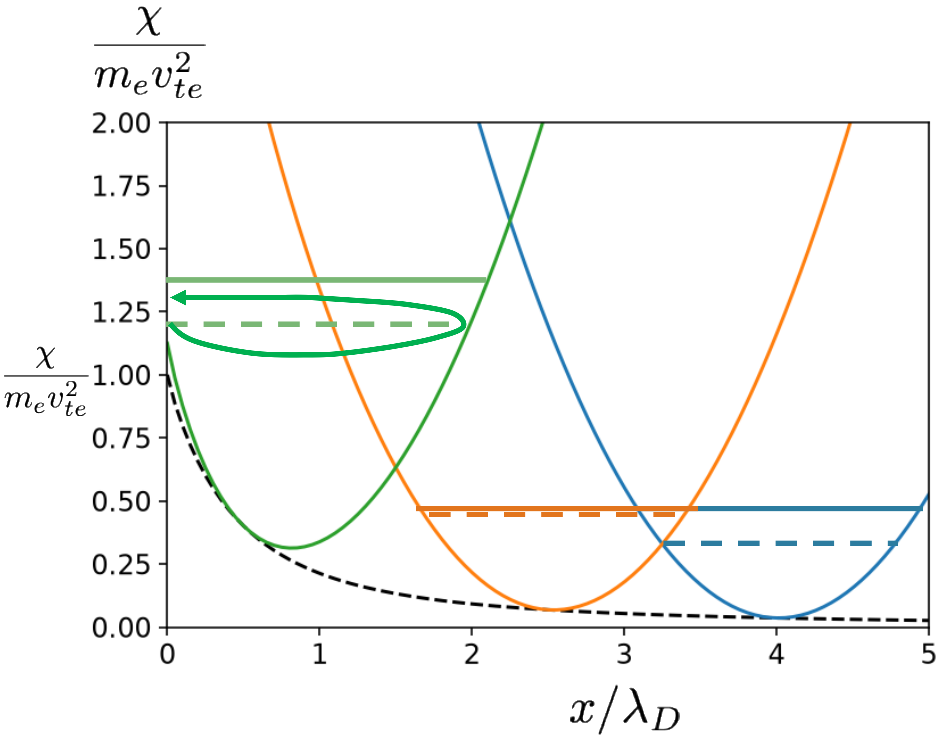


Electron orbits in the Debye sheath

- As electrons approach wall, $U_\perp$ increases.
- If $U_\perp = U$, electrons are reflected.



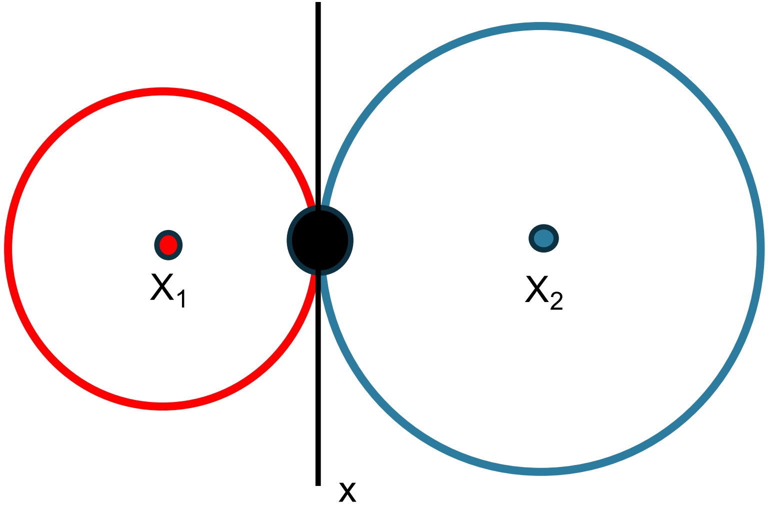
- If $U_\perp = \chi_M$, electrons hit wall.



(Closed) Electron Density in the Debye Sheath

- Particles with $U < U_\perp$ have already been reflected.
- Particles with $U_\perp > \chi_M$ are lost to wall.
- Integral over X represents FLR effects.
- From densities, GYRAZE calculates self-consistent potential using an iterative method.

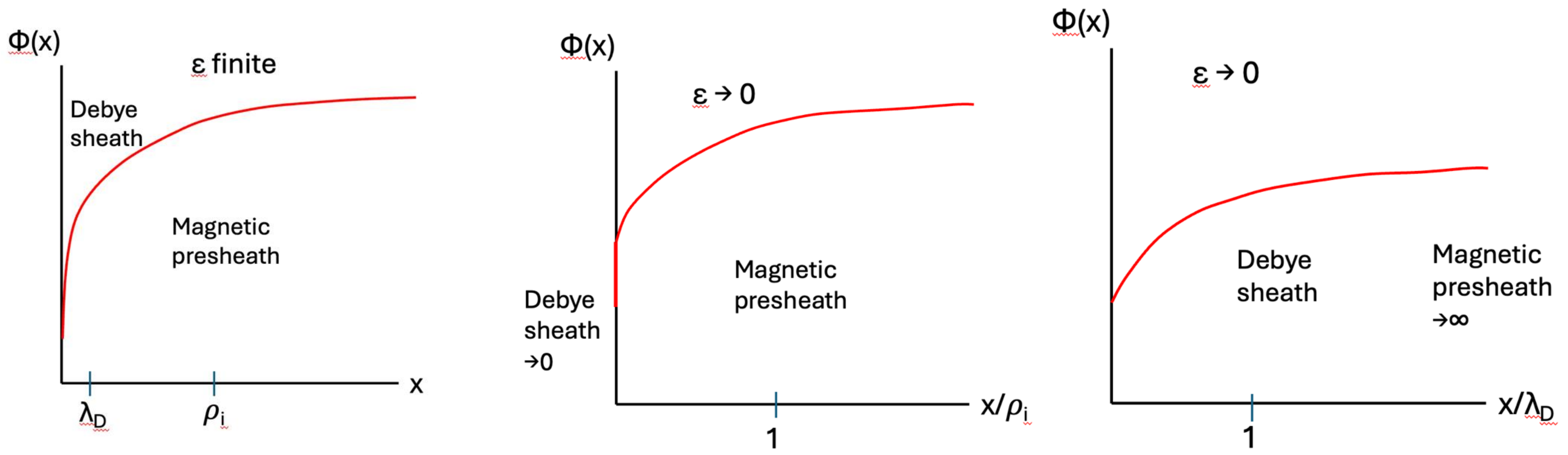
$$n_e = \sum_{\sigma_\parallel} \int_{X_m}^{\infty} dX \int_{\chi}^{\chi_M} \frac{2dU_\perp}{\sqrt{2(U_\perp - \chi)}} \int_{U_\perp}^{\infty} \frac{F\,\mathrm{dU}}{\sqrt{2(U - U_\perp)}}$$



Scale Separated Solution of the Sheath

- Assume solutions to the DS and MP can be obtained separately/iteratively, under the assumption $\varepsilon = \lambda_D/\rho_i \rightarrow 0$.

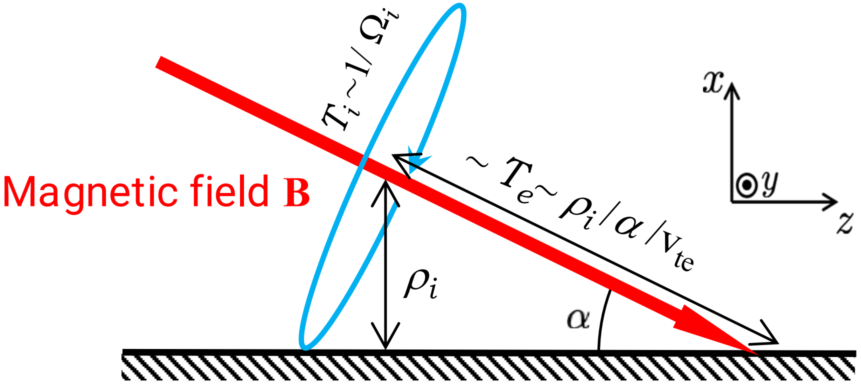
$$\epsilon^2 \frac{d^2\hat{\phi}}{dz^2} = \frac{1}{n_{ref}}\left(n_e[\hat{\phi}] - n_i[\hat{\phi}]\right)$$



Non-monotonic Potential Profiles

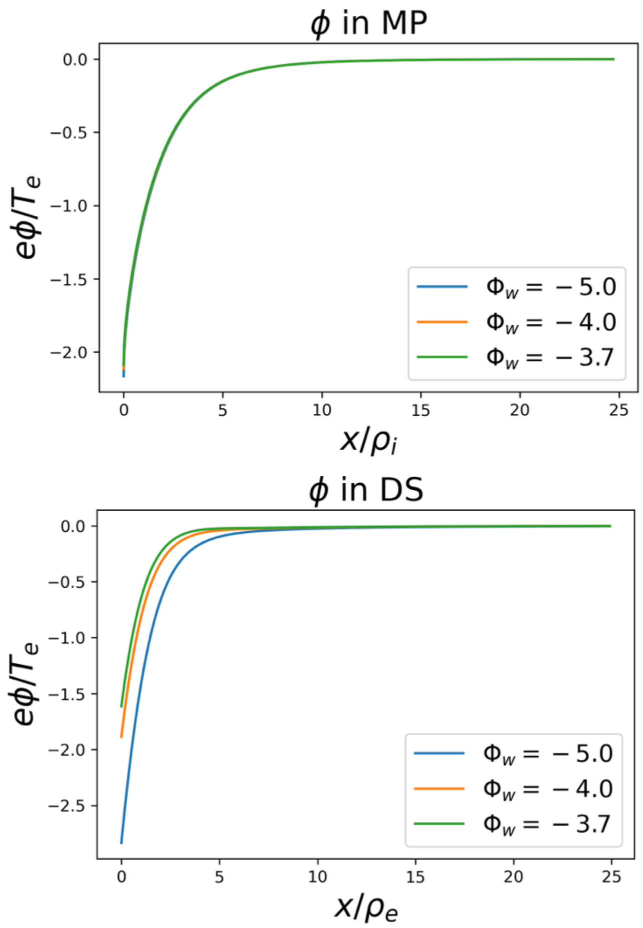
Critical Angles

- Would like to explore smaller α and ϕ_w.
- But potential can become non-monotonic!
- At $\alpha \sim (m_e/m_i)^{1/2}$, ion orbits may cause them to hit the wall before electrons.



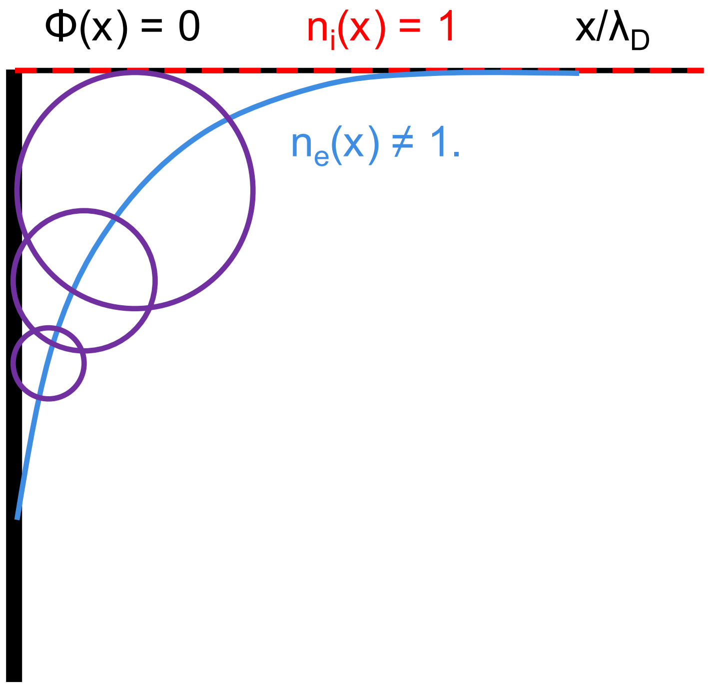
Critical Angles

- For fixed α, presheath drop roughly constant ~ ln(α).
- Decreasing ϕ_w decreases potential drop in DS.



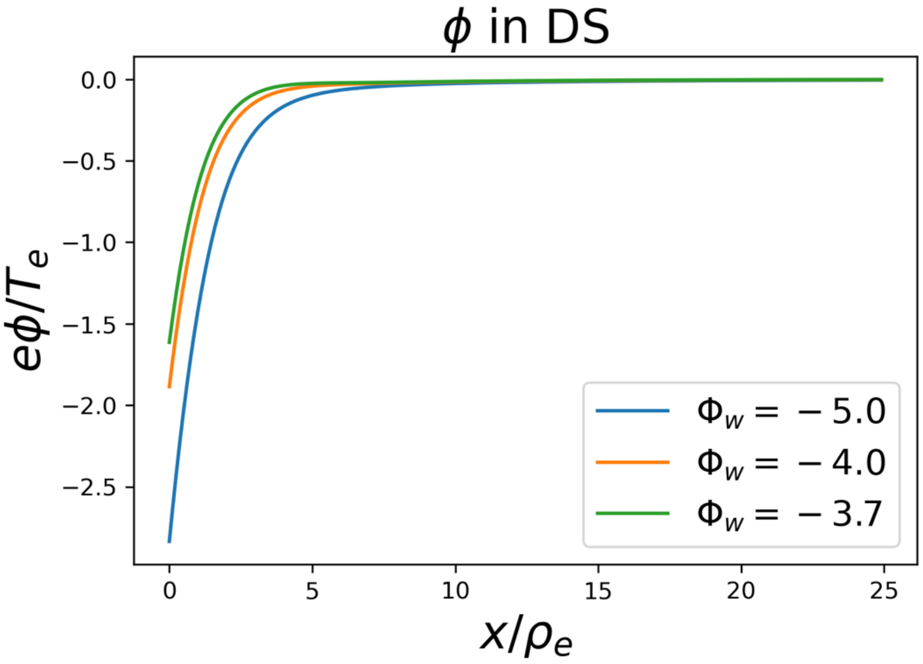
Critical Angles

- For finite ρ_e, there exists minimum DS potential drop ~ γ^2 [2].
- Finite drop caused by decrease in n_e as electrons intersect wall.
- Decreasing $\Delta\phi_{DS}$ below this means ϕ must become positive, and so non-monotonic.



[2] Geraldini et al, 2025

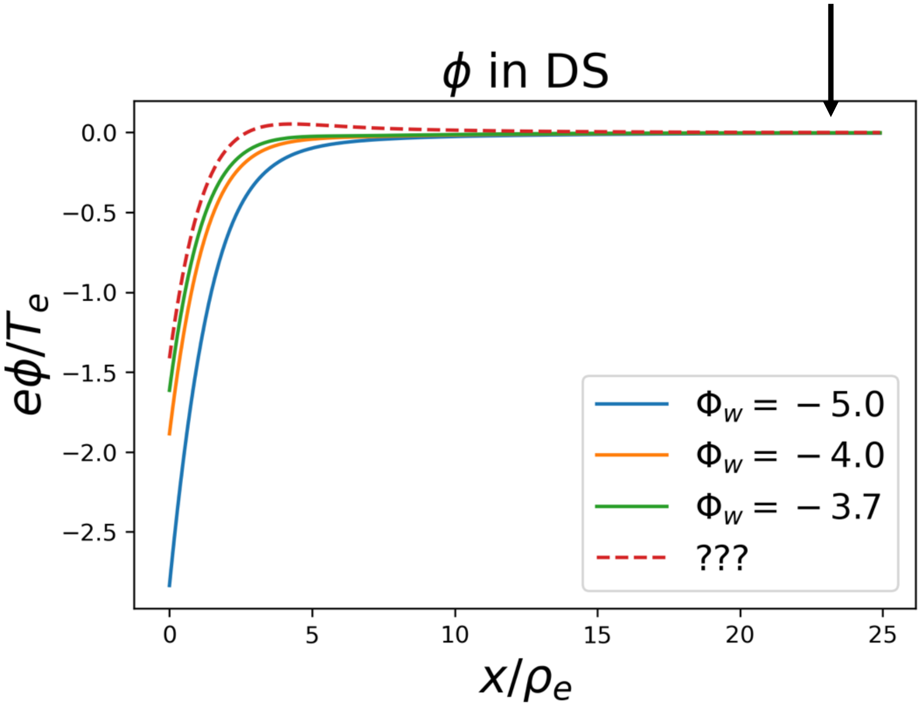
- Numerically, cannot converge below $\alpha_* \sim (\gamma m_e/m_i)^{1/2}$ [2].
- What happens as we approach this point?



[2] Geraldini et al, 2025

Critical Angles

- Numerically, cannot converge below $\alpha_* \sim (\gamma m_e/m_i)^{1/2}$.
- What happens as we approach this point?
- Does ϕ just jump to a positive non-monotonic solution?



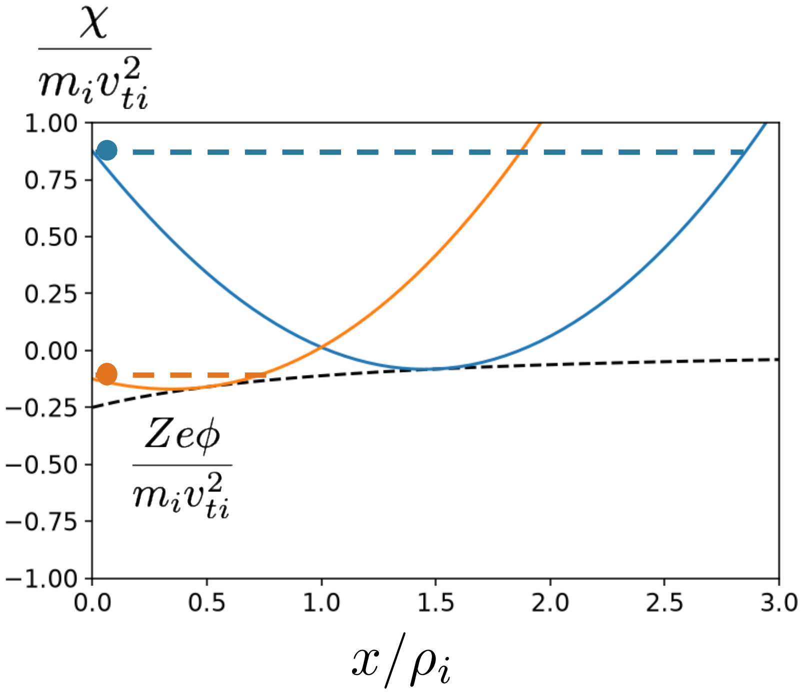
Potential at the Debye Sheath Entrance

- Taylor expanding n_i and n_e at the Debye sheath entrance ($x_{mp} \to 0$) yields [2].

$$n_{e,mp}(x) - n_{e,mp}(0) \sim \phi_{mp}(x) - \phi_{mp}(0)$$

$$n_{i,mp}(x) - n_{i,mp}(0) \sim \sqrt{\chi_{mp}(0) - \chi_{mp}(x)}$$

- Ion density drop larger due to slow ions barely turning around in their gyro-orbits.
- But cannot satisfy quasineutrality.



[2] Geraldini et al, 2025

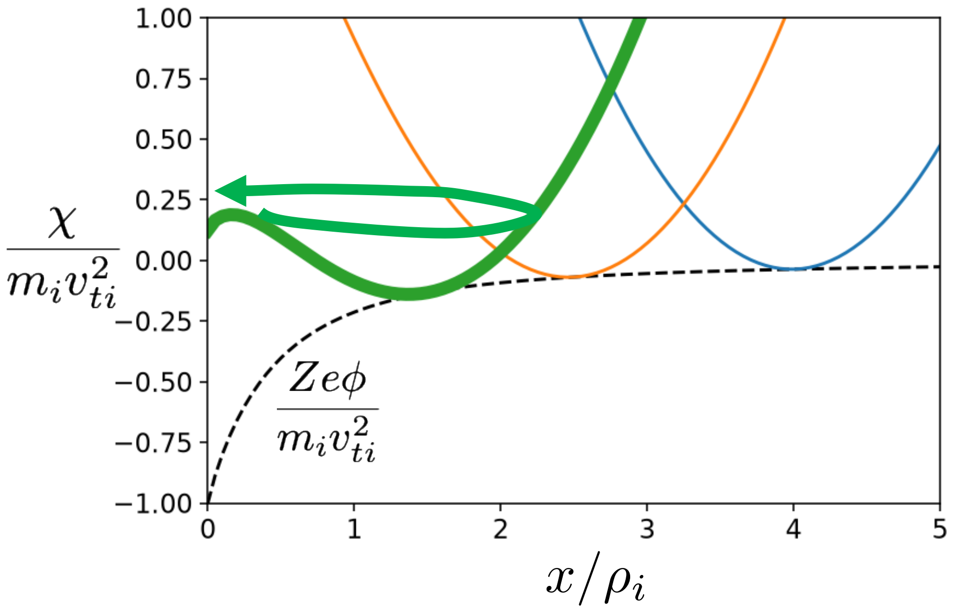
The Kinetic Bohm Condition

- Therefore, must eliminate all slow ions by having a singular electric field [2].

$$\phi_{mp}(x) - \phi_{mp}(0) \sim \sqrt{x}$$

- Only orbits are open orbits as ions no longer gyrate.
- Leads to the kinetic Bohm condition.

$$Z \int d^3\, v f_{i,dse} \frac{v_B^2}{v_x^2} = \frac{T_e}{e} \frac{dn_{e,mp}}{d\phi_{mp}} \Big|_{x=0}$$



[2] Geraldini et al, 2025

Potential at the Debye Sheath Entrance

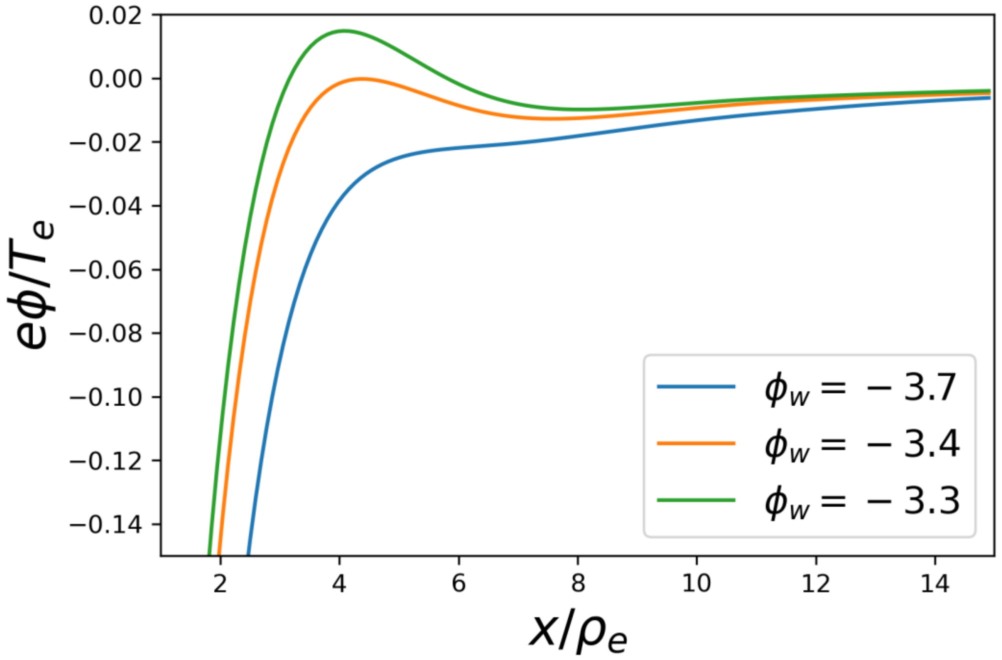
- Can also expand at the DSE from the DS side, taking x → ∞ and ϕ small.
- Asserting kinetic Bohm condition yields

$$\phi_{ds}''(x) \sim \left(Z \int d^3\, v f_{i,dse} \frac{v_B^2}{v_x^2} - \frac{T_e}{e} \frac{dn_{e,mp}}{d\phi_{mp}} \bigg|_{x=0} \right) \phi_{ds} - \phi_{ds}^2 \qquad \phi_{ds}(x) \sim -\frac{1}{(x+c)^2}$$

- Very robust conditions, even at $\alpha \sim (\gamma m_e/m_i)^{1/2}$.
- Even non-monotonic potentials must follow this behavior.

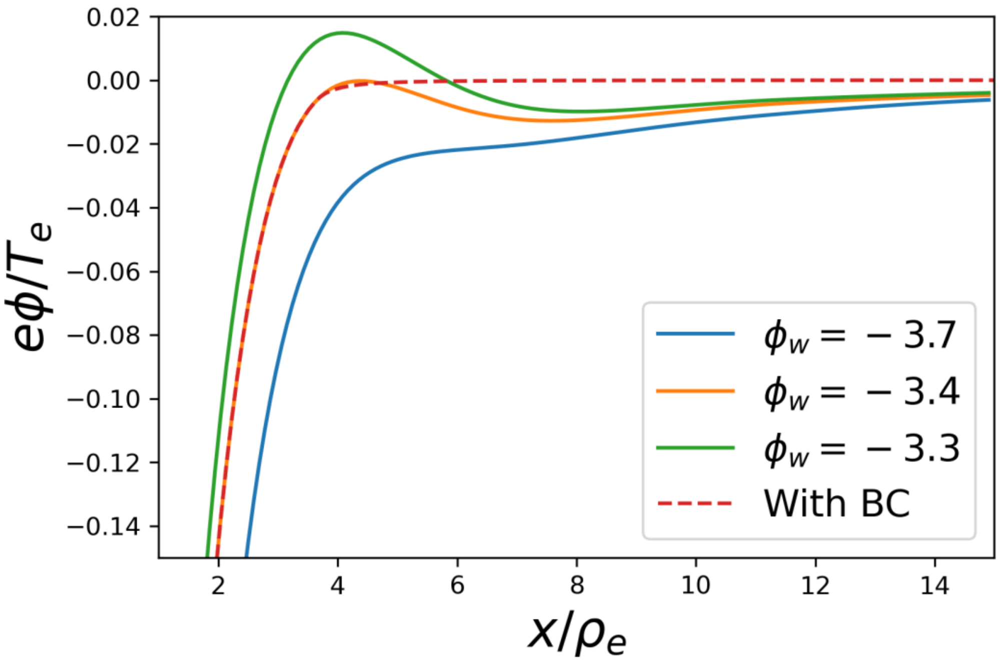
Non-monotonic Potential Profiles

- Carefully running GYRAZE yields non-monotonic solutions near critical angles that remain negative.
- Still satisfies Bohm condition and does not reflect ions (for now).



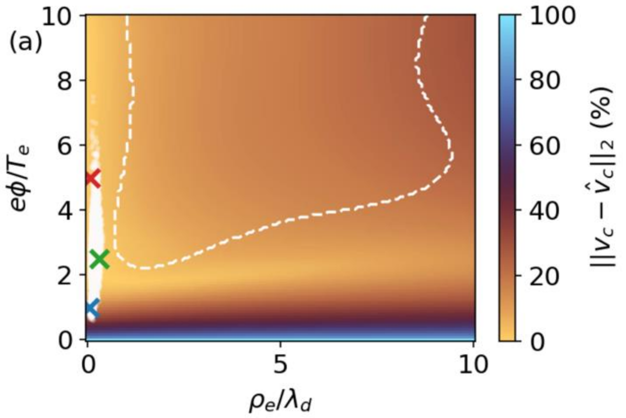
Non-monotonic Potential Profiles

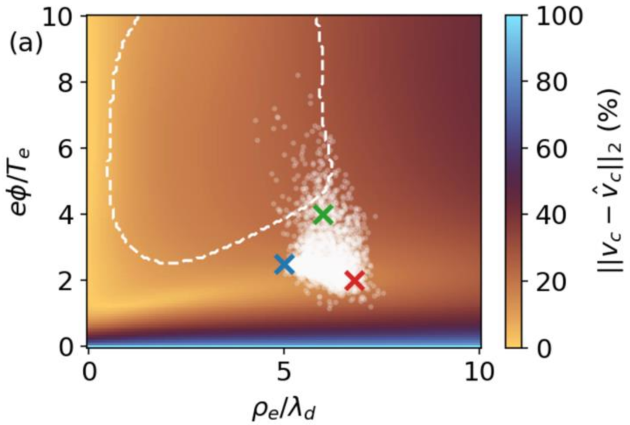
- For numerical efficiency, asymptotic result at large x used when ϕ or 1-n_e too small.
- Applying asymptotic result prematurely can lead to monotonic profiles.
- Still working to understand these solutions, obtain ϕ > 0 solutions and improve robustness of GYRAZE.



Other things going on…

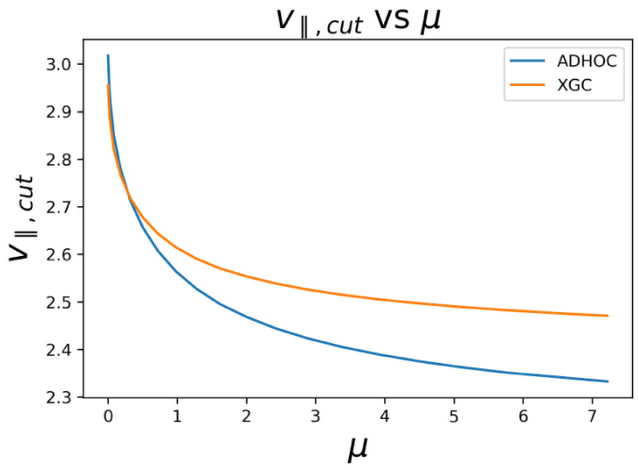
- Surrogate model recently developed to predict electron cutoff model.
- Being coupled with XGC and Gkeyll.
- Most of the simulation parameters being used would not converge in GYRAZE.

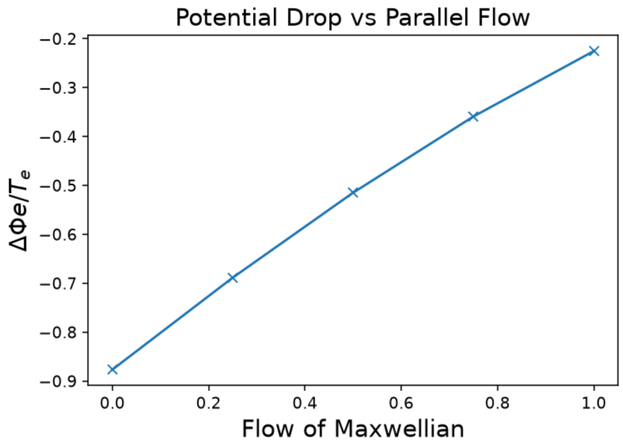




Other things going on…

- Numerical distribution functions do not satisfy the Chodura condition.
- Can apply Felix's time-dependent theory to modify these distribution functions, but can require a large additional potential drop.





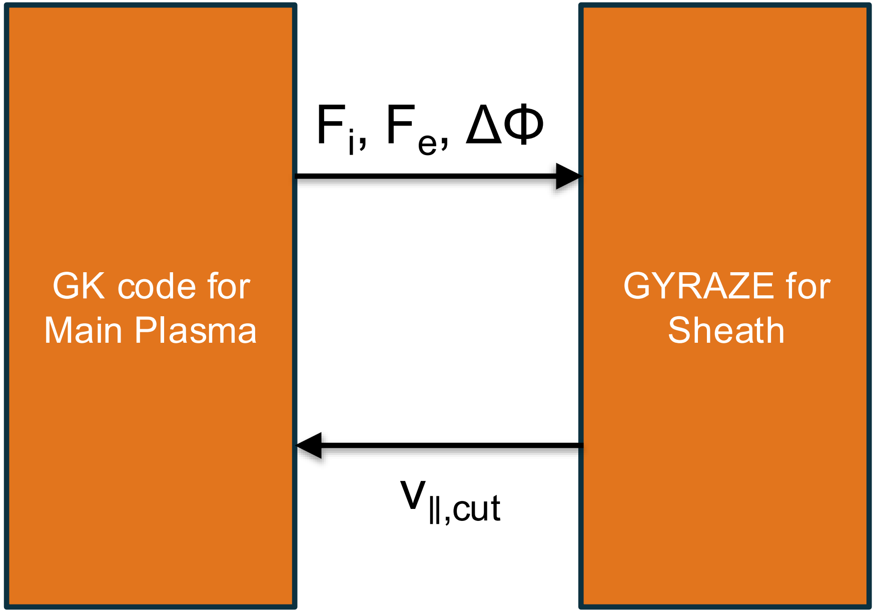
Conclusions

- To better understand divertor heat fluxes and SOL currents, would like to accurate sheath models at small α and ϕ_w.
- But limited by non-monotonic profiles.
- Non-montonic profiles must still obey the kinetic Bohm condition.

Effects of Numerical Distributions and the Chodura Condition

Coupling GYRAZE and Edge Gyrokinetic Cdes

- As input, GYRAZE requires distribution functions F_i, and F_e from the bulk plasma.
- Currently uses adhoc analytical distributions but would ideally obtain from edge codes.
- Edge GK codes require v_{cut} to set boundary condition for electrons.

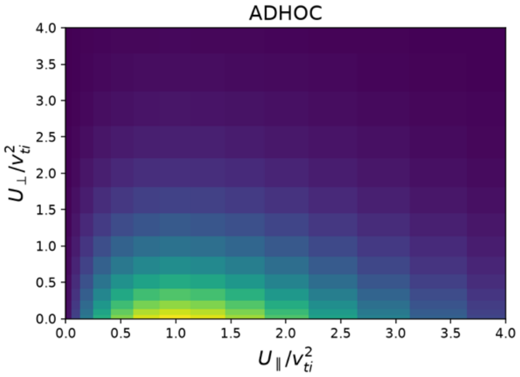


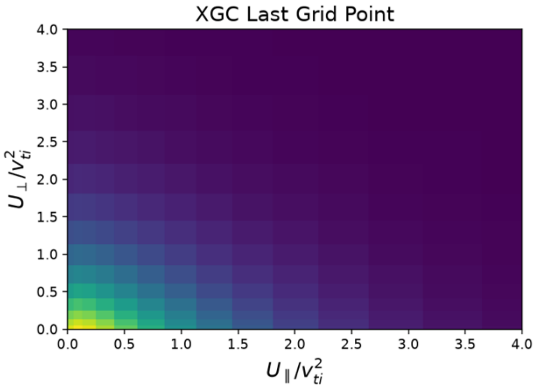
Chodura Condition

- Ion distributions near the magnetic presheath entrance must satisfy the kinetic Chodura condition.

$$Z\int d^3\,v f_{i,mpe}\frac{v_B^2}{v_z^2}=\frac{T_e}{e}\frac{dn_{e,mp}}{d\phi_{mp}}\Big|_{x\to\infty}$$

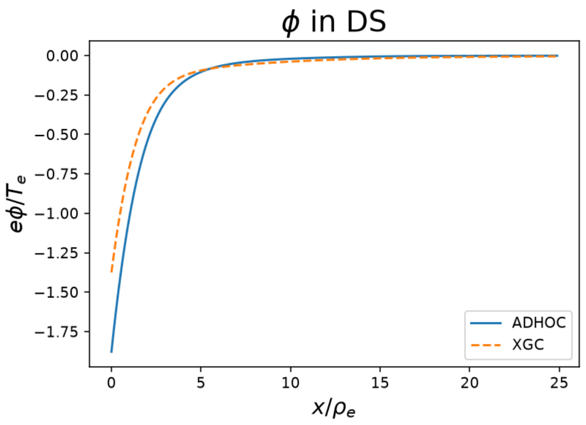
- Cannot have slow ions parallel to **B**.
- But numerical distribution functions may not satisfy.

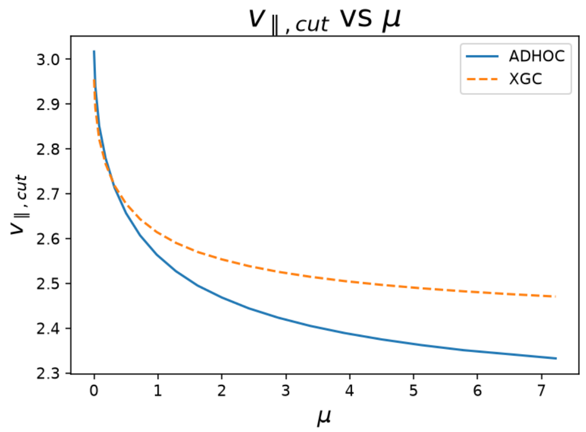




Numerical vs Analytical Distribution Functions

- Different distribution functions can lead to very different results.
- Can affect predictions of heat fluxes, monotonicity, etc.





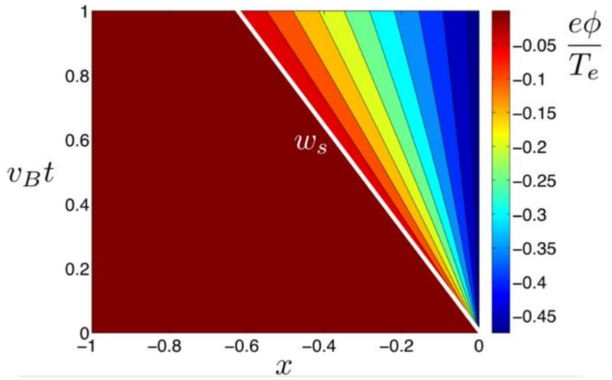
Time-Dependent Solutions and Rarefaction Waves

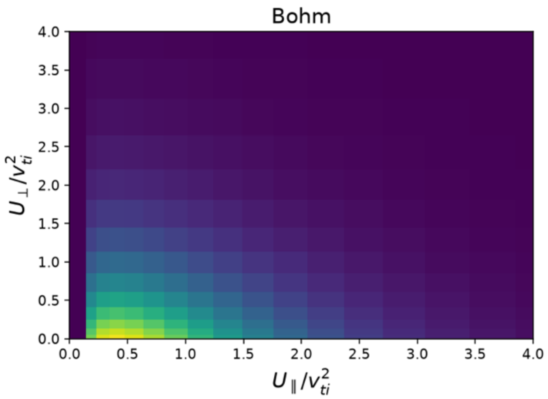
- Can resolve by assuming there is a third layer, where the time-dependent equation is solved.

$$\partial_t F + v_z \partial_z F - \frac{Ze}{m_i} \partial_z \Phi \partial_{v_z} F = 0$$

- Transform to w = z/t

$$(v_z - w)\partial_w F - \frac{Ze\phi'}{T_e} \partial_{v_z} F = 0$$



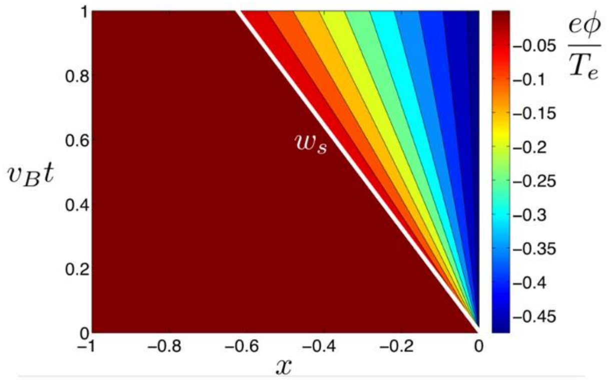


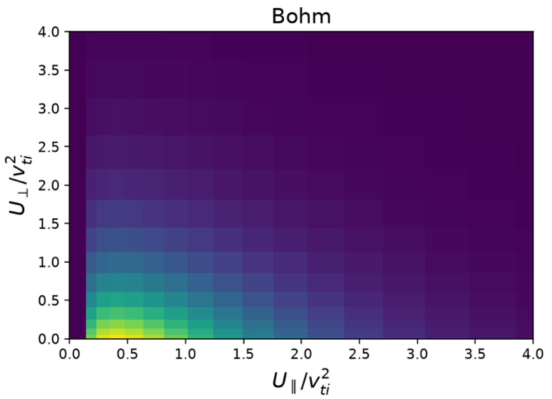
- Substitute kinetic equation into quasineutrality and IBP.

$$Z \int \frac{\partial_{v_z} F}{(v_z - w)^2} = n_{e0} \exp\left(\frac{e\phi}{T_e}\right)$$

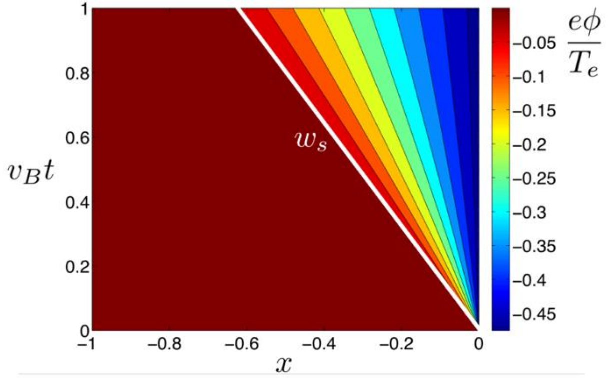
- Satisfies Bohm/Chodura condition at w = 0.
- Evolve F using characteristics

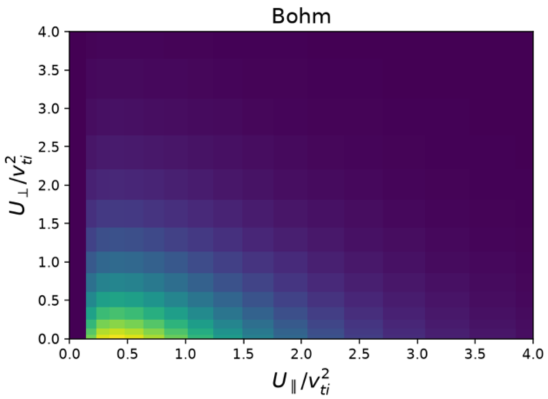
$$V_{zb}' = -\frac{Ze\phi'}{m_i(V_{zb} - w)}$$





- Plasma evolves to satisfy Bohm/Chodura condition by emitting rarefaction waves back through the bulk.





- Results with new distribution functions are similar to those from adhoc distributions.
- Extra potential drop needed to satisfy Chodura condition can be comparable to total potential drop.