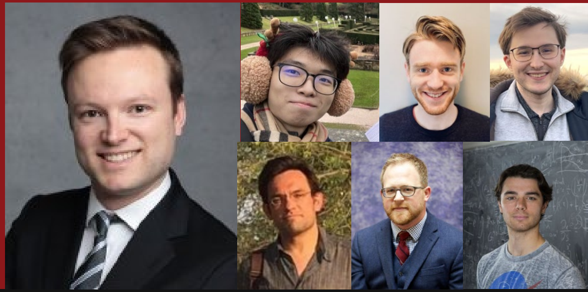
Anomalous cosmic-ray transport in intermittent CGL-MHD turbulence

Robert J. Ewart

Collaborators: Patrick Reichherzer, Archie Bott, Matt Kunz, Stephen Majeski, Michael Nastac, Shuzhe Ren, & Alex Schekochihin



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Motivation: radio emission around clusters

Facts of the case:

- We are interested in understanding the radio emission in and around clusters (Haloes, Relics, etc.)

- The plasma is (mostly) high-β $\implies$ fluid motions engender microscale instabilities (mirror/firehose/ion-cyclotron)

Theoretical Question(s):

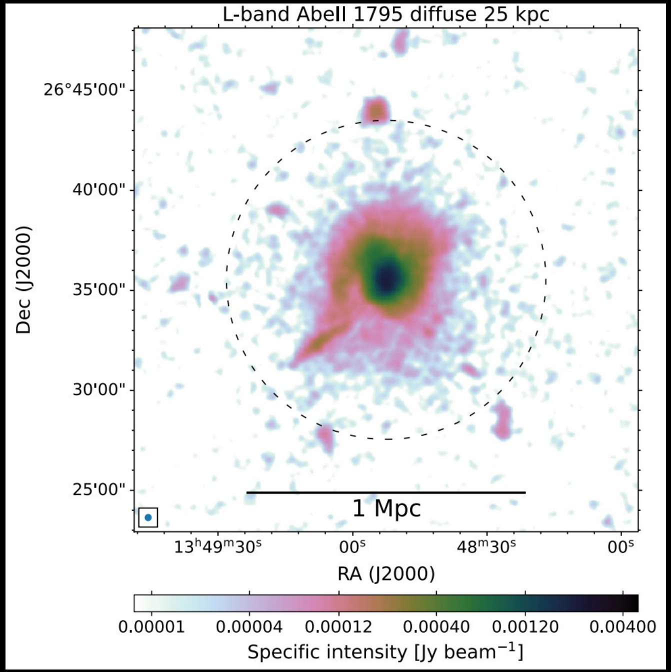
- How do CRs behave in a medium infested with small-scale instabilities?
  - Can vanilla (or vanilla +) MHD keep a halo bright?
  - What transport properties would CRs have?

Conceits:

- Clusters are homogeneous periodic boxes (preferably resolved at 1024^3)

- CRs never feedback

Van Weeren et al. (2026) MNRAS stag054



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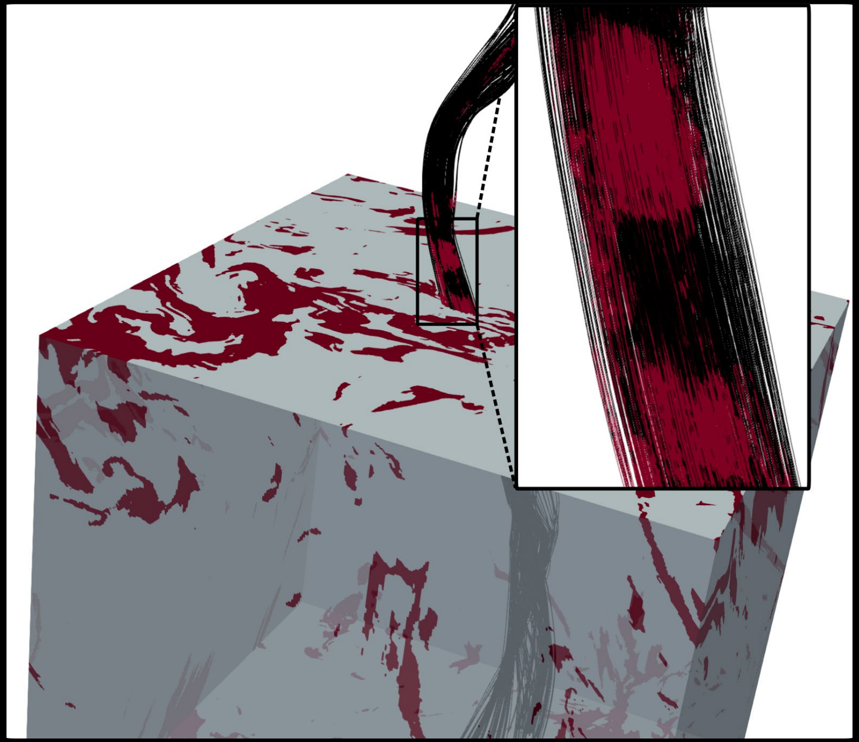

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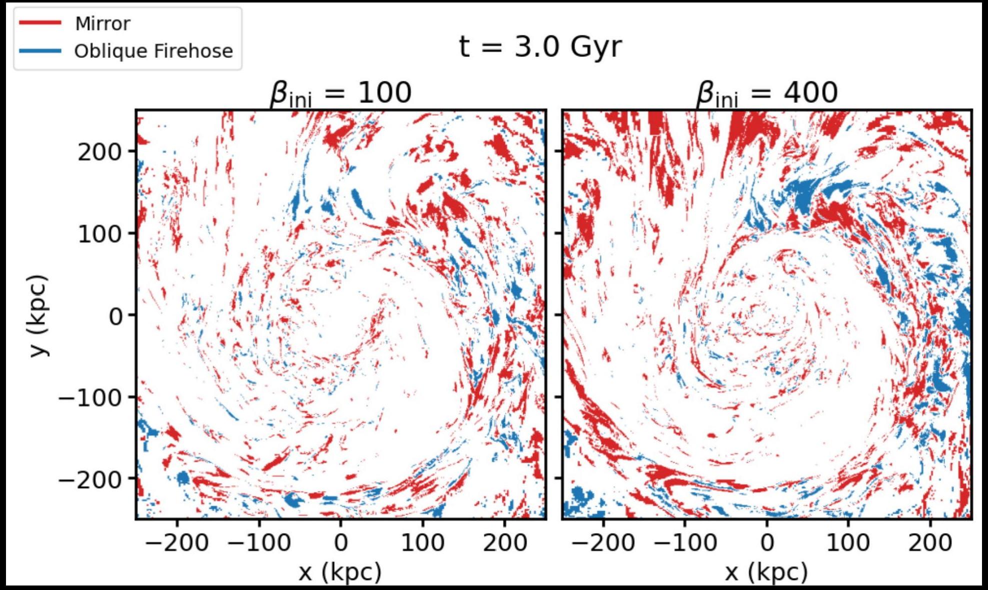

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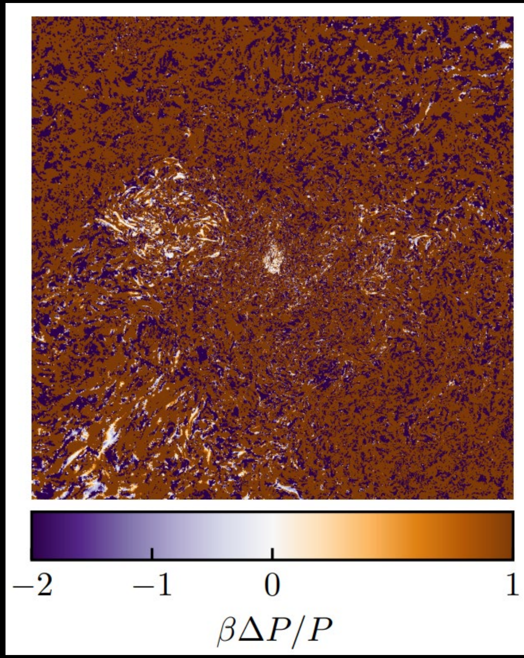

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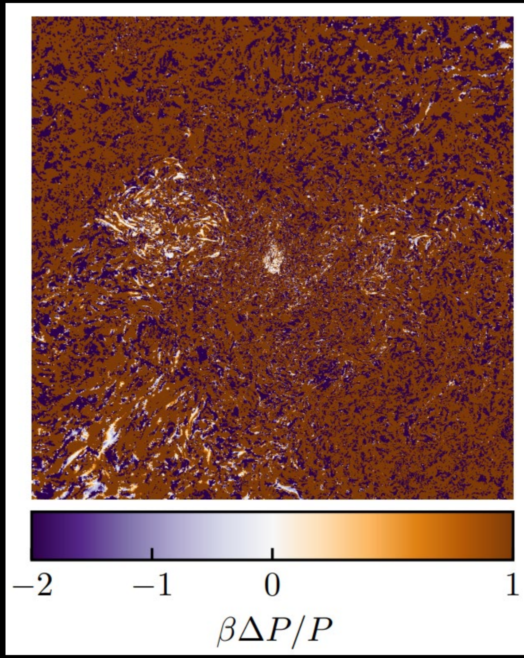

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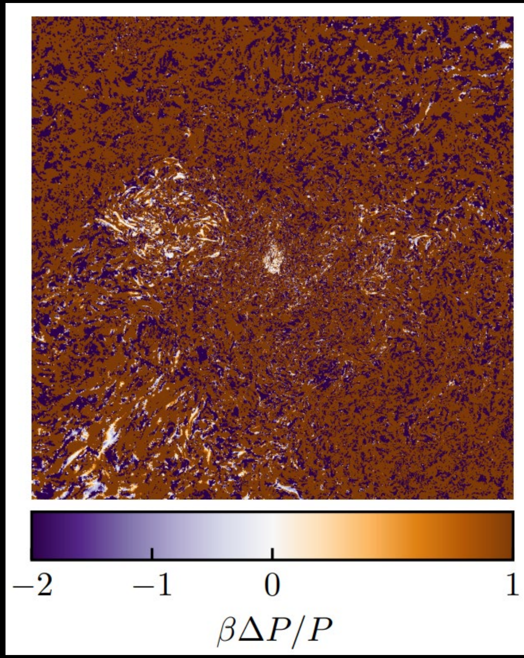

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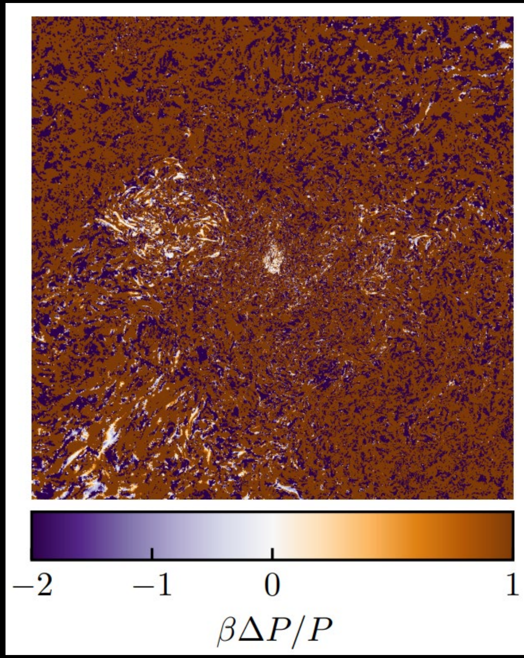
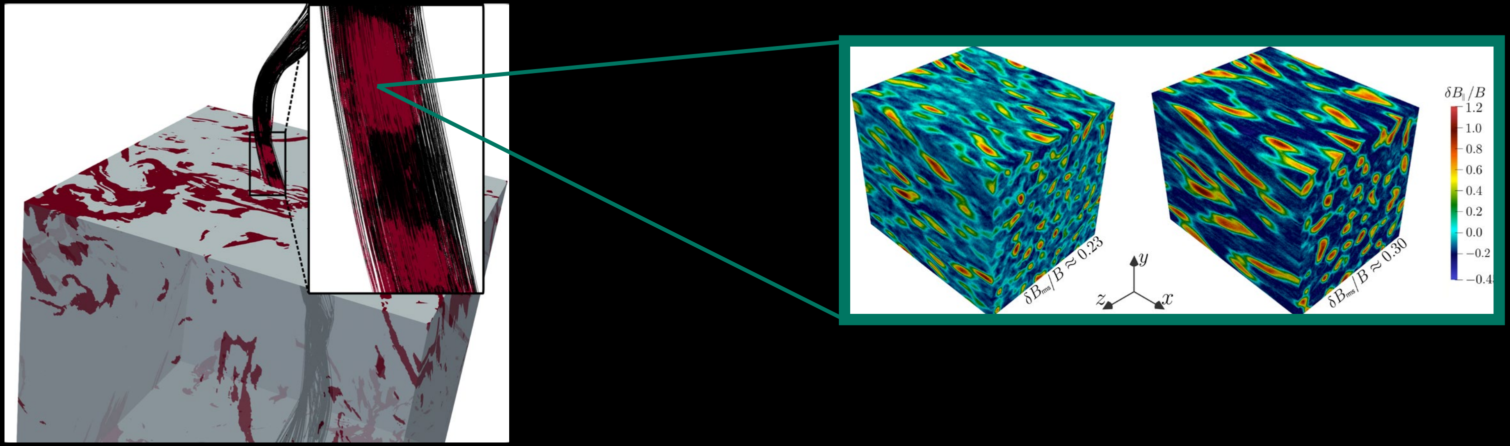


Why do micro-instabilities change things?



- Inside any region that has gone past the mirror threshold, the thermal plasma will have gone violently unstable.
- The mirror instability saturates with $\delta B/B \sim 1$.
- This creates the perfect structure to scatter cosmic rays off.

$$\frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}t} = \frac{q}{m}\frac{\boldsymbol{v}\times\boldsymbol{B}}{c} \implies \nu_{\rm scatter} \sim \frac{c\,l_{\rm mirror}}{r_g^2}\left(\frac{\delta B}{B}\right)^2 \implies \kappa_{\rm mirror} \sim 10^{24}\left(\frac{E}{\rm GeV}\right)^2 \mathrm{cm}^2\,\mathrm{s}^{-1}$$

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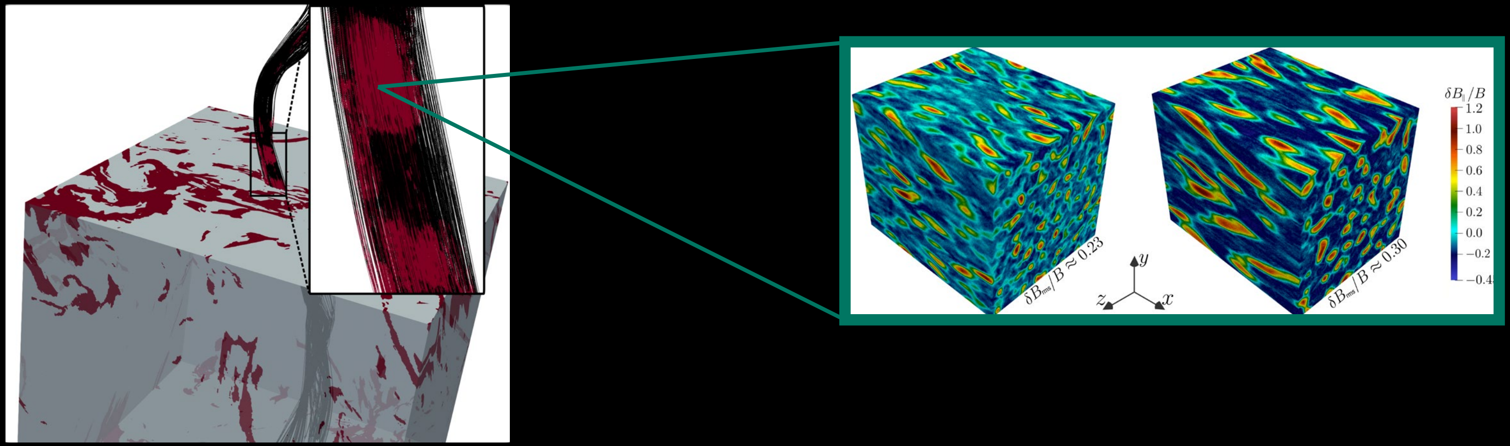

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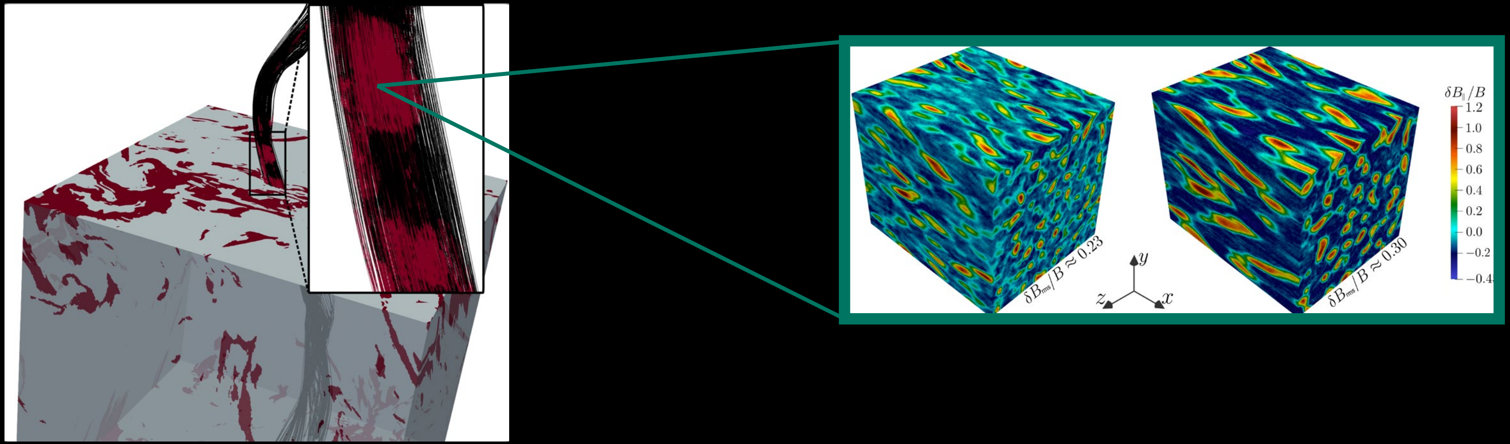

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Patchiness & acceleration

What does vanilla MHD energisation look like?

See talk by Kempski (Tues)

- A cosmic ray will change its energy when it scatters off a large-amplitude fluctuation in the magnetic field.

- If the large-amplitude fluctuation in the magnetic field is associated with a fluid velocity u, then the cosmic ray gets an energy kick $\sim u/c$.

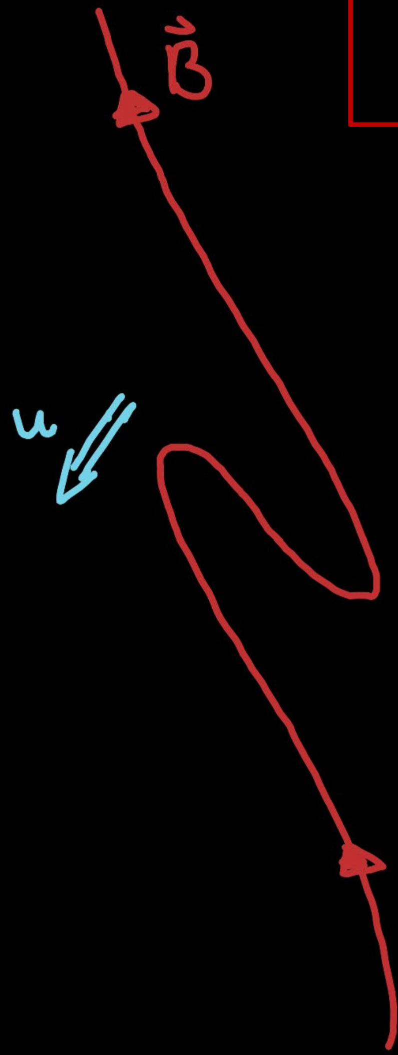
$$\text{After } N \text{ kicks} \qquad \left(\frac{\delta E}{E}\right)^2 \sim N\left(\frac{u}{c}\right)^2$$

- Assume that you encounter such a field reversal at a scale similar to $L_{\rm outer}$ (it's more subtle than this, see talk by Kempski)

$$N \sim \frac{Tc}{L_{\rm outer}} \implies \left(\frac{\delta E}{E}\right)^2 \sim \frac{Tc}{L_{\rm outer}}\left(\frac{U_{\rm outer}}{c}\right)^2$$

- So how long would it take to re-energise a cosmic ray?

$$t_{\rm ene} \sim \frac{L_{\rm outer}}{c}\left(\frac{c}{U_{\rm outer}}\right)^2 = \left(\frac{L_{\rm outer}}{100\,\rm Kpc}\right)\left(\frac{U_{\rm outer}}{100\,\rm Km\,s^{-1}}\right)^{-2} 10^3\,\rm Gyr$$



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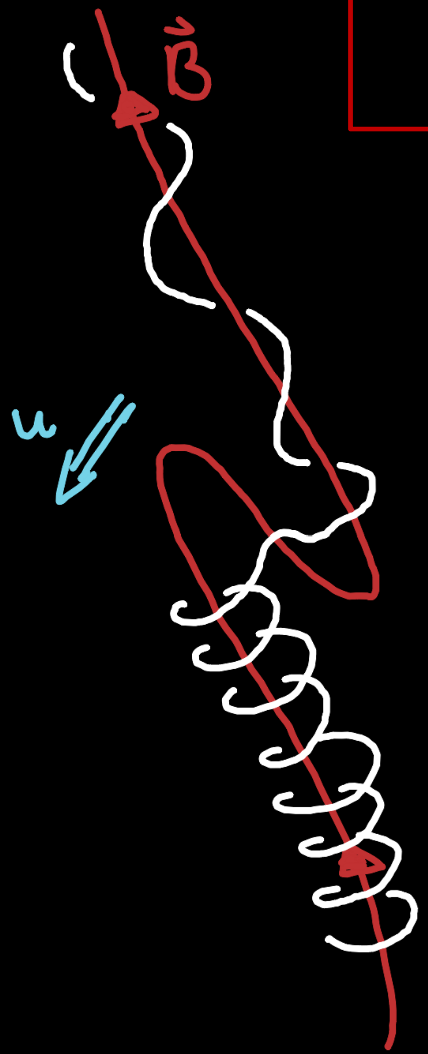

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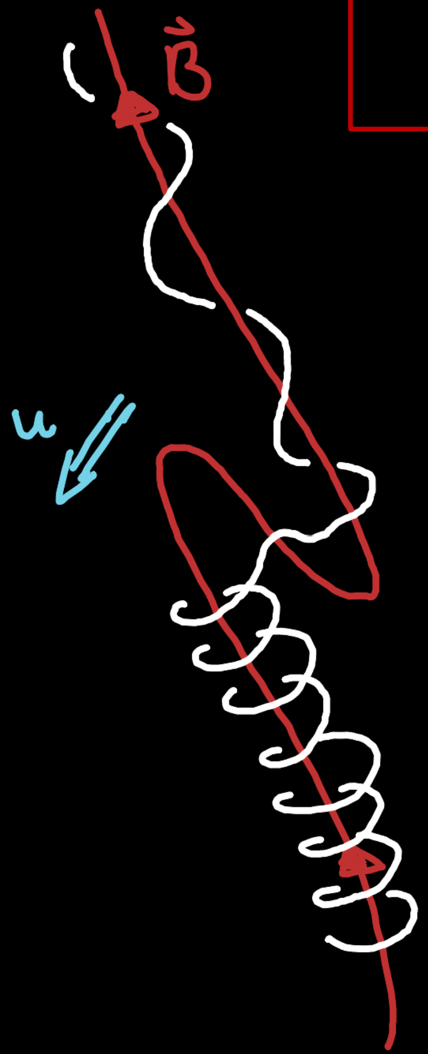

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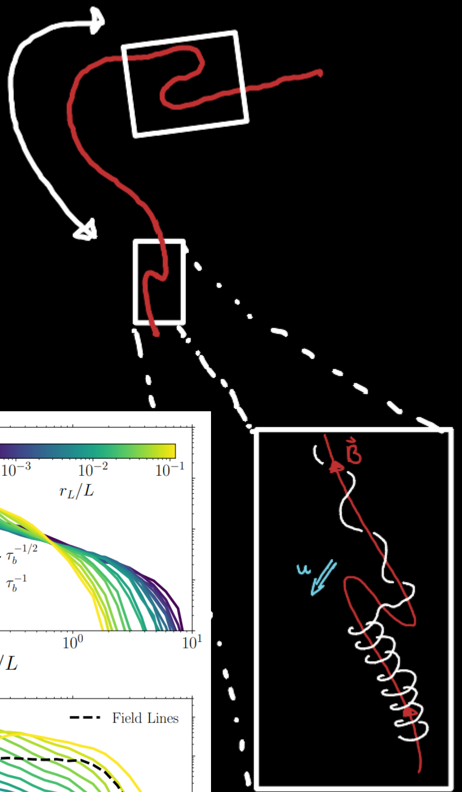

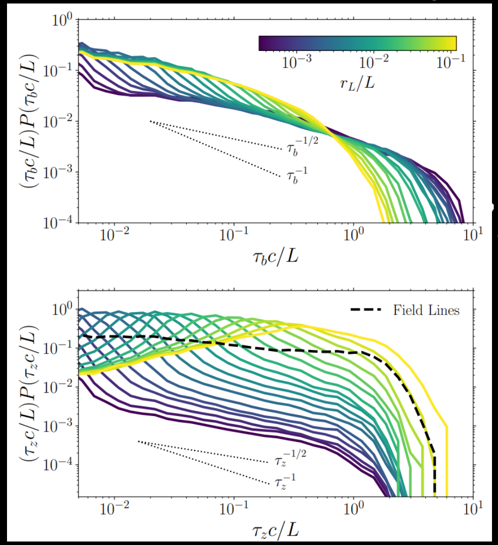

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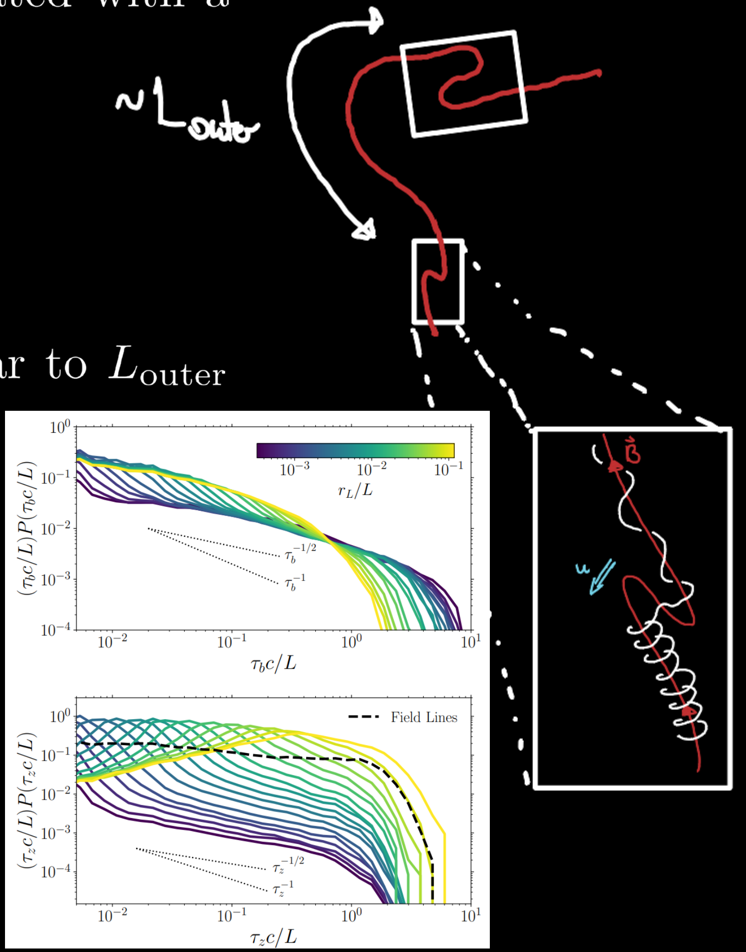

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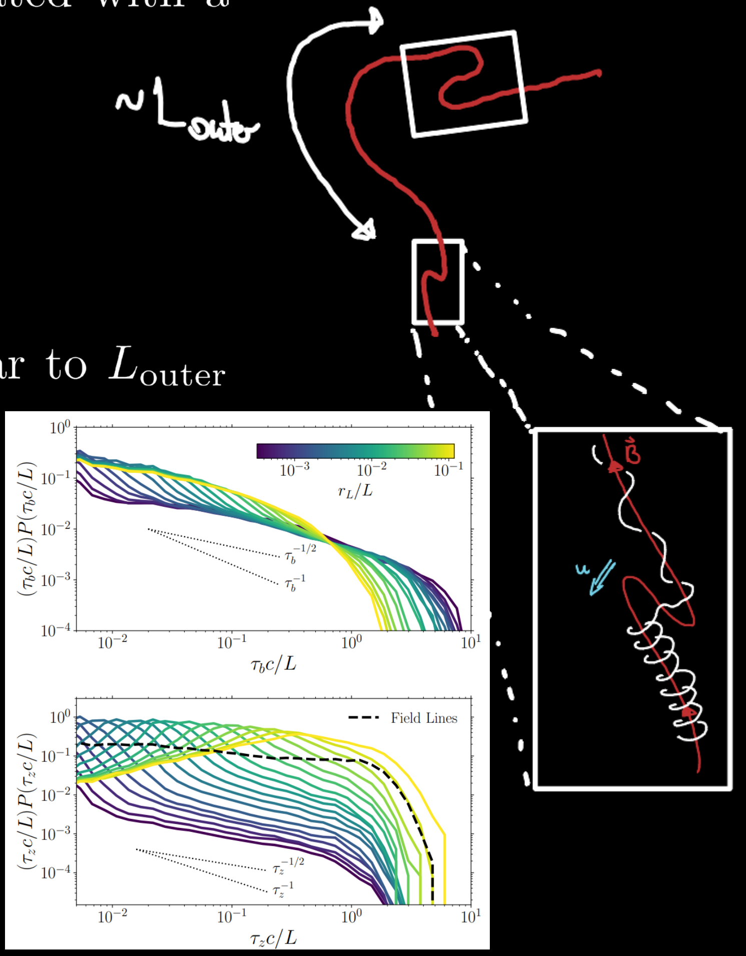

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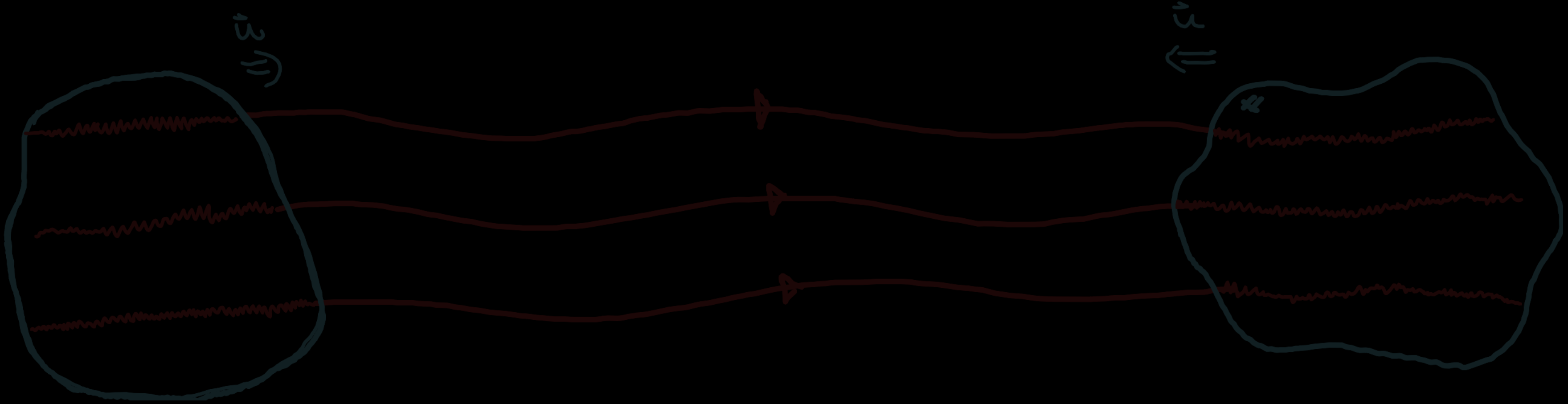
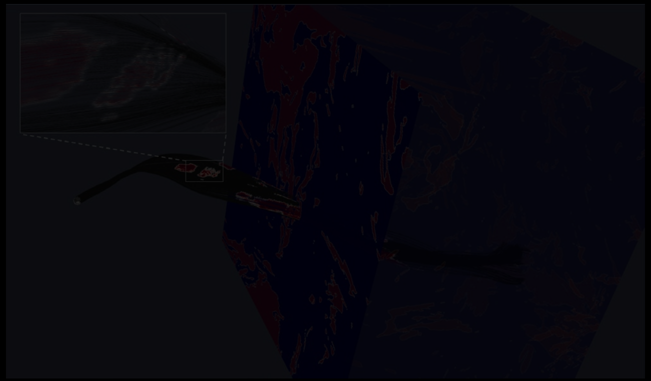
- The killer for energisation in MHD is that the random walk in energy was random: **each kick happened once**.
- The patches present an opportunity for particles to gain energy repeatedly from the same coherent motion (this difference is precisely what makes shocks good accelerators)
- What should such an acceleration mechanism like this depend on?

$$\left(\frac{\delta E}{E}\right)^2 \sim N\left(N_{\mathrm{trap}}\frac{u}{c}\right)^2$$

$$l_{\parallel} \equiv \begin{matrix}\text{size of}\\ \text{the trap}\end{matrix} \qquad \delta u_{\parallel}(l_{\parallel}) \equiv \begin{matrix}\text{speed of}\\ \text{compression}\end{matrix} \qquad N_{\mathrm{trap}} \equiv \begin{matrix}\text{number of}\\ \text{bounces}\end{matrix}$$





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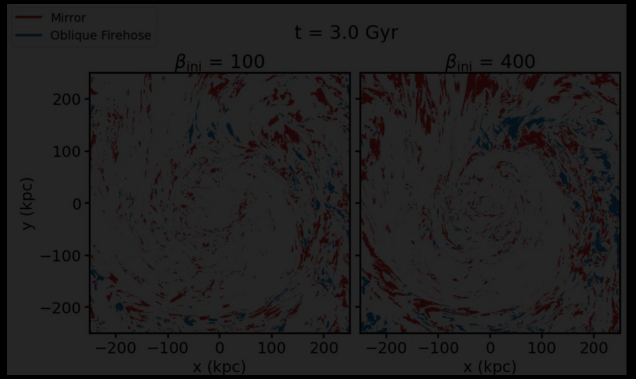

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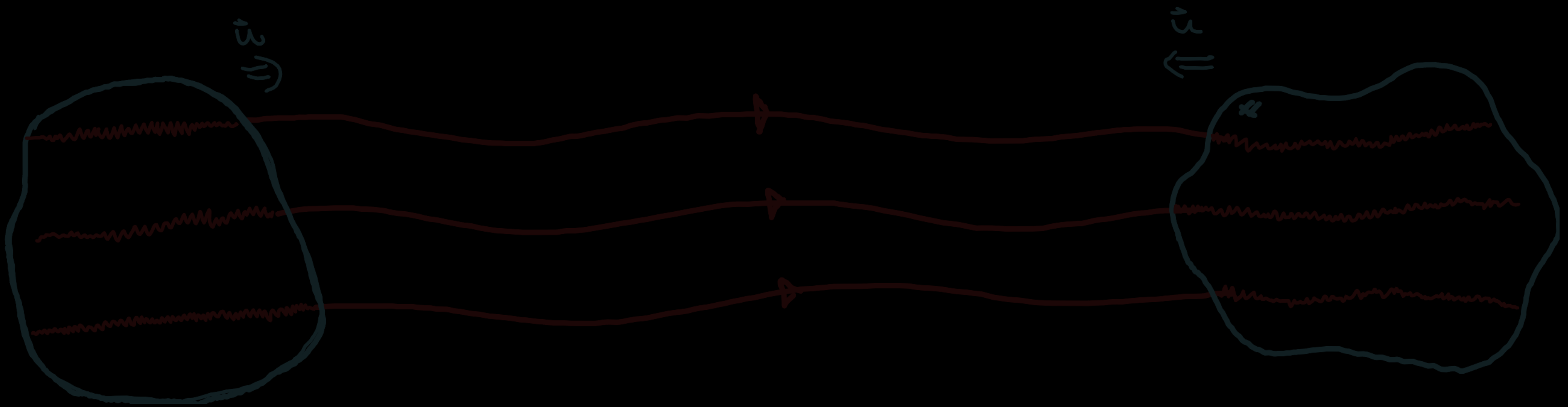

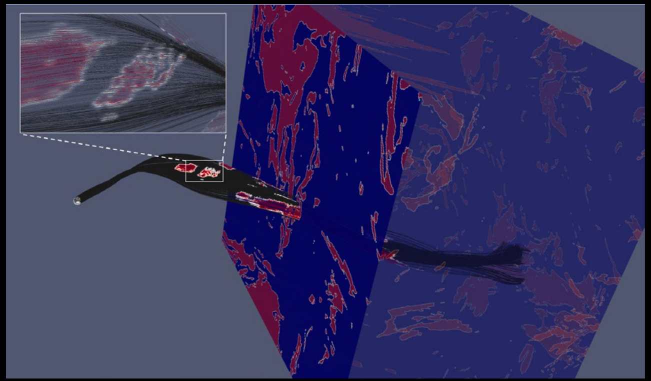

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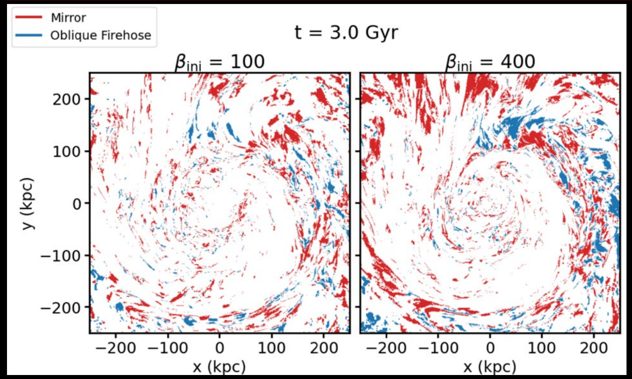

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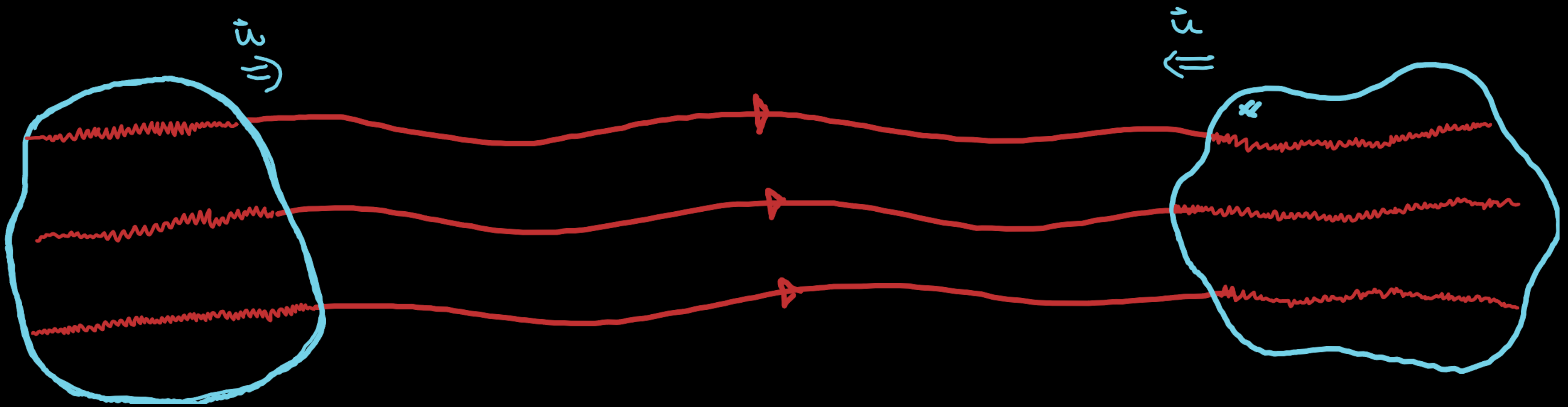

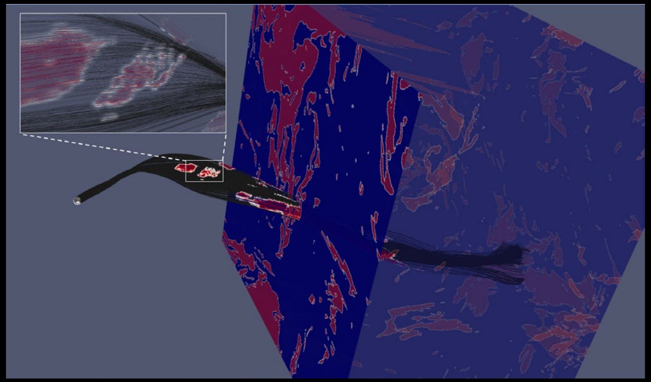

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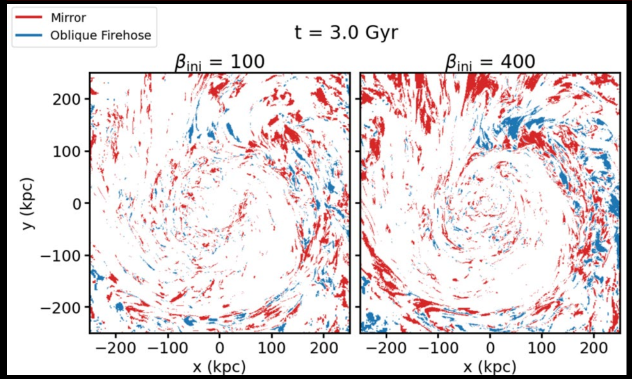

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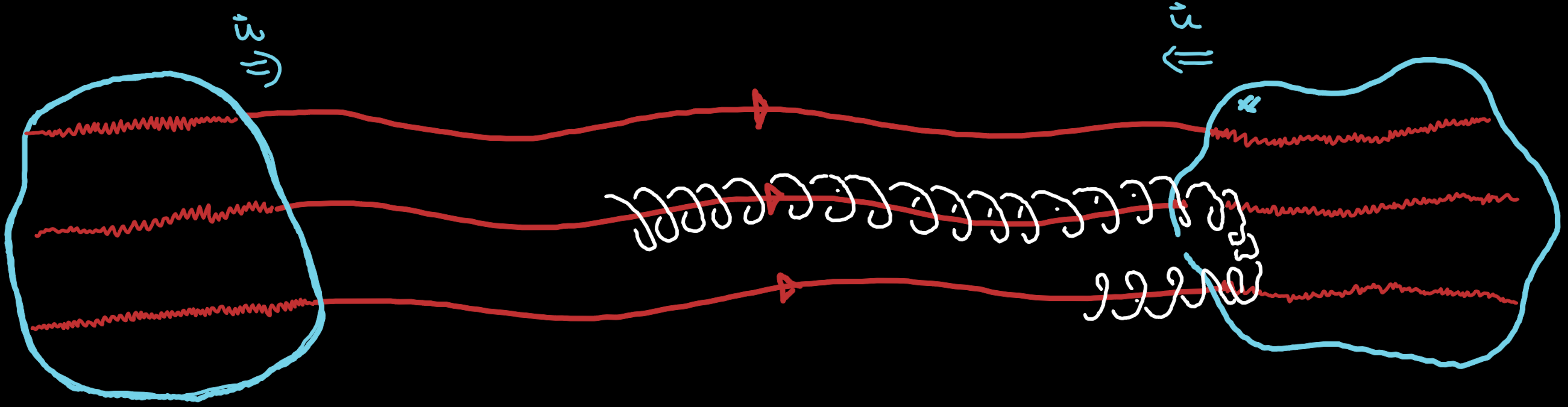

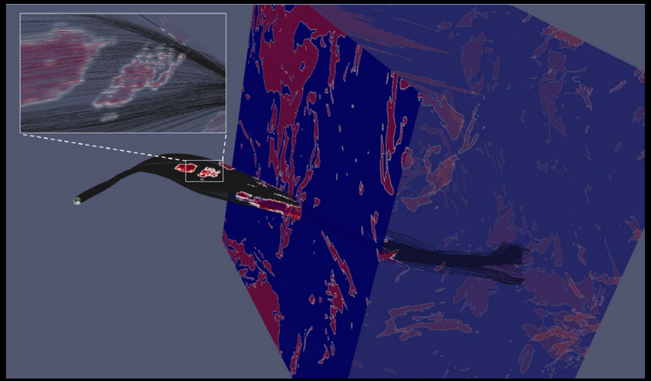

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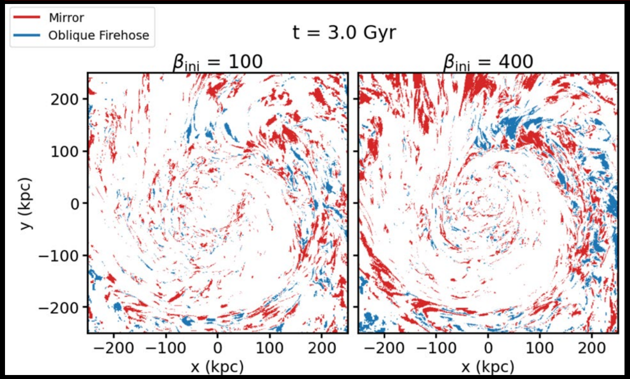

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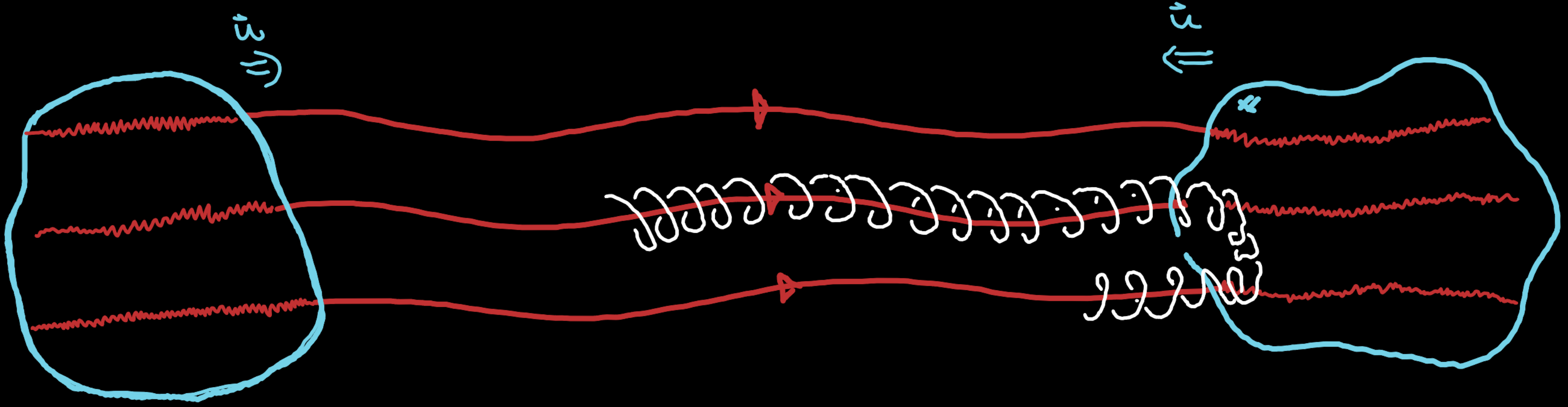
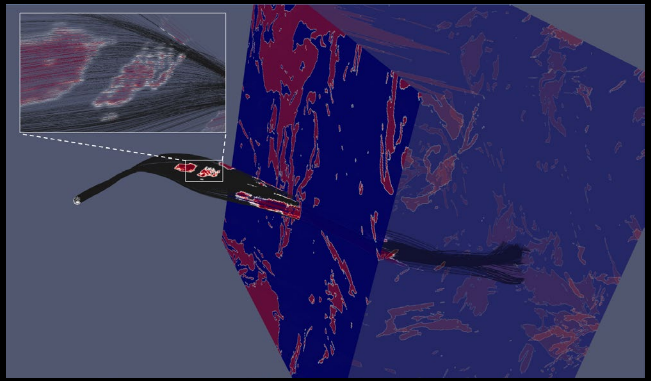
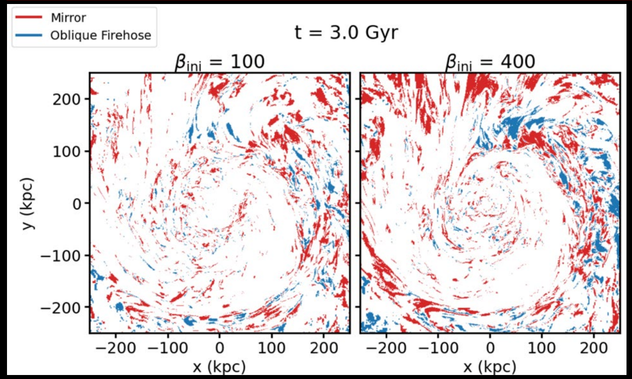




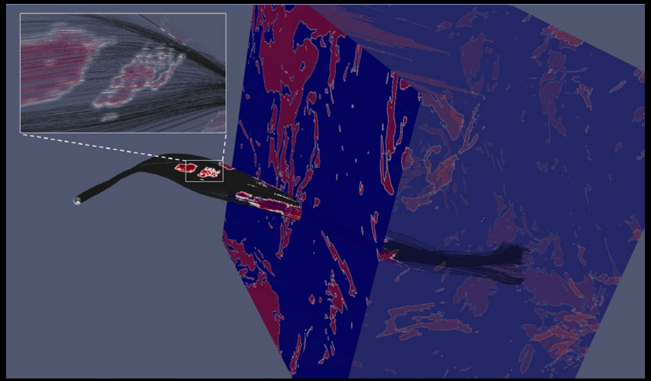
ZuHone et al. (2026) arXiv preprint 2606.17371



Why would patches help?

- The killer for energisation in MHD is that the random walk in energy was random: **each kick happened once**.
- The patches present an opportunity for particles to gain energy repeatedly from the same coherent motion (this difference is precisely what makes shocks good accelerators)
- What should such an acceleration mechanism like this depend on?

$$\left(\frac{\delta E}{E}\right)^2 \sim N\left(N_{\mathrm{trap}}\frac{u}{c}\right)^2$$



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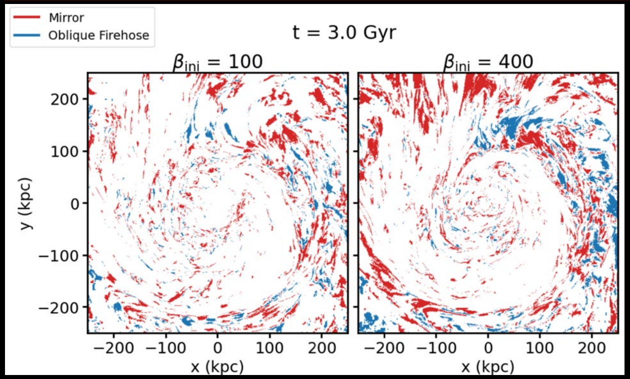
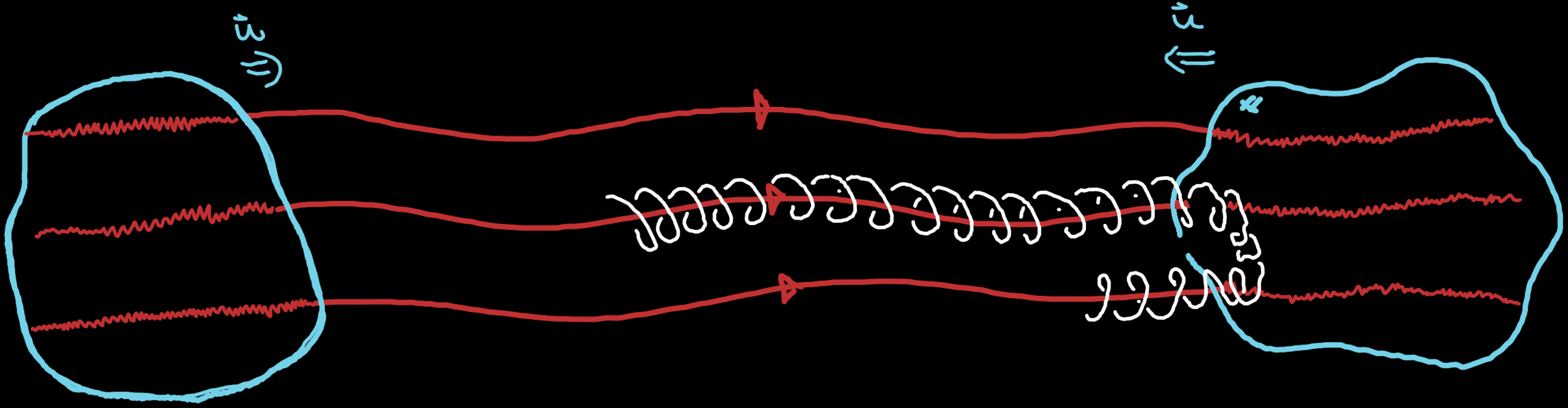


$l_{\parallel} \equiv$ size of the trap

$\delta u_{\parallel}(l_{\parallel}) \equiv$ speed of compression

$\boxed{N_{\mathrm{trap}} \equiv \text{number of bounces}}$



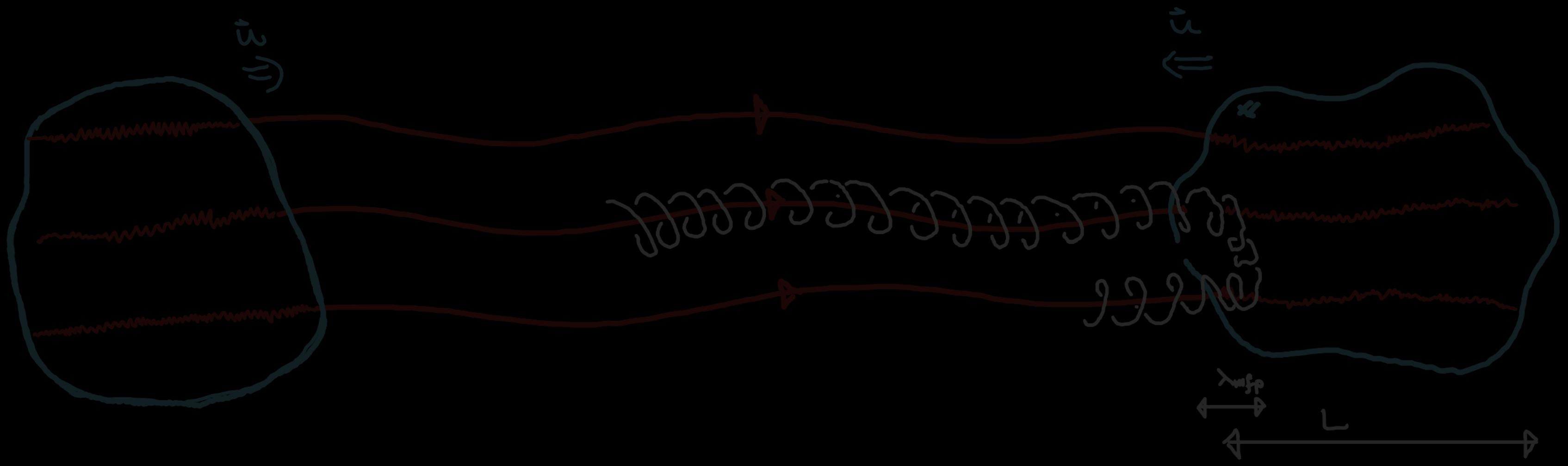
Escape time (CR jail break)

- We've put our cosmic rays in a cage. How do they get out of the jail?

Q: Can you bore through the wall?

A: **No.** The mean free path in mirror-unstable regions is far too short ($\lambda_{\rm mfp} \sim \mu$pc). You exit the way you came in.

Fun paper about nuances here: Kotera & Lemoine (2008) Phys. Rev. D



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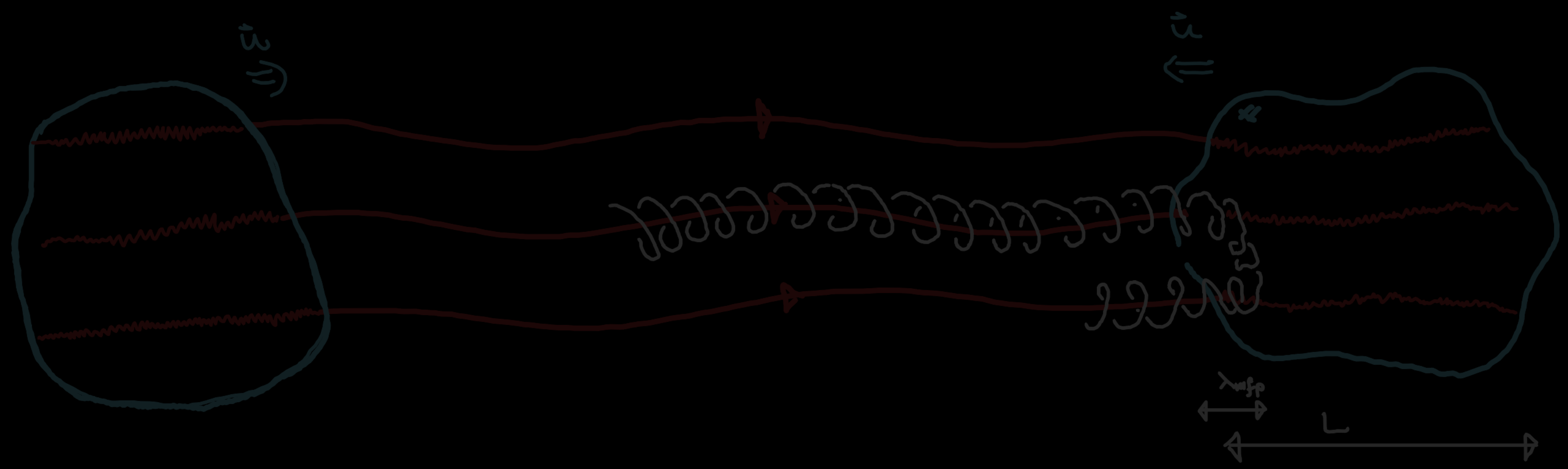

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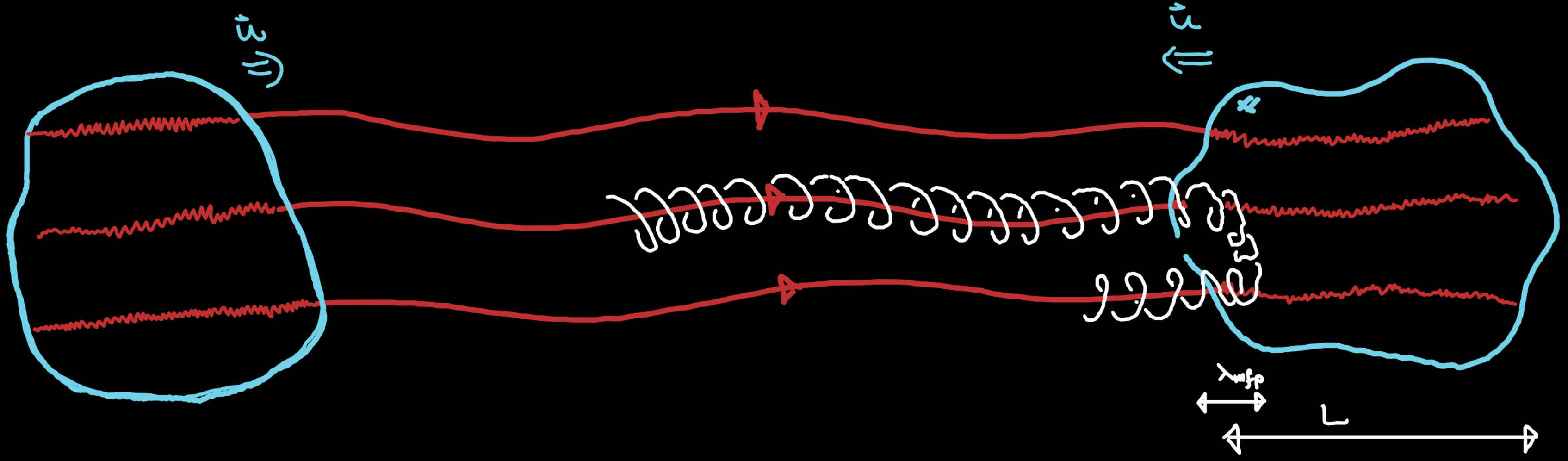

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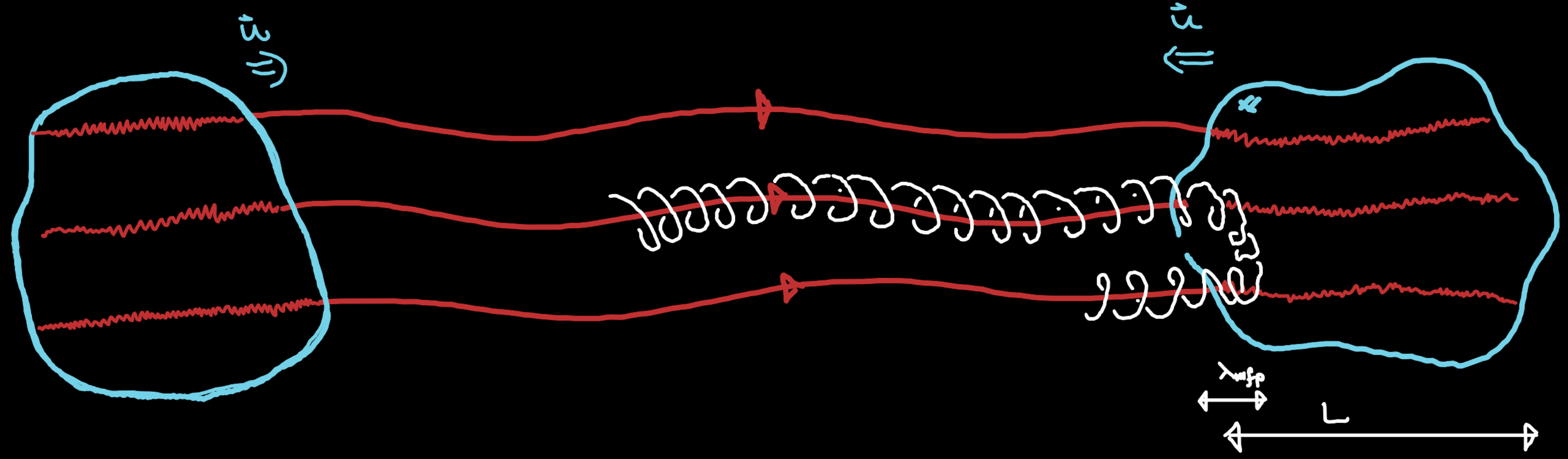

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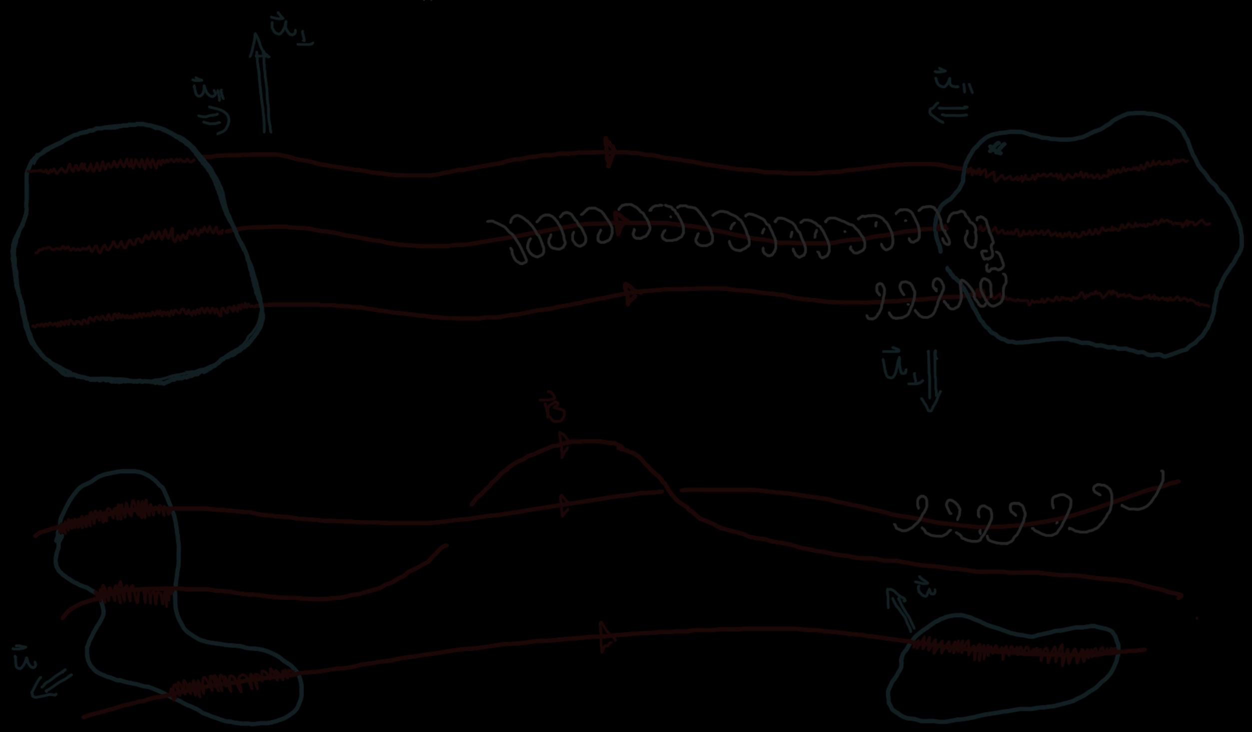
$$N_{\mathrm{trap}}(l_{\parallel}) \sim \frac{l_{\parallel}}{V_A}\frac{c}{l_{\parallel}} \sim \frac{c}{V_A} \qquad \Longrightarrow \qquad t_{\mathrm{esc}} \sim \frac{l_{\parallel}}{V_A}$$



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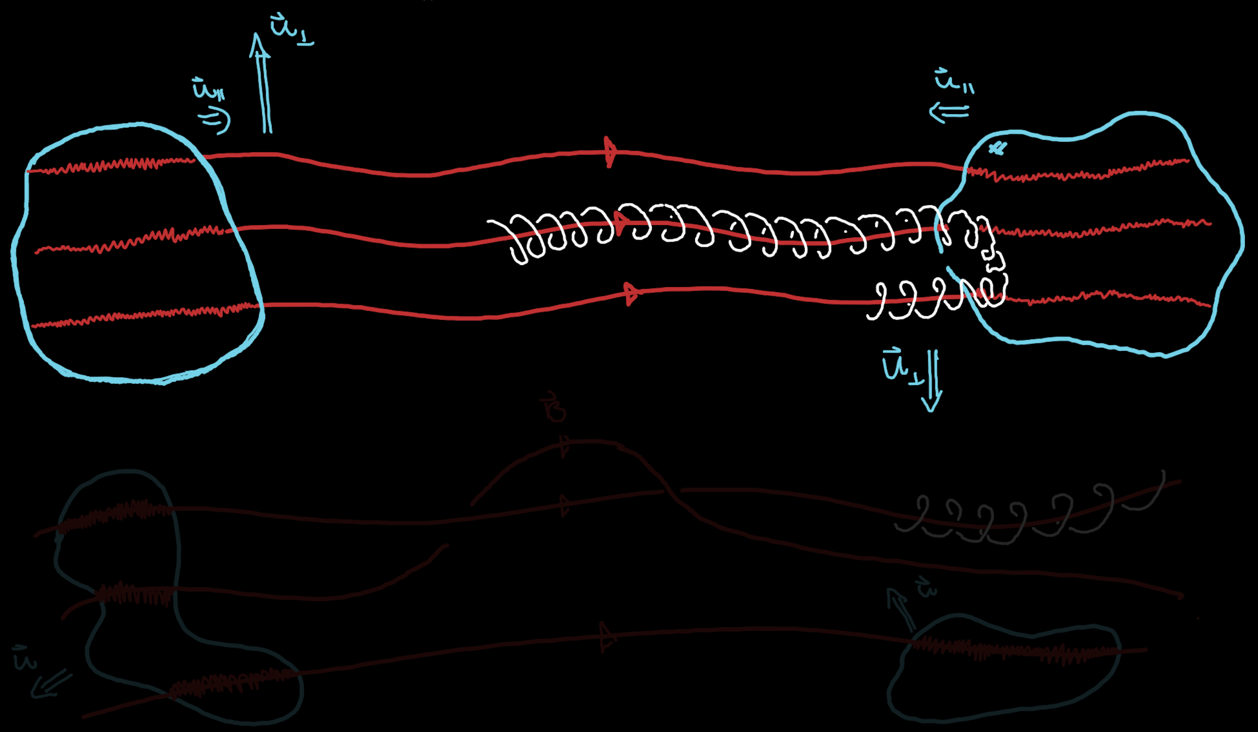

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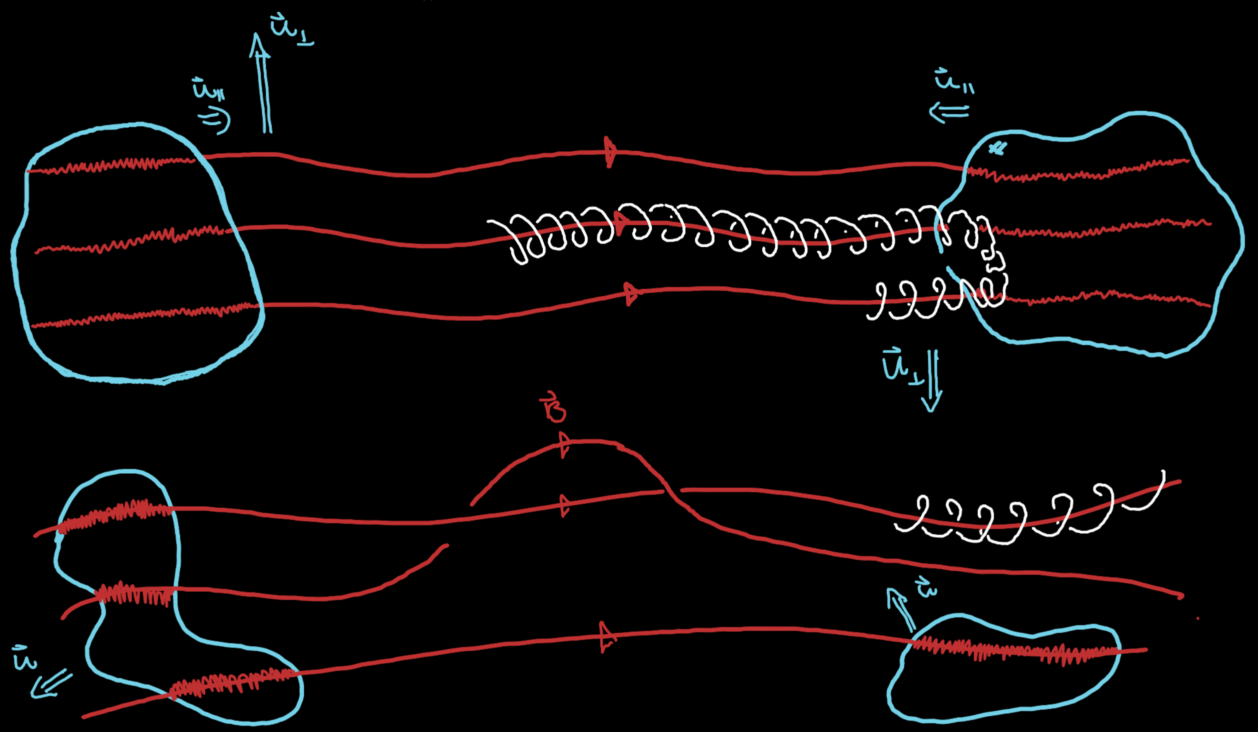

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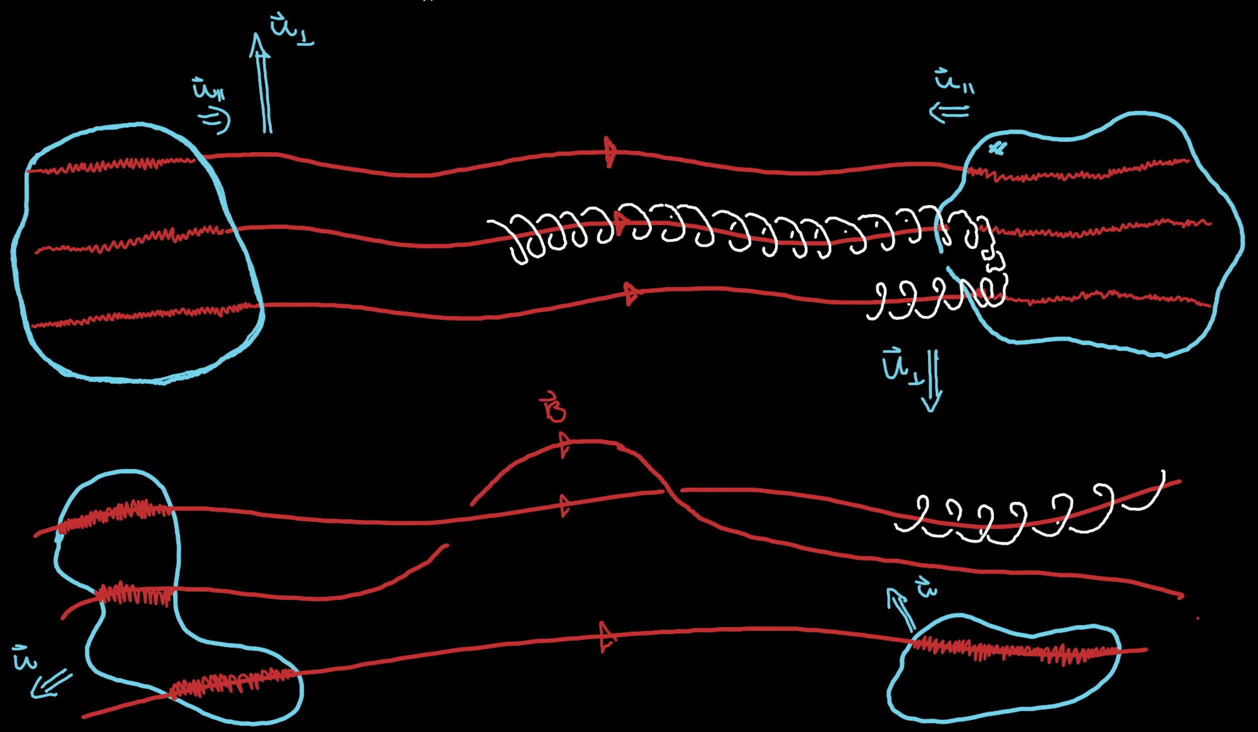

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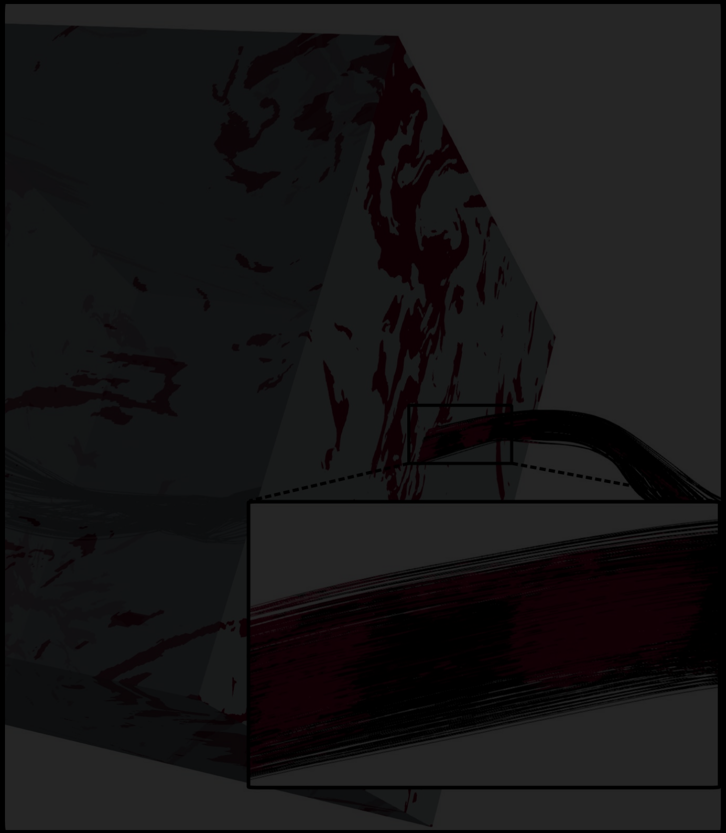

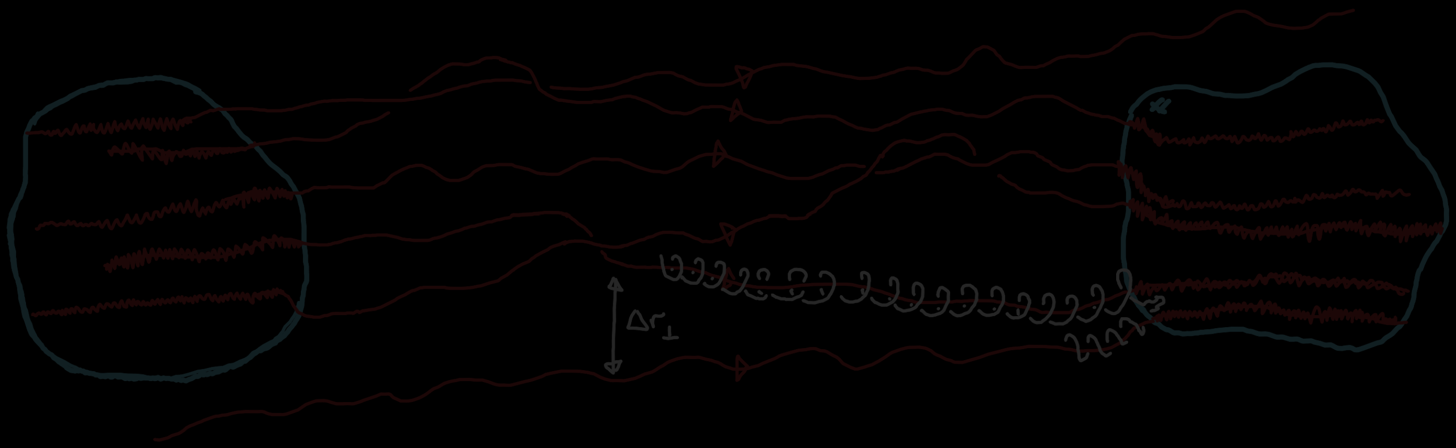

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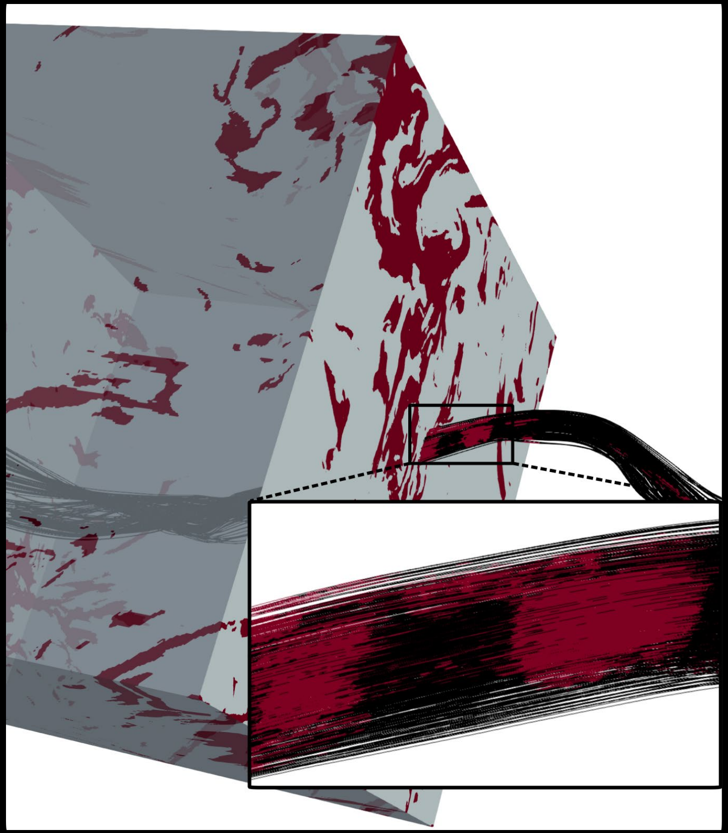

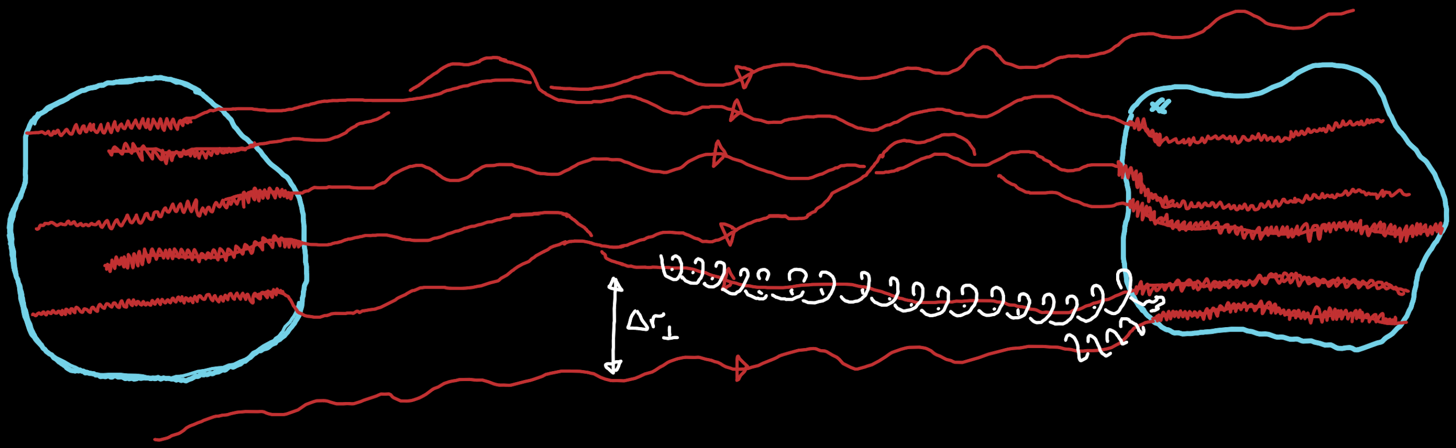

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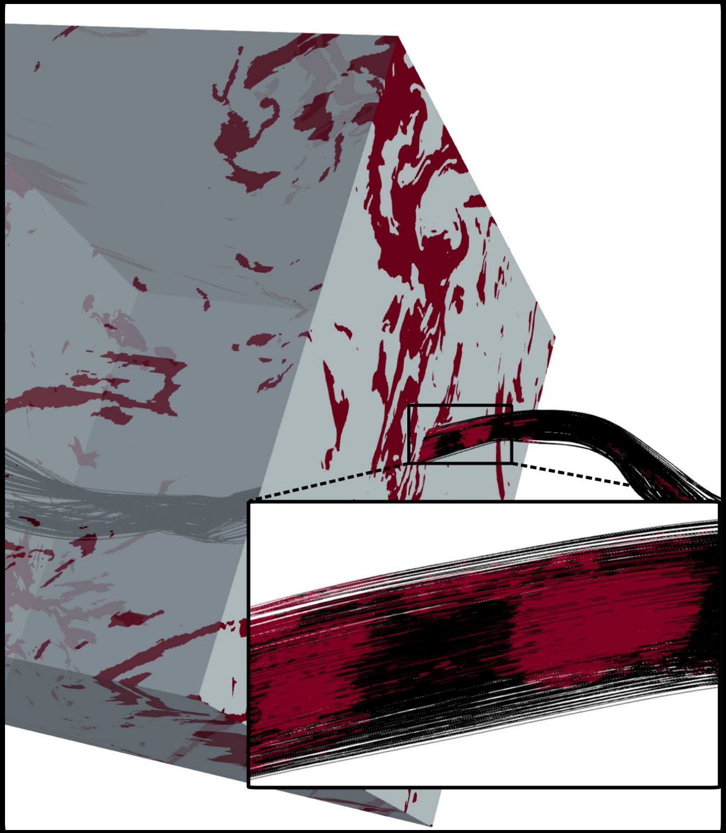

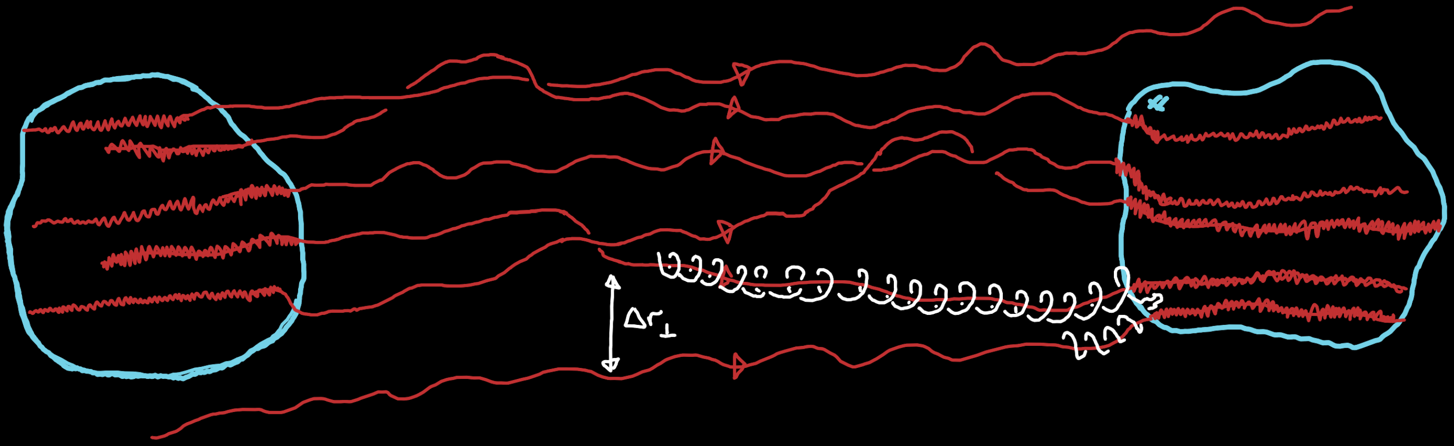

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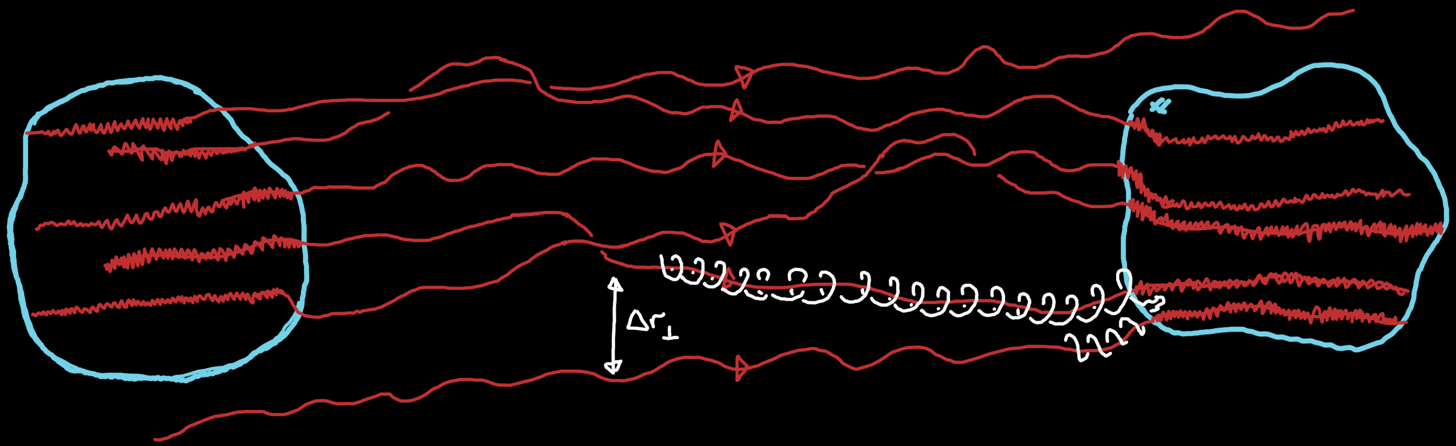

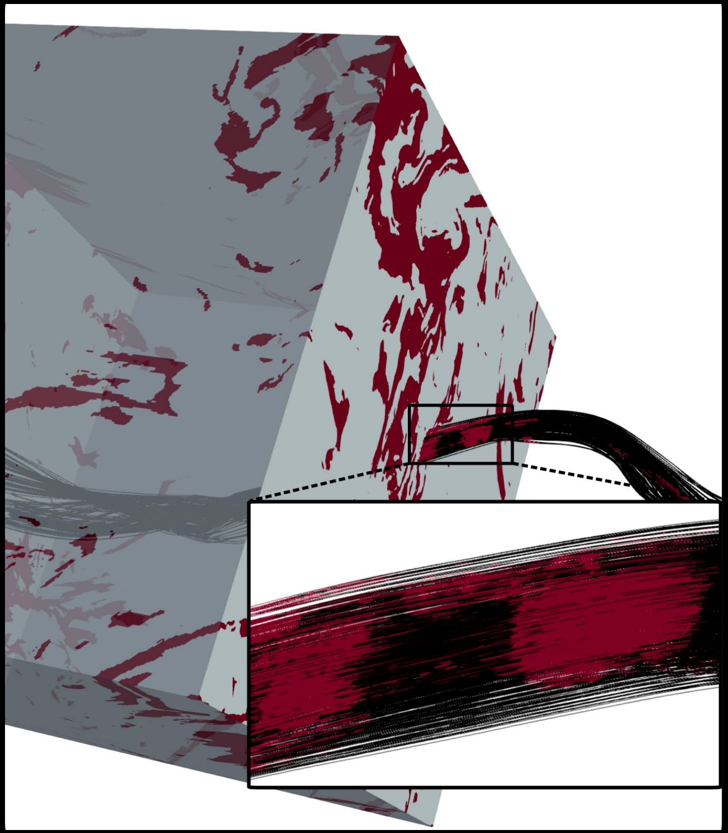

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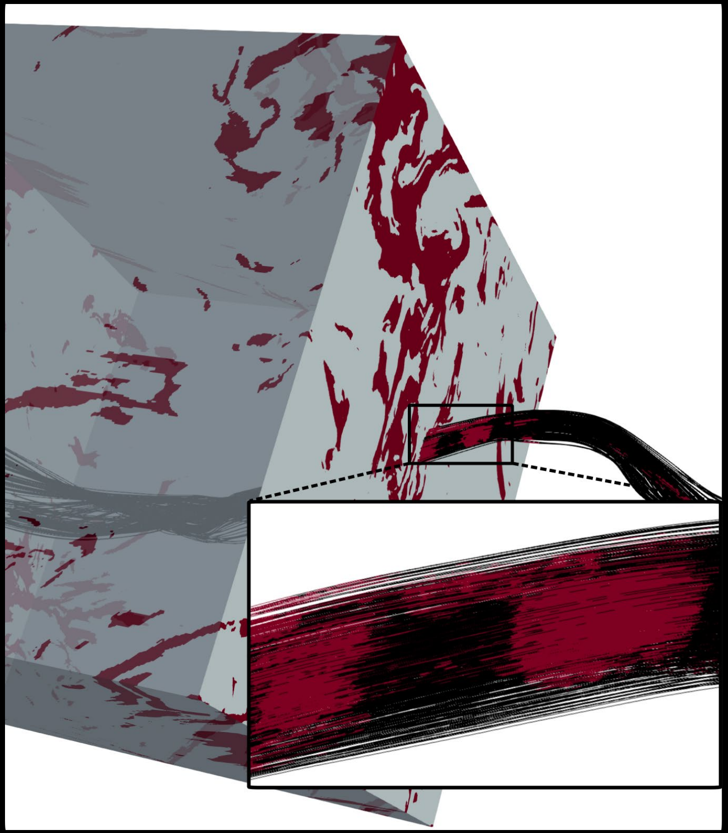

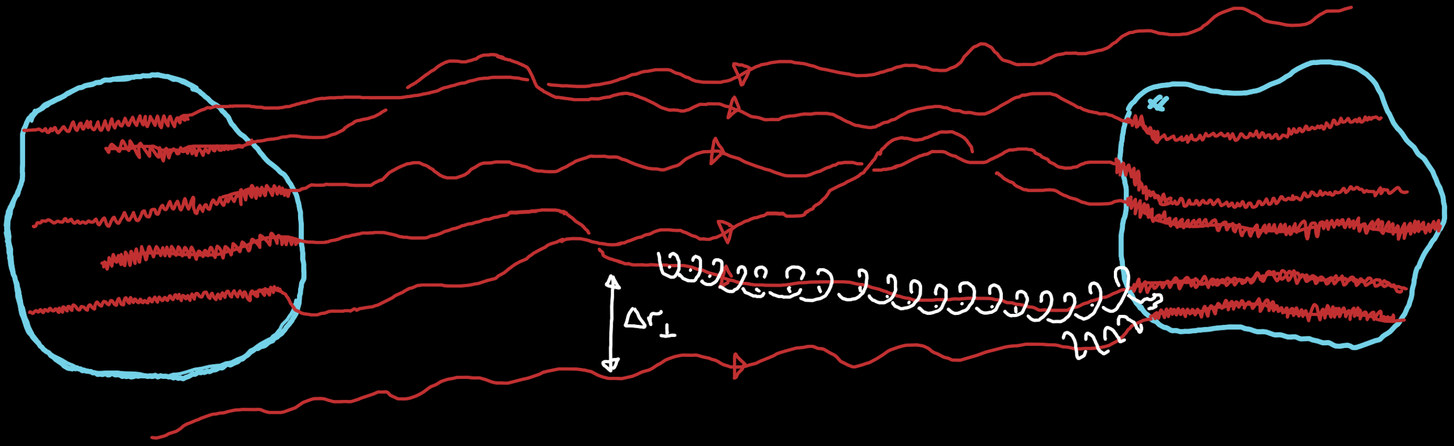

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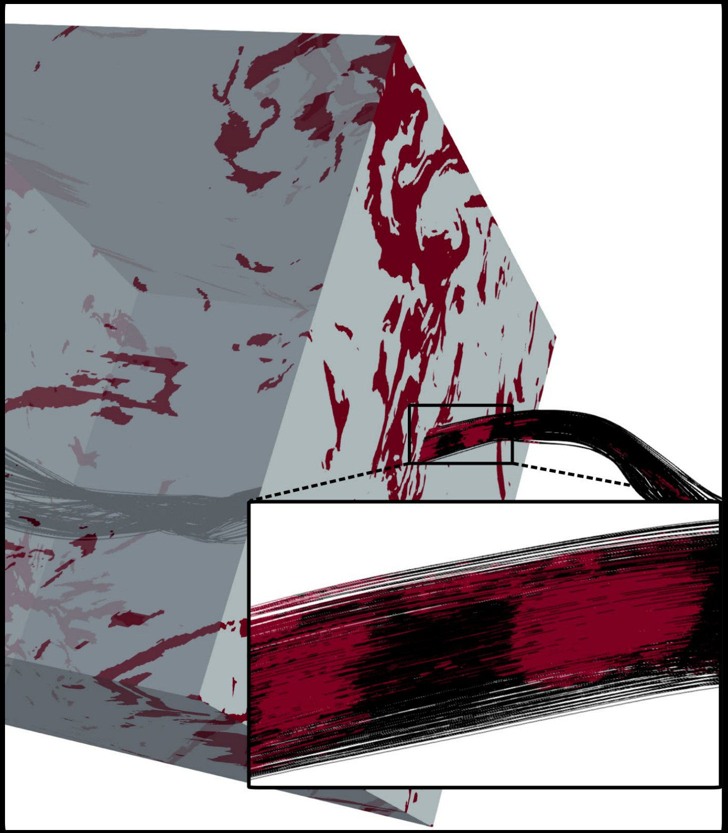

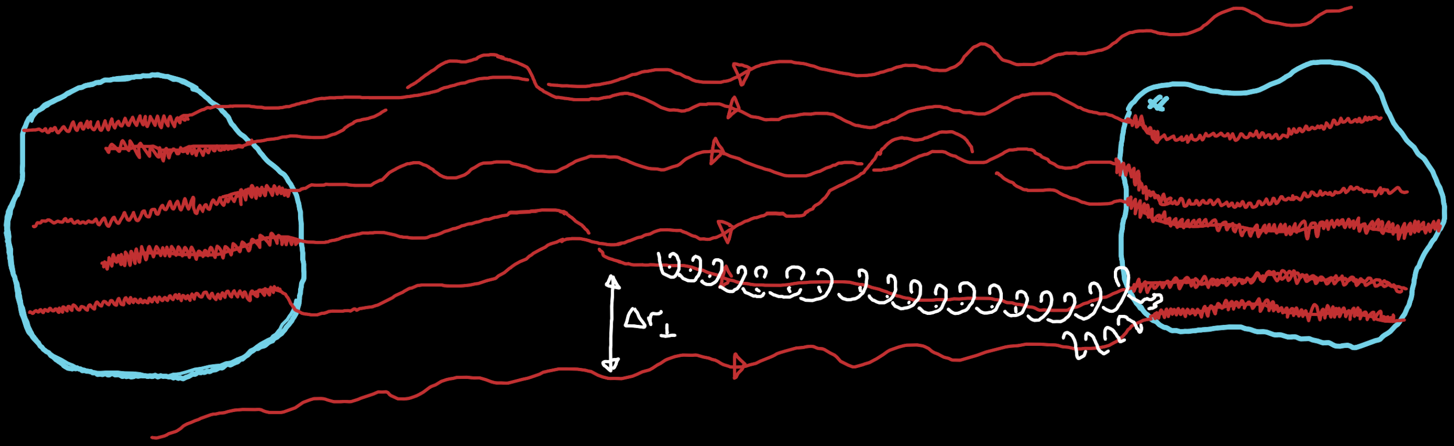

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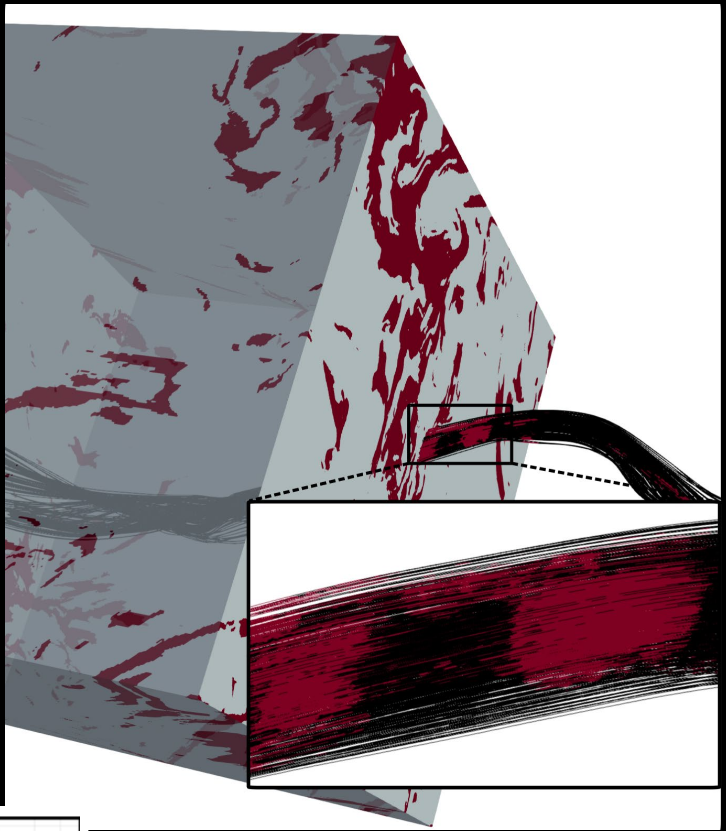

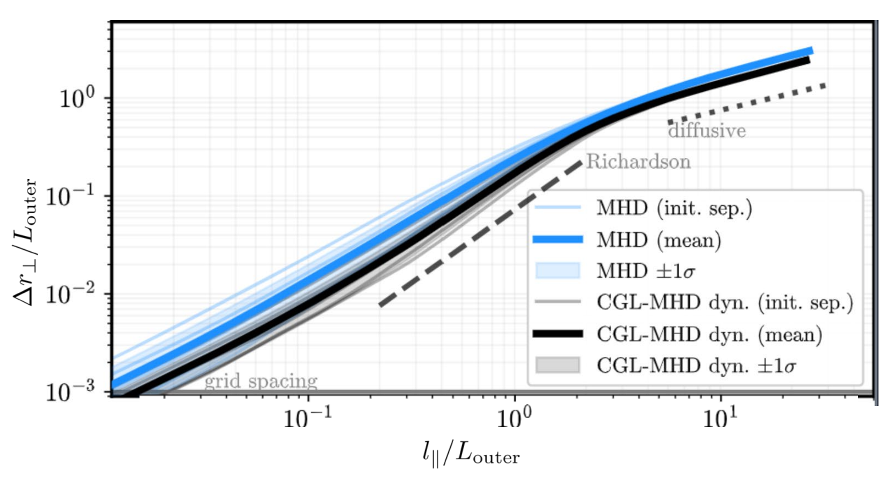

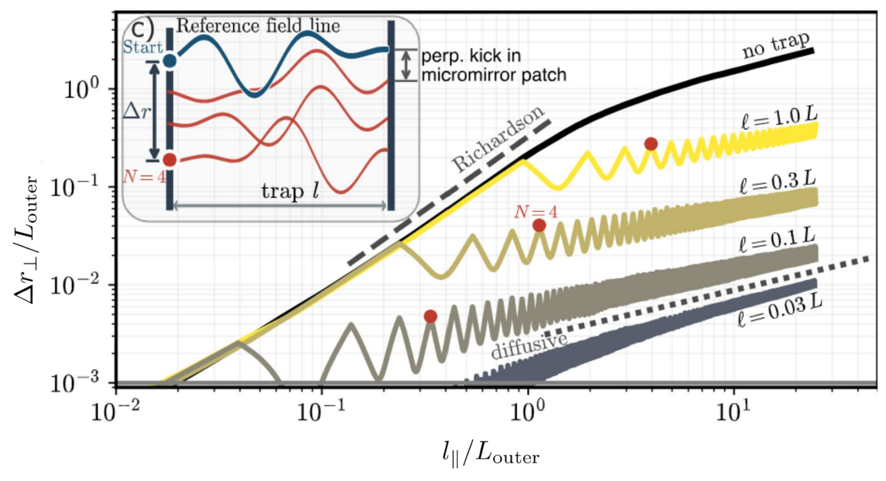

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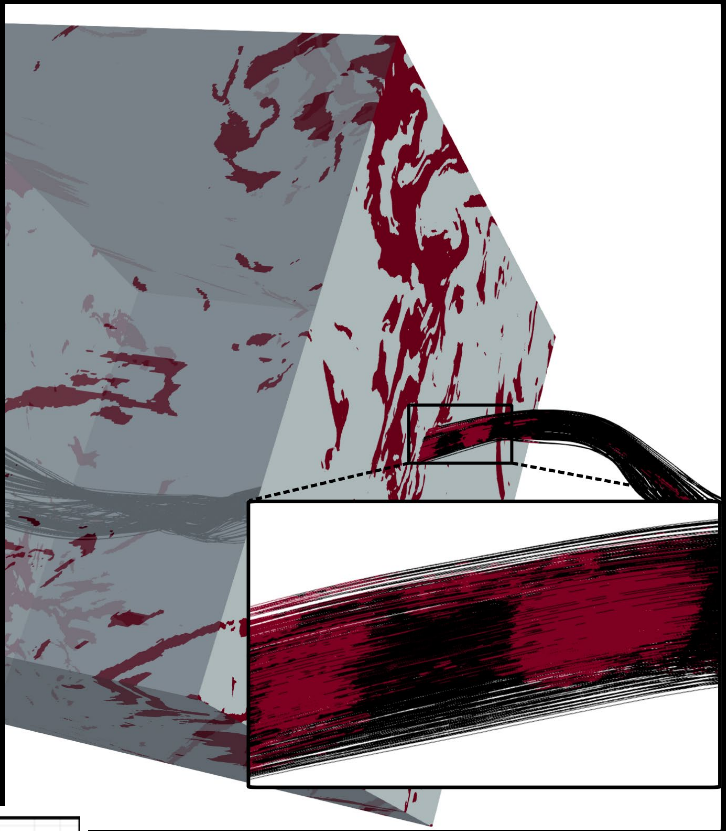

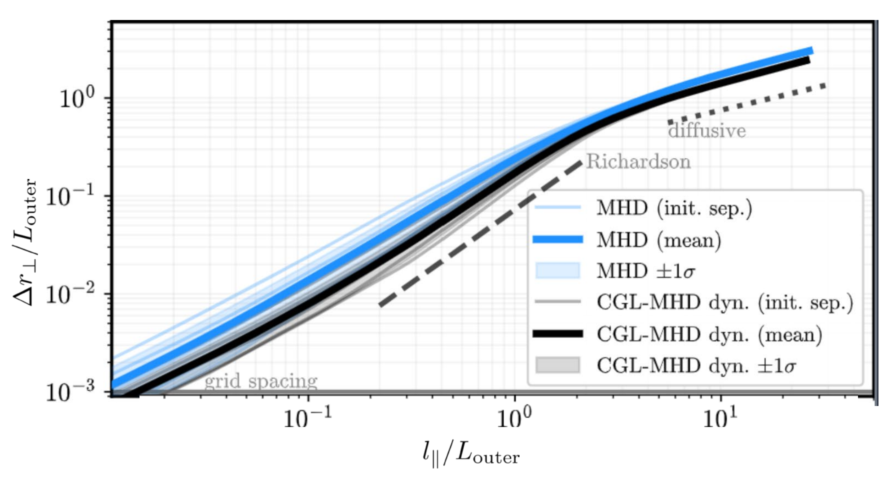

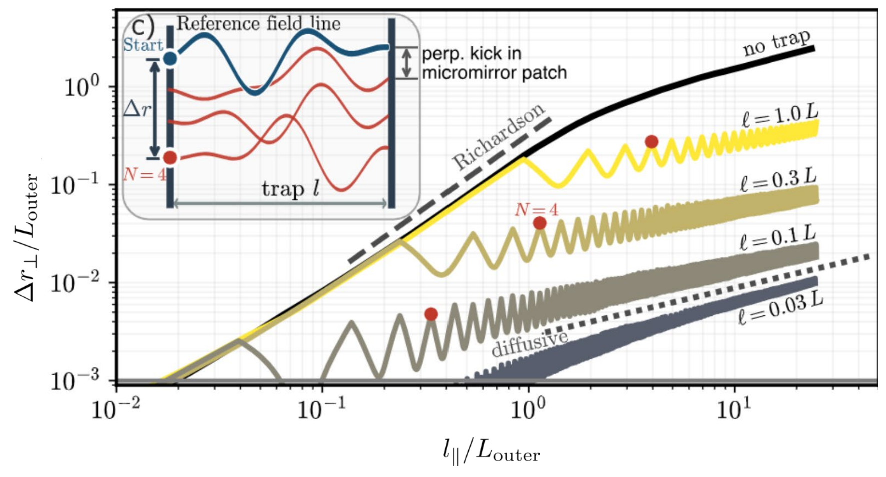

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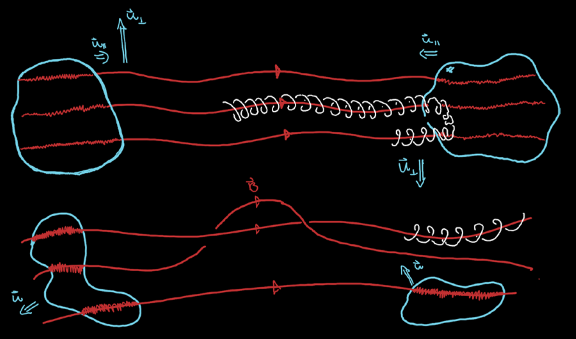

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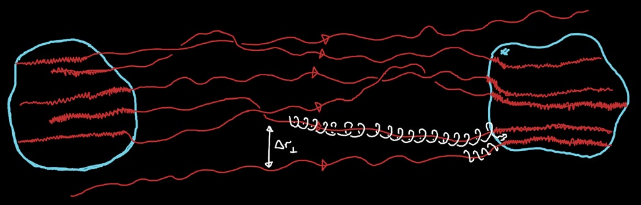

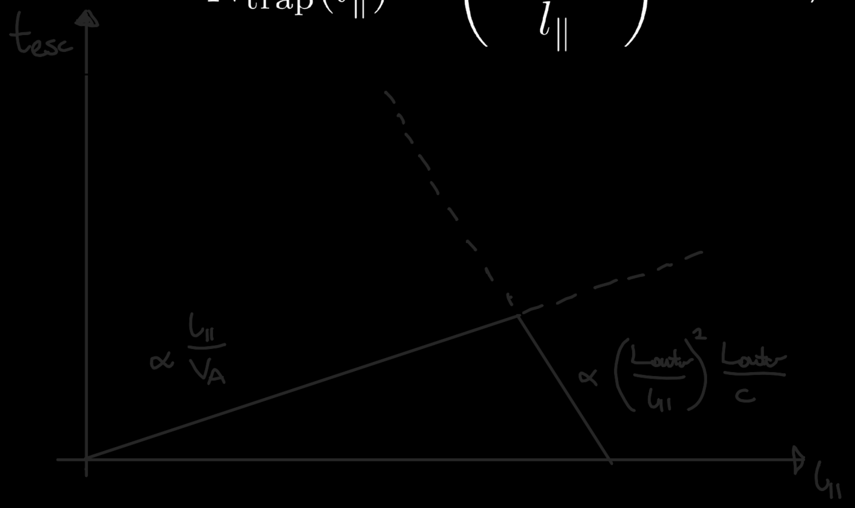

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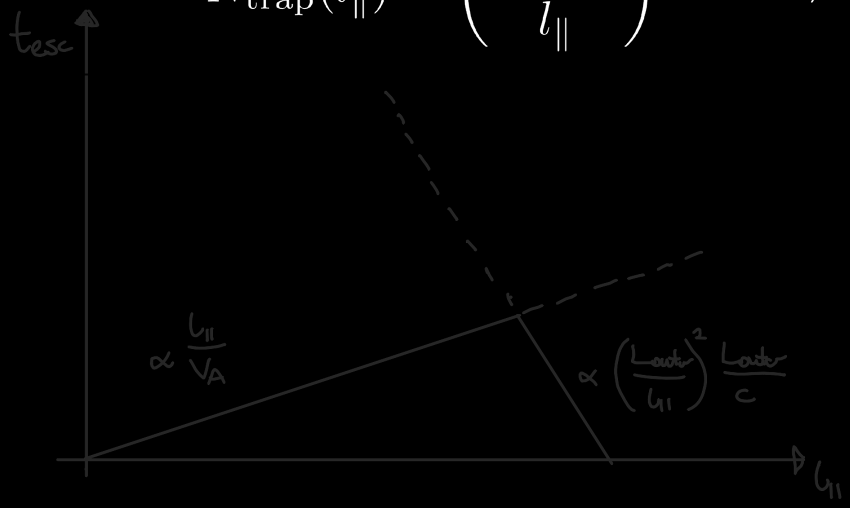

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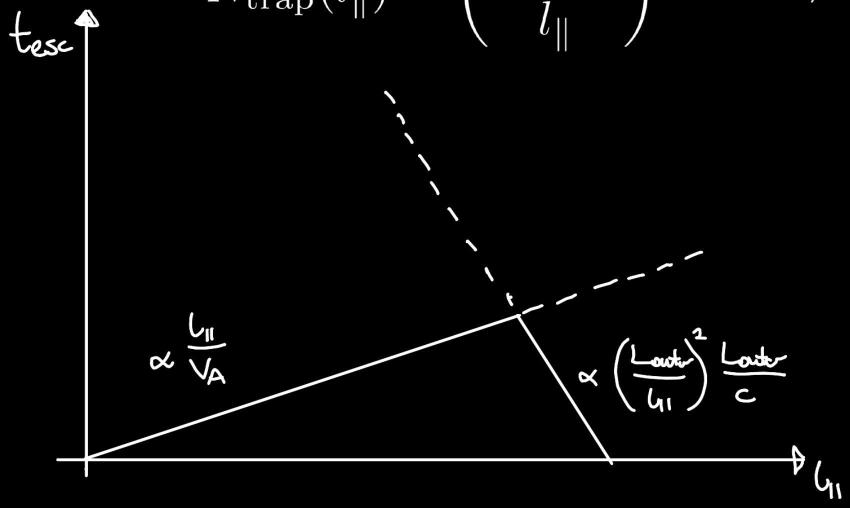

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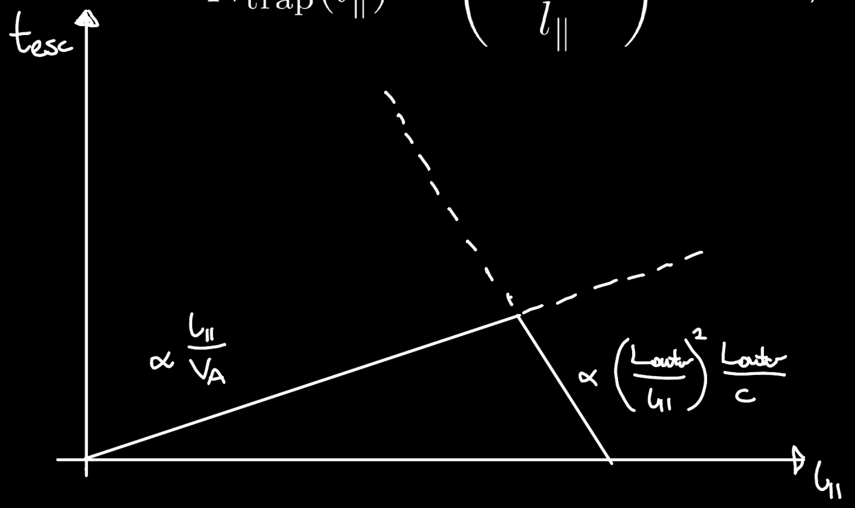

$$\boxed{l_\mathrm{crit} \sim L_\mathrm{outer}\left(\frac{V_A}{c}\right)^{1/3}}$$

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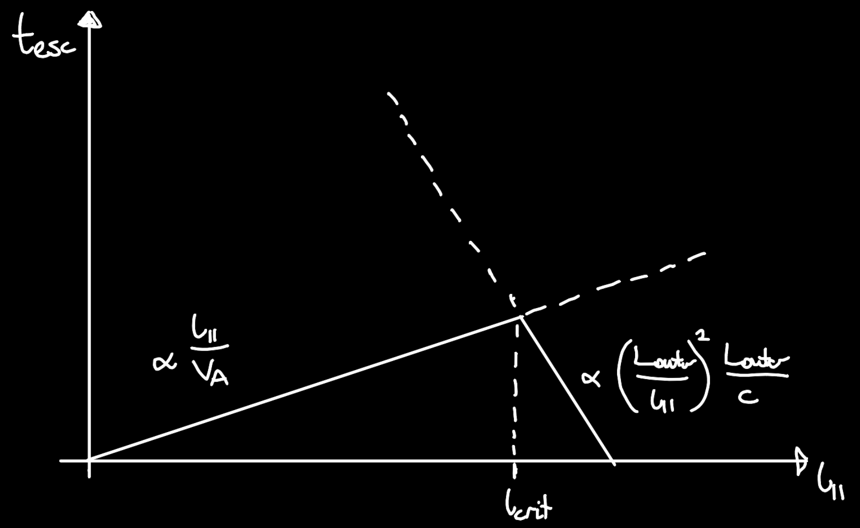

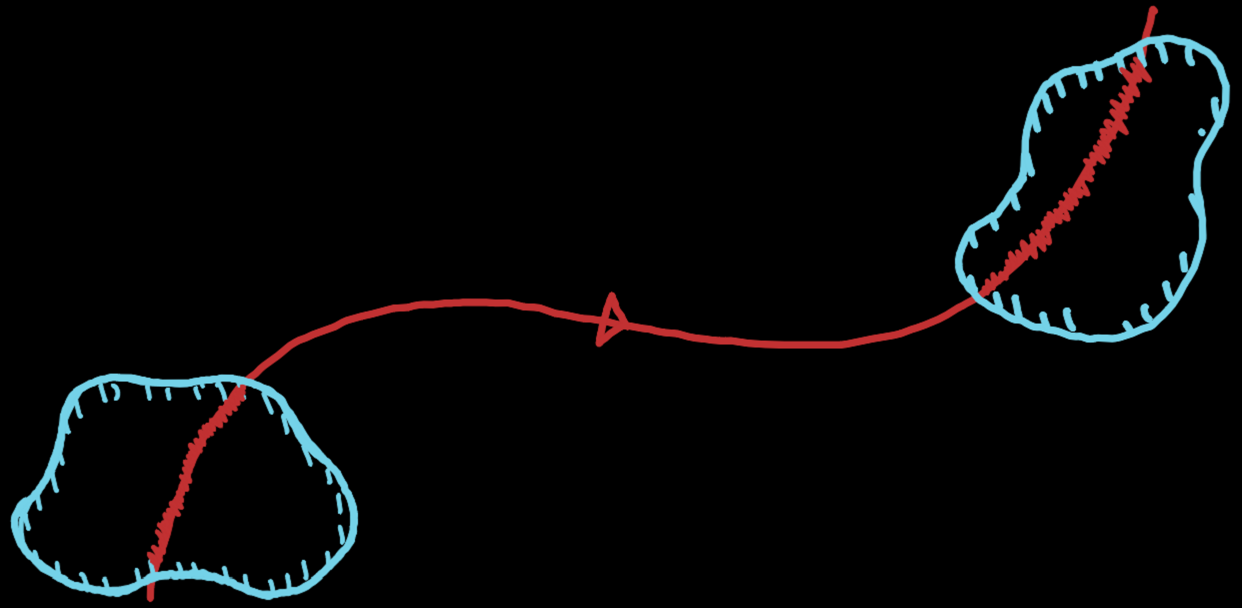

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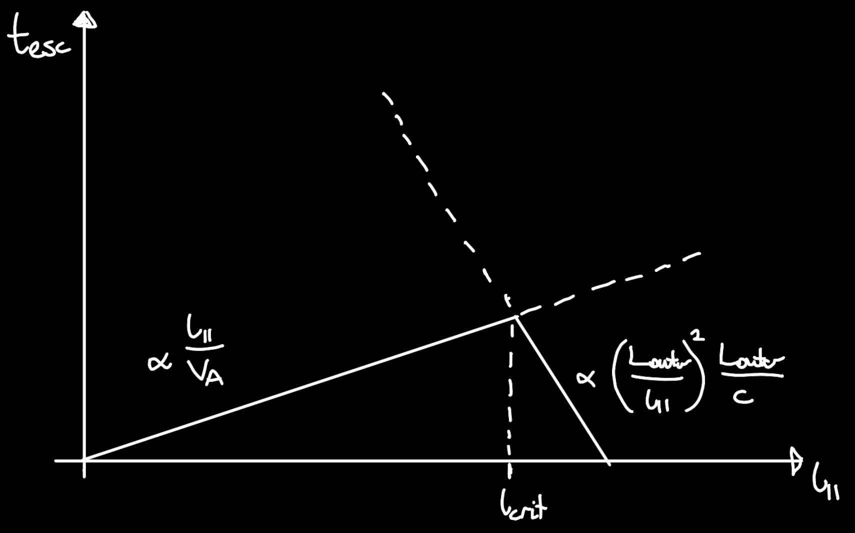

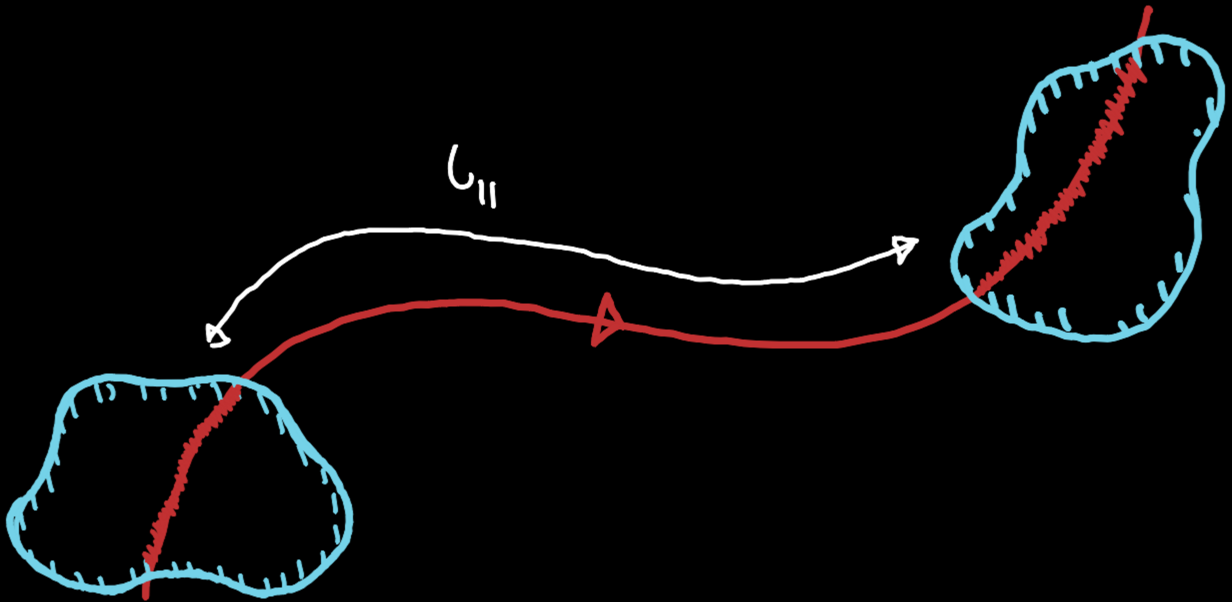

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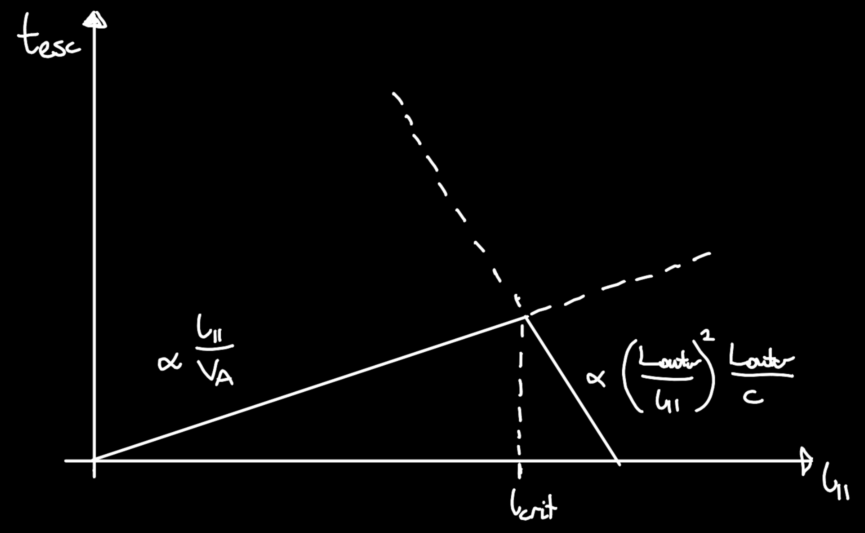
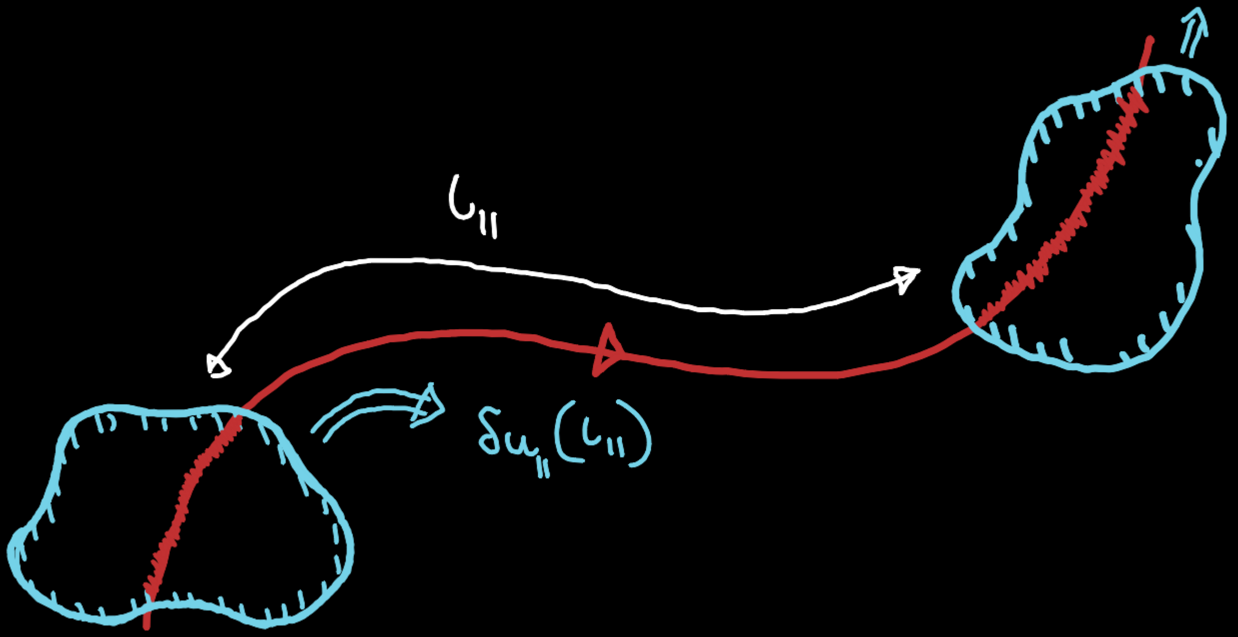




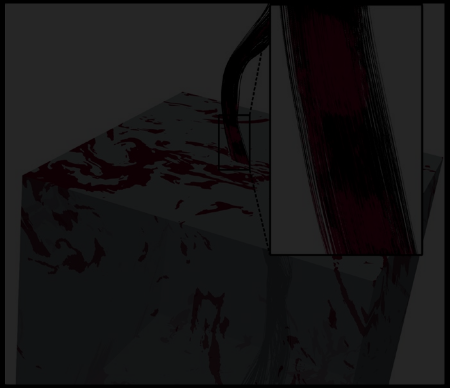
Velocity differences along field lines

See talks by Kunz (mins ago) Majeski (coming up)

- There is some precedent for this to scale as $\delta u_{\|}(l_{\|}) \sim l_{\|}^{1/2}$ (by analogy with RMHD).
- On the flip side, this is *exactly* the thing that magneto-immutability should be reducing.

$$\hat{\boldsymbol{b}}\hat{\boldsymbol{b}} : \nabla \boldsymbol{u} \sim k_{\|}\delta u_{\|}$$

- The simplest thing to do is just go into the simulation and measure it.



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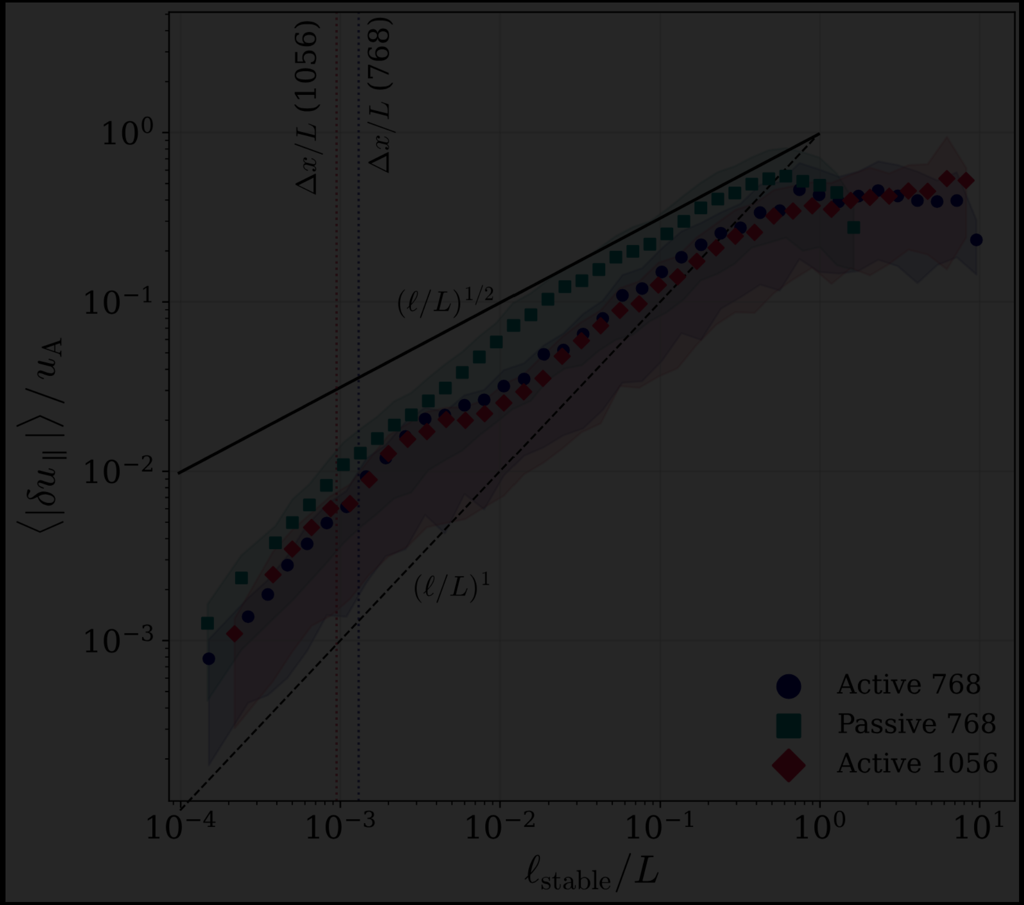

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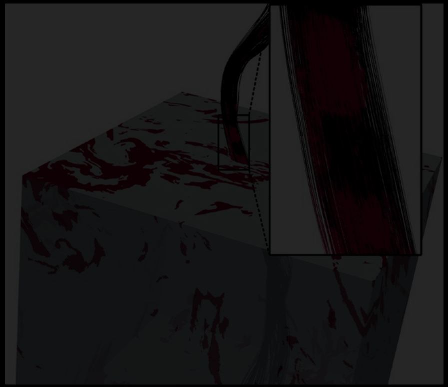

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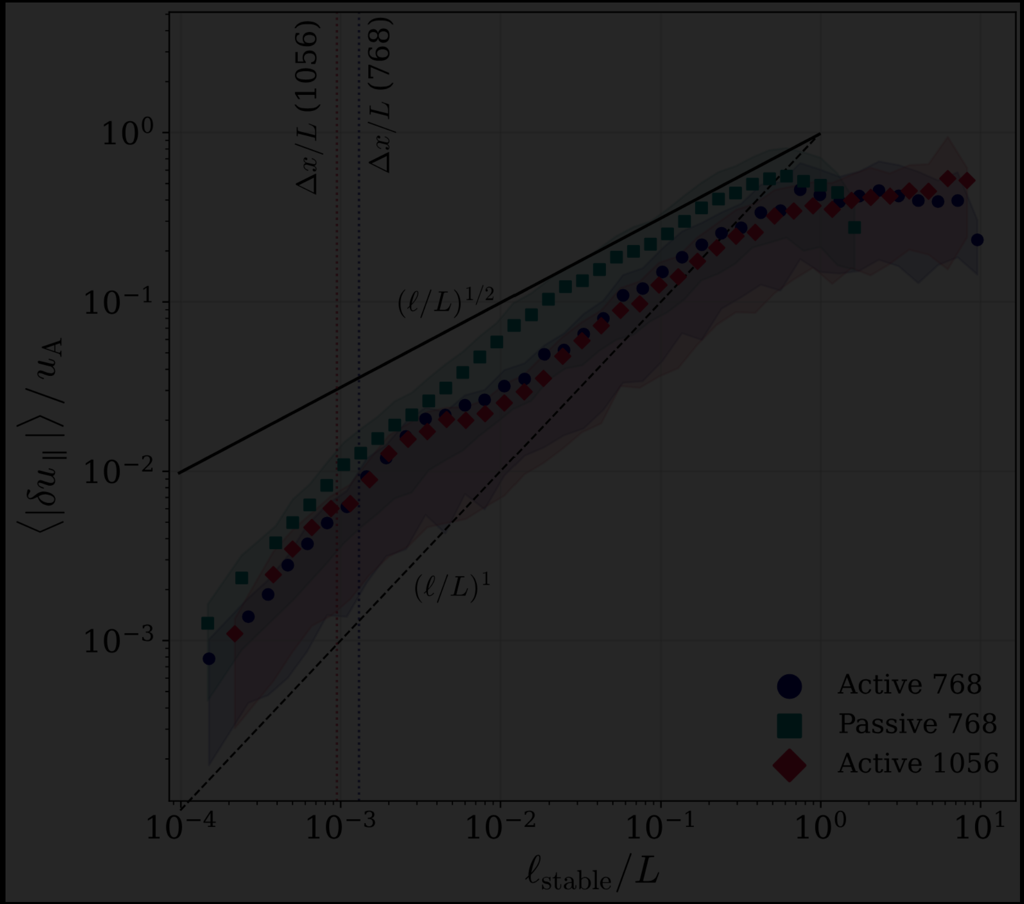

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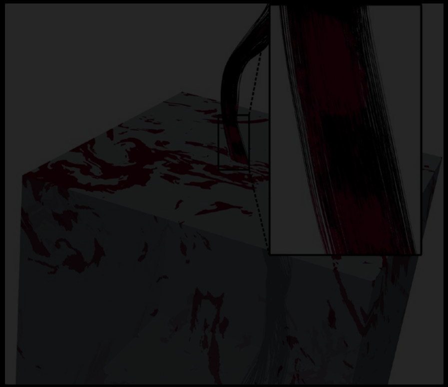

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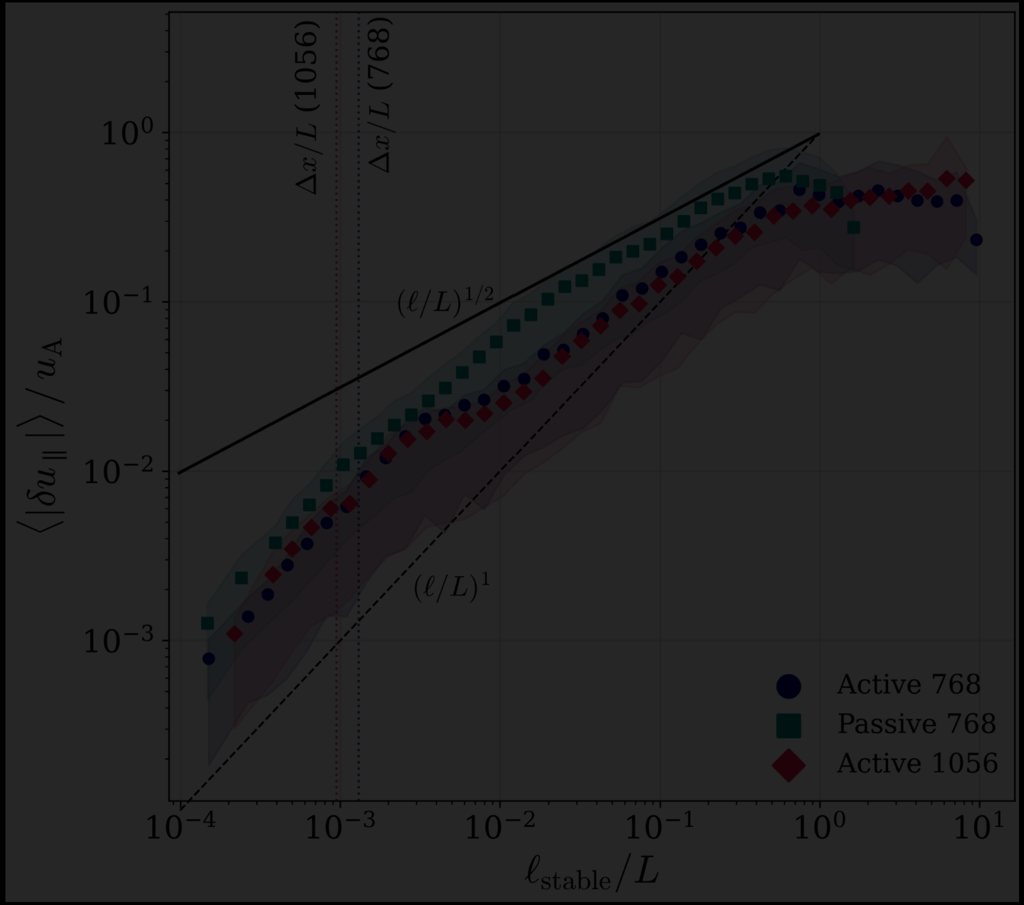

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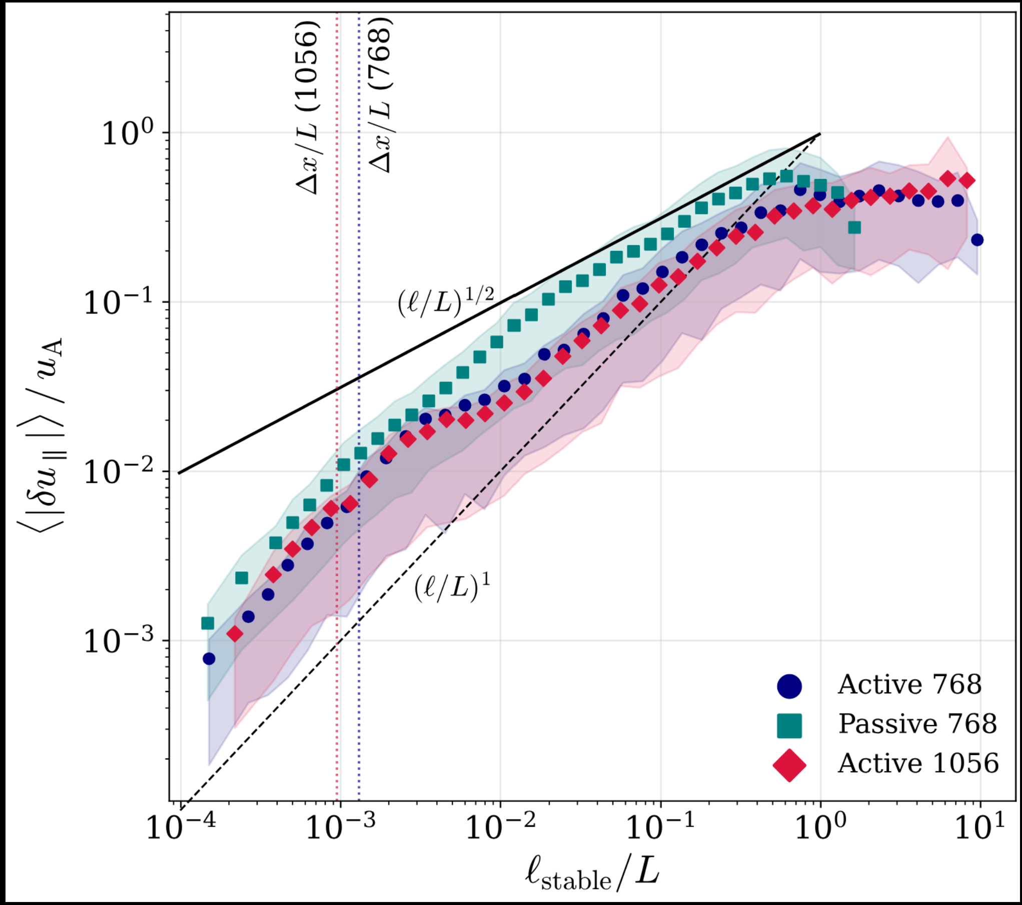

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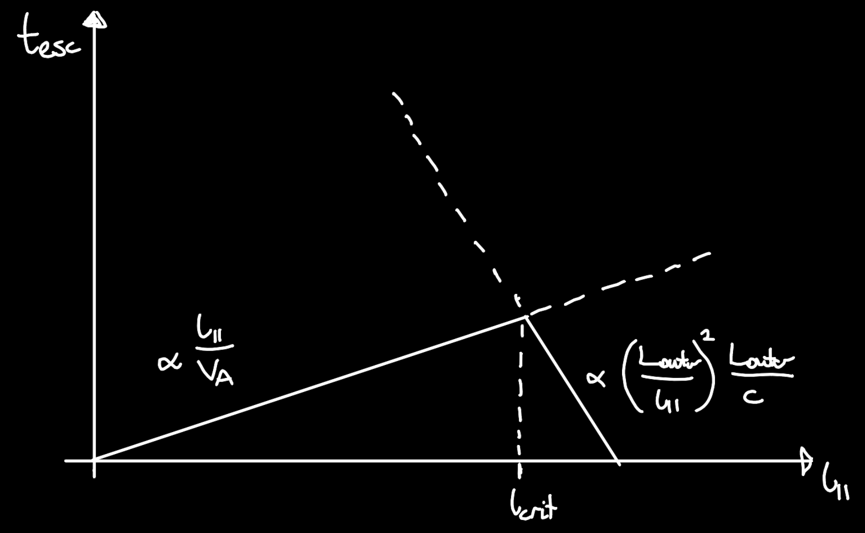

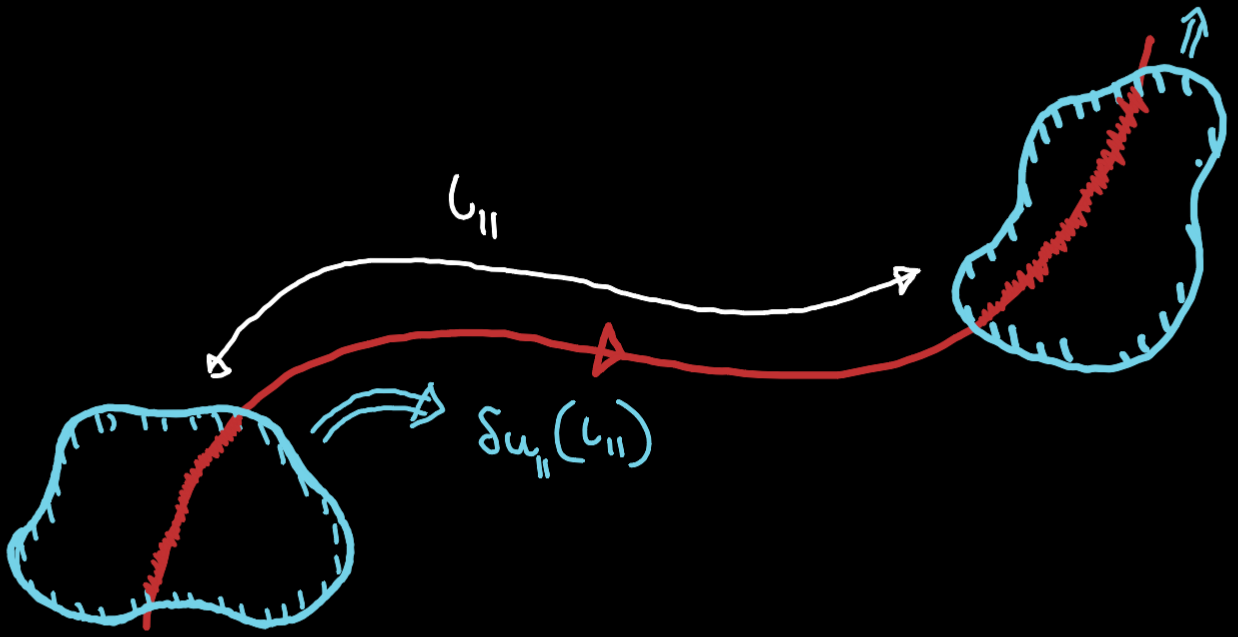

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$$\boxed{P(l_{\|}) \sim l_{\|}^{-3/2}}$$

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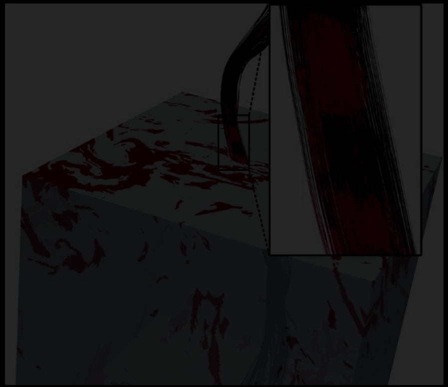

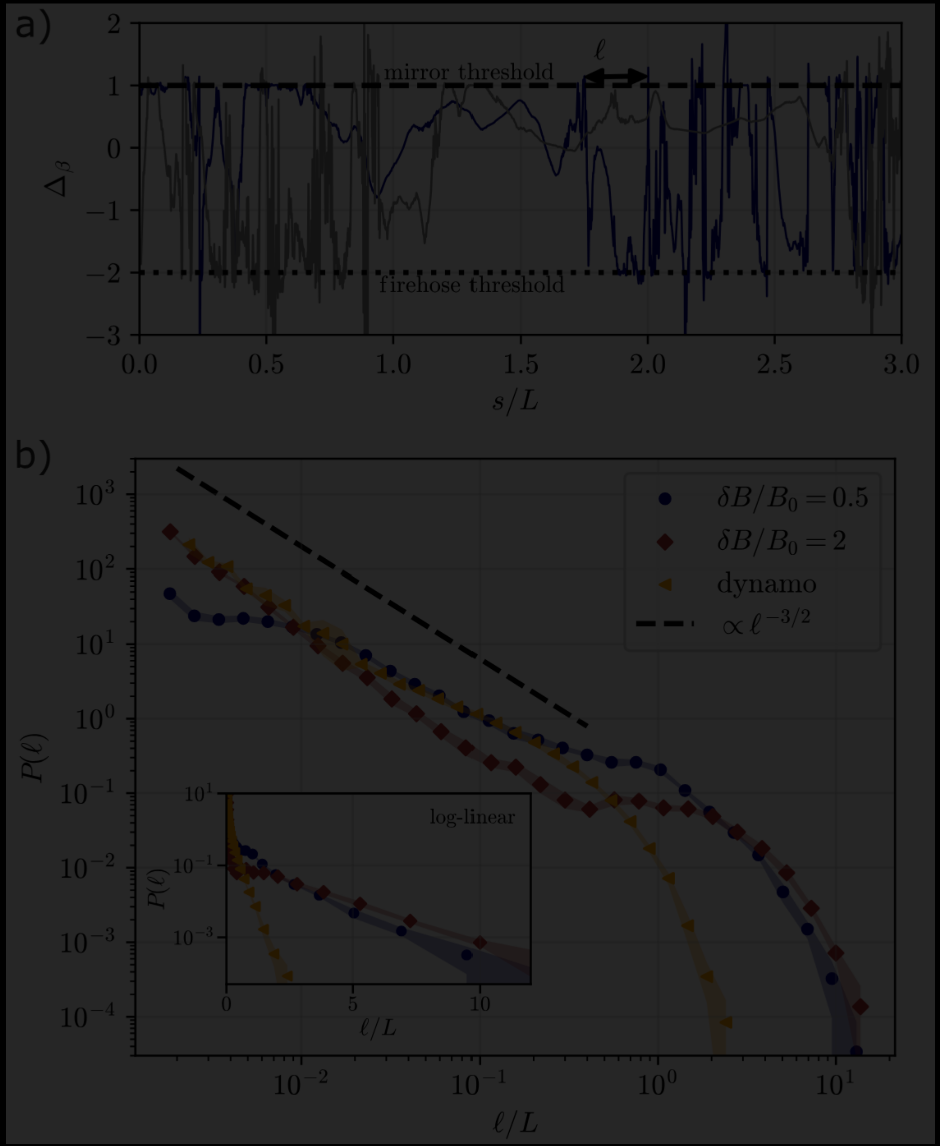

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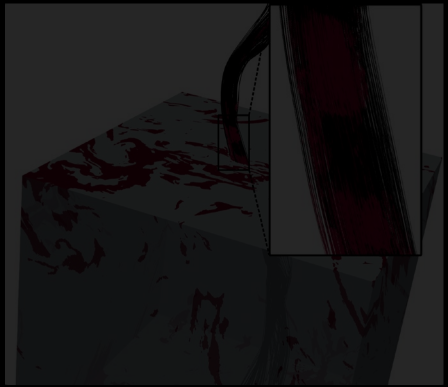

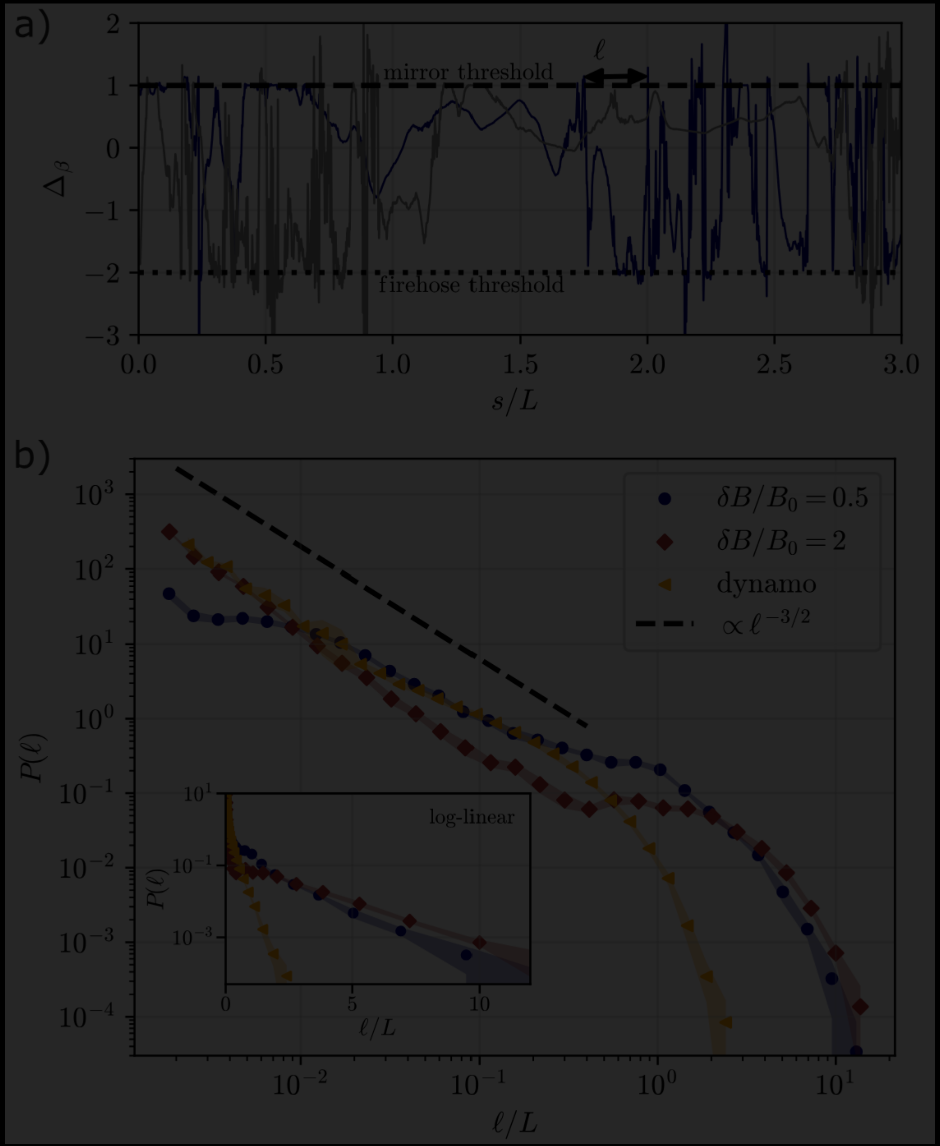

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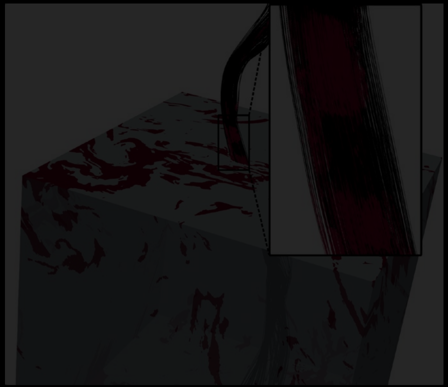

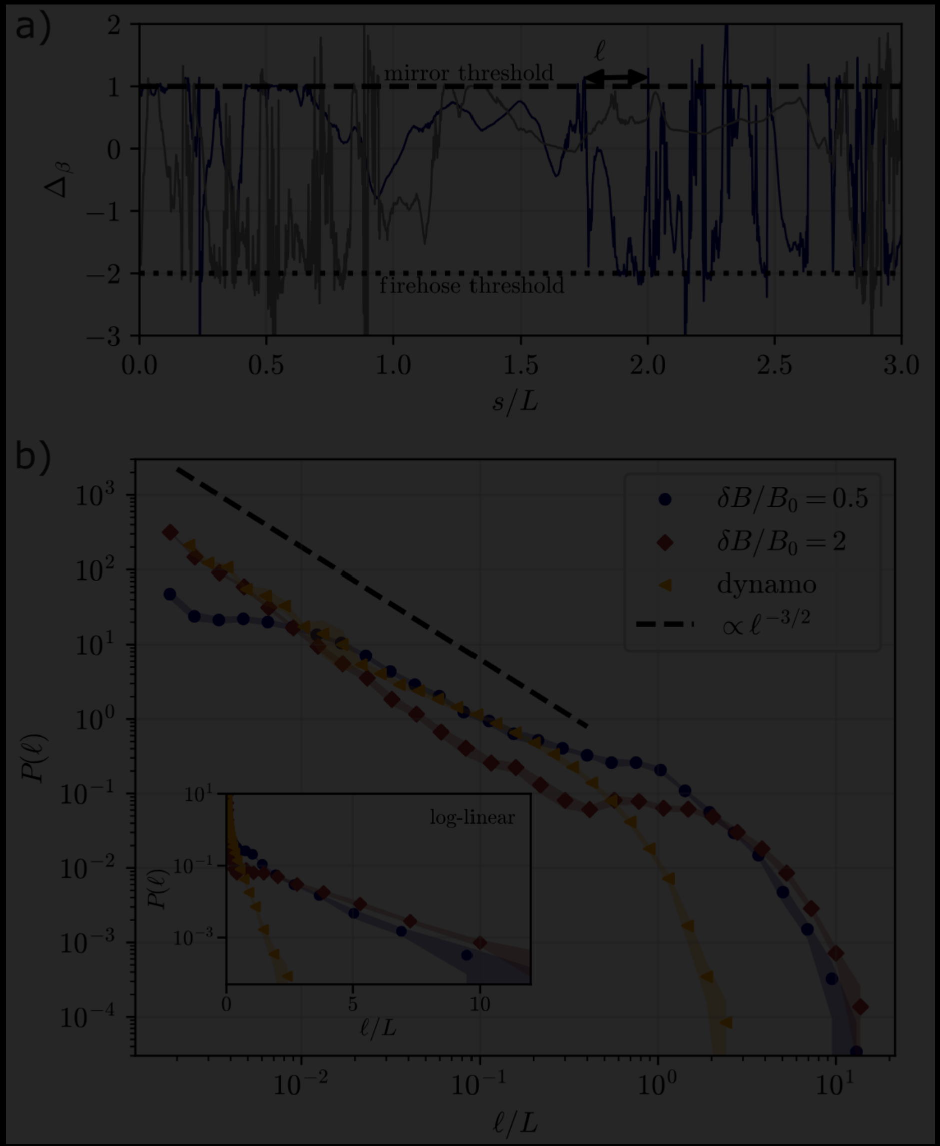

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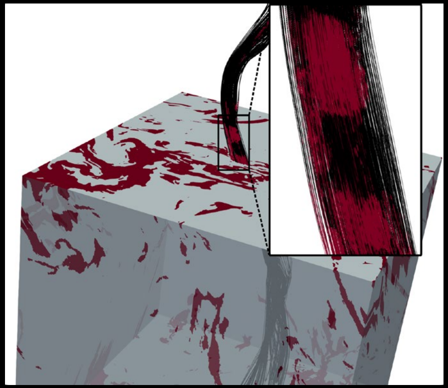

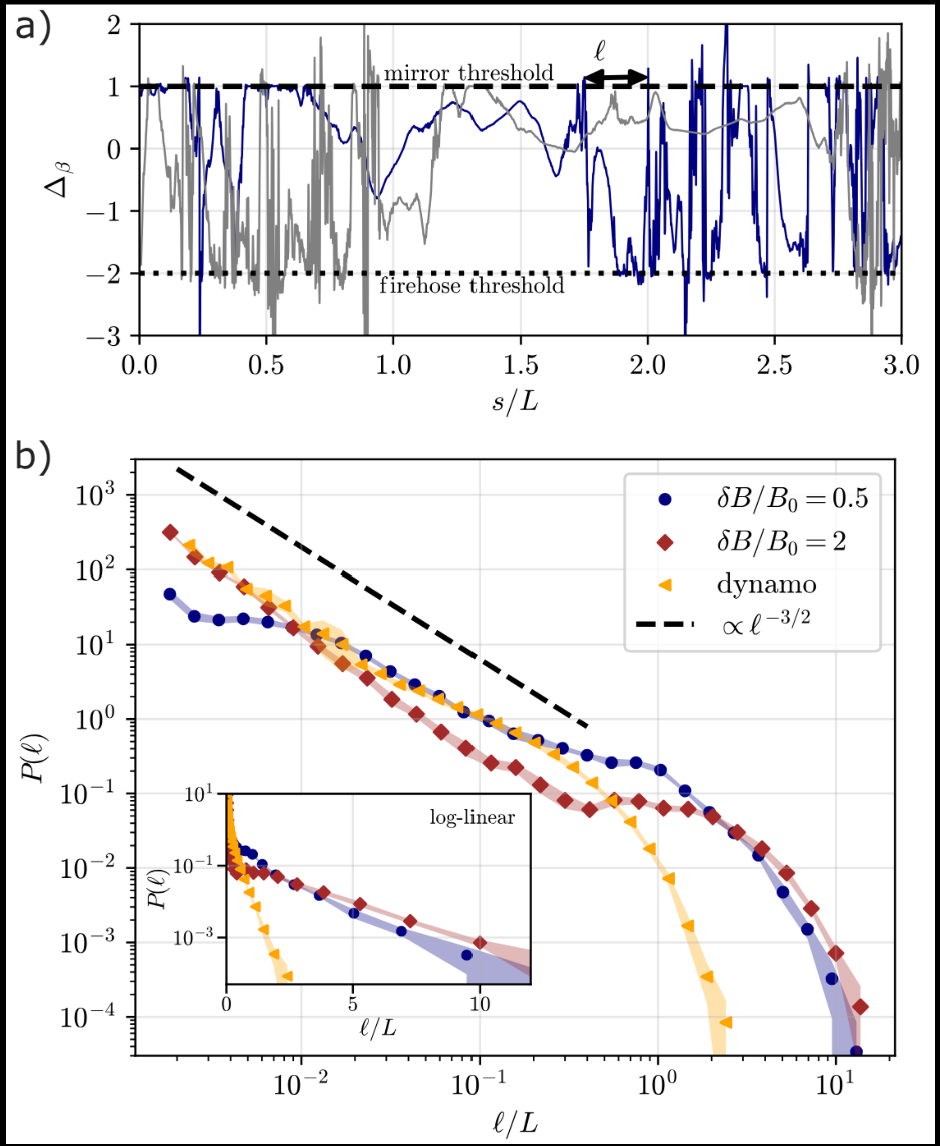

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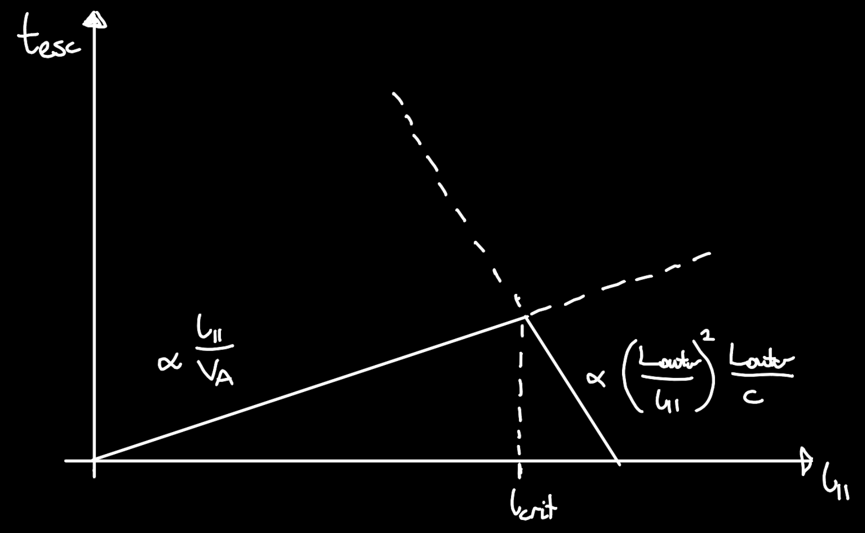

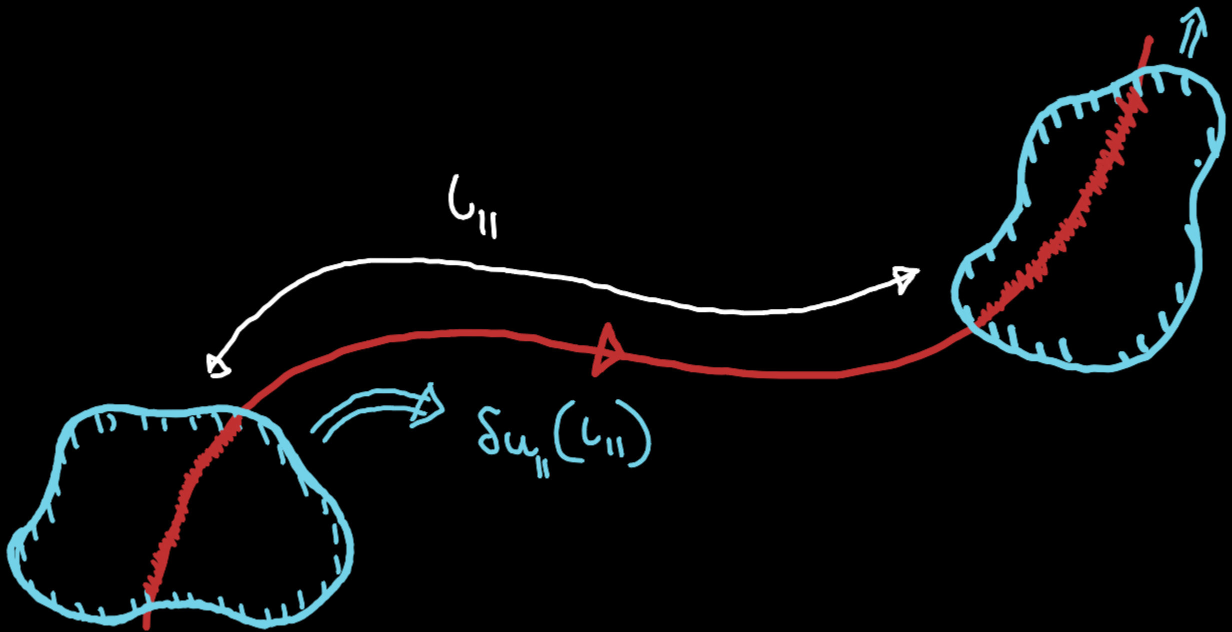

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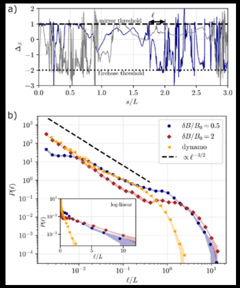
Summary

- Creating acceleration better than Fermi II in regular MHD is hard.

- High-β environments can create regions of micro-unstable plasma that are efficient scatterers.

- If those regions move coherently, it's easy-ish to construct a scenario where you get Fermi I.



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- What type of Fermi acceleration you get depends on details of CGL MHD that are only *just* being resolved at 1000^3
- It's not obvious that CGL MHD *without further bells and whistles* can keep up with what we've been shown this week in radio emission.

- Seeing this Fermi I with a particle pusher is a serious numerical challenge.