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Anomalous transport in weakly collisional, high-β plasmas: where do we stand?

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What's the problem?

Fluid description of plasma dynamics can always be derived from kinetic description…

$$\left.\frac{\mathrm{D}n_s}{\mathrm{D}t}\right|_s = -n_s \nabla \cdot \boldsymbol{V}_s ,$$

Viscosity tensor *Friction force*

$$m_s n_s \left.\frac{\mathrm{D}\boldsymbol{u}_s}{\mathrm{D}t}\right|_s = -\nabla p_s - \nabla \cdot \boldsymbol{\pi}_s + Z_s e n_s \left(\boldsymbol{E} + \frac{\boldsymbol{u}_s \times \boldsymbol{B}}{c}\right) + \sum_{s \neq s'} \boldsymbol{R}_{ss'} ,$$

$$\frac{3}{2} n_s \left.\frac{\mathrm{D}T_s}{\mathrm{D}t}\right|_s = -p_s \nabla \cdot \boldsymbol{u}_s - \nabla \cdot \boldsymbol{q}_s - \boldsymbol{\pi}_s : \nabla \boldsymbol{u}_s + \sum_{s \neq s'} \tilde{W}_{ss'} .$$

Heat flux *Ion-electron energy exchange*

…but these equations are not closed → need ***transport models!*** E.g. in an electron-ion plasma, what are

$$\boldsymbol{q}_i = \boldsymbol{q}_i(n_i, T_i, \boldsymbol{u}_i, \Delta \boldsymbol{u}, \Delta T, \boldsymbol{B}, \nabla n_i, \nabla T_i, \nabla \boldsymbol{u}_i, \dots), \quad \boldsymbol{q}_e = \boldsymbol{q}_e(n_e, T_e, \boldsymbol{u}_e, \Delta \boldsymbol{u}, \Delta T, \boldsymbol{B}, \nabla n_e, \nabla T_e, \nabla \boldsymbol{u}_e, \dots),$$

$$\boldsymbol{\pi}_i = \boldsymbol{\pi}_i(n_i, T_i, \boldsymbol{u}_i, \Delta \boldsymbol{u}, \Delta T, \boldsymbol{B}, \nabla n_i, \nabla T_i, \nabla \boldsymbol{u}_i, \dots), \quad \boldsymbol{\pi}_e = \boldsymbol{\pi}_e(n_e, T_e, \boldsymbol{u}_e, \Delta \boldsymbol{u}, \Delta T, \boldsymbol{B}, \nabla n_e, \nabla T_e, \nabla \boldsymbol{u}_e, \dots),$$

$$\boldsymbol{R}_{ei} = \boldsymbol{R}_{ei}(n_e, T_e, \boldsymbol{u}_e, \Delta \boldsymbol{u}, \Delta T, \boldsymbol{B}, \nabla n_e, \nabla T_e, \nabla \boldsymbol{u}_e, \dots),$$

$$\tilde{W}_{ie} = \tilde{W}_{ie}(n_e, T_e, \boldsymbol{u}_e, \Delta \boldsymbol{u}, \Delta T, \boldsymbol{B}, \nabla n_e, \nabla T_e, \nabla \boldsymbol{u}_e, \dots)?$$

Calculate transport models from kinetic description

Classical transport theory

1950s'/1960s' theories: collisional processes and magnetic fields determine transport properties

Applicability: transport on macroscopic length scales ($L \gg \lambda_e \sim \lambda_i$).

Method: expand kinetic equation order by order in $\epsilon \equiv \lambda_e/L \sim \lambda_i/L$ (Chapman-Enskog expansion), show that

$$f_e \approx f_{\mathrm{M}e} + \underbrace{f_e^{(1)}}_{\mathcal{O}(\epsilon f_e)} + \dots, \quad f_i \approx f_{\mathrm{M}i} + \underbrace{f_i^{(1)}}_{\mathcal{O}(\epsilon f_i)} + \dots, \quad \Longrightarrow \quad \text{compute fluxes from } f_e^{(1)},\ f_i^{(1)}$$

Results: unmagnetised plasma ($\lambda_s \ll \rho_s,\ s = i, e$) has isotropic, diffusive heat and momentum transport:

Heat flux $\quad \boldsymbol{q}_s = -\frac{3}{2} n_s \chi_s \nabla T_s,\ \chi_s = \mathcal{A}_s \lambda_s^2 \nu_s \quad$ *Thermal diffusivity*

Viscosity tensor $\quad \boldsymbol{\pi}_s = -\mu_s \left[\nabla \boldsymbol{V}_s + (\nabla \boldsymbol{V}_s)^{\mathrm{T}} - \frac{2}{3}(\nabla \cdot \boldsymbol{V}_s)\mathbf{I}\right], \quad \mu_s = \mathcal{B}_s m_s n_s \lambda_s^2 \nu_s \quad$ *Dynamic viscosity*

Friction force $\quad \boldsymbol{R}_{ei} = e n_e \frac{\eta c^2}{4\pi} \boldsymbol{j},\ \eta = 0.51 d_e^2 \nu_{ei} \quad$ *Resistivity*

Ion-electron energy exchange $\quad \tilde{W}_{ie} = \frac{3 n_e m_e \nu_{ei}}{m_i}(T_e - T_i).$

Magnetised plasma ($\lambda_s \gg \rho_s,\ s = i, e$) has anisotropic transport, with largest components along magnetic field lines (see Braginskii 1965).

Breakdown of classical transport theory

What if the CE distribution functions underlying classical transport theory are kinetically unstable?

- (Naïve) rebuttal 1: won't an 'almost Maxwellian' distribution function be stable?
- (Naïve) rebuttal 2: doesn't collisional damping prevent kinetic instabilities?

Both rebuttals are overly simplistic…

- *Thresholdless instabilities* (Ramani & Laval 1978, Epperlein 1984) e.g. Weibel instability
- Characteristic scales of kinetic instabilities are *not* fluid scales; in (e.g.) a classical hydrogen plasma,

$$\frac{\lambda_{\rm mfp}}{\lambda_{\rm D}} \sim n_e \lambda_{\rm D}^3 \gg 1 \implies$$ *Non-trivial range of collisionless scales in classical plasma*

…and concern is legitimate. *Presence of kinetic instability could alter transport significantly!*

Major questions to address:

1. When do kinetic instabilities arise?
2. When they do arise, how do they alter transport? What are $\boldsymbol{q}_e$, $\boldsymbol{q}_i$, $\boldsymbol{\pi}_e$, $\boldsymbol{\pi}_i$, $\boldsymbol{R}_{ei}$, $\tilde{W}_{ie}$ in such cases?

It's an old problem… but recent progress!

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Kahn (1964) first to highlight the issue, but quantitative, general solution lacking…

…but mantle taken up by Alex and other collaborators just over two decades ago!

Transverse plasma waves and their instability. Part 2

By F. D. KAHN
Department of Astronomy, University of Manchester

(Received 7 October 1963)

A dispersion relation is set up for transverse waves in a plasma with an ellipsoidal velocity distribution. It is shown that the most unstable wave has its wave-normal parallel to the shortest axis of the ellipsoid and its vector potential parallel or anti-parallel to the longest axis. The maximum possible amplification rate is then calculated. Ellipsoidal velocity distributions arise in any flow with an anisotropic pressure tensor. If the resulting instability leads to a microscopic redistribution of particle velocities, then the effective transport coefficients of the plasma are changed. In particular it is shown that the effective viscosity is decreased, and becomes dependent on the local gradient of the macroscopic velocity.

PLASMA INSTABILITIES AND MAGNETIC FIELD GROWTH IN CLUSTERS OF GALAXIES

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Received 2005 January 17; accepted 2005 April 12

ABSTRACT

We show that under very general conditions, cluster plasmas threaded by weak magnetic fields are subject to very fast growing plasma instabilities driven by the anisotropy of the plasma pressure (viscous stress) with respect to the local direction of the magnetic field. Such an anisotropy will naturally arise in any weakly magnetized plasma that has low collisionality and is subject to stirring. The magnetic field must be sufficiently weak for the instabilities to occur, viz., $\beta \gtrsim \mathrm{Re}^{1/2}$. The instabilities are captured by the extended MHD model with Braginskii viscosity. However, their growth rates are proportional to the wavenumber down to the ion gyroscale, so MHD equations with Braginskii viscosity are not well posed and a fully kinetic treatment is necessary. The instabilities can lead to magnetic fields in clusters being amplified from seed strength of $\sim 10^{-18}$ G to dynamically important strengths of $\sim 10\ \mu$G on cosmologically trivial timescales ($\sim 10^8$ yr). The fields produced during the amplification stage are at scales much smaller than observed. Predicting the saturated field scale and structure will require a kinetic theory of magnetized cluster turbulence.

Subject headings: galaxies: clusters: general — instabilities — magnetic fields — MHD — plasmas — turbulence

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Wolfgang Pauli Institute, Vienna

7th (Astro)Plasma Kinetics Working Group Meeting
24 March - 4 April 2014

Responsible Organisers: M. Kunz (Princeton) & A. Schekochihin (Oxford)

TOPICS:

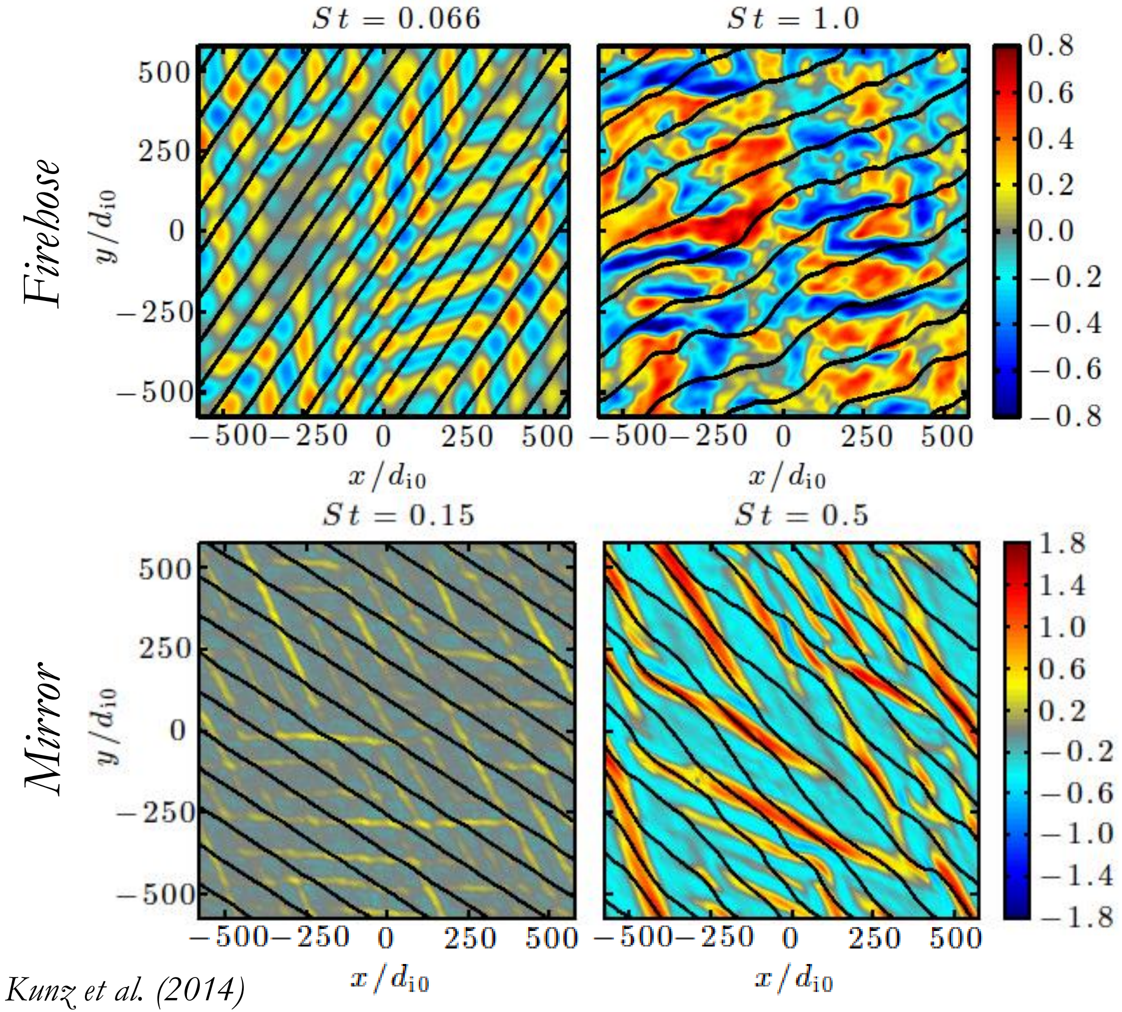
- **PA**: Pressure anisotropies *(weeks 1, 2)*
- **FM**: Firehose, mirror and other instabilities *(week 1)*
- **PD**: Plasma dynamo *(week 1)*
- **KT**: Kinetic turbulence and phase mixing *(weeks 1, 2)*
- **KR**: Kinetic reconnection *(week 2)*
- **GK**: Astrophysical gyrokinetics *(weeks 1, 2)*
- **HE**: Ion and electron heating *(weeks 1, 2)*
- AOB

Note: *All topics to be discussed at all times, but the distribution between weeks 1 & 2 shown above corresponds to the periods of likely concentration of effort (and also reflects the interests of those planning to attend during those weeks).*

WPI workshops have been key part of this effort!

How has progress been made?

Stunning theoretical insight of 21st century plasma theorists… also, we have bigger computers now
→ Key simulation tools have been collisionless particle-in-cell (PIC) simulations or hybrid-PIC



Kunz et al. (2014)

Many new theory/simulation papers in the last two decades investigating different transport physics:

Schekochihin and Cowley (2006), Hellinger (2007), Camporeale & Burgess (2008), Hellinger & Trávníček (2008), Pokhotelov et al. (2008), Quataert (2008), Schekochihin et al. (2008), Hellinger et al. (2009), Lazar et al. (2009), Schekochihin et al. (2009), Camporeale and Burgess (2010), Schekochihin et al. (2010), Pokhotelov & Amariutei (2011), Rosin et al. (2011), Achterberg (2013), Guo et al. (2014), Hellinger et al. (2014), Kunz et al. (2014), Mogavero & Schekochihin (2014), Hellinger and Trávníček (2015), Sironi & Narayan (2015), Kunz et al. (2015), Rincon et al. (2015), Riquelme et al. (2015), Ruyer et al. (2015), Astfalk & Jenko (2016), Komarov et al. (2016), Melville et al. (2016), Riquelme et al. (2016), Roberg-Clark et al. (2016), Riquelme et al. (2017), Squire et al. (2017), Guo et al. (2018), Hellinger & Štverák (2018), Komarov et al. (2018), Kunz et al. (2018), Riquelme et al. (2018), Roberg-Clark et al. (2018a), Roberg-Clark et al. (2018b), Kuzichev et al. (2019), Shaaban et al. (2019), Squire et al. (2019), Bell et al. (2020), Kim et al. (2020), Kunz et al. (2020), Bott et al. (2021), Drake et al. (2021), Arzamasskiy et al. (2023), Squire et al. (2023), Majeski et al. (2023), Ley et al. (2023), Bott et al. (2024), apologies to other papers I may have missed…

What have we learnt so far?

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A1: criteria for when Braginskii transport theory fails (Bott et al. 2024):

Insufficient magnetic stabilisation $\beta \gtrsim \mathrm{Ma}^{-1}\frac{L_V}{\lambda_e} \gg 1$ and $\frac{\lambda_e}{L_V} \gtrsim \left(\frac{d_e}{L_V}\right)^{2/3}\left(\frac{m_i}{m_e}\right)^{1/6}\mathrm{Ma}^{-1/3}$ *Insufficient collisional stabilisation*

→ When classical theory fails, we know which of 16 different kinetic instabilities are dominant!

A2: transport coefficients depend on magnetic field strength and macroscopic evolution (e.g Komarov et al. 2018)

→ Weakly collisional plasmas behave like non-Newtonian, shear-thinning fluids!

$$q_{e\|,\mathrm{eff}} \simeq 2.1\frac{B^2}{4\pi}m_e^{-1/2}T_e^{1/2}, \quad \mu_{\|,\mathrm{eff}} \simeq 0.8\frac{B^2}{4\pi}\left|\left(\hat{\boldsymbol{b}}\hat{\boldsymbol{b}} - \frac{1}{3}\mathbf{I}\right):\nabla\boldsymbol{u}\right|^{-1}.$$

A3: effective collisionality $\nu_{\mathrm{eff}} \sim \beta/\tau$ (usually) larger than Coulomb collisionality

→ Suppressed magnetic-field-aligned transport compared to classical models

A4: distinct transport models necessary depending on β (e.g. ultra-high $\beta \gtrsim \tau\Omega_i$ vs. moderately-high $\beta \ll \tau\Omega_i$)

Some very recent work (2025/26) to consider

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Heat conduction

Viscosity

Experiments

"Collisionless whistler heat-flux instability (WHFI) in ultra-high-β plasmas"

Rhys Davies

DL: large-amplitude fluctuations; transport is dominated by advection, not diffusion (non-Gaussian diffusion?)

"Modelling transport in weakly collisional, magnetised plasmas using thermodynamic forcing"

Prakriti Pal Choudhury

DL: novel, generalisable method for studying instabilities/calculate transport using PIC simulations at reduced cost

"Anomalous viscosity in firehose-susceptible high-β plasmas"

DL: effective collisionality is robustly given by $\nu_{\rm eff} \sim \beta_i/\tau$; effective collision operator for firehoses

"Collisional whistler instability and temperature staircases"

Nicolas Lopez

DL: WHFI transitions to 'fluid' instability as ν_e/Ω_e increases past unity, but can still modify temperature profiles

"Collisionless conduction in a high-β plasma"

Evan Yerger

DL: largest WHFI simulations to date; effective collision operator for WHFI

"Characterising heat conduction in magnetised, weakly collisional plasma on Orion"

Thomas Vincent

DL: we made a high-β plasma using lasers; evidence of spontaneous microturbulence and suppressed conduction

What’s next?

Problem is not solved yet… Many outstanding issues!

1. Effective resistivity: $\boldsymbol{R}_{ei}$ (and $\boldsymbol{\pi}_e$) still not well characterised
   - *See Aikya Banerjee’s talk on Tuesday, Matt Kunz’s on Wednesday*
2. Ion conductivity: $\boldsymbol{q}_i$ drives two instabilities (gyrothermal, slow-hydromagnetic wave) in high-β plasma…
3. Electron-ion equilibration: do high-β plasma instabilities affect $\tilde{W}_{ei}$?
4. Cross-field transport: could be large in ultra-high-β plasmas (or not - non-Gaussian transport…)?
5. Weakly collisional vs. collisionless: when are collisionless simulations a good proxy (or not)?
   - *Lopez et al. (2025) suggests that modifications to transport could be possible even with “strong” collisionality*
   - *Ongoing work (Rhys Davies, Oxford) hints that collisionality could determine* β *at which transport is minimised*
6. Multiple free-energy sources? For example, what happens to kinetic instabilities in the presence of both temperature and bulk-velocity gradients?
   - *Choudhury & Bott (2026) proposed ‘thermodynamic forcing’ methodology to investigate scenario – preliminary studies have shown that dominant instability (and transport) can change qualitatively*
7. Reduced kinetic models of high-β plasma instabilities; effective collision operators?