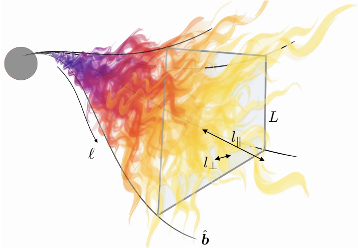
Generalised RMHD: putting solar-wind turbulence on an industrial basis

J. Squire[1], B. D. G. Chandran[2], T. Adkins[3], W. A. Clarke[4,5],
R. Meyrand[2,1], and M. W. Kunz[6,3]



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3
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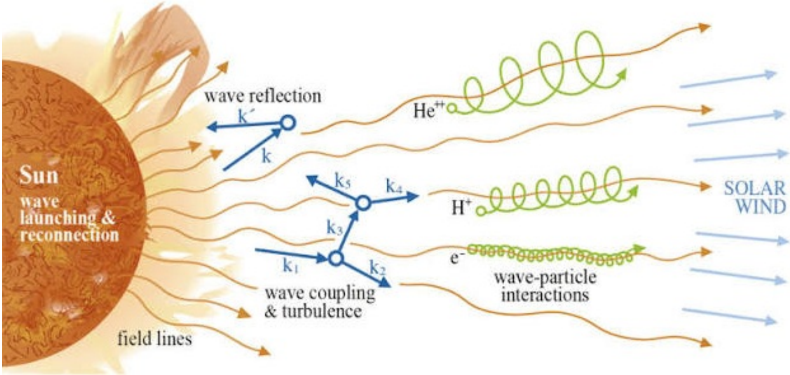
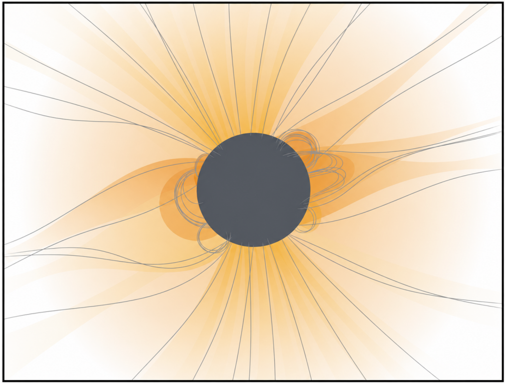
Solar-wind acceleration and heating

Figure due to B. Chandran



- Numerous processes are responsible for the acceleration and heating of the solar wind: reconnection, reflection, Alvénic heating, ...
- Models have typically assumed locally symmetric flux-tube geometries or added additional effects in ad-hoc ways

Solar-wind acceleration and heating



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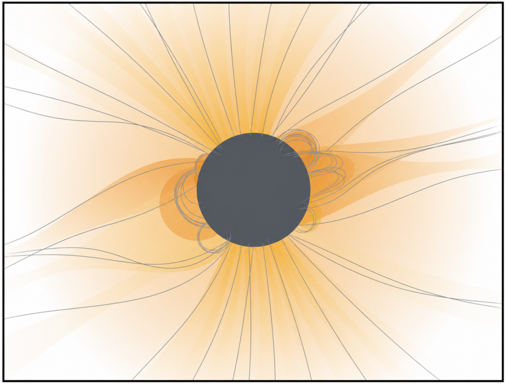

- Numerous processes are responsible for the acceleration and heating of the solar wind: reconnection, reflection, Alvénic heating, ...
- Models have typically assumed locally symmetric flux-tube geometries or added additional effects in ad-hoc ways
- Can we capture (some of) these processes **within a single framework**?

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Our starting point

- Compressible MHD in a (slowly) rotating frame:

$$
\begin{aligned}
&\frac{\partial \rho}{\partial t} + \boldsymbol{\nabla} \cdot (\rho \boldsymbol{u}) = S_\rho, \\
&\rho \left(\frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{u} \cdot \boldsymbol{\nabla} \boldsymbol{u} + 2\boldsymbol{\Omega} \times \boldsymbol{u} \right) + \boldsymbol{\nabla} \left(p + \frac{B^2}{8\pi} \right) - \frac{\boldsymbol{B} \cdot \boldsymbol{\nabla} \boldsymbol{B}}{4\pi} \\
&\qquad\qquad = -\rho \boldsymbol{\nabla} \Phi_{\mathrm{tot}} + \boldsymbol{\nabla} \cdot \boldsymbol{\Pi} + \boldsymbol{S}_{\rho U} - \boldsymbol{u} S_\rho, \\
&\frac{\partial \boldsymbol{B}}{\partial t} = \boldsymbol{\nabla} \times (\boldsymbol{u} \times \boldsymbol{B}) - \boldsymbol{\nabla} \times (\eta \boldsymbol{\nabla} \times \boldsymbol{B}), \\
&\frac{\partial E_{\mathrm{th}}}{\partial t} + \boldsymbol{\nabla} \cdot (\boldsymbol{u} E_{\mathrm{th}}) = -p \boldsymbol{\nabla} \cdot \boldsymbol{u} - \boldsymbol{\nabla} \cdot \boldsymbol{q} - \boldsymbol{\Pi} : \boldsymbol{\nabla} \boldsymbol{u} + \frac{\eta}{4\pi} |\boldsymbol{\nabla} \times \boldsymbol{B}|^2 + S_{\mathrm{th}},
\end{aligned}
$$

where ρ, $\boldsymbol{u}$, $\boldsymbol{B}$, p, $\boldsymbol{\Pi}$, $\boldsymbol{q}$, η have their usual meaning, and

- rotation rate: $\boldsymbol{\Omega}$
- thermal energy: $E_{\mathrm{th}} = p/(\gamma - 1)$
- total potential: $\Phi_{\mathrm{tot}} = \Phi_{\mathrm{grav}} + \Phi_{\mathrm{rot}}$, $\Phi_{\mathrm{rot}} = -|\boldsymbol{\Omega} \times \boldsymbol{r}|^2/2$.
- sources: $S_\rho, \boldsymbol{S}_{\rho U}, S_\rho$

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$$= -\rho \boldsymbol{\nabla} \Phi_{\rm tot} + \boldsymbol{\nabla} \cdot \boldsymbol{\Pi} + \boldsymbol{S}_{\rho U} - \boldsymbol{u} S_\rho,$$

$$\frac{\partial \boldsymbol{B}}{\partial t} = \boldsymbol{\nabla} \times (\boldsymbol{u} \times \boldsymbol{B}) - \boldsymbol{\nabla} \times (\eta \boldsymbol{\nabla} \times \boldsymbol{B}),$$

$$\rho T \left(\frac{\partial}{\partial t} + \boldsymbol{u} \cdot \boldsymbol{\nabla} \right) s = -\gamma c_v T S_\rho - \boldsymbol{\nabla} \cdot \boldsymbol{q} - \boldsymbol{\Pi} : \boldsymbol{\nabla} \boldsymbol{u} + \frac{\eta}{4\pi} |\boldsymbol{\nabla} \times \boldsymbol{B}|^2 + S_{\rm th}$$

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- sources: $S_\rho, \boldsymbol{S}_{\rho U}, S_\rho$
- entropy: $s - s_0 = c_v \log p/\rho^\gamma$, $c_v = R/(\gamma - 1)$

Another multiscale expansion...

- Inspired by fusion gyrokinetics (Frieman & Chen, 1982; Sugama & Horton, 1998; Abel et al., 2013), perform an asymptotic expansion in $\epsilon = k_\parallel / k_\perp \ll 1$:

$$g = \sum_{n \geqslant 0} \epsilon^n g^{(n)}, \quad g^{(n)} = \left\langle g^{(n)} \right\rangle_{\rm turb} + \delta g^{(n)}, \quad \left\langle \delta g^{(n)} \right\rangle_{\rm turb} = 0.$$

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Turbulence average

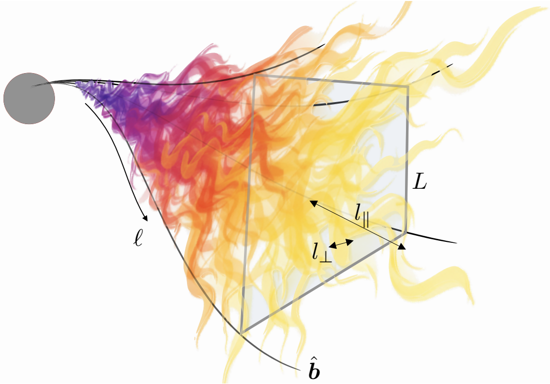
$$\langle \ldots \rangle_{\mathrm{turb}} = \langle \langle \ldots \rangle_{\perp} \rangle_t \,,$$

over intermediate timescales

$$\omega^{-1} \ll T \ll \tau,$$

and large spatial scales

$$\ell_{\perp} \ll L \lesssim \ell_{\parallel}.$$



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- Ordering:

$$\frac{\delta\rho}{\rho} \sim \frac{\delta p}{p} \sim \frac{\delta s}{s} \sim \frac{|\delta \boldsymbol{B}|}{B} \sim \frac{|\delta \boldsymbol{u}|}{v_A} \sim \frac{|\boldsymbol{\kappa}|}{k_\perp} \sim \frac{\omega}{k_\perp v_A} \sim \frac{k_\parallel}{k_\perp} \sim \epsilon.$$

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- Multiple time scales analysis:

$$\frac{\partial}{\partial t} \to \epsilon \frac{\partial}{\partial t} + \epsilon^3 \frac{\partial}{\partial \tau}.$$

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- Field-aligned mean flow:

$$\left\langle \boldsymbol{u}^{(0)} \right\rangle_{\text{turb}} \equiv \boldsymbol{U} = U \boldsymbol{b}.$$

First order: equilibrium and constraints

- Continuity:

$$\text{Equilibrium}: \quad \frac{\partial \log \rho}{\partial \ell} + \frac{\partial \log U}{\partial \ell} - \frac{\partial \log B}{\partial \ell} = (\rho U)^{-1} S_\rho.$$

$$\text{Fluctuations}: \quad \boxed{\nabla \cdot \delta \boldsymbol{u}_\perp = 0.}$$

- Momentum:

$$\text{Equilibrium}: \quad v_A^2(\boldsymbol{\kappa} - \nabla_\perp \log B) - \frac{c_s^2}{\gamma} \nabla_\perp \log p = U^2 \boldsymbol{\kappa} + \nabla_\perp \Phi_{\rm grav},$$

$$-\frac{c^2}{\gamma}\frac{\partial \log p}{\partial \ell} + \boldsymbol{b} \cdot \boldsymbol{S}_{\rho U} - U S_\rho = U^2 \frac{\partial U}{\partial \ell} + \boldsymbol{b} \cdot \nabla \Phi_{\rm grav}.$$

$$\text{Fluctuations}: \quad \boxed{\frac{\delta p}{p} = -\gamma \frac{v_A^2}{c_s^2} \frac{\delta B_\parallel}{B}.}$$

- Induction:

Equilibrium : no constraint due to $\boldsymbol{U}$ being parallel to $\boldsymbol{b}$.

$$\text{Fluctuations}: \quad \boxed{\nabla \cdot \delta \boldsymbol{B}_\perp = 0.}$$

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$$\text{Fluctuations}: \quad \boxed{\frac{\delta p}{p} = -\gamma \frac{v_A^2}{c_s^2} \frac{\delta B_\parallel}{B}.}$$

- Entropy/energy:

$$\text{Equilibrium}: \quad \frac{pU}{\gamma - 1} \frac{\partial s}{\partial \ell} = S_{\mathrm{th}} - \gamma c_v T S_\rho - \boldsymbol{\nabla} \cdot \boldsymbol{q}.$$

$$\text{Fluctuations}: \quad \boxed{\frac{\delta s}{c_v} = \frac{\delta p}{p} - \gamma \frac{\delta \rho}{\rho}.}$$

Second order: generalised RMHD

- The averaged equations at second order are homogenous in $\left\langle g^{(1)} \right\rangle_{\mathrm{turb}} = 0$.
- In addition to one for the entropy δs, one obtains fluctuating equations for the Elsässer fields

$$\boldsymbol{z}^{\pm} = \delta \boldsymbol{u}_{\perp} \mp \frac{\delta \boldsymbol{B}_{\perp}}{\sqrt{4\pi\rho}},$$

and the slow-mode variables

$$z_{\parallel}^{\pm} = \delta u_{\parallel} \mp \delta V_{\parallel}, \quad \delta V_{\parallel} = \frac{v_A^2}{v_S} \frac{\delta B_{\parallel}}{B}, \quad v_S = \frac{v_A}{\sqrt{1 + v_A^2/v_S^2}}.$$

- The total fluctuating energy

$$W_{\mathrm{tot}} = \frac{1}{4}\rho \left\langle |\boldsymbol{z}^{+}|^2 + |\boldsymbol{z}^{-}|^2 + |z_{\parallel}^{+}|^2 + |z_{\parallel}^{-}|^2 \right\rangle_{\perp} + \frac{1}{2}\frac{p}{\gamma(\gamma-1)} \left\langle \left(\frac{\delta s}{c_v} \right)^2 \right\rangle_{\perp},$$

obeys a conservation law similar to that seen in gyrokinetics: injected by equilibrium gradients, and dissipated by non-ideal effects (e.g., resistivity).

Third order: transport equations

- Transport-timescale induction equation:

$$\frac{\partial \boldsymbol{B}}{\partial \tau} = \boldsymbol{\nabla} \times \left[\left(\boldsymbol{U}_{\perp 2} + \boldsymbol{V}_{\psi} \right) \times \boldsymbol{B} \right] = \boldsymbol{\nabla} \times \left(\tilde{\boldsymbol{V}}_{\psi} \times \boldsymbol{B} \right),$$

  where

$$\boldsymbol{V}_{\psi} = \left\langle \delta \boldsymbol{u}_{\perp} \frac{\delta B_{\parallel}}{B} - \frac{\delta \boldsymbol{B}_{\perp}}{B} \delta u_{\parallel} \right\rangle_{\mathrm{turb}}.$$

- $\boldsymbol{U}_{\perp 2} = \left\langle \boldsymbol{u}^{(2)} \right\rangle_{\mathrm{turb}}$ is the second-order perpendicular mean flow that is set up by the fluctuations in order to maintain perpendicular force balance. It is required to ensure that the equilibrium remains stationary on $\sim \omega^{-1}$.

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- Magnetic energy evolution:

$$\frac{\partial}{\partial \tau} \left(\frac{B^2}{8\pi} \right) = -\frac{B^2}{4\pi} \tilde{\boldsymbol{V}}_\psi \cdot \left(\boldsymbol{\kappa} - \frac{\boldsymbol{\nabla}_\perp B}{B} \right) - \boldsymbol{\nabla} \cdot \left(\frac{B^2}{4\pi} \tilde{\boldsymbol{V}}_\psi \right).$$

  A force free field ($\boldsymbol{\kappa} = \boldsymbol{\nabla}_\perp B / B$) provides no source of free-energy $\Rightarrow$ equivalent to the statement that there is no drive purely from magnetic field gradients (Abel et al., 2013).

Third order: transport equations

- Transport equations

$$\boxed{\left.\frac{\mathrm{d}}{\mathrm{d}\tau}\right|_{\psi} G + \boldsymbol{\nabla}\cdot\boldsymbol{\Gamma}_G = S_G,}$$

  where $G = (\rho, \rho U, E_{\mathrm{th}})$, and $\mathrm{d}/\mathrm{d}\tau|_{\psi}$ is the time-derivative in the magnetic-field frame (following a control volume whose boundary moves with $\tilde{\boldsymbol{V}}_{\psi}$).
- Turbulent fluxes:

$$\begin{aligned}
\boldsymbol{\Gamma}_{\rho} &= \rho\left(\boldsymbol{V}_{\rho} - \boldsymbol{V}_{\psi}\right)\\
\boldsymbol{\Gamma}_{\rho U} &= \rho U\left(\boldsymbol{V}_{U} + \boldsymbol{V}_{\rho} - \boldsymbol{V}_{\psi}\right)\\
\boldsymbol{\Gamma}_{\mathrm{th}} &= E_{\mathrm{th}}\left(\boldsymbol{V}_{p} - \gamma\boldsymbol{V}_{\psi}\right)
\end{aligned}$$

- Turbulent velocities:

$$\boldsymbol{V}_{\rho} = \left\langle \frac{\delta\rho}{\rho}\delta\boldsymbol{u}_{\perp}\right\rangle_{\mathrm{turb}}, \quad \boldsymbol{V}_{\psi} = \left\langle \delta\boldsymbol{u}_{\perp}\frac{\delta B_{\parallel}}{B} - \frac{\delta\boldsymbol{B}_{\perp}}{B}\delta u_{\parallel}\right\rangle_{\mathrm{turb}},$$

$$\boldsymbol{V}_{p} = \left\langle \frac{\delta p}{p}\delta\boldsymbol{u}_{\perp}\right\rangle_{\mathrm{turb}}, \quad \boldsymbol{V}_{U} = \left\langle \frac{\delta u_{\parallel}\delta\boldsymbol{u}_{\perp}}{U} - \frac{\delta B_{\parallel}\delta\boldsymbol{B}_{\perp}}{4\pi\rho U}\right\rangle_{\mathrm{turb}}.$$

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Alfvénic dynamics

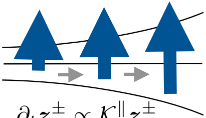
$$\boxed{\frac{\partial \boldsymbol{z}^{\pm}}{\partial t} + (U \pm v_A)\,(\boldsymbol{b} \cdot \boldsymbol{\nabla} \qquad)\,\boldsymbol{z}^{\pm} + \boldsymbol{z}^{\mp} \cdot \boldsymbol{\nabla} \boldsymbol{z}^{\pm} + \frac{\boldsymbol{\nabla}_{\perp} \tilde{p}}{\rho} = 0}$$

Alfvénic dynamics

$$\boxed{\frac{\partial \boldsymbol{z}^{\pm}}{\partial t} + (U \pm v_A)\left(\boldsymbol{b} \cdot \boldsymbol{\nabla} + \boldsymbol{b}\,\boldsymbol{\kappa}\ \cdot\right)\boldsymbol{z}^{\pm} + \boldsymbol{z}^{\mp} \cdot \boldsymbol{\nabla} \boldsymbol{z}^{\pm} + \frac{\boldsymbol{\nabla}_{\perp}\tilde{p}}{\rho} = 0}$$

Alfvénic dynamics

(i) WKB Growth



$$\partial_t \boldsymbol{z}^{\pm} \propto \mathcal{K}^{\parallel} \boldsymbol{z}^{\pm}$$
$$\partial_t z_{\parallel}^{\pm} \propto \mathcal{K}^{\parallel} z_{\parallel}^{\pm}$$

$$\frac{\partial \boldsymbol{z}^{\pm}}{\partial t} + (U \pm v_A)\left(\boldsymbol{b}\cdot\boldsymbol{\nabla} + \boldsymbol{b}\,\boldsymbol{\kappa}\ \cdot\right)\boldsymbol{z}^{\pm} + \boldsymbol{z}^{\mp}\cdot\boldsymbol{\nabla}\boldsymbol{z}^{\pm} + \frac{\boldsymbol{\nabla}_{\perp}\tilde{p}}{\rho}$$
$$= -\,(U \mp v_A)\left[\qquad\qquad - \frac{1}{4}\left(\boldsymbol{z}^{\pm}\qquad\qquad\right)\frac{\partial \log\rho}{\partial \ell}\right]$$

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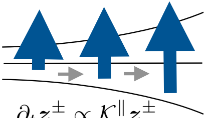

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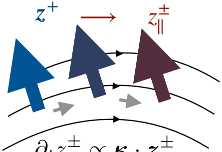
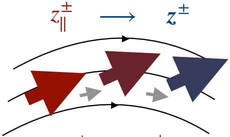
$$\boxed{\begin{aligned} &\frac{\partial \boldsymbol{z}^{\pm}}{\partial t} + (U \pm v_A)\left(\boldsymbol{b}\cdot\boldsymbol{\nabla} + \boldsymbol{b}\,\boldsymbol{\kappa}\ \cdot\ \right)\boldsymbol{z}^{\pm} + \boldsymbol{z}^{\mp}\cdot\boldsymbol{\nabla}\boldsymbol{z}^{\pm} + \frac{\boldsymbol{\nabla}_{\perp}\tilde{p}}{\rho} \\ &= -(U \mp v_A)\left[\qquad\qquad -\frac{1}{4}\left(\boldsymbol{z}^{\pm}\qquad\qquad\right)\frac{\partial \log \rho}{\partial \ell}\right] - 2\boldsymbol{\kappa}\left(U\delta u_{\parallel} - \frac{B\delta B_{\parallel}}{4\pi\rho}\right) \end{aligned}}$$

(iv) Curvature 1



$\partial_t z_{\parallel}^{\pm} \propto \boldsymbol{\kappa}\cdot\boldsymbol{z}^{\pm}$

(v) Curvature 2



$\partial_t \boldsymbol{z}^{\pm} \propto \boldsymbol{\kappa}\, z_{\parallel}^{\pm}$

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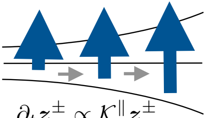

$$\partial_t \boldsymbol{z}^\pm \propto \mathcal{K}^\| \boldsymbol{z}^\pm$$
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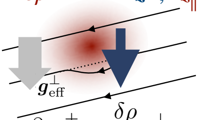
(vi) Gravity

$\delta\rho \longrightarrow \boldsymbol{z}^\pm,\ z_\|^\pm$



$$\partial_t \boldsymbol{z}^\pm \propto \frac{\delta\rho}{\rho}\boldsymbol{g}_{\rm eff}^\perp$$

$$\frac{\partial \boldsymbol{z}^\pm}{\partial t} + (U \pm v_A)\,(\boldsymbol{b}\cdot\boldsymbol{\nabla} + \boldsymbol{b}\,\boldsymbol{\kappa}\ \cdot)\,\boldsymbol{z}^\pm + \boldsymbol{z}^\mp\cdot\boldsymbol{\nabla}\boldsymbol{z}^\pm + \frac{\boldsymbol{\nabla}_\perp \tilde{p}}{\rho}$$
$$= -(U \mp v_A)\left[\qquad\qquad - \frac{1}{4}\left(\boldsymbol{z}^\pm \qquad\qquad\right)\frac{\partial \log\rho}{\partial \ell}\right] - 2\boldsymbol{\kappa}\left(U\delta u_\| - \frac{B\delta B_\|}{4\pi\rho}\right) + \frac{\delta\rho}{\rho}\boldsymbol{g}_{\rm eff}^\perp$$

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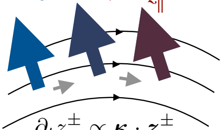
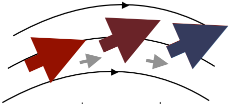
$\boldsymbol{z}^+ \longrightarrow z_\|^\pm$



$$\partial_t z_\|^\pm \propto \boldsymbol{\kappa}\cdot\boldsymbol{z}^\pm$$

(v) Curvature 2

$z_\|^\pm \longrightarrow \boldsymbol{z}^\pm$



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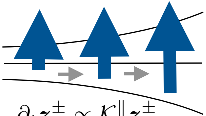

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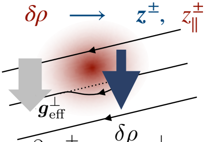

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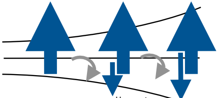
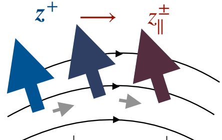
$$= -(U \mp v_A)\left[\boldsymbol{z}^{\mp}\cdot\boldsymbol{\nabla}\boldsymbol{b} - \frac{1}{4}\left(\boldsymbol{z}^{\pm} - \boldsymbol{z}^{\mp}\right)\frac{\partial \log\rho}{\partial \ell}\right] - 2\boldsymbol{\kappa}\left(U\delta u_{\parallel} - \frac{B\delta B_{\parallel}}{4\pi\rho}\right) + \frac{\delta\rho}{\rho}\boldsymbol{g}_{\text{eff}}^{\perp}$$

(ii) Reflection



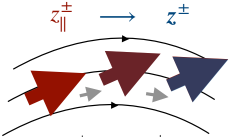
$$\partial_t \boldsymbol{z}^{\mp} \propto \mathcal{K}^{\parallel} \boldsymbol{z}^{\pm}$$
$$\partial_t z_{\parallel}^{\mp} \propto \mathcal{K}^{\parallel} z_{\parallel}^{\pm}$$

(iv) Curvature 1



$$\partial_t z_{\parallel}^{\pm} \propto \boldsymbol{\kappa}\cdot\boldsymbol{z}^{\pm}$$

(v) Curvature 2



$$\partial_t \boldsymbol{z}^{\pm} \propto \boldsymbol{\kappa}\, z_{\parallel}^{\pm}$$

Reflection due to field line ‘squashing’

$$\frac{\partial \boldsymbol{z}^{\pm}}{\partial t} = \cdots - (U \mp v_A) \left(\boldsymbol{\nabla b} + \frac{1}{4} \frac{\partial \log \rho}{\partial \ell} \mathsf{I} \right) \cdot \boldsymbol{z}^{\mp}.$$

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- Consider the perpendicular part:

$$(\boldsymbol{\nabla b})_{\perp} = \frac{1}{2} (\boldsymbol{\nabla} \cdot \boldsymbol{b}) \, \mathsf{I}_{\perp} + \mathsf{S} + \mathsf{A}.$$

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  – First term gives the familiar reflection term.

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  - First term gives the familiar reflection term.
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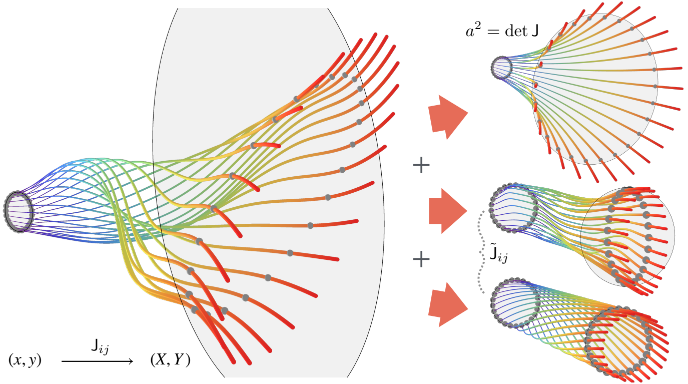
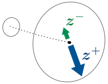
  - The symmetric, traceless tensor S represents reflection caused by area-preserving elliptical deformations of the flux surface:

$$\boldsymbol{z}^{\mp} \cdot \mathsf{S} = |\mathsf{S}|\left(z_l^{\mp}\boldsymbol{e}_l - z_s^{\mp}\boldsymbol{e}_s\right), \quad |\mathsf{S}| = \left(\frac{1}{2}\mathsf{S}_{ij}\mathsf{S}_{ij}\right)^{1/2}.$$

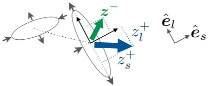
Reflection due to field line 'squashing'



Expansion reflection:
$\partial_t \boldsymbol{z}^- = \frac{1}{2}(U + v_A)\mathcal{K}^{\parallel}_{v_A} \boldsymbol{z}^+ + \ldots$



Q-reflection (squashing):
$\partial_t \boldsymbol{z}^- = -(U + v_A)\mathsf{S} \cdot \boldsymbol{z}^+ + \ldots$
$\propto -|\mathsf{S}| z_l^+ \hat{\boldsymbol{e}}_l + |\mathsf{S}| z_s^+ \hat{\boldsymbol{e}}_s$



Twist:
No contribution in RMHD

– The symmetric, traceless tensor S represents reflection caused by area-preserving elliptical deformations of the flux surface:

$$\boldsymbol{z}^{\mp} \cdot \mathsf{S} = |\mathsf{S}| \left(z_l^{\mp} \boldsymbol{e}_l - z_s^{\mp} \boldsymbol{e}_s \right), \quad |\mathsf{S}| = \left(\frac{1}{2} \mathsf{S}_{ij} \mathsf{S}_{ij} \right)^{1/2} .$$

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- Consider the perpendicular part:

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- Consider the perpendicular part:

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A geometry-dependent competition

- Assume $\boldsymbol{z}^+$ is energetically dominant, $z^+ \gg z^-$, where $z^\pm \sim \left\langle |\boldsymbol{z}^\pm|^2 \right\rangle_{\text{turb}}^{1/2}$.
- Rate of heating due to reflection:

$$Q^{\text{exp}} \sim \frac{W_\perp^+}{t_{\text{nl}}^+} \sim \left(k_\perp^{\text{o}} z^-\right) W_\perp^+.$$

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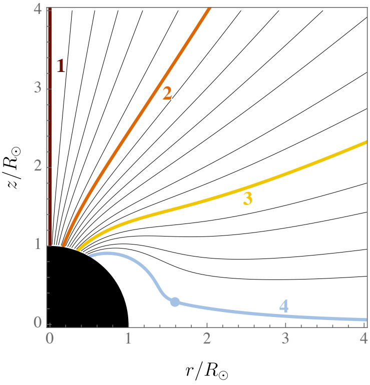

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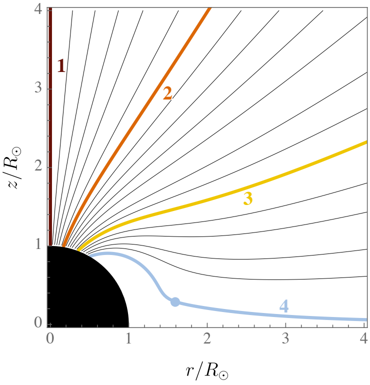

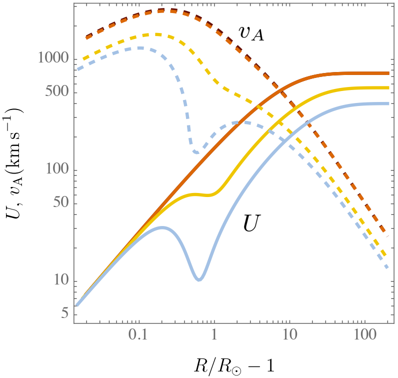

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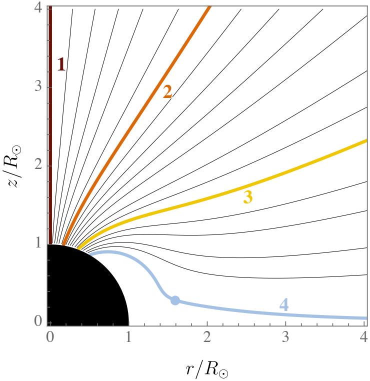

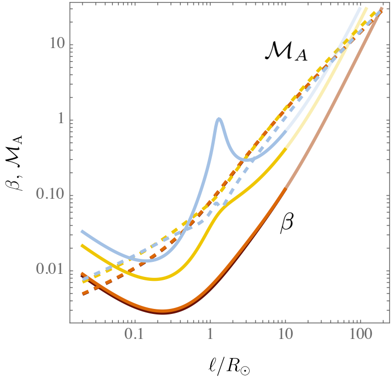

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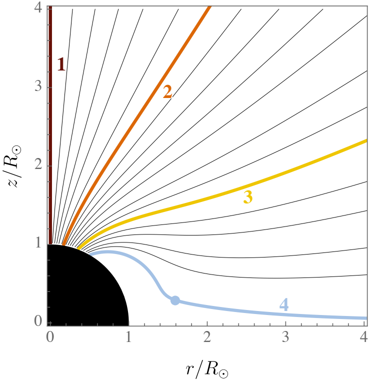

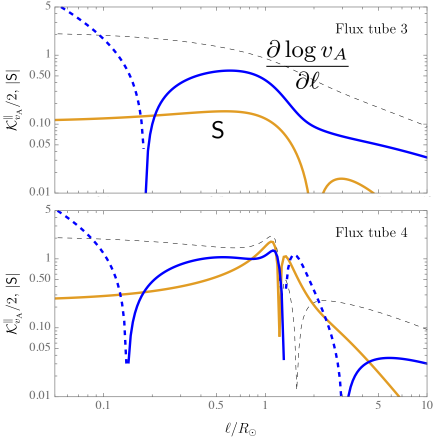

Table of Contents

Density transport

- The equilibrium density evolves as

$$\left.\frac{\partial \rho}{\partial \tau}\right|_{\psi} + \boldsymbol{\nabla} \cdot \boldsymbol{\Gamma}_{\rho} = S_{\rho}, \quad \boldsymbol{\Gamma}_{\rho} = \rho(\boldsymbol{V}_{\rho} - \boldsymbol{V}_{\psi}),$$

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Fuelling the slow wind

$$\boxed{\mathbf{\Gamma}_\rho = -\eta_{\mathrm{turb}} \left[\boldsymbol{\nabla}_\perp \rho - 2\rho \left(\frac{\boldsymbol{\nabla}_\perp B}{B} + \mathcal{M}_A \frac{\boldsymbol{\nabla}_\perp U}{U} \right) \right]}$$

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- Cumulative mass flux along tube of width $L_{\perp,\rho}(\ell)$:

$$\dot{M}_\perp(\ell) \sim \int_0^{\min(\ell,\ell_{\mathrm{tip}})} \mathrm{d}\ell' \left|\boldsymbol{\Gamma}_\rho(\ell')\right| L_{\perp,\rho}(\ell).$$

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- Compare to an equivalent mass flux due to transport along the tube:

$$\dot{M}_{\rm base} = \rho(\ell)U(\ell)L_{\perp,\rho}(\ell).$$

Fuelling the slow wind

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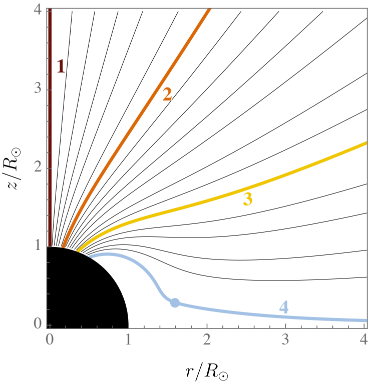
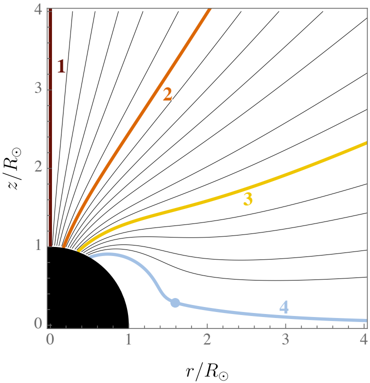


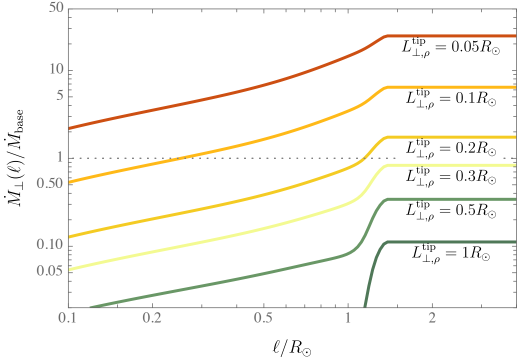
Fuelling the slow wind

$$\boxed{\mathbf{\Gamma}_\rho = -\eta_{\text{turb}} \left[\boldsymbol{\nabla}_\perp \rho - 2\rho \left(\frac{\boldsymbol{\nabla}_\perp B}{B} + \mathcal{M}_A \frac{\boldsymbol{\nabla}_\perp U}{U} \right) \right]}$$

- Cumulative mass flux along tube of width $L_{\perp,\rho}(\ell)$:

$$\dot{M}_\perp(\ell) \sim \int_0^{\min(\ell,\ell_{\text{tip}})} \mathrm{d}\ell' \left|\mathbf{\Gamma}_\rho(\ell')\right| L_{\perp,\rho}(\ell).$$

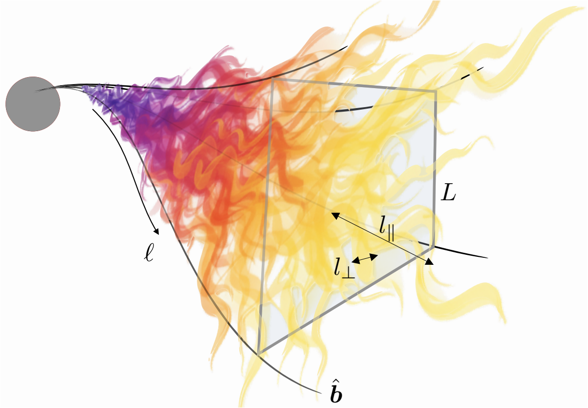




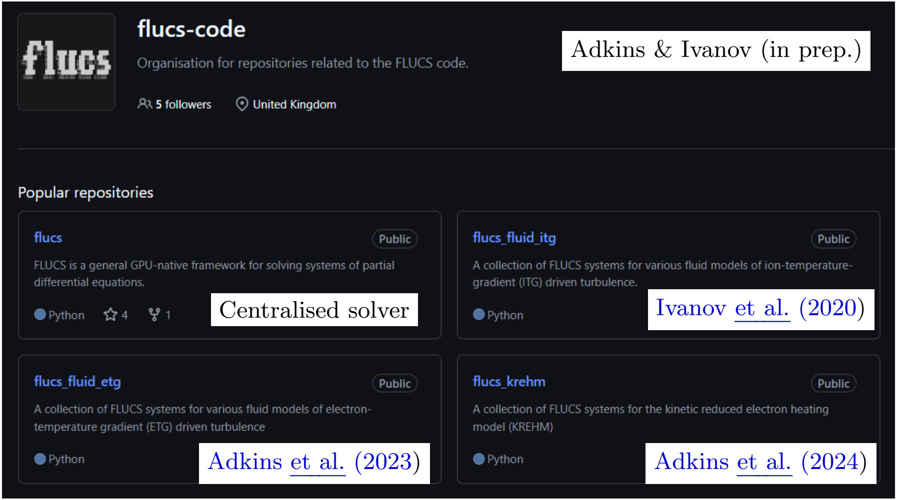
Summary

- We have derived a geometry-complete, multiscale transport theory for RMHD turbulence in the sub-Alfvénic solar corona, including:
  - 'Squashing' factors, transverse gradients, curvature, and gravity
  - Fast anisotropic fluctuations coupled to a slowly-evolving background
  - Coupling of Alfvénic and compressive sectors through gradients
  - and much more...

arXiv



A potentially useful tool



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