Neoclassical transport in the pedestal: theory and numerical comparison

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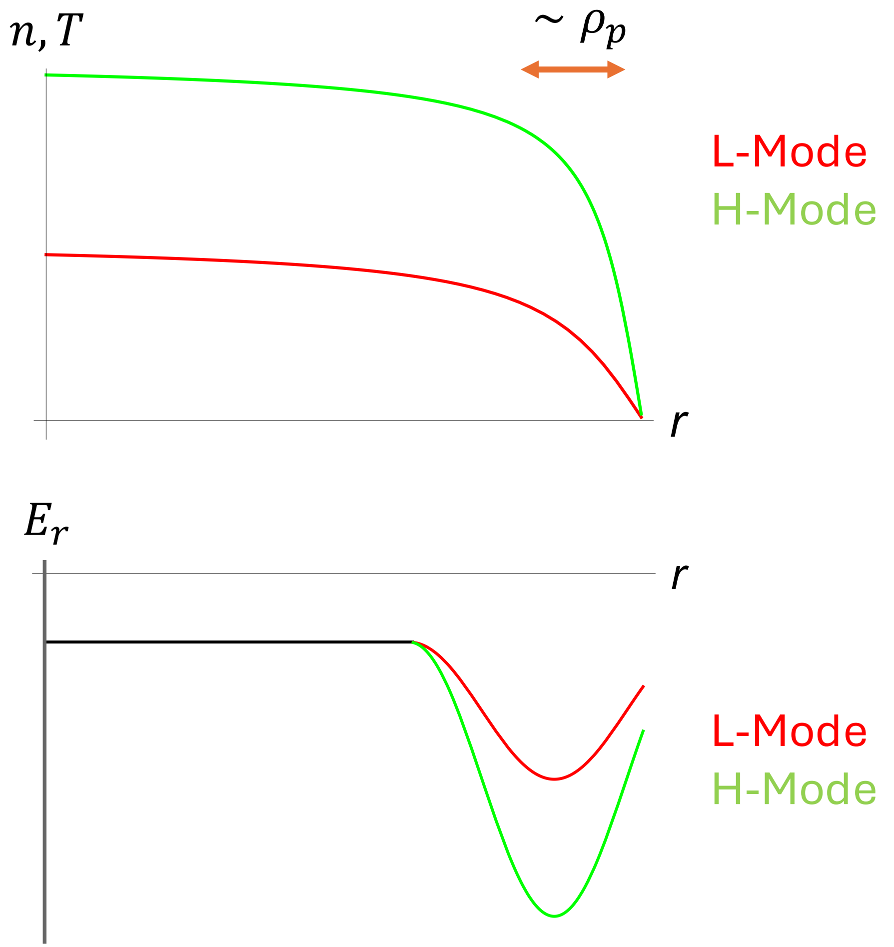
The Pedestal

- Improved confinement through shearing of turbulence
  – "transport barrier"
- strong decrease of density and temperature + radial electric field well in H-mode plasma
- Pedestal width is of the order of ρ_p[1,2]



[1] E. Viezzer et al 2018 *Nucl. Fusion* **58** 026031 [2] R. M. McDermott et al 2009 *PoP* **16**, 056103

Motivation

Neoclassical transport relevant in strong gradient regions due to reduced turbulence[1]

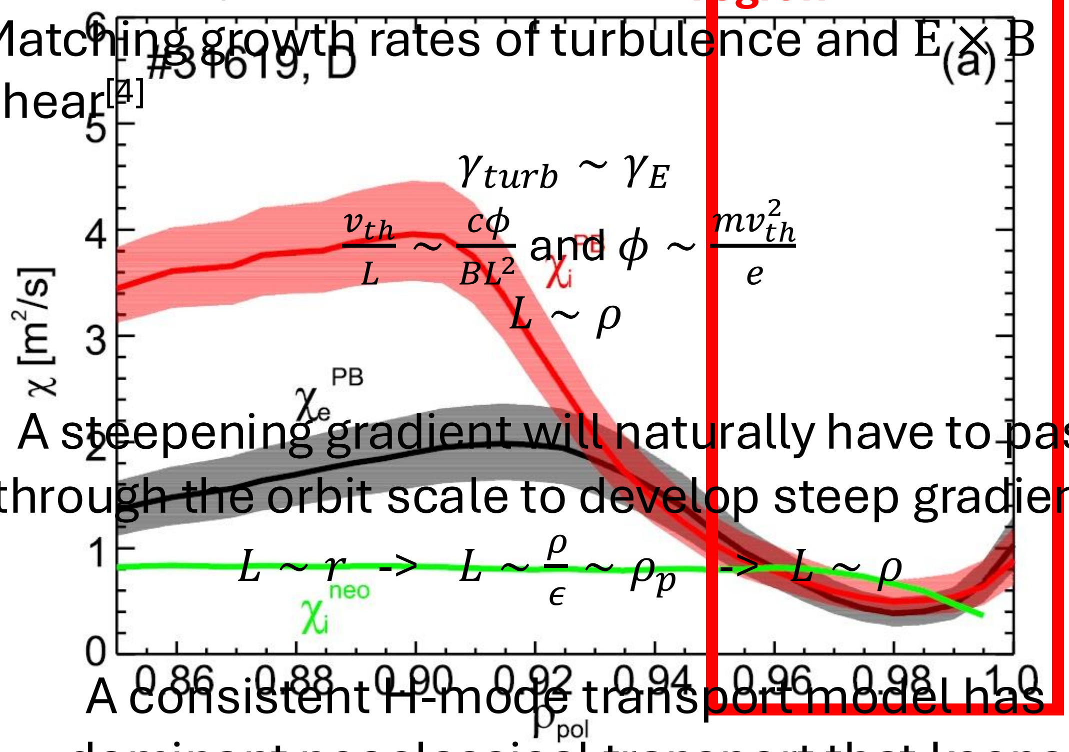
- Pedestal width is of the order of ρ_p[1,2]
- Problem: "standard" neoclassical theory requires weak gradients

$$\frac{\rho_p}{L} \ll 1$$

We need to extend neoclassical theory into regions of strong gradients: $L \sim \rho_p$

Experimentally:

Strong gradient region



Heat diffusivities[1]

Theoretically:

Matching growth rates of turbulence and E × B shear[4]

$$\gamma_{turb} \sim \gamma_E$$

$$\frac{v_{th}}{L} \sim \frac{c\phi}{BL^2} \text{ and } \phi \sim \frac{mv_{th}^2}{e}$$

$$L \sim \rho$$

A steepening gradient will naturally have to pass through the orbit scale to develop steep gradients:

$$L \sim r \text{ -> } L \sim \frac{\rho}{\epsilon} \sim \rho_p \text{ -> } L \sim \rho$$

A consistent H-mode transport model has dominant neoclassical transport that keeps turbulent perturbation stable

GOAL: Extend neoclassical theory into regions where turbulence is reduced (pedestal)

and study if the resulting profiles describes stable low transport states (H-mode)

[1] E. Viezzer *et al* 2018 *Nucl. Fusion* **58** 026031
[2] R. M. McDermott *et al* 2009 *PoP* **16**, 056103
[4] Waltz *et al* 1994 Phys. Plasmas **1**, 2229

Structure

1. Orderings and transport equations

2. Turbulence-free pedestal
   - Transport
   - XGC comparison

3. Low turbulence pedestal
   - Transport
   - Stability analysis

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Model set up and orderings

Large aspect ratio:

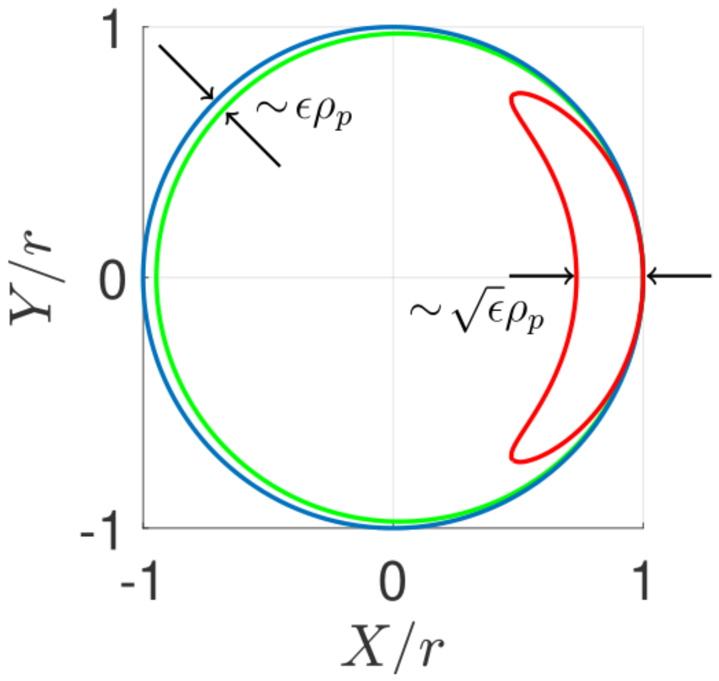
$$\epsilon \equiv \frac{r}{R} \ll 1$$

Scale separation:

$$L_{n,\Phi,T} \sim \rho_p \equiv \rho \frac{B}{B_p} \sim \rho \frac{q}{\epsilon}$$

$$\rho \ll \rho_p$$

$\Rightarrow$ Drift kinetics



Circular flux surfaces:

Slim orbit width:
Many orbits within one gradient length scale
$\Rightarrow f = f_M + g$

Poloidal variation:

$$\Phi - \phi(\psi) = \phi_\theta(\theta, \psi) \sim \epsilon \frac{T}{e}$$

Strong gradients:

$$\rho_* \equiv \frac{\rho}{L} \sim \frac{\rho}{\rho_p} \sim \epsilon$$

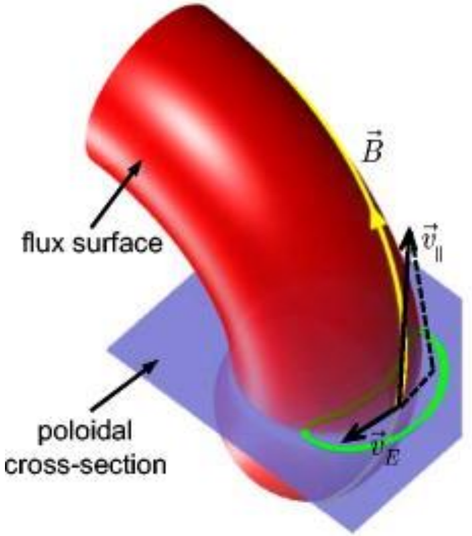
Weak gradients:

$$\rho_* \equiv \frac{\rho}{L} \sim \frac{\rho}{r} \ll \epsilon$$

Previous work assumed small temperature gradients[5-7], *small mean parallel flow gradients*[8-10] *and were inconsistent in the poloidal variation and the mean parallel flow*

[5] G. Kagan et al 2009 *PoP* **16**, 056105
K.Shaing et al 2012 *PoP* **19**
[6] P. Catto et al 2011 *Pl Phys Contl Fus* **53**
[7] P. Catto et al 2013 *Pl Phys Contl Fus* **55**
[8] K.Shaing et al 1992 *Phys Fluids* **4**
[9]
[10] J.

Shift of trapped particle region



G. Kagan et al 2009 *PoP* **16**, 056105

Trapped particles:

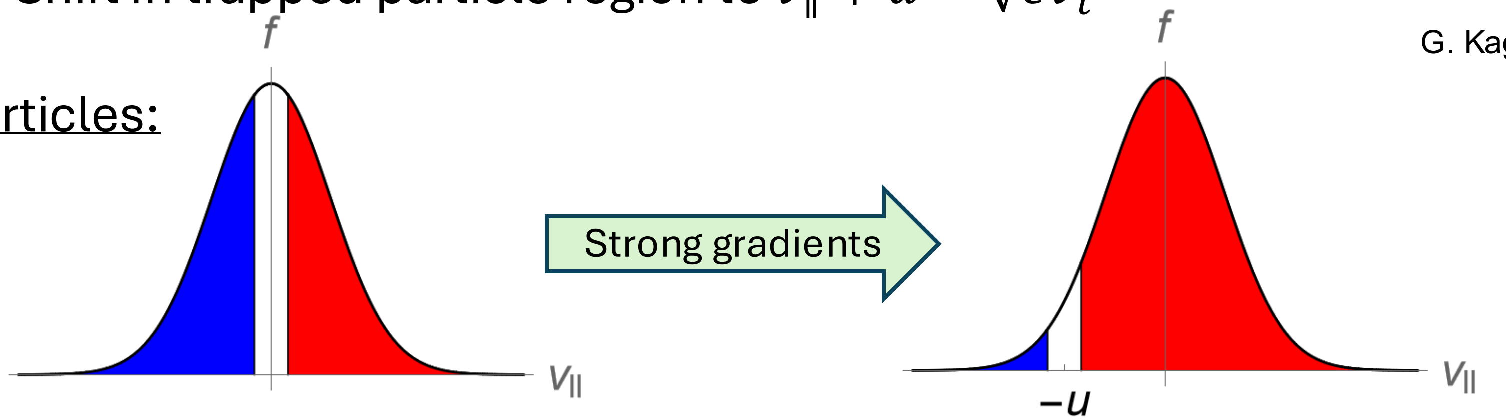
Poloidal velocity:

$$\dot{\theta} = \left(v_{\parallel}\hat{b} + v_{E\times B}\right)\cdot\nabla\theta = \left(v_{\parallel} + \frac{cI}{B}\frac{\partial\Phi}{\partial\psi}\right)\hat{b}\cdot\nabla\theta \equiv (v_{\parallel} + u)\hat{b}\cdot\nabla\theta$$

Poloidal components of parallel velocity and $E \times B$ – drift balance[4,7]

$\Rightarrow$ Shift in trapped particle region to $v_{\parallel} + u \sim \sqrt{\epsilon}v_t$

Passing particles:



Shift in trapped particle region causes asymmetry in passing particle number:

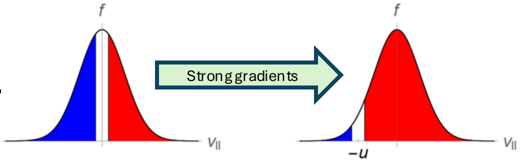
more red particles $(v_{\parallel} + u > 0)$ than blue particles $(v_{\parallel} + u < 0)$

[5] G. Kagan et al 2009 *PoP* **16**, 056105

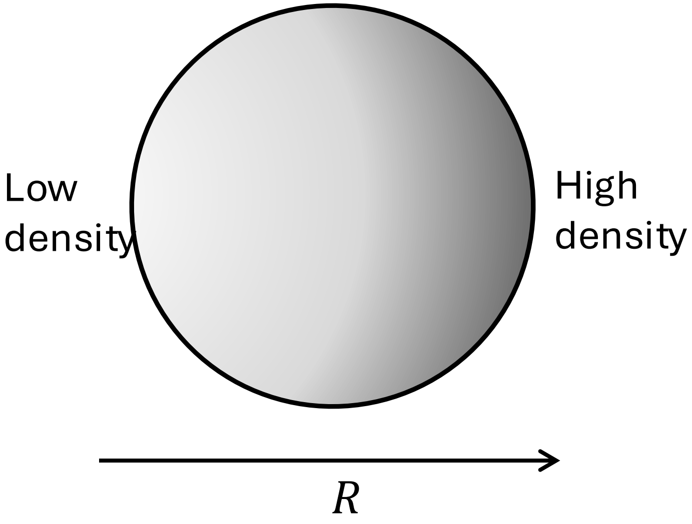
[8] K. C. Shaing et al 1992 *Physics of Fluids B: Plasma physics* **4**

Poloidal Variation

- Shift in Trapped Particle Region causes asymmetry in passing particle nu more red particles $(v_{\parallel} + u > 0)$ than blue particles $(v_{\parallel} + u < 0)$



- Centrifugal forces
- Mean parallel flow gradient
- Orbit width asymmetry



$\Rightarrow$ Poloidal Variation within a flux surface in density, potential, flow, and temperature [1*,2*]

$\Rightarrow$ Particles can be trapped on the inboard side

[1*] S. Trinczek et al 2023 **89** *JPP*
[2*] S. Trinczek et al 2025 **91** *JPP*

Transport equations

Ion neoclassical particle and energy fluxes in the banana regime:

$$\Gamma_{\mathrm{i}} = -1.1\sqrt{\frac{r}{R}}\frac{\nu I^2 p_i}{|S|^{3/2} m_i \Omega_{\mathrm{i}}^2}\left\{\left[\frac{\partial}{\partial\psi}\ln p_i - \frac{m_i(u+V_\parallel)}{T_i}\left(\frac{\partial V_\parallel}{\partial\psi} - \frac{\Omega}{I}\right)\right] G_1(u, V_\parallel, \phi_c) - 1.17\frac{\partial}{\partial\psi}\ln T_i \, G_2(u, V_\parallel, \phi_c)\right\}$$

$$Q_i = \frac{m_i u^2}{2}\Gamma_{\mathrm{i}} - 1.46\sqrt{\frac{r}{R}}\frac{\nu I^2 p_i T_i}{|S|^{3/2} m_i \Omega_i^2}\left\{\left[\frac{\partial}{\partial\psi}\ln p_i - \frac{m_i(u+V_\parallel)}{T_i}\left(\frac{\partial V_\parallel}{\partial\psi} - \frac{\Omega}{I}\right)\right] H_1(u, V_\parallel, \phi_c) - 0.25\frac{\partial}{\partial\psi}\ln T_i \, H_2(u, V_\parallel, \phi_c)\right\}$$

- Modification of transport coefficient by poloidal dependence of the potential
- Transport driven by gradient of mean parallel flow
- Orbit squeezing[5]
- Explicit dependence on mean parallel flow

Orbit squeezing:

$$S = 1 + \frac{cI^2}{\Omega B}\frac{\partial^2\Phi}{\partial\psi^2}$$

Trapped particle velocity:

$$u = \frac{cI}{B}\frac{\partial\Phi}{\partial\psi}$$

[8] K. C. Shaing et al 1992 *Physics of Fluids B: Plasma physics* **4**

Transport equations

$$\underbrace{\Gamma_i}_{\to 0} = -1.1\sqrt{\frac{r}{R}}\frac{\nu I^2 p}{\underbrace{|S|^{3/2}}_{\to 1} m\Omega^2}\left\{\left[\frac{\partial}{\partial\psi}\ln p - \frac{m(u+V_\parallel)}{T}\left(\underbrace{\frac{\partial V_\parallel}{\partial\psi}}_{\to 0} - \frac{\Omega}{I}\right)\right]\underbrace{G_1(u, V_\parallel, \phi_c)}_{\to 1} - 1.17\frac{\partial}{\partial\psi}\ln T\,\underbrace{G_2(u, V_\parallel, \phi_c)}_{\to 1}\right\}$$

$$Q = \underbrace{\frac{mu^2}{2}\Gamma_i}_{\to 0} - 1.46\sqrt{\frac{r}{R}}\frac{\nu I^2 pT}{\underbrace{|S|^{3/2}}_{\to 1} m\Omega^2}\left\{\left[\frac{\partial}{\partial\psi}\ln p - \frac{m(u+V_\parallel)}{T}\left(\underbrace{\frac{\partial V_\parallel}{\partial\psi}}_{\to 0} - \frac{\Omega}{I}\right)\right]\underbrace{H_1(u, V_\parallel, \phi_c)}_{\to 1} - 0.25\frac{\partial}{\partial\psi}\ln T\,\underbrace{H_2(u, V_\parallel, \phi_c)}_{\to 1}\right\}$$

Electron particle transport: $\Gamma_e = (\dots)$
Electron energy transport: $Q_e = (\dots)$
Ion momentum transport: $\gamma = (\dots)$

Poloidal variation from QN: $\phi_c = (\dots)$

Bootstrap current: $j^B = (\dots)$

For Banana [1*,2*] and for Plateau regime

Weak gradient limit

Orbit squeezing:

$$S = 1 + \frac{cI^2}{\Omega B}\frac{\partial^2\Phi}{\partial\psi^2}$$

Trapped particle velocity:

$$u = \frac{cI}{B}\frac{\partial\Phi}{\partial\psi}$$

[1*] S. Trinczek et al 2023 **89** *JPP*

[2*] S. Trinczek et al 2025 **91** *JPP*

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The turbulence free pedestal: Neoclassical Ambipolarity

In practice: Take input profiles of density, temperature and mean flow and calculate transport quantities

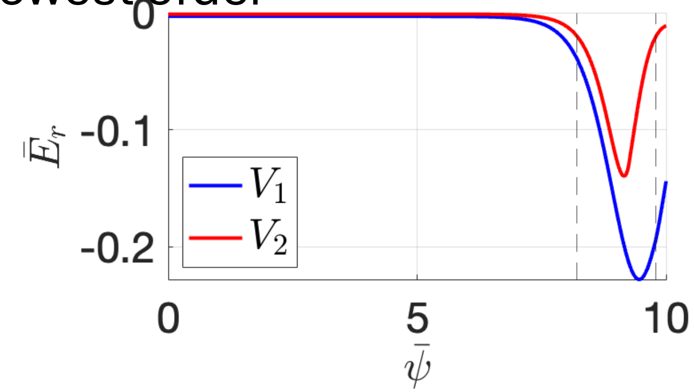
No turbulence:

- Consistent with ambipolarity: $\Gamma_i = \Gamma_e$

$$\frac{\Gamma_e^{neo}}{\Gamma_i^{neo}} \sim \sqrt{\frac{m_e}{m_i}}$$

- We must impose $\Gamma_i \simeq 0$ to lowest order

Banana NA



Give $n_i, T_i, T_e, V_\parallel$ profiles

No turbulence

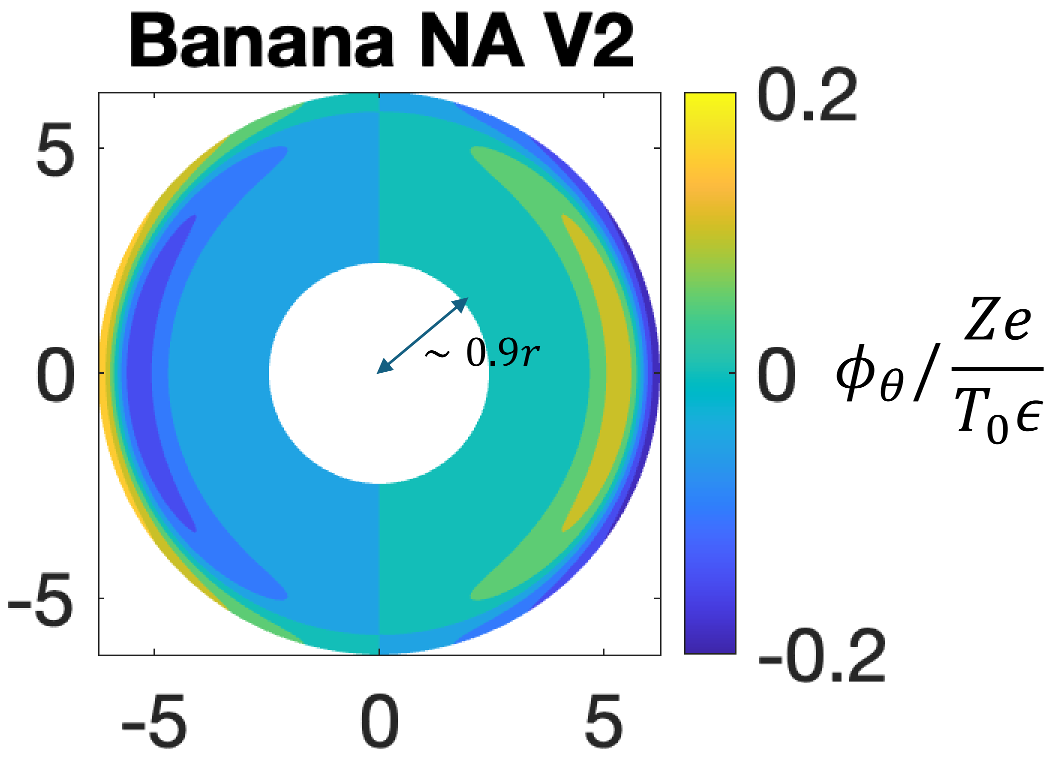
nonlinear

Solve $\Gamma_i = (\dots) = 0$ for E_r

Moment Equations

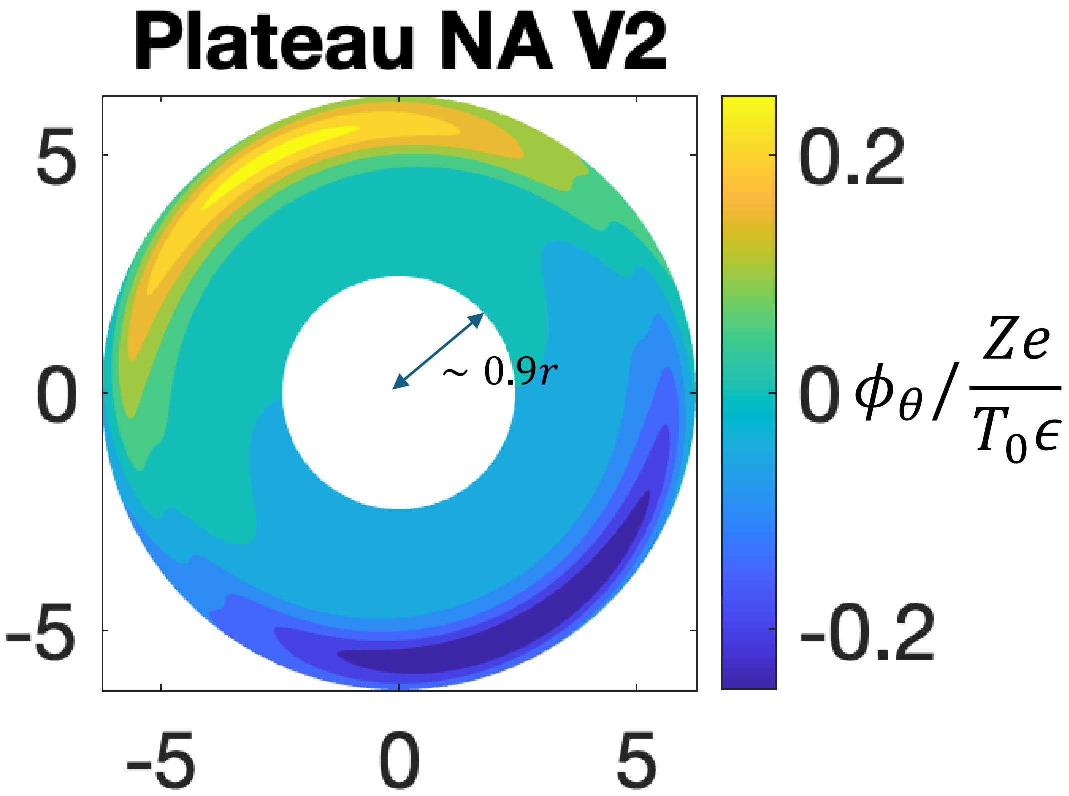
Get $Q_i, \Gamma_e, Q_e, \gamma, \phi_\theta, j^B$ profiles

Results: Poloidal variation



In-out asymmetry

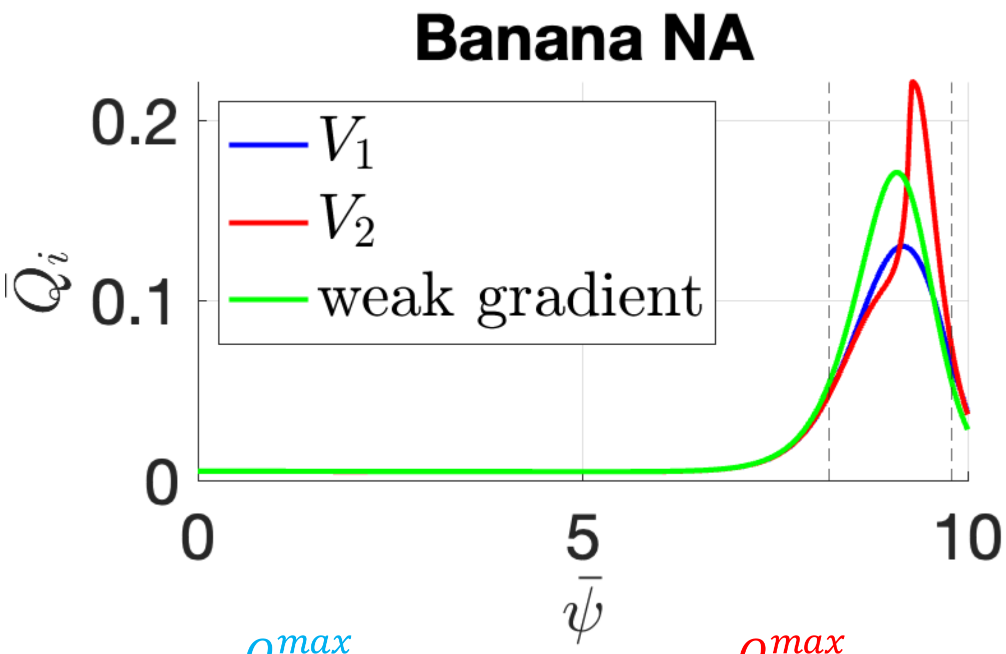
Not true to scale in radius

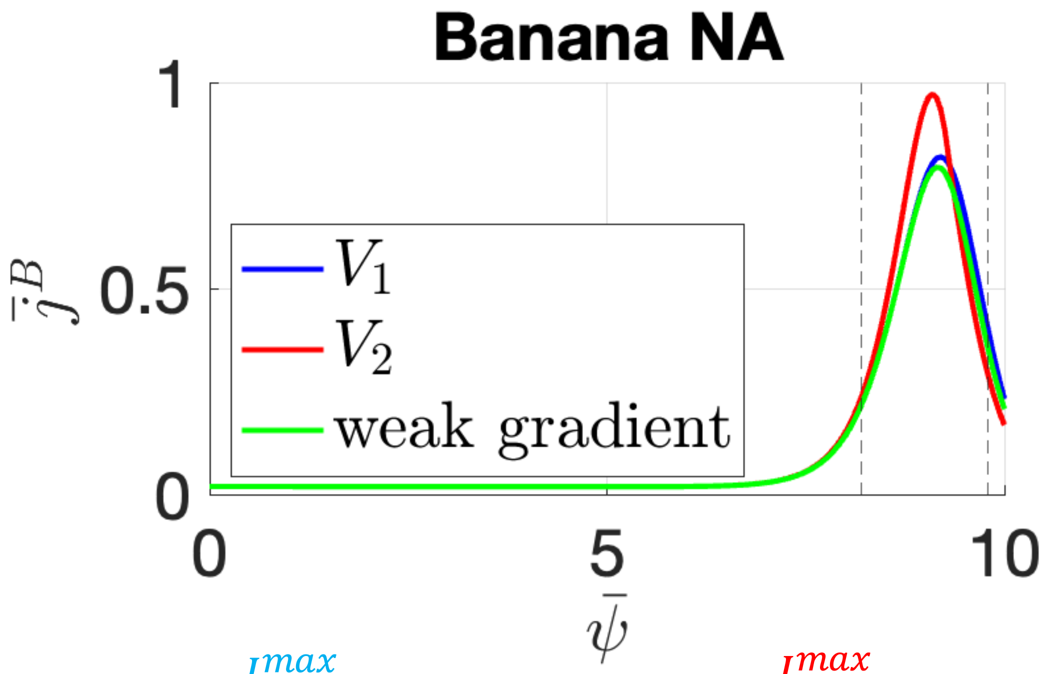


In-out and up-down asymmetry

Not true to scale in radius

Results: Banana regime



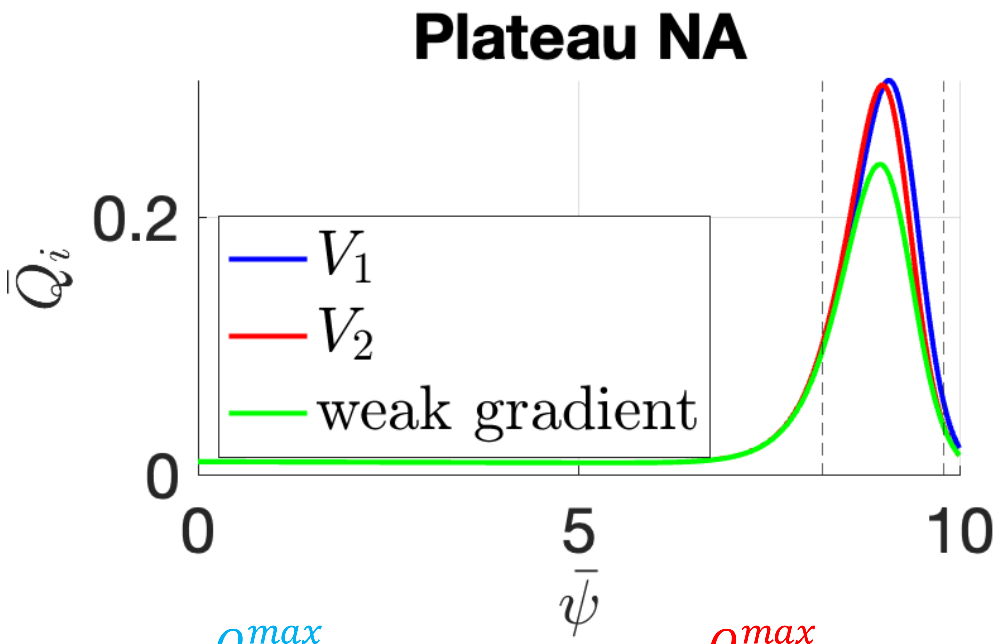


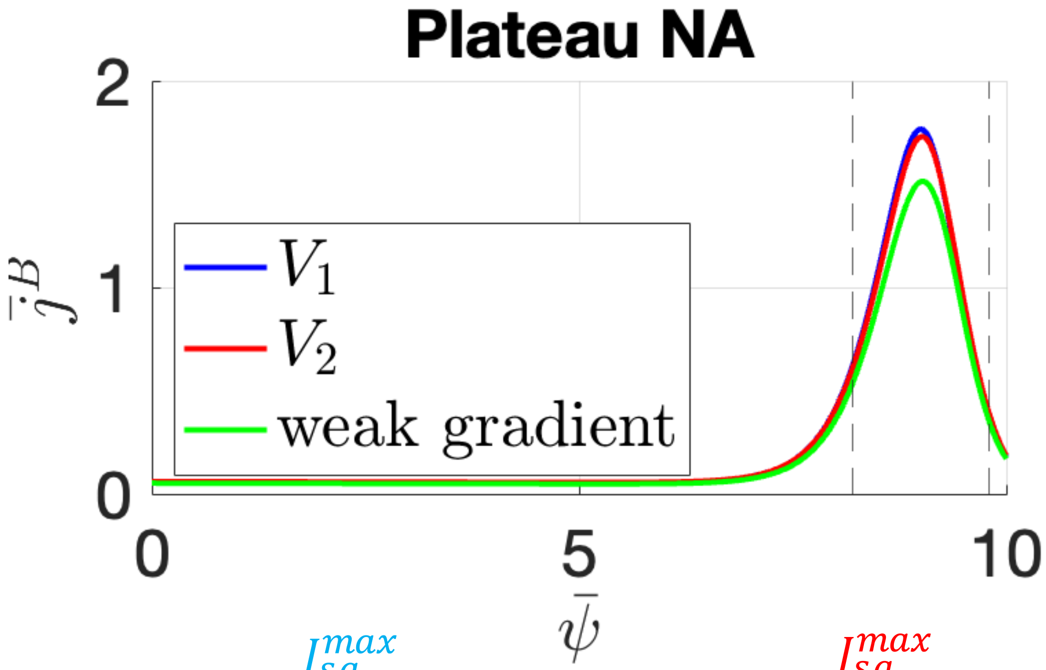
$\frac{Q_{sg}^{max}}{Q_{wg}^{max}} \simeq 0.76$ $\frac{Q_{sg}^{max}}{Q_{wg}^{max}} \simeq 1.29$ $\frac{J_{sg}^{max}}{J_{wg}^{max}} \simeq 1.03$ $\frac{J_{sg}^{max}}{J_{wg}^{max}} \simeq 1.22$

Strong gradient neoclassical theory predicts a larger or smaller energy flux, depending on the flow

Strong gradient neoclassical theory predicts larger or similar bootstrap current, depending on the flow

Results: Plateau regime





$\frac{Q_{sg}^{max}}{Q_{wg}^{max}} \simeq 1.27$ $\frac{Q_{sg}^{max}}{Q_{wg}^{max}} \simeq 1.26$ $\frac{J_{sg}^{max}}{J_{wg}^{max}} \simeq 1.17$ $\frac{J_{sg}^{max}}{J_{wg}^{max}} \simeq 1.14$

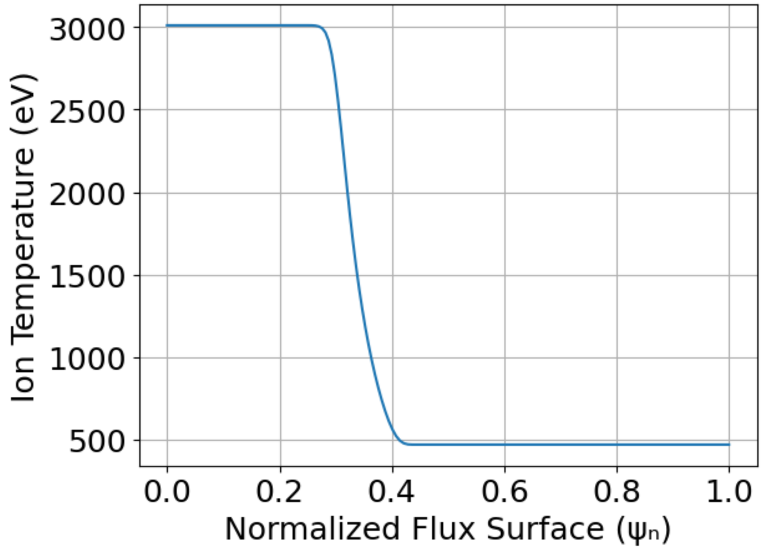
Strong gradient neoclassical theory predicts larger energy flux and bootstrap current

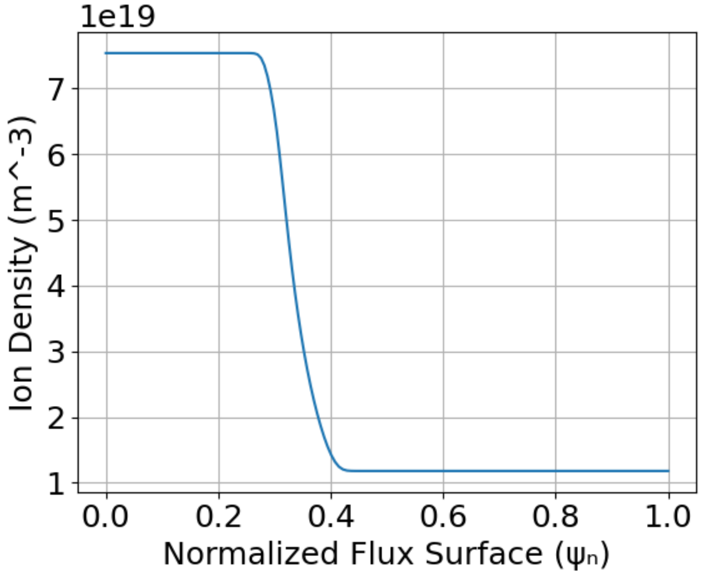
Choice of mean parallel flow is less important

Work in progress: XGC comparison

- XGC is a gyrokinetic particle-in-cell code with a nonlinear Fokker-Planck collision operator
- XGCa is the axisymmetric version of XGC that has been successfully benchmarked to weak gradient neoclassical theory[11]
- Objective: Compare fluxes, poloidal variation and bootstrap current modifications

Simulation setup: Strong density and temperature gradient profiles with heat sources to maintain temperature gradient



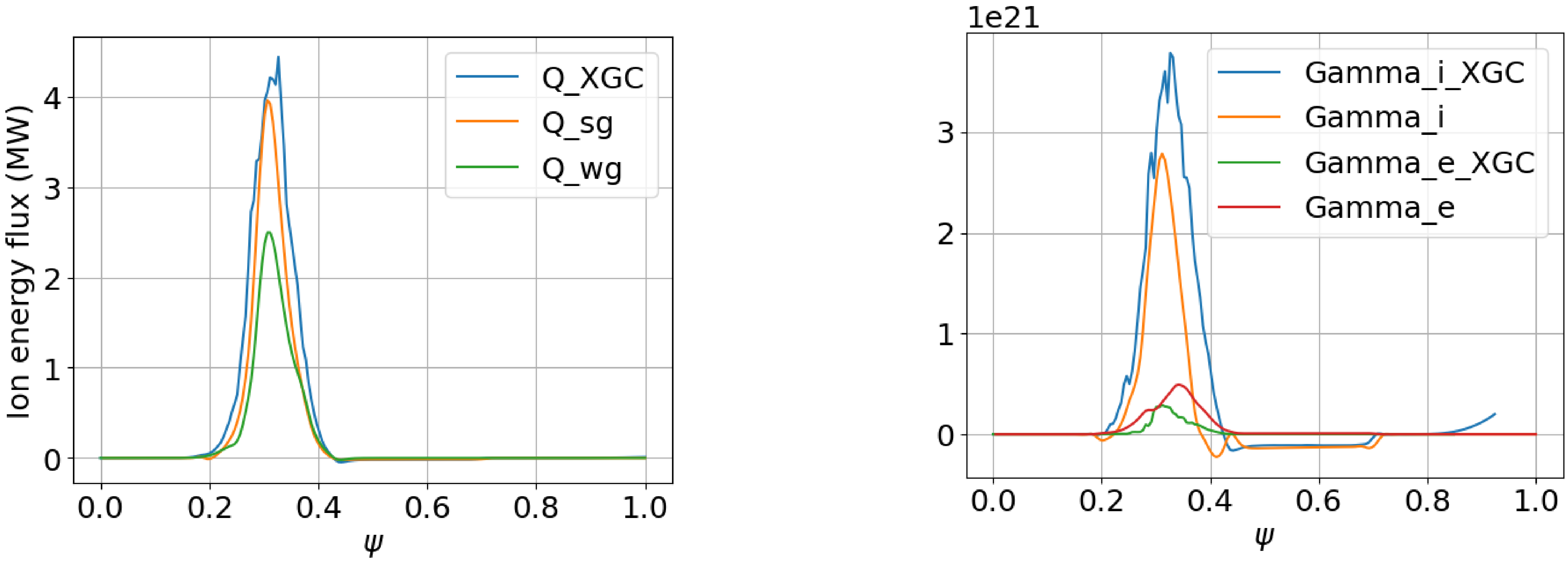


Let profiles evolve for about $3\tau_i$

[11] R. Hager et al 2019 *PoP* **26**

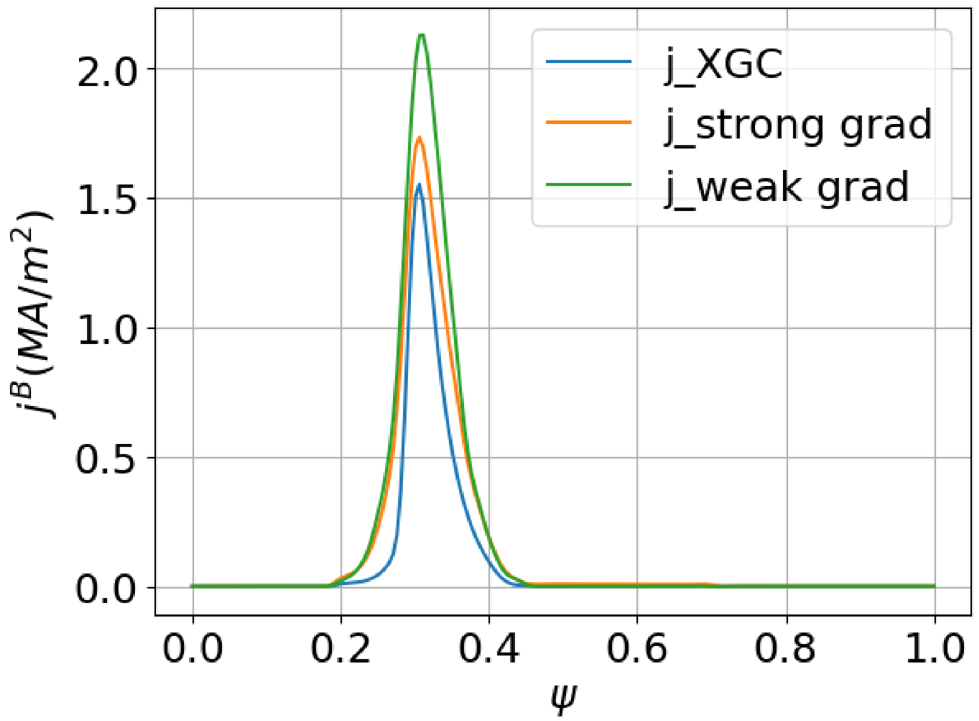
Work in progress: XGC comparison

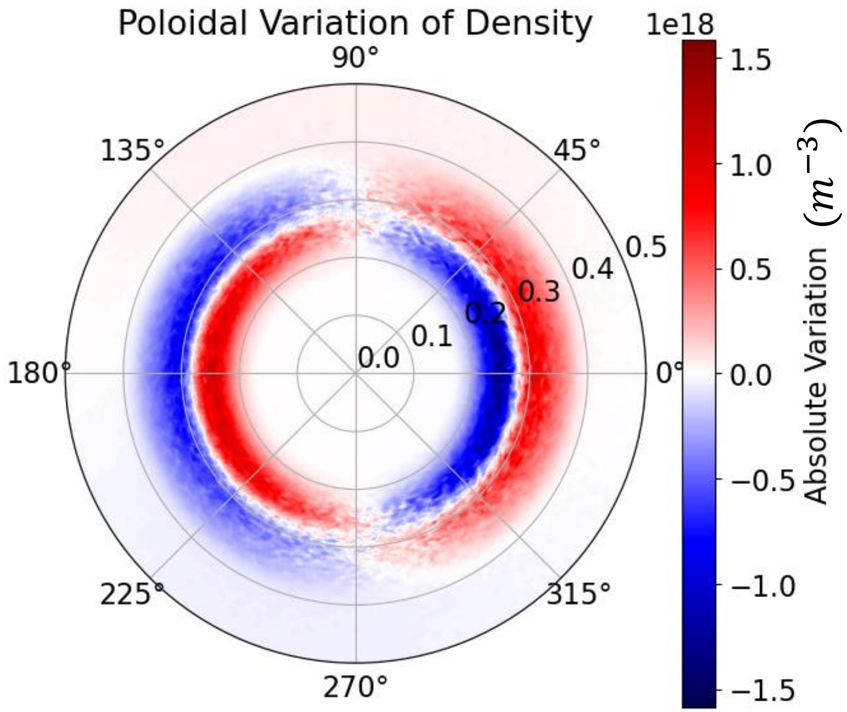
Preliminary results: ion energy flux and ion particle flux



Work in progress: XGC comparison

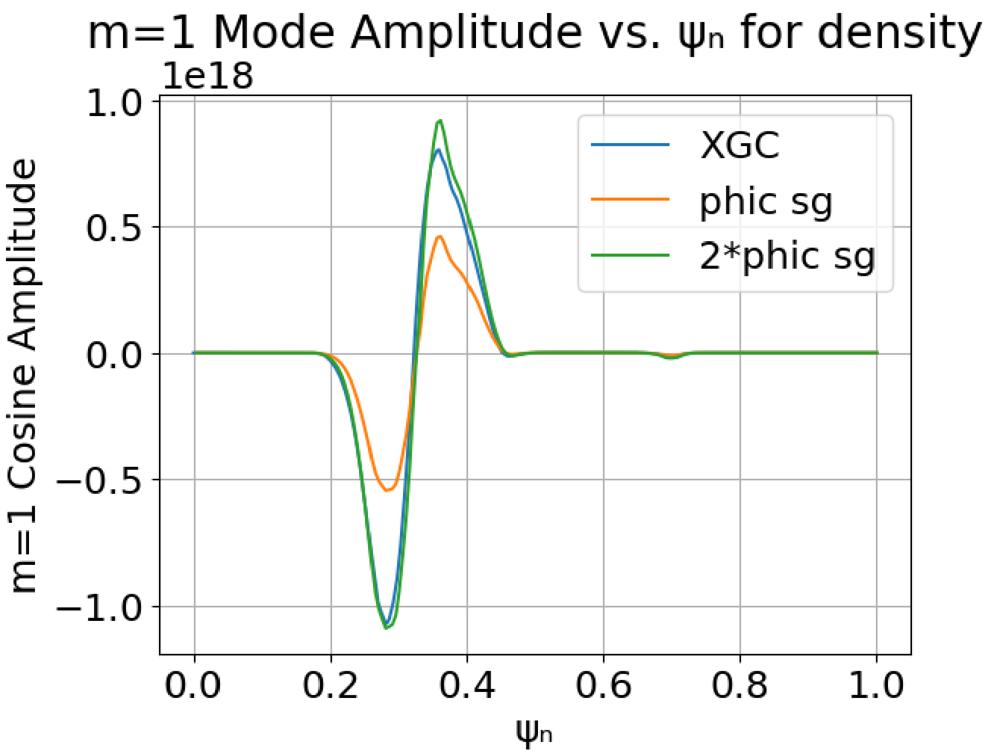
Preliminary results: Bootstrap current and poloidal variation

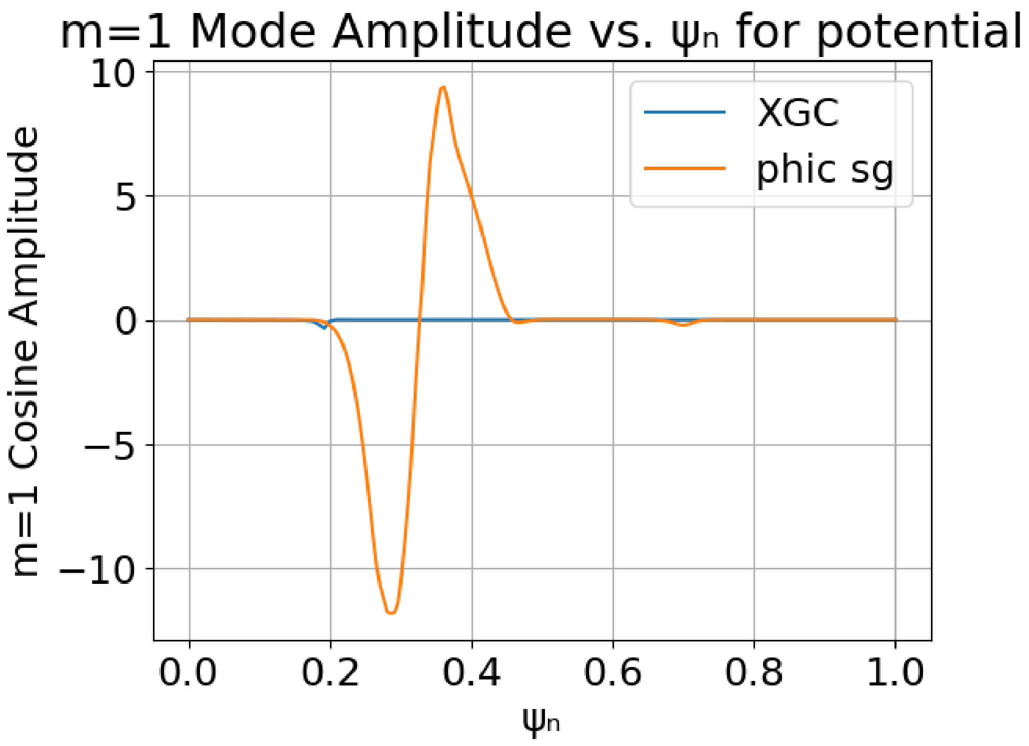




Work in progress: XGC comparison

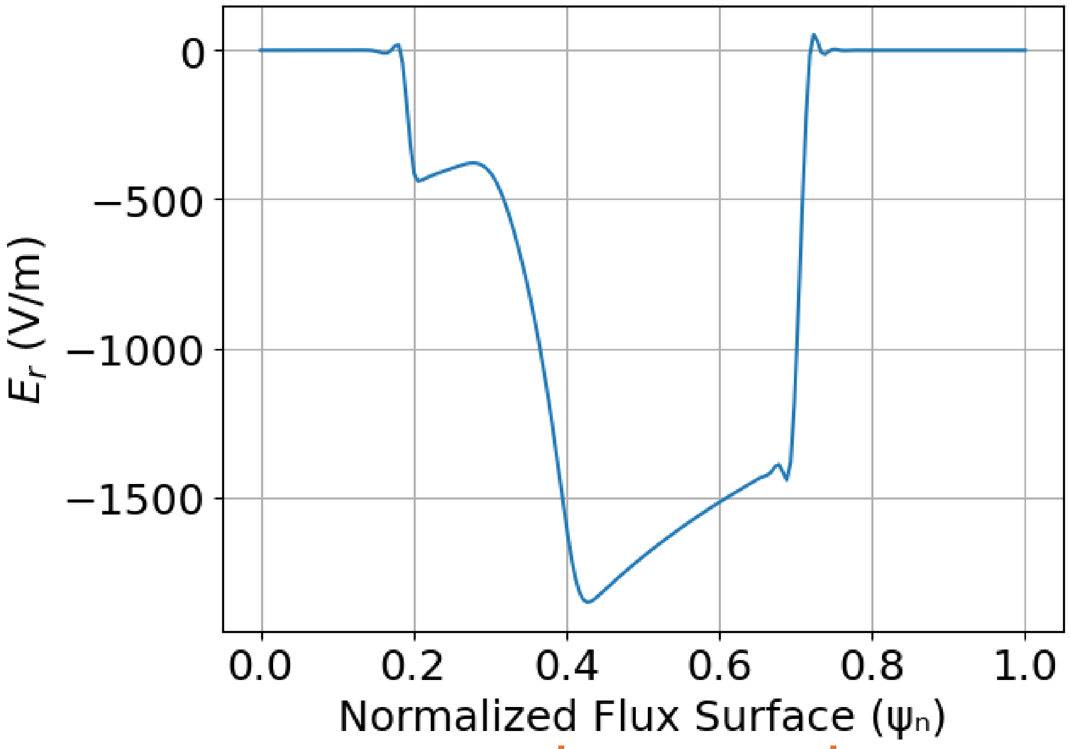
Problems: Poloidal variation prediction does not agree



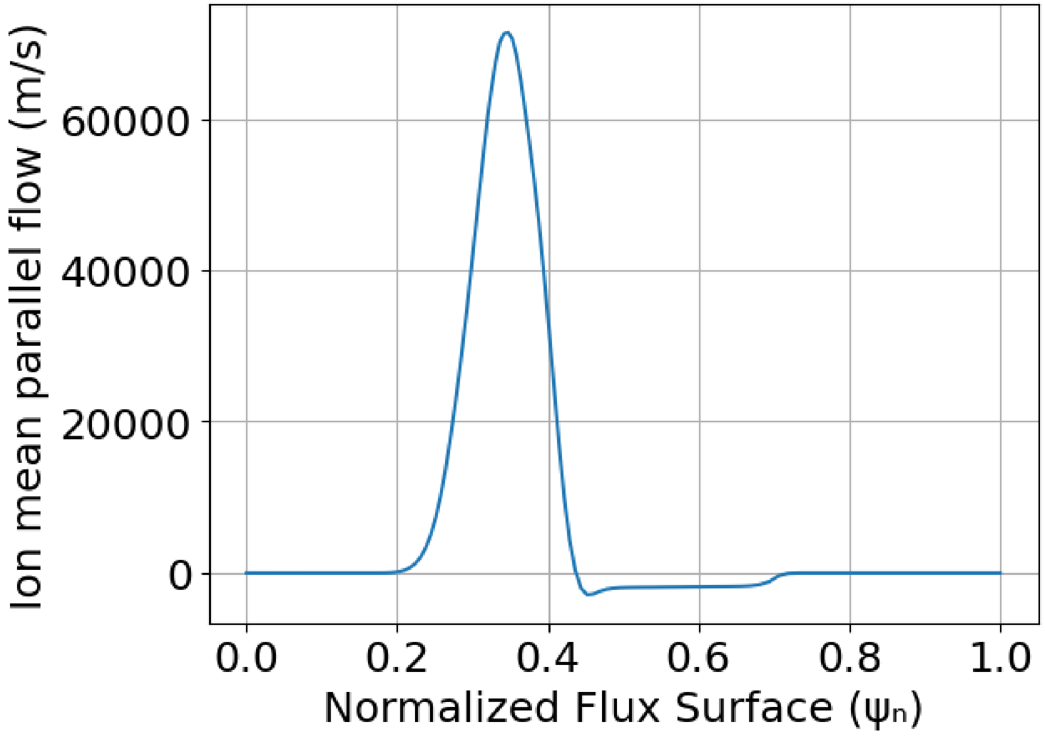


Work in progress: XGC comparison

Problems: Radial electric field and mean parallel flow show “artificial torque”



flat profiles



flat profiles

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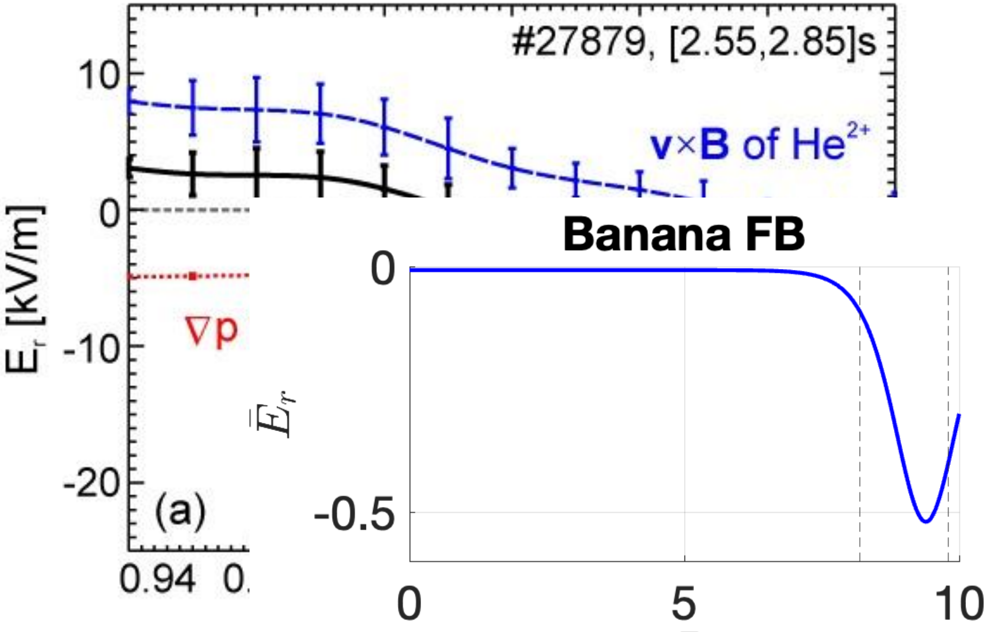
Low turbulence pedestal: Radial force balance

- Assumption: the radial electric field balances the pressure gradient[12]



[12] E. Viezzer et al 2013 *Nucl Fusion* **53**

$$en \frac{\partial \Phi}{\partial \psi} = \frac{\partial p}{\partial \psi}$$

Give $n_i, T_i, T_e, V_\parallel$ profiles

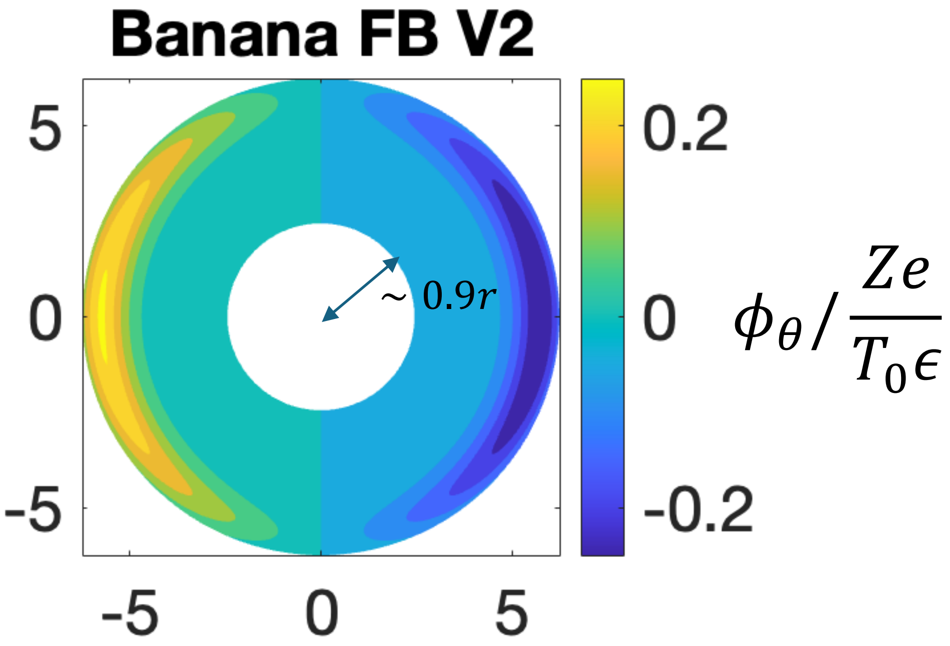
Radial force balance

$Zen \frac{\partial \Phi}{\partial \psi} = \frac{\partial p}{\partial \psi}$ gives E_r

Moment Equations

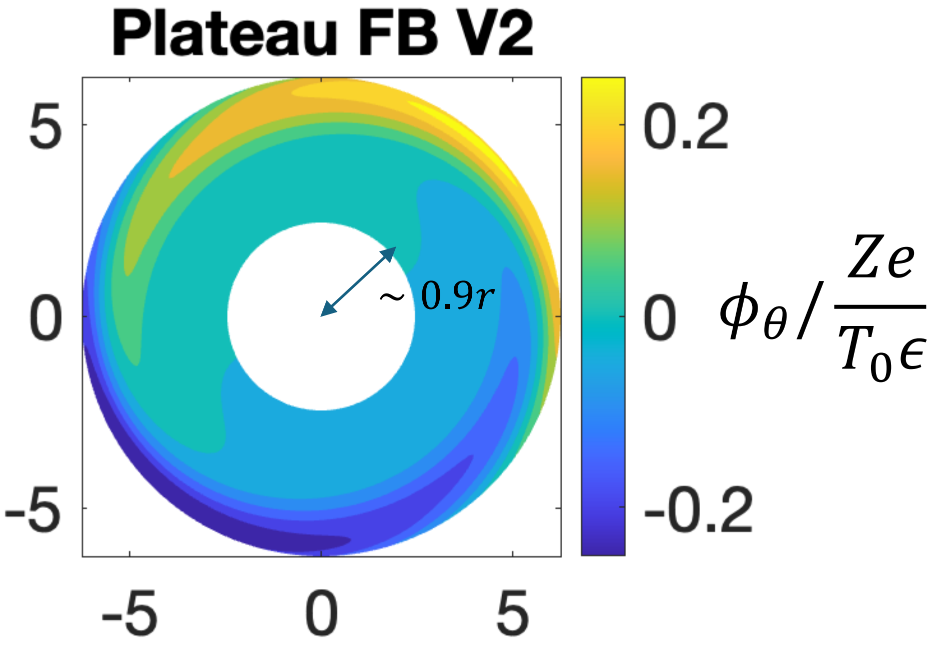
Get $\Gamma_i, Q_i, \Gamma_e, Q_e, \gamma,$ ϕ_θ, j^B profiles

Results: Poloidal variation



In-out asymmetry

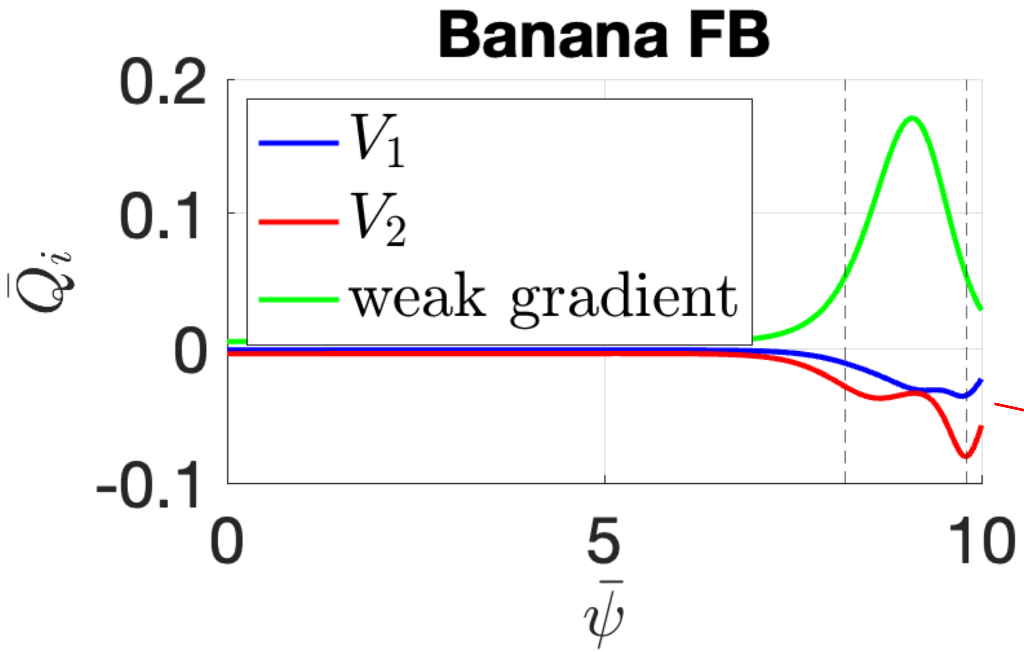
Not true to scale in radius



In-out and up-down asymmetry

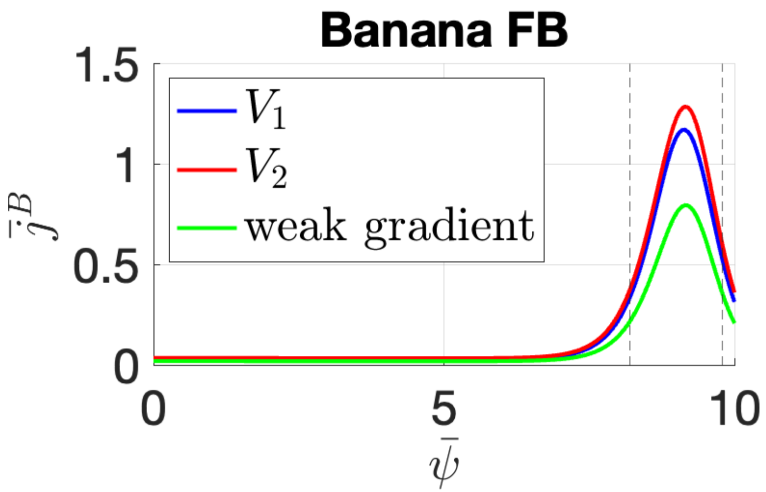
Not true to scale in radius

Results: Banana regime



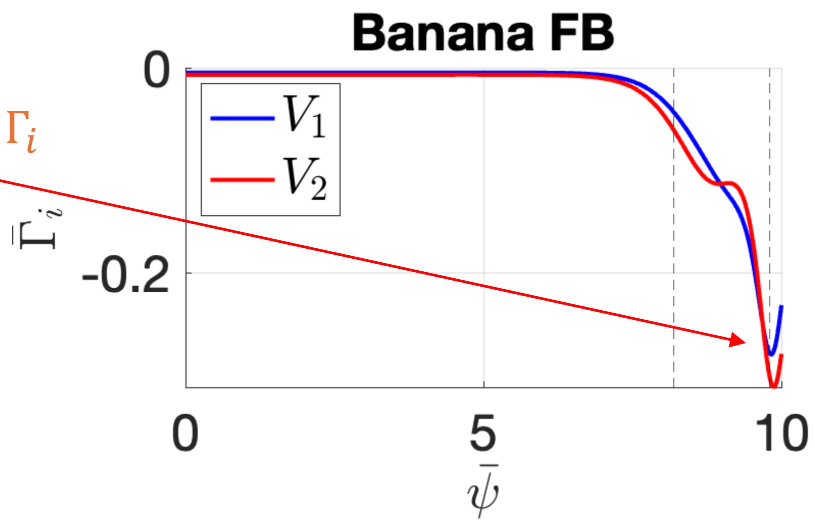
$\sim u^2\Gamma_i$

$$\frac{|Q_{sg}^{max}|}{Q_{wg}^{max}} \simeq 0.21 \qquad \frac{|Q_{sg}^{max}|}{Q_{wg}^{max}} \simeq 0.48$$

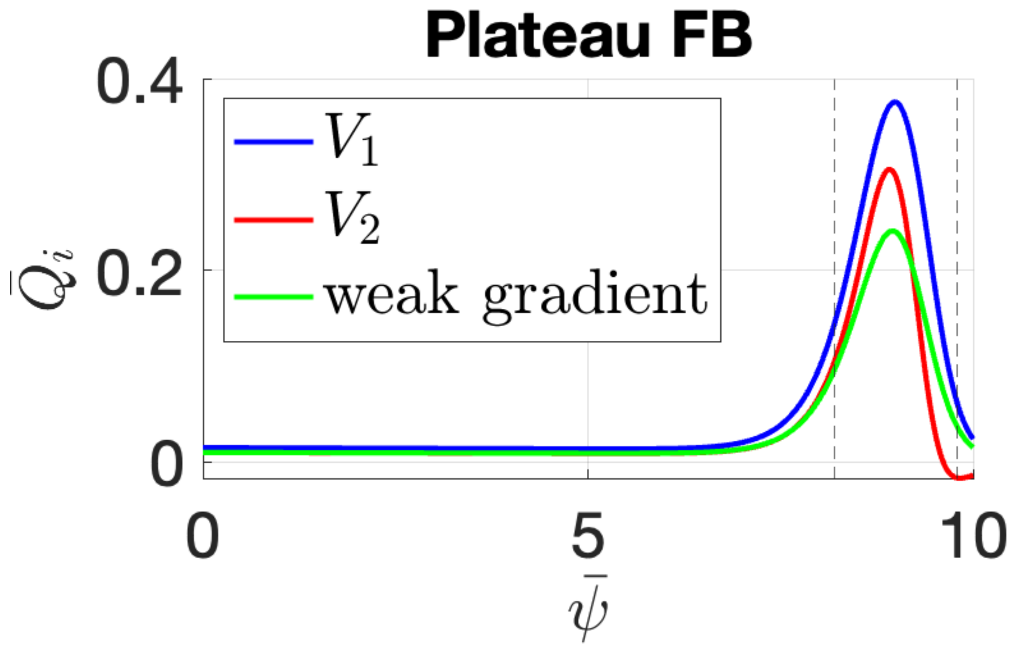


$$\frac{J_{sg}^{max}}{J_{wg}^{max}} \simeq 1.47$$

$$\frac{J_{sg}^{max}}{J_{wg}^{max}} \simeq 1.62$$

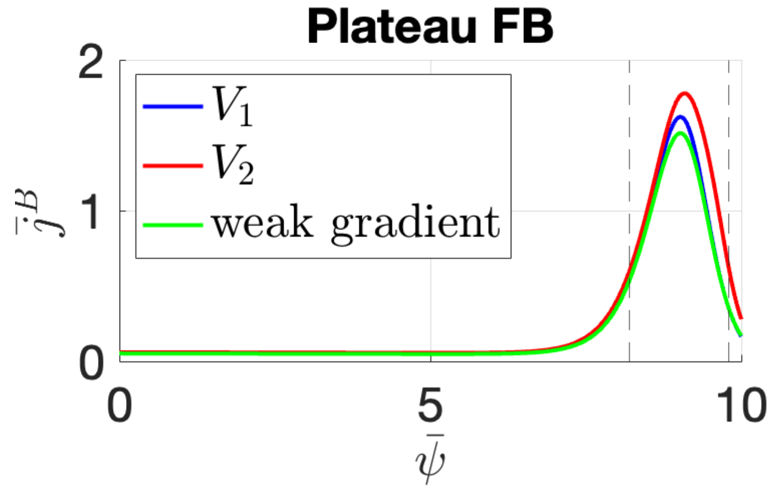


Results: Plateau regime



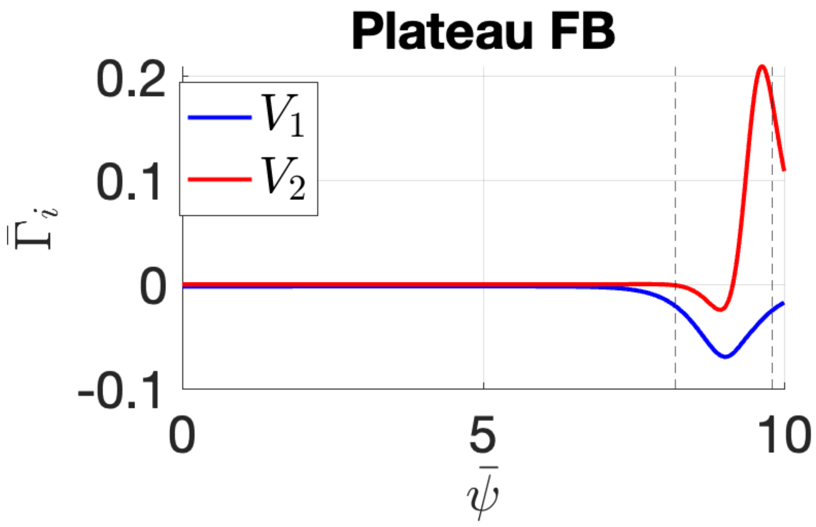
$$\frac{Q_{sg}^{max}}{Q_{wg}^{max}} \simeq 1.56$$

$$\frac{Q_{sg}^{max}}{Q_{wg}^{max}} \simeq 1.27$$



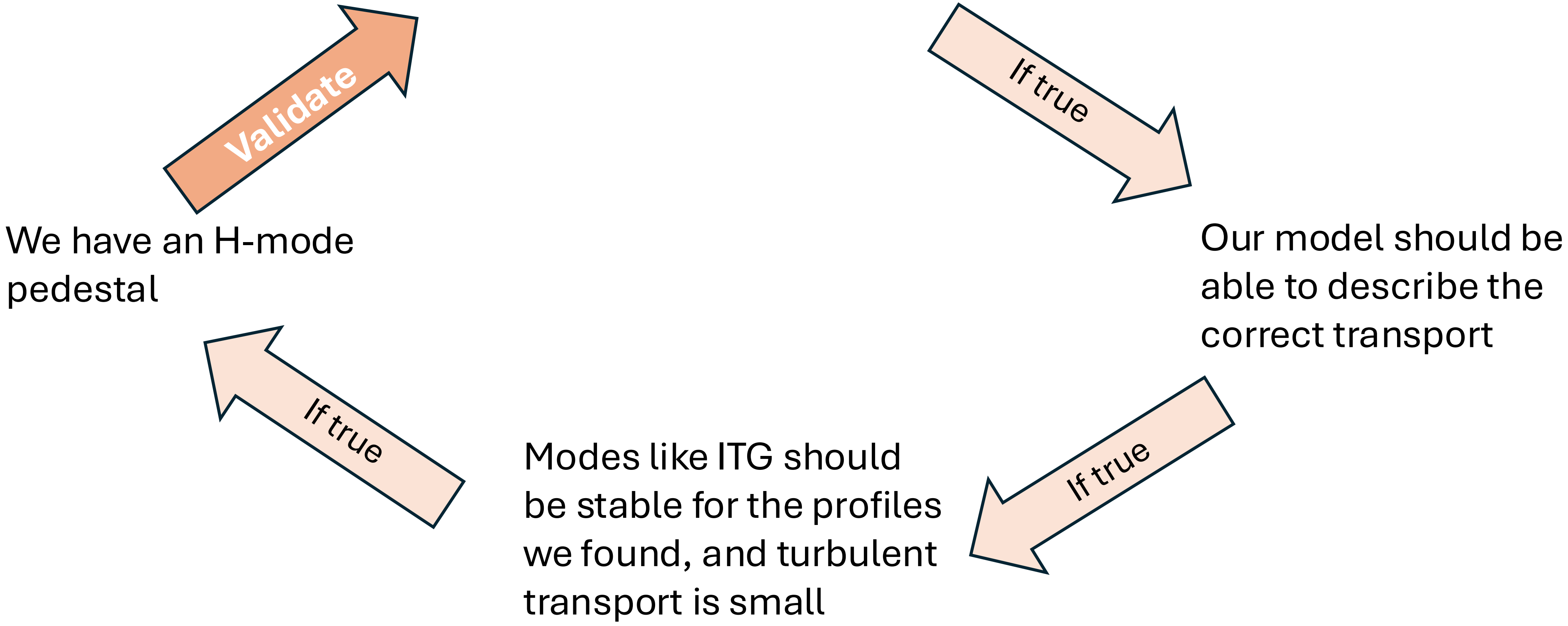
$$\frac{J_{sg}^{max}}{J_{wg}^{max}} \simeq 1.07$$

$$\frac{J_{sg}^{max}}{J_{wg}^{max}} \simeq 1.17$$



Stability analysis

Assumption: neoclassical transport is dominant in H-mode pedestals



Stability analysis

In practice: How do things like

- $u \sim v_t$
- $V_\parallel \sim v_t$
- ϕ_θ causing trapped particles on the inboard side

affect, for example, ITG, TEM and KBM in a regime where

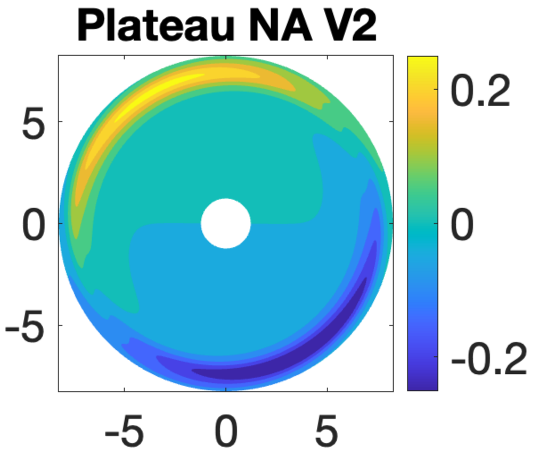
- $\rho_* \sim \epsilon$
- $\nabla\Phi \sim \frac{\Phi}{\rho_p}$

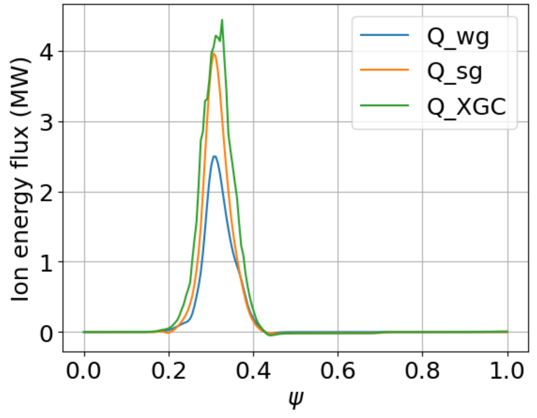
Bonus question: Can we find a threshold when profiles become unstable (H-L transition)?

Conclusions

We extend **neoclassical theory** into regions of **strong gradients** to describe **H-mode pedestals** and find

- Modifications to transport in banana and plateau regime due to
  - Poloidal variation
  - Explicit dependence on mean parallel flow
  - Orbit squeezing
- Predictions for turbulence-free and low-turbulence scenarios





Remaining questions:

- Can we get good agreement with **XGC simulations**?
- Are the solutions **stable** and describe an H-mode pedestal?