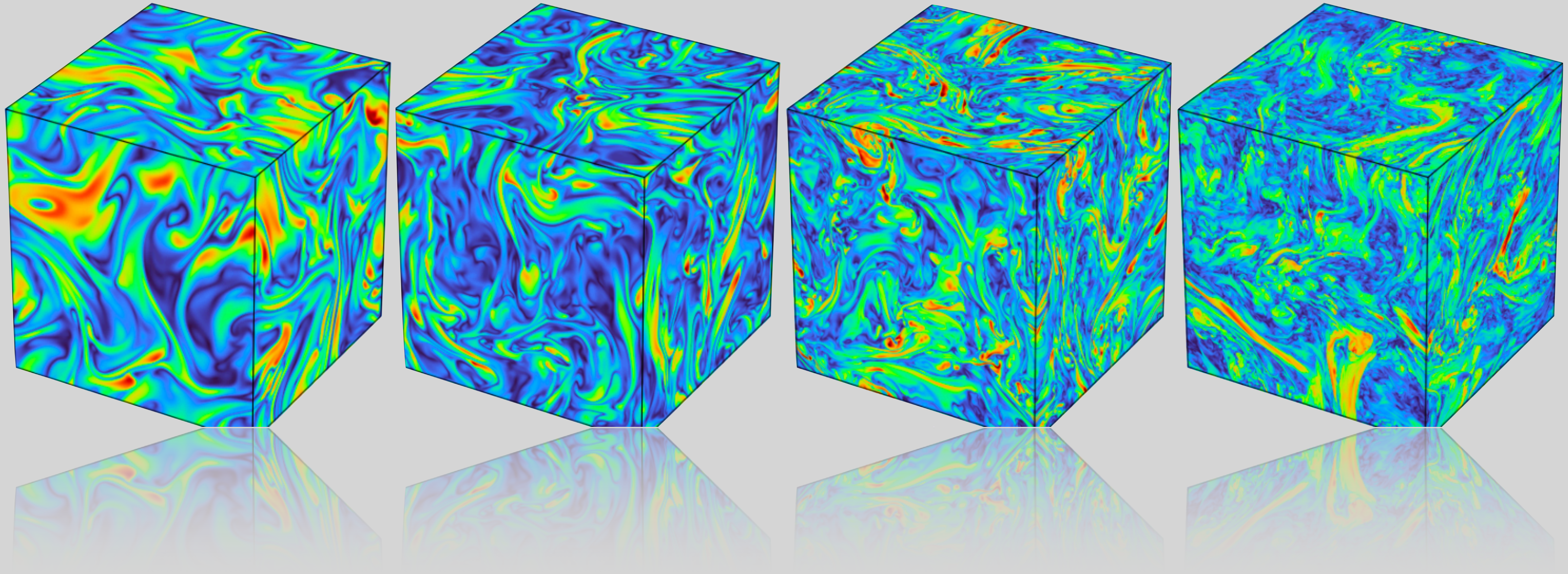


DECAY OF DYNAMO-INITIALIZED MAGNETIC FIELDS

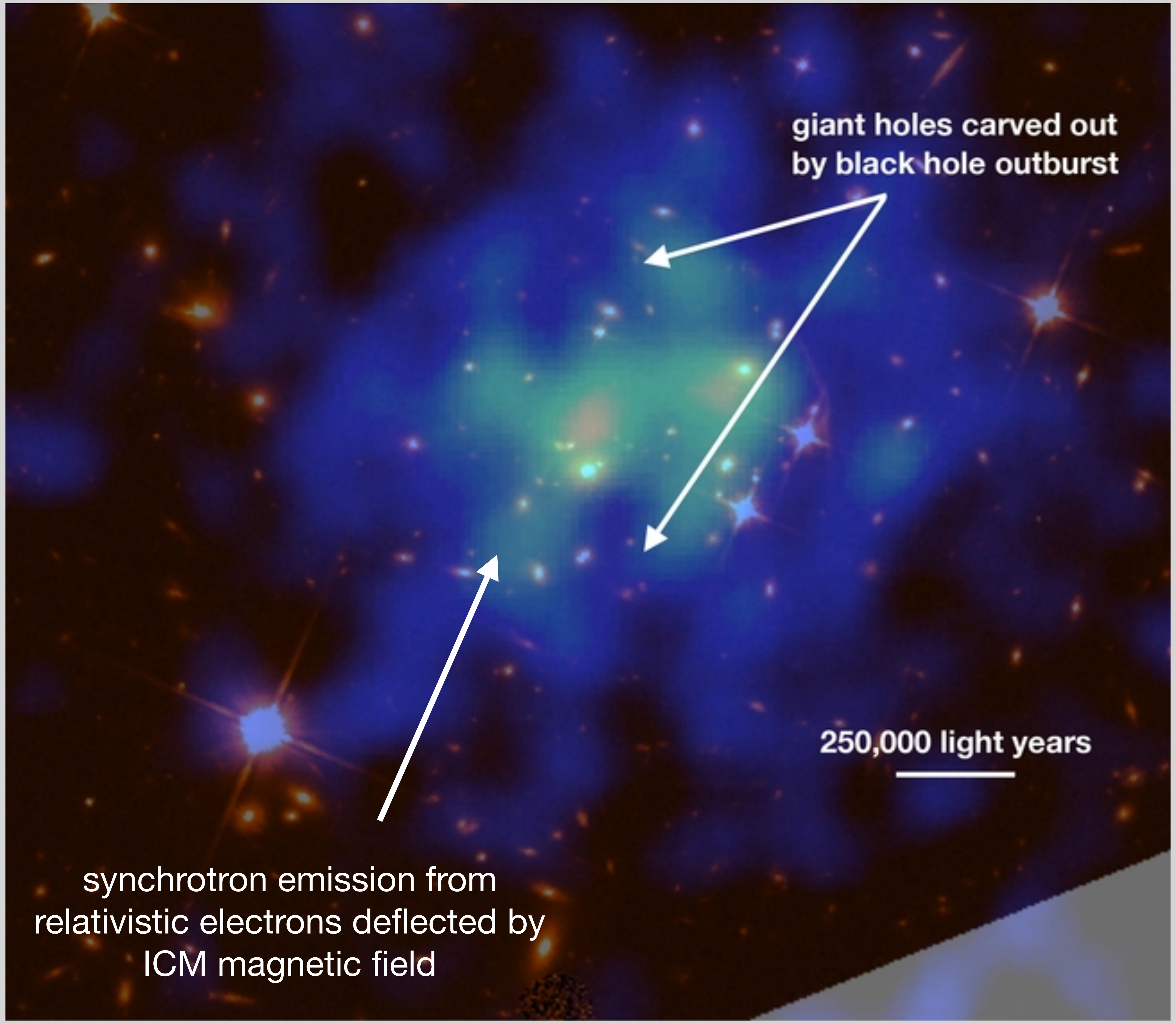
Zach Hemler (Princeton)
with Prof. Matt Kunz and Dr. David Hosking

16th Plasma Kinetics Working Meeting, 07/22/25

figure adapted from Galishnikova, Kunz, & Schekochihin (2022)
color corresponds to magnetic-field strength B

Motivation

Astrophysical dynamos with intermittent forcing



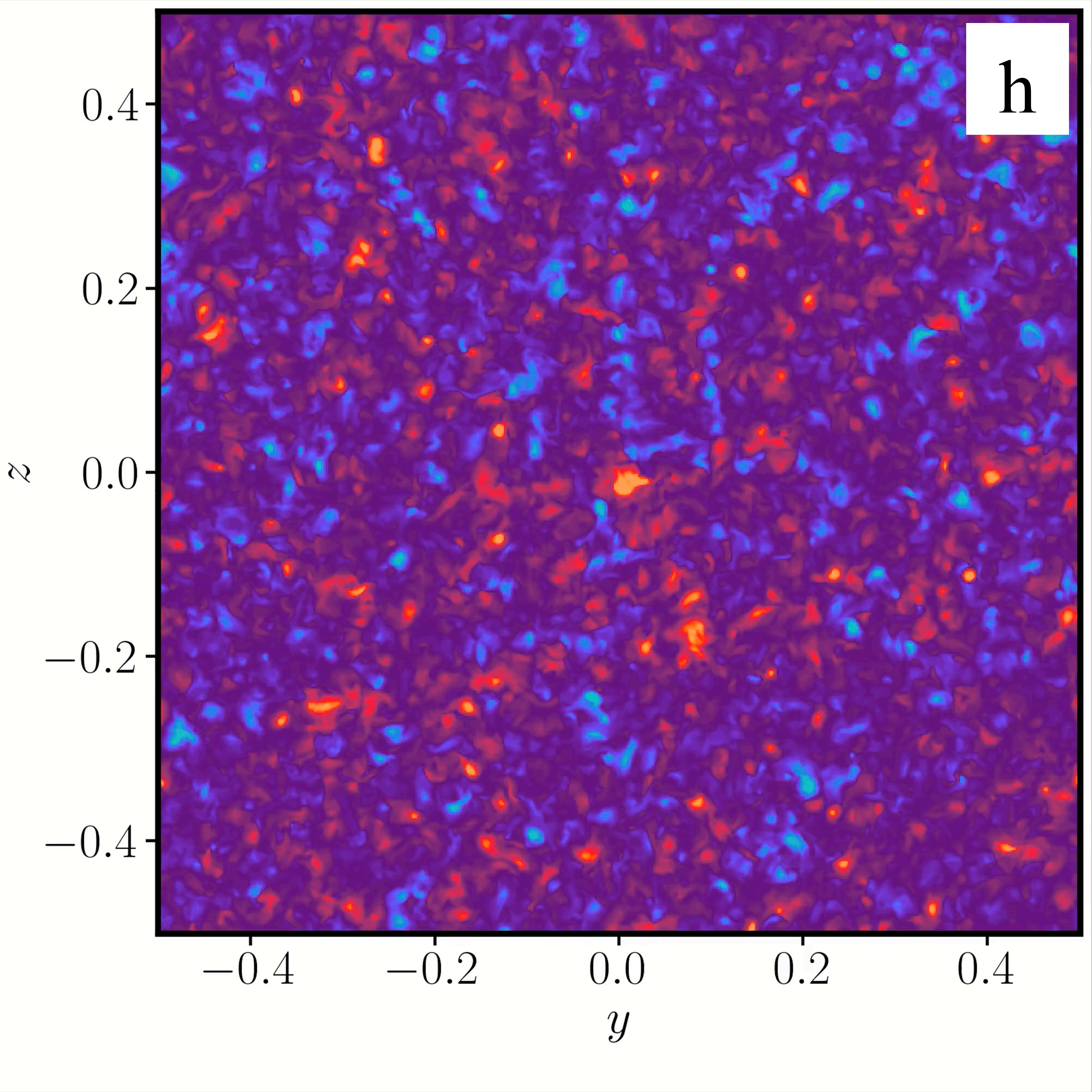
intracluster medium (ICM)

Phenomenology of decaying fields

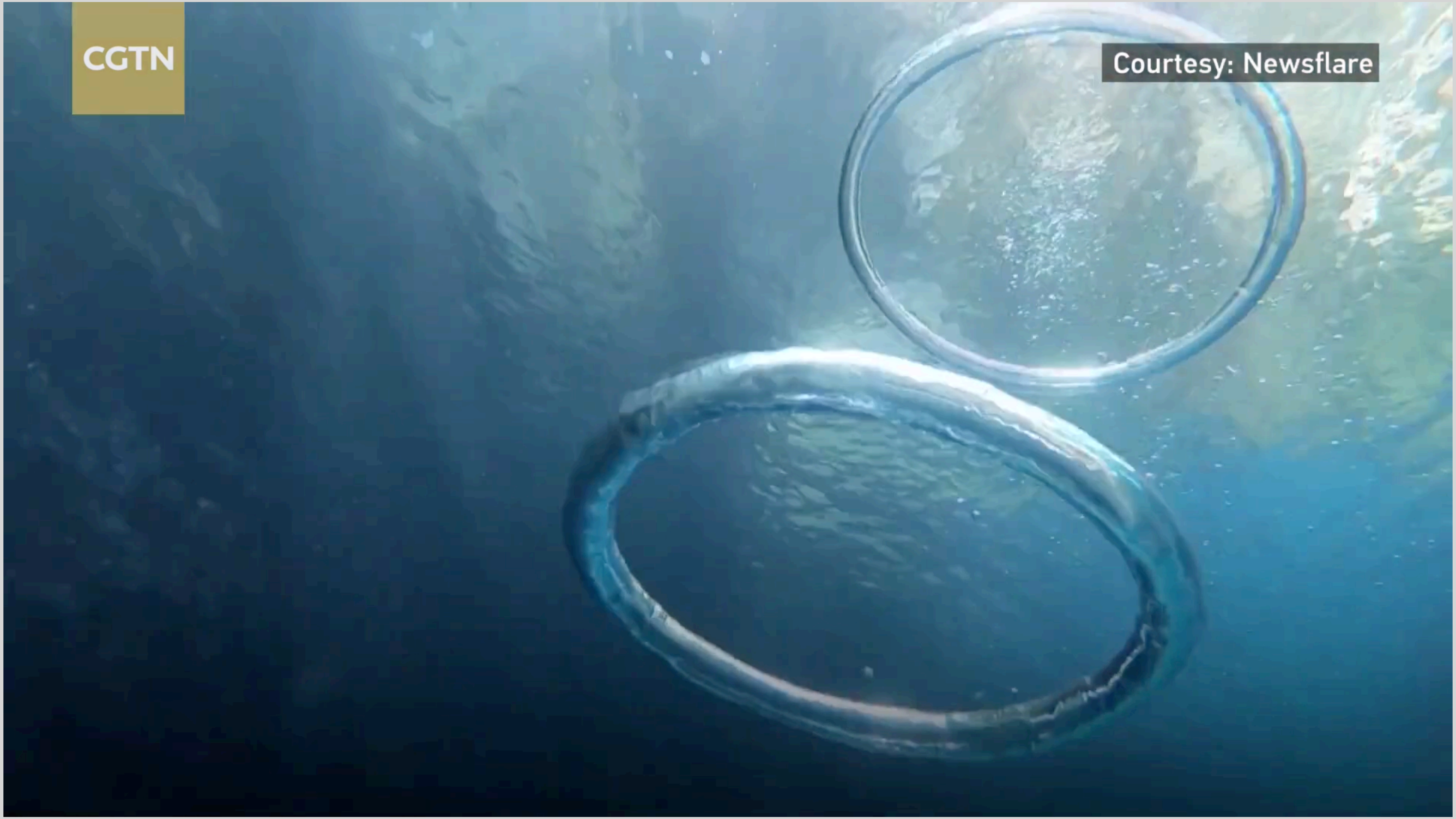
- In the absence of forcing...

  $B \downarrow$: magnetic field decays resistively

  $L \uparrow$: magnetic structures merge/grow via reconnection
- Merger dynamics can be constrained by...
  - conservation of helicity
  - conservation of magnetic flux



Phenomenology of decaying fields

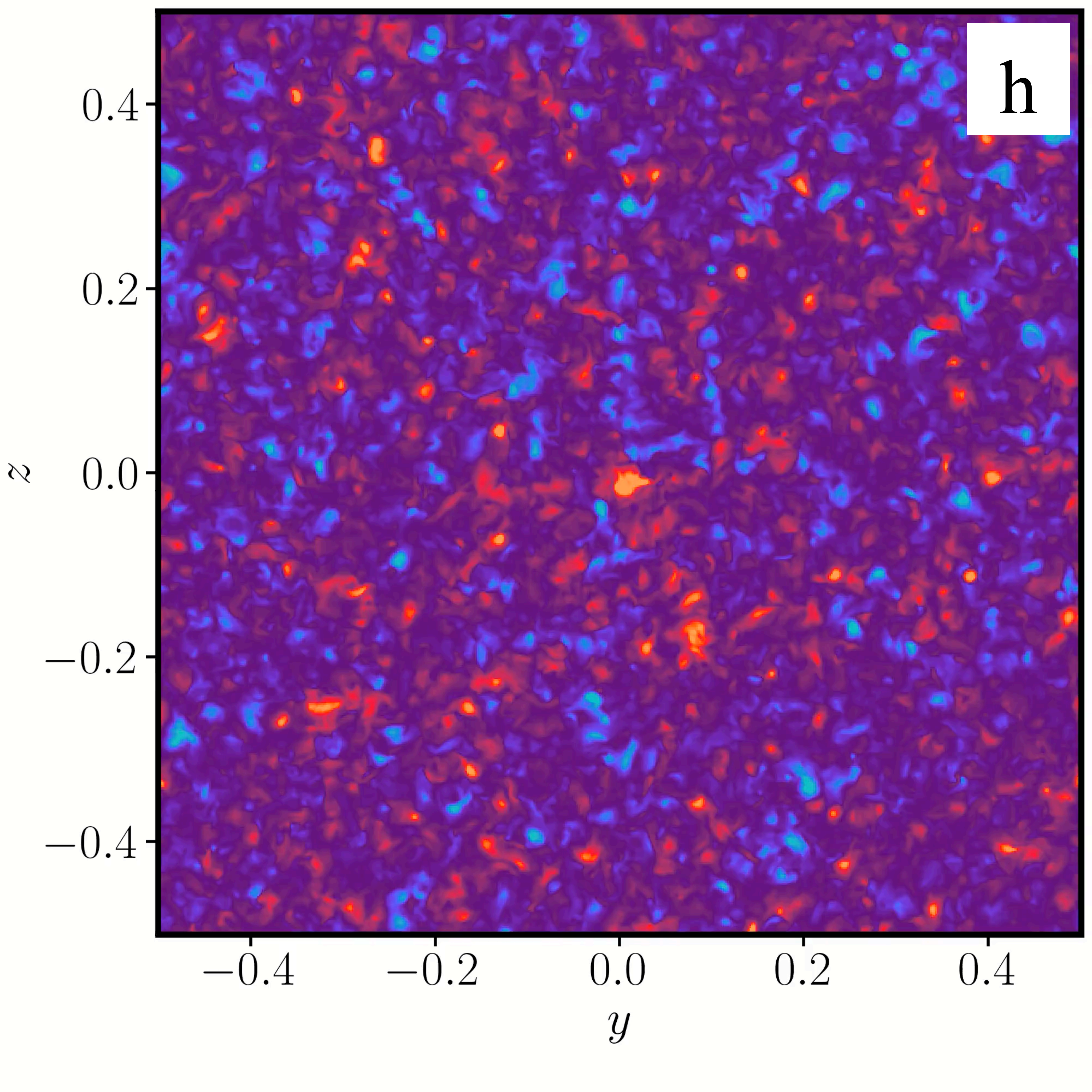


Phenomenology of decaying fields

- In the absence of forcing...

  $B \downarrow$: magnetic field decays resistively

  $L \uparrow$: magnetic structures merge/grow via reconnection
- Merger dynamics can be constrained by...
  - conservation of helicity
  - conservation of magnetic flux
- Merger-constraining quantity sets statistical invariant
- Statistical invariant sets...
  - statistical scaling ($B^\alpha L \sim$ const)
  - decay laws
  - presence or absence of inverse cascade



Phenomenology of decaying fields

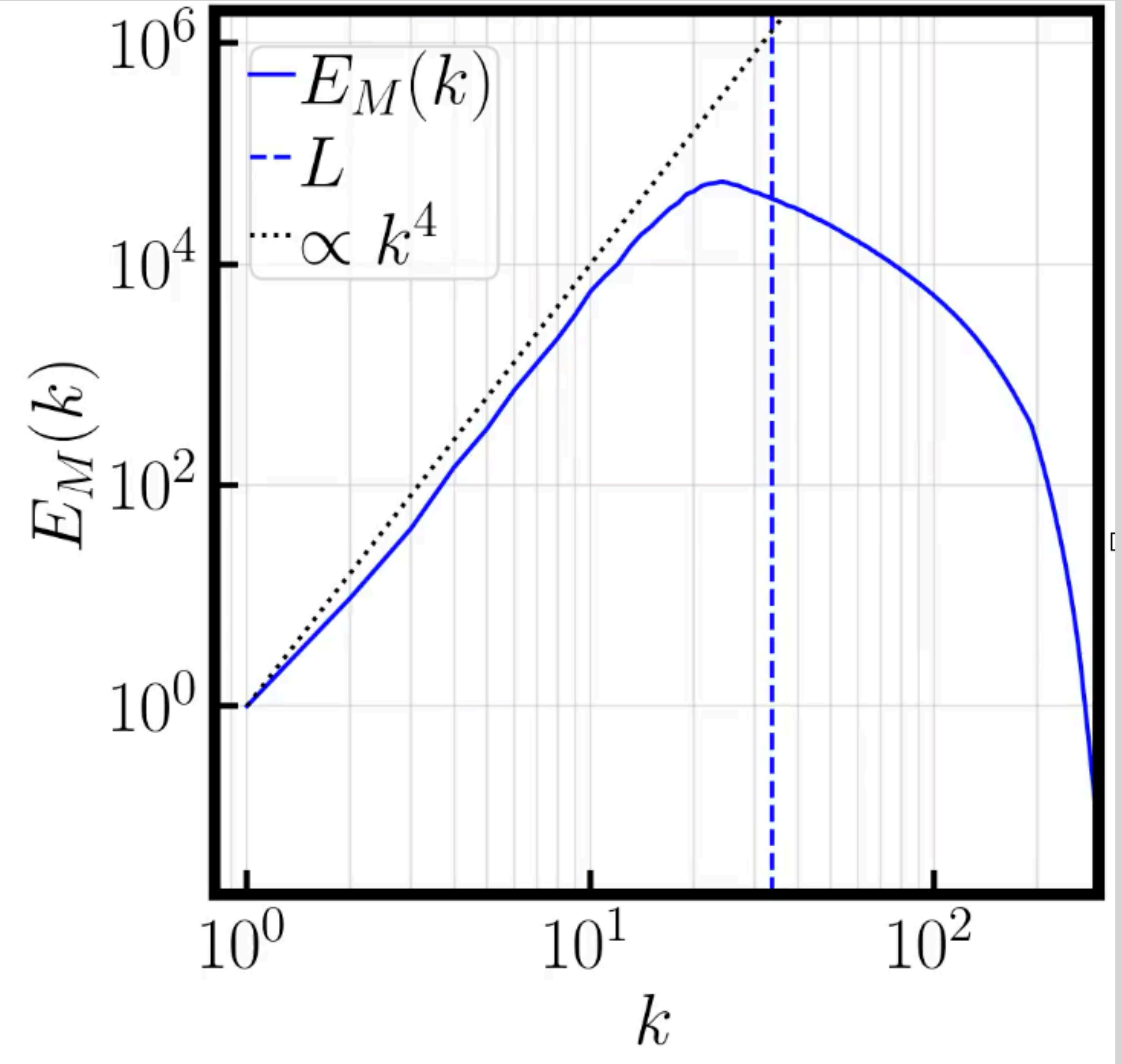
do structures contain significant net flux? → **NO**

↓

field like ensemble of quadrupoles; infrared $E_M(k) \propto k^4$

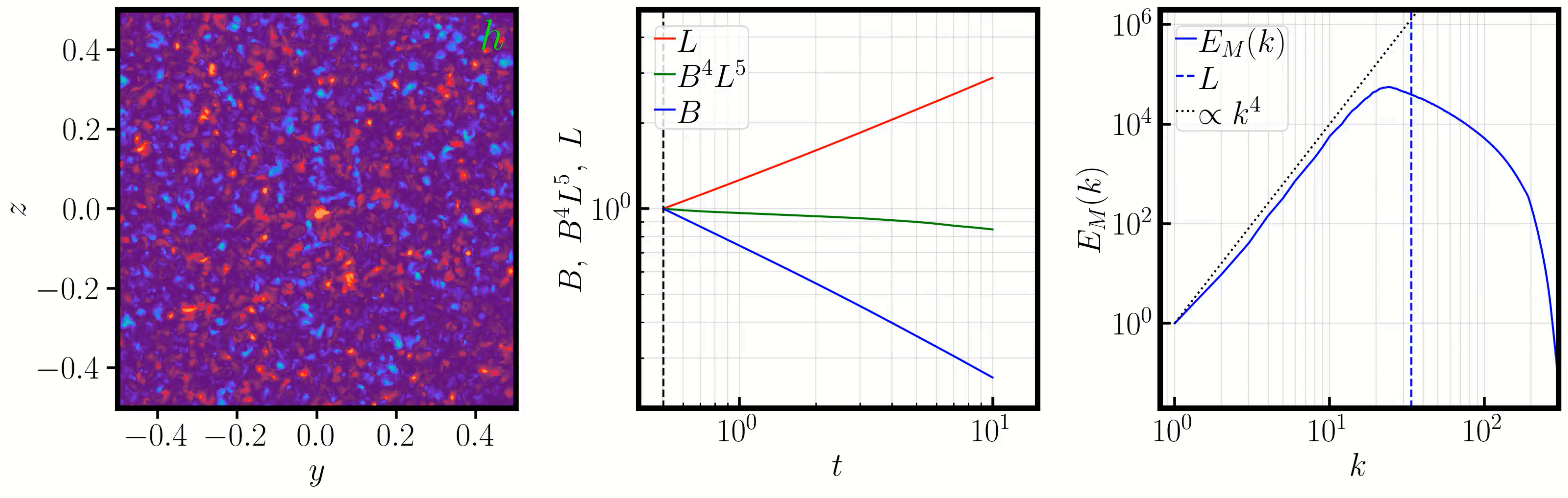
↓

merger dynamics solely constrained by helicity conservation



Phenomenology of decaying fields

Helicity-constrained merger dynamics, I_H



helicity-constrained merger dynamics, with some assumptions $\Longrightarrow$ global statistical scaling $B^4L^5 \sim$ const

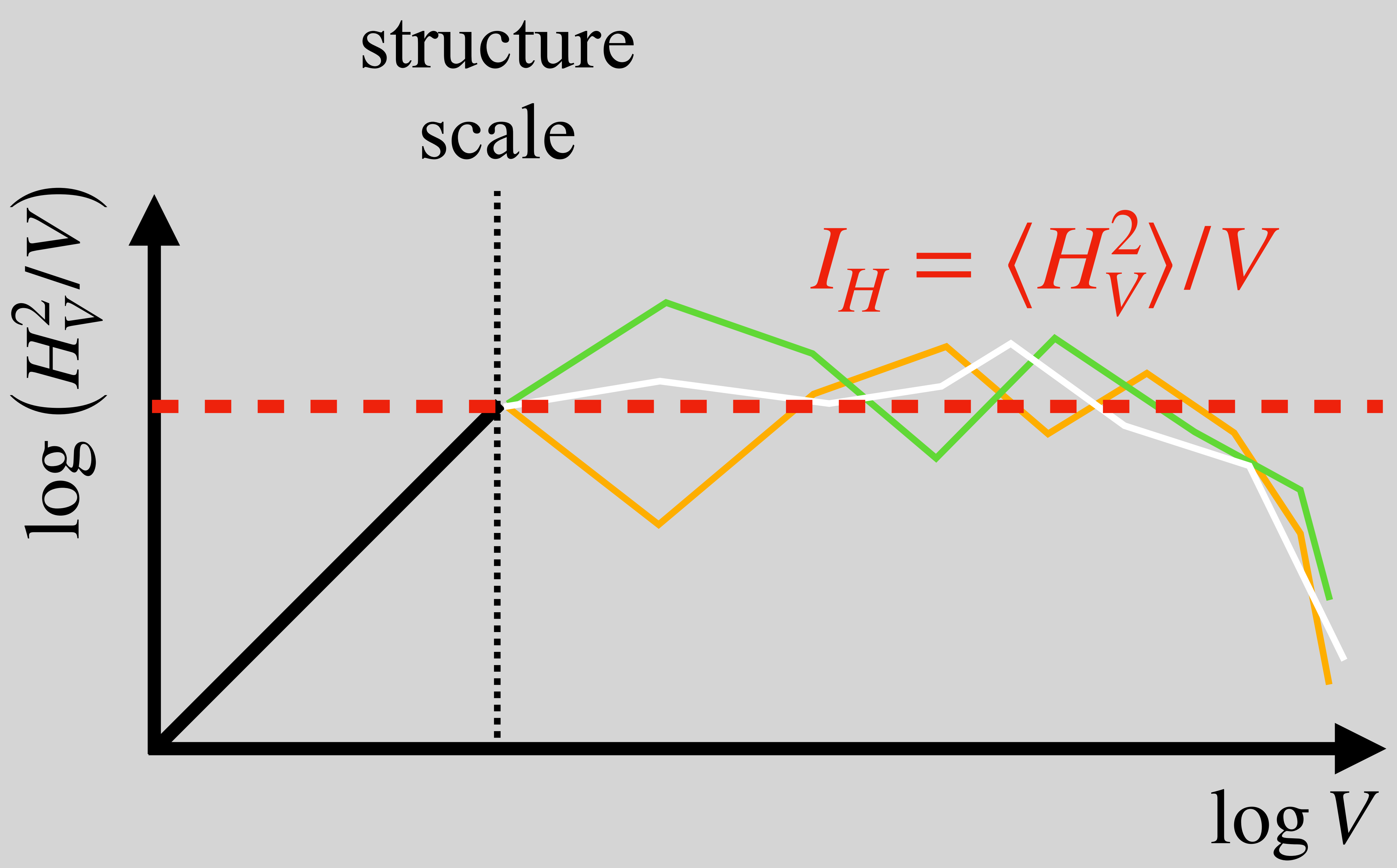
$$I_H \equiv \int \langle h(\boldsymbol{x})\, h(\boldsymbol{x}+\boldsymbol{r})\rangle \, d^3\boldsymbol{r} \qquad [I_H] = B^4L^5 \sim \text{const}$$

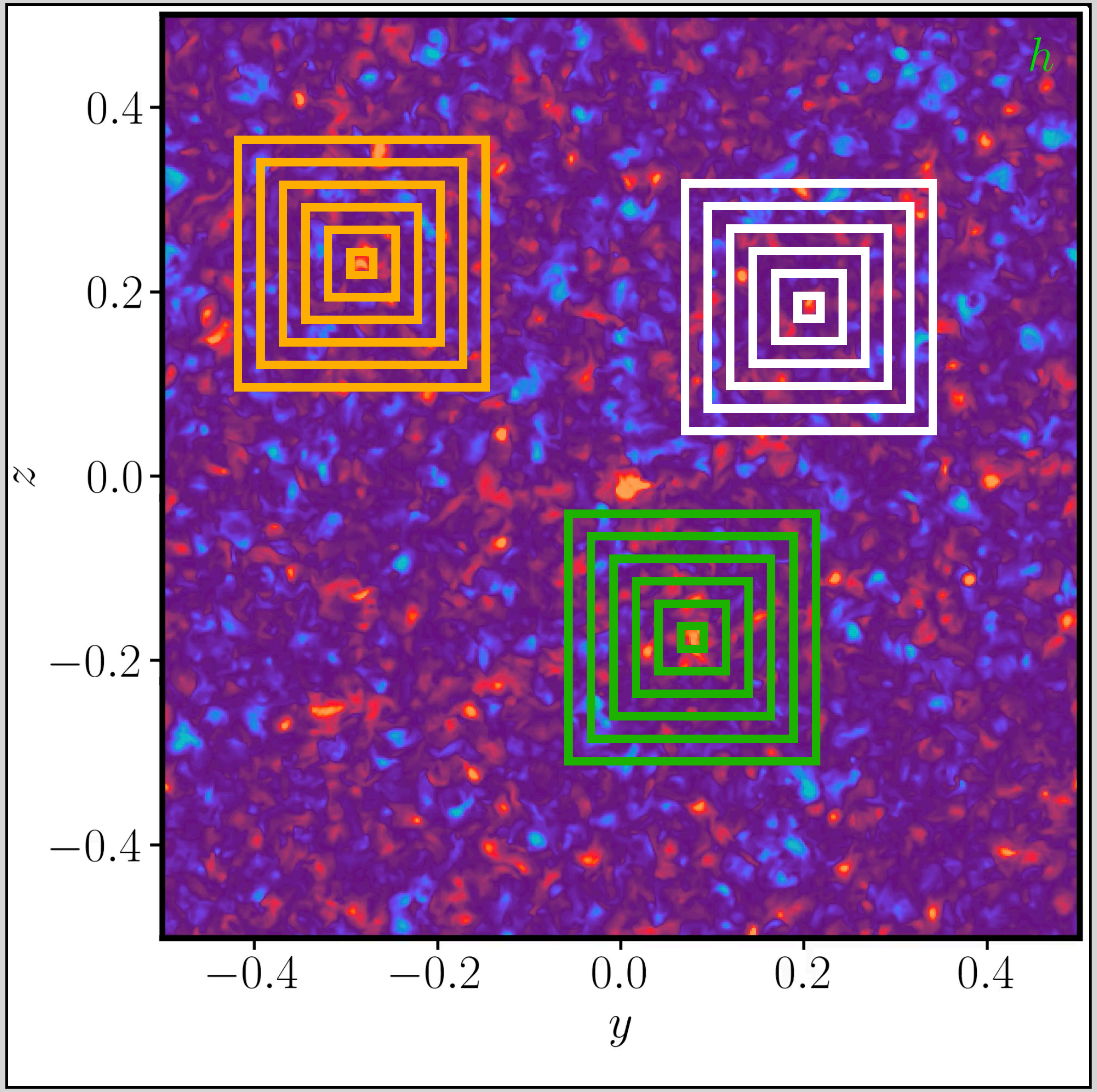
physically, I_H is roughly the characteristic squared net helicity per unit volume within structures

Phenomenology of decaying fields

Helicity-constrained merger dynamics, I_H

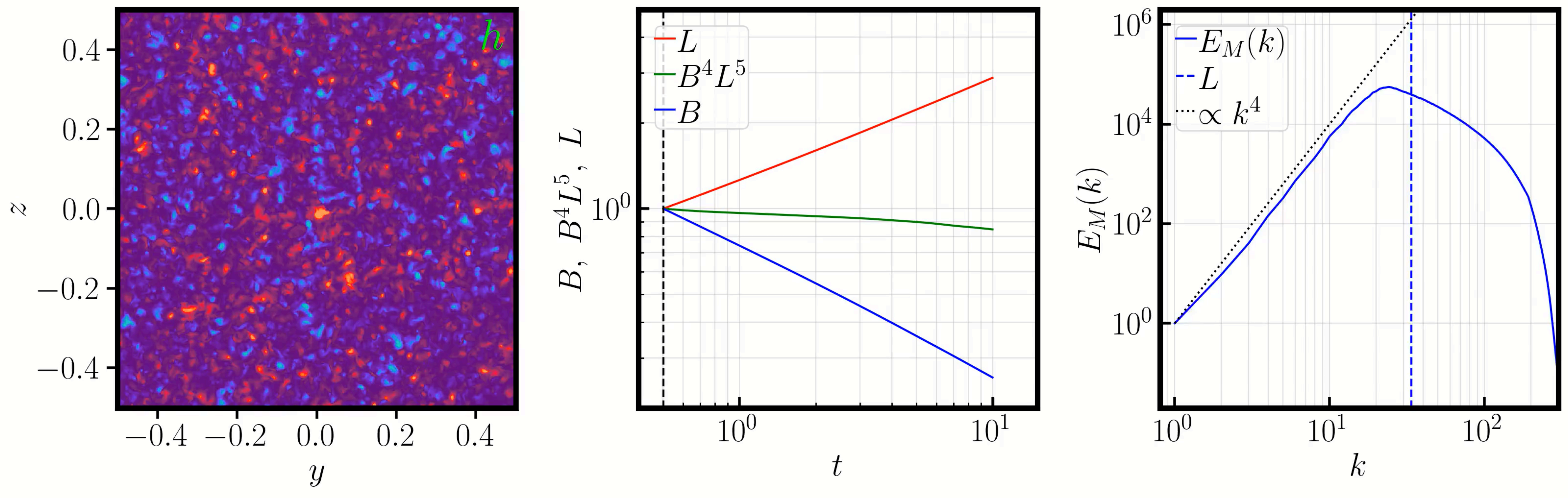
$$H_V \equiv \int_V d^3\boldsymbol{x}\, h(\boldsymbol{x})$$





Phenomenology of decaying fields

Helicity-constrained merger dynamics, I_H

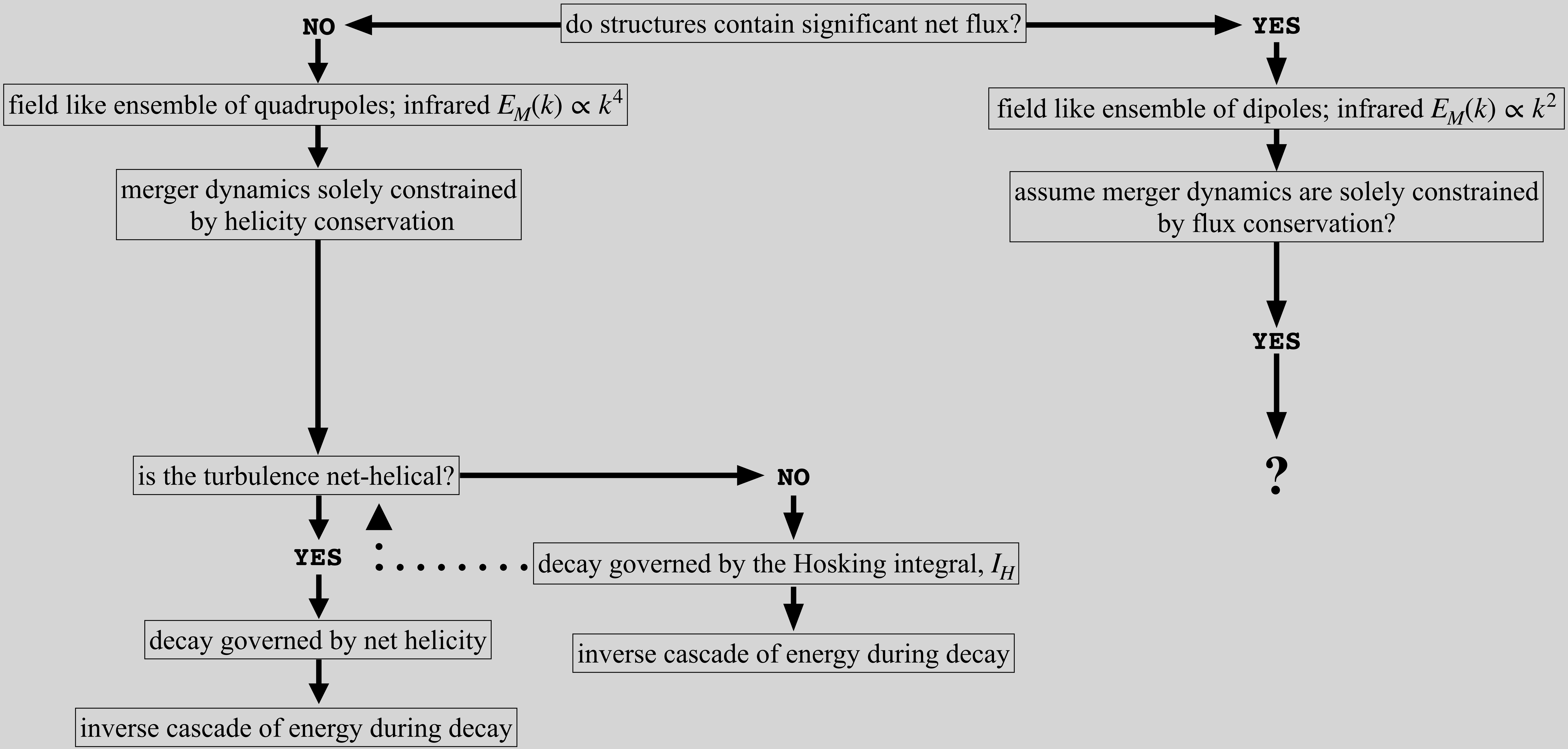


structures **do not** contain significant net flux $\Longrightarrow E_M(k < k_{\rm IR}) \propto k^4$

$$E_M(k < k_{\rm IR}) \sim B(t)^2 L(t)^5 k^4 \sim I_H B(t)^{-2} k^4 \sim B(t)^{-2} k^4$$

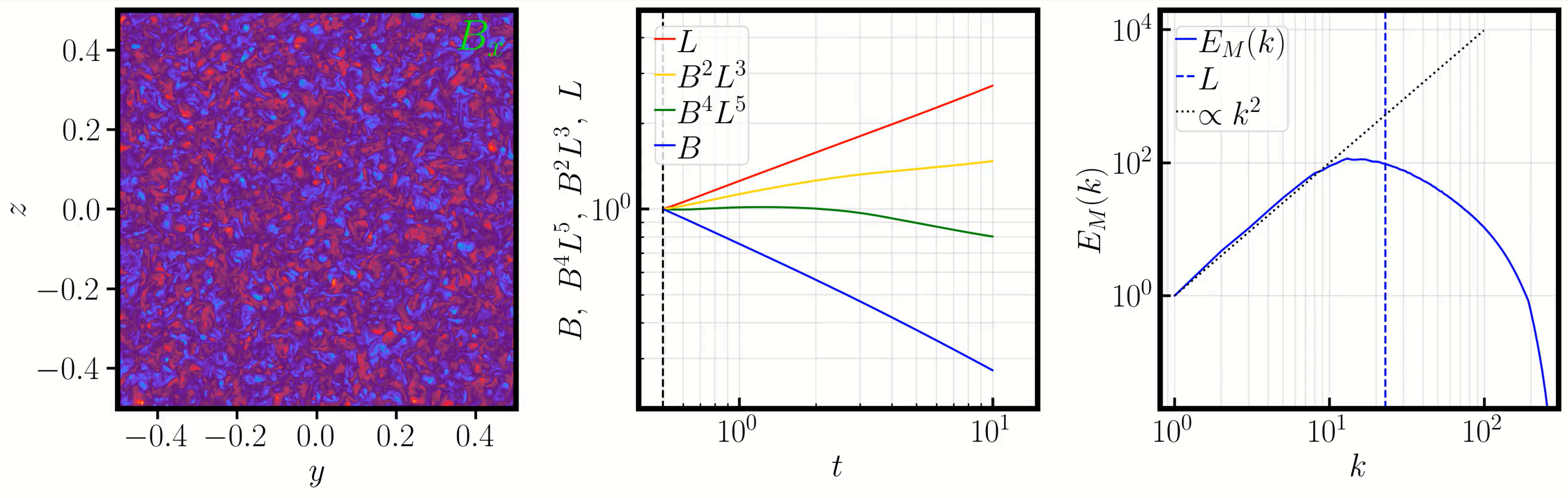
$$B \downarrow \Longrightarrow \; E_M(k < k_{\rm IR}) \uparrow \Longrightarrow \text{inverse cascade!}$$

Phenomenology of decaying fields



Phenomenology of decaying fields

Flux-constrained merger dynamics, I_B



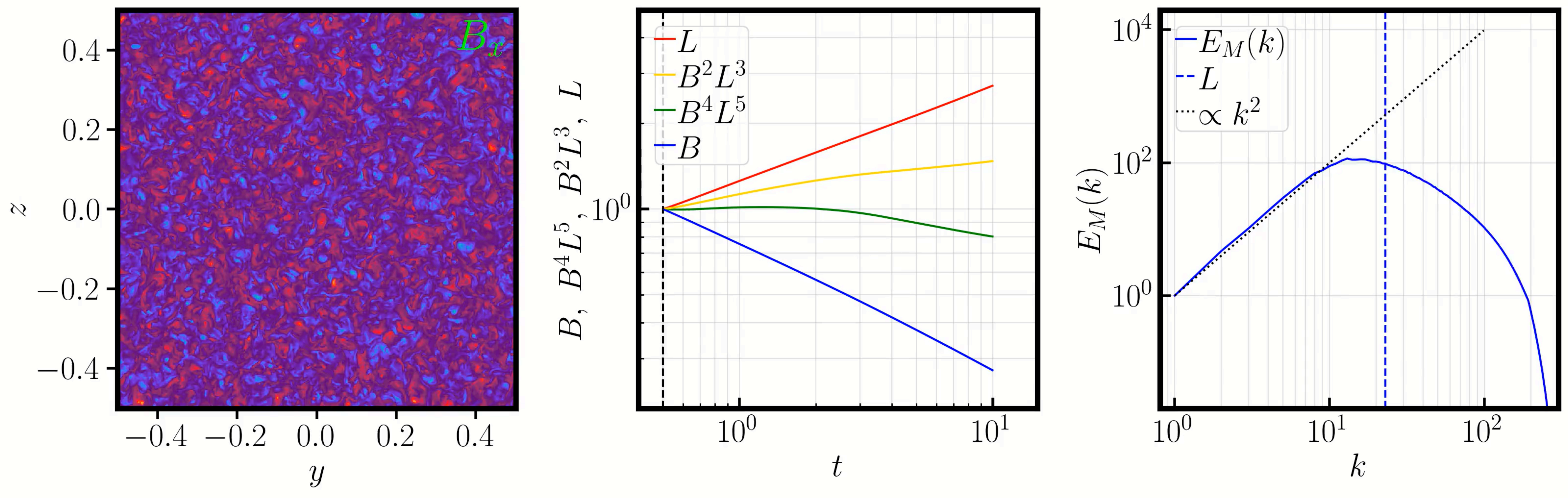
flux-constrained merger dynamics, with some assumptions $\Longrightarrow$ global statistical scaling $B^2L^3 \sim \text{const}$

$$I_B \equiv \int \langle \boldsymbol{B}(\boldsymbol{x}) \cdot \boldsymbol{B}(\boldsymbol{x}+\boldsymbol{r}) \rangle \, d^3\boldsymbol{r} \qquad [I_B] = B^2L^3 \sim \text{const}$$

physically, I_B is roughly the characteristic squared net flux per unit volume within structures
or, more accurately, the mean-square fluctuation level of net flux per unit volume over large control volumes

Phenomenology of decaying fields

Flux-constrained merger dynamics, I_B



structures **do** contain significant net flux $\Longrightarrow E_M(k < k_{\rm IR}) \propto k^2$

$$E_M(k < k_{\rm IR}) \sim B(t)^2 L(t)^3 k^2 \sim I_B k^2 \sim k^2$$

$$B \downarrow \Longrightarrow\ E_M(k < k_{\rm IR}) \sim \text{const}\ \Longrightarrow\ \text{permanence of large scales}$$

Phenomenology of decaying fields

Flux-constrained merger dynamics, I_B

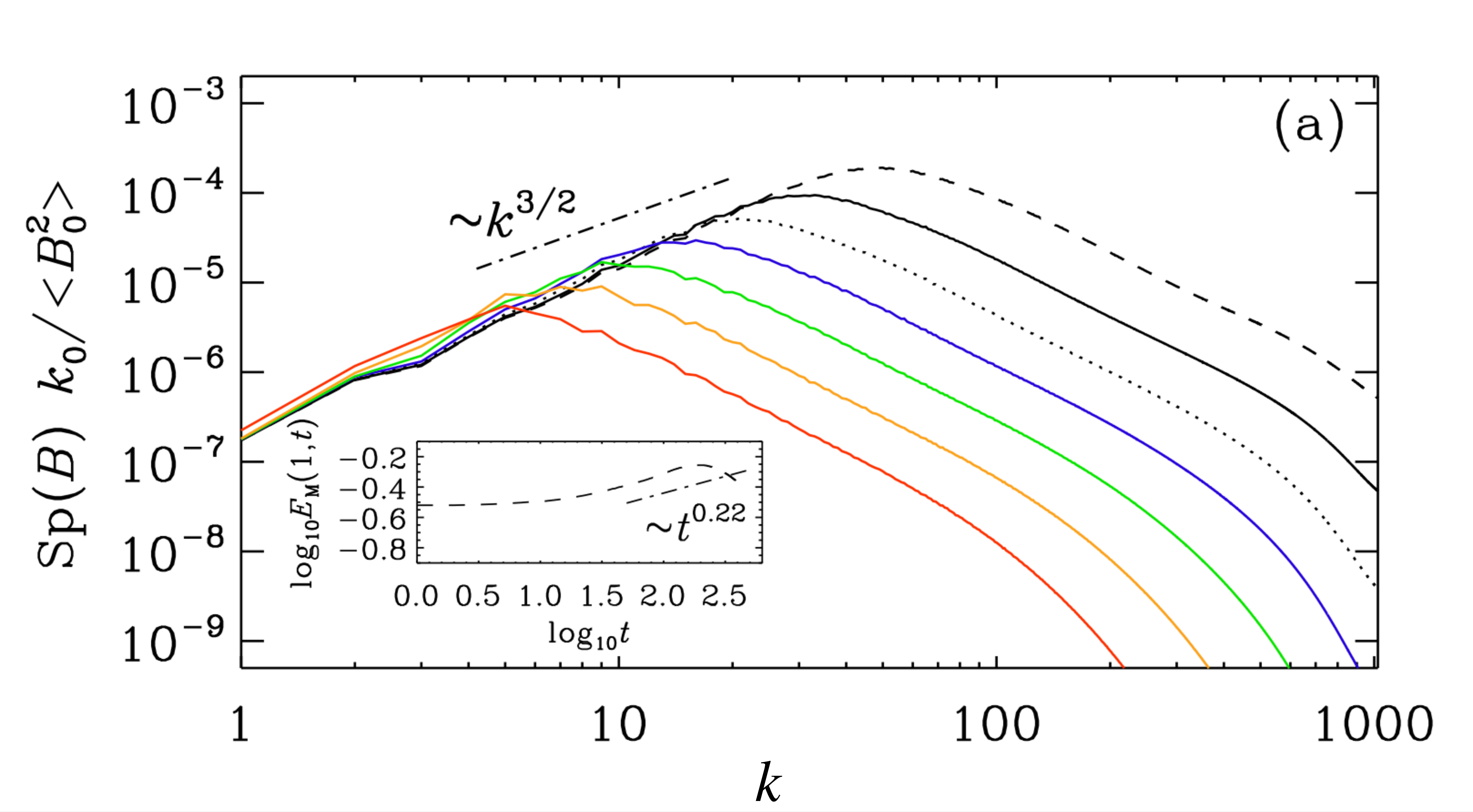
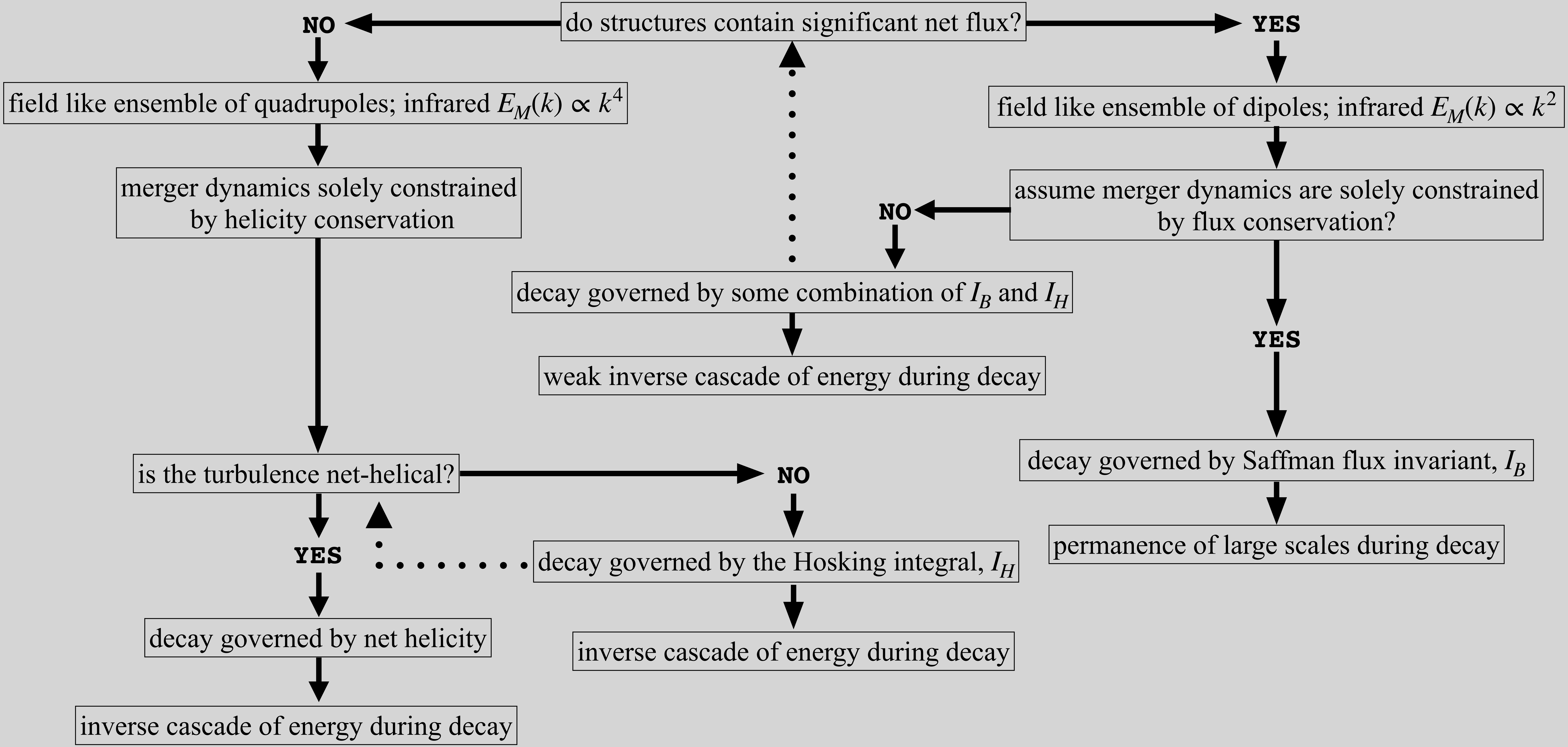
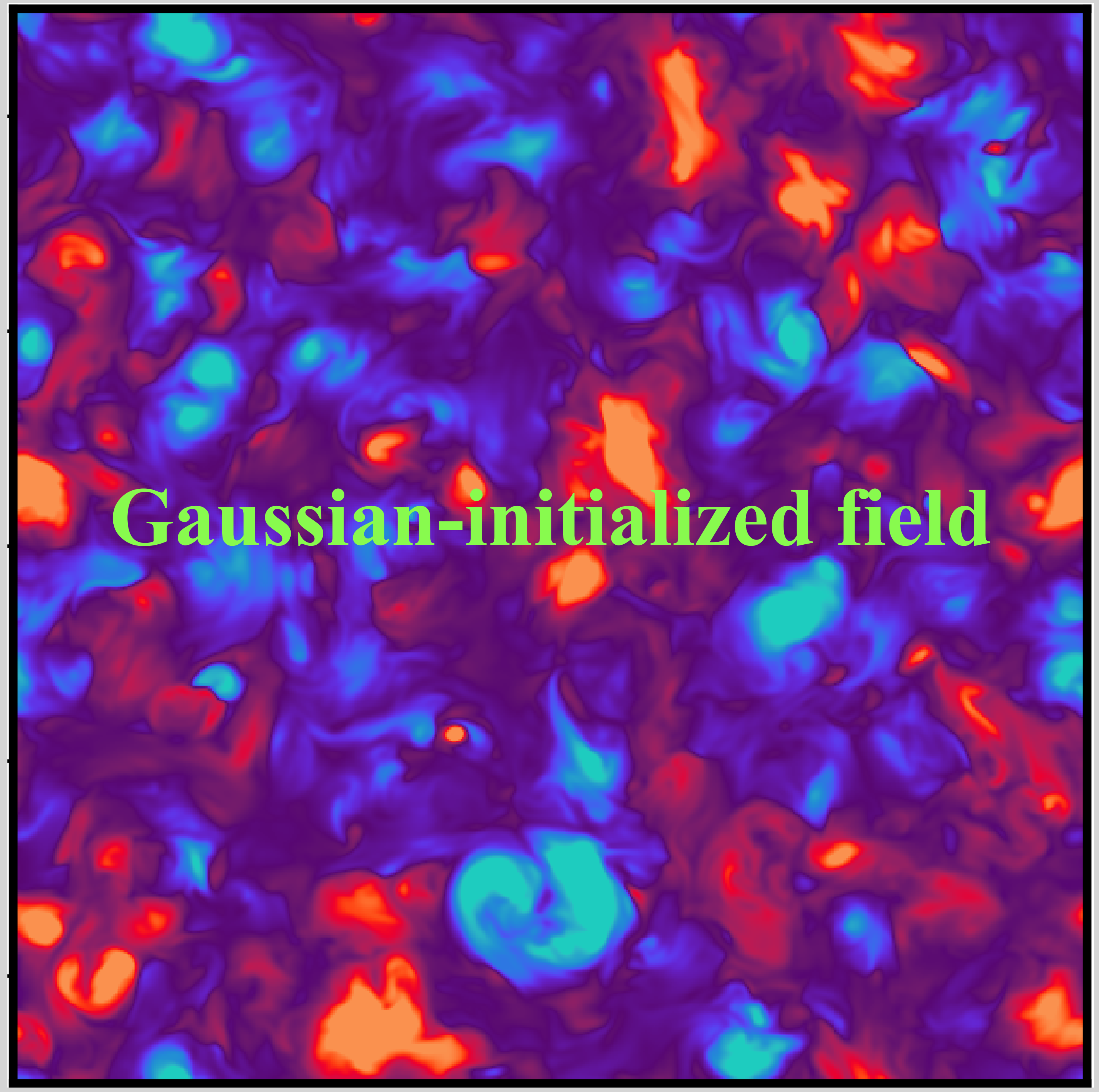


figure courtesy of Brandenburg, Sharma, & Vachaspati (2023)

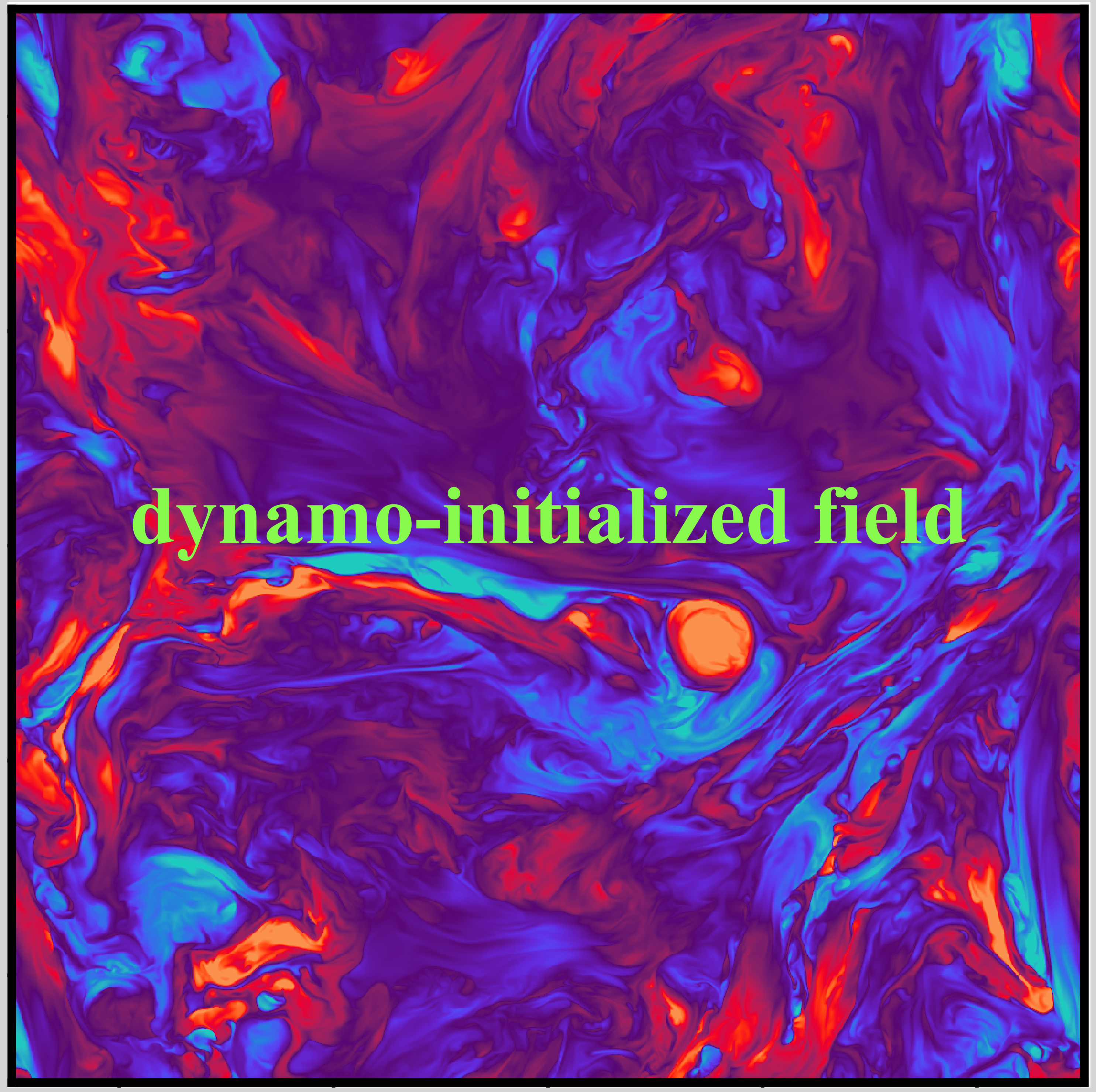
Phenomenology of decaying fields



Gaussian-initialized field vs. dynamo-initialized field



- Artificially initialized (non-physical?)
- Magnetically dominated ($B_{\mathrm{rms}} \gg u_{\mathrm{rms}}$)
- Structures under less tension (i.e., blobs)
- Governing invariant: I_H or I_B

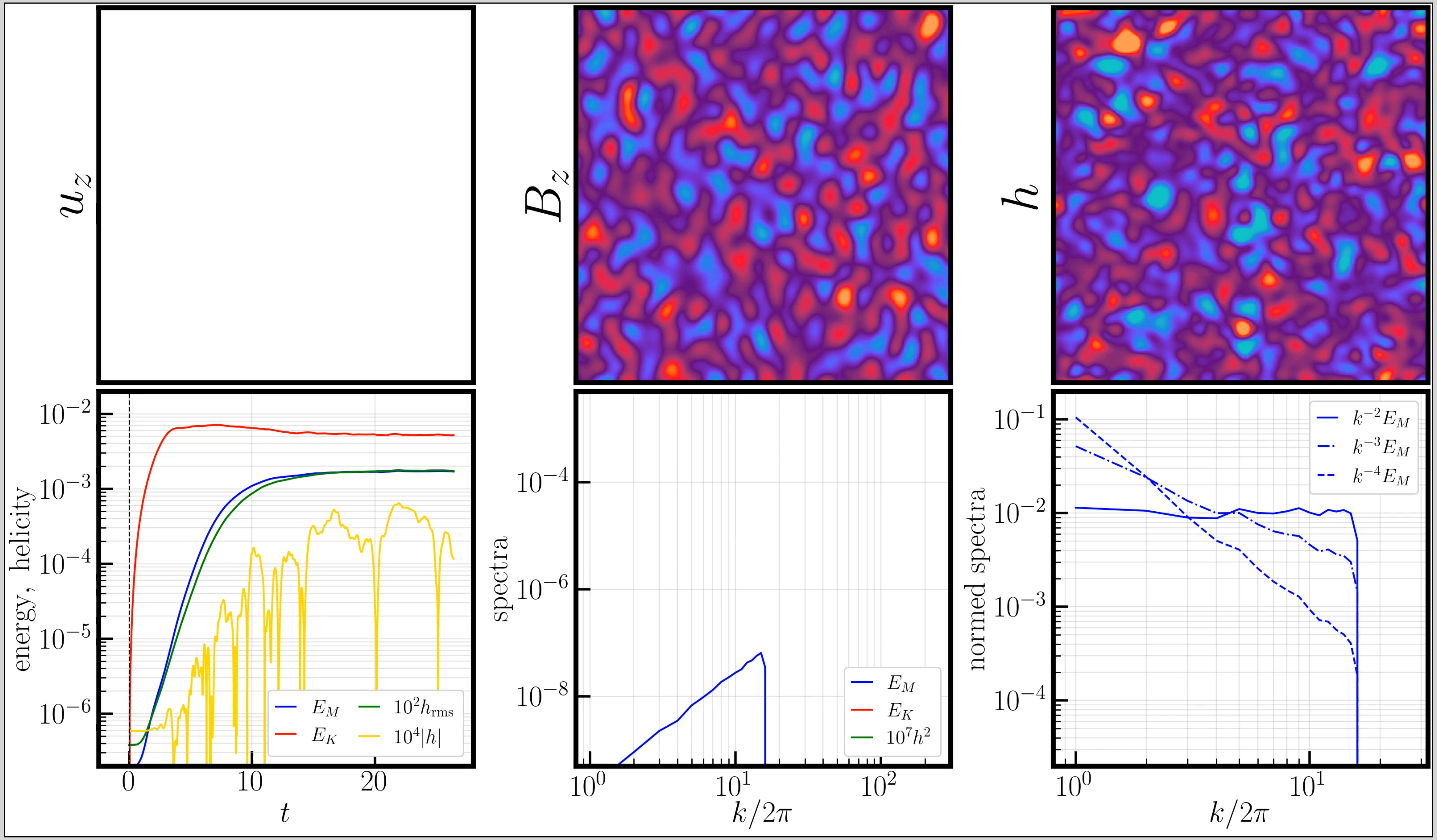


- Develops naturally from field amplification
- Initial energy equipartition ($B_{\mathrm{rms}} \sim u_{\mathrm{rms}}$)
- Structures under more tension (i.e., folds)
- Governing invariant: ???

Dynamo phase: field amplification

576^3, $\mathrm{Rm}_4 \approx 1.4\mathrm{E}6$, $\mathrm{Pm} = 1$, $n_F \in (8, 16)$, 4th-order hyperdissipation

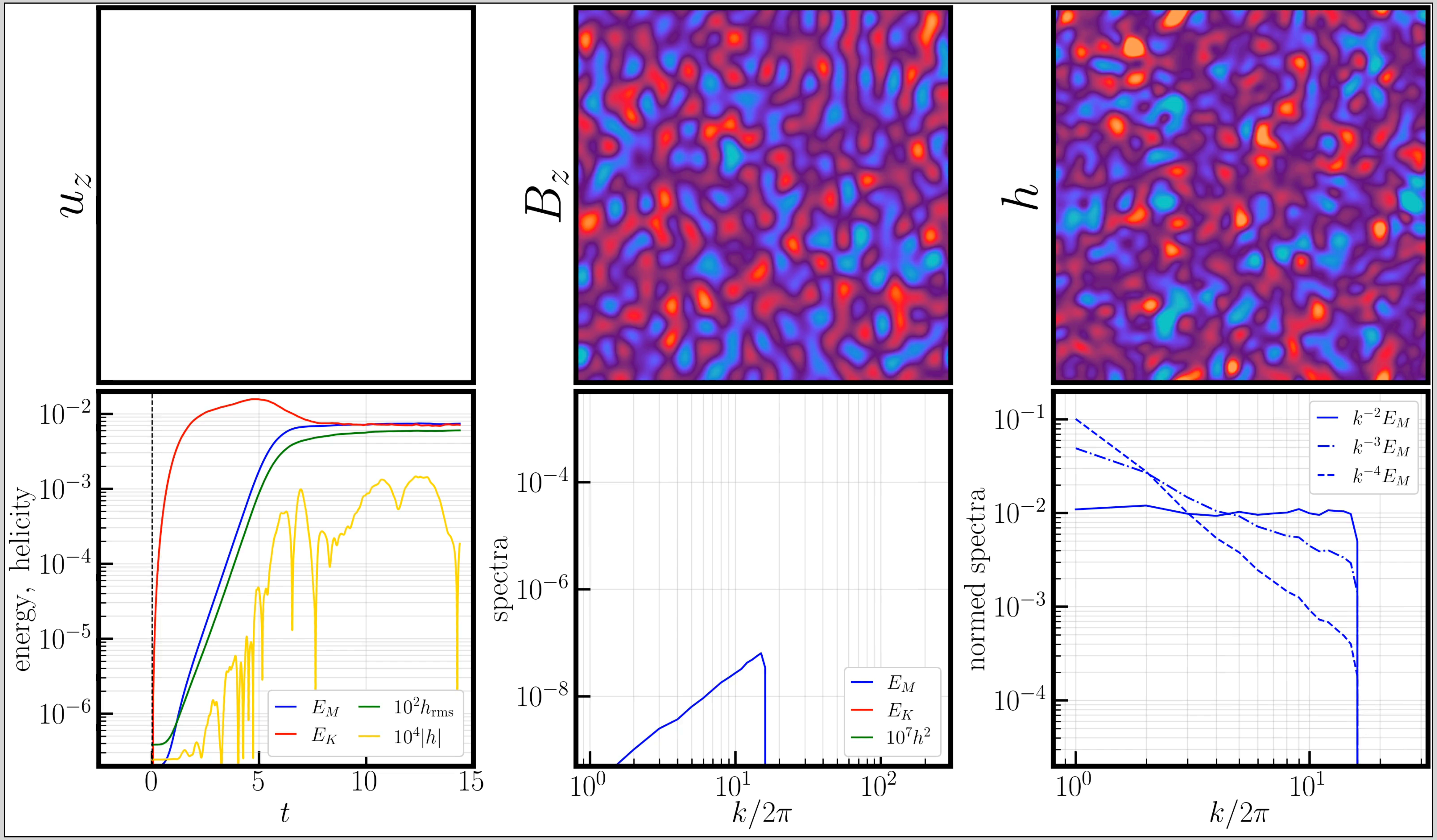
At saturation, magnetic-energy spectrum exhibits $\propto k^2$ infrared spectrum (i.e., fluxy structures)



Dynamo phase: field amplification

1152^3, $\mathrm{Rm}_4 \approx 2.8\mathrm{E}6$, $\mathrm{Pm} = 100$, $n_F \in (8, 16)$, 4th-order hyperdissipation

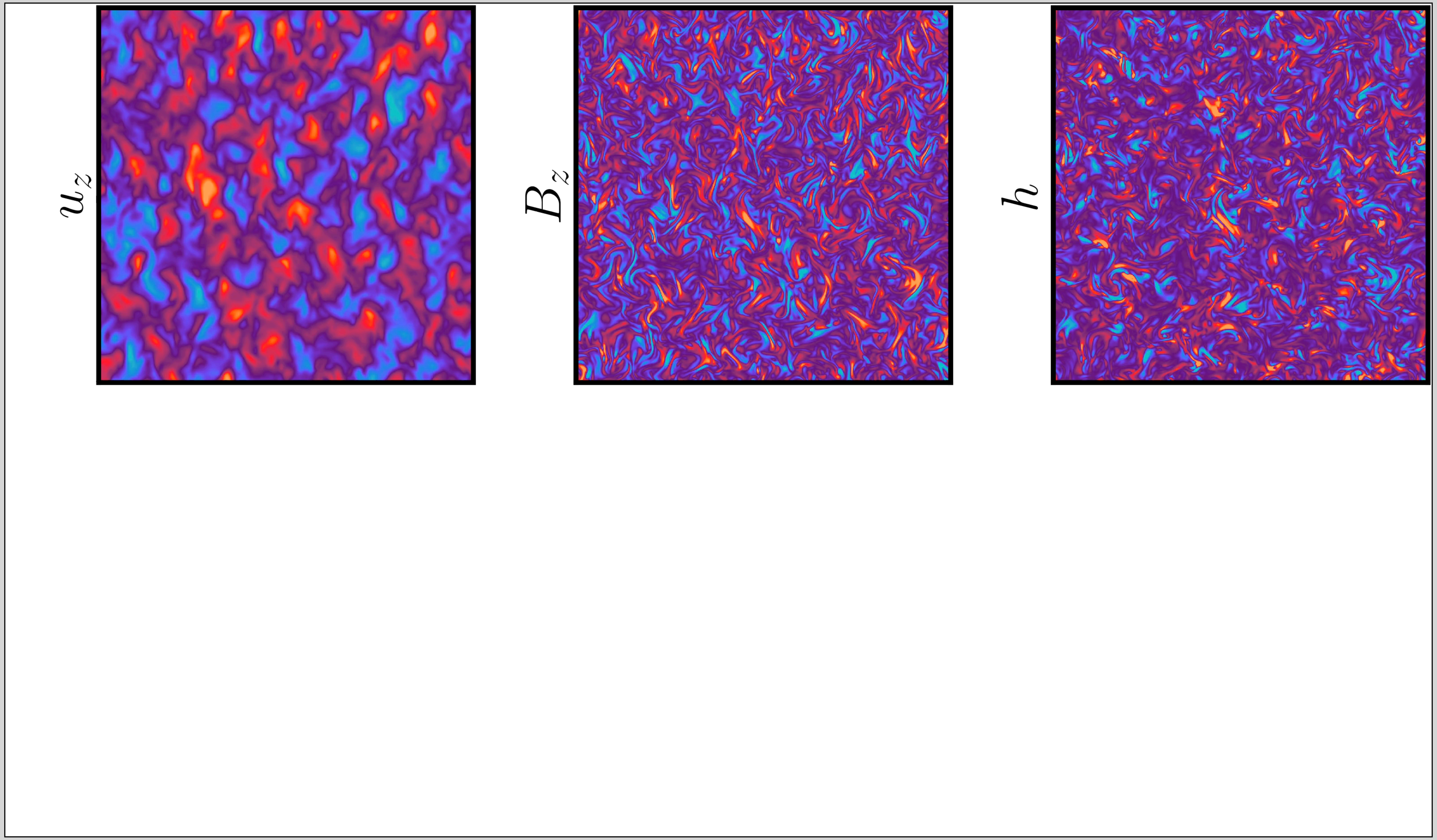
At saturation, magnetic-energy spectrum exhibits $\propto k^2$ infrared spectrum (i.e., fluxy structures)



Decay phase: structure evolution

1152^3, $\mathrm{Rm}_4 \approx 2.8\mathrm{E}6$, $\mathrm{Pm} = 100$, $n_F \in (8, 16)$, 4th-order hyperdissipation

dynamo-
init
field



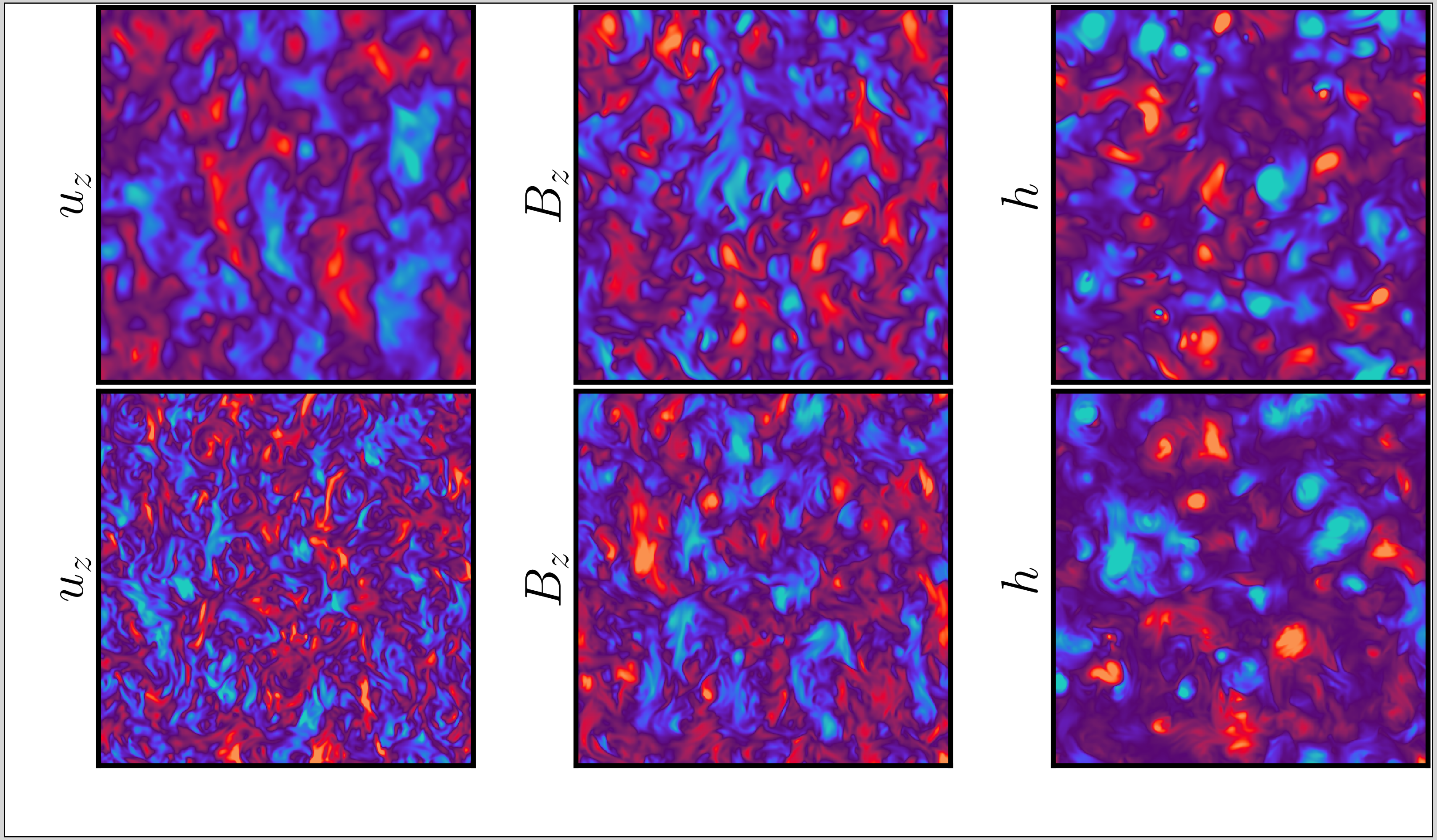
Decay phase: structure evolution

1152^3, $\mathrm{Rm}_4 \approx 2.8\mathrm{E}6$, $\mathrm{Pm} = 100$, $n_F \in (8, 16)$, 4th-order hyperdissipation

Sufficiently decayed dynamo fields appear qualitatively similar to decayed random fields!

dynamo-init field

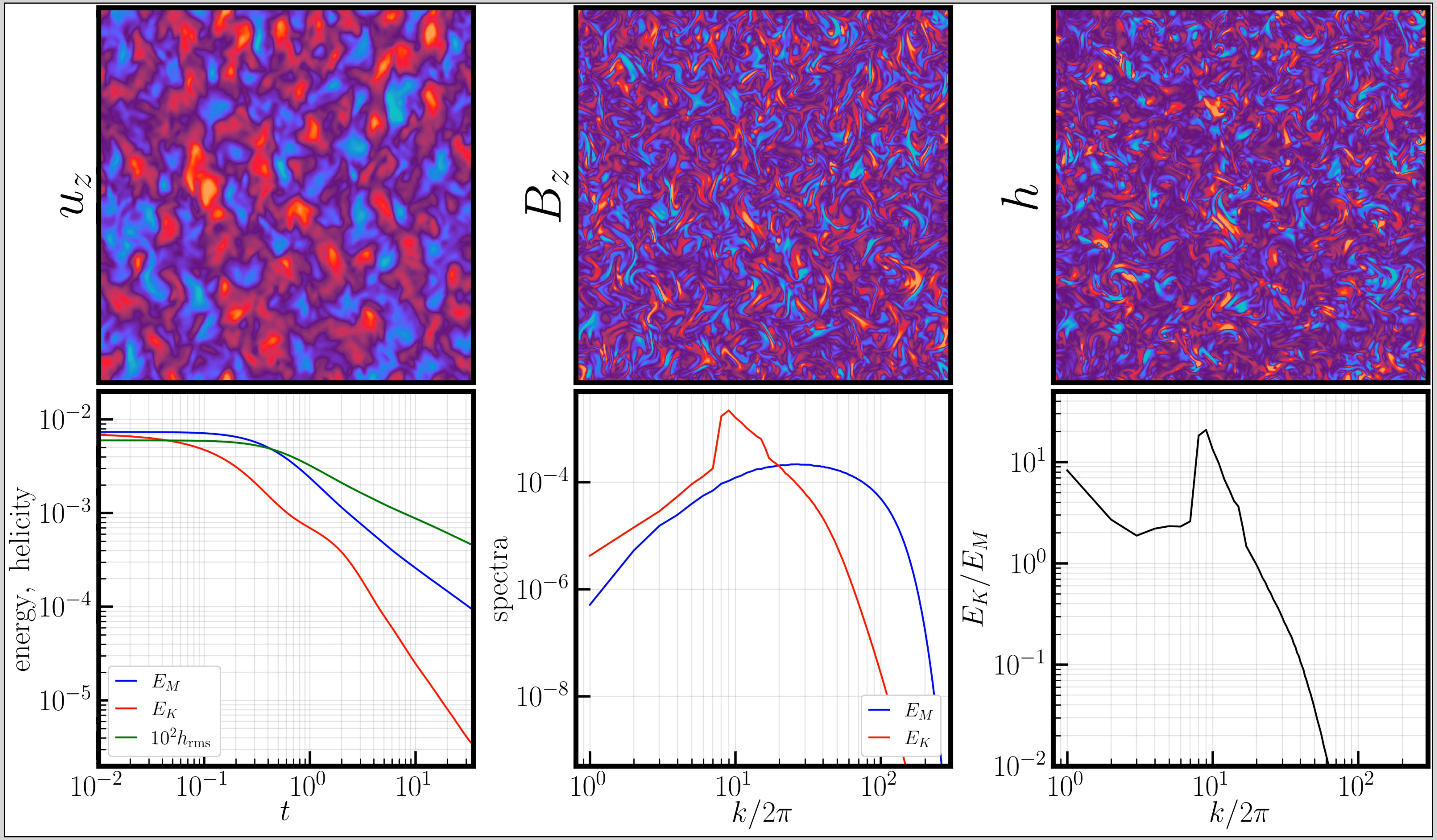
Gaussian-init field



Decay phase: structure evolution

1152^3, $\mathrm{Rm}_4 \approx 2.8\mathrm{E}6$, $\mathrm{Pm} = 100$, $n_F \in (8, 16)$, 4th-order hyperdissipation

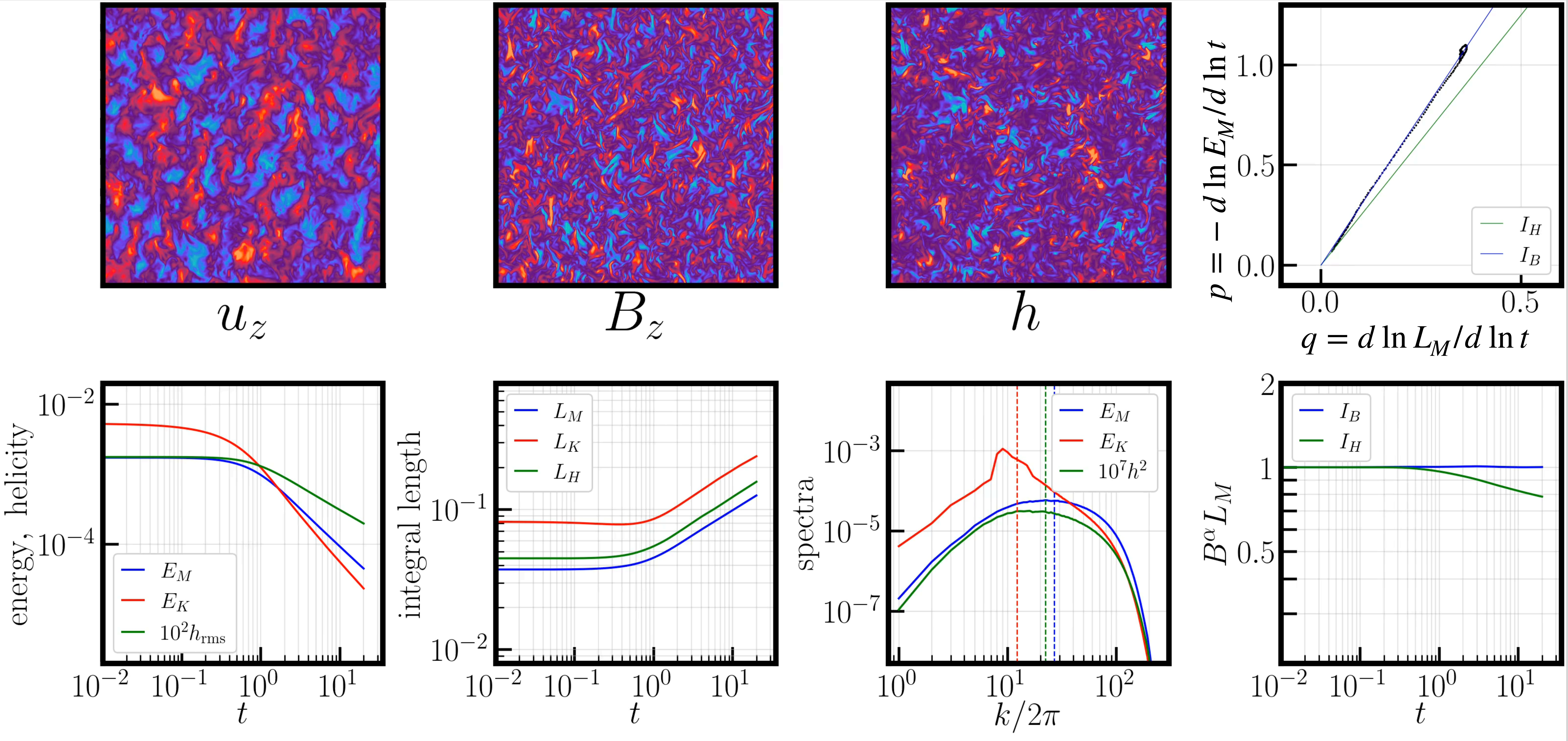
Spectra show evidence that magnetic folds unwrap and drive forcing-scale flows $(\tau_A < \tau_{\rm rec})$



Decay phase: governing invariants

576^3, $\mathrm{Rm}_4 \approx 1.4\mathrm{E}6$, $\mathrm{Pm} = 1$, $n_F \in (8, 16)$, 4th-order hyperdissipation

Diagnostics show strong evidence for I_B-governed decay prior to structures hitting the box scale



Decay phase: governing invariants

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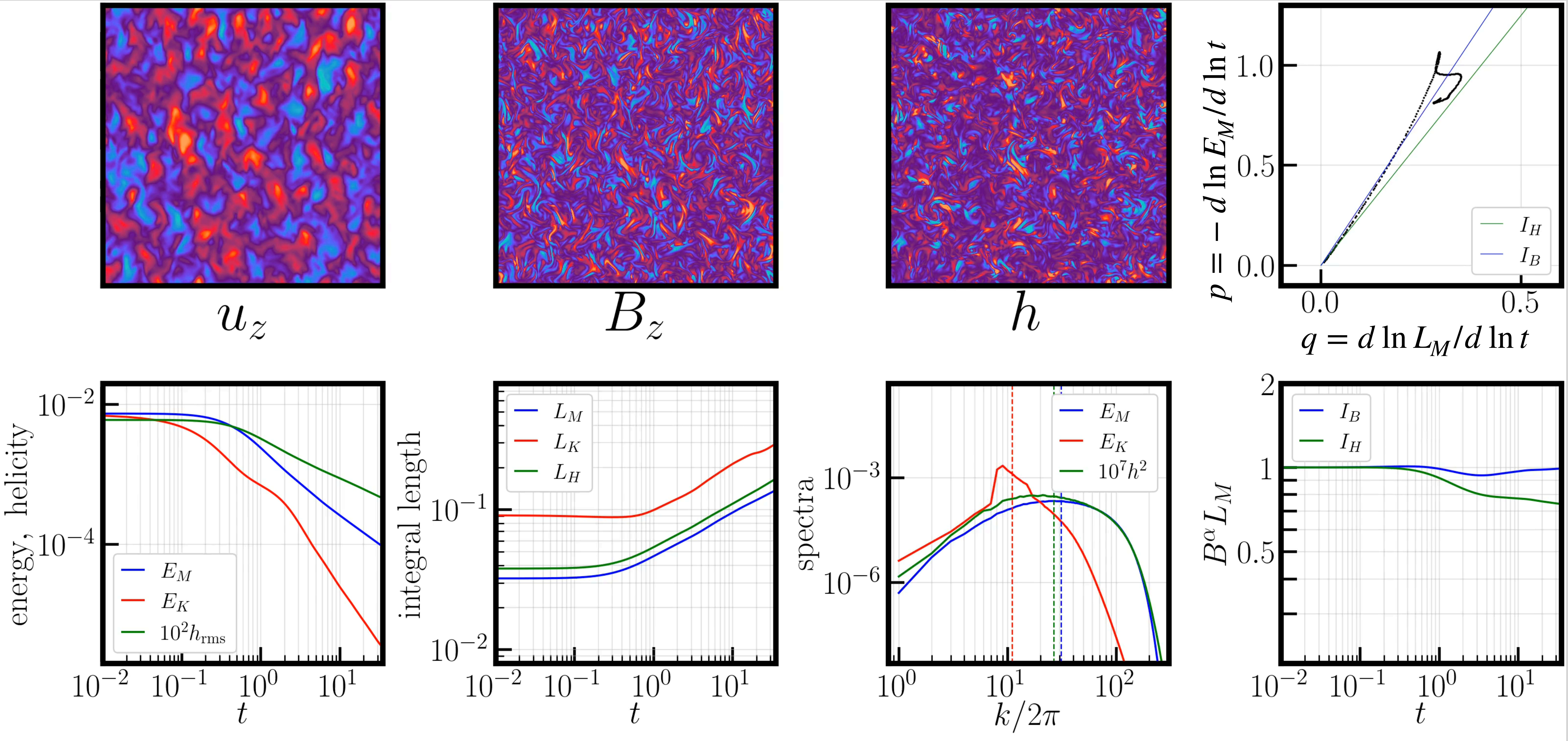


Decay phase: late-time decay must be I_H-governed?

Figure courtesy of Hosking & Schekochihin (2023)

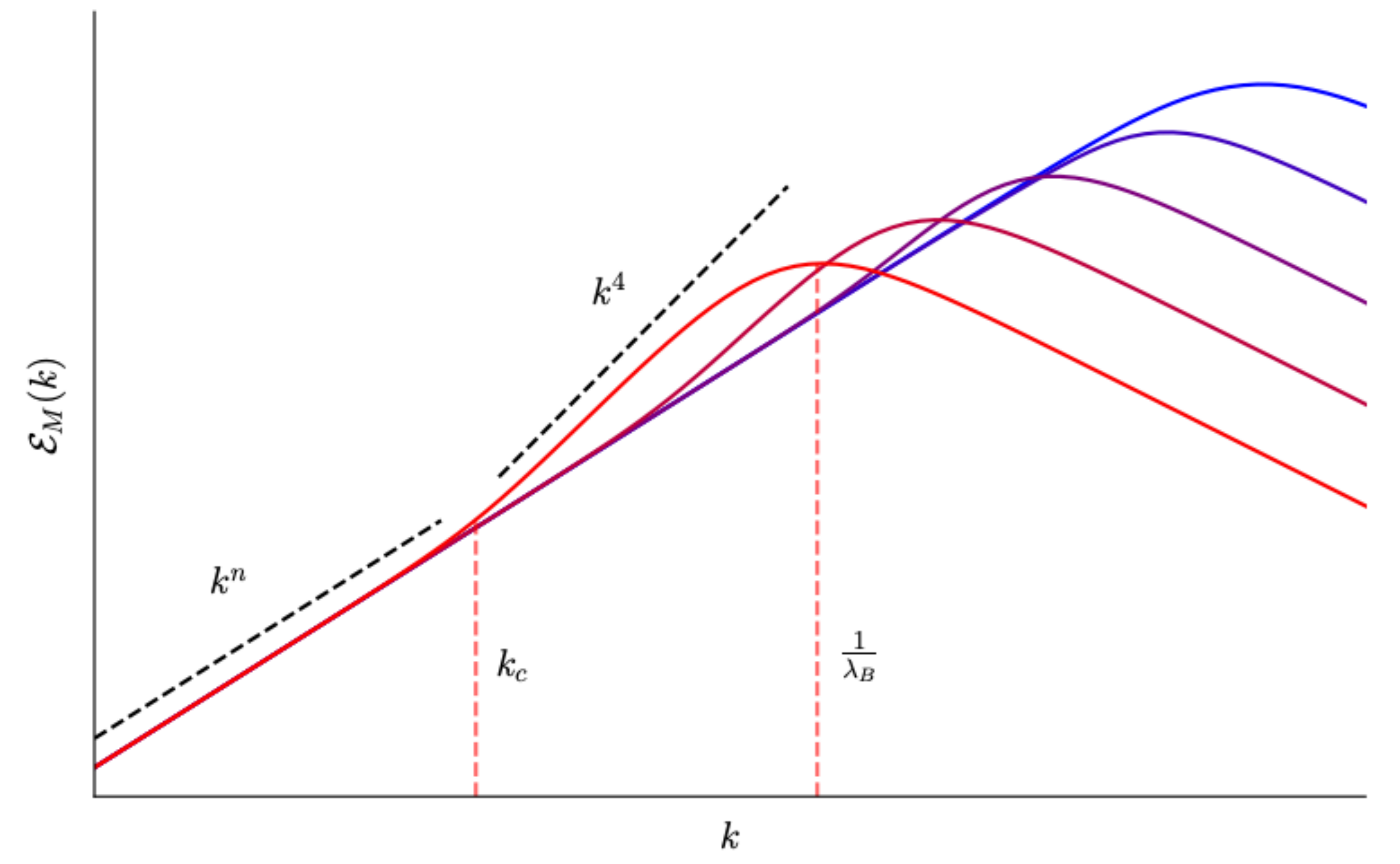


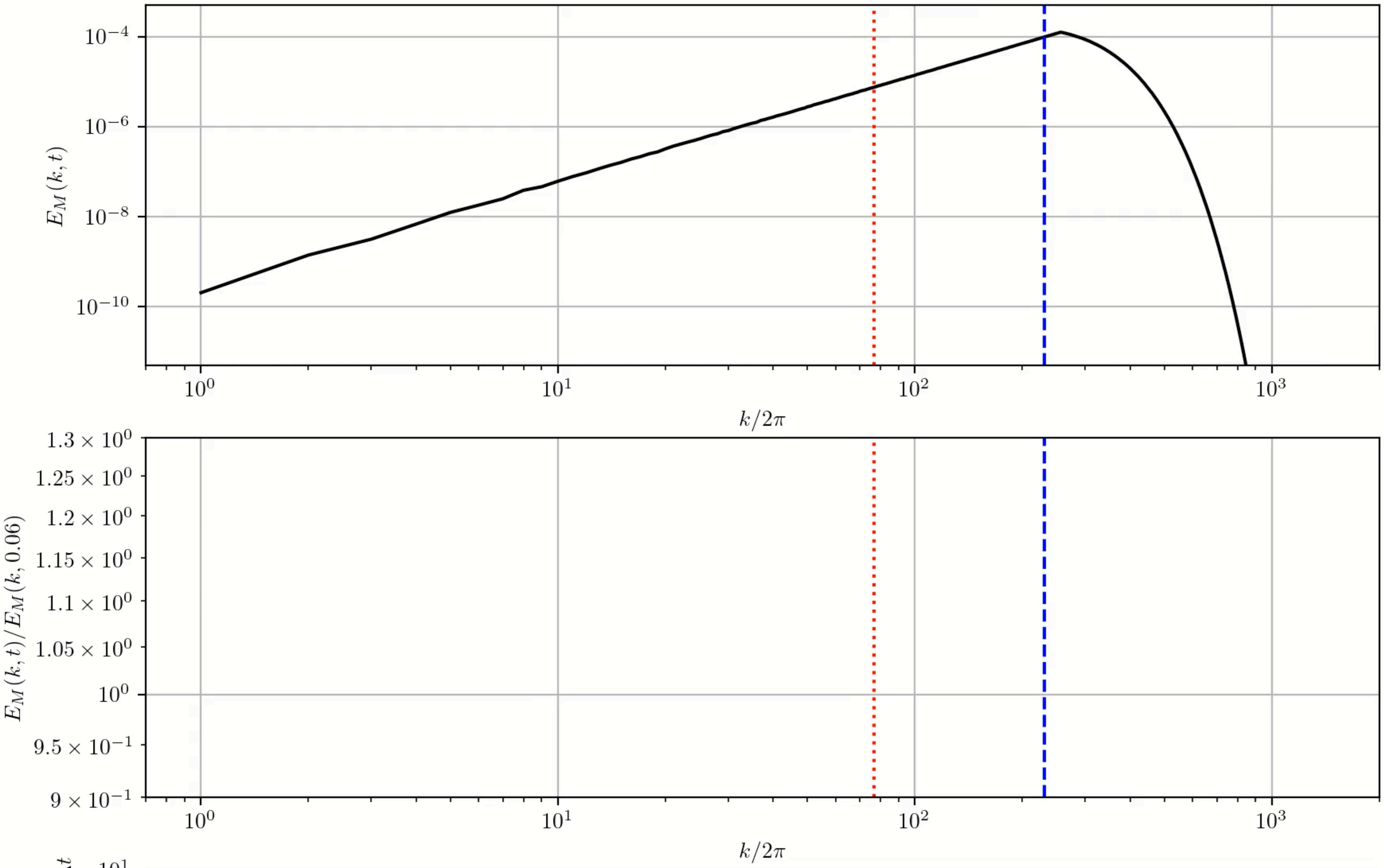
Figure 6. **Schematic of the evolution of $\mathcal{E}_M(t,k)$ for $\mathcal{E}_M(t=0,k\to 0)\propto k^n$.** For $n \leq 3$, the $k \to 0$ asymptotic of $\mathcal{E}_M(t,k)$ is preserved as the turbulence decays; this is a consequence of magnetic-flux conservation. Nonetheless, the inverse-transfer effect persists, though it only occurs for $k > k_c(t)$, where k_c is given by Eq. (77). The progression of time is from blue to red in this figure.

HS23 argues that net flux in structures is dissipated faster than net helicity in structures

Given a sufficiently wide infrared range, I_B-governed decay could transition to I_H-governed decay

If astrophysical dynamos develop fluxy structures, is the infrared range sufficiently wide to dissipate this flux?

If not, how can energy be inverse-transferred to large scales?


10^{-4}
10^{-6}
10^{-8}
10^{-10}
$E_M(k,t)$
10^0
10^1
10^2
10^3
$k/2\pi$
1.3×10^0
1.25×10^0
1.2×10^0
1.15×10^0
1.1×10^0
1.05×10^0
10^0
9.5×10^{-1}
9×10^{-1}
$E_M(k,t)/E_M(k,0.06)$
10^0
10^1
10^2
10^3
$k/2\pi$

The story so far...

1. A field generated from a turbulent dynamo (with non-intermittent, spatially homogeneous forcing) develops fluxy structures at saturation.

2. At saturation, the field is organized into magnetic folds under tension. Once forcing ceases, $E_K(k_F)$ decays rapidly and equilibrates with $E_M(k_F)$. Once $E_M(k_F) \sim E_K(k_F)$, the magnetic folds unwrap and drive forcing-scale flows.

3. Once folds unwrap, the decayed dynamo field appears qualitatively similar to a decayed random field. Thus, the decay of a random field could inform the decay of a dynamo field at this stage.

4. It appears that decay I_B governs the decay of (saturated) dynamo fields prior to structures hitting the box scale. However, given a large enough box, the decaying field might become uncorrelated and transition to I_H-governed decay.

5. Supplementary high-res simulations of a decaying random field show some evidence for a transition from I_B-governed decay to I_H-governed decay.