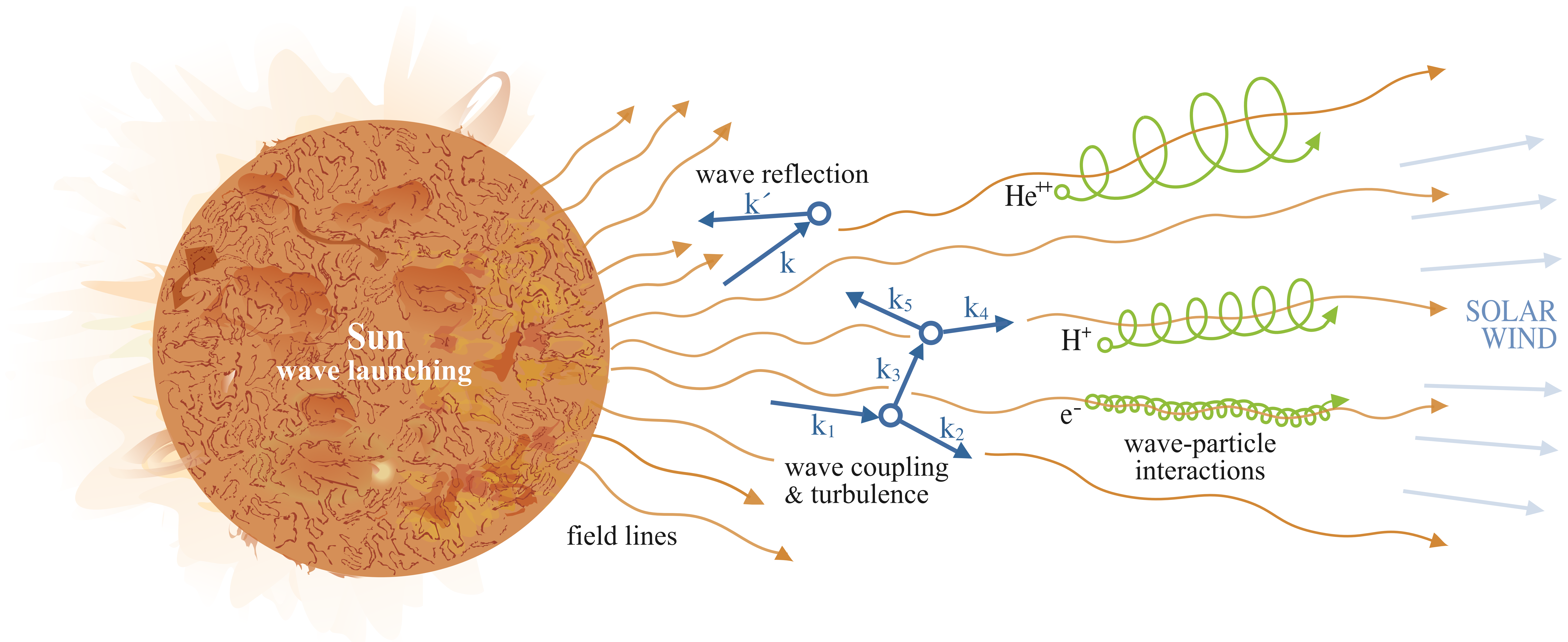
Intermittency in Reflection-Driven Turbulence

B. Chandran, N. Sioulas, S. Bale, T. Bowen, V. David, R. Meyrand, E. Yerger, JPP, 91, E57 (2025)

16th Plasma Kinetics Working Meeting, Vienna, July, 2025

Solar-Wind Energization by Turbulent Alfvén Waves

(E.g., Parker 1965, Velli et al 1989, Zhou & Matthaeus 1989, Matthaeus et al 1999, Cranmer et al 2007, Halekas et al 2023)



- Photospheric motions and/or reconnection launch Alfven waves along open magnetic-field lines
- These outward-propagating waves undergo partial reflection
- Counter-propagating waves interact to produce turbulence, which causes wave energy to ‘cascade’ from long wavelengths to short wavelengths
- Short-wavelength waves dissipate, heating the plasma.

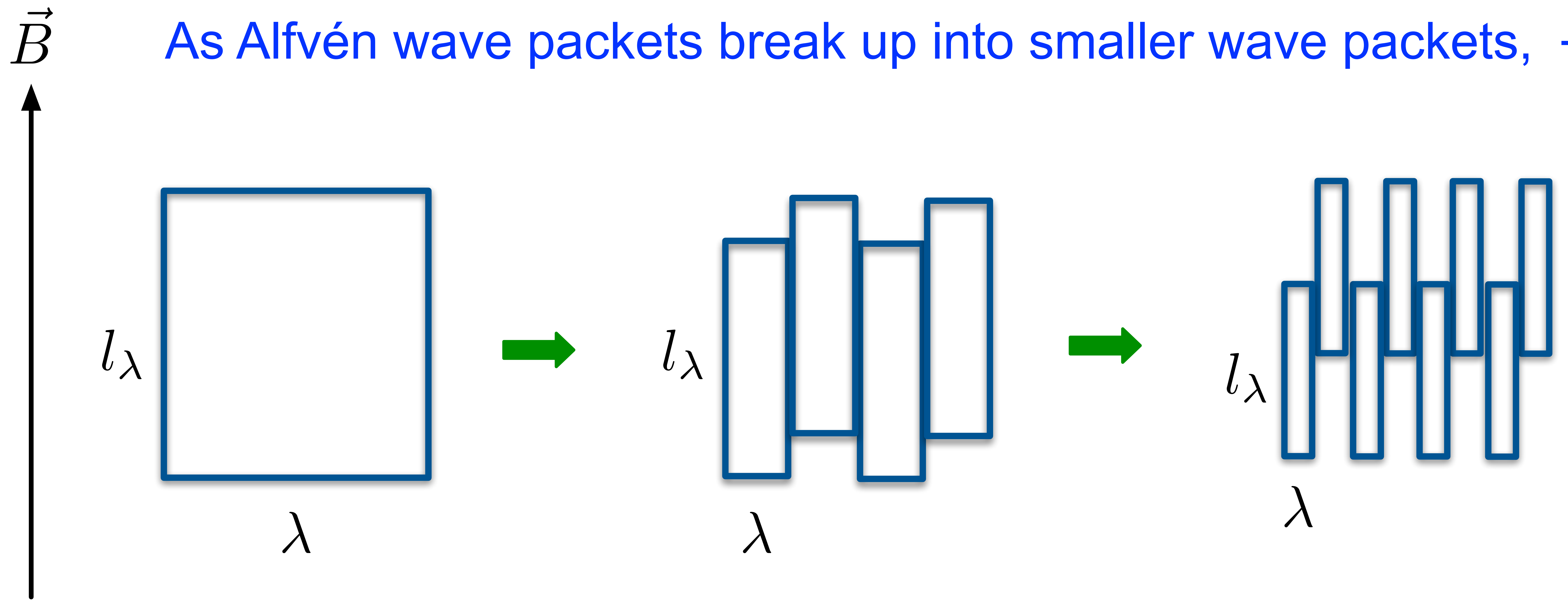
Elsasser Variables

- $$\boldsymbol{z}^{\pm} = \delta\boldsymbol{v} \pm \frac{\delta\boldsymbol{B}}{\sqrt{4\pi\rho}} \qquad \boldsymbol{B} = \boldsymbol{B}_0 + \delta\boldsymbol{B} \qquad \boldsymbol{v}_{\mathrm{A}} = \frac{\boldsymbol{B}_0}{\sqrt{4\pi\rho}}$$
- $\boldsymbol{z}^{+}$ represents Alfvén-wave (AW) fluctuations that propagate at velocity $-\boldsymbol{v}_{\mathrm{A}}$
- $\boldsymbol{z}^{-}$ represents AW fluctuations that propagate at velocity $\boldsymbol{v}_{\mathrm{A}}$
- $$\frac{\partial \boldsymbol{z}^{\pm}}{\partial t} + \left(\mp\boldsymbol{v}_{\mathrm{A}} + \boldsymbol{z}^{\mp}\right)\cdot\nabla\boldsymbol{z}^{\pm} = -\,\nabla\Pi, \qquad \Pi = \left(p + B^2\right)/\rho$$
- Only counter-propagating AWs interact
- $\boldsymbol{z}^{\pm}$ follows the field lines of $\boldsymbol{B}_0$ plus the part of $\delta\boldsymbol{B}$ from $\delta\boldsymbol{z}^{\mp}$. This is the nonlinear interaction (Maron & Goldreich 2001).

Anisotropic Energy Cascade in Alfvénic Turbulence

(E.g., Montgomery & Turner 1981, Shebalin et al 1983, Goldreich & Sridhar 1995, Chen et al 2012, Sioulas et al 2024)

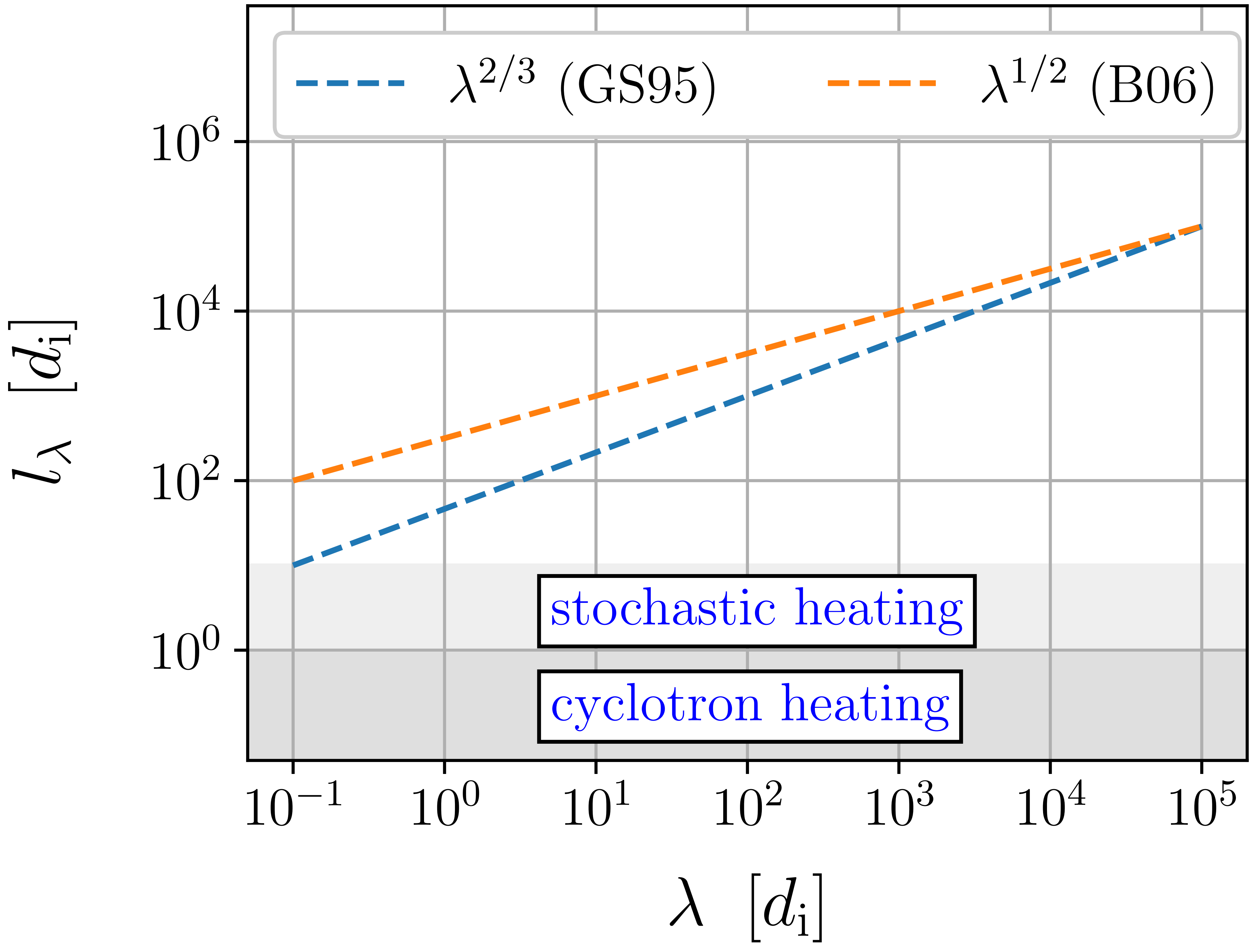
As Alfvén wave packets break up into smaller wave packets, $\frac{l_\lambda}{\lambda}$ increases



Two things we need to know to figure out how turbulence dissipates at small λ:

1. The anisotropy ratio $\frac{l_\lambda}{\lambda}$. Determines the characteristic frequency $k_\| v_A \sim \frac{v_A}{l_\lambda}$ at small λ
2. The fluctuation amplitudes — determines rates of nonlinear heating mechanisms like stochastic ion heating. (1 and 2 are related by critical balance.)

Goldreich & Sridhar 1995 (GS95) vs Boldryev 2006 (B06)



GS95: $$\frac{l_\lambda}{v_A} \sim \frac{\lambda}{\delta z_\lambda}$$

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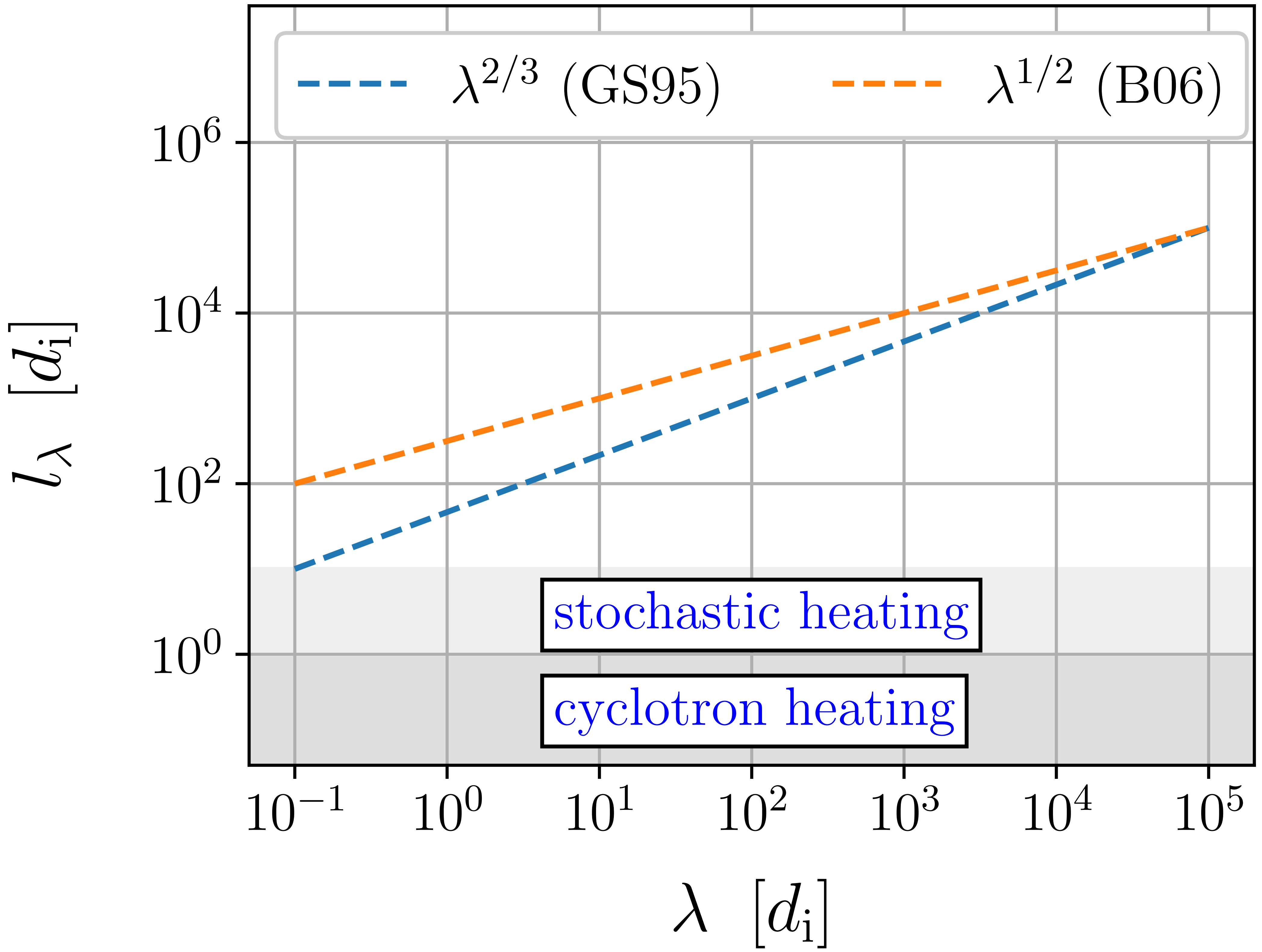

GS95: $\dfrac{l_\lambda}{v_A} \sim \dfrac{\lambda}{\delta z_\lambda}$

$$\epsilon \sim \frac{(\delta z_\lambda)^3}{\lambda} \propto \lambda^0 \longrightarrow \delta z_\lambda \propto \lambda^{1/3}$$

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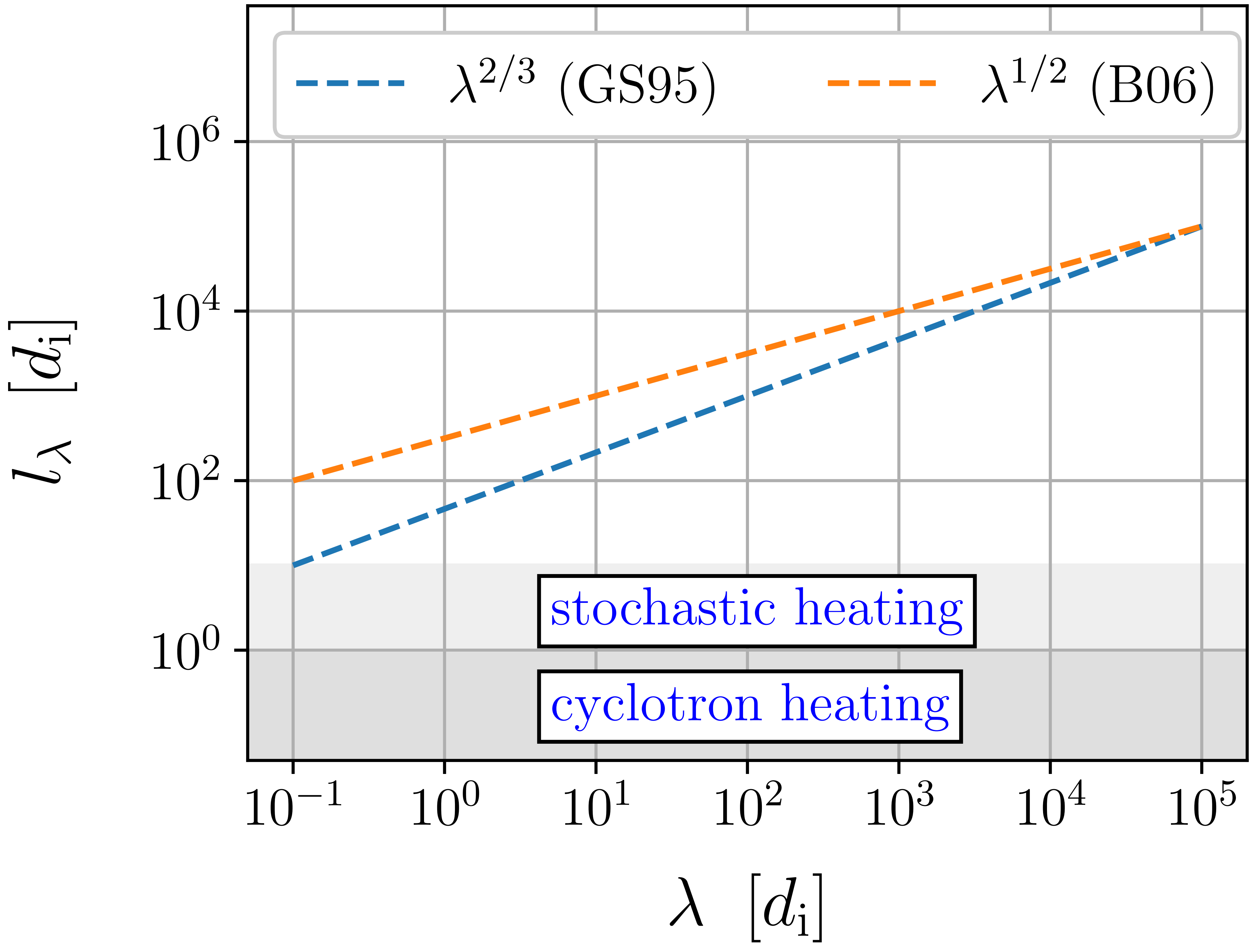

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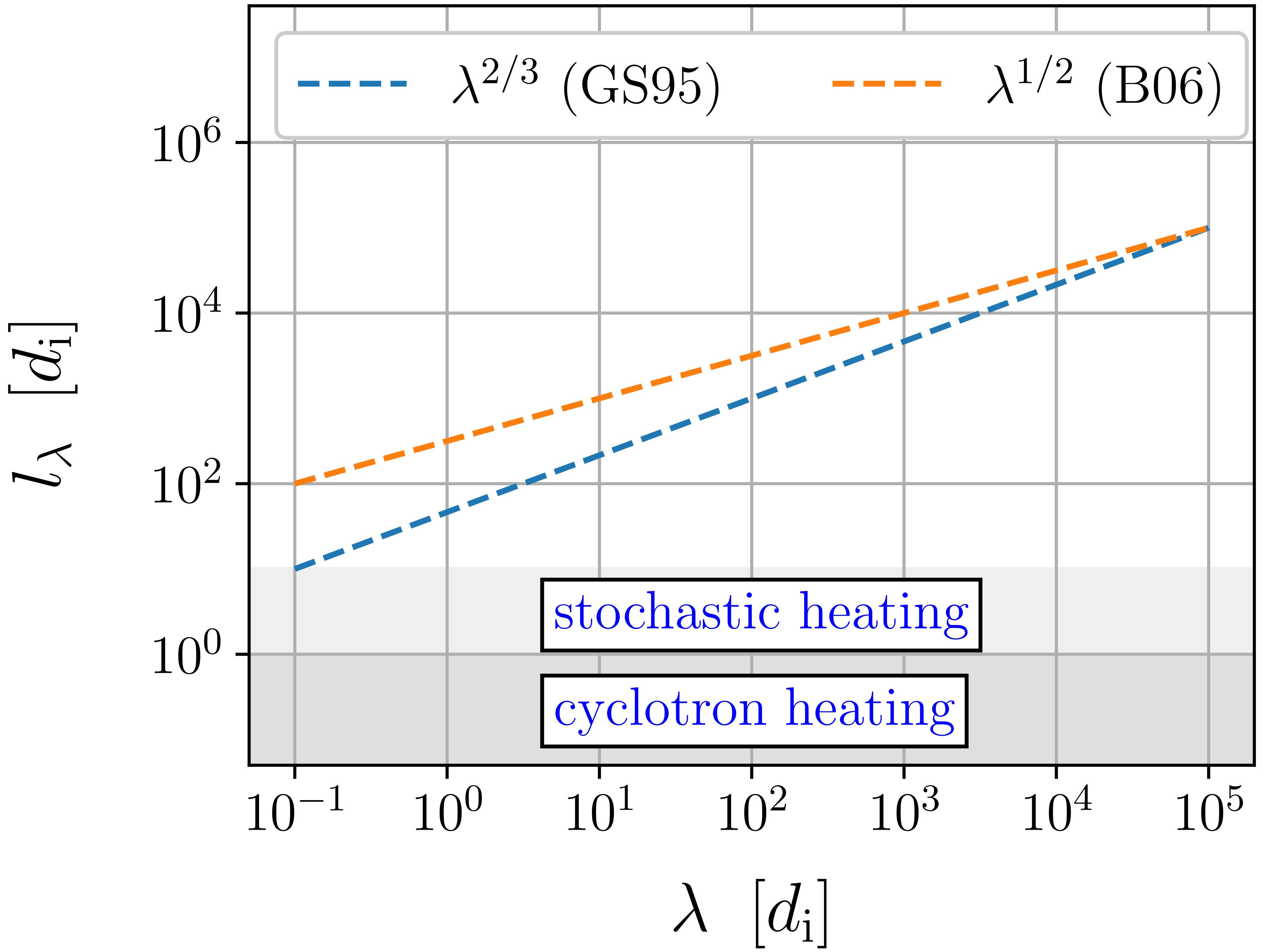

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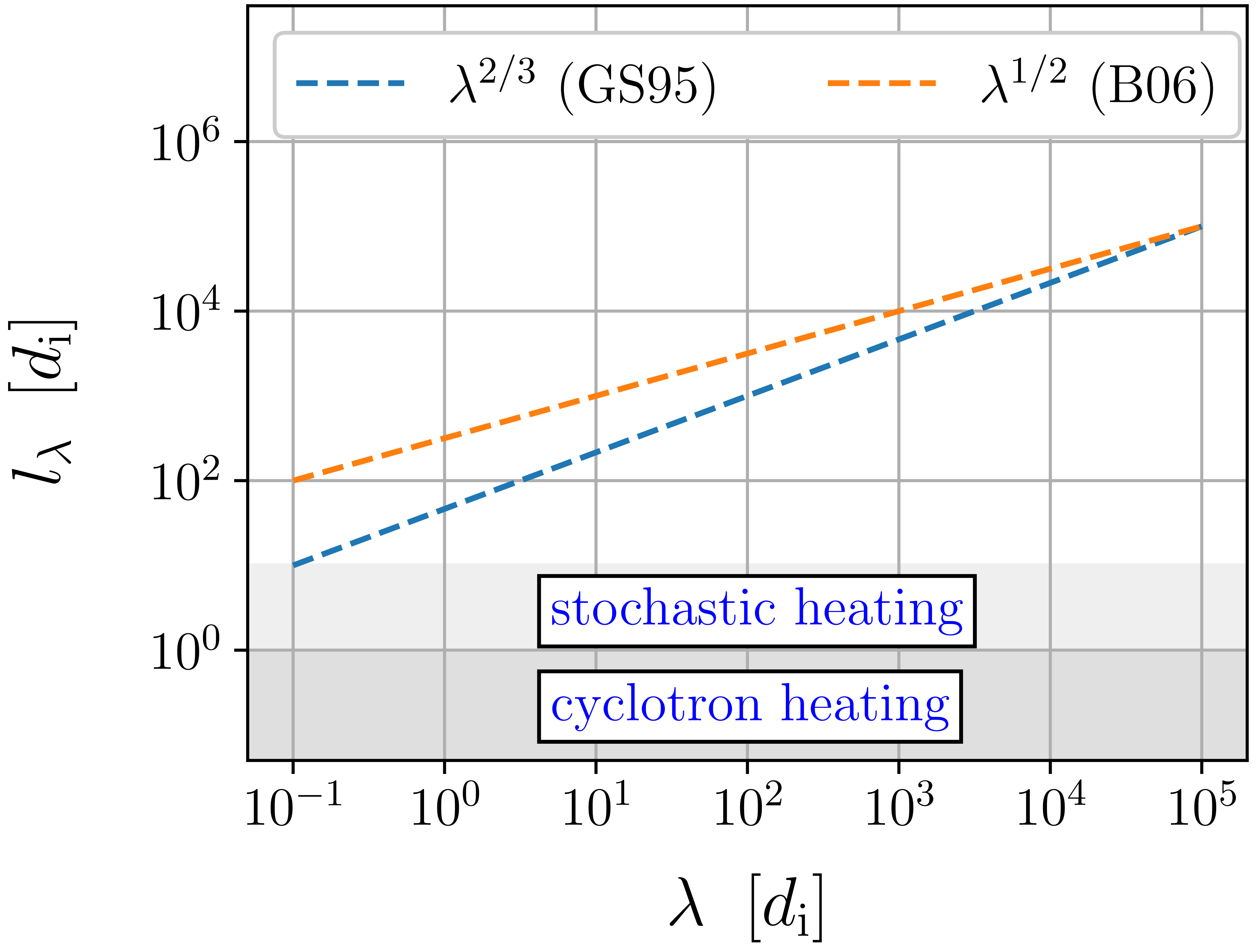

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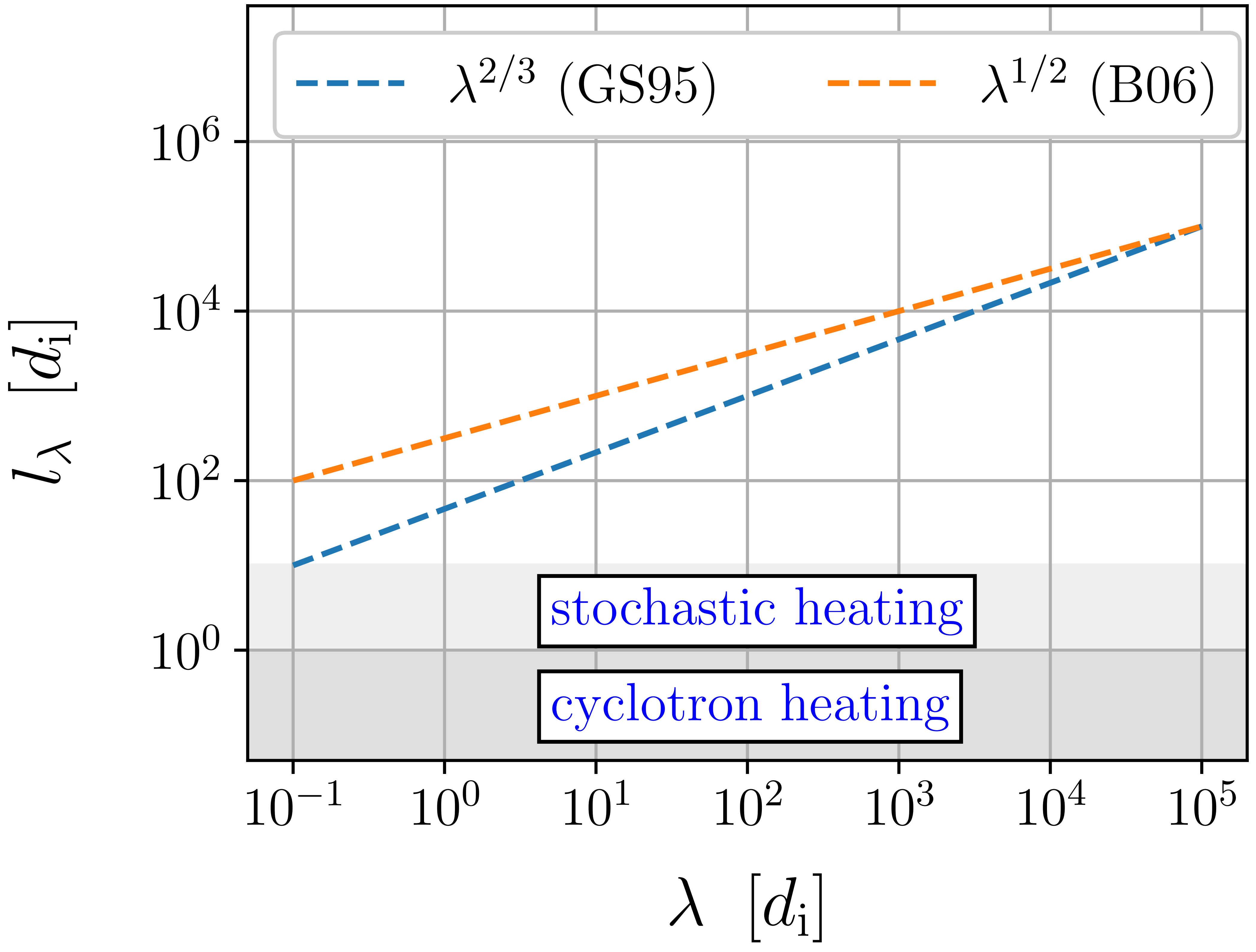

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Note: for Alfvén waves, if $\omega = k_\| v_A \sim \frac{v_A}{l_\lambda} \sim \Omega_i$, then $l_\lambda \sim \frac{v_A}{\Omega_i} = d_i$

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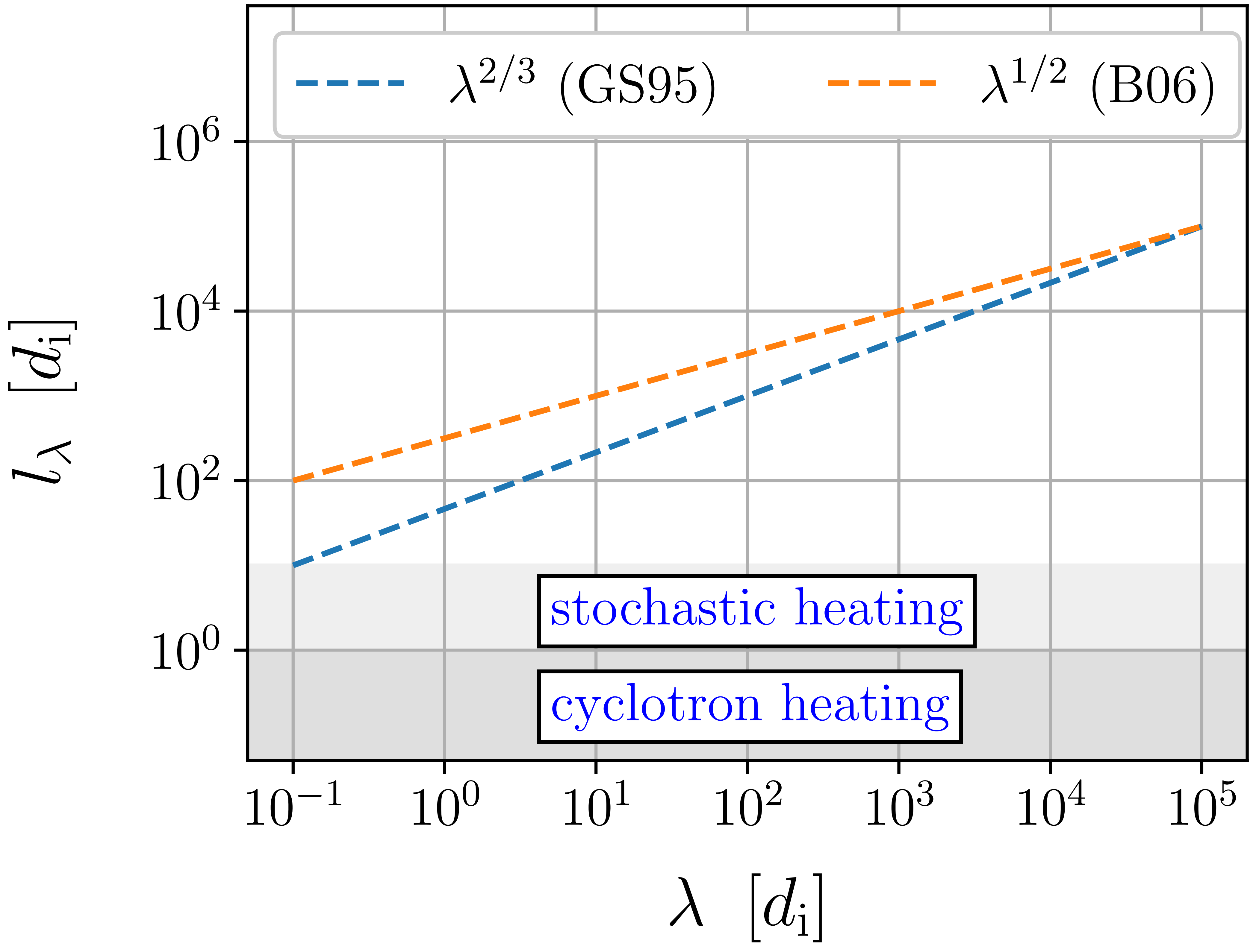
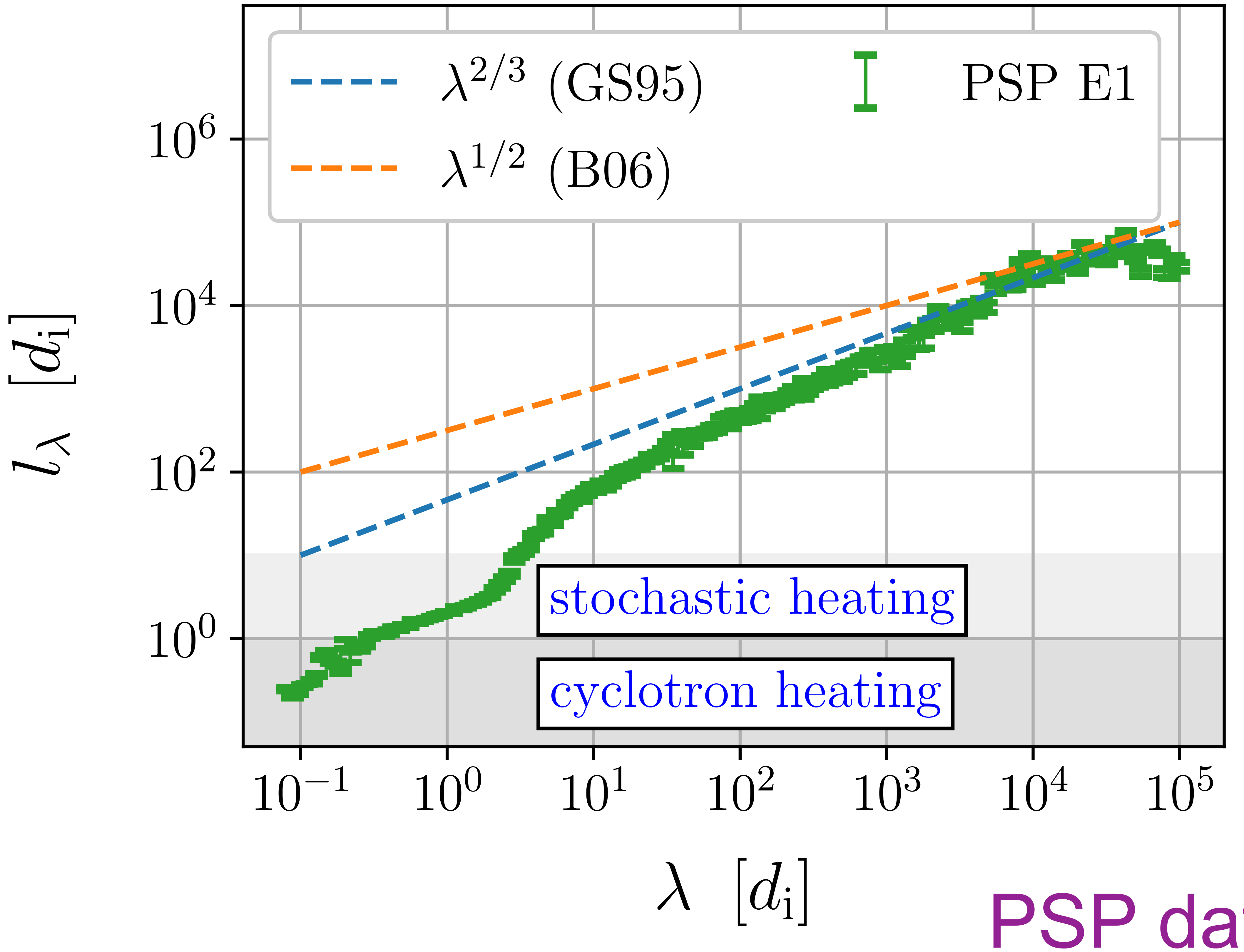


These theories are for balanced turbulence, in which $z^+_\mathrm{rms} \simeq z^-_\mathrm{rms}$.

What about imbalanced turbulence, in which $z^-_\mathrm{rms} \ll z^+_\mathrm{rms}$, near the Sun?

Note: for Alfvén waves, if $\omega = k_\parallel v_\mathrm{A} \sim \dfrac{v_\mathrm{A}}{l_\lambda} \sim \Omega_i$, then $l_\lambda \sim \dfrac{v_\mathrm{A}}{\Omega_\mathrm{i}} = d_\mathrm{i}$

What Parker Solar Probe (PSP) Tells Us About Anisotropy



PSP data from Nikos Sioulas

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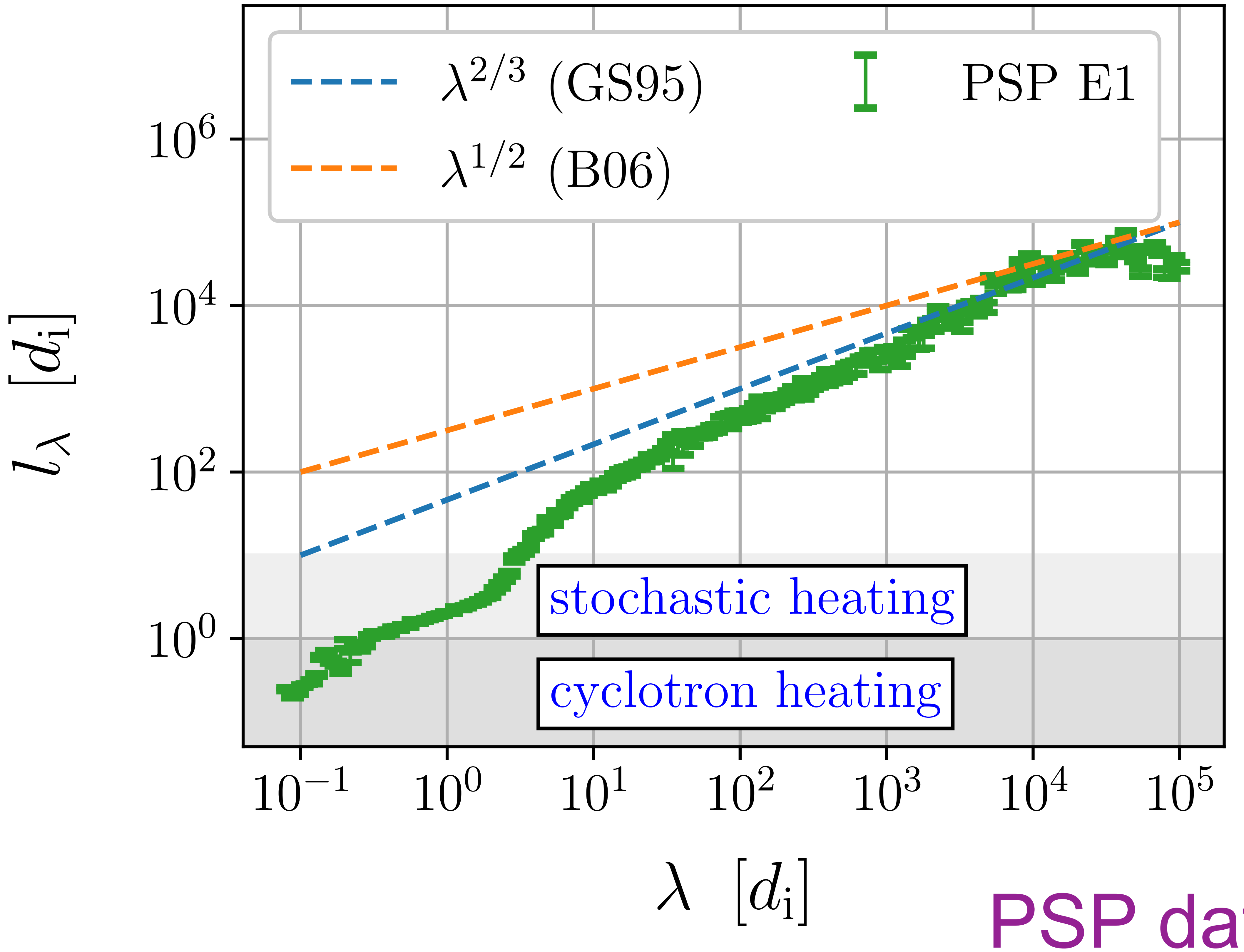
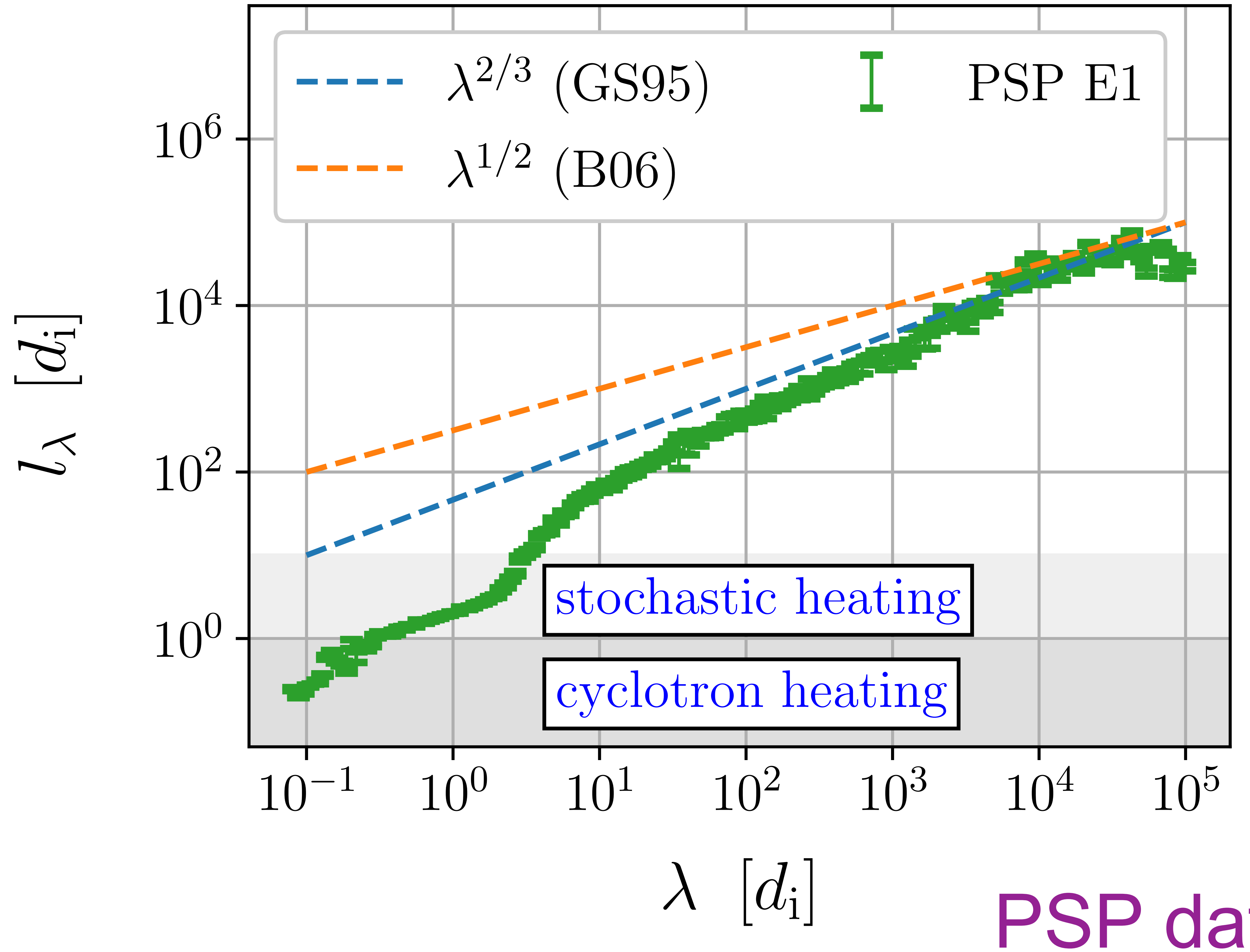
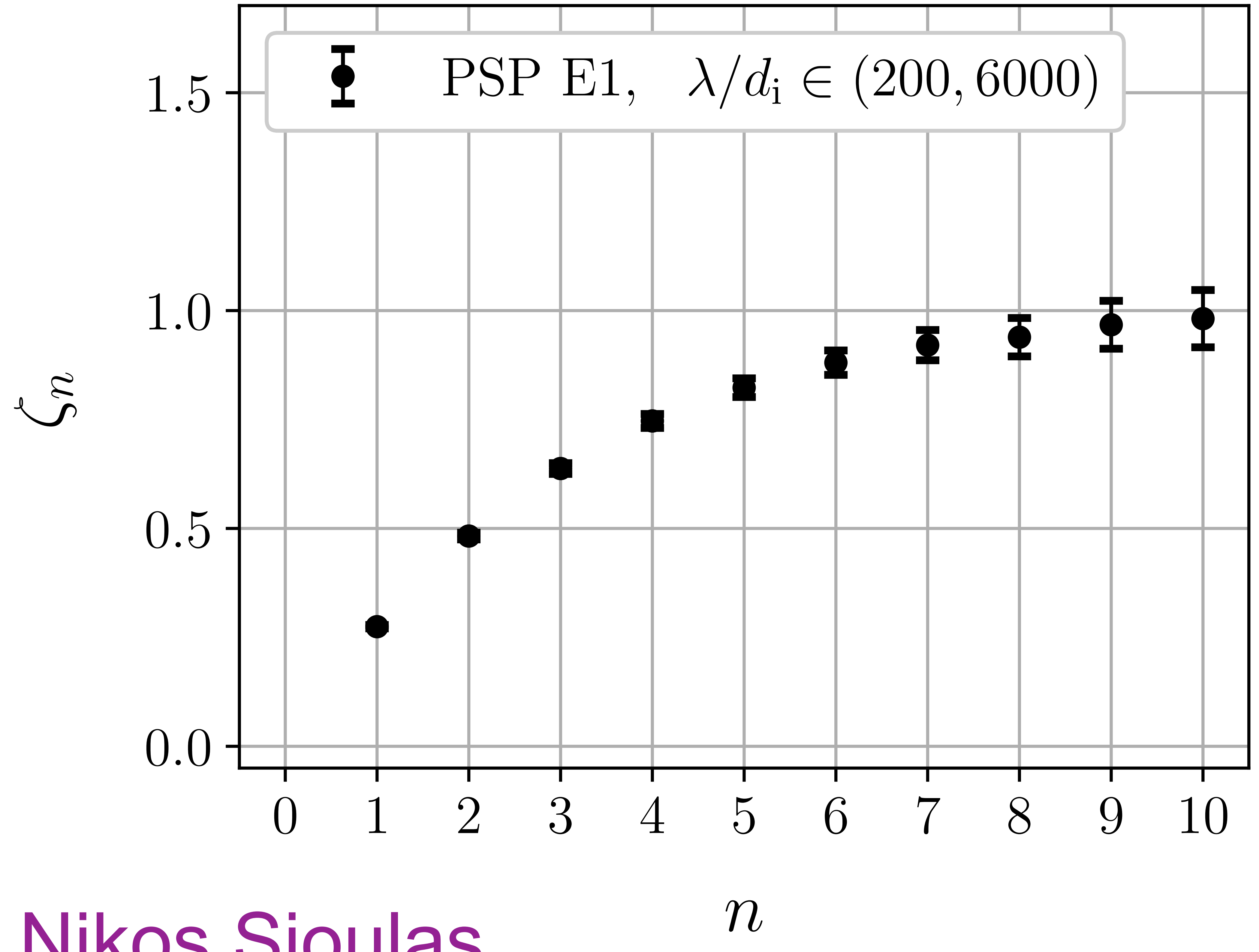


PSP data from Nikos Sioulas

- ⟶ we need a new turbulence model for the near-Sun solar wind

Hint from PSP: Solar-Wind Turbulence is Also Intermittent

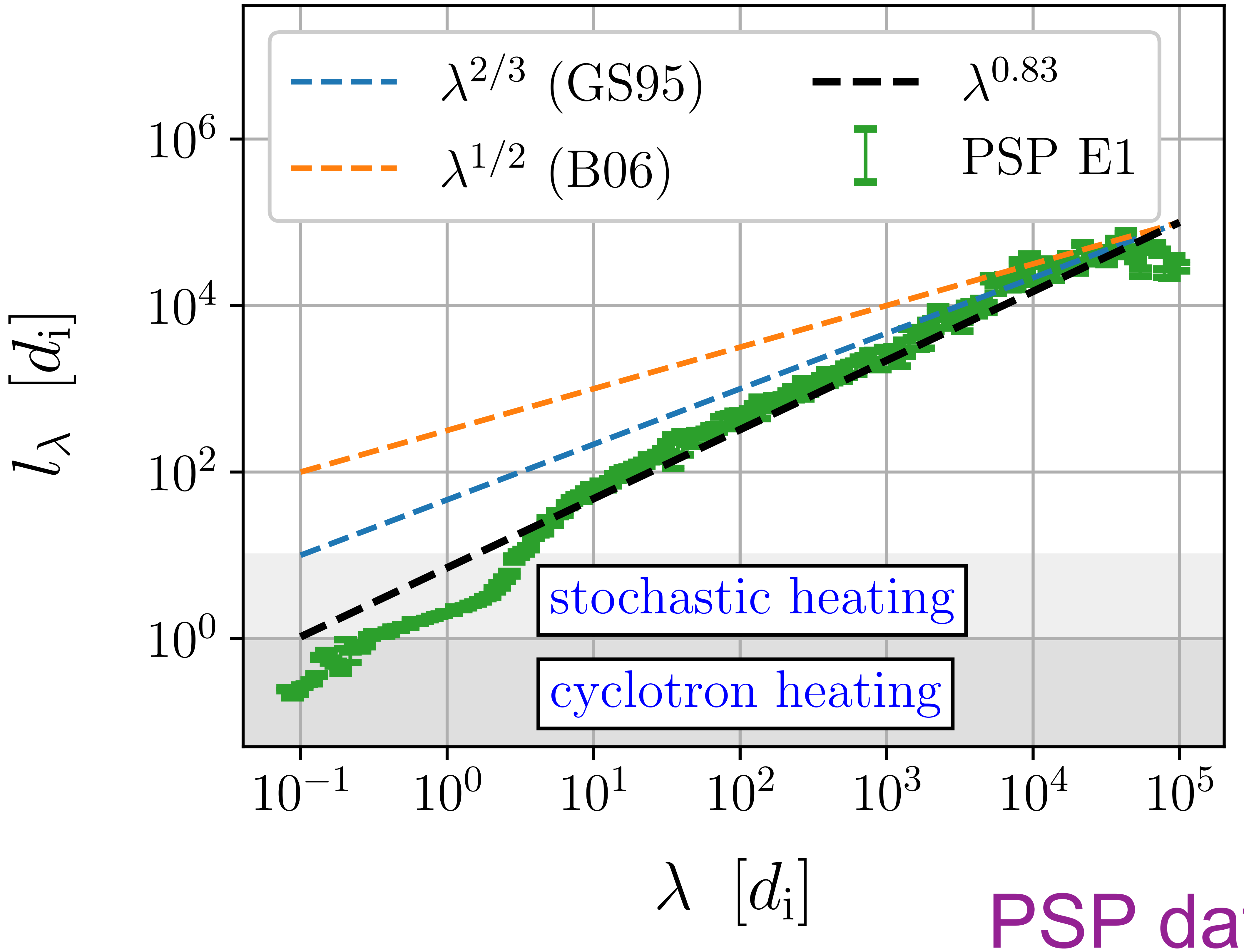


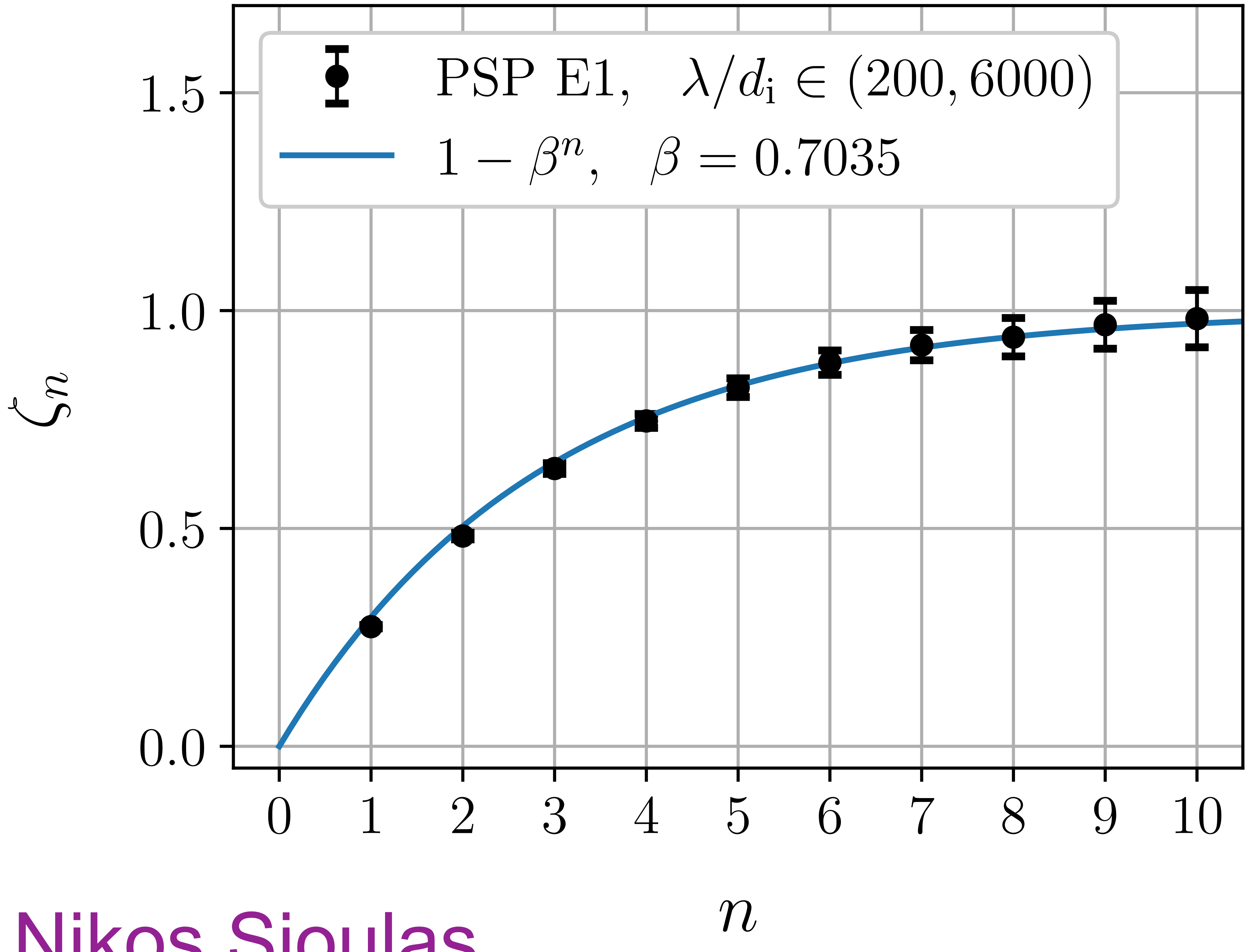


PSP data from Nikos Sioulas

- $\longrightarrow$ we need a new turbulence model for the near-Sun solar wind

What PSP Tells Us About Anisotropy and Intermittency





PSP data from Nikos Sioulas

- ⟶ we need a new turbulence model for the near-Sun solar wind
- We have developed such a model that offers an explanation for these observations

Model for the Probability Distribution Function of $\delta z_\lambda^\pm$

$$\delta z_\lambda^\pm = \overline{z}^\pm \beta^q$$

$\beta \in (0,1)$ is a constant, q is a random integer, $\overline{z}^\pm$ is a scale-independent random number that is independent of q,

$$P(q) = \frac{e^{-\mu}\mu^q}{q!}$$

Poisson distribution with mean μ, which is a function of λ

These assumptions are a little like assuming a power-law form for the energy spectrum. We need some model for how the PDF of fluctuation amplitudes broaden as λ decreases. Note: we don't require the full PDF to be log-Poisson.

To complete the model, we need to evaluate $\mu(\lambda)$ and β

Same approach in Chandran, Schekochihin, & Mallet 2015 (CSM15), Mallet & Schekochihin 2017 (MS17). Similar approach in She & Leveque (1994) and Dubrulle (1995).

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Poisson distribution with mean μ, which is a function of λ

$$P(0) = e^{-\mu} \propto \lambda$$

filling factor of strongest fluctuations is $\propto \lambda$. Same as in CSM15 and MS17.

$$\longrightarrow \quad \mu = \ln\left(\frac{L_\perp}{\lambda}\right)$$

$L_\perp$ is the perpendicular outer scale

$$w_\lambda^\pm = \left\langle \overline{z}^\pm \right\rangle \beta^\mu \propto e^{\mu \ln\beta} \propto \lambda^{-\ln\beta}$$

the median or typical fluctuation amplitude at scale λ

Higher-Order Structure Functions

$$\delta z_\lambda^\pm = \bar{z}^\pm \beta^q$$

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$$\delta z_\lambda^\pm = \overline{z}^\pm \beta^q \qquad P(q) = \frac{e^{-\mu}\mu^q}{q!}$$

$$\left\langle \left(\delta z_\lambda^\pm\right)^n \right\rangle = \int_0^\infty \mathrm{d}\overline{z}^\pm\, Q(\overline{z}^\pm) \sum_{q=0}^{\infty} P(q)[\overline{z}^\pm \beta^q]^n$$

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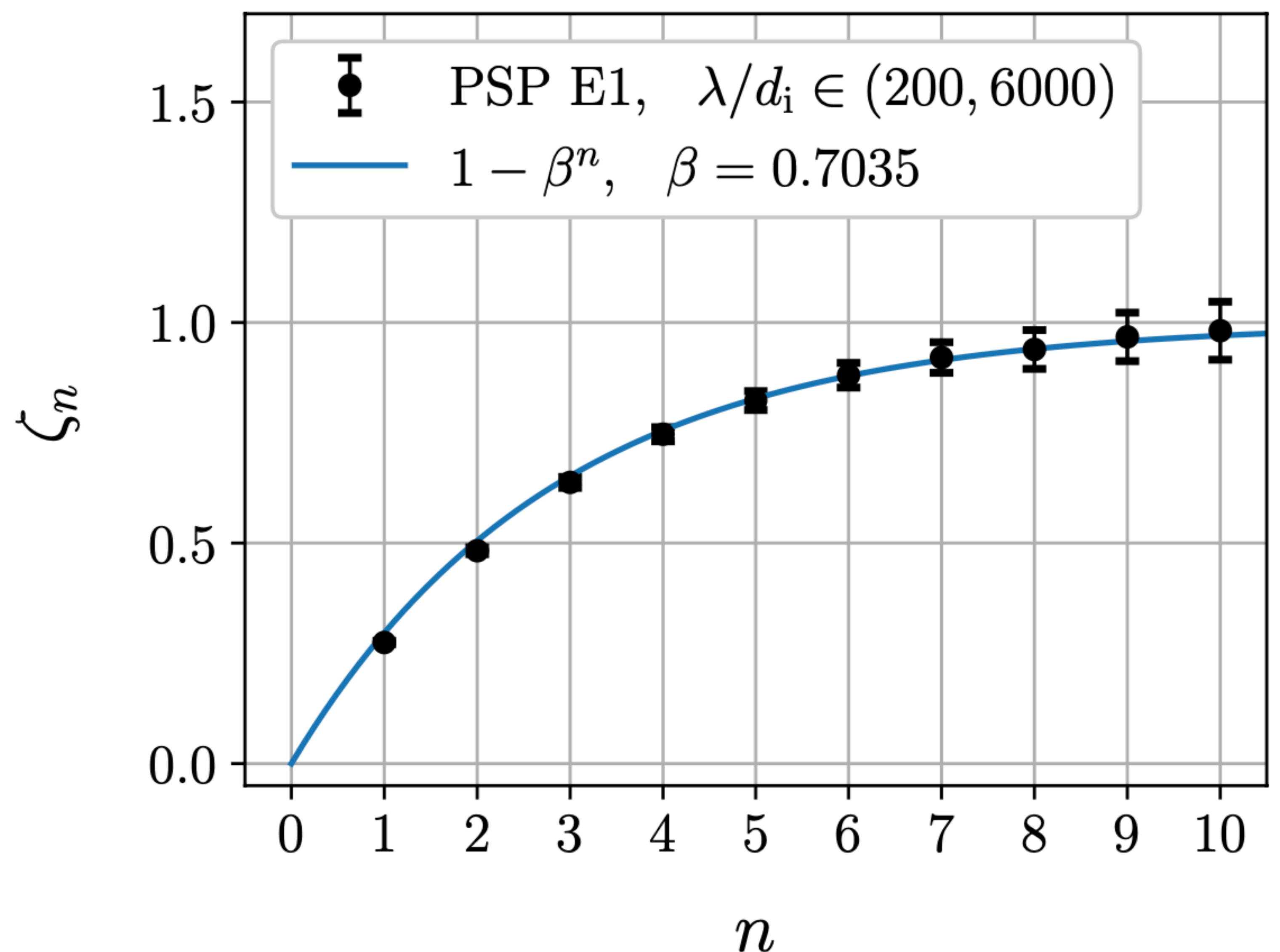
$$e^{-\mu} \propto \lambda \quad \longrightarrow \quad \left\langle \left(\delta z_\lambda^\pm\right)^n \right\rangle \propto \lambda^{\zeta_n} \quad \text{with} \quad \zeta_n = 1 - \beta^n$$

Same formula as in CSM15 and MS17.

Data from Sioulas et al (2024)



Lithwick, Goldreich, & Sridhar (LGS) 2007 Model of Strong Imbalanced MHD Turbulence

Assume $\delta z^+_{\rm rms} \gg \delta z^-_{\rm rms}$ and that $\chi^+_\lambda \equiv \dfrac{\delta z^+_\lambda l^+_\lambda}{\lambda v_{\rm A}} \gtrsim 1,$ where $l^\pm_\lambda$ is the parallel correlation length of a fluctuation of perpendicular correlation length λ.

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l_λ^- is the distance δz_λ^- propagates before cascading: $l_\lambda^- \sim \dfrac{v_{\rm A}\lambda}{\delta z_\lambda^+}$

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l^-_λ is the distance δz^-_λ propagates before cascading: $l^-_\lambda \sim \dfrac{v_{\rm A}\lambda}{\delta z^+_\lambda}$

δz^+_λ is cascaded by δz^-_λ: $\rightarrow l^+_\lambda \sim l^-_\lambda$

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$$\chi^+_\lambda = \frac{\delta z^+_\lambda l^+_\lambda}{\lambda v_{\rm A}} \sim 1$$

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$$\chi^+_\lambda = \frac{\delta z^+_\lambda l^+_\lambda}{\lambda v_{\rm A}} \sim 1 \qquad\qquad \chi^-_\lambda = \frac{\delta z^-_\lambda l^-_\lambda}{\lambda v_{\rm A}} \ll 1$$

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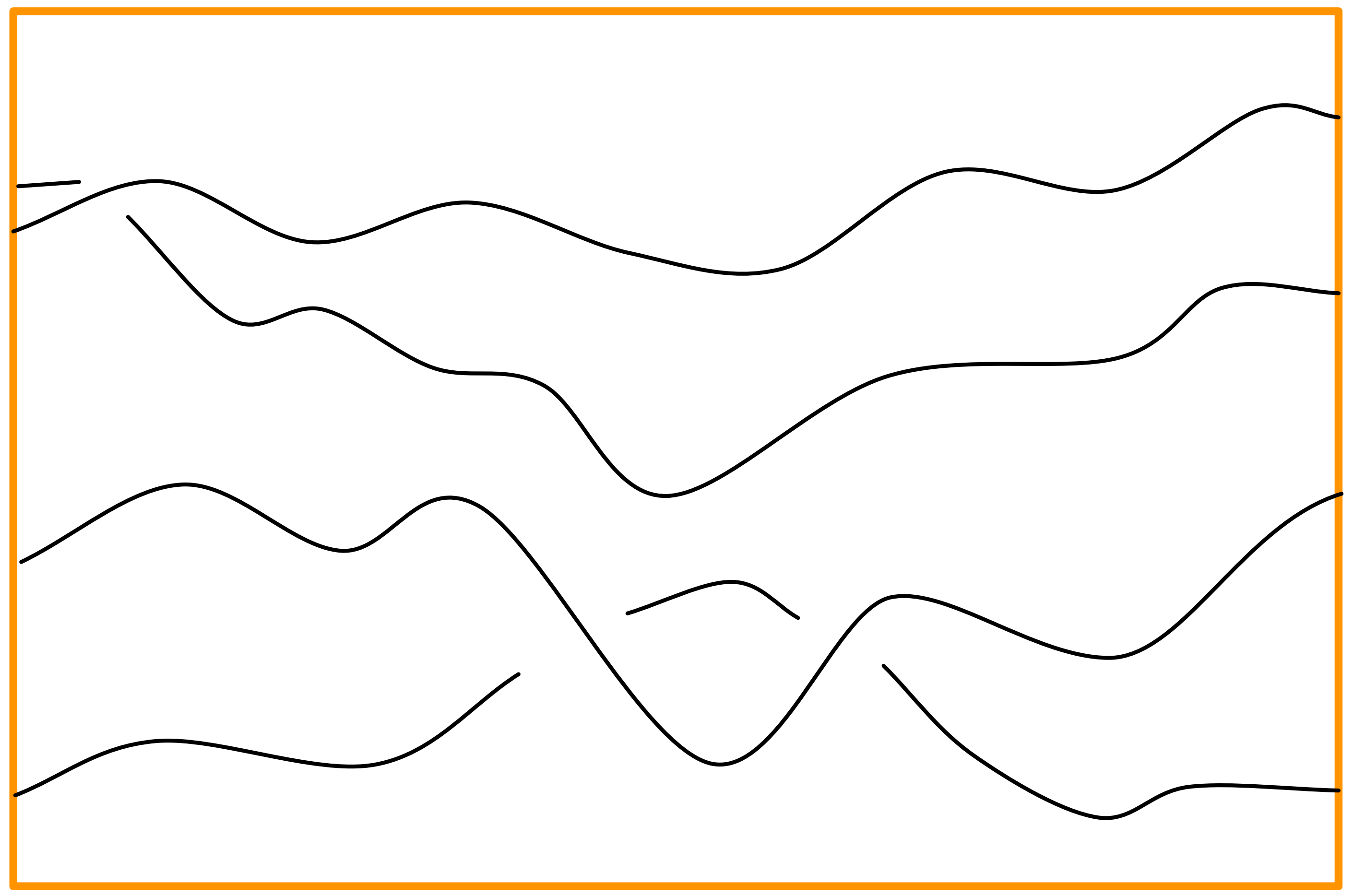
$\chi^+_\lambda = \dfrac{\delta z^+_\lambda l^+_\lambda}{\lambda v_{\rm A}} \sim 1$ $\chi^-_\lambda = \dfrac{\delta z^-_\lambda l^-_\lambda}{\lambda v_{\rm A}} \ll 1$ And yet $\tau^+_{{\rm casc},\lambda} \sim \dfrac{\lambda}{\delta z^-_\lambda}$ (!)

Why Is $\tau^{+}_{\mathrm{casc},\lambda} \sim \dfrac{\lambda}{\delta z^{-}_{\lambda}}$ Even Though $\chi^{-}_{\lambda} = \dfrac{\delta z^{-}_{\lambda} l^{-}_{\lambda}}{\lambda v_{\mathrm{A}}} \ll 1$?

LGS07 thought experiment: let z^{+} have a broadband power spectrum, but let z^{-} be infinitesimal and 'injected' (forced) with infinite coherence time in the `z^{+} frame' that propagates along $\boldsymbol{B}_0$ with the z^{+} fluctuations at speed v_{A}. Then the z^{-} vector field becomes time-independent in the z^{+} frame at all scales, and the (infinitesimal) shearing by z^{-} has an infinite coherence time in the z^{+} frame.



Why Is $\tau^+_{\mathrm{casc},\lambda} \sim \frac{\lambda}{\delta z^-_\lambda}$ Even Though $\chi^-_\lambda = \frac{\delta z^-_\lambda l^-_\lambda}{\lambda v_{\mathrm{A}}} \ll 1$?

Now, let z^- increase to a finite value but remain $\ll z^+$. Let z^- be injected with a coherence time, as measured in the z^+ frame, that is at least as long as the lifetime of the z^+ eddies at the forcing scale (`anomalous coherence'). How long do you have to wait until the shearing of a z^+ wave packet at scale λ by a z^- wave packet at scale λ changes at a fixed location in the z^+ frame?

Well, if the z^+ packet doesn't change, then the z^- wave packet doesn't change. So you have to wait until the z^+ wave packets change before the shearing that they experience changes. $\longrightarrow$ $\tau^+_{\mathrm{casc},\lambda} \sim \frac{\lambda}{\delta z^-_\lambda}$

NOTE: reflection-driven turbulence yields anomalous coherence

Energy Cascade Rate

$$\epsilon_\lambda^+ = \frac{\left(\delta z_\lambda^+\right)^2}{\tau_{\mathrm{casc},\lambda}^+}$$

In inertial range, $\left\langle \epsilon_\lambda^+ \right\rangle \propto \lambda^0$

$$\tau_{\mathrm{casc},\lambda}^+ \sim \frac{\lambda}{\delta z_\lambda^-} \quad \longrightarrow \quad \left\langle \frac{(\delta z_\lambda^+)^2 \delta z_\lambda^-}{\lambda} \right\rangle \propto \lambda^0$$

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Assertion: in the sub-volume that dominates $\left\langle \epsilon_\lambda^+ \right\rangle$, in which $\delta z_\lambda^+ > w_\lambda^+$, the driving of δz_λ^- is uniform, but the damping time scale of δz_λ^- is $\propto 1/\delta z_\lambda^+$.

$$\longrightarrow \delta z_\lambda^- \propto \frac{1}{\delta z_\lambda^+} \longrightarrow \delta z_\lambda^- = \frac{w_\lambda^+ w_\lambda^-}{\delta z_\lambda^+}$$

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$$\longrightarrow \delta z_\lambda^- \propto \frac{1}{\delta z_\lambda^+} \longrightarrow \delta z_\lambda^- = \frac{w_\lambda^+ w_\lambda^-}{\delta z_\lambda^+}$$

$$\left\langle \epsilon_\lambda^+ \right\rangle \sim \frac{\left\langle \delta z_\lambda^+ \right\rangle w_\lambda^+ w_\lambda^-}{\lambda} \propto \lambda^{-\beta - 2\ln\beta} \qquad \longrightarrow \beta = -2\ln\beta \qquad \longrightarrow \beta = 2W_0(1/2) = 0.7035$$

$$\longrightarrow \left\langle \left(\delta z_\lambda^+\right)^2 \right\rangle \propto \lambda^{1-\beta^2} = \lambda^{0.505} \qquad \longrightarrow E(k_\perp) \propto k_\perp^{-1.51}$$

Anisotropy of the Energetically Dominant Small-Scale Fluctuations

Recall: $\delta z_\lambda^\pm = \overline{z}^\pm \beta^q \qquad P(q) = \dfrac{e^{-\mu}\mu^q}{q!}$

$$\left\langle \left(\delta z_\lambda^\pm\right)^n \right\rangle = \left\langle \left(\overline{z}^\pm\right)^n \right\rangle e^{-\mu} \sum_{q=0}^{\infty} \frac{\left(\mu\beta^n\right)^q}{q!} = \left\langle \left(\overline{z}^\pm\right)^n \right\rangle e^{-\mu+\mu\beta^n} \qquad \left(\text{using } \sum_{q=0}^{\infty} \frac{x^q}{q!} = e^x\right)$$

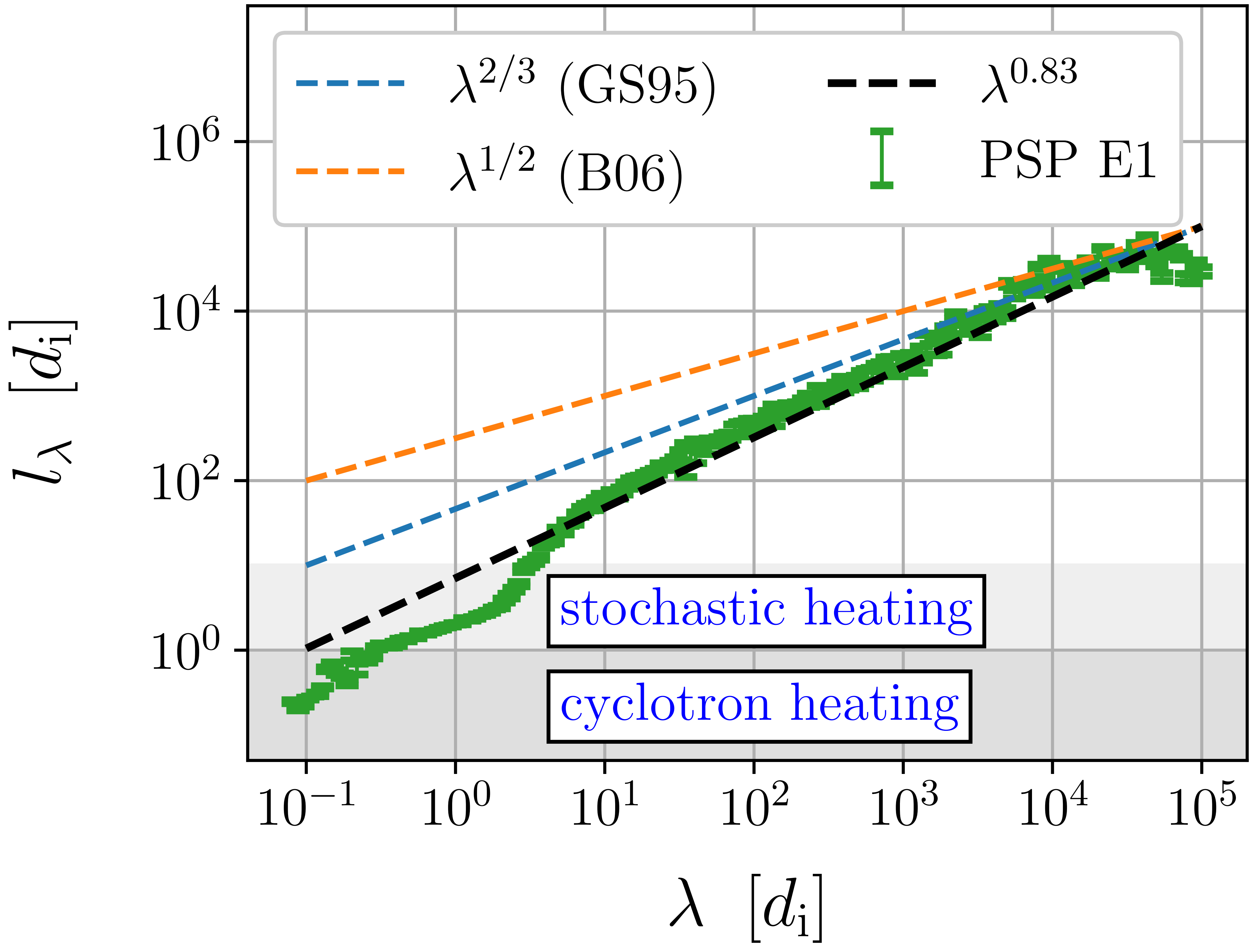
This sum is dominated by $q \simeq \mu\beta^n$, and by fluctuations with amplitude

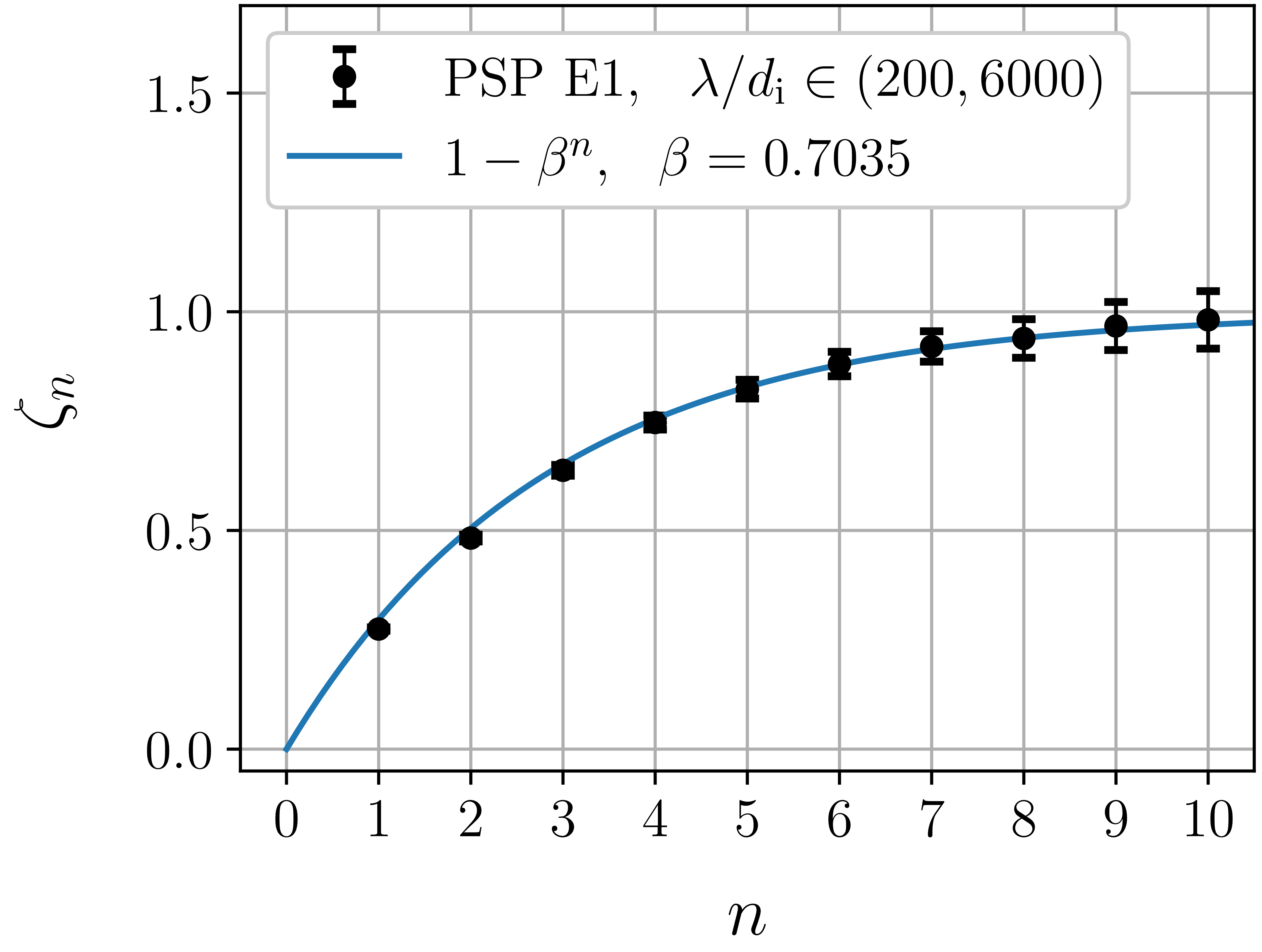
$\delta z_{(n),\lambda}^\pm = \overline{z}^\pm \beta^{\mu\beta^n} \propto \lambda^{-\beta^n \ln\beta}$ that have parallel correlation lengths $l_{(n),\lambda} = \dfrac{v_A \lambda}{\delta z_{(n),\lambda}^+}$

Recall: $e^{-\mu} \propto \lambda$

$$\longrightarrow \quad \delta z_{(2),\lambda}^\pm \propto \lambda^{-\beta^2 \ln\beta} = \lambda^{0.174} \qquad \text{and} \qquad l_{(2),\lambda} \equiv \frac{v_A \lambda}{\delta z_{(2),\lambda}^+} \propto \lambda^{1+\beta^2 \ln\beta} = \lambda^{0.826}$$

Comparison with PSP E1 Observations from Sioulas et al (2024)





Voilà — cyclotron heating.

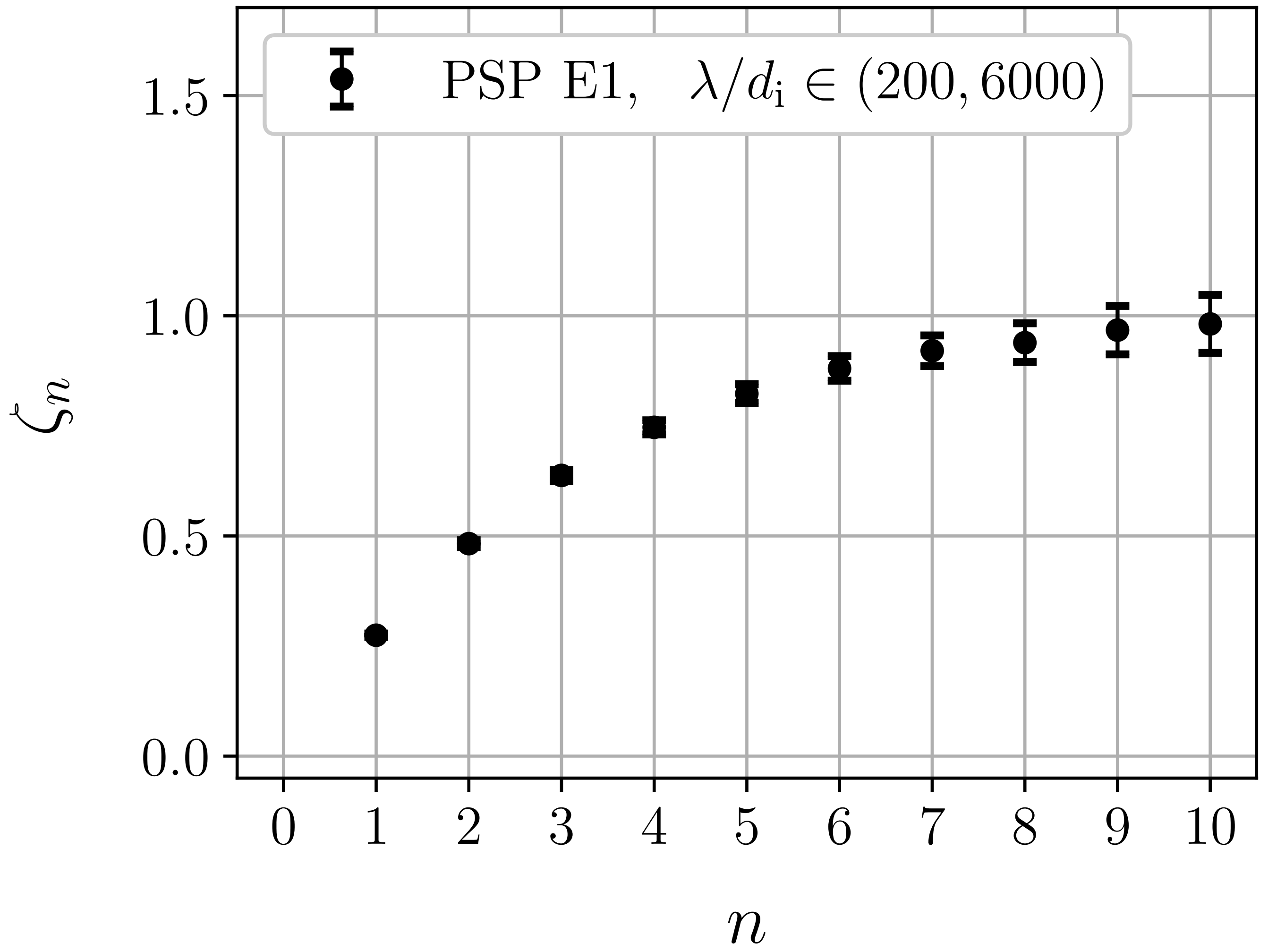
Not so fast! Are these sheets, or tubes?

$P(0) = e^{-\mu} \propto \lambda$ — this means sheets

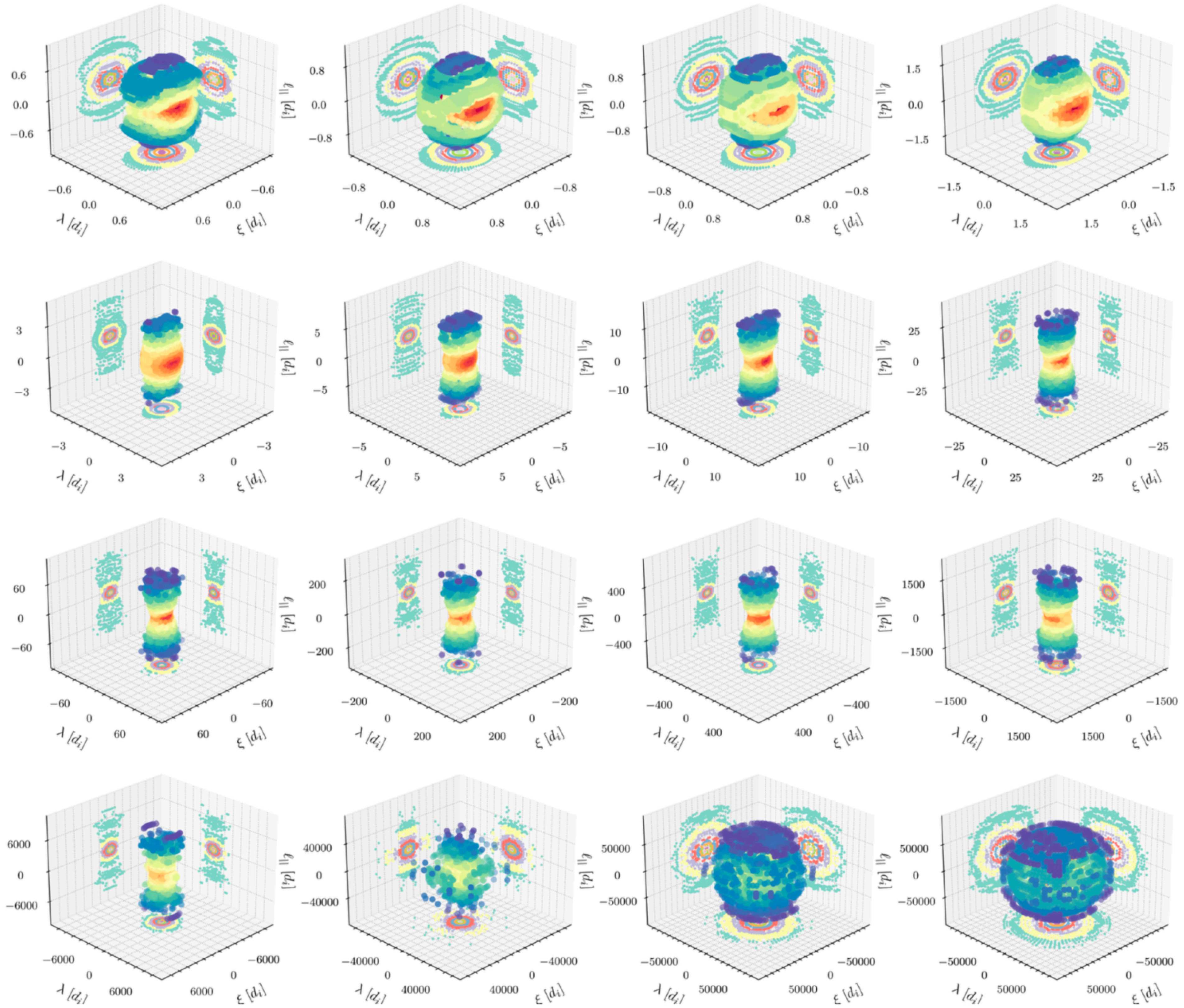
Assertion: in the sub-volume that dominates $\langle \epsilon_\lambda^+ \rangle$, in which $\delta z_\lambda^+ > w_\lambda^+$, the driving of δz_λ^- is uniform, but the damping time scale of δz_λ^- is $\propto 1/\delta z_\lambda^+$.

$$\longrightarrow \delta z_\lambda^- \propto \frac{1}{\delta z_\lambda^+} \longrightarrow \delta z_\lambda^- = \frac{w_\lambda^+ w_\lambda^-}{\delta z_\lambda^+}$$ — this means tubes

Okay - so let's use the observations to decide.



— this means sheets. As $n \to \infty$, $\left\langle (\delta z_\lambda^+)^n \right\rangle \to f_\lambda^{(\infty)} (\delta z_{\infty,\lambda}^+)^n$



— this means tubes

Conclusion

- In intermittent reflection-driven turbulence, inertial-range fluctuations are tube-like, and so stronger fluctuations have shorter parallel length scales via critical balance, $l_\lambda^+ = \dfrac{\lambda v_{\mathrm{A}}}{\delta z_\lambda^+}$.

- The fluctuations that dominate the energy and energy cascade rate are unusually strong, and hence have unusually large frequencies, which enhances perpendicular ion heating.

- Lingering questions over how to reconcile tube-like inertial-range fluctuations with measurements of higher-order structure functions.