Scalings for short-wavelength ITG turbulence

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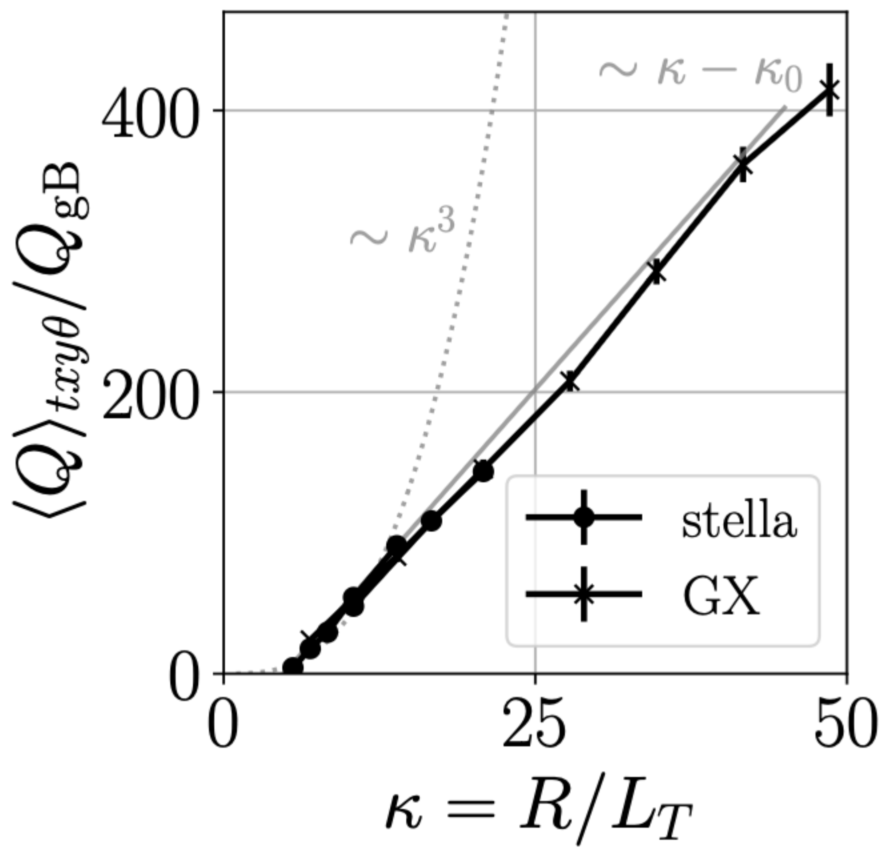
What happens to ITG stability/transport for large plasma gradients?



$$\eta = \frac{L_n}{L_T} \gg 1$$

Nies *et al.*, arXiv:2409.02283

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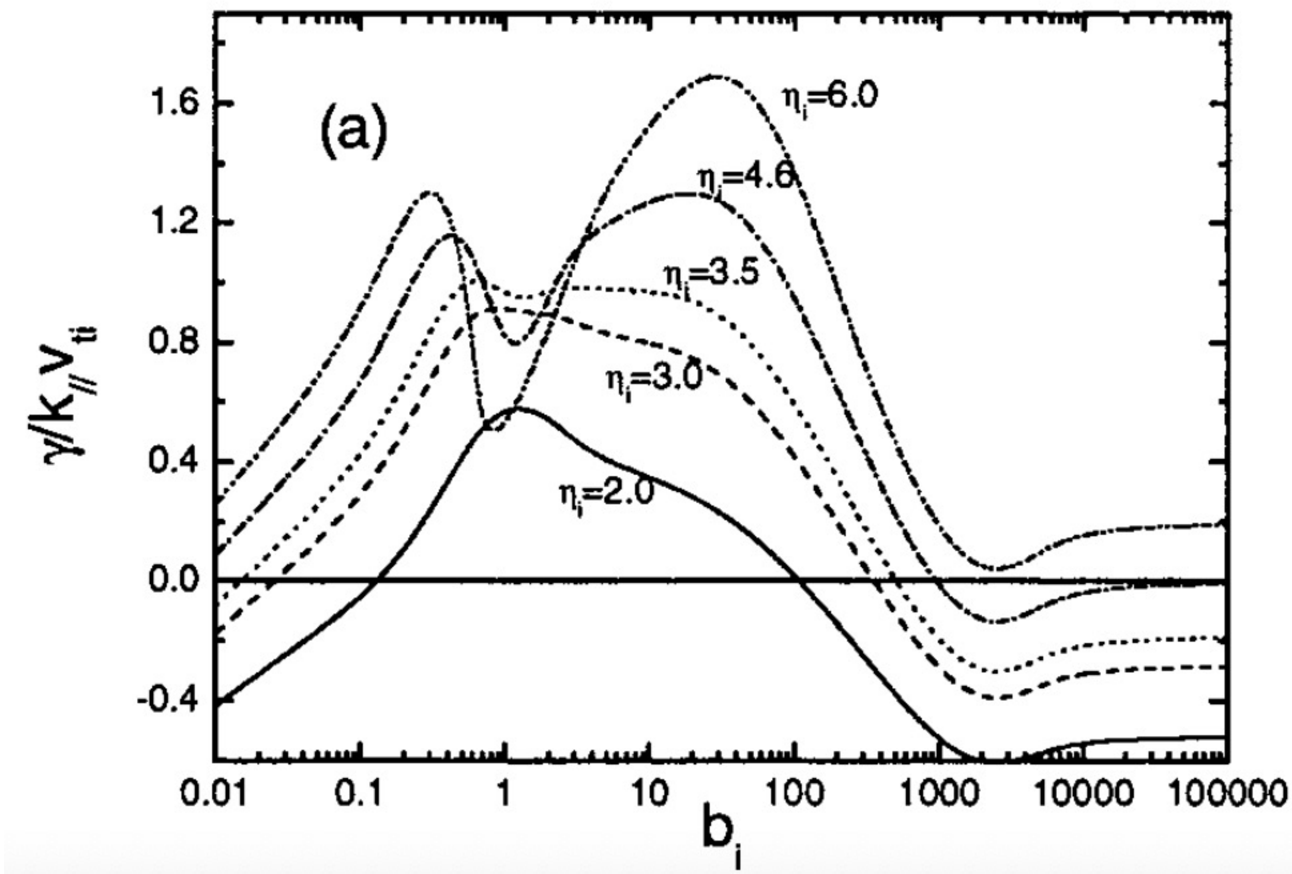


Sheared slab ITG

Gao *et al*., PoP 2003

Hassam PoF 1990, Smolyakov PRL 2003, Chowdhury PoP 2009, …

Heuristic fluid model

2D gyrokinetics in a Z-pinch, with Botlzmann electrons and mode frequency much larger than the drift frequency

Density moment: $\delta N_{\mathbf{k}} \equiv \int d^3\mathbf{v}\ J_0(\alpha_k)\hat{h}_{\mathbf{k}}$

$$\omega\left(\frac{\delta N_{\mathbf{k}}}{n_i} - \frac{\Gamma_0(b_k)}{\tau}\frac{e\hat{\varphi}_{\mathbf{k}}}{T_e}\right) - \omega_d\frac{\delta p_{\mathbf{k}}}{n_i T_i} + \left[\Gamma_0(b_k) + \eta b_k\left(\Gamma_1(b_k) - \Gamma_0(b_k)\right)\right]\frac{\omega_*}{\tau}\frac{e\hat{\varphi}_{\mathbf{k}}}{T_e} = 0,$$

Pressure moment: $\delta p_{\mathbf{k}} \equiv \int d^3\mathbf{v}\ \frac{m}{2}\left(2v_\parallel^2 + v_\perp^2\right) J_0(\alpha_k)\hat{h}_{\mathbf{k}}$

$$\omega\left(\frac{\delta p_{\mathbf{k}}}{n_i T_i} - \frac{K_1(b_k)}{\tau}\frac{e\hat{\varphi}_{\mathbf{k}}}{T_e}\right) + \left(K_1(b_k) + \eta K_2(b_k)\right)\frac{\omega_*}{\tau}\frac{e\hat{\varphi}_{\mathbf{k}}}{T_e} = 0.$$

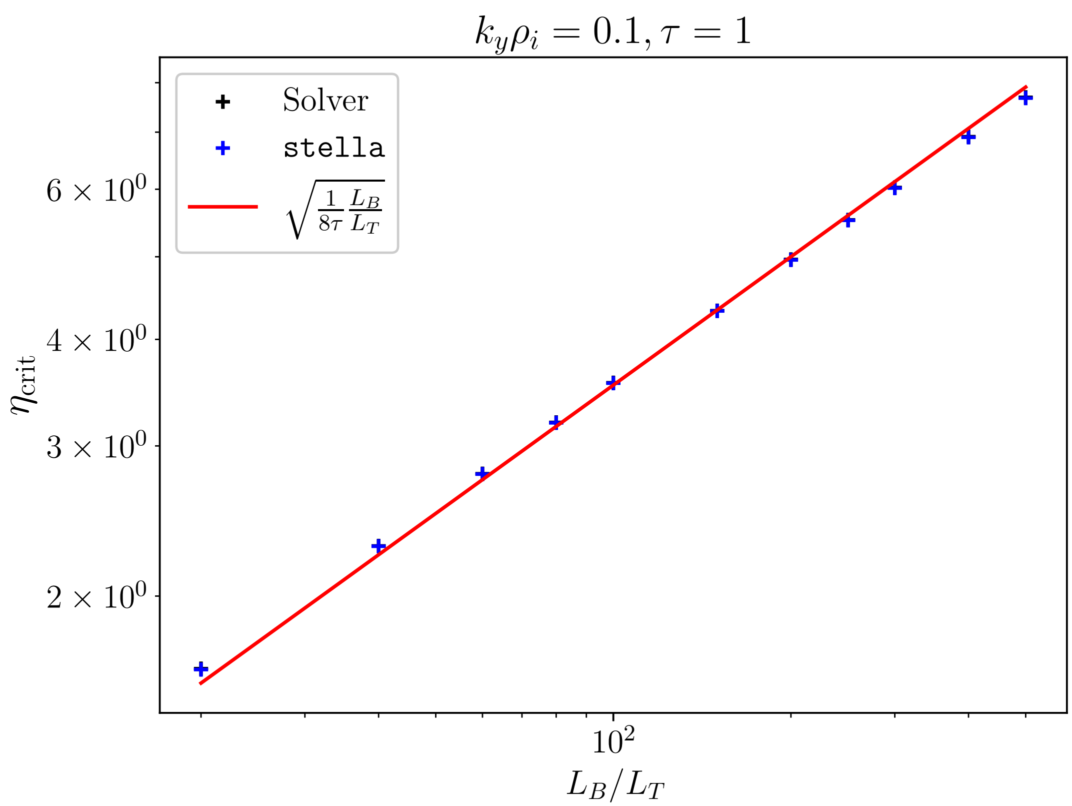
$$\Gamma_j(b_k) = e^{-b_k} I_j(b_k) \qquad b_k = \frac{k_\perp^2 \rho_i^2}{2}$$

Limiting cases with $L_B/L_T \gg 1$

Long wavelength: $\Gamma_0(b_k) \approx 1 \qquad \Gamma_1(b_k) \approx 0.$

$\eta \omega_d \tau \gg \omega_*,$ $\qquad \omega_{\pm}^{\mathrm{lw}} \approx \pm \mathrm{i} k_y \rho_i \frac{c_s}{\sqrt{L_B L_p}}$

$\eta \omega_d \tau \sim \omega_*$ $\qquad \eta_{\mathrm{crit}} \approx \sqrt{\frac{1}{8\tau} \frac{L_B}{L_T}}$



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$$\eta\omega_d\tau \sim \omega_* \qquad \eta_{\mathrm{crit}} \approx \sqrt{\frac{1}{8\tau}\frac{L_B}{L_T}}$$

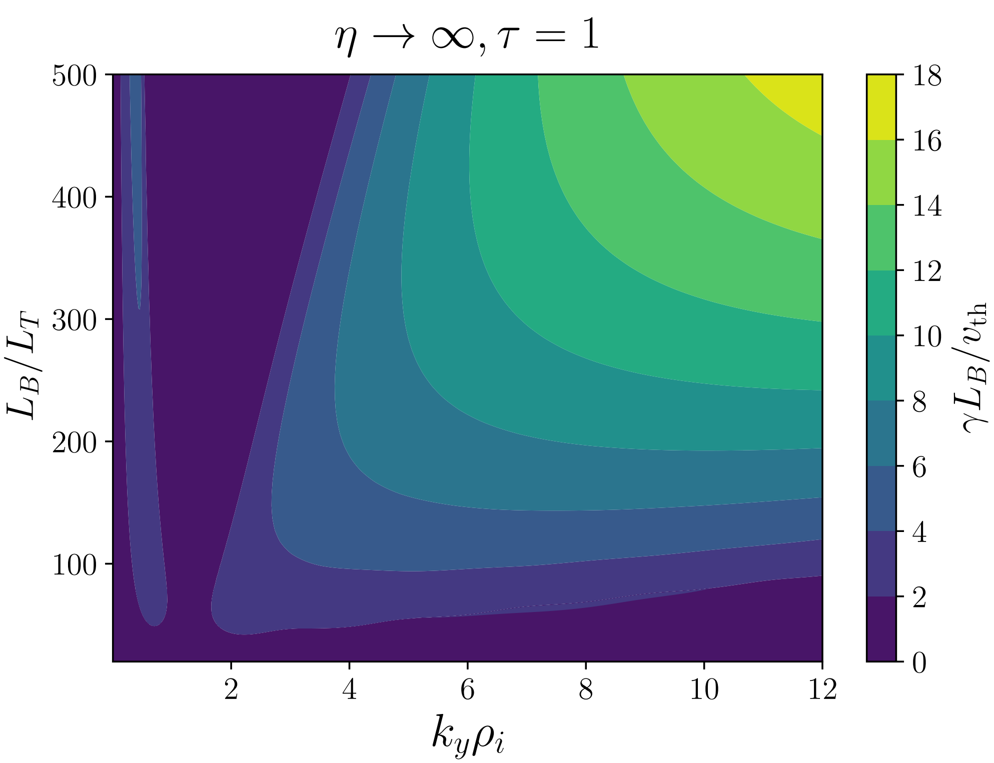
Short wavelength:

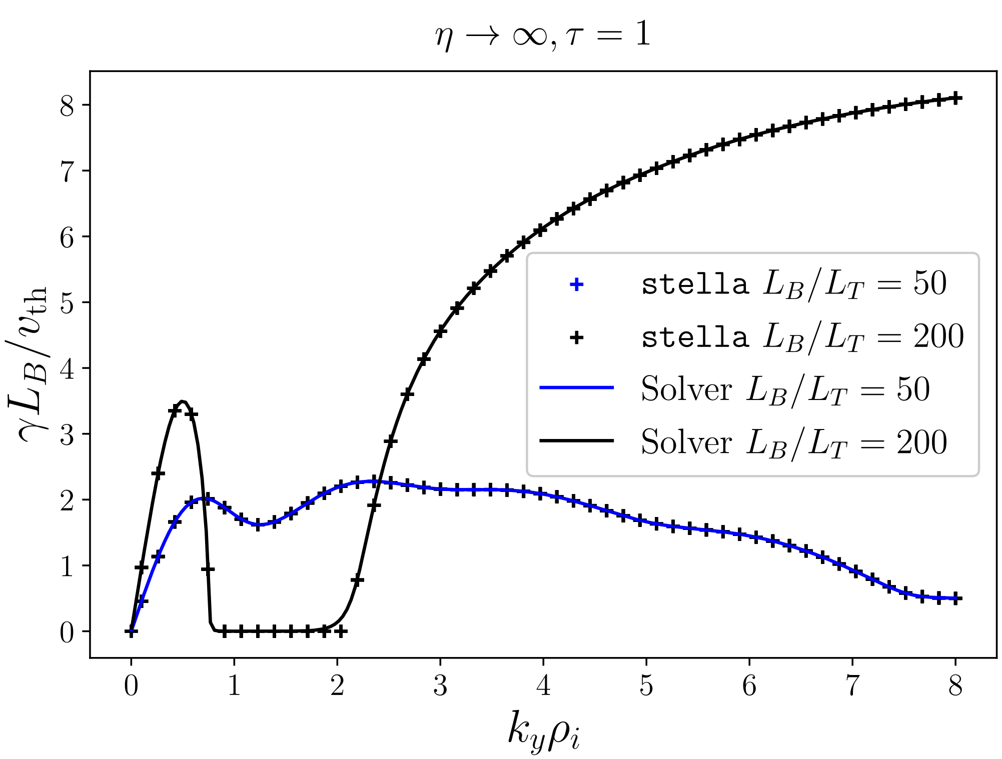
$$\Gamma_0(b_k) \approx \frac{1}{\sqrt{\pi}}\frac{1}{k_\perp \rho_i}\left(1 + \frac{1}{4}\frac{1}{k_\perp^2 \rho_i^2}\right), \quad \Gamma_1(b_k) \approx \frac{1}{\sqrt{\pi}}\frac{1}{k_\perp \rho_i}\left(1 - \frac{3}{4}\frac{1}{k_\perp^2 \rho_i^2}\right)$$

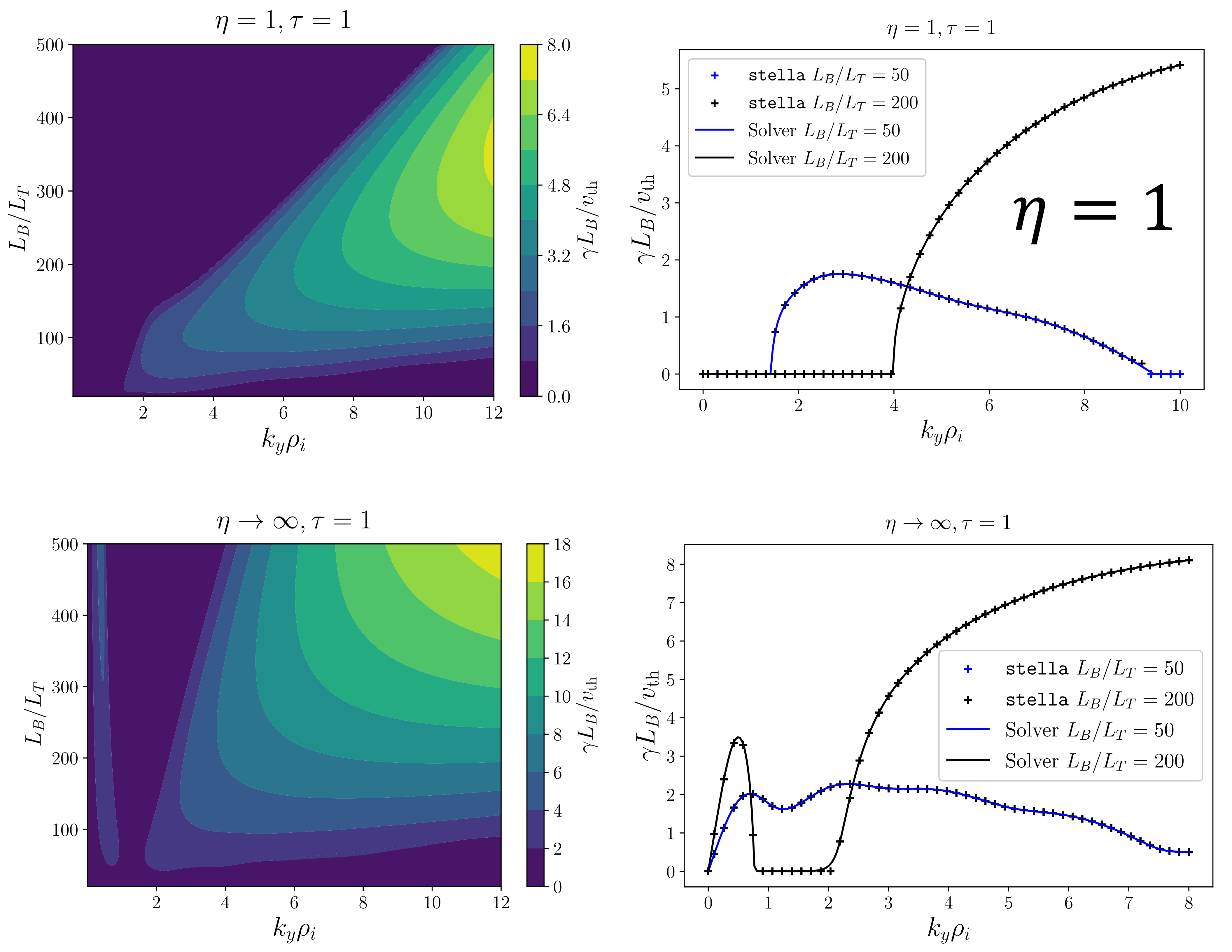
Stable for wavenumbers smaller than $k_\perp^o \rho_i = \dfrac{(2-\eta)^2}{(2+\eta)}\dfrac{L_B}{12 L_n^{\mathrm{eff}}}$

Numerical growth rate spectra

$$\eta \gg 1$$







$\eta = 1, \tau = 1$
L_B/L_T
$k_y\rho_i$
$\gamma L_B/v_{\mathrm{th}}$
0.0
1.6
3.2
4.8
6.4
8.0
$\eta = 1, \tau = 1$
stella $L_B/L_T = 50$
stella $L_B/L_T = 200$
Solver $L_B/L_T = 50$
Solver $L_B/L_T = 200$
$\eta = 1$
$\eta \to \infty, \tau = 1$
L_B/L_T
$k_y\rho_i$
$\gamma L_B/v_{\mathrm{th}}$
$\eta \to \infty, \tau = 1$
stella $L_B/L_T = 50$
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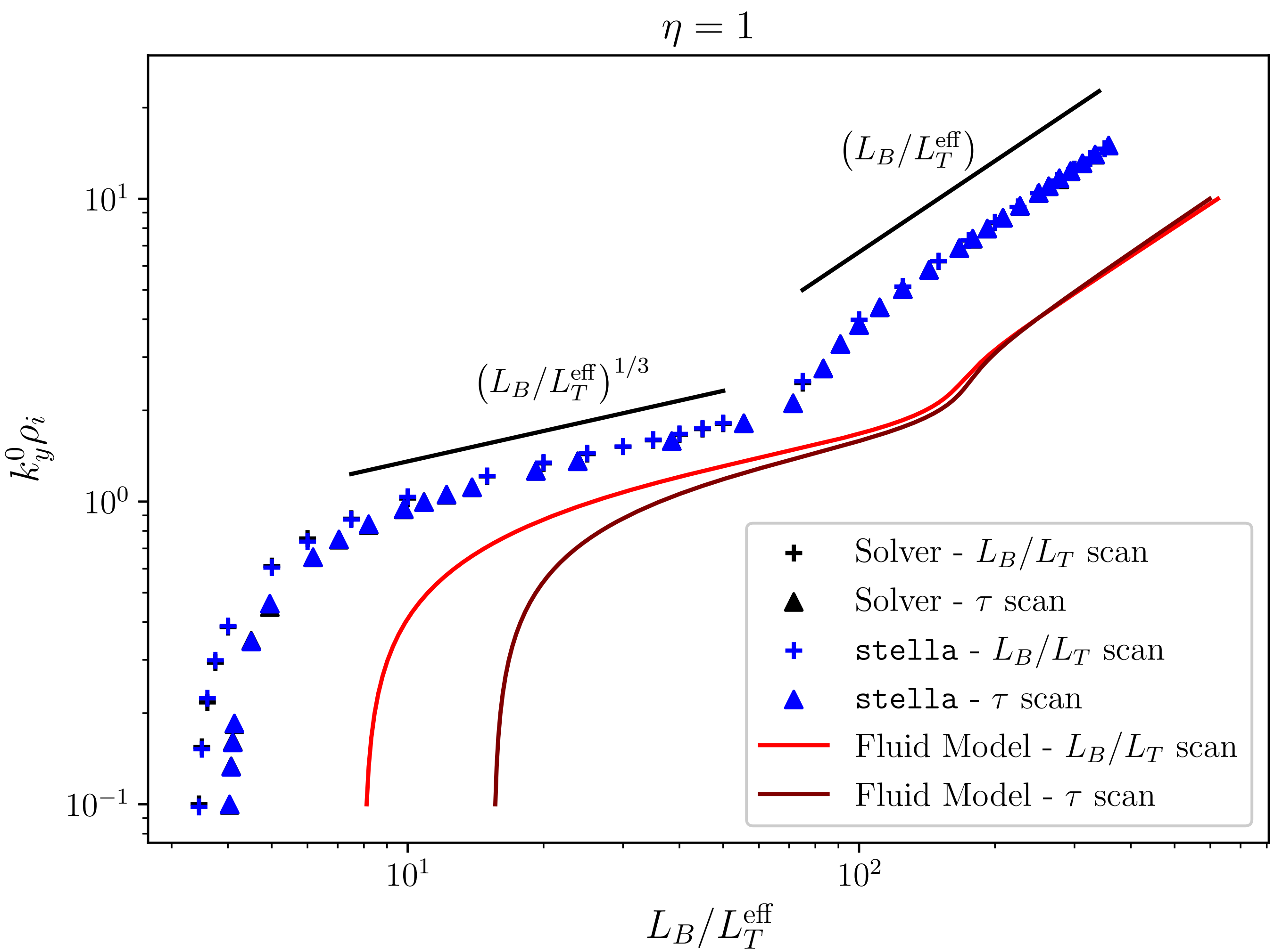
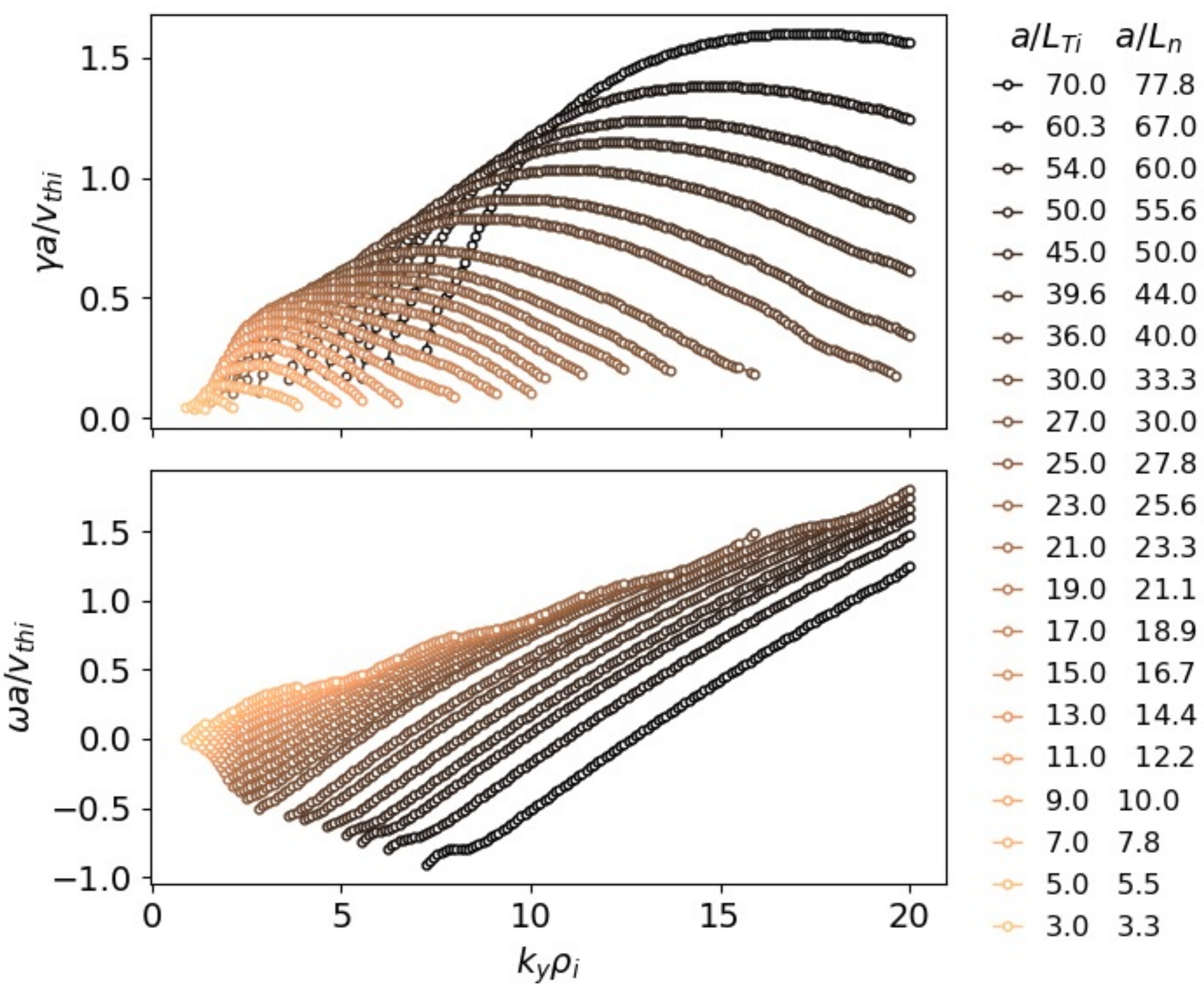
Solver $L_B/L_T = 50$
Solver $L_B/L_T = 200$

Minimum unstable wavenumber increases with driving gradient



Cyclone Base Case (CBC) linear results



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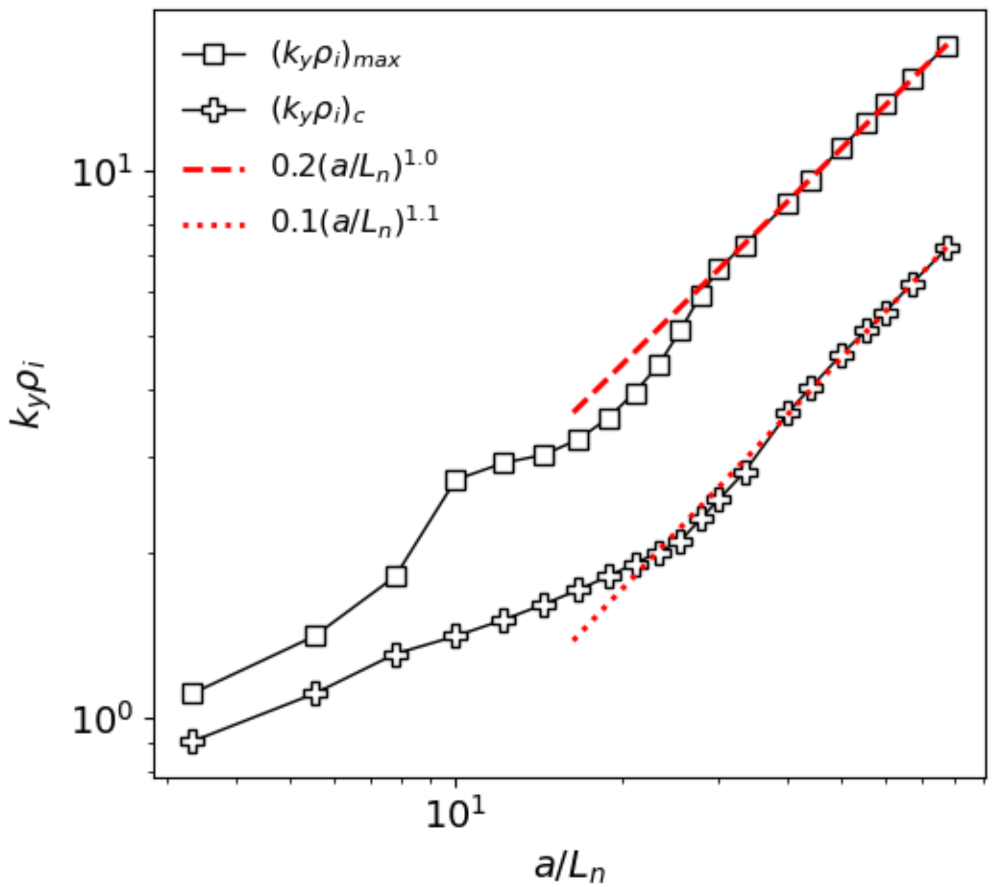
SWITG wavenumber scalings



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Nonlinear scalings

Diffusive estimate: $\chi \sim \frac{1}{\left(k_\perp^o\right)^2 \tau_{\rm nl}^o}$

Nonlinear time: $\frac{\partial}{\partial t} \int d^3v \; J_0(\alpha_k) \hat{h}_{\mathbf{k}}^o \sim \left(\int d^3v \langle \mathbf{v}_E \rangle_{\mathbf{R}} \cdot \nabla h \right)_{\mathbf{k}^o} \equiv \frac{e\hat{\varphi}_{\mathbf{k}}^o}{T} \frac{n}{\tau_{\rm nl}^o}$

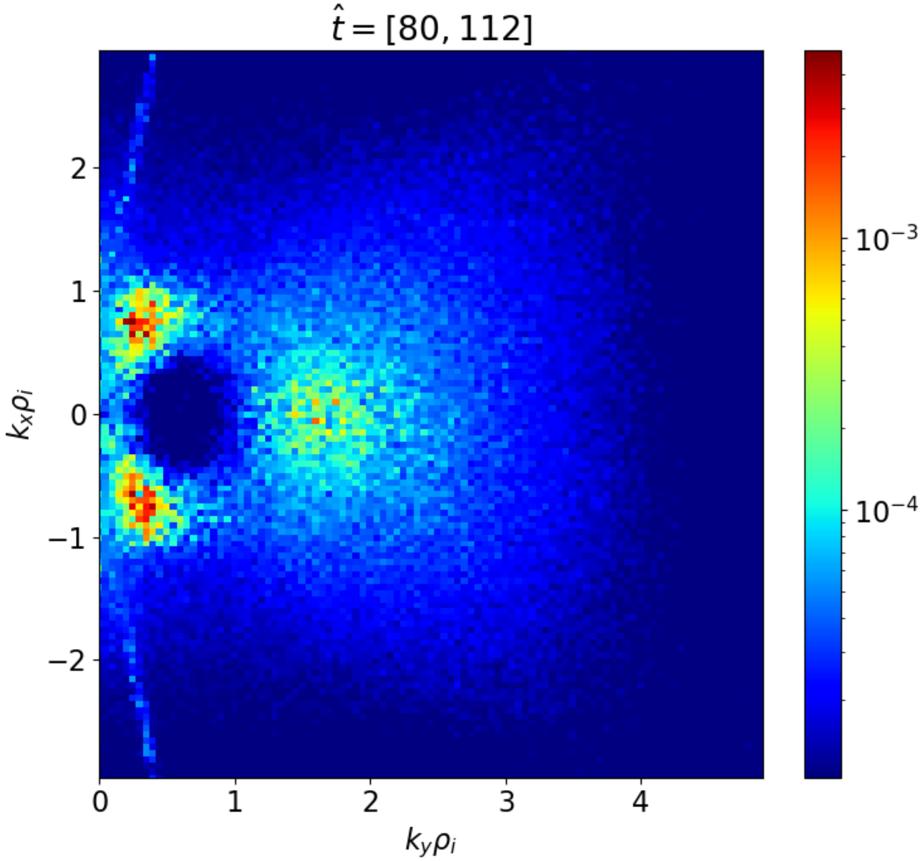
Time at outer scale: $\frac{e\hat{\varphi}_{\mathbf{k}}^o}{T} \frac{n}{\tau_{\rm nl}^o} \sim \Gamma_0(b_k^o) \omega_*^o \eta \frac{e\hat{\varphi}_{\mathbf{k}}^o}{T} n,$

$$\chi \sim \frac{\Gamma_0(b_k)}{k_\perp^o} \frac{v_{\rm th}}{L_T} \sim \left(\frac{1}{k_\perp^o}\right)^2 \frac{v_{\rm th}}{L_T} \sim \rho_i^2 \frac{v_{\rm th}}{L_B} \left(\frac{L_T}{L_B}\right)$$ ➡ Q constant

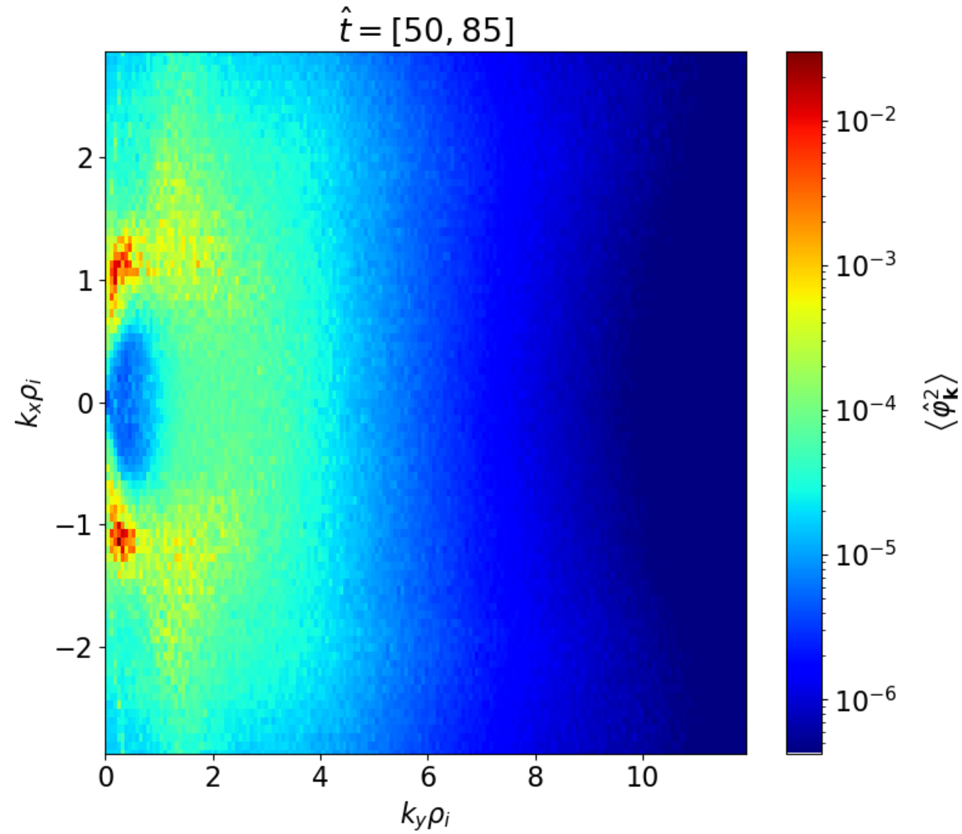
Critical balance: $k_\parallel^o \sim \frac{1}{L_T}$

CBC density spectra

$$\frac{a}{L_n} = 15$$



$$\frac{a}{L_n} = 35$$



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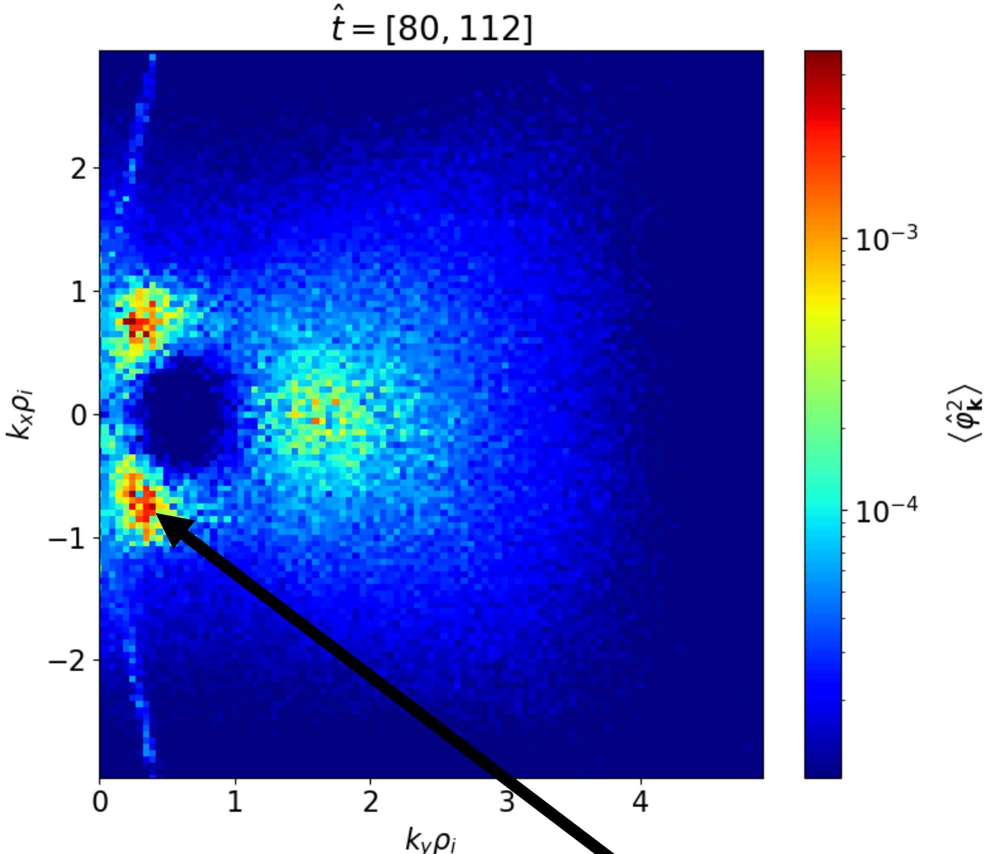
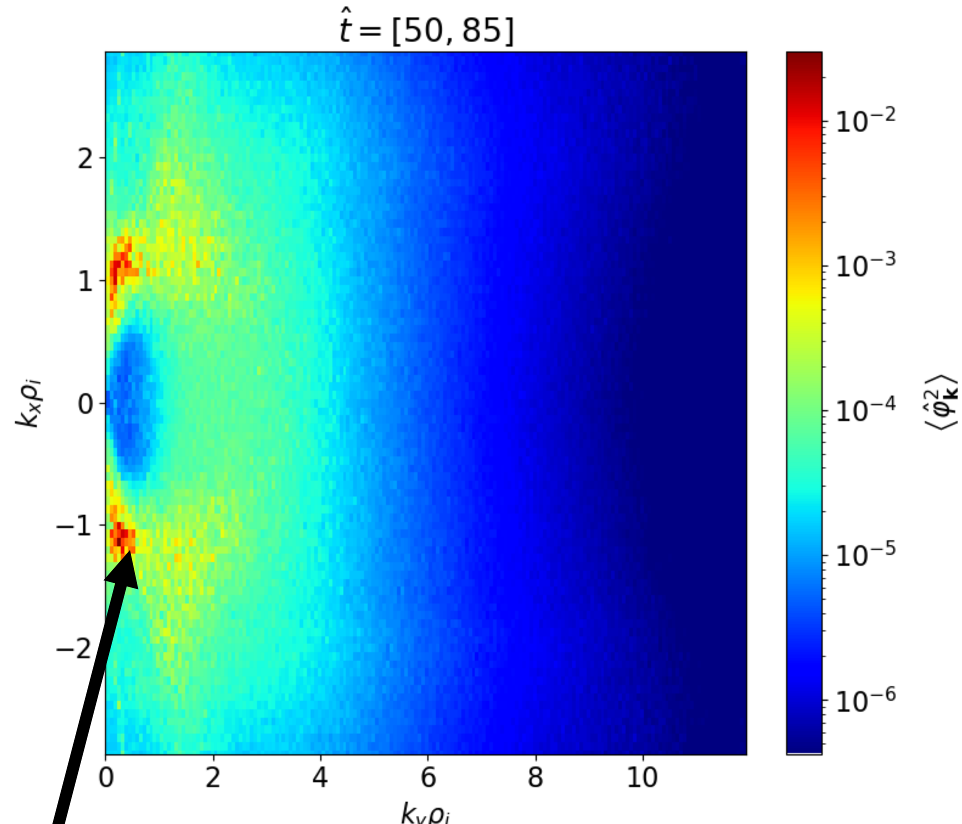
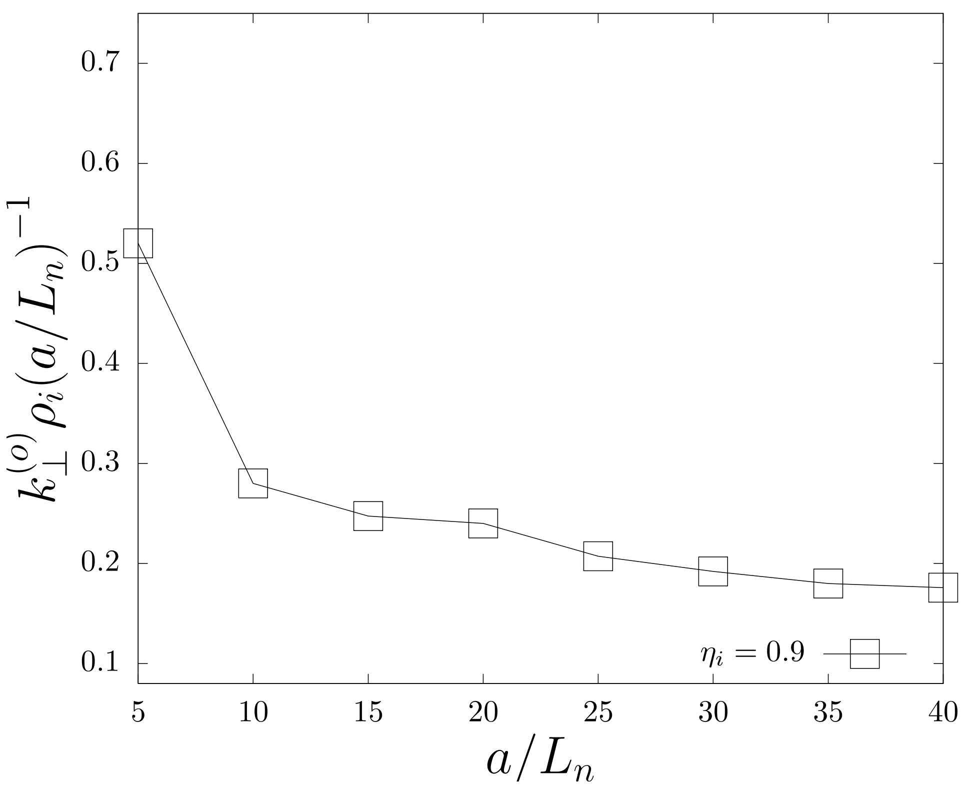
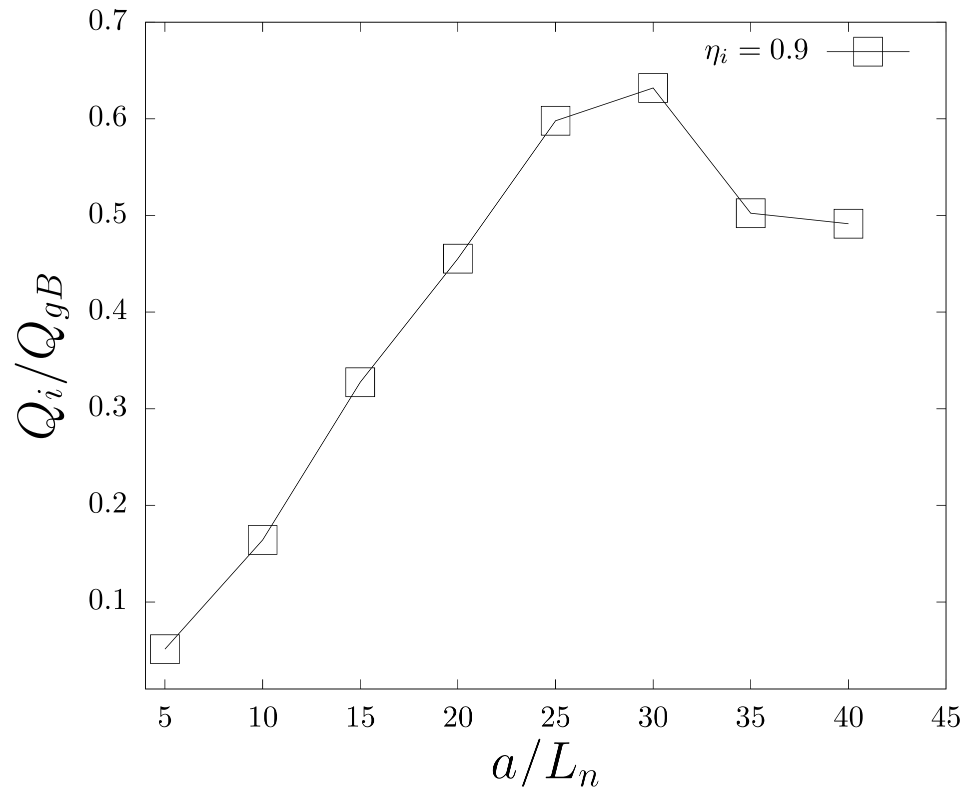




J. Parisi *et al.*, NF 2020

Outer scale and heat flux scalings





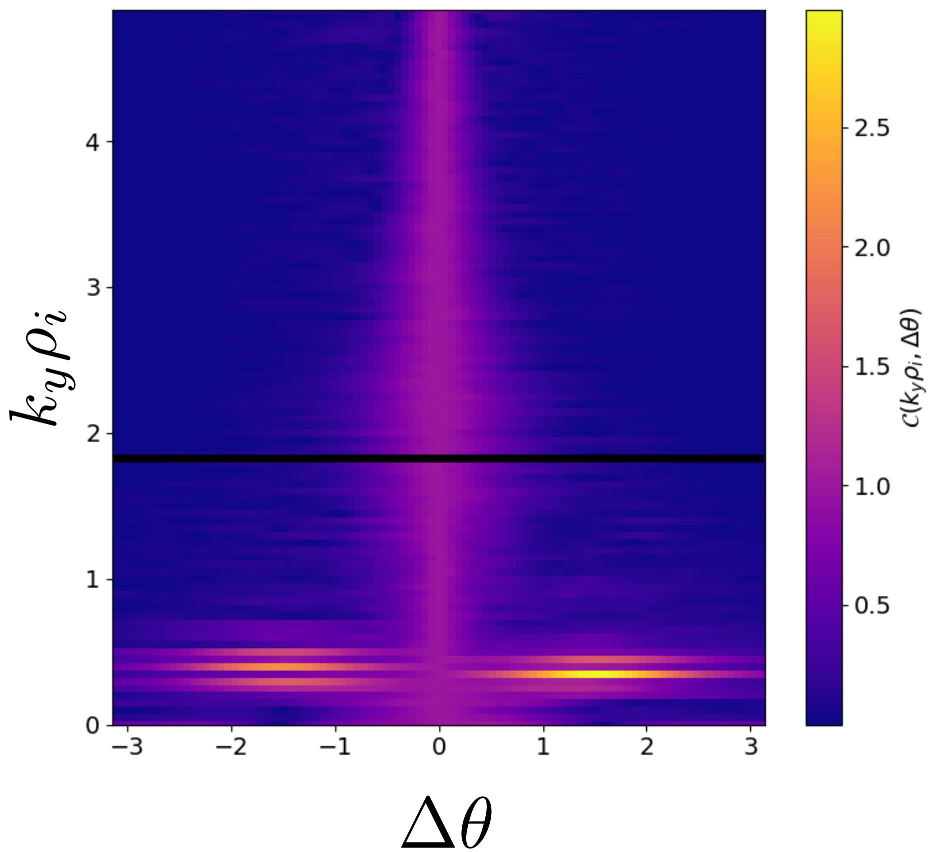
$$k_\perp^o \rho_i \sim \frac{a}{L_n}$$

$$Q \sim \text{constant}$$

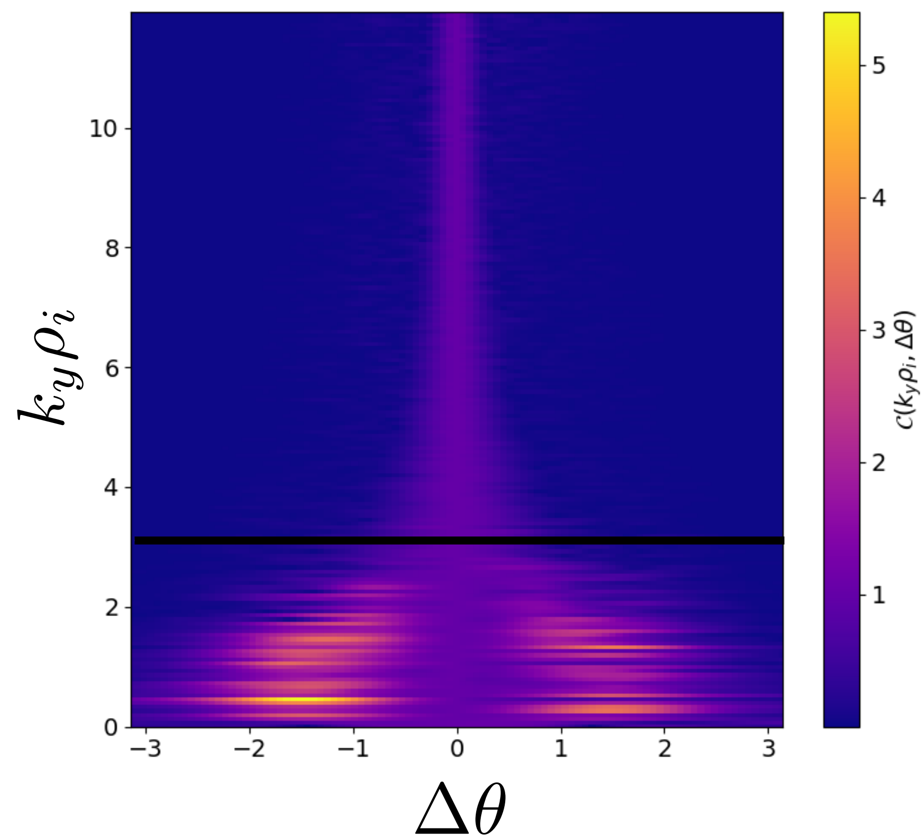
Parallel correlation lengths

Expect $k_\parallel^o a \sim \frac{a}{L_n}$

$\frac{a}{L_n} = 15$



$\frac{a}{L_n} = 35$



Summary and open questions

- For comparable density and temperature gradients, increasing ITG leads to a regime where the ion heat flux becomes independent of the ITG
- This is associated with a shift to sub-Larmor scales, where kinetic electron physics likely important
- Possible to access this regime where TEM are suppressed? Stellarators or tokamaks with particle trapping on inboard?