The good, the bad, and the curvy

A tale of curvature and instability

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- One can show that ITG is
  - unstable if $L_B L_{T_i} > 0$ (*bad curvature*),
  - a stable wave if $L_B L_{T_i} < 0$ (*good curvature*).

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- Are all curvature-driven instabilities unstable only in bad curvature?
- If not, what distinguishes a good-curvature instability form a bad-curvature one?

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Linear gyrokinetics

$$\frac{\partial}{\partial t}\left(h_s - \frac{q_s\langle\chi\rangle_{\boldsymbol{R}_s}}{T_{0s}} f_{0s}\right) + \left(v_\parallel \hat{\boldsymbol{b}}_0 + \boldsymbol{V}_{\mathrm{d}s}\right)\cdot\boldsymbol{\nabla} h_s + \langle\boldsymbol{V}_\chi\rangle_{\boldsymbol{R}_s}\cdot\boldsymbol{\nabla} f_{0s} = 0, \qquad (2)$$

$$\chi = \phi - \frac{\boldsymbol{v}\cdot\delta\boldsymbol{A}}{c}. \qquad (3)$$

Linear drive (advection of equilibrium):

$$\langle\boldsymbol{V}_\chi\rangle_{\boldsymbol{R}_s}\cdot\boldsymbol{\nabla} f_{0s} = -\frac{c}{B_0}\left(\hat{\boldsymbol{b}}_0 \times \frac{\partial\langle\chi\rangle_{\boldsymbol{R}_s}}{\partial\boldsymbol{R}_s}\right)\cdot\boldsymbol{\nabla} x\left[\frac{1}{L_{n_s}} + \frac{1}{L_{T_s}}\left(\frac{v^2}{v_{\mathrm{th}s}^2} - \frac{3}{2}\right)\right] f_{0s}. \qquad (4)$$

where the gradients are

$$L_{n_s} \equiv -\partial \ln n_{0s}/\partial x, \quad L_{T_s} \equiv -\partial \ln T_s/\partial x. \qquad (5)$$

Magnetic drifts:

$$\boldsymbol{V}_{\mathrm{d}s} = \frac{\hat{\boldsymbol{b}}_0}{\Omega_s}\times\left(v_\parallel^2 \hat{\boldsymbol{b}}_0\cdot\boldsymbol{\nabla}\hat{\boldsymbol{b}}_0 + \frac{1}{2}v_\perp^2\boldsymbol{\nabla}\log B_0\right). \qquad (6)$$

In what follows x, y, and z are the radial, poloidal, and parallel coordinate, respectively.

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Field equations:

$$\sum_s \frac{q_s^2 n_{0s}}{T_{0s}}\phi = \sum_s q_s \int \mathrm{d}^3\boldsymbol{v}\,\langle h_s\rangle_{\boldsymbol{r}}\,, \qquad (8)$$

$$\boldsymbol{\nabla}_\perp^2 \delta A_\parallel = -\frac{4\pi}{c}\sum_s q_s \int \mathrm{d}^3\boldsymbol{v}\; v_\parallel \langle h_s\rangle_{\boldsymbol{r}}\,, \qquad (9)$$

$$\boldsymbol{\nabla}_\perp^2 \delta B_\parallel = -\frac{4\pi}{B_0}\boldsymbol{\nabla}_\perp\boldsymbol{\nabla}_\perp : \sum_s m_s \int \mathrm{d}^3\boldsymbol{v}\;\langle \boldsymbol{v}_\perp\boldsymbol{v}_\perp h_s\rangle_{\boldsymbol{r}}\,. \qquad (10)$$

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Much of what follows is equally valid in proper toroidal geometry.

However, the above assumptions will allow us to get some nice analytical results for the instabilities.

GK conservation laws

- GK conserves (nonlinearly) the free energy

$$W = \sum_s \int \mathrm{d}^3\boldsymbol{r} \int \mathrm{d}^3\boldsymbol{v} \, \frac{T_{0s}\delta f_s^2}{2 f_{0s}} + \int \mathrm{d}^3\boldsymbol{r} \, \frac{|\delta\boldsymbol{B}|^2}{8\pi}, \tag{12}$$

whose time evolution is

$$\frac{\mathrm{d}W}{\mathrm{d}t} = \sum_s \left(\frac{1}{L_{n_s}} - \frac{3}{2}\frac{1}{L_{T_s}} \right) T_{0s}\varGamma_s + \sum_s \frac{1}{L_{T_s}} Q_s, \tag{13}$$

where $\varGamma_s$ and Q_s are the particle and heat (energy) fluxes, respectively.

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- $W > 0 \implies$ **the right-hand side of (13) must be positive for an instability.**

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- $W > 0 \implies$ **the right-hand side of (13) must be positive for an instability.**
- This says nothing about the magnetic-field curvature...

GK conservation laws

- One can construct other nonlinear invariants using

$$I_s(\boldsymbol{v}) = \int \mathrm{d}^3\boldsymbol{R}_s \, \frac{T_{0s}}{2f_{0s}} \left(h_s - \frac{q_s \langle \chi \rangle_{\boldsymbol{R}_s}}{T_{0s}} f_{0s} \right)^2 . \tag{14}$$

that satisfies

$$\frac{\mathrm{d}I_s}{\mathrm{d}t} = \int \mathrm{d}^3\boldsymbol{R}_s \, \left[q_s \langle \chi \rangle_{\boldsymbol{R}_s} v_\parallel \hat{\boldsymbol{b}}_0 \cdot \boldsymbol{\nabla} h_s + q_s \langle \chi \rangle_{\boldsymbol{R}_s} \boldsymbol{V}_{\mathrm{d}s} \cdot \boldsymbol{\nabla} h_s - h_s \boldsymbol{V}_\chi \cdot \boldsymbol{\nabla} \ln f_{0s} \right] . \tag{15}$$

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There are (at least) two reasons why Y is interesting...

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- Y depends only on the EM fields, not the distribution functions:

$$Y = -\sum_{\boldsymbol{k},s} \frac{q_s^2 n_{0s}}{2T_{0s}} \left(1 - \Gamma_{0s} + \frac{\Gamma_{1s}}{2}\right) \left|\phi_{\boldsymbol{k}}\right|^2 + V \sum_{\boldsymbol{k}} \left(\frac{k_\perp^2}{8\pi} + \sum_s \frac{\Gamma_{0s}}{8\pi d_s^2}\right) \left|\delta A_{\parallel \boldsymbol{k}}\right|^2$$
$$+ V \sum_{\boldsymbol{k}} \left[\left(\frac{\left|\delta B_{\parallel \boldsymbol{k}}\right|^2}{8\pi} + \sum_s n_{0s} T_{0s} \Gamma_{1s} \left|\frac{\delta B_{\parallel \boldsymbol{k}}}{B_0} + \frac{q_s \phi_{\boldsymbol{k}}}{2T_{0s}}\right|^2\right)\right], \qquad (17)$$

where

$$\Gamma_{0s} = \mathrm{I}_0(\alpha_s)\mathrm{e}^{-\alpha_s}, \quad \Gamma_{1s} = \left[\mathrm{I}_0(\alpha_s) - \mathrm{I}_1(\alpha_s)\right] \mathrm{e}^{-\alpha_s}. \qquad (18)$$

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- The evolution equation for Y does not have the usual ‘injection’ terms:

$$\frac{\mathrm{d}Y}{\mathrm{d}t} = \sum_s \int \mathrm{d}^3\boldsymbol{R}_s \int \mathrm{d}^3\boldsymbol{v}\; q_s \langle\chi\rangle_{\boldsymbol{R}_s} v_\parallel \hat{\boldsymbol{b}}_0 \cdot \boldsymbol{\nabla} h_s$$
$$- \frac{1}{R}\sum_s 2Q_{\parallel s} - \frac{1}{L_B}\sum_s Q_{\perp s}, \tag{19}$$

where $Q_{\parallel s}$ and $Q_{\perp s}$ are the radial fluxes of $v_\parallel$ and $v_\perp$ energy.

Probably the most important slide

- Consider unstable modes that satisfy two assumptions:
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- Expect electrostatic modes to be unstable when $L_B L_{T_s} > 0$, i.e., bad curvature.
- **If anything is unstable in good curvature, then it is likely electromagnetic!**

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Low-beta electromagnetic fluctuations at the d_e scale

Consider fluctuations with

$$\frac{m_e}{m_i} \ll \beta_e \sim k_\perp^2 \rho_e^2 \ll k_\perp^2 d_e^2 \sim 1 \ll k_\perp^2 \rho_i^2 \tag{23}$$

and

$$\omega \sim k_\parallel v_{\mathrm{the}} \sim \omega_{*e} \sim \omega_{Te} \sim \omega_{\mathrm{de}}, \tag{24}$$

where

$$\omega_{*e} = \frac{\rho_e v_{\mathrm{the}} k_y}{2L_{n_e}}, \quad \omega_{Te} = \frac{\rho_e v_{\mathrm{the}} k_y}{2L_{T_e}}, \quad \omega_{\mathrm{de}} = \frac{\rho_e v_{\mathrm{the}} k_y}{2L_B}. \tag{25}$$

Low-beta electromagnetic fluctuations at the d_e scale

- Defining

$$\varphi \equiv \frac{e\phi}{T_{0e}}, \quad \mathcal{A} \equiv \frac{\delta A_\parallel}{\rho_e B_0}, \tag{26}$$

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GK reduces to electron drift kinetics (DK):

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- The field equations are

$$-(1+\tau^{-1})\varphi = \frac{1}{n_{0e}}\int \mathrm{d}^3\boldsymbol{v}\; h_e, \tag{28}$$

$$d_e^2 \nabla_\perp^2 \mathcal{A} = \frac{1}{n_{0e}}\int \mathrm{d}^3\boldsymbol{v}\; \frac{v_\parallel}{v_{\rm the}} h_e = \frac{u_{\parallel e}}{v_{\rm the}}, \tag{29}$$

where $u_{\parallel e}$ is the perturbed parallel flow, $\tau \equiv eT_{0i}/q_i T_{0e}$ is the temperature ratio, $d_e = \rho_e/\sqrt{\beta_e}$ is the electron skin depth.

2D low-beta fluctuations

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- For now, consider only the 2D case. **In this case, electrostatic and electromagnetic fluctuations decouple!**

- Even (in $v_\parallel$) part of the DK equation

$$\frac{\partial}{\partial t}\left[h_e^{(\text{even})} + \varphi f_{0e}\right] + \frac{\rho_e v_{\text{the}}}{2L_B}\left(\frac{2v_\parallel^2}{v_{\text{the}}^2} + \frac{v_\perp^2}{v_{\text{the}}^2}\right)\frac{\partial h_e^{(\text{even})}}{\partial y} + \frac{\rho_e v_{\text{the}}}{2}\left[\frac{1}{L_{n_e}} + \frac{1}{L_{T_e}}\left(\frac{v^2}{v_{\text{ths}}^2} - \frac{3}{2}\right)\right]\frac{\partial \varphi}{\partial y} f_{0e} = 0, \qquad (30)$$

together with quasineutrality

$$-(1+\tau^{-1})\varphi = \frac{1}{n_{0e}}\int \mathrm{d}^3\boldsymbol{v}\; h_e^{(\text{even})}. \qquad (31)$$

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- Has a curvature-driven instability **only if the curvature is bad**.

2D electromagnetic fluctuations

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- Contains only fluctuations that are **odd in $v_\parallel$ moments of h_e**.
- E.g., has **no density or temperature fluctuations**.
- Contains a novel instability, **the magnetic-drift mode**, that is **unstable only if the curvature is good.**

Conservation laws

The conserved quantities are

$$W = \int \mathrm{d}^3\boldsymbol{r} \int \mathrm{d}^3\boldsymbol{v}\ \frac{T_{0e}h_e^2}{2f_{0e}} + n_{0e}T_{0e}\int \mathrm{d}^3\boldsymbol{r}\ \left(d_e^2\left|\boldsymbol{\nabla}_\perp\mathcal{A}\right|^2 - \frac{1+\tau^{-1}}{2}\varphi^2\right), \tag{34}$$

$$Y = n_{0e}T_{0e}\int \mathrm{d}^3\boldsymbol{r}\ \left(\mathcal{A}^2 + d_e^2\left|\boldsymbol{\nabla}_\perp\mathcal{A}\right|^2 - \frac{\tau^{-1}}{2}\varphi^2\right), \tag{35}$$

and they satisfy

$$\frac{\mathrm{d}W}{\mathrm{d}t} = \frac{Q_{\parallel e} + Q_{\perp e}}{L_{T_e}}, \tag{36}$$

$$\frac{\mathrm{d}Y}{\mathrm{d}t} = -\frac{2Q_{\parallel e} + Q_{\perp e}}{L_B}, \tag{37}$$

where

$$Q_{\parallel e} = \rho_e v_{\mathrm{th}e} T_{0e}\int \mathrm{d}^3\boldsymbol{r} \int \mathrm{d}^3\boldsymbol{v}\ \frac{v_\parallel^2}{v_{\mathrm{th}e}^2}\left(-\frac{1}{2}\frac{\partial\varphi}{\partial y}h_e + \frac{\partial\mathcal{A}}{\partial y}\frac{v_\parallel}{v_{\mathrm{th}e}}h_e\right), \tag{38}$$

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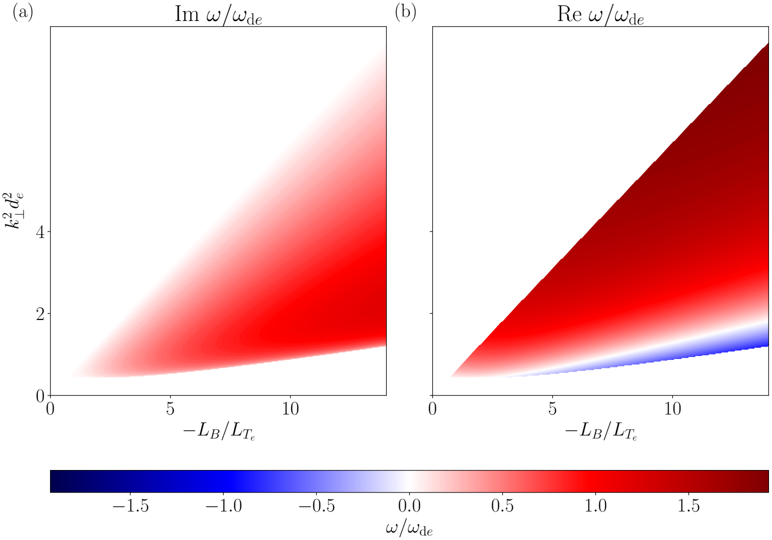
The magnetic-drift mode



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- The MDM dispersion relation is

$$
\begin{aligned}
0 = & -k_\perp^2 d_e^2 + \frac{\omega - \omega_{*e}}{\omega_{\rm de}} \left[2 + 2\sqrt{\frac{\omega}{2\omega_{\rm de}}} \mathrm{Z}\left(\sqrt{\frac{\omega}{2\omega_{\rm de}}}\right) + \frac{1}{2}\mathrm{Z}\left(\sqrt{\frac{\omega}{2\omega_{\rm de}}}\right)^2 \right] \\
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- Seems to be related to a magnetic drift wave propagating against the magnetic drifts.

The magnetic drift wave

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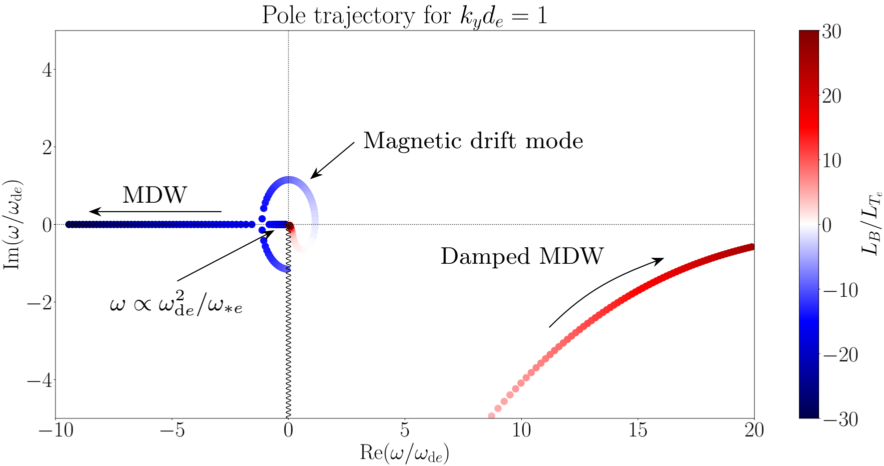
- This wave **exists if and only if there are curvature drifts**.



Pole trajectory for $k_y d_e = 1$
Magnetic drift mode
MDW
$\omega \propto \omega_{\mathrm{de}}^2/\omega_{*e}$
Damped MDW
$\mathrm{Im}(\omega/\omega_{\mathrm{de}})$
$\mathrm{Re}(\omega/\omega_{\mathrm{de}})$
L_B/L_{T_e}
4
2
0
−2
−4
−10
−5
0
5
10
15
20
30
20
10
0
−10
−20
−30

The magnetic-drift mode in 3D

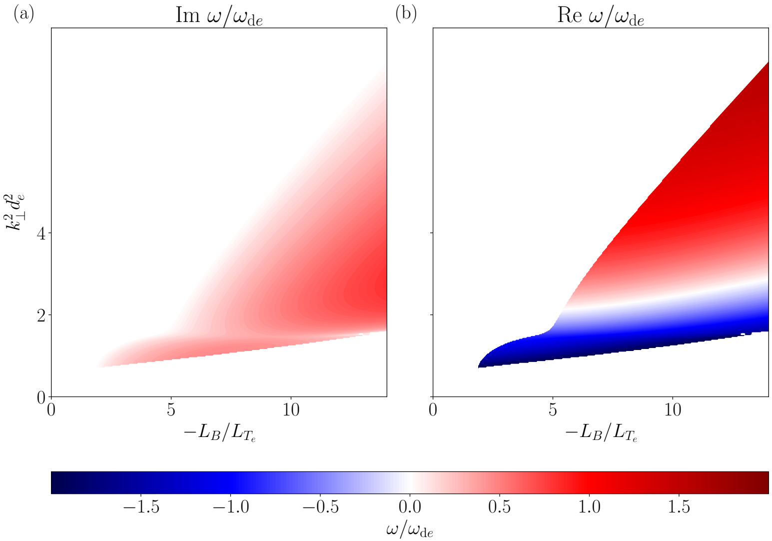


Figure 1: MDM growth rate and curvature at $k_\parallel L_B/\sqrt{\beta_e} = 2$.

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- Maybe weird parameter regime?
- **Maybe we have seen it** — related to other known instabilities?

Conclusions

- The conserved quantity

$$Y = -\sum_{\boldsymbol{k},s} \frac{q_s^2 n_{0s}}{2T_{0s}} \left(1 - \Gamma_{0s} + \frac{\Gamma_{1s}}{2}\right) |\phi_{\boldsymbol{k}}|^2 + V \sum_{\boldsymbol{k}} \left(\frac{k_\perp^2}{8\pi} + \sum_s \frac{\Gamma_{0s}}{8\pi d_s^2}\right) \left|\delta A_{\parallel \boldsymbol{k}}\right|^2$$
$$+ V \sum_{\boldsymbol{k}} \left[\left(\frac{\left|\delta B_{\parallel \boldsymbol{k}}\right|^2}{8\pi} + \sum_s n_{0s} T_{0s} \Gamma_{1s} \left|\frac{\delta B_{\parallel \boldsymbol{k}}}{B_0} + \frac{q_s \phi_{\boldsymbol{k}}}{2T_{0s}}\right|^2\right)\right] \quad (45)$$

  seems important for understanding curvature-driven instabilities.
- Such instabilities can be driven both in good and bad curvature.
- Electrostatic instabilities can be shown to live in bad curvature.
- We have at least one example of a good-curvature electromagnetic instability.

Here be more slides…

Fluxes

$$\mathit{\Gamma}_s^E \equiv \int \mathrm{d}^3\boldsymbol{r} \int \mathrm{d}^3\boldsymbol{v}\ (\boldsymbol{V}_E \cdot \boldsymbol{\nabla} x)\, \delta f_s = \int \mathrm{d}^3\boldsymbol{r} \int \mathrm{d}^3\boldsymbol{v}\ (\boldsymbol{V}_E \cdot \boldsymbol{\nabla} x)\langle h_s\rangle_{\boldsymbol{r}}\,, \tag{46}$$

$$\mathit{\Gamma}_s^{\|} \equiv n_{0s} \int \mathrm{d}^3\boldsymbol{r}\ u_{\|s} \frac{\delta B_x}{B_0}, \tag{47}$$

$$\mathit{\Gamma}_s^{\boldsymbol{\nabla} B} \equiv \int \mathrm{d}^3\boldsymbol{r} \int \mathrm{d}^3\boldsymbol{v}\ \langle \delta \boldsymbol{V}_{\boldsymbol{\nabla} B} h_s\rangle_{\boldsymbol{r}} \cdot \boldsymbol{\nabla} x, \tag{48}$$

$$Q_{\|s}^E \equiv \int \mathrm{d}^3\boldsymbol{r} \int \mathrm{d}^3\boldsymbol{v}\ m_s v_\|^2\, (\boldsymbol{V}_E \cdot \boldsymbol{\nabla} x)\langle h_s\rangle_{\boldsymbol{r}}\,, \tag{49}$$

$$Q_{\perp s}^E \equiv \int \mathrm{d}^3\boldsymbol{r} \int \mathrm{d}^3\boldsymbol{v}\ \frac{1}{2} m_s v_\perp^2\, (\boldsymbol{V}_E \cdot \boldsymbol{\nabla} x)\langle h_s\rangle_{\boldsymbol{r}}\,, \tag{50}$$

$$Q_{\|s}^{\|} \equiv \int \mathrm{d}^3\boldsymbol{r} \int \mathrm{d}^3\boldsymbol{v}\ m_s v_\|^2 \frac{\delta B_x}{B_0} v_\| \langle h_s\rangle_{\boldsymbol{r}}\,, \tag{51}$$

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$$Q_{\|s}^{\boldsymbol{\nabla} B} \equiv \int \mathrm{d}^3\boldsymbol{r} \int \mathrm{d}^3\boldsymbol{v}\ m_s v_\|^2 \langle \delta \boldsymbol{V}_{\boldsymbol{\nabla} B} h_s\rangle_{\boldsymbol{r}} \cdot \boldsymbol{\nabla} x, \tag{53}$$

$$Q_{\perp s}^{\boldsymbol{\nabla} B} \equiv \int \mathrm{d}^3\boldsymbol{r} \int \mathrm{d}^3\boldsymbol{v}\ \frac{1}{2} m_s v_\perp^2 \langle \delta \boldsymbol{V}_{\boldsymbol{\nabla} B} h_s\rangle_{\boldsymbol{r}} \cdot \boldsymbol{\nabla} x, \tag{54}$$

where

$$\delta \boldsymbol{V}_{\boldsymbol{\nabla} B} = \frac{1}{\Omega_s} (\hat{\boldsymbol{b}}_0 \times \boldsymbol{v}) \frac{\boldsymbol{v} \cdot \boldsymbol{\nabla}_\perp \delta B_\|}{B_0}. \tag{55}$$

Hermite-Laguerre fluid approach

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- Instead of solving the Landau initial-value problem, we can directly look for the normal modes of the system.

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- One way is to consider a truncated expansion

$$g_{l,m} = \frac{1}{n_{0e}} \int \mathrm{d}^3\boldsymbol{v}\ (-1)^l \frac{\mathrm{L}_l(v_\perp^2/v_{\mathrm{the}}^2)\mathrm{H}_m(v_\parallel/v_{\mathrm{the}})}{\sqrt{2^m m!}} \delta f_e. \tag{56}$$

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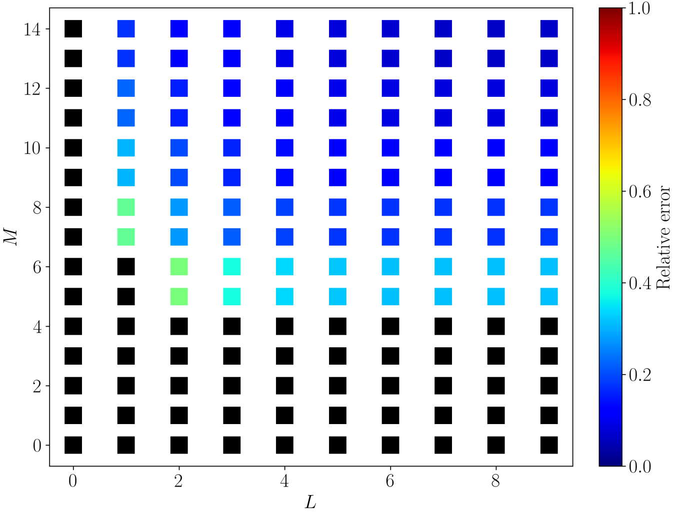
- Maybe we can capture the instability with just a handful of modes? E.g.,

$$u_{\parallel e} = v_{\mathrm{th}e} \frac{g_{0,1}}{\sqrt{2}}, \tag{57}$$

$$\delta q_{\parallel e} = n_{0e} T_{0e} v_{\mathrm{th}e} \sqrt{3} g_{0,3}, \tag{58}$$

$$\delta q_{\perp e} = n_{0e} T_{0e} v_{\mathrm{th}e} \frac{g_{0,3}}{\sqrt{2}}. \tag{59}$$

Hermite-Laguerre fluid approach



The dispersion relation

$$L_{\phi\phi}L_{AA} + 2L_{A\phi}^2 = 0, \tag{60}$$

where

$$L_{\phi\phi} = -1 - \tau^{-1} - \left[\zeta - \zeta_* + \zeta_T\left(\partial_a + \partial_b + \frac{3}{2}\right)\right] I_{a,b}\big|_{a=b=1}, \tag{61}$$

$$L_{AA} = -k_\perp^2 d_e^2 - 2\left[\zeta - \zeta_* + \zeta_T\left(\partial_a + \partial_b + \frac{3}{2}\right)\right] \partial_a I_{a,b}\big|_{a=b=1}, \tag{62}$$

$$L_{A\phi} = -\left[\zeta - \zeta_* + \zeta_T\left(\partial_a + \partial_b + \frac{3}{2}\right)\right] J_{a,b}\big|_{a=b=1}, \tag{63}$$

$$\tag{64}$$

and also

$$I_{a,b} = \frac{1}{\sqrt{\pi}}\int_{-\infty}^{+\infty} \mathrm{d}u \int_0^{+\infty} \mathrm{d}\mu \, \frac{e^{-au^2-b\mu}}{u - \zeta + \zeta_\mathrm{d}(2u^2+\mu)}, \tag{65}$$

$$J_{a,b} = \frac{1}{\sqrt{\pi}}\int_{-\infty}^{+\infty} \mathrm{d}u \int_0^{+\infty} \mathrm{d}\mu \, \frac{ue^{-au^2-b\mu}}{u - \zeta + \zeta_\mathrm{d}(2u^2+\mu)}. \tag{66}$$

$$\tag{67}$$

The dispersion relation

$$I_{a,b}\big|_{a=b=1} = -\frac{1}{2\zeta_\mathrm{d}} \mathrm{Z}_+\mathrm{Z}_- \tag{68}$$

$$\partial_a I_{a,b}\big|_{a=b=1} = -\frac{1}{2\zeta_\mathrm{d}} \left[2 + \frac{1}{\zeta_\mathrm{d}} (\mathrm{Z}_+ - \mathrm{Z}_-) + \zeta_+\mathrm{Z}_- + \zeta_-\mathrm{Z}_+ - \left(\frac{1}{4\zeta_\mathrm{d}^2} - \frac{1}{2}\mathrm{Z}_+\mathrm{Z}_-\right)\right] \tag{69}$$

$$J_{a,b}\big|_{a=b=1} = -\frac{1}{2\zeta_\mathrm{d}} \left(\mathrm{Z}_+ - \mathrm{Z}_- - \frac{\mathrm{Z}_+\mathrm{Z}_-}{2\zeta_\mathrm{d}}\right) \tag{70}$$

$$\left(\partial_a + \partial_b + \frac{3}{2}\right) I_{a,b}\big|_{a=b=1} = \frac{1}{2\zeta_\mathrm{d}} \left[\zeta_+\mathrm{Z}_- + \zeta_-\mathrm{Z}_+ + \left(\frac{\zeta}{\zeta_\mathrm{d}} + \frac{1}{4\zeta_\mathrm{d}^2} - 1\right)\mathrm{Z}_+\mathrm{Z}_-\right] \tag{71}$$

$$\left(\partial_a + \partial_b + \frac{3}{2}\right) \partial_a I_{a,b}\big|_{a=b=1} = \frac{1}{2\zeta_\mathrm{d}} \left[\frac{1}{2\zeta_\mathrm{d}^2} + \frac{\zeta}{\zeta_\mathrm{d}} - \frac{1}{2\zeta_\mathrm{d}} (\mathrm{Z}_+ - \mathrm{Z}_-) - \frac{1}{4\zeta_\mathrm{d}^2} (\zeta_+\mathrm{Z}_- + \zeta_-\mathrm{Z}_+) \right.$$
$$\left. + \frac{1}{\zeta_\mathrm{d}} \left(\zeta_+^2\mathrm{Z}_+ - \zeta_-^2\mathrm{Z}_-\right) + \frac{\zeta}{2\zeta_\mathrm{d}} (\zeta_+\mathrm{Z}_+ + \zeta_-\mathrm{Z}_-) + \frac{1}{4\zeta_\mathrm{d}^2} \left(\frac{3}{2} - \frac{1}{4\zeta_\mathrm{d}^2} - \frac{\zeta}{\zeta_\mathrm{d}} + 2\zeta\zeta_\mathrm{d}\right)\mathrm{Z}_+\mathrm{Z}_-\right] \tag{72}$$

$$\left(\partial_a + \partial_b + \frac{3}{2}\right) J_{a,b}\big|_{a=b=1} = \frac{1}{2\zeta_\mathrm{d}} \left[\frac{1}{2\zeta_\mathrm{d}} - \frac{1}{2} (\mathrm{Z}_+ - \mathrm{Z}_-) - \frac{1}{2\zeta_\mathrm{d}} (\zeta_+\mathrm{Z}_- + \zeta_-\mathrm{Z}_+) \right.$$
$$\left. + \zeta_+^2\mathrm{Z}_+ - \zeta_-^2\mathrm{Z}_- + \frac{1}{2\zeta_\mathrm{d}} \left(1 - \frac{1}{4\zeta_\mathrm{d}^2} - \frac{\zeta}{\zeta_\mathrm{d}}\right)\mathrm{Z}_+\mathrm{Z}_-\right], \tag{73}$$

where

$$\zeta_\pm \equiv \frac{\sqrt{1 + 8\zeta_\mathrm{d}\zeta} \pm 1}{4\zeta_\mathrm{d}} \tag{74}$$

and $\mathrm{Z}_\pm = \mathrm{Z}(\zeta_\pm)$.

Electrostatic 2D dispersion

$$
\begin{aligned}
0 = & -1 - \tau^{-1} + \frac{\omega - \omega_{*e}}{2\omega_{\mathrm{de}}} \mathrm{Z}\left(\sqrt{\frac{\omega}{2\omega_{\mathrm{de}}}}\right)^{2} \\
& - \frac{\omega_{Te}}{2\omega_{\mathrm{de}}}\left[2\sqrt{\frac{\omega}{2\omega_{\mathrm{de}}}}\mathrm{Z}\left(\sqrt{\frac{\omega}{2\omega_{\mathrm{de}}}}\right) + \left(\frac{\omega}{\omega_{\mathrm{de}}} - 1\right)\mathrm{Z}\left(\sqrt{\frac{\omega}{2\omega_{\mathrm{de}}}}\right)^{2}\right],
\end{aligned}
\tag{75}
$$

Ivanov, P.g. & Adkins, T. 2023 An analytical form of the dispersion function for local linear gyrokinetics in a curved magnetic field. *J. Plasma Phys.* **89** (2).

Plunk, G. G., Cowley, S. C., Schekochihin, A. A. & Tatsuno, T. 2010 Two-dimensional gyrokinetic turbulence. *J. Fluid Mech.* **664**, 407–435.

Romanelli, F. 1989 Ion temperature-gradient-driven modes and anomalous ion transport in tokamaks. *Phys. Fluids B* **1** (5), 1018–1025.

Schekochihin, A. A., Cowley, S. C., Dorland, W., Hammett, G. W., Howes, G. G., Quataert, E. & Tatsuno, T. 2009 Astrophysical gyrokinetics: Kinetic and fluid turbulent cascades in magnetized weakly collisional plasmas. *Astrophys. J. Suppl. Ser.* **182** (1), 310–377.

Terry, P., Anderson, W. & Horton, W. 1982 Kinetic effects on the toroidal ion pressure gradient drift mode. *Nucl. Fusion* **22** (4), 487–497.

Zocco, A., Xanthopoulos, P., Doerk, H., Connor, J. W. & Helander, P. 2018 Threshold for the destabilisation of the ion-temperature-gradient mode in magnetically confined toroidal plasmas. *Journal of Plasma Physics* **84** (1).