MAX-PLANCK-INSTITUT
FÜR PLASMAPHYSIK

Designing stellarators with a transport barrier

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EUROfusion

Motivation

- **A strongly sheared ExB flow is believed to cause suppress turbulence.**

- **Is it possible to design a stellarator so that this occurs at some pre-defined location?**

Radial electric field

Radial current in gyrokinetics

- According to standard gyrokinetics, the radial current from small-scale fluctuations vanishes to lowest order.

$$\left.\begin{aligned}
f_{a1} &= -\frac{e_a \delta\phi(\boldsymbol{r}, t)}{T_a} f_{a0} + g_a(\boldsymbol{R}, H, \mu, t), \\
\sum_a \frac{n_a e_a^2}{T_a}\delta\phi &= \sum_a e_a \int g_a J_0 \, \mathrm{d}^3 v, \\
\delta A_\parallel &= \frac{\mu_0}{k_\perp^2} \sum_a e_a \int v_\parallel g_a J_0 \, \mathrm{d}^3 v, \\
\delta B_\parallel &= -\frac{\mu_0}{k_\perp} \sum_a e_a \int v_\perp g_a J_1 \, \mathrm{d}^3 v, \\
\chi &= \delta\phi - \boldsymbol{v}\cdot\delta\boldsymbol{A} \\
\delta\Gamma_a \cdot \nabla r &= \int g_a \frac{\mathbf{b}\times\nabla\langle\chi\rangle_{\mathbf{R}}}{B}\cdot\nabla r \, d^3 v,
\end{aligned}\right\} \Longrightarrow \quad \delta\mathbf{J}\cdot\nabla r = \sum_a e_a \delta\boldsymbol{\Gamma}_a \cdot \nabla r = 0,$$

Sugama et al. 1996

Parra & Catto, 2008

Radial current from neoclassical transport

- Neoclassical radial particle flux of each species σ

$$\Gamma_\sigma = -D_\sigma(E_r) n_\sigma \left(\frac{d \ln n_\sigma}{dr} - \frac{e_\sigma E_r}{T_\sigma} + \delta_\sigma \frac{d \ln T_\sigma}{dr} \right).$$

- In most stellarators, this flux is ambipolar, $\sum_\sigma e_\sigma \Gamma_\sigma = 0,$ only for one or a few values of E_r.

- This condition determines E_r even if most of the transport is turbulent!

- Exceptions:
  - unnecessarily well neoclassically-optimised fields $(D_\sigma \sim \rho_* D_{\mathrm{gB}})$
  - axisymmetric and (perhaps) quasisymmetric fields
  - small scales: zonal flows

Radial electric field

Neoclassical ambipolarity equation is nonlinear

$$\Gamma_i(E_r) = \Gamma_e(E_r)$$

Usually $E_r < 0$ (ion root) since $D_e < D_i$.

- Causes strong inward neoclassical transport for highly charged impurities.

$E_r > 0$ (electron root) has been observed in low-density plasmas with $T_e > T_i$.

- Beneficial for impurity expulsion
- Hitherto thought to be impossible in reactors since $T_e = T_i$.

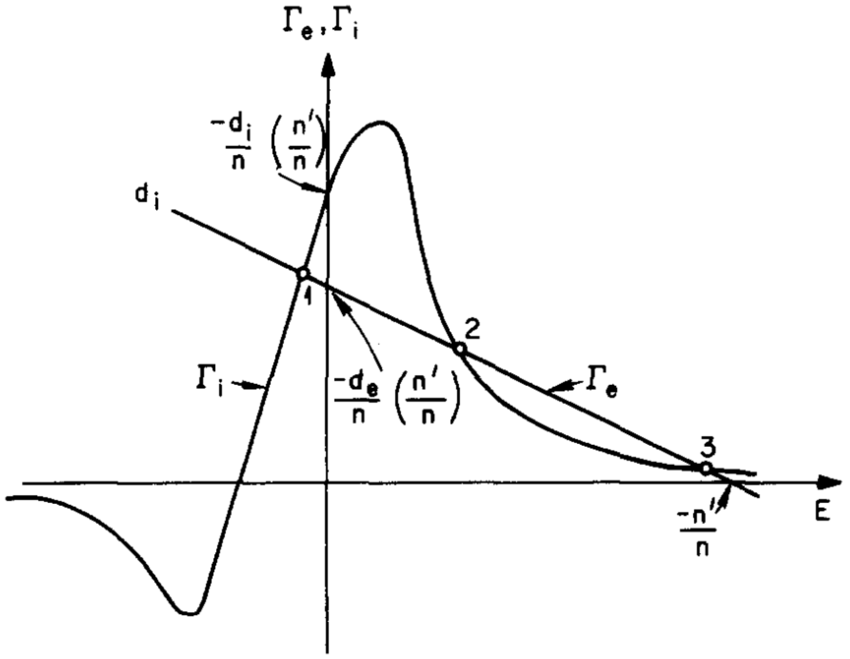


FIG. 1. Electron and ion flux against electric field for model problem.

Hastings, Nucl. Fusion 1986

Experimental results

- In most stellarators, the radial electric field broadly follows the predictions from neoclassical theory.

- Electron roots predicted and observed in LHD, CHS, W7-AS and TJ-II at low density when $T_i < T_e$.
  - accompanied by steep T_e profiles (transport barrier) in the core.
  - expected hysteresis observed in W7-AS (Stroth PRL 2001).

- Electron root not expected nor observed
  - in any present-day stellarator at moderate or high densities, where $T_i = T_e$,
  - or in HSX although $T_i \ll T_e$.

- Further verification of theory underway in W7-X.

Neoclassical transport of electrons and ions

The diffusion coefficient for a particle of speed v depends on two dimensionless parameters:

$$\nu^* = \frac{\nu a}{v} \quad \text{and} \quad \text{Ma} = \frac{E_r}{Bv}$$

Small-Ma limit

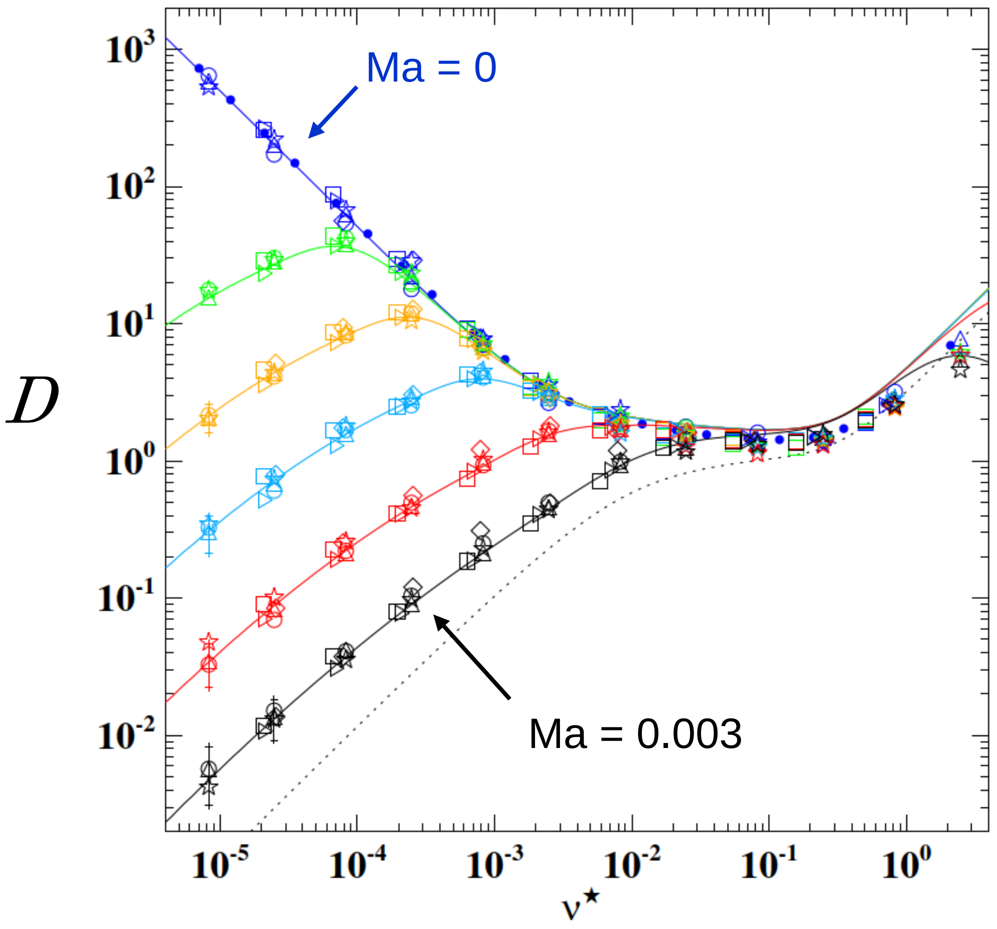
$$D^{1/\nu} \sim \frac{\epsilon_{\text{eff}}^{3/2} v^2 \rho_*^2}{\nu},$$

Larger Ma:

$$D^{\sqrt{\nu}} \sim \frac{\nu^{1/2} v^2 \rho_*^2}{\omega_E^{3/2}} \sqrt{\ln\left(\frac{\omega_E}{\nu}\right)}$$

$$\omega_E = \frac{E_r}{aB}$$

Galeev et al. 1969



Beidler et al. 2011

Neoclassical transport of electrons and ions

- In the ion root, the diffusion coefficients are given by

$$D_e^{1/\nu} \sim \frac{\epsilon_{\text{eff}}^{3/2} v_{de}^2}{\nu_e}, \qquad D_i^{\sqrt{\nu}} \sim \epsilon_i^{3/2} \frac{\nu_i^{1/2} \rho_{*i}^2 v_{Ti}^2}{\omega_E^{3/2}} \sqrt{\ln\left(\frac{\omega_E}{\nu_i}\right)}$$

where ε_i and ε_{eff} are coefficients depending only on the B-field geometry.

- Ion transport (determined by ε_i) is controlled mostly by shallowly trapped particles.
- Electron transport (determined by ε_{eff}) depends on all trapped particles.

Notation:

$$\omega_E = \frac{E_r}{aB}$$

$$\rho_{*i} = \frac{v_{Ti}}{a\Omega_i}$$

$$\nu_{*i} = \frac{\nu_i a}{v_{Ti}}$$

Neoclassical theory of electron root optimisation

- Onset of electron root approximately when

$$D_e^{1/\nu} > D_i^{\sqrt{\nu}} \qquad \Rightarrow \qquad \frac{T_e}{T_i} \geq \left(\frac{m_i}{m_e}\right)^{1/7} \left(\frac{\epsilon_i \nu_{*i}}{\epsilon_{\rm eff} \rho_{*i}}\right)^{3/7}$$

- Electron root thus possible by targetted (de)-optimisation
  - Decrease the ratio $\epsilon_i / \epsilon_{\rm eff}$.
  - Improve confinement of shallowly trapped particles, degrade it for deeply trapped ones.

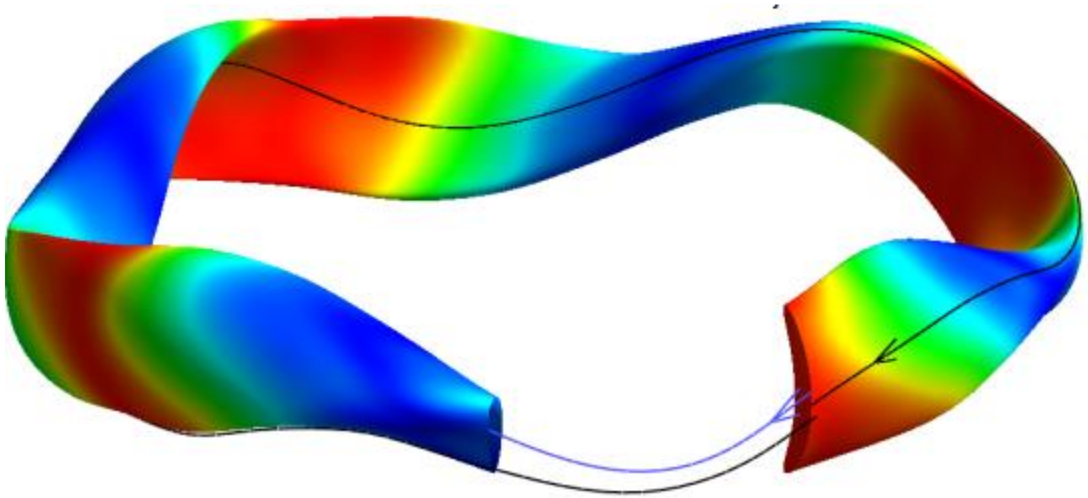
A concrete example

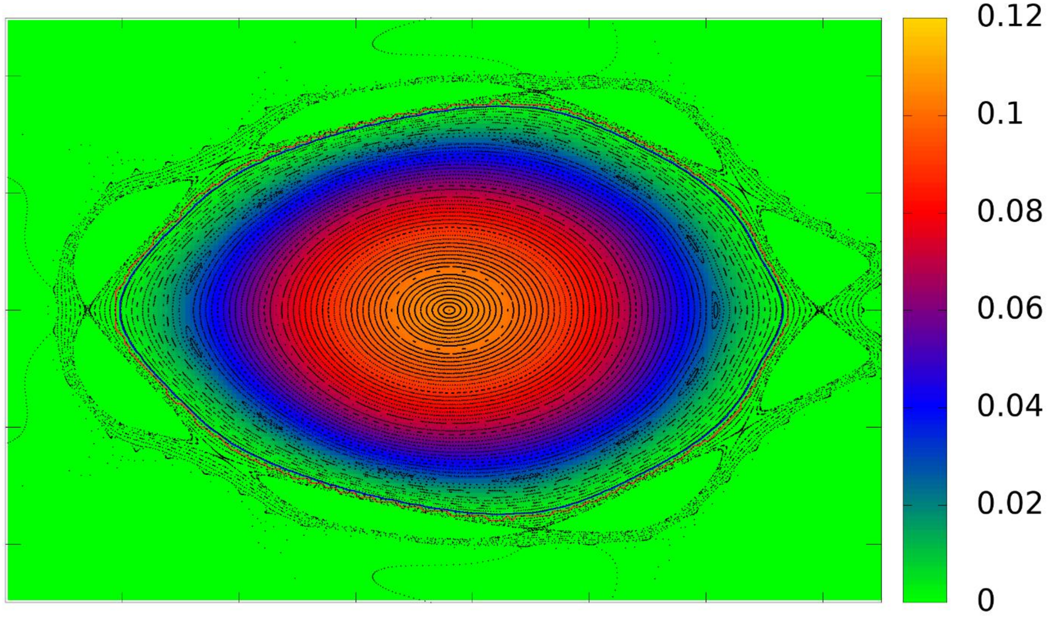
Optimisation goals

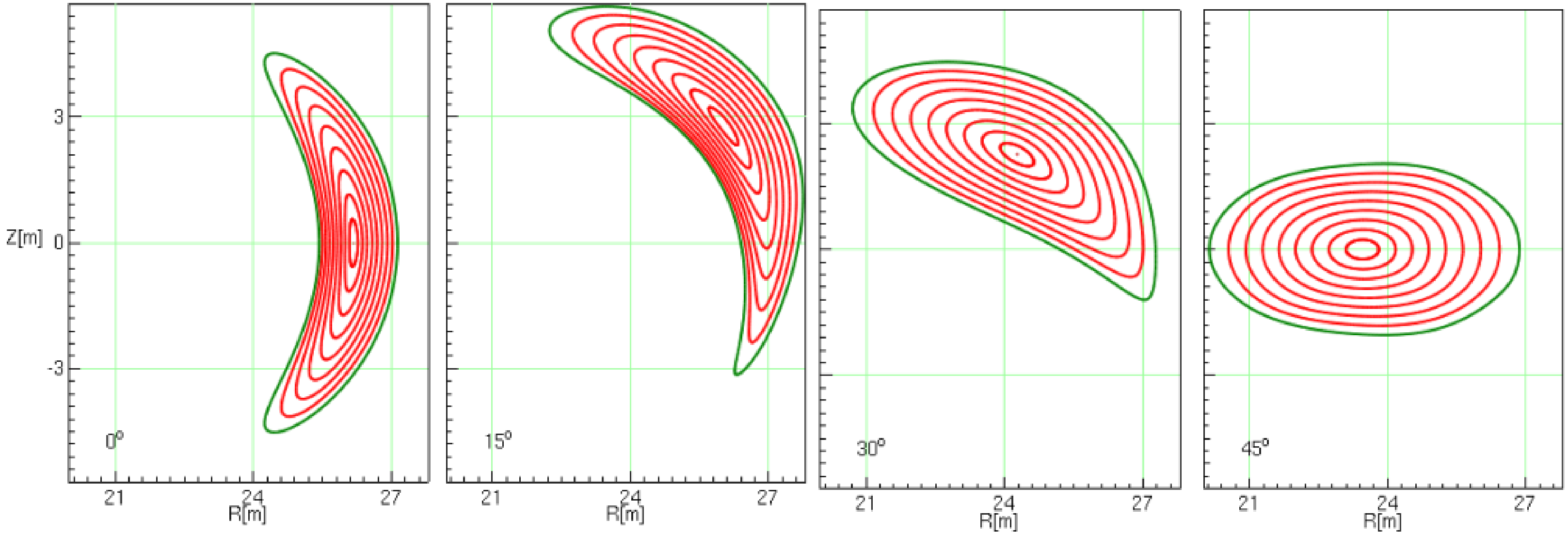
- Quasi-isodynamic magnetic field, implying
    - Good fast-ion confinement
    - Small neoclassical transport
    - Negligible bootstrap current
- Reduced ITG- and TEM-driven turbulence
- MHD stable up to some target β
    - maximum-J property at this β
- Edge islands for divertor operation
- Coils simpler than, or comparable to, those of W7-X

Until very recently, these goals seemed incompatible with each other, but not anymore

SQuID: stable quasi-isodynamic design







A=10, N = 4

Considerable interest from stellarator startups

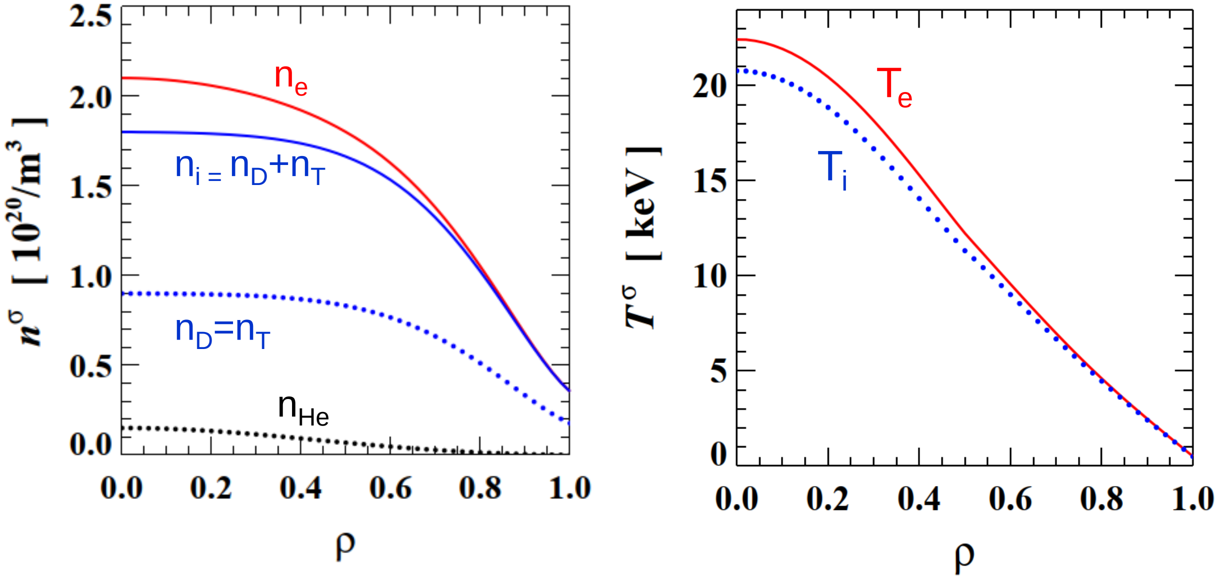
Goodman et al ,2024

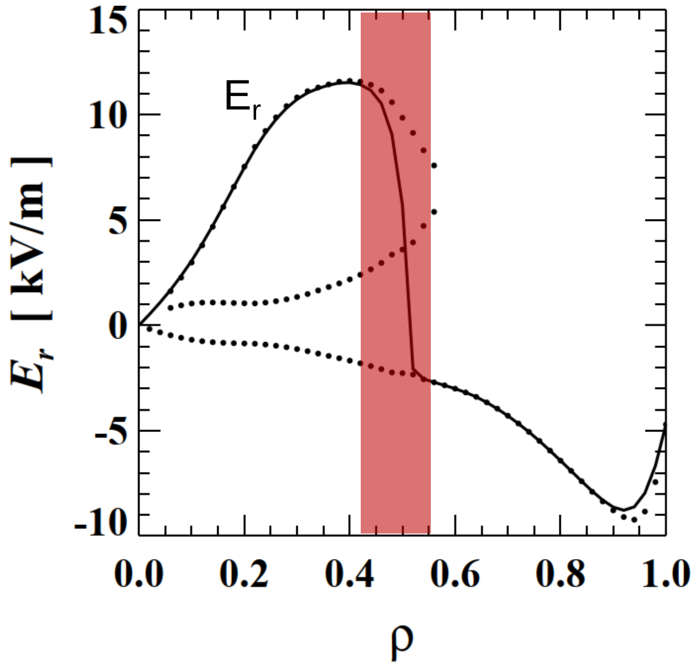
Electron root in SQuID

Reactor case: V=1450 m^3, R = 20.13 m, a = 1.91 m, B=5.4 T

Transition region:

- Strongly sheared electric field
- Transport barrier?

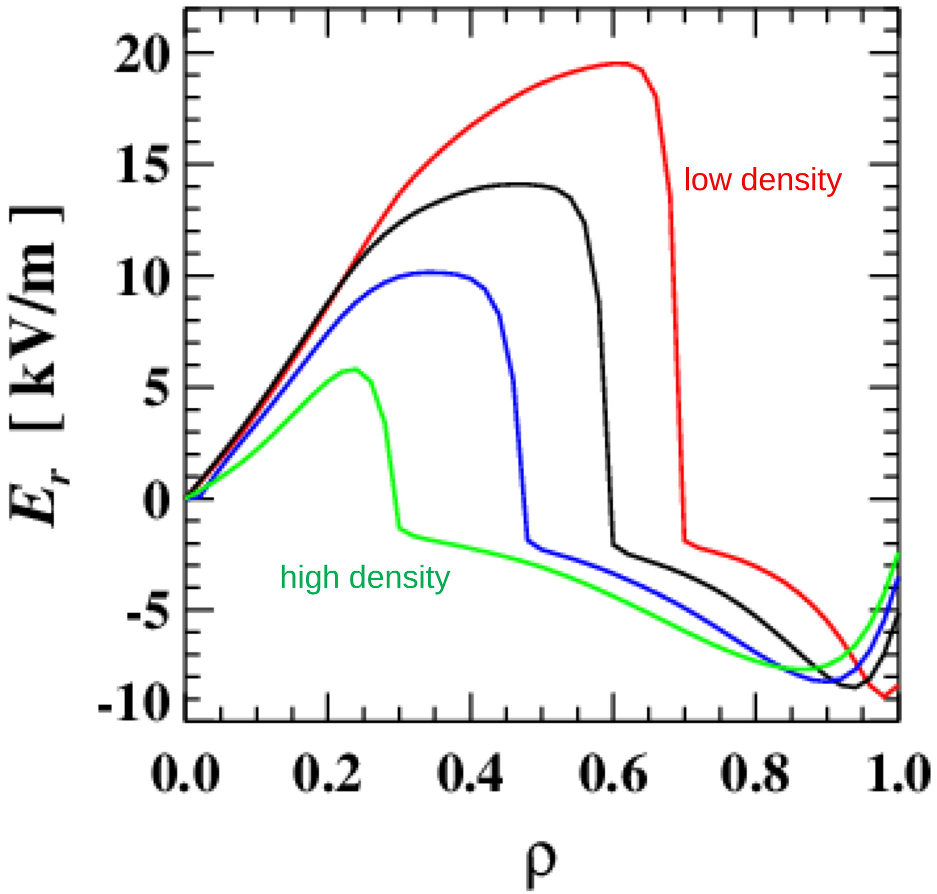




Electric field

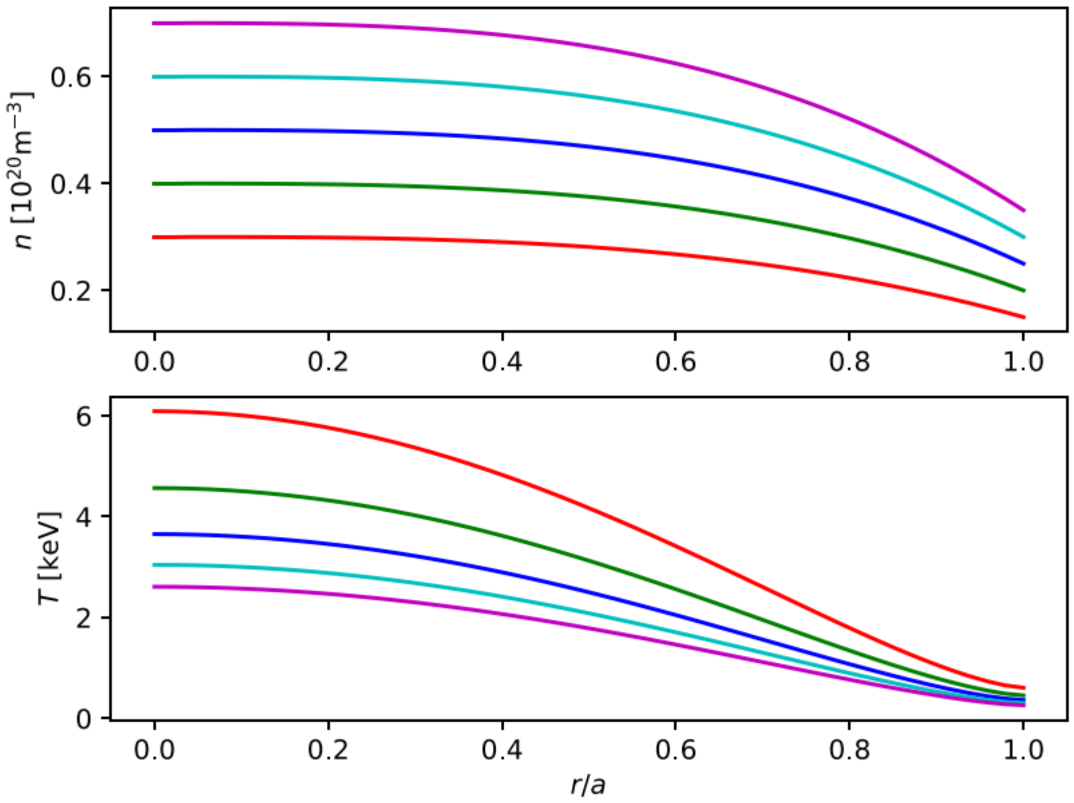
Density scan

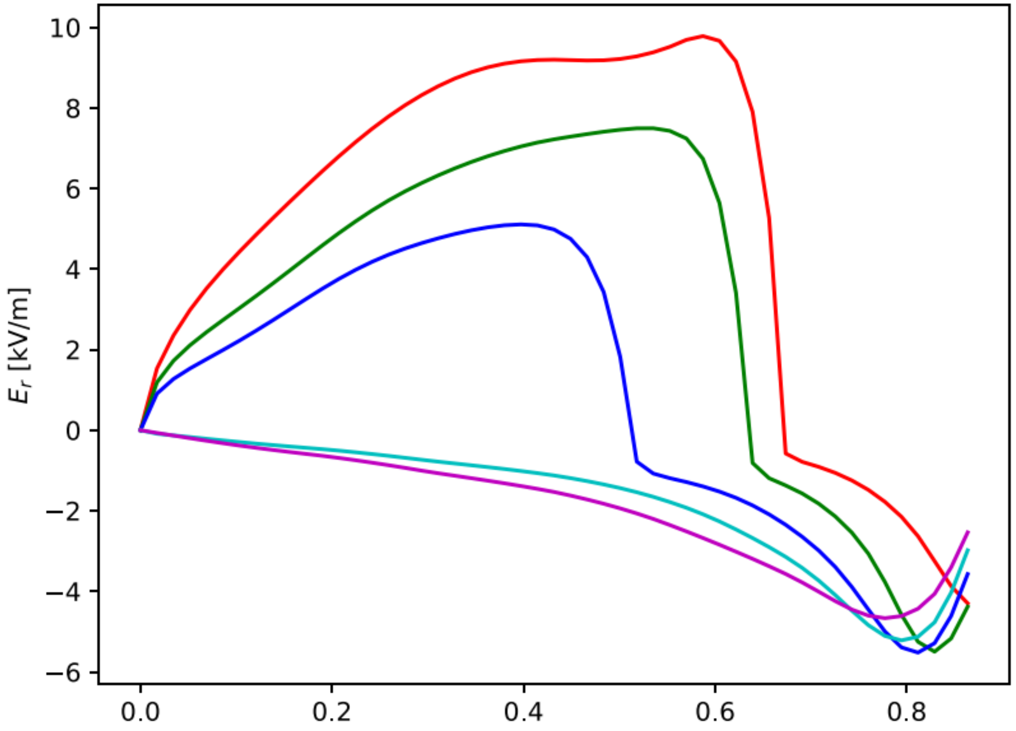
Central density varied from $n_e(0)= 1.4{\cdot}10^{20}$ m^{-3} to $n_e = 2.4{\cdot}10^{20}$ m^{-3}



Electron root in W7X-size device with $T_e = T_i$

- Scaled to W7-X volume and field strength
- Density and temperature profiles such that $\langle\beta\rangle = 2\%$

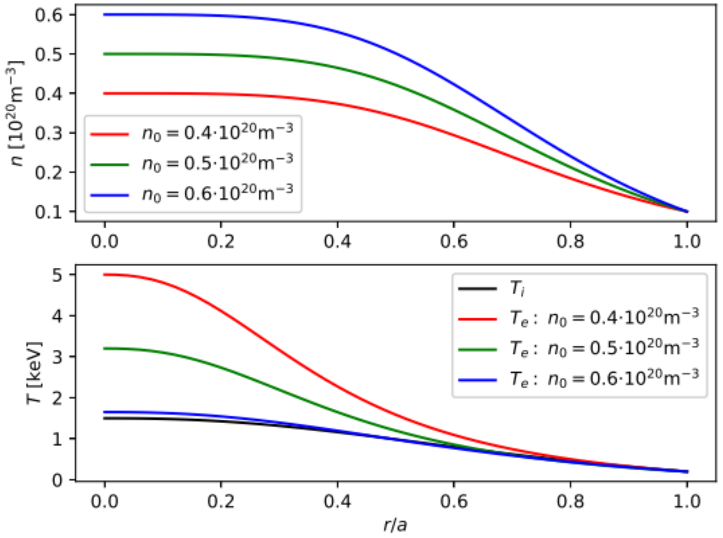




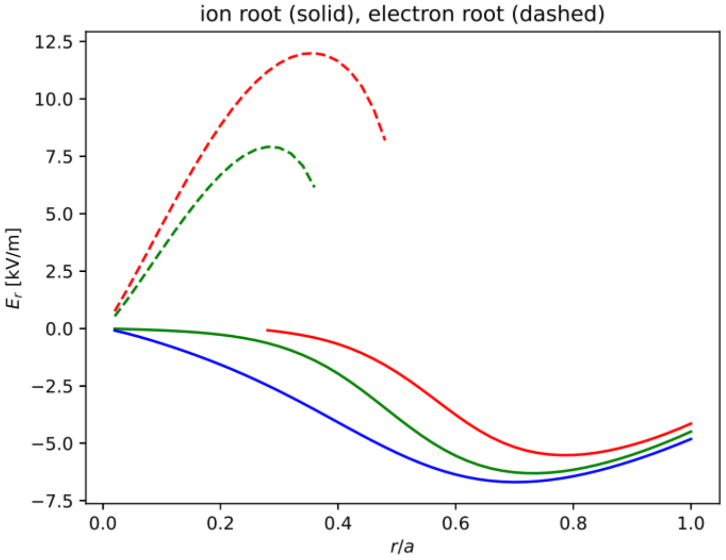
Testing predictions in W7-X

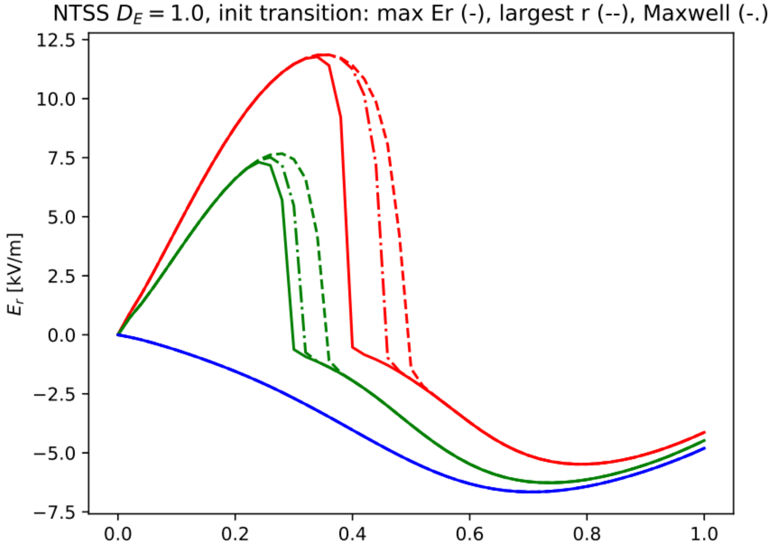
- A central feature of the SQuID electron root with $T_i = T_e$ is the simulataneous presence of three roots in the plasma core.
    - Will the plasma ”choose” the electron root?

- Could be tested in the high-mirror configuration of W7-X with $T_i < T_e$.



$$\langle\beta\rangle = 0.4\%$$





Transport barrier?

- ExB flow can suppress tubulence when (Waltz 1994, Ivanov et al, 2023)

$$\frac{1}{B}\frac{dE_r}{dr} > \gamma_{\max}$$

- For electrostatic instabilities with $k_\perp \rho_i = O(1)$

$$\gamma_{\max} = \frac{\alpha v_{Ti}}{L_\perp}, \qquad \alpha \sim 0.02$$

- If the width of the transition region is w and $E_r \sim T_i / e L_\perp$, a transport barrier should arise if

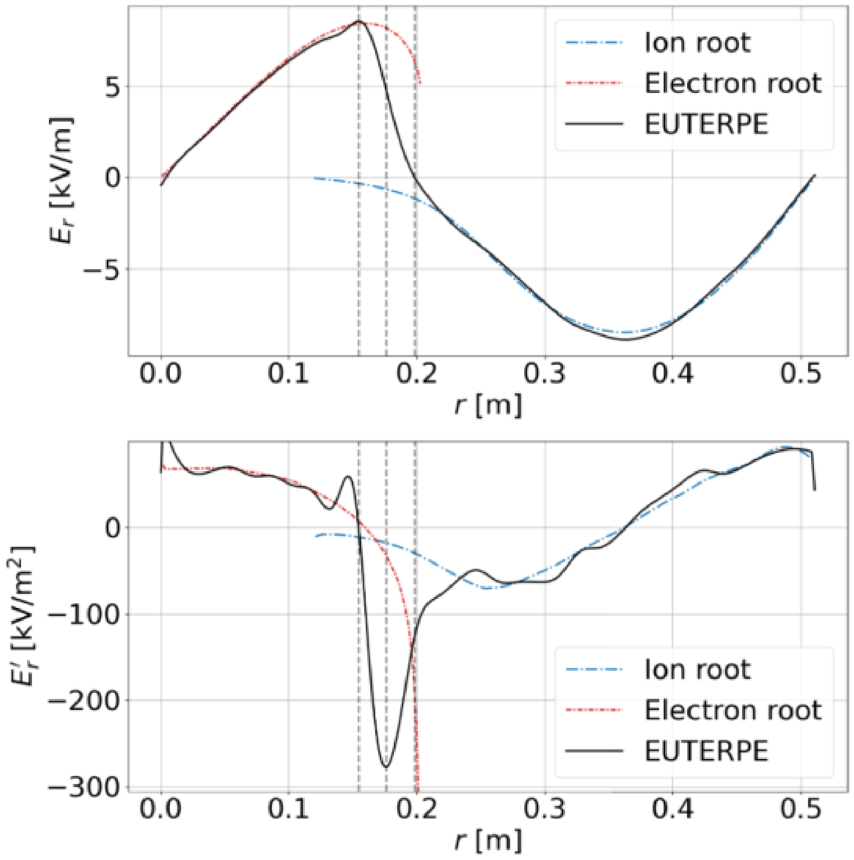
$$w \leq \frac{\rho_i}{\alpha}.$$

and increase the core temperature by at least

$$\frac{\Delta T}{T} \sim w \left| \frac{d \ln T}{dr} \right| \sim \frac{w}{L_\perp} \leq \frac{\rho_i}{\alpha L_\perp}.$$

Theoretical issues

- The electron-ion-root-transition region cannot be described by standard local neoclassical theory.
  - In the figures above instead modelled by a cruder model in the NTSS transport code.
  - Has also recently been simulated with the global gyrokinetic EUTERPE code without turbulence.
- In order to assess the strength of a transport barrier, global simulations of simultaneous neoclassical and turbulent transport should be carried out.



Kuczynski et al, 2024

Summary

- In non-quasisymmetric stellarators, the radial electric field is determined by neoclassical transport, even if most of the energy transport is turbulent.

- It is possible to tailor the magnetic field so that $E_r > 0$ in the core and $E_r < 0$ in the edge, even if $T_e=T_i$.

- Strong ExB shear arises in the transition region, probably causing a transport barrier.