A new quasi-linear transport model for electromagnetic turbulence in high-beta spherical tokamaks and its application to STEP

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Outline

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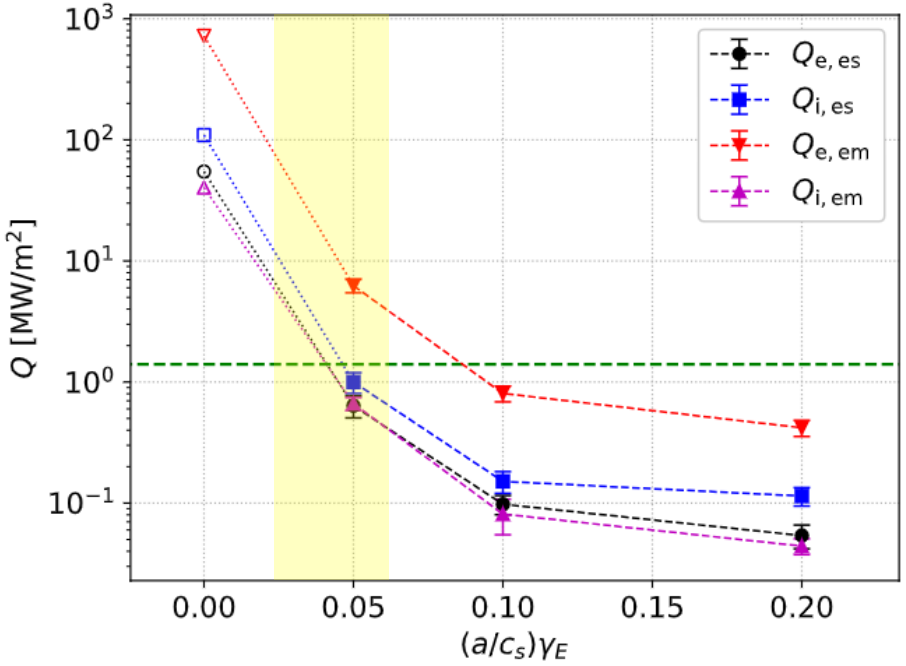
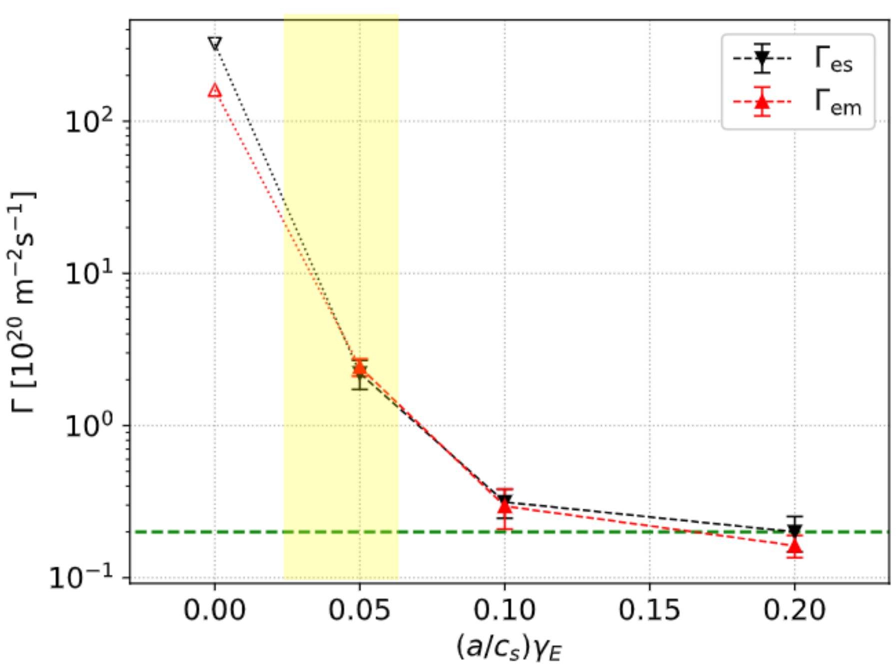
- Why building a reduced transport model for STEP?
- The quasi-linear model
  - The quasi-linear metric Λ
  - Quasi-linear fluxes
- Transport simulations with T3D
  - Implementation of the quasi-linear model in T3D
  - First flux-driven simulations of a STEP flat-top operating point
- Summary

For details see Giacomin et al arXiv:2404.17453v1, accepted for publication in JPP.

Why do we need a new reduced transport model?

D. Kennedy's talk showed that the STEP flat-top operating point obtained with JINTRAC modelling is <u>not</u> in kinetic equilibrium: gyrokinetic simulations predict large electromagnetic turbulent fluxes driven by kinetic ballooning modes.





Why do we need a new reduced transport model?

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What are our options?

1. Use other existing saturation rule implemented in available transport models such as TGLF [1] or QuaLiKiz [2, 3].

   Fast solution but inaccurate for electromagnetic turbulence such as hybrid-KBMs in STEP.

2. Use high-fidelity transport models based on nonlinear gyrokinetic simulations, such as Tango-GENE [4], PORTALS-CGYRO [5], T3D-GX [6].

   Very accurate solution but too expensive computationally.

3. Build an electromagnetic reduced transport model to specifically addressed STEP-like regimes.

[1] G.M. Stabler *et al.* Phys. Plasmas 14 (2007).
[2] C. Bourdelle *et al.* Plasma Phys. Control. Fusion 58 (2015).
[3] J. Citrin *et al.* Plasma Phys. Control. Fusion 58 (2017).
[4] A. Di Siena *et al*. Nucl. Fusion 62 (2022).
[5] P. Rodriguez-Fernandez *et al.* Nucl. Fusion 62 (2022).
[6] M. Barnes *et al.* Phys. Plasmas 17 (2010); T. Qian *et al.* Bulletin of APS (2022).

A reduced transport model for high-β ST

Computational cost / accuracy →

Fast, low-cost integrated models (not accurate for STEP)	Reduced transport model for STEP	High-fidelity GK turbulent transport models (too expensive for STEP)

$$Q \sim \delta p k_y \delta\phi \sim \gamma \frac{\delta p^2}{\partial_x p} \sim (\gamma/k_\perp^2)\partial_x p \propto \gamma/k_\perp^2 \quad \Rightarrow \quad Q_{\rm ql} = Q_0 \Lambda^\alpha$$

The quasi-linear reduced model is composed of:

- The quantity Λ ⟵ General function
- The coefficients Q_0 and α ⟵ Specific to STEP

Building the function Λ

- Perpendicular wavenumber averaged along θ

$$\langle k_\perp^2 \rangle_{k_y,\theta_0,i} = \frac{\int \mathrm{d}\theta \; k_\perp^2(\theta, k_y, \theta_0)\, J(\theta)\, |\chi_i(\theta)|^2}{\int \mathrm{d}\theta \; J(\theta) |\chi_i(\theta)|^2}$$

where $\chi_i \in \{e\delta\phi/T_e, \delta A_\| / (\rho_s B_0), \delta B_\| / B_0\}$

- Summing $\gamma/\langle k_\perp^2 \rangle$ over all the three fields

$$\hat{\Lambda}(k_y, \theta_0) = \sum_{i=1}^{3} \frac{\max_\theta |\chi_i(k_y, \theta_0, \theta)|}{\max_\theta |\chi_1(k_y, \theta_0, \theta)|} \, \frac{\gamma(k_y, \theta_0)}{\langle k_\perp^2(k_y, \theta_0, \theta) \rangle_{\theta,i}}$$

- Integrating over θ_0

$$\bar{\Lambda}(k_y) = \frac{1}{\theta_{0,\max}(k_y, \gamma_E)} \int_0^{\theta_{0,\max}(k_y,\gamma_E)} \hat{\Lambda}(k_y, \theta_0)\, \mathrm{d}\theta_0$$

The reduced model retains the effect of the equilibrium flow shear

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- A linearly growing mode in presence of flow shear is “tilted” in time, $k_x = \gamma_E k_y \Delta t$.
- The value of k_x increases to $k_x = \gamma_E k_y / \gamma$ after a growth time.
- k_x is related to θ_0 as $k_x = \hat{s}\theta_0 k_y$ (in ballooning space)
- The value of θ_0 reached after a growth time (and limited to π) is therefore

$$\theta_{0,\mathrm{max}} = \min\left(\frac{\gamma_E}{\hat{s}\gamma},\ \pi\right)$$

- Small values of γ_E are sufficient to reduce the value of Λ if $\gamma(\theta_0)$ decreases with θ_0.
- The average over $\theta_0 \in [0, \theta_{0,max}]$ reduces to the value at $\theta_0 = 0$ at $\gamma_E = 0$.

Building the function Λ

- Perpendicular wavenumber averaged along θ

$$\langle k_\perp^2 \rangle_{k_y,\theta_0,i} = \frac{\int \mathrm{d}\theta\ k_\perp^2(\theta, k_y, \theta_0)\, J(\theta)\, |\chi_i(\theta)|^2}{\int \mathrm{d}\theta\ J(\theta)|\chi_i(\theta)|^2}$$

where $\chi_i \in \{e\delta\phi/T_e, \delta A_\| /(\rho_s B_0), \delta B_\| / B_0\}$

- Summing $\gamma/\langle k_\perp^2 \rangle$ over all the three fields

$$\hat{\Lambda}(k_y, \theta_0) = \sum_{i=1}^{3} \frac{\max_\theta |\chi_i(k_y, \theta_0, \theta)|}{\max_\theta |\chi_1(k_y, \theta_0, \theta)|}\ \frac{\gamma(k_y, \theta_0)}{\langle k_\perp^2(k_y, \theta_0, \theta)\rangle_{\theta,i}}$$

- Integrating over θ_0

$$\bar{\Lambda}(k_y) = \frac{1}{\theta_{0,\max}(k_y, \gamma_E)} \int_0^{\theta_{0,\max}(k_y,\gamma_E)} \hat{\Lambda}(k_y, \theta_0)\, \mathrm{d}\theta_0$$

- Finally integrating over k_y

$$\Lambda = \int \mathrm{d}k_y\, \bar{\Lambda}(k_y)$$

$$Q_{\mathrm{ql}} = Q_0 \Lambda^\alpha$$

The quasi-linear model

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Linear flux ratio (species over total) as quasi-linear weight (similar for particle flux)

$$Q_{\mathrm{ql},s} = \frac{Q_0 \Lambda^{\alpha-1}}{\rho_* c_s} \int \mathrm{d}k_y \, \frac{1}{\theta_{0,\max}} \int_0^{\theta_{0,\max}} \mathrm{d}\theta_0 \, \frac{Q_{1,s}(k_y,\theta_0)}{Q_1(k_y,\theta_0)} \sum_{i=1}^{3} \frac{\max_\theta |\chi_i(k_y,\theta_0,\theta)|}{\max_\theta |e\delta\phi(k_y,\theta_0,\theta)/T_e|} \frac{\gamma(k_y,\theta_0)}{\langle k_\perp^2(k_y,\theta_0,\theta)\rangle_{\theta,i}}$$

Normalizing factor

Equilibrium flow shear suppression

Weighted sum over each field contribution

$$\Lambda = \frac{1}{\rho_* c_s} \int \mathrm{d}k_y \, \frac{1}{\theta_{0,\max}} \int_0^{\theta_{0,\max}} \mathrm{d}\theta_0 \sum_{i=1}^{3} \frac{\max_\theta |\chi_i(k_y,\theta_0,\theta)|}{\max_\theta |e\delta\phi(k_y,\theta_0,\theta)/T_e|} \frac{\gamma(k_y,\theta_0)}{\langle k_\perp^2(k_y,\theta_0,\theta)\rangle_{\theta,i}} \quad \Rightarrow \quad \sum_s Q_{\mathrm{ql,s}} = Q_0 \Lambda^\alpha$$

The coefficients Q_0 and α

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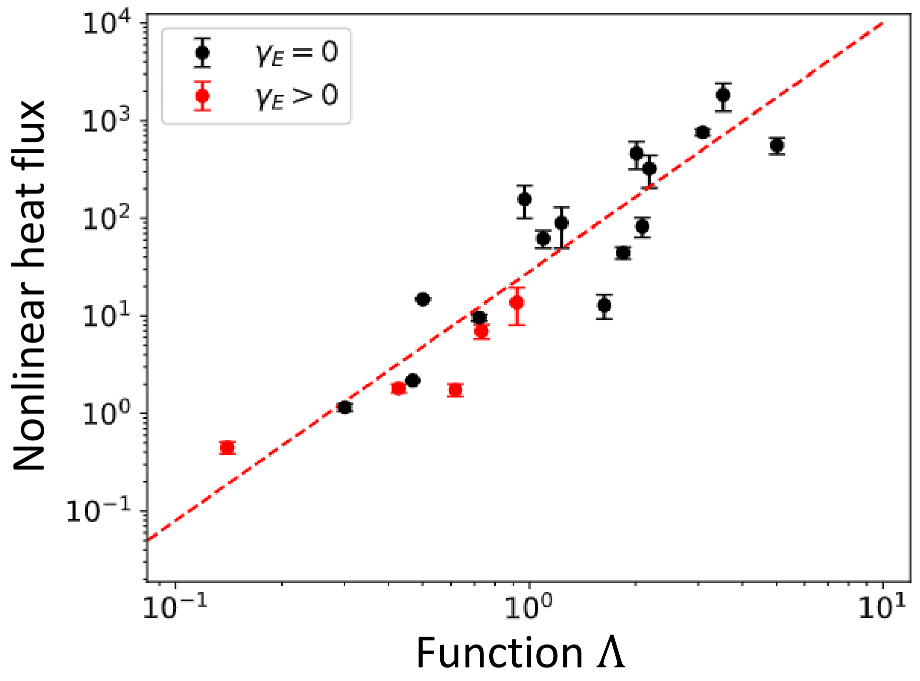
- The function $\Lambda(\mathrm{k_y}, \theta_0)$ is computed directly from linear GK simulations.
- The coefficients Q_0 and α link Λ to the quasi-linear total heat flux.
- How are these coefficients computed? From a database of nonlinear GK STEP simulations!

The database includes heat flux values from about 20 nonlinear simulations with and without the equilibrium flow shear, different pressure gradients, β (and β'), $\hat{s}$ and q.



Implementation in T3D

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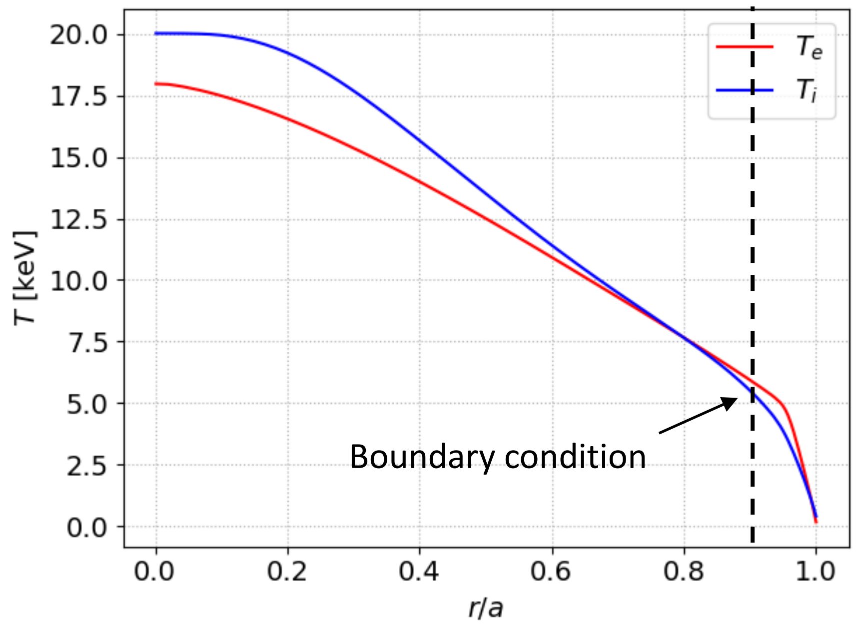
- T3D [1] is an improved version of Trinity [2], written in Python with a highly modular structure.
- Transport equations solved by T3D:

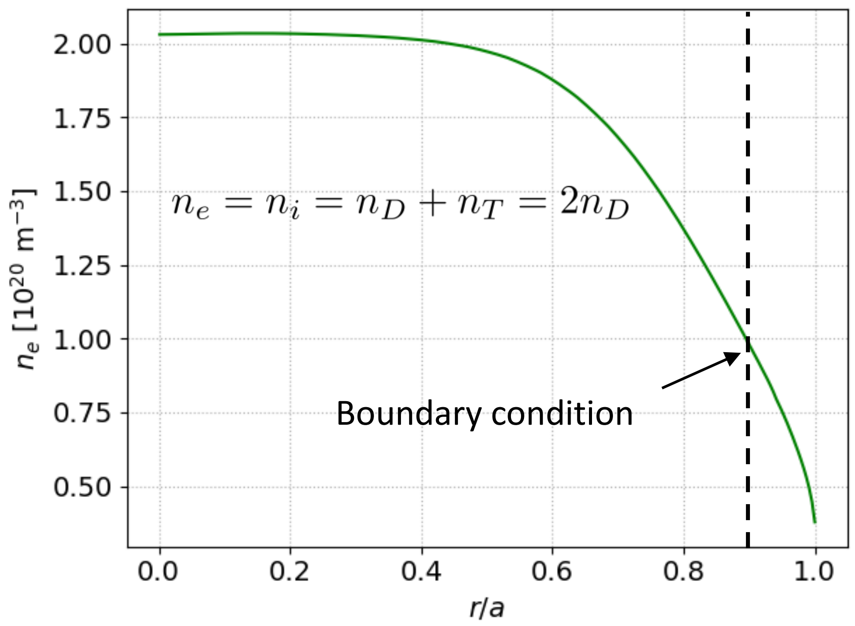
$$\frac{\partial n_s}{\partial t} + \frac{1}{V'}\frac{\partial}{\partial \Psi}(V'\overline{\Gamma_s}) = \overline{S_{n,s}}$$
$$\frac{3}{2}\frac{\partial p_s}{\partial t} + \frac{1}{V'}\frac{\partial}{\partial \Psi}(V'\overline{Q_s}) = \frac{3}{2} n_s \sum_u \nu_{su}(T_u - T_s) + \overline{S_{p,s}}$$

- Transport equations require the value of the fluxes Γ_s and Q_s, which are computed from the reduced transport model.
- T3D is coupled to GS2 to compute the function Λ.
- Different neoclassical models are available in T3D. Here we use NEO [3].

[1] T. Qian *et al.* Bulletin of APS (2022).
[2] M. Barnes *et al.* Phys. Plasmas 17 (2010).
[3] E.A. Belli & J. Candy *et al.* Plasma Phys. Control. Fusion 50 (2008).

Numerical setup and initial state





T3D-GS2 numerical parameters	
N_r	6
r_{edge}/a	0.9
N_p	3
n_{k_y}	12
n_{θ_0}	6
n_{min}	2
n_θ	33
n_λ	24
n_ϵ	10

- The initial state is the STEP-EC-HD from JINTRAC-JETTO [1].
- 150 MW ECHR, 340 MW radiated power, 360 MW initial alpha heating.
- Particle source from pellet injection.
- Shearing rate from neoclassical radial electric field (JINTRAC-JETTO).
- $(N_p+1)(N_r-1)n_{k_y}n_{\theta_0} = 1440$ single mode linear GS2 simulations per T3D iteration.
- Finite Dirichlet boundary condition at r/a = 0.9.

[1] E. Tholerus *et al*. Submitted to Nucl. Fusion (arXiv:2403.09460).

There are three main approximations

1. Only density and temperature profiles of electrons and a single thermal ion species are evolved. The quasi-neutrality is enforced on the ion species, such that $n_i = n_D + n_T = n_e$, where $n_D = n_T = n_i/2$. **Impurities and fast α particles are neglected**.

2. The geometrical shaping parameters, q and $\hat{s}$ are considered constant while evolving the pressure profile. **The parameter β' is evolved with the pressure profile**.

3. The **equilibrium flow** shear is computed from the initial radial electric field evaluated from the ion pressure gradient and neoclassical flows, and it **is kept constant** as the kinetic profiles evolve. Sensitivity to γ_E is evaluated.

T3D steady state solution

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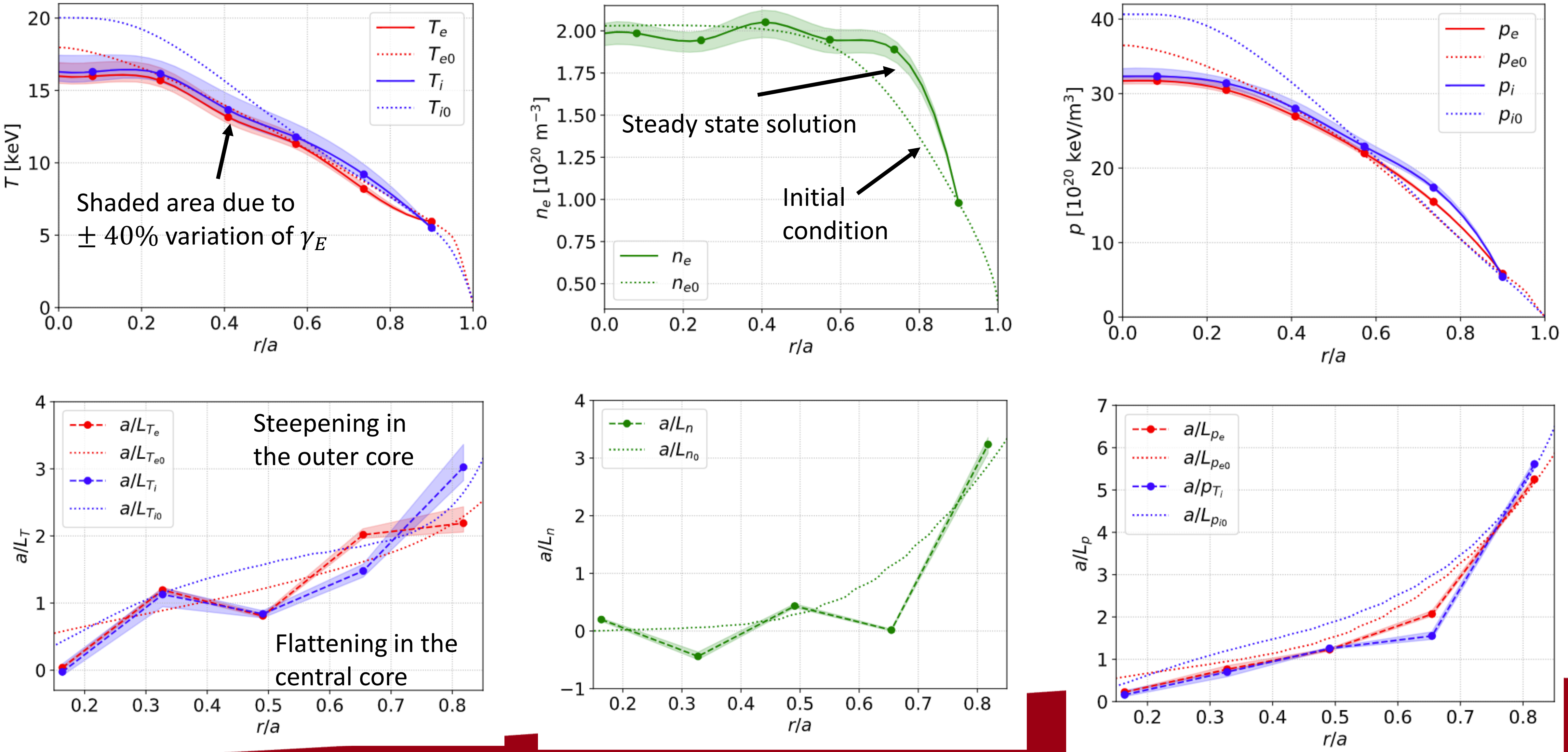
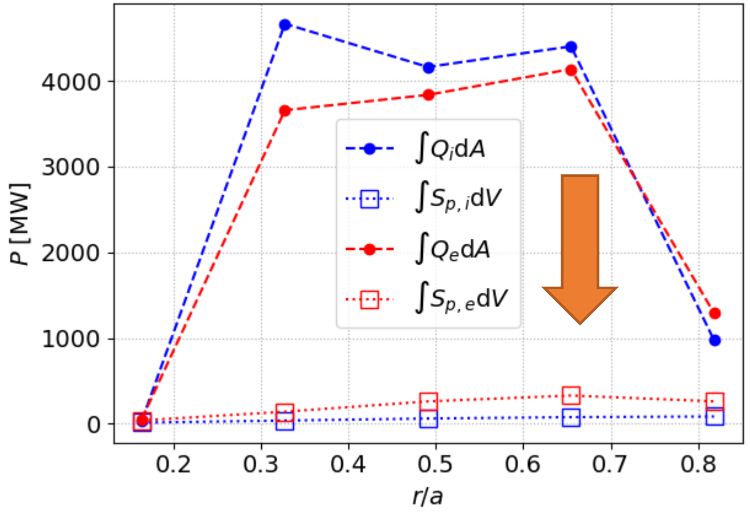
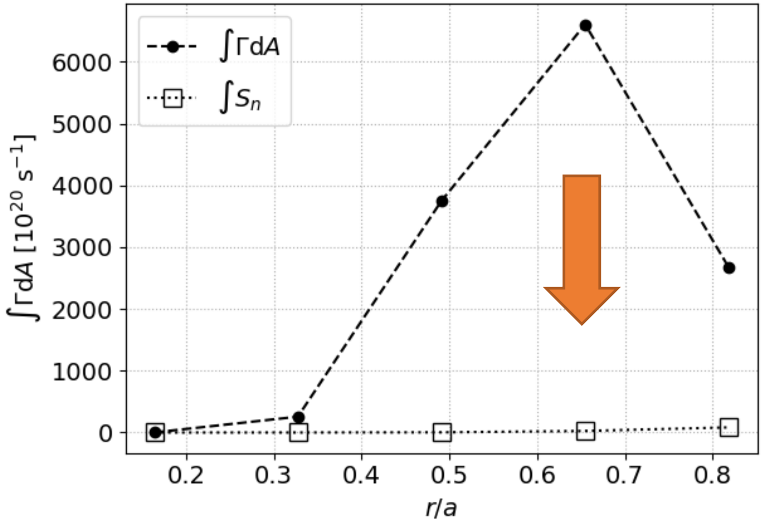


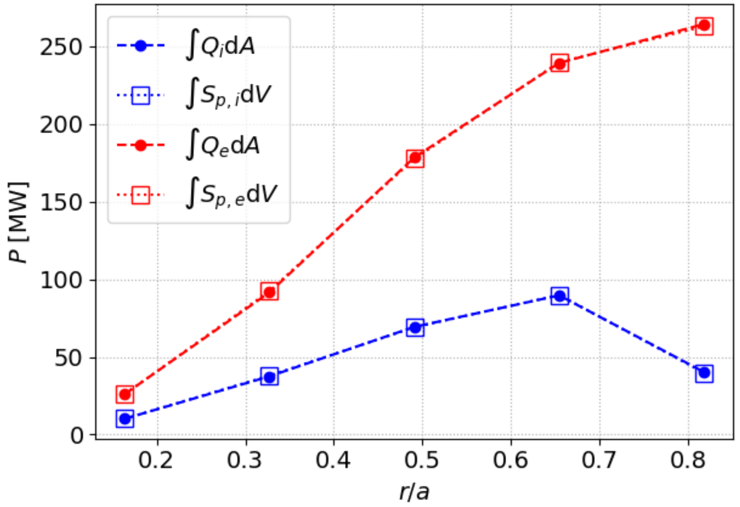
Initial strong power and particle imbalance

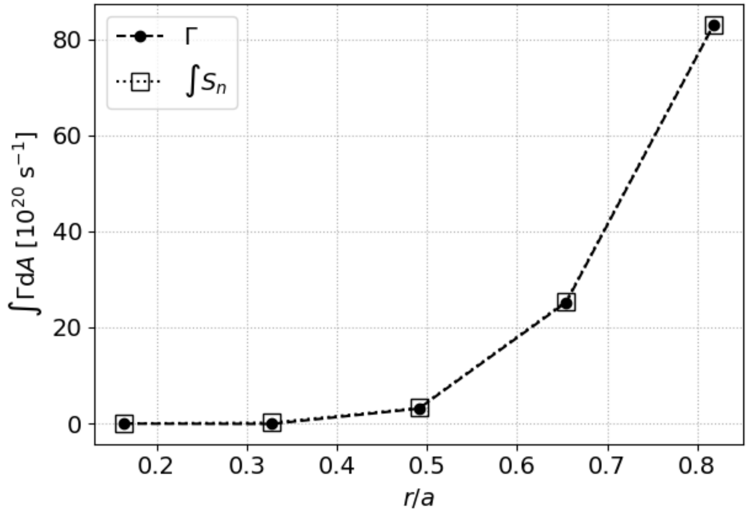
Initial power and particle balance
Power and particle losses largely exceed available sources.



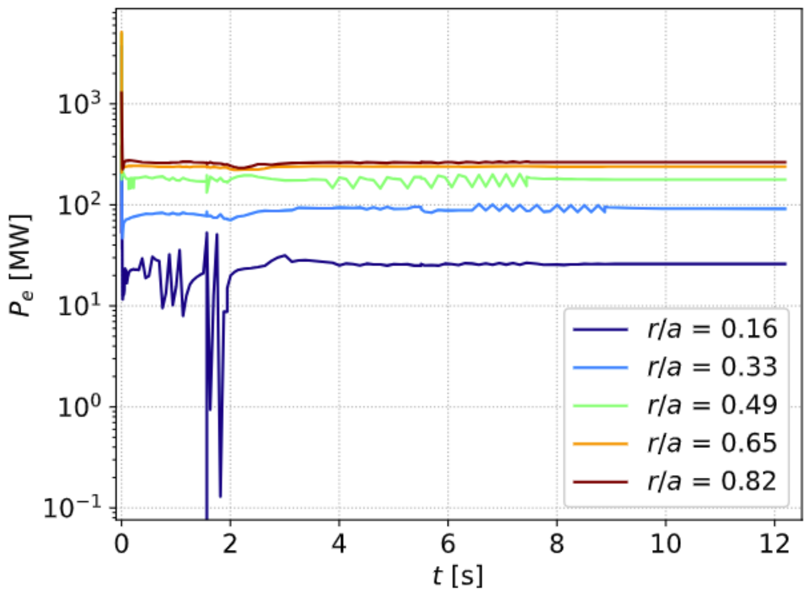


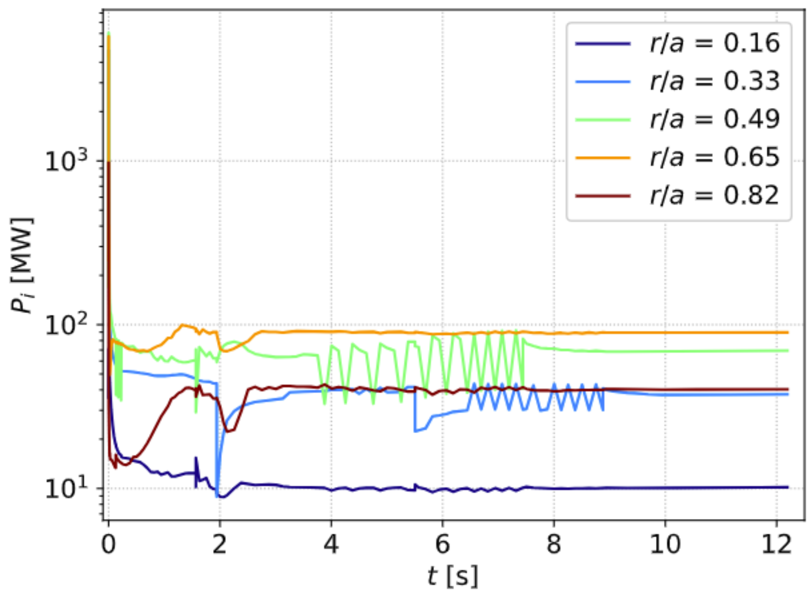
Final power and particle balance
Excellent agreement between power and particle losses and available sources.

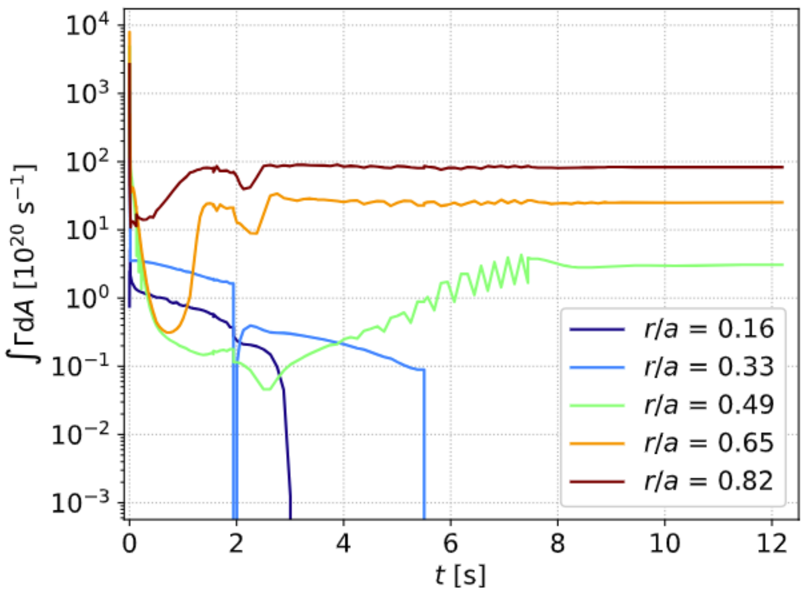




Time evolution of power and particle loss

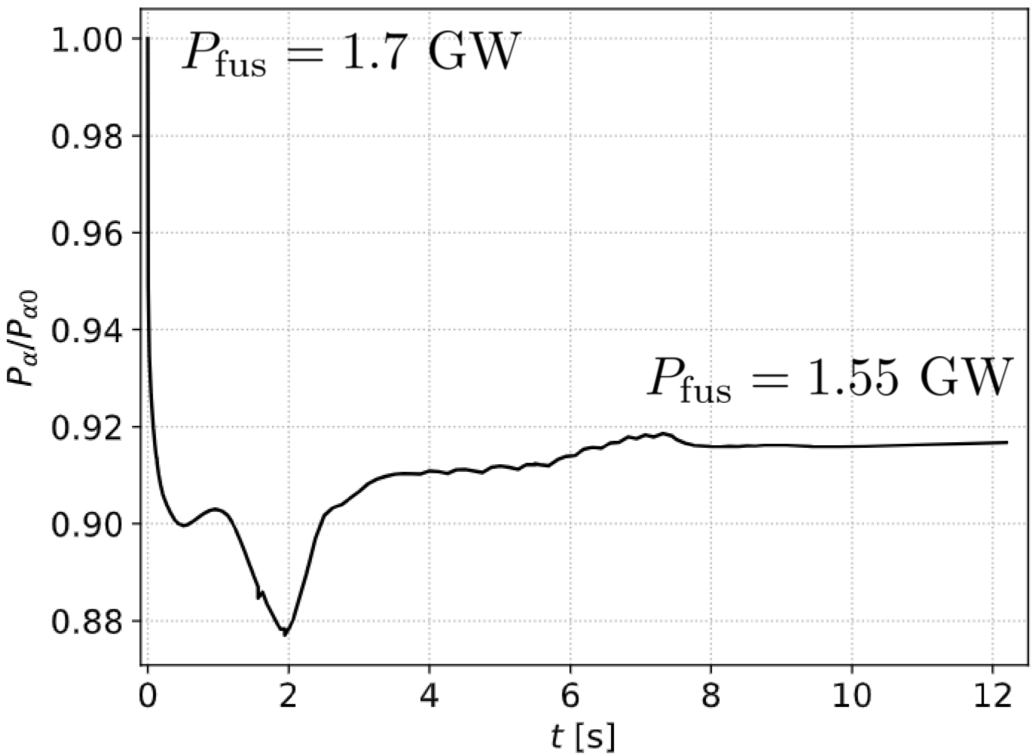






- Evolution divided in three phases:
  - Short initial phase ($t < 0.1$ s, 50 iterations): power and particle losses decrease suddenly.
  - Long second phase ($t < 9$ s, 100 iterations): particle and power losses converge to available sources.
  - Final phase ($t > 9$ s, 40 iterations): particle and power losses are constant.
- Presence of some ripple from low k_y modes being stabilized/destabilized.
- Particle flux vanishes at $r/a \leq 0.33$.
- Long time is required to relax to a steady state solution (particle confinement time is 12.5 s).

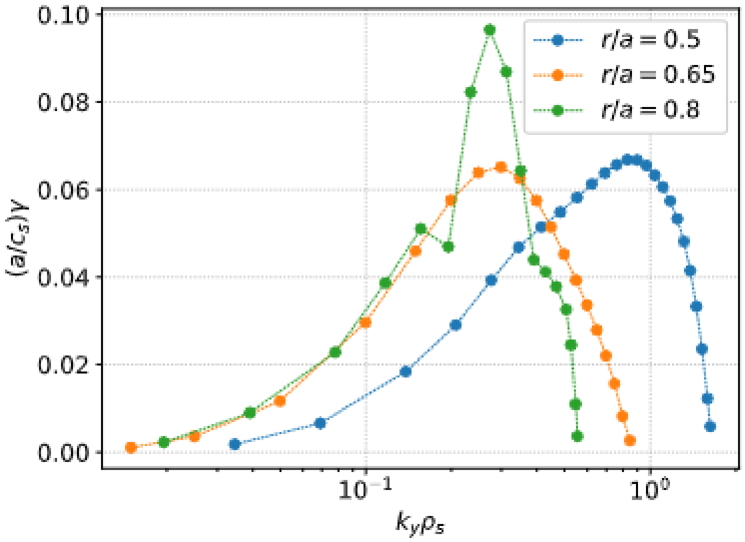
Fusion power reduced by 10%

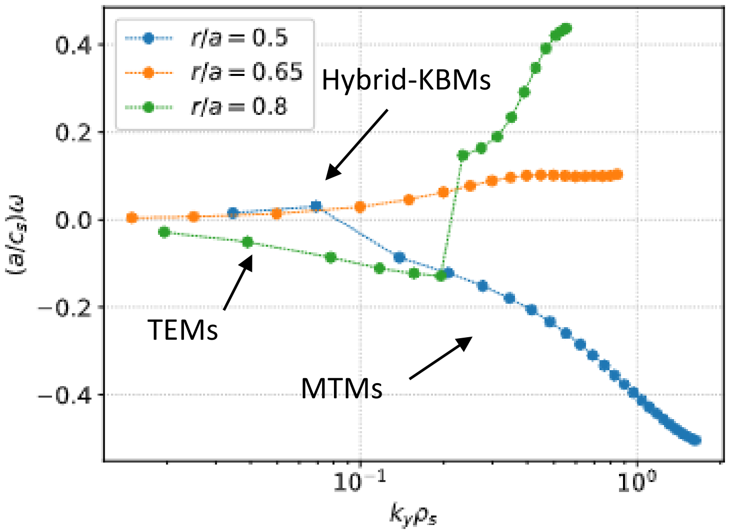


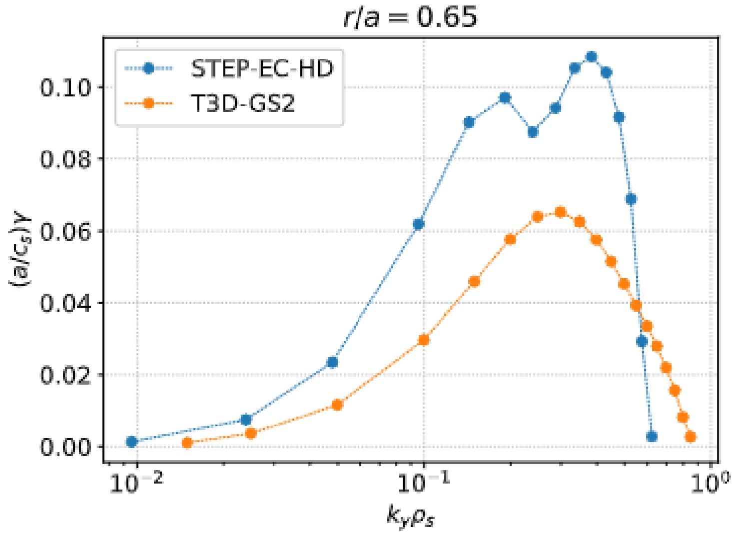
- Fusion power is 10% smaller in the T3D solution than in the initial STEP operating point. The fusion power drops is just an indicative value. **Approximations may be important.**
- Energy and confinement time are similar to STEP-EC-HD.

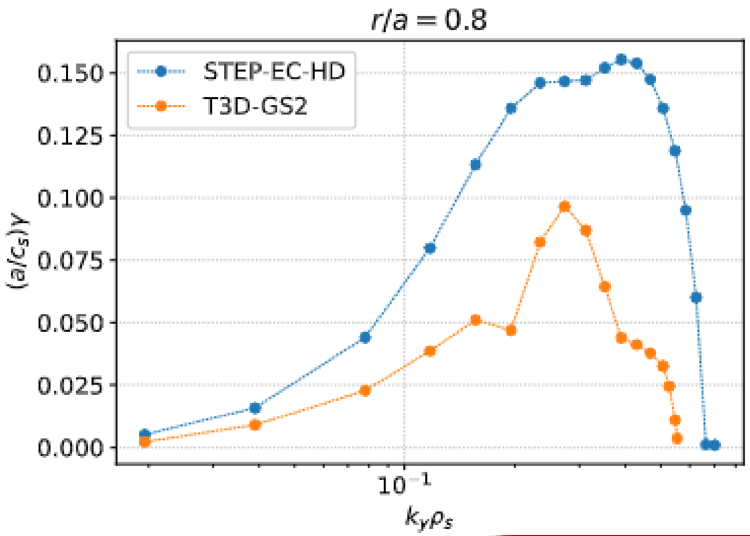
Are the T3D-GS2 final profiles a steady state solution for GK turbulent transport?

Linear GK analysis is reassuring







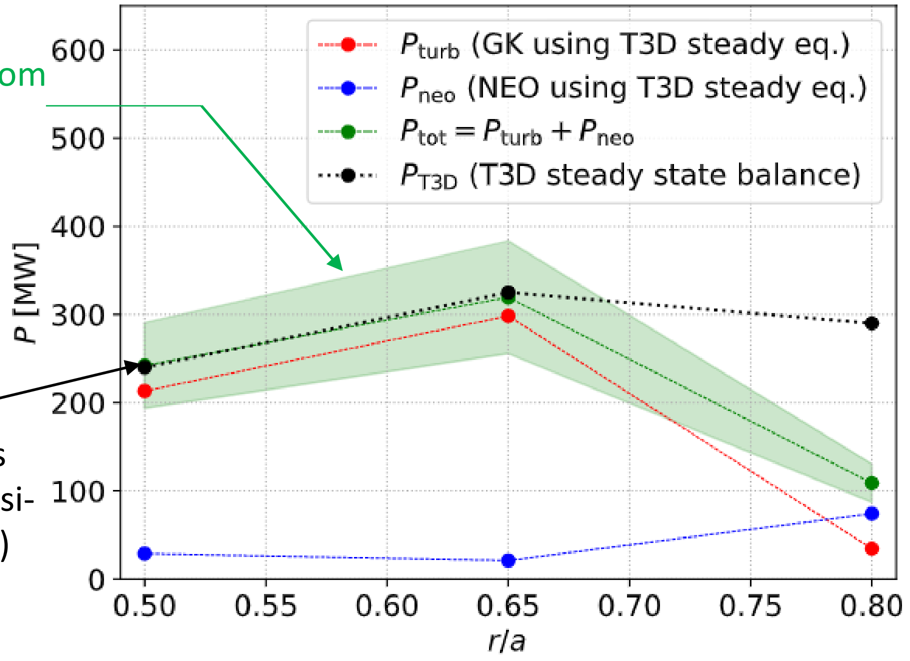


The T3D steady state is characterized by GK instabilities similar to STEP-EC-HD.

Growth rate values are in general smaller, especially as low k_y.

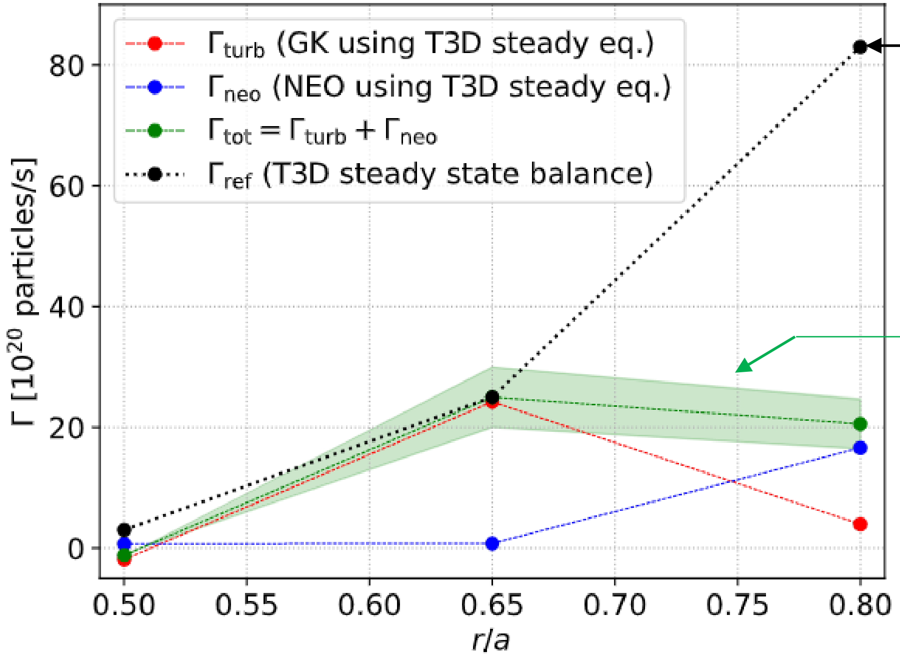
The T3D solution is a steady state for GK

Nonlinear GK simulations predict heat and particle fluxes that are equal or below the available sources in the T3D steady state solution



Power losses from high-fidelity simulations

Power losses from T3D (quasi-linear model)



Particle losses from T3D (quasi-linear model)

Particle losses from high-fidelity simulations

Summary

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- Confinement prediction in STEP requires modelling turbulent transport from hybrid-KBMs, but GK turbulent transport calculations are prohibitively expensive.
- A new quasi-linear transport model is built for hybrid-KBM transport in STEP-like regimes and implemented in the T3D transport code.
- The first STEP T3D simulations return plasma profiles similar to the initial condition with fusion power reduced by 10%.
- A number of approximations are considered: no fast α particles, no impurities and constant equilibrium.
- GK nonlinear simulations show that the T3D solution is compatible with the available STEP sources.

Future work

- Effect of impurities and fast α particles.
- Stochastic transport from MTMs.
- Sensitivity on the safety factor or evolution of the equilibrium with kinetic profiles.
- Sensitivity to boundary condition.
- Experimental validation in high-β spherical tokamaks.
- Surrogate model for Λ.