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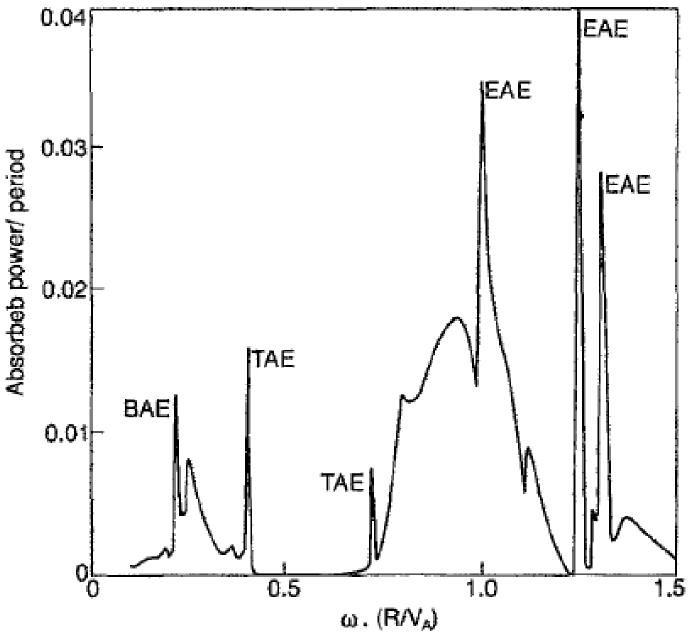
Nonlinear Alfven wave behavior on tokamaks

Michael Fitzgerald

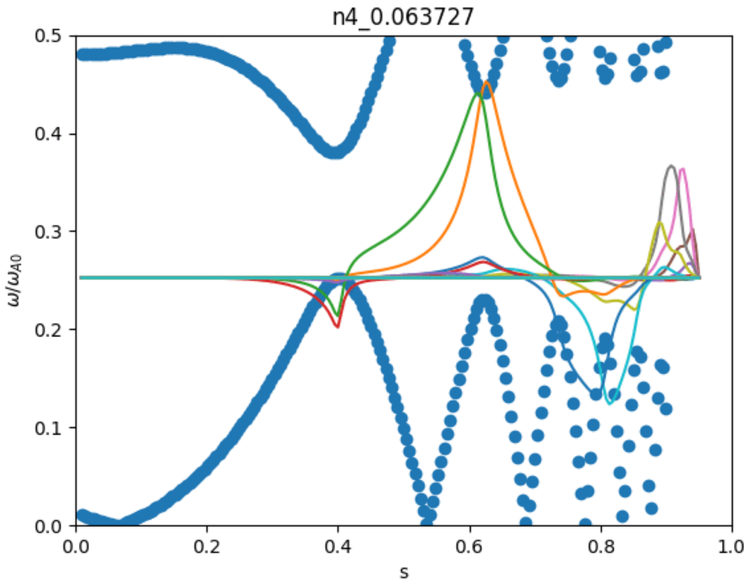
Alfven eigenmodes in toroidal geometry

A tokamak acts as a resonant cavity supporting Alfven eigenmodes

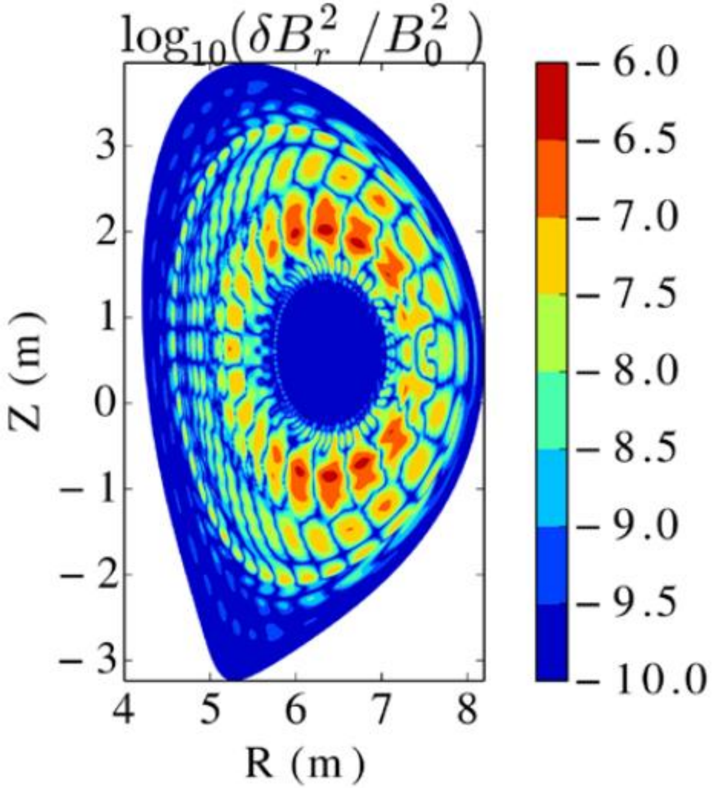


[Huysmans et al. 1995 Phys. Plasmas 2 1605]

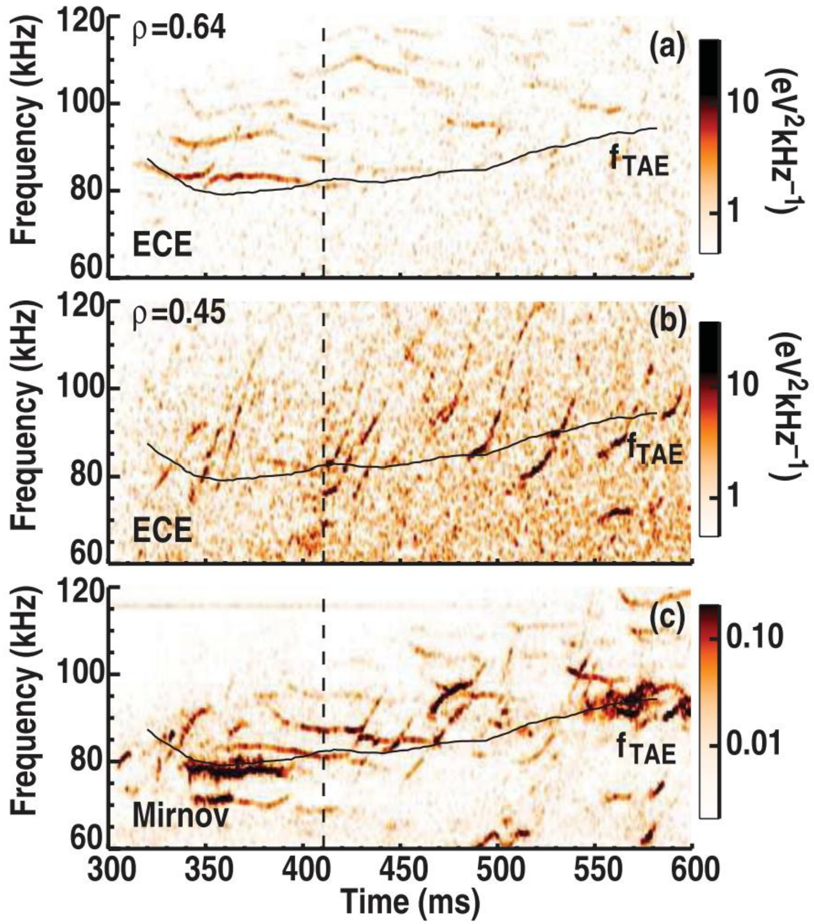
Many different TAEs are possible



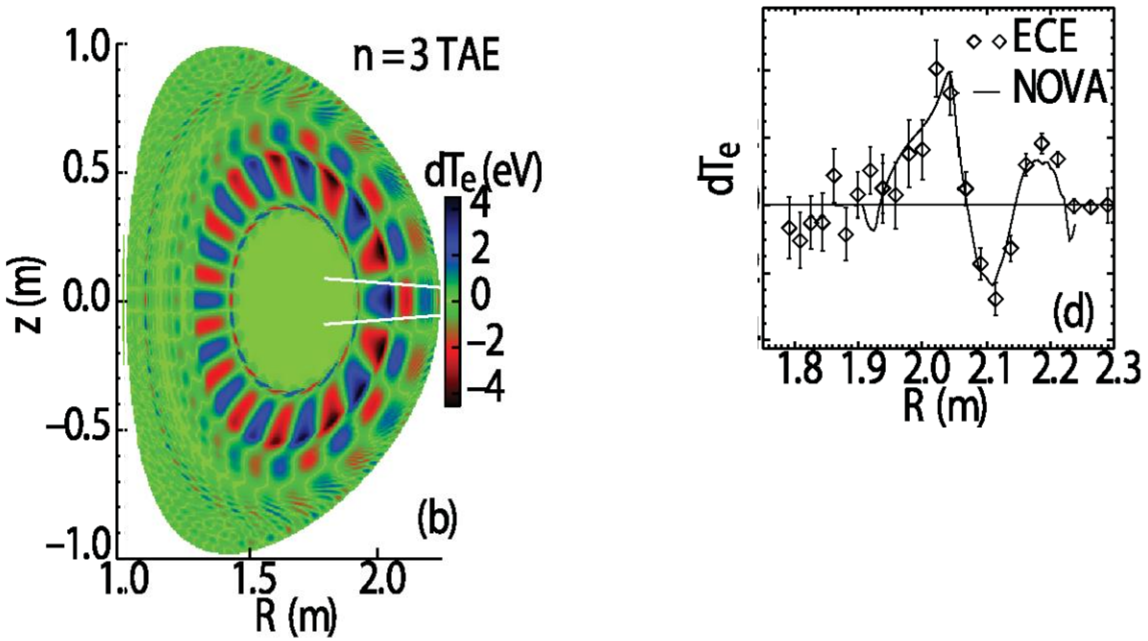
Burning plasma predicted to have alpha driven TAEs



Alfven eigenmodes driven by resonant interactions with fast particles



The oscillations are characterised by mainly sloshing of the bulk thermal plasma



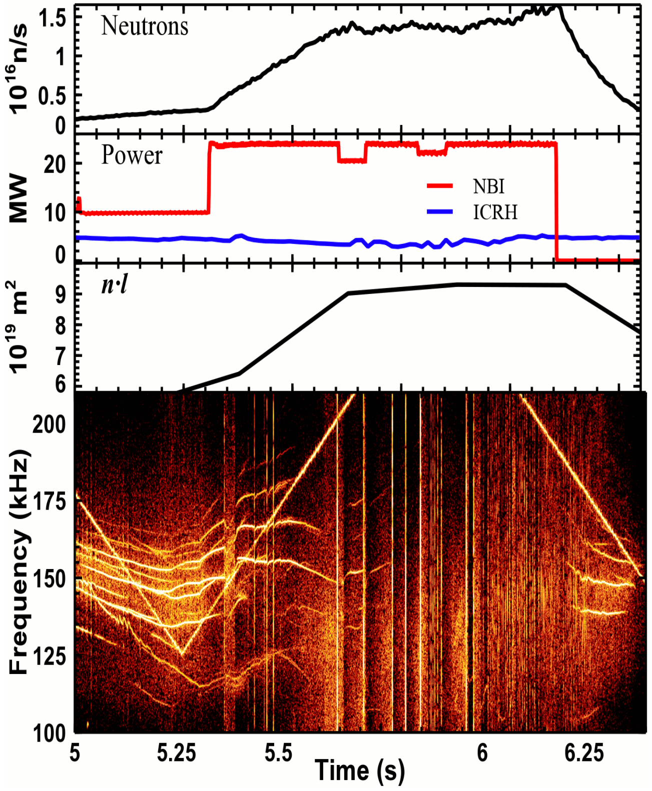
PRL 97, 135001 (2006) PHYSICAL REVIEW LETTERS week ending 29 SEPTEMBER 2006

Radial Structure of Alfvén Eigenmodes in the DIII-D Tokamak through Electron-Cyclotron-Emission Measurements

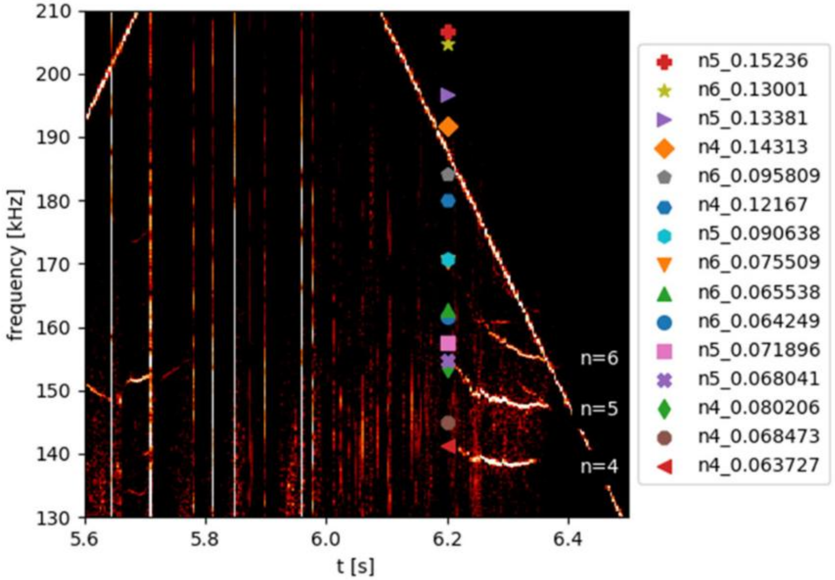
M. A. Van Zeeland,[1,*] G. J. Kramer,[2] M. E. Austin,[3] R. L. Boivin,[4] W. W. Heidbrink,[5] M. A. Makowski,[6] G. R. McKee,[7] R. Nazikian,[2] W. M. Solomon,[2] and G. Wang[8]

TAEs on JET fit incompressible model

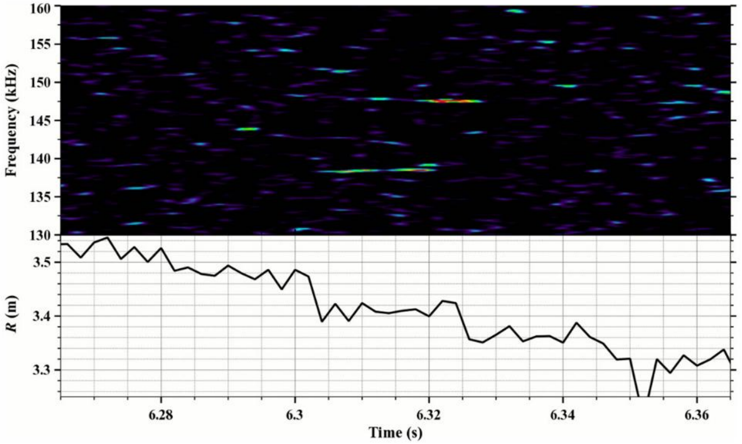
JET shot 92416



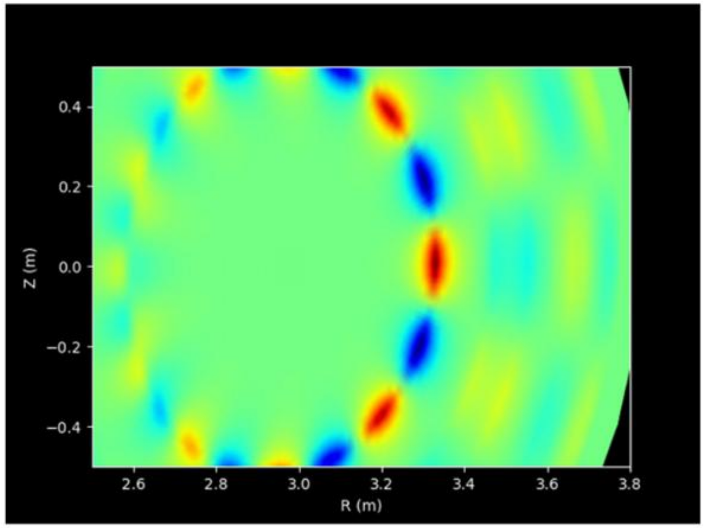
Mirnov 92416



Reflectometry 92416

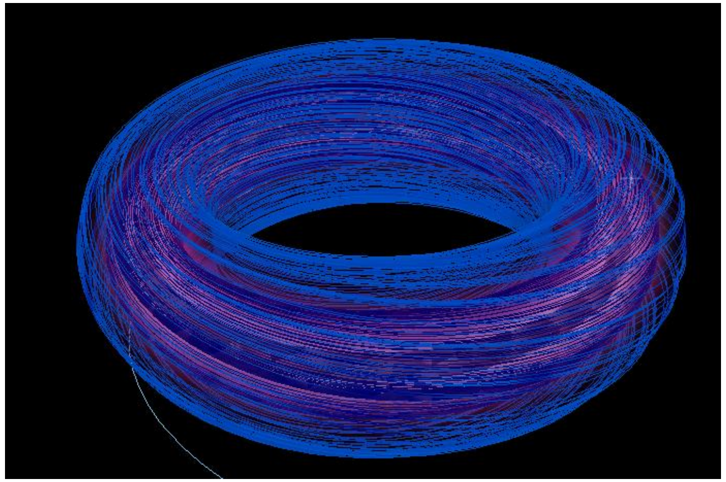


Prediction

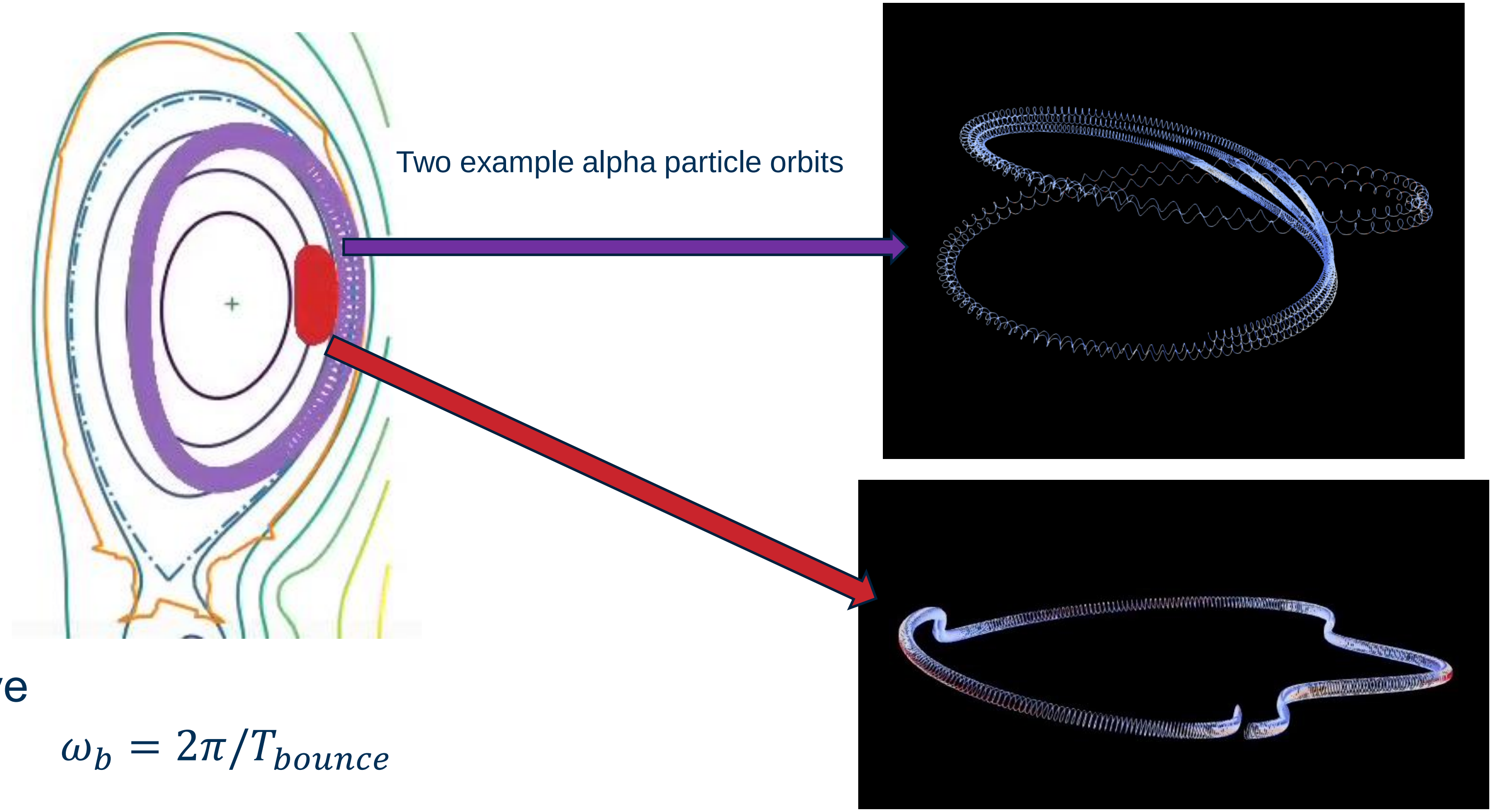


Resonance in a tokamak

The **eigenmodes move in the toroidal direction** with a wavenumber n/R and frequency ω **and are stuck on the magnetic surfaces**



Particles move both in the poloidal and toroidal directions **and drift away from surfaces**



Particles resonate with the wave if the wave period matches an integer number $\boldsymbol{p}$ of particle periods

$$\omega_b = 2\pi/T_{bounce}$$

$$n\langle\dot{\phi}\rangle + p\omega_b - \omega = 0$$

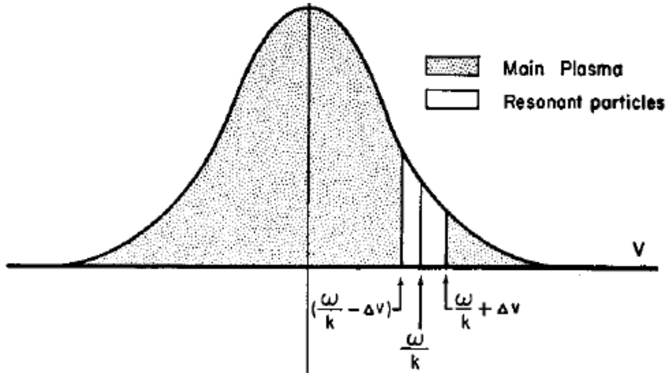
Resonant wave-particle interaction with eigenmodes

Alfven eigenmodes are weakly driven and damped

$$\gamma_L = \int d^3x d^3v \sum_\sigma \sum_p \frac{\delta\gamma(x,v;p,\sigma)}{n\langle\omega_\phi\rangle + p\omega_\theta - \omega}$$

$$\delta\gamma(x,v;p,\sigma) \propto (\omega - n\omega_*)\left(\frac{\partial F}{\partial E}\right)_{\mu,P_\phi}$$

$$\omega_* \equiv \left(\frac{\partial F}{\partial P_\phi}\right)_{E,\mu} \Big/ \left(\frac{\partial F}{\partial E}\right)_{\mu,P_\phi}$$



THE PHYSICS OF FLUIDS VOLUME 4, NUMBER 7 JULY, 1961
On Landau Damping
JOHN DAWSON

$\omega > n\omega_*$ then you get Landau damping

- JET NBI
- Thermal Ions

$\omega < n\omega_*$ then you get inverse Landau damping (drive)

- Alpha particles
- ICRH
- MAST NBI, ITER NBI

Why shear Alfven eigenmodes in experiment resemble linear MHD solutions

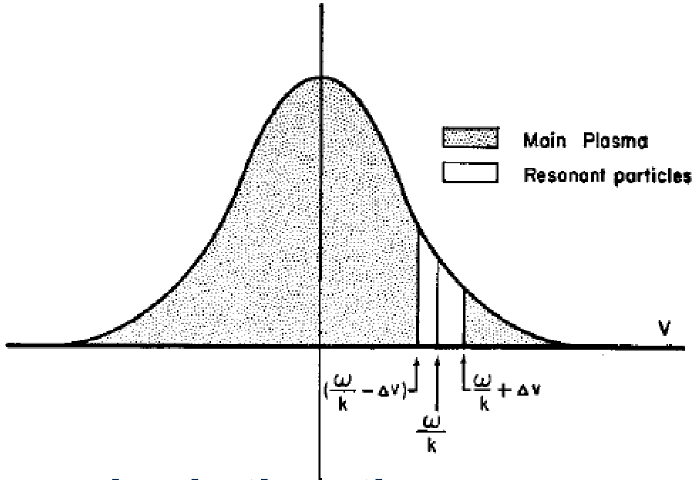
$$-\frac{c^2}{\omega^2}\nabla\times\nabla\times\tilde{\mathbf{E}}(\mathbf{x},\omega)+\tilde{\mathbf{E}}(\mathbf{x},\omega)$$
$$=-\frac{i\mu_0 c^2}{\omega}\left[\int dx'\,\boldsymbol{\sigma}(x,x',\omega)\tilde{\mathbf{E}}(x',\omega)\ +\tilde{\mathbf{J}}_{NL}(\mathbf{x},\omega)+\tilde{\mathbf{J}}_{free}(\mathbf{x},\omega)+\tilde{\mathbf{J}}_{\bar{\sigma}}(\mathbf{x},\omega)\right]$$

This bulk fluid motion has an associated nonlinearity, using cold plasma dispersion this can be estimated from the 1st order polarization drift

$$\frac{\left|\tilde{\mathbf{J}}_{NL,bulk}\right|}{\left|\boldsymbol{\sigma}_{bulk}\tilde{\mathbf{E}}\right|}=\frac{\delta B^2}{B^2}+\cdots$$

The "fluid nonlinearity" from plasma sloshing is thus tiny

$$\frac{\delta B}{B}\sim 1\times 10^{-3}$$



The main nonlinearity in Alfven eigenmodes is then the behaviour of the resonant population, not the bulk

Coherent nonlinear physics – nonlinear Landau damping "Berk/Breizman" theory

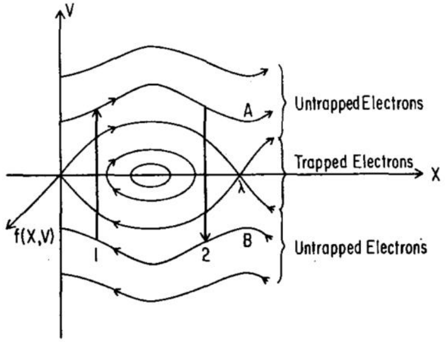
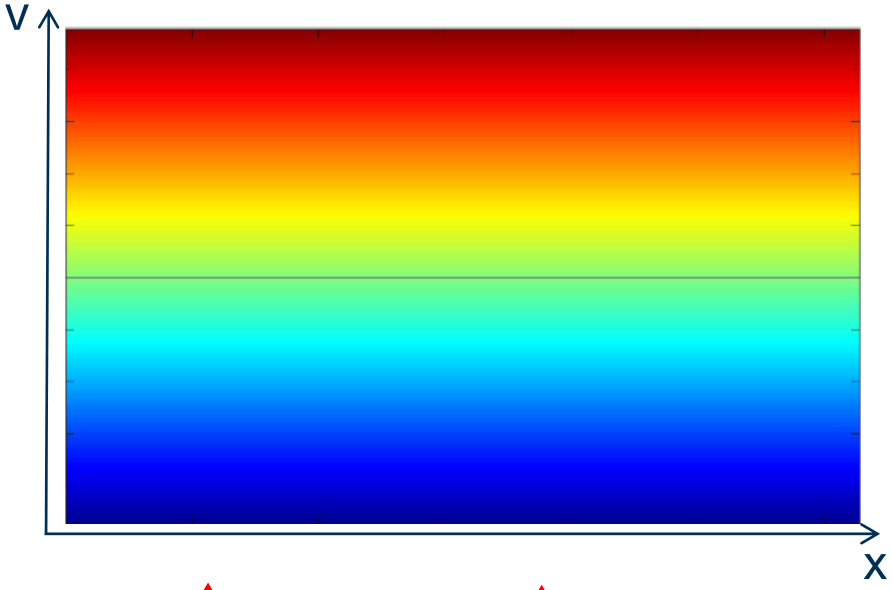
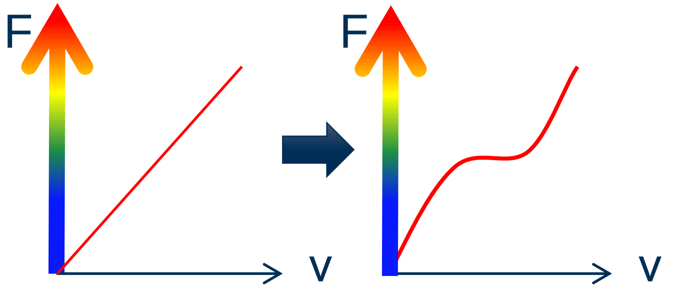
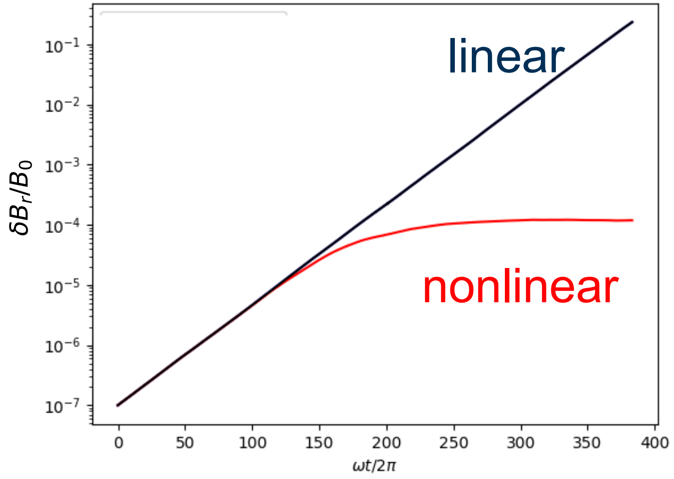


FIG. 2. The phase trajectories of the resonant electrons.

O'Neil T 1965 Collisionless Damping of Nonlinear Plasma Oscillations Phys. Fluids 8 2255







Nonlinear bounce frequency

$$\omega_B = \left(\frac{eEk}{m_i}\right)^{\frac{1}{2}}$$

resonant particles have $\left(\frac{\delta B}{B}\right)^{\frac{1}{2}}$ response

Saturation when $\omega_B \sim \gamma_L$

Berk, H. L., Breizman, B. N., & Ye, H. (1992). Scenarios for the nonlinear evolution of alpha-particle-induced Alfvén wave instability. Physical Review Letters, 68(24), 3563–3566.

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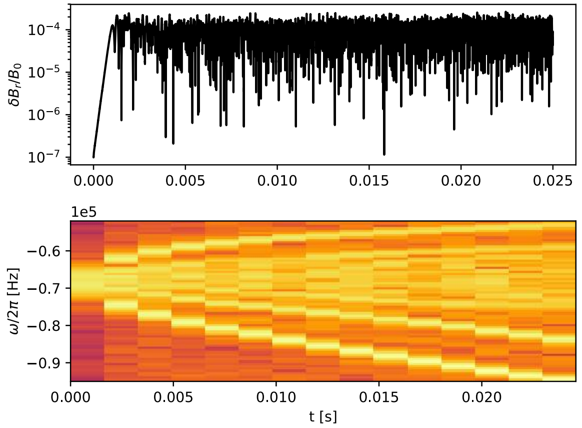
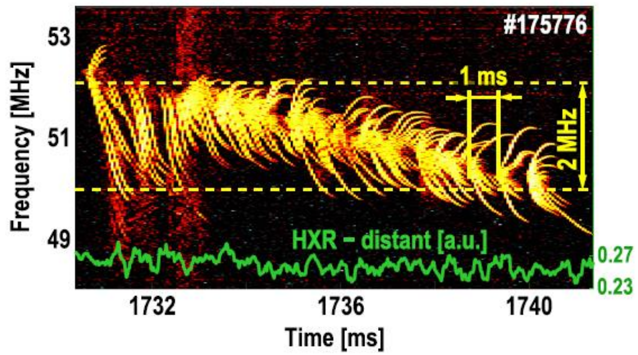
Coherent nonlinear physics – chirping BGK waves in Berk/Breizman theory

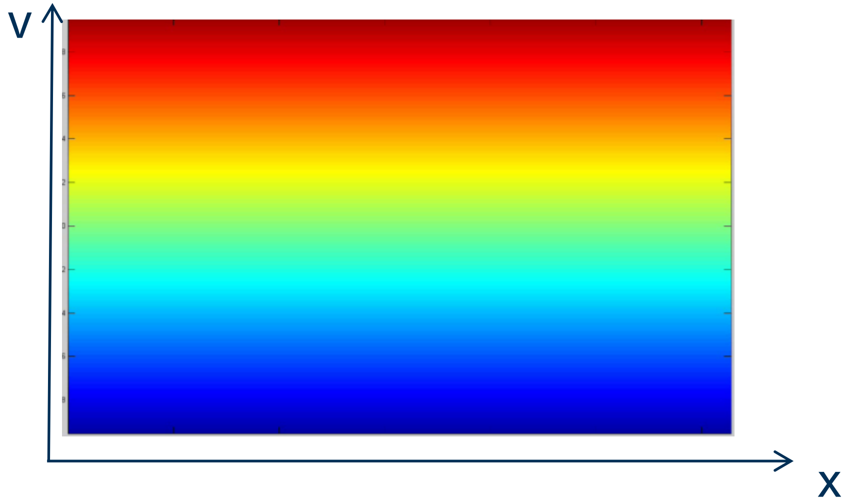
HALO 3D modelling of TAE



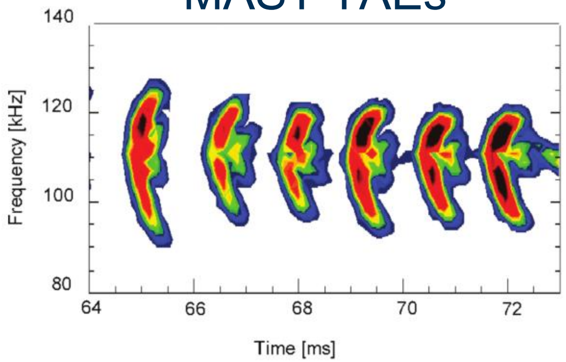
DIIID CAEs



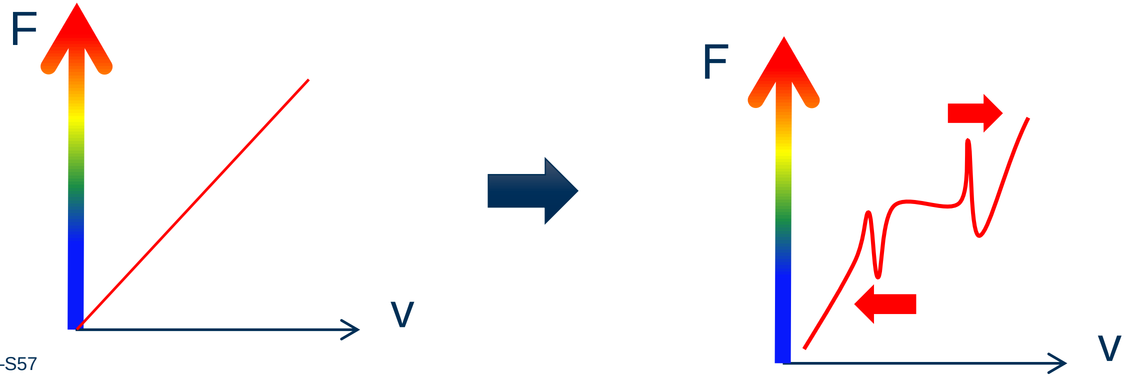
Lvovskiy, A. …. (2019). Nuclear Fusion, 59(12), 124004.



MAST TAEs



Pinches, S.…PPCF, 46(7), S47–S57



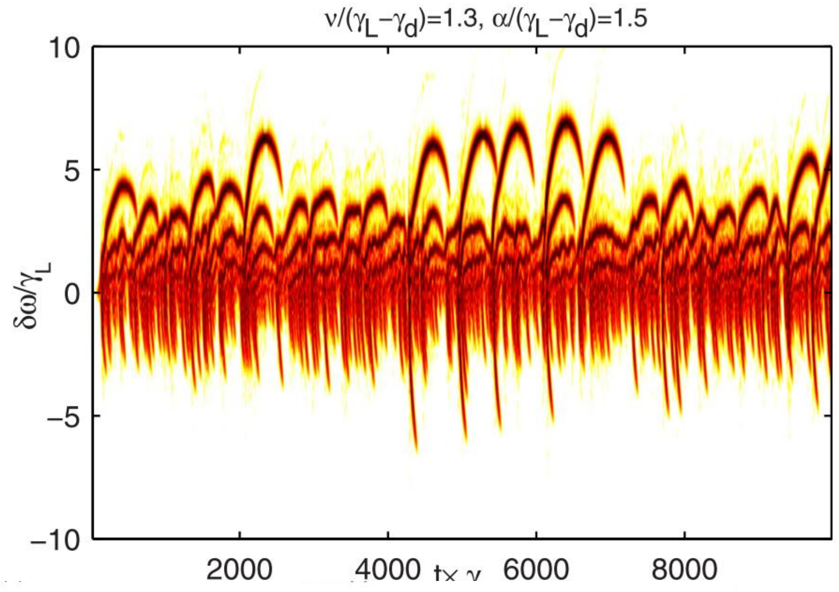
Berk, H. L., Breizman, B. N., Candy, J., Pekker, M., & Petviashvili, N. V. (1999). Spontaneous hole–clump pair creation. Physics of Plasmas, 6(8), 3102.

1D bump on tail modelling vs experiment

PHYSICS OF PLASMAS 17, 092305 (2010)

Effect of dynamical friction on nonlinear energetic particle modes

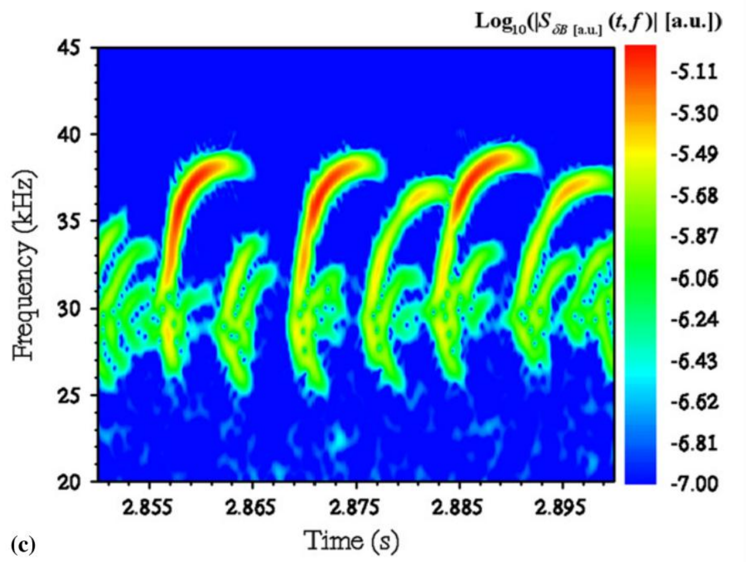
M. K. Lilley,[1,a)] B. N. Breizman,[2] and S. E. Sharapov[3]



Nucl. Fusion 46 (2006) S888–S897

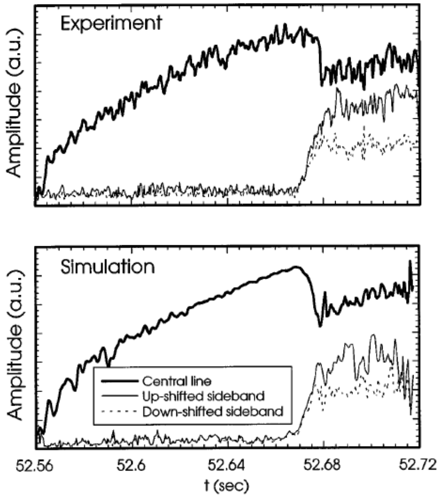
Explanation of the JET $n = 0$ chirping mode

H.L. Berk[1], C.J. Boswell[2], D. Borba[3,4], A.C.A. Figueiredo[3], T. Johnson[5], M.F.F. Nave[3], S.D. Pinches[6], S.E. Sharapov[7] and JET EFDA contributors[8]



Inclusion of collisions changes the nonlinear evolution of the BGK waves

JET TAEs



VOLUME 81, NUMBER 25 PHYSICAL REVIEW LETTERS 21 DECEMBER 1998

Nonlinear Splitting of Fast Particle Driven Waves in a Plasma: Observation and Theory

A. Fasoli,[1,2] B. N. Breizman,[3] D. Borba,[1,4] R. F. Heeter,[1,5] M. S. Pekker,[3] and S. E. Sharapov[1]

Integrable orbits for isolated resonances behave like 1D

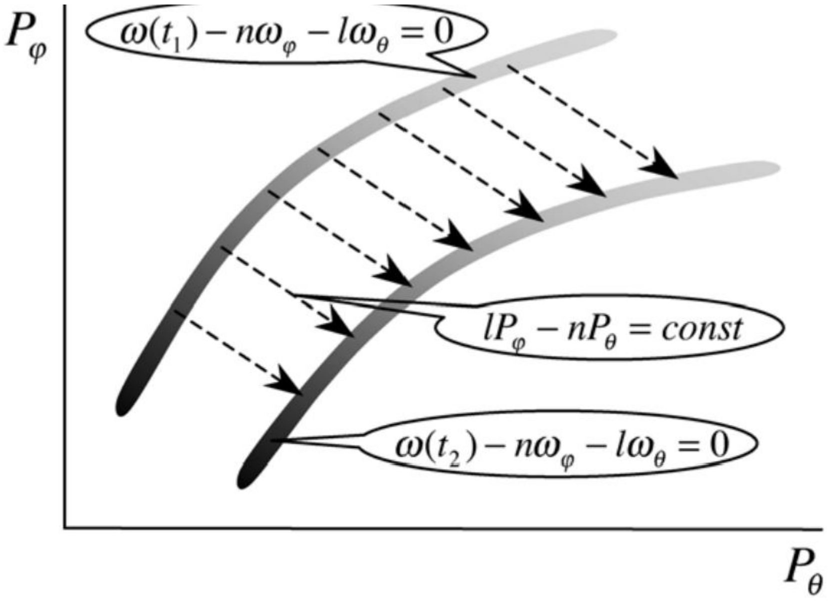


Figure 6. Transport of resonant particles during frequency sweeping. The shaded areas are snapshots of the moving resonant region in the momentum space. The shades of gray mark different values of the particle distribution function. The trapped resonant particles form a locally flat distribution across the resonance and preserve the value of their distribution function when the resonance carries them along the dashed lines.

Three constants of the motion are P_ϕ, μ, E so the equilibrium motion is completely integrable in 3D (Liouvile-Arnold theorem)

Low frequency modes approximately preserve E, μ
Individual modes have a

$$\omega - n\omega_\varphi\left(P_\varphi;\, P_\theta;\, P_\psi\right) - l\omega_\theta\left(P_\varphi;\, P_\theta;\, P_\psi\right) = 0,$$

For a single isolated mode

$$E - \frac{\omega}{n} P_\phi = \text{constant}$$

Nucl. Fusion **50** (2010) 084014 (6pp) doi:10.1088/0029-5515/50/8/084014

Nonlinear travelling waves in energetic particle phase space

Boris N. Breizman

Early simulations showed signs of fluid nonlinearity – zonal flows

UK Atomic Energy Authority

Nonlinear evolution of the toroidal Alfvén instability using a gyrofluid model*

D. A. Spong,† B. A. Carreras, and C. L. Hedrick
Oak Ridge National Laboratory, Oak Ridge, Tennessee 37831-8071

Phys. Plasmas, Vol. 1, No. 5, May 1994

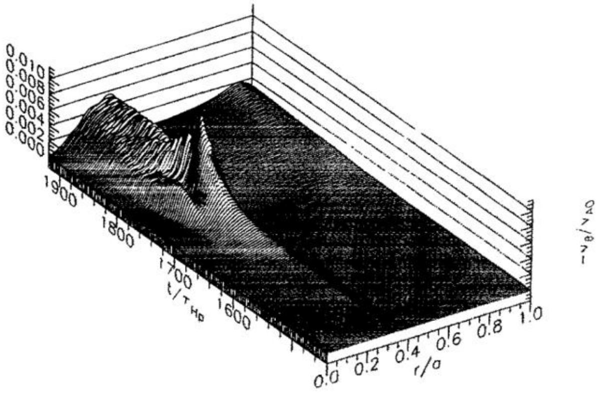


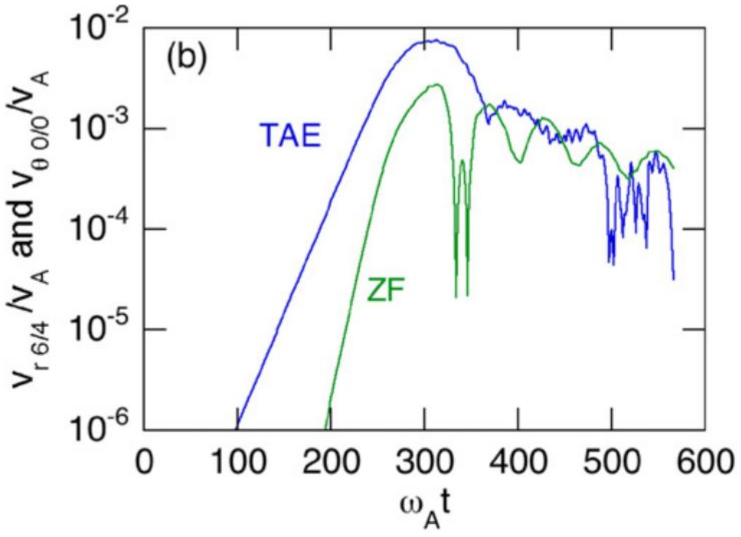
FIG. 8. Radial and time variation of nonlinearly generated poloidal E×B flow velocity.

Nucl. Fusion **50** (2010) 084016 (9pp)
doi:10.1088/0029-5515/50/8/084016

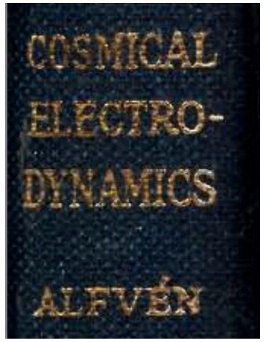
Nonlinear magnetohydrodynamic effects on Alfvén eigenmode evolution and zonal flow generation

Y. Todo[1,2], H.L. Berk[3] and B.N. Breizman[3]

response of the energetic particles. Specifically, we studied the evolution of an $n = 4$ TAE mode destabilized by its resonant interaction with energetic particles in a tokamak plasma. When the TAE saturation level is $\delta B/B \leqslant 10^{-3}$ no significant difference was found between the results of the linear-MHD simulation and the nonlinear MHD simulations. On the other hand, when in the linear-MHD simulation the TAE saturation level is $\delta B/B \sim 10^{-2}$, the saturation level in the nonlinear MHD case is found to be reduced to half the result of the linear-MHD simulation. We found that the nonlinearly generated $n = 0$ and the higher-n ($n \geqslant 8$) modes provide increased energy dissipation that appears crucial for achieving a reduced TAE saturation level.



"Breaking the pure Alfvenic state"



The general solution encounters mathematical difficulties. For the case of an *incompressible fluid with constant density* ρ *in a homogeneous magnetic field* $\mathbf{H}_0$ a solution has been given by Walén (1944). In this case we have

$$\operatorname{div} \mathbf{v} = 0, \tag{5}$$

$$\operatorname{grad} H_0 = 0. \tag{6}$$

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}\,\mathrm{grad})\mathbf{v} + \frac{\mu}{4\pi\rho}[\mathbf{H}\,\mathrm{curl}\,\mathbf{H}] = \mathbf{G} - \frac{1}{\rho}\,\mathrm{grad}\,p.$$

The magnetic field $\mathbf{H} = \mathbf{H}_0 + \mathbf{h}$,

$$\pm\ (\mathbf{H}_0\,\mathrm{grad})\mathbf{h} = \left(\frac{4\pi\rho}{\mu}\right)^{\frac{1}{2}} \frac{\partial \mathbf{h}}{\partial t}.$$

In the calculations no second-order terms have been neglected. Consequently the result holds even if $h > H_0$.

PHYSICS OF PLASMAS **20**, 055402 (2013)

On nonlinear physics of shear Alfvén waves[a)]

Liu Chen[1,2,b)] and Fulvio Zonca[3,1]
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[3]*Associazione EURATOM-ENEA sulla Fusione, CP 65-00044 Frascati, Italy*

Specifically, we have examined three effects: finite ion compressibility, non-ideal kinetic effects, and the tokamak geometry, keeping them separate for the sake of clarity. In realistic situations, all these three effects must be considered on the same footing and, depending on the specific problem under investigation, may concur in various extents to the breaking of the pure Alfvénic state and, hence, to the nonlinear system behavior. Some examples of such practical appli-

Transverse incompressible waves that satisfy $\pm k_{\parallel} v_A = \omega$ can have arbitrary amplitude and propagate with no nonlinearity in uniform plasma – the "pure Alfvenic state"

“Breaking the pure Alfvenic state” – particle gyration

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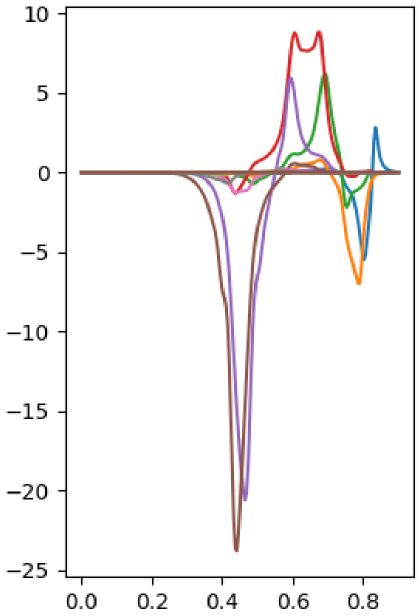
Linear version

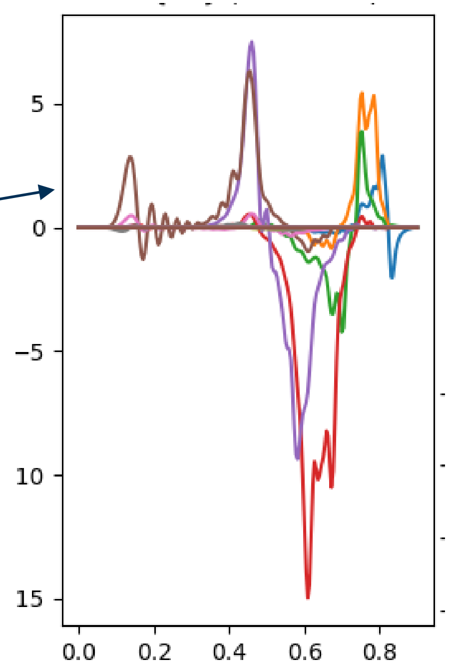


$$\mathscr{L}_m \simeq \frac{d}{dr}\left(\frac{\omega^2}{v_A^2}-k_{\| m}^2\right)\frac{d}{dr}-\frac{m^2}{r^2}\left(\frac{\omega^2}{v_A^2}-k_{\| m}^2\right).$$

$$\left(\mathscr{L}_m+\bar{\rho}^2\frac{d^4}{dr^4}\right)\phi_m+\epsilon(r)\frac{\omega^2}{v_A^2}\frac{d^2}{dr^2}(\phi_{m+1}+\phi_{m-1})=0,$$

$$\bar{\rho}^2\equiv\rho_i^2\left(\frac{3}{4}\frac{\omega^2}{v_A^2}+\tau k_{\| m}^2\right) \quad \text{and} \quad \tau\equiv\frac{T_e}{T_i}(1-i\delta)$$

[Candy 1994 Phys. Plasmas 1 356–72]



MHD picture breaks down on ion gyration length scale

Linear mode conversion to Kinetic Alfven wave (like flavour mixing with neutrinos - a linear process).

"Breaking the pure Alfvenic state" – geometric effects

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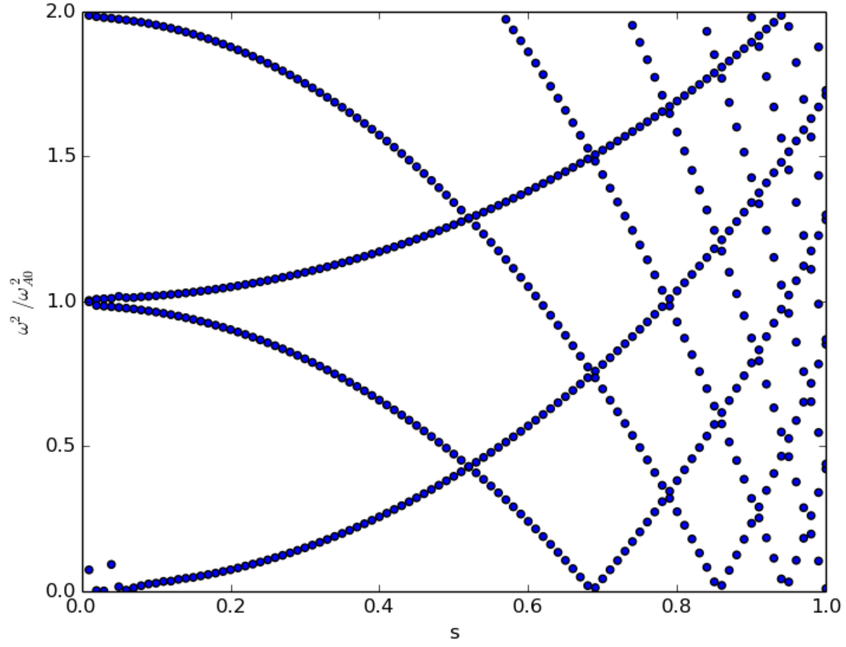
cylinder $\omega = \pm k_{\|m} v_A$



$$\frac{d}{dr}\left[r^3\left(\frac{\omega^2}{v_A^2}-k_{\|m}^2\right)\frac{dE_m}{dr}\right]+\omega^2 r^2 E_m \frac{d}{dr}\left(\frac{1}{v_A^2}\right)$$

$$-(m^2-1)\left(\frac{\omega^2}{v_A^2}-k_{\|m}^2\right) rE_m = 0$$

PHYSICS OF PLASMAS 13, 042504 (2006)

Nonlinearly driven second harmonics of Alfvén cascades

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B. N. Breizman
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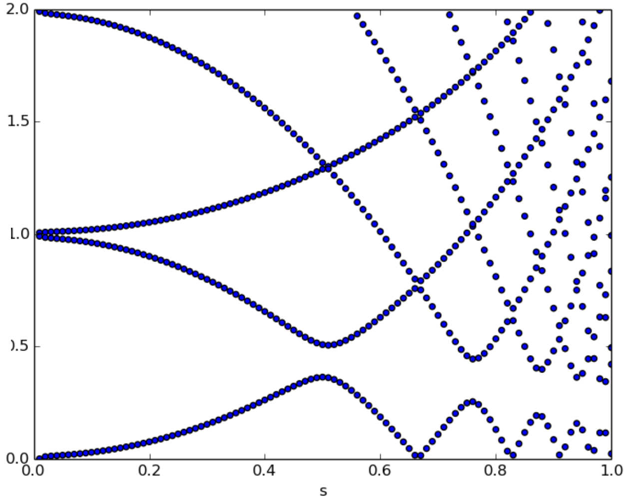
M. Lisak and D. Anderson
Department of Radio and Space Science, Chalmers University of Technology, SE-412 96 Göteborg, Sweden

shear Alfvén wave. For shear Alfvén perturbations in a uniform equilibrium magnetic field, the quadratic terms $4\pi\rho(\mathbf{v}\cdot\nabla)\mathbf{v}$ and $(\mathbf{B}\cdot\nabla)\mathbf{B}$ tend to cancel in the momentum balance equation. For this reason, extreme care is needed to properly include magnetic curvature effects and to evaluate the coupling between shear Alfvén perturbations and compressional perturbations.

$$\frac{\rho_2}{\rho_1} \sim \frac{\rho_{\Phi_2}}{\rho_1} \sim \frac{m^2}{r^2}\frac{\Phi_1}{B_0} \sim \frac{mq}{\varepsilon}\frac{|\mathbf{B}_{\Phi_1}|}{B_0}.$$

Toroidicity added $\omega \neq \pm k_{\|m} v_A$



$$\frac{d}{dr}\left[r^3\left(\frac{\omega^2}{v_A^2}-k_{\|m}^2\right)\frac{dE_m}{dr}\right]+r^2E_m\frac{d}{dr}\left(\frac{\omega}{v_A}\right)^2$$

$$-(m^2-1)\left(\frac{\omega^2}{v_A^2}-k_{\|m}^2\right)rE_m$$

$$+\frac{d}{dr}\left[r^3\left(\frac{\omega}{v_A}\right)^2\left(\Delta'+\frac{2r}{R_0}\right)\left(\frac{dE_{m+1}}{dr}+\frac{dE_{m-1}}{dr}\right)\right.$$

$$\left.-r^3\Delta' k_{\|m}\left(k_{\|m+1}\frac{dE_{m+1}}{dr}+k_{\|m-1}\frac{dE_{m-1}}{dr}\right)\right]=0.$$

Berk, H. L., Van Dam, J. W., Guo, Z., & Lindberg, D. M. (1992). *Physics of Fluids B: Plasma Physics*, *4*(7), 1806.

“Breaking the pure Alfvenic state” – finite beta

Nucl. Fusion 59 (2019) 066031 (11pp)

https://doi.org/10.1088/1741-4326/ab1285

Nonlinear excitation of a geodesic acoustic mode by toroidal Alfvén eigenmodes and the impact on plasma performance

Zhiyong Qiu[1,a], Liu Chen[1,2], Fulvio Zonca[1,3] and Wei Chen[4]

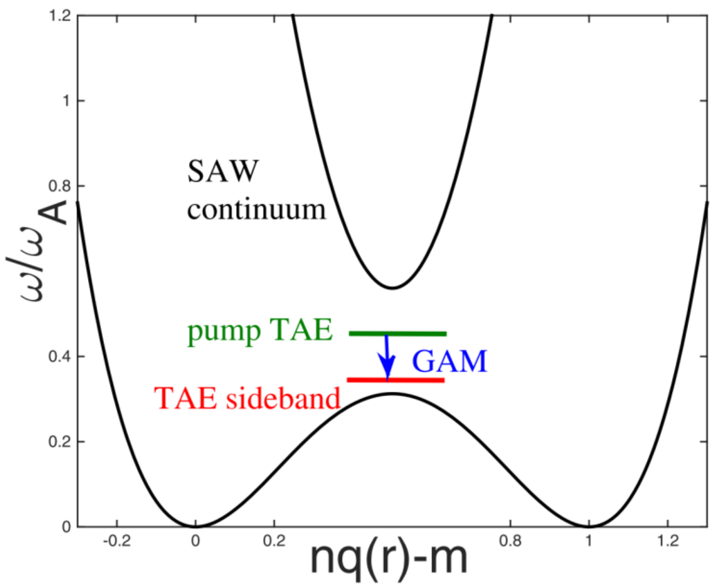


Figure 1. Cartoon of TAE decay into GAM and TAE lower sideband in the low-β_i limit.

The effect of nonlinear mode coupling on the stability of toroidal Alfvén eigenmodes

C. N. Lashmore-Davies and A. Thyagaraja
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R. A. Cairns
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In this paper we have considered a mode coupling mechanism for the saturation of a TAE instability. This mechanism provides a channel for the finite-amplitude TAE wave, which obtains its energy from the fusion alpha particles, to transfer this energy to other modes of the plasma. This will occur once the TAE wave has reached the threshold amplitude for modulational instability. As a result, the amplitude of the TAE wave will saturate close to the modulational threshold level so that additional energy flowing into the TAE wave from the alpha particles will be transferred to these other fluctuations.

Outlook on nonlinear Alfven waves

turbulence crowd

fast particle crowd

- Wave-particle nonlinearity of trapped population well understood and perturbative modelling very advanced
- Wave-wave nonlinearity for global Alfvenic modes a newer topic with relatively inaccessible analytical theory. Plenty black-box simulations giving mixed picture.
- Room for toy modelling to bridge the gap. Identification of dominant wave-wave contributions would help guide experimental scenario design.
- Nonlinear predictions should depend on amplitude – need amplitude predictions for the onset of different non-linearities.