Issues on simple modeling of fast ion effect on zonal flow self-generation

<u>G.J. Choi</u>[1]

with T.S. Hahm[1], S.J. Park[1] Y.-S. Na[1], P.H. Diamond[2]

[1] Seoul National University

[2] University of California, San Diego

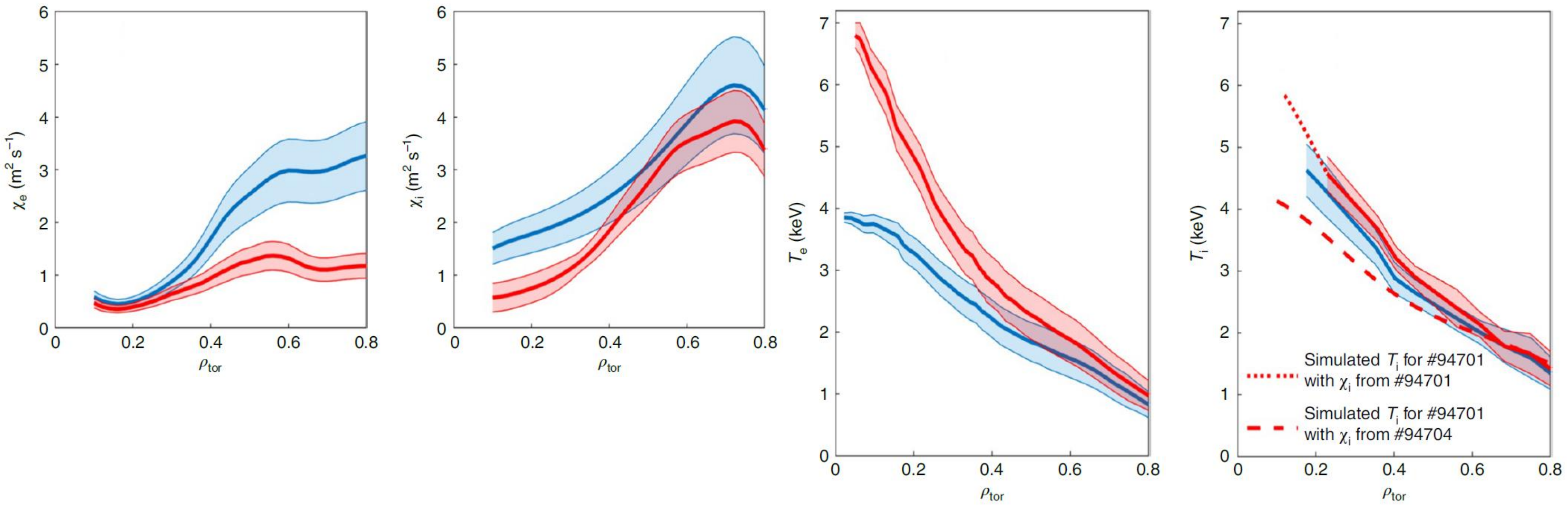
Presented at 15th Plasma Kinetics Working Meeting,
Wolfgang Pauli Institute, Vienna, August 5, 2024

Outline

- Fast ion effects on tokamak confinement
- Modified Hasegawa-Mima equation for zonal flow generation
- Fast ion effects on zonal flow-turbulence system
- Ongoing work toward a 3-animal system

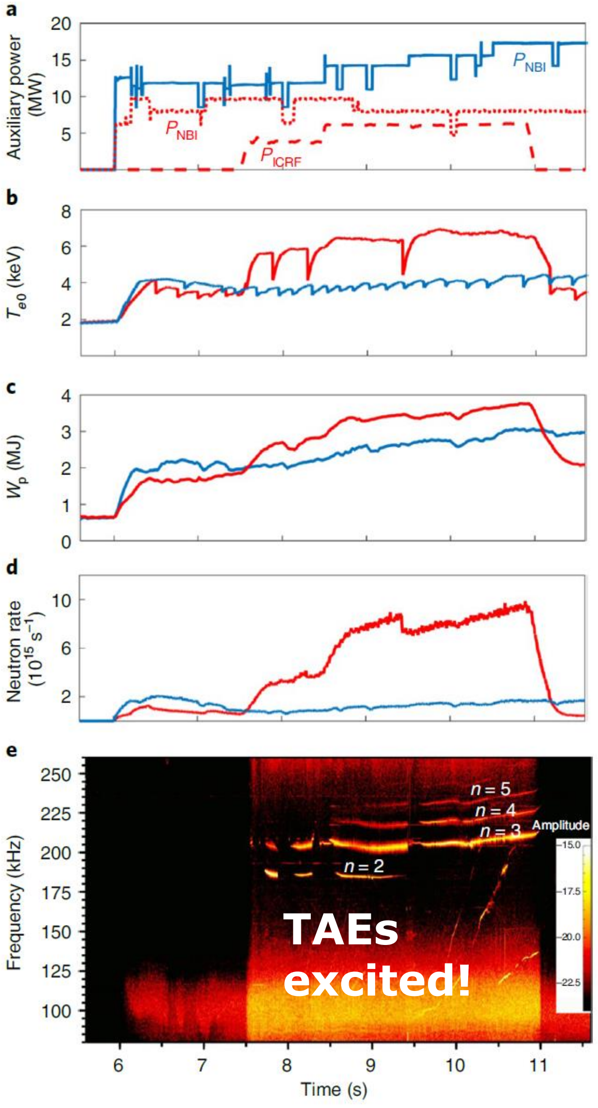
Fast ion-induced TAE-mediated transport barrier

- **JET** has reported thermal confinement enhancement with **ICRH** mixed with NB, in comparison to the case of NB-only having similar heating power.
- Nonlinear turbulence suppression by **unstable TAEs** excited by ICRH-induced MeV **fast ions** has been addressed as the working mechanism.
- Thermal ion heat conductivity χ_i was considerably reduced in the core region.



Simulated T_i for #94701 with χ_i from #94701

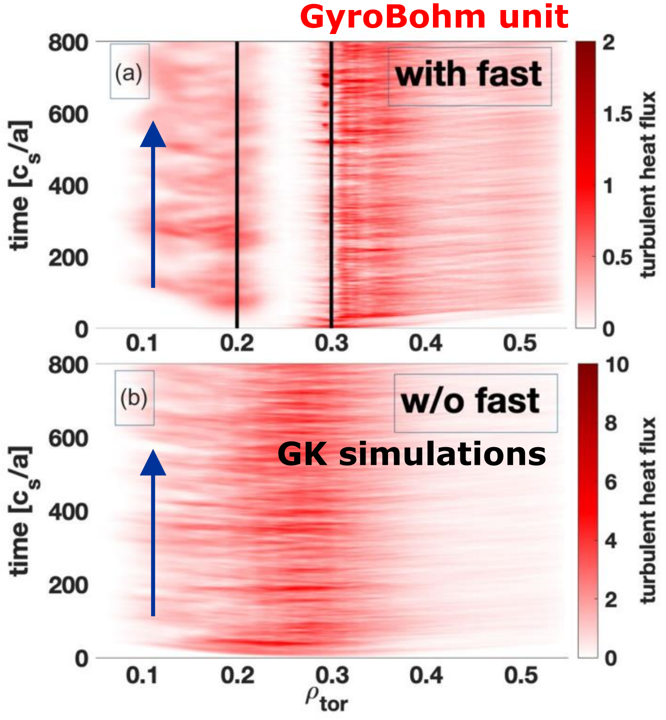
Simulated T_i for #94701 with χ_i from #94704



[S. Mazzi *et al.*, *Nat. Phys.* **18**, 776 (2022)]

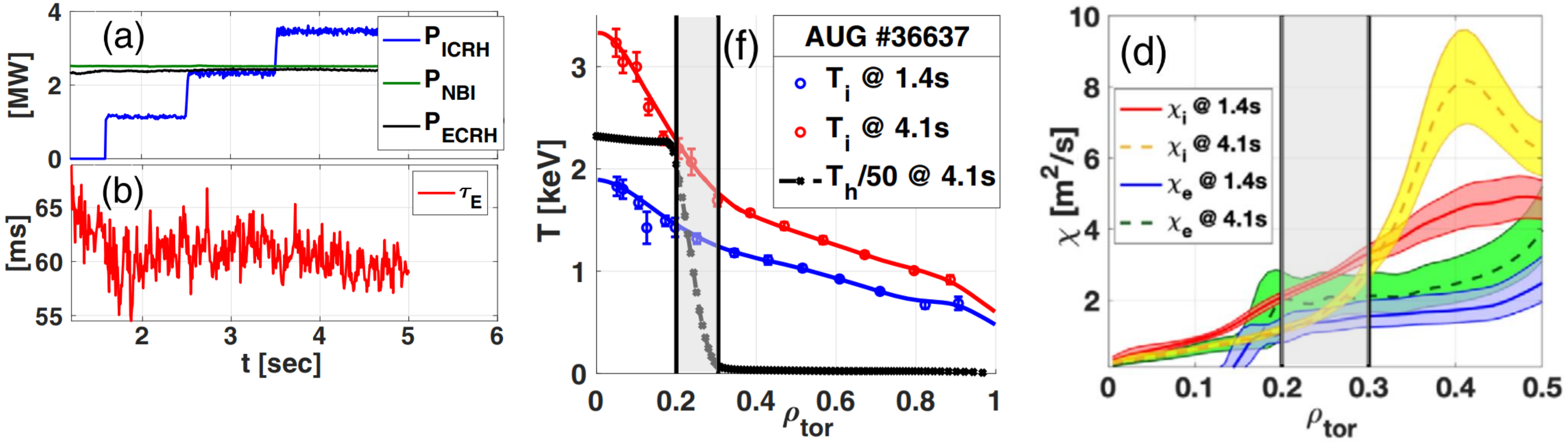
F-ATB (Fast ion-induced Anomalous Transport Barrier)

- **ASDEX-U** has found a **fast ion-induced ITB (F-ATB) by the inclusion of ICRH** in addition to the background NB and ECRH, **without AE**. ⇒ **Direct fast ion effect**.
- Ion heat conductivity χ_i was reduced by half in the ITB, despite of ~40% increased auxiliary heating.
- Gyrokinetic simulations yield clear difference in the radial profiles of total ion heat fluxes with and without fast ions.



GyroBohm unit

GK simulations

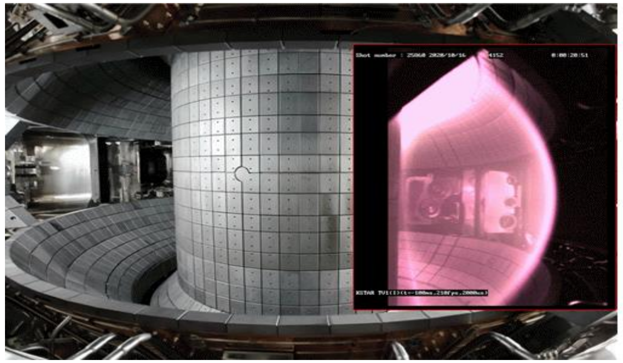


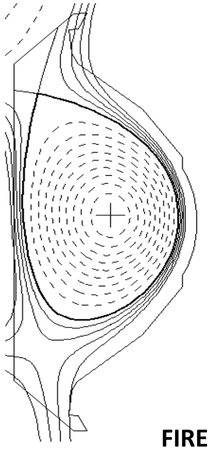
[A. Di Siena *et al*., *Phys. Rev. Lett.* **127**, 025002 (2021)]

FIRE (Fast Ion Regulated Enhancement) mode

- **Stationary ITB discharges** have been established in **NB-only** plasmas in a diverted configuration at q_{95} ~ 4–5 on **KSTAR**.
- L-H transition was avoided by keeping low density ($\bar{n}_e \sim 1.5 \times 10^{19}\ m^{-3}$) and unfavorable ∇B single-null diverted configuration.
- Fast ions have significant roles in this new regime, so it is coined to "Fast-Ion-Regulated Enhancement (**FIRE**)."

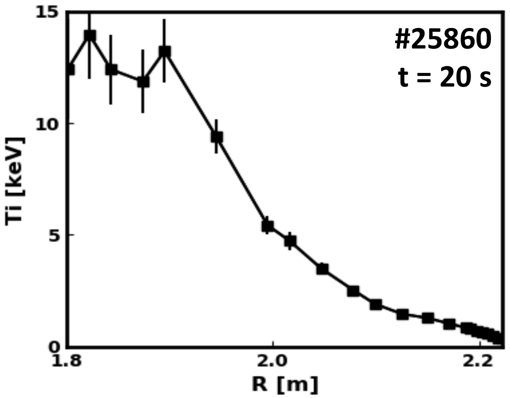
<Camera Image of KSTAR FIRE mode >



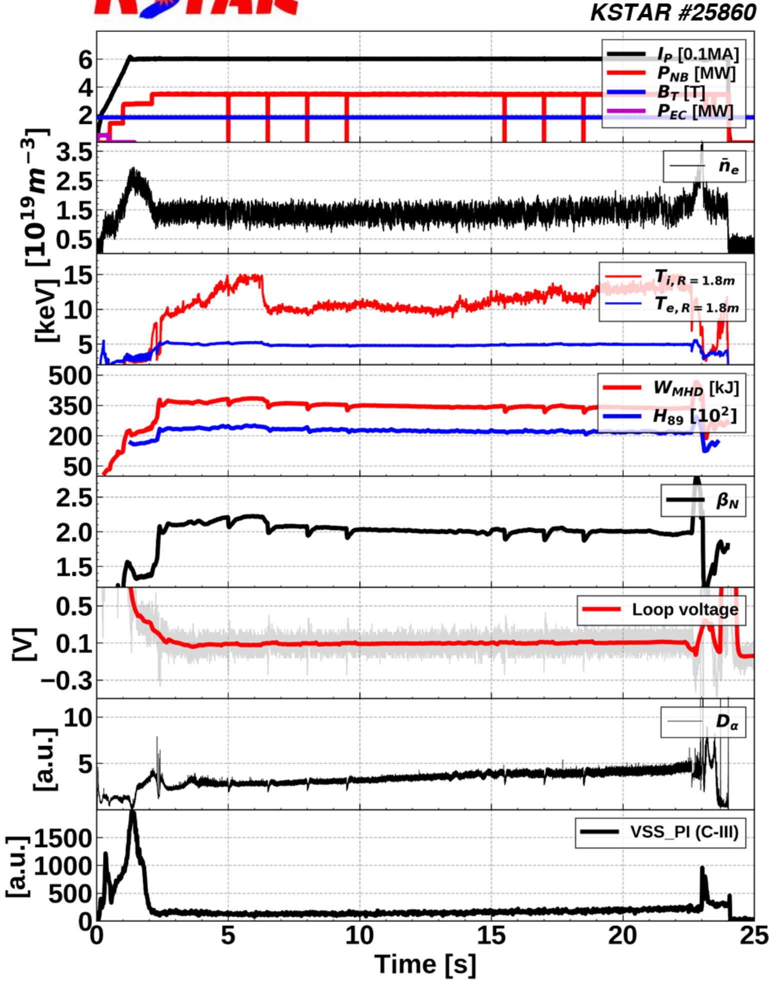


FIRE #25860
EFIT construction at 20 s

<Ion Temperature Profile of FIRE mode>



KSTAR

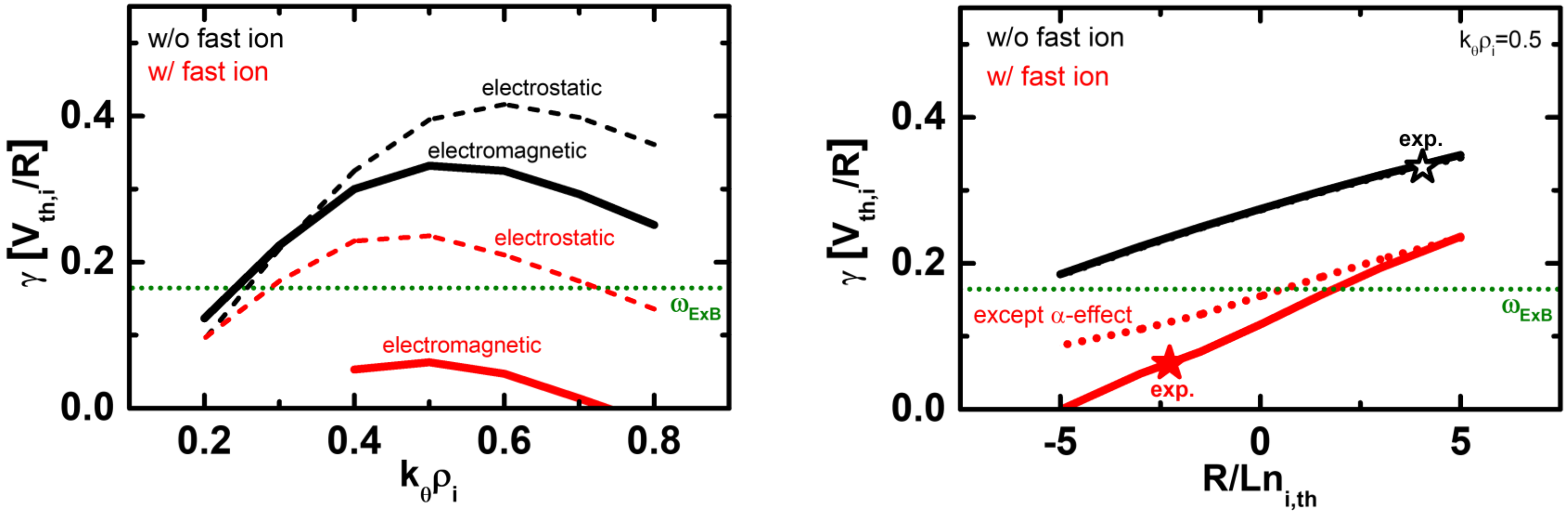


[H. Han, S.J. Park and Y.-S. Na *et al.*, *Nature* **609**, 269 (2022)]

Courtesy: Y.-S. Na

Linear turbulence stability in FIRE mode

- Thermal ion density gradient $\nabla n_i < 0$ in the core in FIRE mode due to dilution by centrally peaked fast ions.
  ⇒ **Strong linear stabilization of ITG turbulence by dilution**.
- ExB shear stabilization is also substantial.
  (But even in its absence, significant reduction of turbulence growth is expected.)
- Shafranov shift and electromagnetic effects contribute to further linear stabilization.



Linear gyrokinetic
simulations
using GKW code

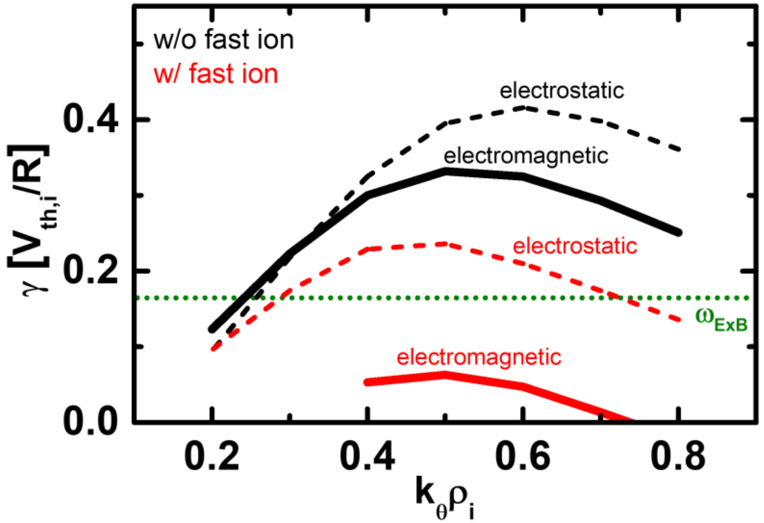
Courtesy: S.J. Park

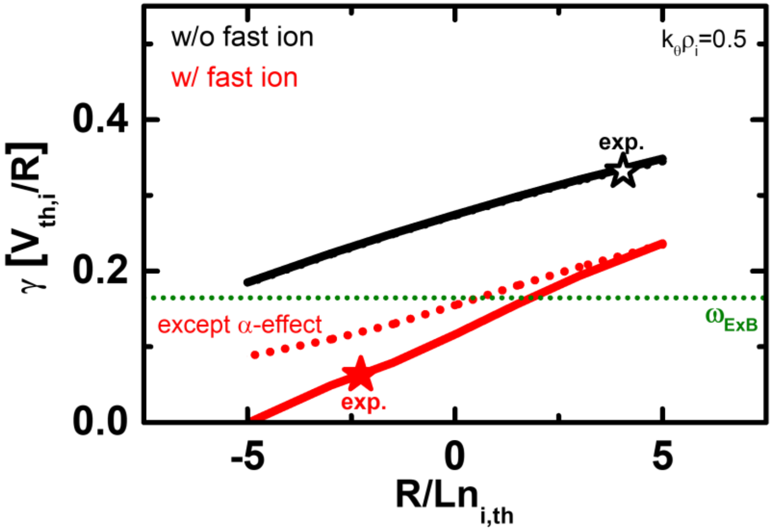
Dedicated CGYRO simulations: [D. Kim *et al*., *Nucl. Fusion* **63**, 124001 (2023), **64**, 066013 (2024)]

Linear turbulence stability in FIRE mode

- Thermal ion density gradient $\nabla n_i < 0$ in the core in FIRE mode due to dilution by centrally peaked fast ions.
  $\Rightarrow$ **Strong linear stabilization of ITG turbulence by dilution**.
- ExB shear stabilization is also substantial.
  (But even in its absence, significant reduction of turbulence growth is expected.)
- Shafranov shift and electromagnetic effects contribute to further linear stabilization.

$\Rightarrow$ So, what would happen to transport level by nonlinear saturated turbulence? $\chi_i \propto \gamma$ drops?





Linear gyrokinetic
simulations
using GKW code

Courtesy: S.J. Park

Dedicated CGYRO simulations: [D. Kim *et al*., *Nucl. Fusion* **63**, 124001 (2023), **64**, 066013 (2024)]

Lesson from simple turbulence-zonal flow model

- Simplest 0D nonlinear model for the coupled turbulence-zonal flow system is as follows.

$$\partial_t \mathcal{E} = 2\gamma\mathcal{E} - \Delta\omega(\mathcal{E})\mathcal{E} - \alpha\mathcal{E}u^2 \qquad \partial_t u^2 = \alpha\mathcal{E}u^2 - \gamma_d u^2$$

Turbulence energy (Prey)

Zonal flow energy (Predator)

Predator-Prey System

Lesson from simple turbulence-zonal flow model

- Simplest 0D nonlinear model for the coupled turbulence-zonal flow system is as follows.

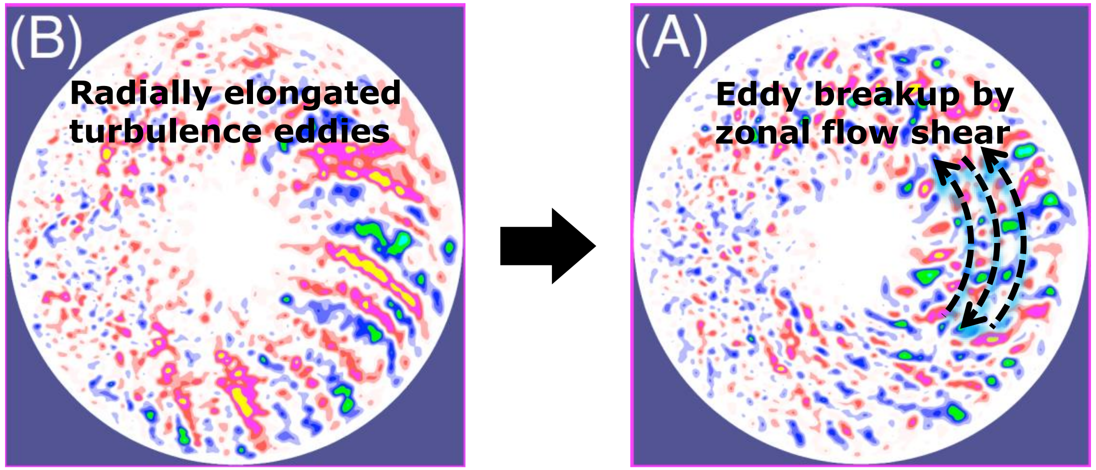
$$\partial_t \mathcal{E} = 2\gamma\mathcal{E} - \Delta\omega(\mathcal{E})\mathcal{E} \boxed{- \alpha\mathcal{E}u^2} \qquad \partial_t u^2 = \boxed{\alpha\mathcal{E}u^2} - \gamma_d u^2$$

Turbulence energy (Prey)

Zonal flow energy (Predator)

Predator-Prey System

Energy transfer from turbulence to zonal flow by nonlinear coupling



Zonal flow is nonlinearly self-generated from turbulence

⇒ Turbulence amplitude and radial size is reduced

"**Turbulence self-regulation**"

⇒ Turbulent transport is reduced. $D \sim \frac{\Delta r^2}{\Delta t}$ ⬇

The triggering mechanism of transition to an enhanced confinement regime with accompanied transport barrier formation in many cases.

Lesson from simple turbulence-zonal flow model

- Simplest 0D nonlinear model for the coupled turbulence-zonal flow system is as follows.

$$\partial_t \mathcal{E} = 2\gamma\mathcal{E} - \Delta\omega(\mathcal{E})\mathcal{E} - \alpha\mathcal{E}u^2 \qquad \partial_t u^2 = \alpha\mathcal{E}u^2 - \gamma_d u^2$$

Turbulence energy (Prey) **Zonal flow** energy (Predator)

Predator-Prey System

- Steady-state solution w/o zonal flow : $0 = 2\gamma\mathcal{E} - \Delta\omega(\mathcal{E})\mathcal{E}$

Then, with $\Delta\omega(\mathcal{E}) = k_\perp^2 D_k(\mathcal{E}) \sim k_\perp^2 D(\mathcal{E})$ we have

$$\boxed{D(\mathcal{E}) \sim \frac{\gamma}{k_\perp^2}}$$

"Mixing-Length Argument" $\Rightarrow D \sim \chi_i \propto \gamma$

- Steady-state solution **with zonal flow** : $0 = \alpha\mathcal{E}u^2 - \gamma_d u^2$

$$\boxed{D(\mathcal{E}) \propto \mathcal{E} = \frac{\gamma_d}{\alpha}}$$

Linear growth of turbulence γ **DOESN'T** have direct impact on transport level!

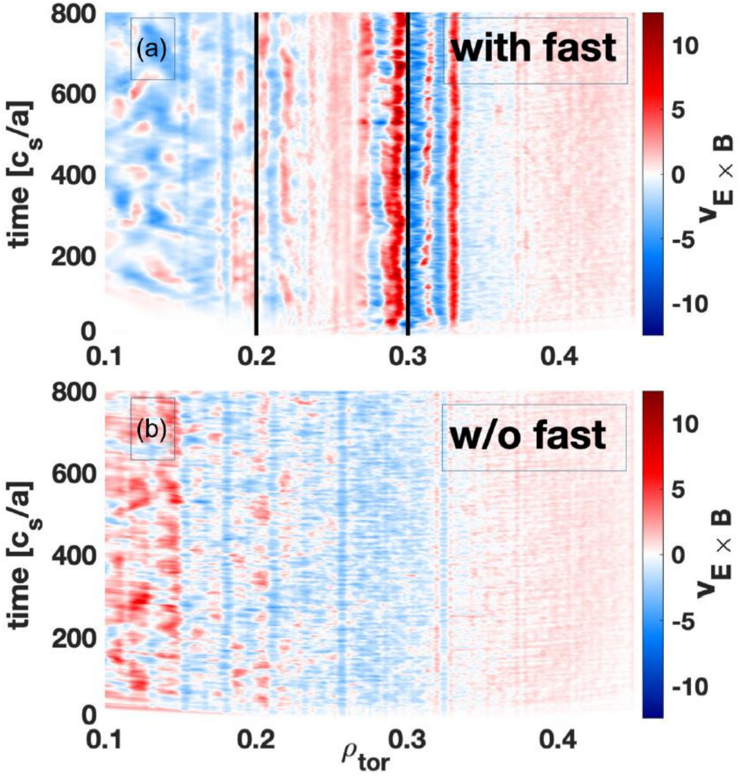
Zonal flow physics determines the turbulent transport level.

⇒ **"Fast Ion Effect on Zonal Flow" is the key**
to understand confinement enhancement in FIRE mode

Robust zonal flow with fast ions from simulations

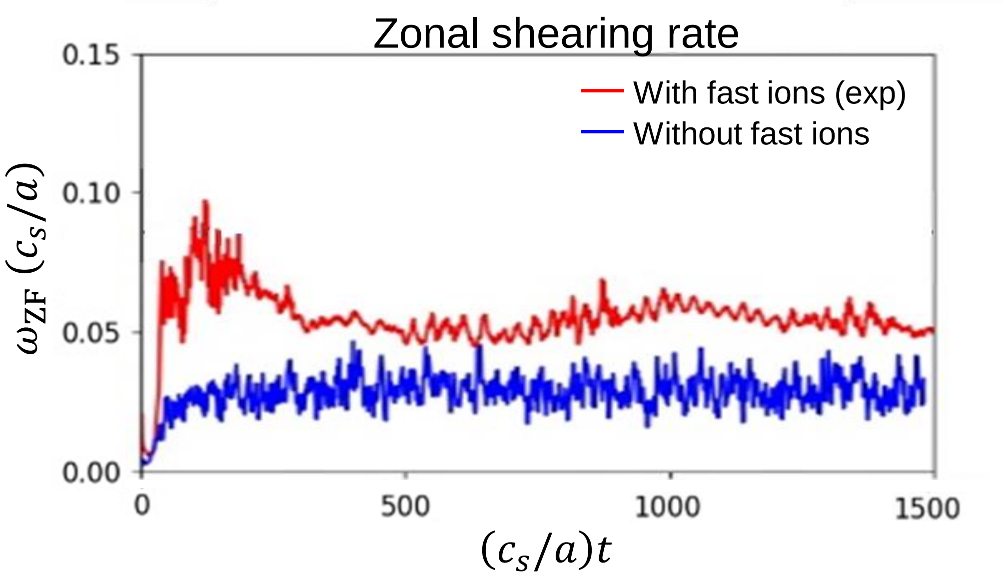
- Nonlinear gyrokinetic simulations for F-ATB in ASDEX-U and FIRE mode in KSTAR both have shown enhancement of zonal flows in the ITB region in the presence of fast ions.

GENE simulation of F-ATB discharge in AUG



[A. Di Siena *et al*., *Phys. Rev. Lett.* **127**, 025002 (2021)]

CGYRO simulation of FIRE mode in KSTAR



[D. Kim *et al*., Nucl. Fusion **63**, 124001 (2023)]

Outline

- Fast ion effects on tokamak confinement
- Modified Hasegawa-Mima equation for zonal flow generation
- Fast ion effects on zonal flow-turbulence system
- Ongoing work toward a 3-animal system

Modified Hasegawa-Mima equation

- **Modified Hasegawa-Mima equation**:
  The paradigm model for the zonal flow(ZF)-drift wave(DW) system in magnetized plasmas.

$$\frac{\partial}{\partial t}\left(\underline{\tilde{\phi}-\rho_s^2\nabla_\perp^2\phi}\right)+\rho_s c_s\left[\phi,\underline{\tilde{\phi}-\rho_s^2\nabla_\perp^2\phi}\right]+\rho_s c_s\frac{1}{n_0}\left(\frac{\partial n_0}{\partial x}\right)\frac{\partial\phi}{\partial y}=0 \quad \text{with} \quad \phi\to\frac{e\phi}{T_e},\quad \mathbf{B}=B\hat{z}$$

Potential Vorticity

which can be derived from electron continuity equation and vorticity equation, with adiabatic electron response $\delta n_e/n_0=e\tilde{\phi}/T_e$. Linearizing it, we obtain the electron DW dispersion relation

$$\omega=\frac{\omega_{*e}}{1+k_\perp^2\rho_s^2}, \quad \text{where} \quad \omega_{*e}=\rho_s c_s\frac{k_y}{L_n}.$$

- The fully normalized version of the modified Hasegawa-Mima equation is

$$\frac{\partial}{\partial t}\left(\tilde{\phi}-\nabla_\perp^2\phi\right)+\left[\underline{\phi,}\,\tilde{\phi}-\nabla_\perp^2\phi\right]-\frac{\partial\phi}{\partial y}=0 \qquad \text{where} \quad \phi\to\frac{L_n}{\rho_s}\frac{e\phi}{T_e},\quad x\to\frac{x}{\rho_s},\quad t\to\frac{L_{ne}}{c_s}t.$$

Hasegawa-Mima nonlinearity

E×B nonlinearity

Modified Hasegawa-Mima equation

- Decomposing the modified Hasegawa-Mima equation into the zonal and the drift wave parts

ZF : $\frac{\partial}{\partial t}\nabla_\perp^2\langle\phi\rangle - \left\langle \boldsymbol{\nabla}\tilde{\phi}\times\hat{z}\cdot\boldsymbol{\nabla}\nabla_\perp^2\tilde{\phi}\right\rangle = 0,$ where $\langle\cdots\rangle$ is the flux-surface average,

HM nonlinearity!

DW : $\frac{\partial}{\partial t}(\tilde{\phi}-\nabla_\perp^2\tilde{\phi}) - \boldsymbol{\nabla}\langle\phi\rangle\times\hat{z}\cdot\boldsymbol{\nabla}(\tilde{\phi}-\nabla_\perp^2\tilde{\phi}) + \boldsymbol{\nabla}\tilde{\phi}\times\hat{z}\cdot\boldsymbol{\nabla}\nabla_\perp^2\langle\phi\rangle + \frac{\partial}{\partial y}\tilde{\phi} = 0.$

- Note that from the above equations we can readily obtain the radially local energy equations

ZF : $\frac{\partial}{\partial t}\frac{1}{2}|\boldsymbol{\nabla}_\perp\langle\phi\rangle|^2 + \boldsymbol{\nabla}\cdot\langle\text{flux terms}\rangle = \boldsymbol{\nabla}_\perp\langle\phi\rangle\cdot\cancel{\left\langle\boldsymbol{\nabla}\tilde{\phi}\times\hat{z}\cdot\nabla_\perp^2\tilde{\phi}\right\rangle}$

DW : $\frac{\partial}{\partial t}\frac{1}{2}\left\langle\tilde{\phi}^2 + \left|\boldsymbol{\nabla}_\perp\tilde{\phi}\right|^2\right\rangle + \boldsymbol{\nabla}\cdot\langle\text{flux terms}\rangle = \cancel{\left\langle\boldsymbol{\nabla}_\perp\tilde{\phi}\cdot\boldsymbol{\nabla}\langle\phi\rangle\times\hat{z}\cdot\nabla_\perp^2\tilde{\phi}\right\rangle}$

which demonstrates the **energy conservation** in the ZF-DW interaction.

Modulational zonal flow growth

- The 3+3-wave-interaction yields the following expression of the modulational zonal flow growth rate Γ.

$$\Gamma^2 = \underbrace{\gamma_{\mathrm{mod}}^2}_{\text{from Reynolds Stress Drive}} - \underbrace{\Delta_{\mathrm{mm}}^2}_{\text{from Frequency Mismatch}}$$

[Diamond-Itoh-Itoh-Hahm, PPCF ‘05]
[Chen-Lin-White, PoP ‘00] toroidal kinetic extension

where

$$\gamma_{\mathrm{mod}}^2 \cong 2k_y^2 q_x^2 \left|\tilde{\phi}_0\right|^2$$

$$\Delta_{\mathrm{mm}}^2 \equiv \left\{\frac{1}{2}\left(\underbrace{(\omega_0 - \omega_+) + (\omega_0 + \omega_-)}\right)\right\}^2 \cong k_y^2 q_x^4$$

Finite threshold by frequency mismatch between the primary drift frequency ω_0 and the zonal flow-modulated sideband drift wave’s characteristic frequency $\omega_\pm$.

Outline

- Fast ion effects on tokamak confinement
- Modified Hasegawa-Mima equation for zonal flow generation
- Fast ion effects on zonal flow-turbulence system
- Ongoing work toward a 3-animal system

Gyrokinetic derivation of modified Hasegawa-Mima

- While the modified H-M equation can be derived by a combination of electron continuity equation and vorticity equation, it could also be derived from the Gyrokinetics. (Even conceptually more natural.)

$$\delta N_i + \delta n_{i\mathrm{pol}} = \delta n_e$$

Quasi-neutrality [Brizard-Hahm, RMP '07], [Wang-Hahm, PoP '06]

$$\Rightarrow \delta N_i + N_{i0}\rho_s^2\nabla_\perp^2 \frac{e\phi}{T_e} = n_{e0}\frac{e\tilde{\phi}}{T_e}$$

Ion polarization density (long-wavelength limit)

Adiabatic electron response

$$\Rightarrow \frac{\delta N_i}{n_{e0}} = \frac{e\tilde{\phi}}{T_e} - \rho_s^2\nabla_\perp^2\frac{e\phi}{T_e}$$

Ion gyrocenter density δN_i is the gyrokinetic realization of the potential vorticity. [Hahm et al., PoP '23]

$$\frac{\partial N_i}{\partial t} - \frac{1}{B}\nabla\phi \times \hat{z}\cdot\nabla N_i = 0$$

Ion gyrocenter continuity equation

$$\Rightarrow \frac{\partial}{\partial t}\left(\tilde{\phi} - \rho_s^2\nabla_\perp^2\phi\right) + \rho_s c_s\left[\phi, \tilde{\phi} - \rho_s^2\nabla_\perp^2\phi\right] + \rho_s c_s \frac{1}{n_{e0}}\left(\frac{\partial N_{i0}}{\partial x}\right)\frac{\partial\phi}{\partial y} = 0 \quad \text{with} \quad \phi \to \frac{e\phi}{T_e}.$$

Simplest gyrokinetic model of fast ion response

- For drift waves, with $\omega, \omega_{*f} \ll k_{\parallel} v_{T_f}$ and Maxwellian fast ion F_0, the linearized gyrokinetic equation

$$-i\left(\omega - k_{\parallel} v_{\parallel f}\right)\delta\tilde{F}_f - i\left(\omega_{*f} - k_{\parallel} v_{\parallel f}\right)\frac{e\tilde{\phi}}{T_f} J_0\left(k_{\perp}\rho_f\right)F_{0f} = 0$$

yields a fast ion gyrocenter density response

$$\frac{\delta\tilde{N}_f}{N_{f0}} = -\Gamma_0\left(k_{\perp}^2\rho_{Tf}^2\right)\frac{e\tilde{\phi}}{T_f}$$

which together with fast ion polarization density

$$\frac{\delta n_{f\mathrm{pol}}}{N_{f0}} = -\left[1 - \Gamma_0\left(k_{\perp}^2\rho_{Tf}^2\right)\right]\frac{e\phi}{T_f}$$

gives adiabatic fast ion response

$$\frac{\delta\tilde{n}_f}{N_{f0}} = -\frac{e\tilde{\phi}}{T_f}.$$

- Therefore, **fast ion response to DW is negligible** compared to the electron response due to $T_f \gg T_e$.

Simplest gyrokinetic model of fast ion response

- Meanwhile, for the fast ion response to zonal flows, since we have $\langle \delta N_f \rangle = 0$,

$$\frac{\langle \delta n_f \rangle}{N_{f0}} = \frac{\langle \delta n_{f\mathrm{pol}} \rangle}{N_{f0}} = -\left[1 - \Gamma_0\left(k_\perp^2 \rho_{Tf}^2\right)\right] \frac{e\langle \phi \rangle}{T_f} \rightarrow -k_\perp^2 \rho_s^2 \frac{e\langle \phi \rangle}{T_e} \quad \text{for} \quad k_\perp \rho_f \ll 1$$

The same with $\frac{\langle \delta n_{i\mathrm{pol}} \rangle}{N_{i0}}$ **!**

- As a result, in the long-wavelength limit $k_\perp \rho_f \ll 1$, the potential vorticity with fast ions becomes

(e.g. KSTAR FIRE mode : T_f/T_e ~ 10)

$$\frac{\delta N_i}{n_{e0}} = \frac{\delta n_e}{n_{e0}} - \frac{\delta n_{i\mathrm{pol}}}{n_{e0}} - \frac{\delta n_f}{n_{e0}}$$

$$= \frac{e\tilde{\phi}}{T_e} \underbrace{- (1-f)\rho_s^2 \nabla_\perp^2 \frac{e\tilde{\phi}}{T_e}}_{\text{DW vorticity reduced by fast ion}} \underbrace{- \rho_s^2 \nabla_\perp^2 \frac{e\langle \phi \rangle}{T_e}}_{\text{ZF vorticity unchanged by fast ions}} \qquad \text{where} \quad f \equiv \frac{n_{f0}}{n_{e0}} \quad \text{fast ion population}$$

[T.S. Hahm, G.J. Choi, S.J. Park and Y.-S. Na, *Phys. Plasmas* **30**, 072501 (2023)]

Modified Hasegawa-Mima with fast ions

- Substituting the expression to the thermal ion gyrocenter continuity equation, we obtain the modified Hasegawa-Mima equation as follows.

$$\frac{\partial}{\partial t}\left\{\tilde{\phi} - (1-f)\nabla_\perp^2\tilde{\phi} - \nabla_\perp^2\langle\phi\rangle\right\} + \left[\phi, \tilde{\phi} - (1-f)\nabla_\perp^2\tilde{\phi} - \nabla_\perp^2\langle\phi\rangle\right] - \eta_n\frac{\partial\phi}{\partial y} = 0 \quad \text{where} \quad \eta_n \equiv \frac{L_{ni}}{L_{ne}}$$

- Note that E×B nonlinearity is unchanged, and Hasegawa-Mima nonlinearity is reduced by $(1-f)$.

- The electron DW eigenfrequency

$$\omega = \frac{(1-f)\eta_n}{1+(1-f)k_\perp^2}\omega_* \qquad \Downarrow$$

is **considerably decreased** by thermal ion dilution $(1-f)$ and profile gradient reduction $\eta_n < 1$.

[T.S. Hahm, G.J. Choi, S.J. Park and Y.-S. Na, *Phys. Plasmas* **30**, 072501 (2023)]

Modulational zonal flow growth with fast ions

- Therefore, with fast ions, the modulational zonal flow growth rate Γ becomes

$$\Gamma^2 = \underbrace{\gamma_{\mathrm{mod}}^2}_{\text{from Reynolds Stress Drive}} - \underbrace{\Delta_{\mathrm{mm}}^2}_{\text{from Frequency Mismatch}}$$

where

$$\gamma_{\mathrm{mod}}^2 \cong 2(1-f)k_y^2 q_x^2 \left|\tilde{\phi}_0\right|^2$$

Reynolds stress drive is reduced

$$\Delta_{\mathrm{mm}}^2 \equiv \left\{\frac{1}{2}\left((\omega_0 - \omega_+) + (\omega_0 + \omega_-)\right)\right\}^2 \cong (1-f)^4 \eta_n^2 k_y^2 q_x^2$$

Frequency mismatch is reduced much more strongly!

- Therefore, we have significant reduction of threshold for zonal flow growth by fast ions.
  In other words, we have an **easier zonal flow generation** with fast ions!

[T.S. Hahm, G.J. Choi, S.J. Park and Y.-S. Na, *Phys. Plasmas* **30**, 072501 (2023)]

Modulational zonal flow growth with fast ions

- Therefore, with fast ions, the modulational zonal flow growth rate Γ becomes

$$\Gamma^2 = \underbrace{\gamma_{\mathrm{mod}}^2}_{\text{from Reynolds Stress Drive}} - \underbrace{\Delta_{\mathrm{mm}}^2}_{\text{from Frequency Mismatch}}$$

where

$$\gamma_{\mathrm{mod}}^2 \cong 2(1-f)k_y^2 q_x^2 \left|\tilde{\phi}_0\right|^2$$

Reynolds stress drive is reduced

$$\Delta_{\mathrm{mm}}^2 \equiv \left\{\frac{1}{2}\left((\omega_0-\omega_+)+(\omega_0+\omega_-)\right)\right\}^2 \cong (1-f)^4 \eta_n^2 k_y^2 q_x^2$$

Frequency mismatch is reduced much more strongly!

- Therefore, we have significant reduction of threshold for zonal flow growth by fast ions.
  In other words, we have an **easier zonal flow generation** with fast ions!

⇒ **Saturated drift wave turbulence level with zonal flow?**

Zonal flow growth from broadband turbulence

- Using wave-kinetic equation and zonal flow vorticity equation, a standard calculation with fast ions yields

$$-i\Omega = -(1-f)^2 q^2 \eta_n \sum_{\mathbf{k}} \frac{k_y^2 \omega_*}{\left[1+(1-f)k_\perp^2\right]^2} R_q k_x \frac{\partial \langle N \rangle}{\partial k_x}, \qquad R_q^{-1} \simeq -i\left(\Omega - q v_{gx}\right) + 2\gamma$$

where $N(\mathbf{x}, \mathbf{k}, t) = \frac{\mathcal{E}_\mathbf{k}}{\omega_\mathbf{k}} = \frac{\left[1+(1-f)k_\perp^2\right]}{\omega_\mathbf{k}} \left|\tilde{\phi}_\mathbf{k}\right|^2$ Wave action density

- We have two limiting forms of the zonal flow dispersion relation as follows.

1. Strong turbulence (resonant) regime

$$\Gamma \simeq -(1-f)^2 q^2 \eta_n \sum_{\mathbf{k}} \frac{k_y^2 \omega_*}{\left[1+(1-f)k_\perp^2\right]^2} \frac{1}{2\gamma} k_x \frac{\partial \langle N \rangle}{\partial k_x}$$

2. Weak turbulence (non-resonant) regime

$$\Omega \simeq (1-f)^2 q^2 \eta_n \sum_{\mathbf{k}} \frac{k_y^2 \omega_*}{\left[1+(1-f)k_\perp^2\right]^2} \frac{1}{\Omega - q v_{gx}} k_x \frac{\partial \langle N \rangle}{\partial k_x}$$

⇒ **Recover the 3+3-wave calculation with**

$$q v_{gx} = -\frac{2(1-f)^2 \eta_n \omega_* q k_x}{\left[1+(1-f)k_\perp^2\right]^2}$$

Continuum version of Frequency Mismatch

[G.J. Choi, P.H. Diamond and T.S. Hahm, *Nucl. Fusion* **64**, 016029 (2024)]

Zonal flow growth from broadband turbulence

- Using wave-kinetic equation and zonal flow vorticity equation, a standard calculation with fast ions yields

$$-i\Omega = -(1-f)^2 q^2 \eta_n \sum_{\mathbf{k}} \frac{k_y^2 \omega_*}{\left[1+(1-f)k_\perp^2\right]^2} R_q k_x \frac{\partial\langle N\rangle}{\partial k_x}, \qquad R_q^{-1} \simeq -i\left(\Omega - q v_{gx}\right) + 2\gamma$$

where $N(\mathbf{x},\mathbf{k},t) = \dfrac{\mathcal{E}_{\mathbf{k}}}{\omega_{\mathbf{k}}} = \dfrac{\left[1+(1-f)k_\perp^2\right]}{\omega_{\mathbf{k}}}\left|\tilde{\phi}_{\mathbf{k}}\right|^2$ Wave action density

Likely relevant to core confinement enhancement

- We have two limiting forms of the zonal flow dispersion relation as follows.

1. Strong turbulence (resonant) regime

$$\Gamma \simeq -(1-f)^2 q^2 \eta_n \sum_{\mathbf{k}} \frac{k_y^2 \omega_*}{\left[1+(1-f)k_\perp^2\right]^2} \frac{1}{2\gamma} k_x \frac{\partial\langle N\rangle}{\partial k_x}$$

2. Weak turbulence (non-resonant) regime

$$\Omega \simeq (1-f)^2 q^2 \eta_n \sum_{\mathbf{k}} \frac{k_y^2 \omega_*}{\left[1+(1-f)k_\perp^2\right]^2} \frac{1}{\Omega - q v_{gx}} k_x \frac{\partial\langle N\rangle}{\partial k_x}$$

⇒ **Recover the 3+3-wave calculation with**

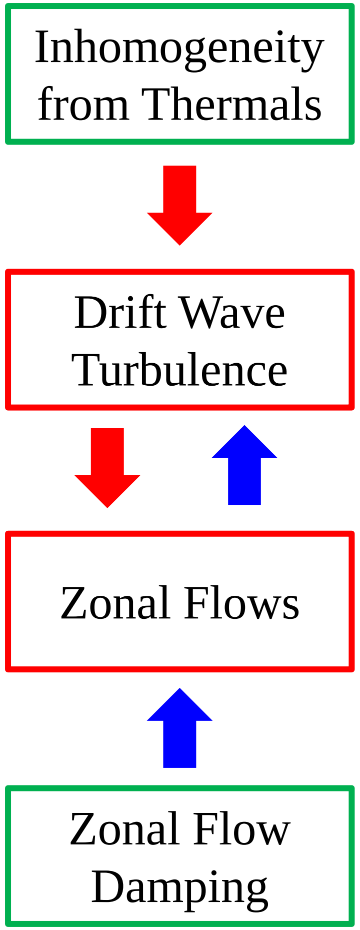
$$q v_{gx} = -\frac{2(1-f)^2 \eta_n \omega_* q k_x}{\left[1+(1-f)k_\perp^2\right]^2}$$

Continuum version of Frequency Mismatch

[G.J. Choi, P.H. Diamond and T.S. Hahm, *Nucl. Fusion* **64**, 016029 (2024)]

Fast Ion Effects on Regulation of Drift Wave Turbulence and Zonal flow

Usual Story without Fast Ions



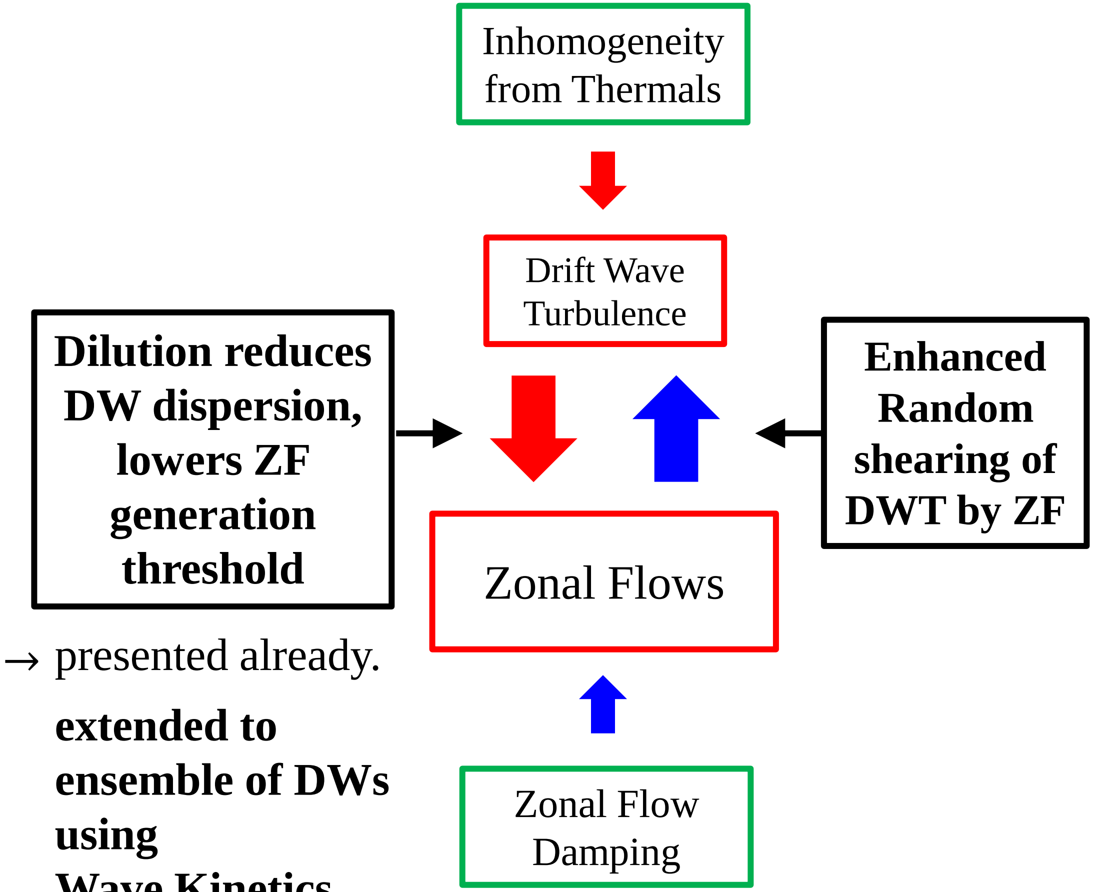
Inhomogeneity from Thermals

Drift Wave Turbulence

Zonal Flows

Zonal Flow Damping

With Dilution from Fast Ions



Inhomogeneity from Thermals

Drift Wave Turbulence

Dilution reduces DW dispersion, lowers ZF generation threshold

Enhanced Random shearing of DWT by ZF

Zonal Flows

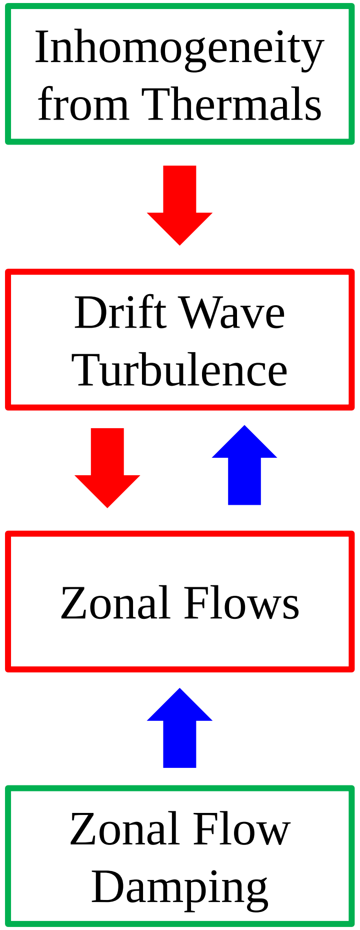
Zonal Flow Damping

→ presented already.

extended to ensemble of DWs using Wave Kinetics.

Fast Ion Effects on Regulation of Drift Wave Turbulence and Zonal flow

Usual Story without Fast Ions



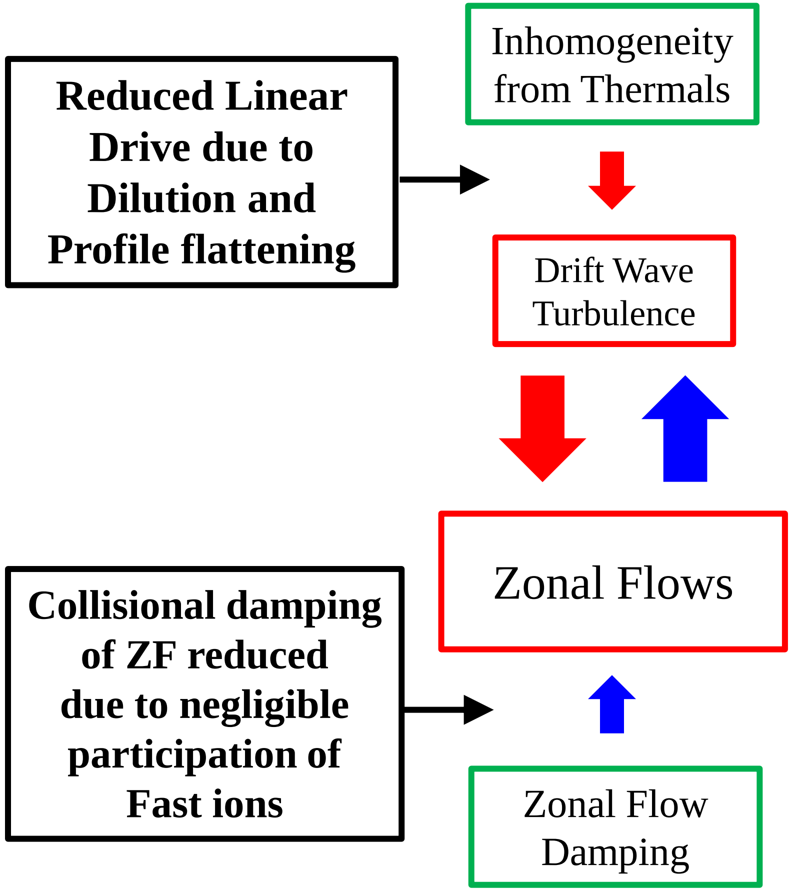
Inhomogeneity from Thermals

Drift Wave Turbulence

Zonal Flows

Zonal Flow Damping

With Dilution from Fast Ions



Reduced Linear Drive due to Dilution and Profile flattening

Inhomogeneity from Thermals

Drift Wave Turbulence

Zonal Flows

Collisional damping of ZF reduced due to negligible participation of Fast ions

Zonal Flow Damping

Predator-Prey model for ZF-DWT with fast ions

- Put everything together, for weak turbulence regime relevant to core turbulence,

ZF : $$\partial_t u^2 = \sqrt{\gamma_{\mathrm{mod}}^2 - \Delta_{\mathrm{mm}}^2}\,H(\gamma_{\mathrm{mod}} - \Delta_{\mathrm{mm}})u^2 - (1-f)\gamma_{d(0)}u^2$$

DWT : $$\partial_t \mathcal{E} = 2\gamma\mathcal{E} - \sqrt{\gamma_{\mathrm{mod}}^2 - \Delta_{\mathrm{mm}}^2}\,H(\gamma_{\mathrm{mod}} - \Delta_{\mathrm{mm}})u^2 - (1-f)B\mathcal{E}^2$$

- The general expression for the nontrivial fixed point : $\boxed{\gamma_{\mathrm{mod}}^2 = \Delta_{\mathrm{mm}}^2 + \gamma_d^2}$

  That is, either DW frequency mismatch or collisional ZF damping provide the threshold for ZF generation.

⇒ **Collisionless limit relevant to core confinement enhancement**:

$$\mathcal{E} \approx \frac{(1-f)^3\eta_n^2\Delta_{\mathrm{mm}(0)}^2}{A'}$$

⬇ **Significant reduction by fast ion-induced dilution**

which is determined by a balance between

$\gamma_{\mathrm{mod}}^2 = (1-f)A'\mathcal{E}$: modulational zonal flow drive

$\Delta_{\mathrm{mm}}^2 = (1-f)^4\eta_n^2\Delta_{\mathrm{mm}(0)}^2$: frequency mismatch

[G.J. Choi, P.H. Diamond and T.S. Hahm, *Nucl. Fusion* **64**, 016029 (2024)]

Conclusion

- Theory suggests a strong effect of dilution $\sim(1-f)^3\eta_n^2$ in weak turbulence regime relevant to core, leading to greatly reduced saturated turbulence level.

- The reduction in turbulence level is completely due to the change in the zonal flow dynamics, rather than change in the linear stability of turbulence.

- These in turn reduce the level of transport, and so improve confinement. Dilution is thus seen as a likely cause of confinement improvement in recent experiments with a large fraction of energetic particles.

[G.J. Choi, P.H. Diamond and T.S. Hahm, *Nucl. Fusion* **64**, 016029 (2024)]

Conclusion

- Theory suggests a strong effect of dilution $\sim(1-f)^3\eta_n^2$ in weak turbulence regime relevant to core, leading to greatly reduced saturated turbulence level.

- The reduction in turbulence level is completely due to the change in the zonal flow dynamics, rather than change in the linear stability of turbulence.

- These in turn reduce the level of transport, and so improve confinement. Dilution is thus seen as a likely cause of confinement improvement in recent experiments with a large fraction of energetic particles.

But it is of course far from the end of the whole story

Outline

- Fast ion effects on tokamak confinement

- Modified Hasegawa-Mima equation for zonal flow generation

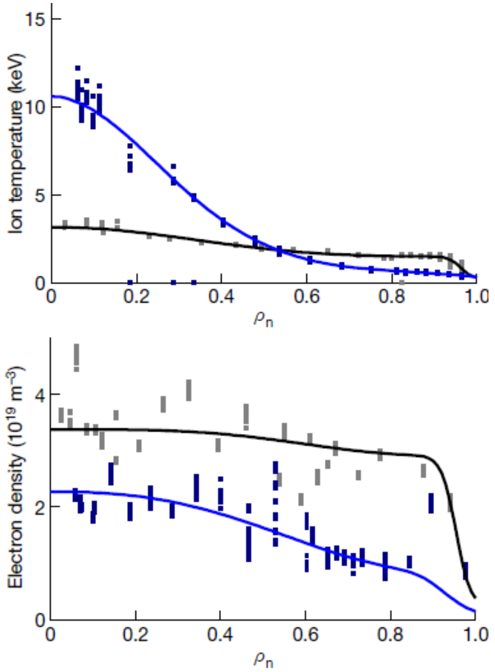
- Fast ion effects on zonal flow-turbulence system

- Ongoing work toward a 3-animal system

Various routes of fast ion effect on confinement

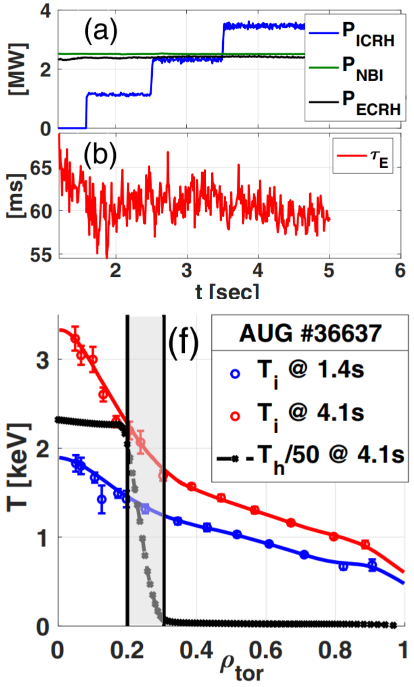
- The mechanisms of fast ion-induced confinement enhancement achieved in recent tokamak experiments seem to be <u>completely different</u>!
- **More comprehensive understanding of fast ion physics ⇒ Novel operation conditions!**

KSTAR (KOREA)



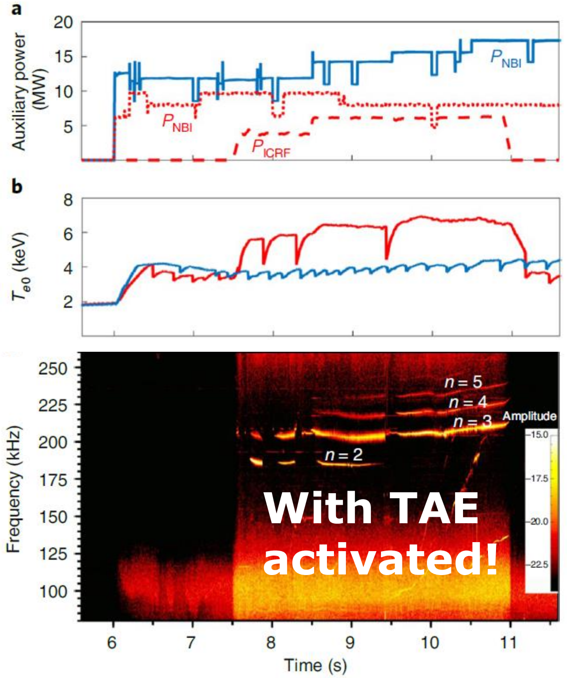
[H. Han *et al.*, *Nature* '22]

AUG (Germany)



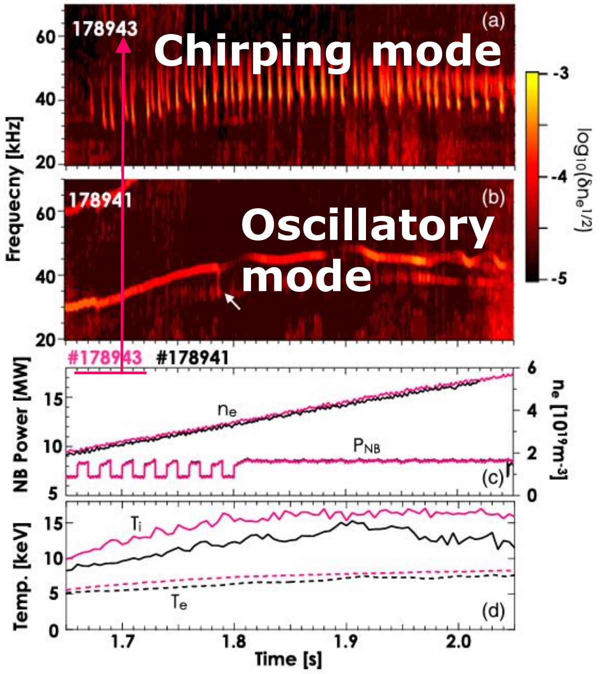
[A. Di Siena *et al.*, *Phys. Rev. Lett.* '21]

JET (UK)



[S. Mazzi *et al.*, *Nat. Phys.* '22]

DIII-D (US)

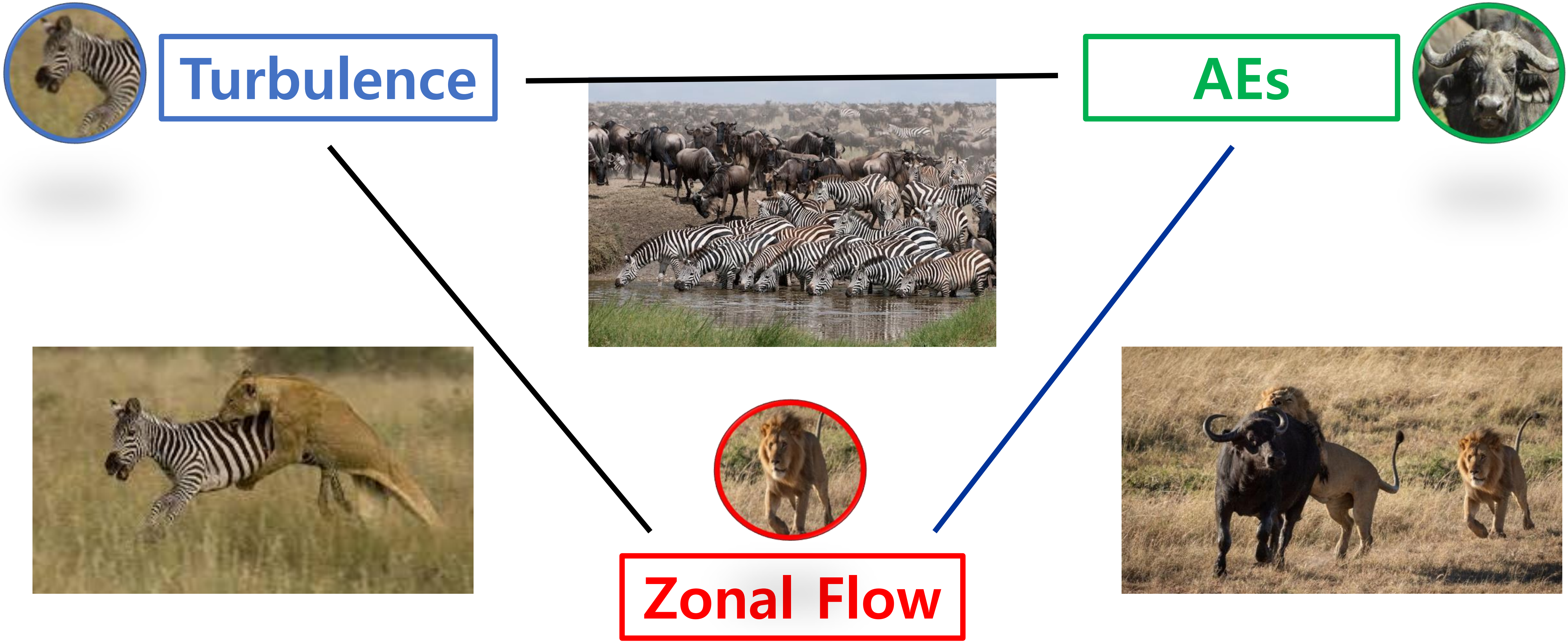


[X.D. Du *et al.*, *Phys. Rev. Lett.* '21]

→ Complexity

Ongoing Work

Heading to 3-Animal Problem!



Turbulence

AEs

Zonal Flow

Zonal Flow from Shear Alfvén Wave?

- Low-frequency Alfvén eigenmodes (AEs) are versions of the shear Alfvén wave (SAW).

  ⇒ **Zonal Flow self-generation from shear Alfvén wave?**

- **Issue**: shear Alfvén wave is in the **"Alfvén State"** $\omega = \pm k_\parallel v_\mathrm{A}, \quad \delta\mathbf{u} = \mp v_\mathrm{A}\delta\mathbf{b}$

  $$\Rightarrow \quad \frac{\partial\delta\mathbf{u}}{\partial t} - v_\mathrm{A}^2\nabla_\parallel\delta\mathbf{b} = \cancel{-\delta\mathbf{u}\cdot\boldsymbol{\nabla}\delta\mathbf{u} + v_\mathrm{A}^2\delta\mathbf{b}\cdot\boldsymbol{\nabla}\delta\mathbf{b}} \qquad \frac{\partial\delta\mathbf{b}}{\partial t} - \nabla_\parallel\delta\mathbf{u} = \cancel{\boldsymbol{\nabla}\times(\delta\mathbf{u}\times\delta\mathbf{b})}$$

  No nonlinear evolution!

- **How to break the Alfvén state?**

  ⇒ Need a **deviation of $\boldsymbol{\omega}$ from $\pm\boldsymbol{\omega}_\mathbf{A}$**, which can be achieved by

  1. Magnetic field inhomogeneity: inside a frequency gap, we have $\omega \neq$ shear Alfvén continuum $\Delta\omega/\omega_\mathrm{A} \sim \epsilon$
  2. Finite Larmor radius (FLR): from ion polarization, heading to "kinetic Alfvén wave" $\Delta\omega/\omega_\mathrm{A} \sim k_\perp^2\rho_s^2$
  3. Plasma inhomogeneity: coupling of shear Alfvén wave with drift wave ("drift-Alfvén wave") $\Delta\omega \sim \omega_{*e}$

- For a shear Alfvén wave in a typical toroidal fusion plasma, we have $\omega_{*e}/\omega_\mathrm{A} \sim (k_y\rho_s)\, q\sqrt{\beta}/\epsilon \sim \epsilon$

  ⇒ **We address ZF generation from SAW by FLR and DW coupling in a reduced system.**

Reduced Equations for Drift-Alfvén Wave

- Since we focus on the finite ZF generation from SAW by the coupling with DW, an electromagnetic extension of the modified Hasegawa-Mima equation is enough.

$$\frac{dN_i}{dt} = 0, \quad \frac{d\delta\sigma_{\mathrm{pol}}}{dt} + \mathbf{b}\cdot\nabla\delta\mathbf{j}_\parallel = 0, \quad \underline{\frac{\partial\delta A_\parallel}{\partial t} = \mathbf{b}\cdot\nabla(\delta\phi_{\mathrm{eff}} - \delta\phi),}$$

where $\mathbf{b} = \hat{z} - \frac{\hat{z}\times\nabla\delta A_\parallel}{B_0}.$

$\Rightarrow \frac{\partial\delta\mathbf{b}}{\partial t} = \nabla\times(\delta\mathbf{u}\times\mathbf{b})$ in the zero parallel electric field limit $\delta\phi_{\mathrm{eff}} = 0$

$$\delta N_i = \underline{\delta n_e} - \delta n_{ip}$$

Assume **adiabatic response** for a simple modeling

$$\boxed{\delta n_e = \underline{\frac{e\delta\phi_{\mathrm{eff}}}{T_e}n_{e0}} - \underline{\frac{\delta x}{L_{ne}}n_{e0}}}$$

$\Leftarrow$ from the full parallel electron force balance $0 = -\mathbf{b}\cdot\nabla\delta\phi_{\mathrm{eff}} - \mathbf{b}\cdot\nabla P_e$

δx is from the Clebsh form $\mathbf{b} = \nabla(x+\delta x)\times\nabla(y+\delta y)$

[Lin-Chen, PoP ’01], [Nishimura-Lin-Wang, PoP ‘07]

Full parallel electric force

Radial thermal force involved due to magnetic field line bending

Frequency Shift of Shear Alfvén Wave

- From the three reduced equations we obtain the linear dispersion relation of drift-Alfvén wave.

$$1 + k_\perp^2 \rho_s^2 - \frac{\omega_{*i}}{\omega} = \frac{\omega(\omega - \omega_{*e})}{\omega_{\mathrm{A}}^2}$$

Recall $\frac{dN_i}{dt} = 0, \quad \delta n_e = \frac{e\delta\phi_{\mathrm{eff}}}{T_e} n_{e0} - \frac{\delta x}{L_{ne}} n_{e0}$

Here, $\omega_{*i} = \omega_{*e} \frac{n_{i0}}{n_{e0}} \frac{L_{ne}}{L_{ni}}$ shouldn't be confused with ion diamagnetic frequency.

- The frequency modification of the SAW by FLR and DW coupling is given by

$$\Delta\omega \simeq \pm\frac{1}{2}\omega_{\mathrm{A}} k_\perp^2 \rho_s^2 + \frac{1}{2}(\omega_{*e} - \omega_{*i})$$

$= 0$ in the absence of fast ions

- But the point is that the two ω_* have different physics origins, one from the parallel electron thermal force and the other from the E×B advection of the thermal ion gyrocenter density.

Simple Modeling of Fast Ions: Issues

- As before, for a simple fast ion modeling, from the gyrokinetics

$$v_\parallel [\![\mathbf{b}_{\mathrm{gc}}]\!] \cdot \nabla(F_0 + \delta F) + [\![\mathbf{b}_{\mathrm{gc}}]\!] \cdot \nabla [\![\delta\phi_{\mathrm{eff,gc}}]\!] v_\parallel \frac{e}{T} F_0 = 0 \qquad [\![\cdots]\!] \text{ Gyroaverage}$$

we obtain the adiabatic fast ion gyrocenter response and fast ion polarization density

$$\frac{\delta\tilde{N}_f}{n_{f0}} = -\Gamma_0\left(k_\perp^2\rho_{Tf}^2\right)\frac{e\delta\tilde{\phi}_{\mathrm{eff}}}{T_f} - \Gamma_0\left(k_\perp^2\rho_{Tf}^2\right)\frac{\delta\tilde{x}}{L_{nf}}, \qquad \frac{\delta n_{f\mathrm{pol}}}{n_{f0}} = -\left[1 - \Gamma_0\left(k_\perp^2\rho_{Tf}^2\right)\right]\frac{e_f\delta\tilde{\phi}}{T_f}.$$

$$\Rightarrow \frac{\delta\tilde{n}_f}{n_{f0}} = -\cancel{\frac{e_f\delta\tilde{\phi}_{\mathrm{eff}}}{T_f}} \underbrace{+ \left[1 - \Gamma_0\left(k_\perp^2\rho_{Tf}^2\right)\right]\frac{e_f}{T_f}(\mathbf{b}\cdot\nabla)^{-1}\frac{\partial\delta\tilde{A}_\parallel}{\partial t}}_{\textbf{???}} - \Gamma_0\left(k_\perp^2\rho_{Tf}^2\right)\frac{\delta\tilde{x}}{L_{nf}}$$

The same order with $\frac{\delta\tilde{n}_{i\mathrm{pol}}}{n_{f0}}$ in the long-wavelength limit,

in the direction that adds up with the thermal ion polarization density

⇒ SAW vorticity is not significantly affected by fast ions?

Simple Modeling of Fast Ions: Issues

- As before, for a simple fast ion modeling, from the gyrokinetics

$$v_\parallel [\![\mathbf{b}_{\mathrm{gc}}]\!] \cdot \nabla (F_0 + \delta F) + [\![\mathbf{b}_{\mathrm{gc}}]\!] \cdot \nabla [\![\delta \phi_{\mathrm{eff,gc}}]\!] v_\parallel \frac{e}{T} F_0 = 0$$

$[\![\cdots]\!]$ Gyroaverage

we obtain the adiabatic fast ion gyrocenter response and fast ion polarization density

$$\frac{\delta \tilde{N}_f}{n_{f0}} = -\Gamma_0\left(k_\perp^2 \rho_{Tf}^2\right) \frac{e \delta \tilde{\phi}_{\mathrm{eff}}}{T_f} - \Gamma_0\left(k_\perp^2 \rho_{Tf}^2\right) \frac{\delta \tilde{x}}{L_{nf}}, \qquad \frac{\delta n_{f\mathrm{pol}}}{n_{f0}} = -\left[1 - \Gamma_0\left(k_\perp^2 \rho_{Tf}^2\right)\right] \frac{e_f \delta \tilde{\phi}}{T_f}.$$

$$\Rightarrow \frac{\delta \tilde{n}_f}{n_{f0}} = \cancel{- \frac{e_f \delta \tilde{\phi}_{\mathrm{eff}}}{T_f}} + \left[1 - \Gamma_0\left(k_\perp^2 \rho_{Tf}^2\right)\right] \frac{e_f}{T_f} (\mathbf{b} \cdot \nabla)^{-1} \frac{\partial \delta \tilde{A}_\parallel}{\partial t} \underline{- \Gamma_0\left(k_\perp^2 \rho_{Tf}^2\right) \frac{\delta \tilde{x}}{L_{nf}}}$$

Decreases the effect of the electron density gradient, in a way different from just dilution due to Γ_0.

⇒ Non-cancellation of the two ω_* in the eigenfrequency expected in the presence of fast ions

BUT in the long-wavelength regime its effect is much smaller than FLR, that is, polarization effect on the frequency shift of SAW…

Summary of the Ongoing Work

- We revisited a reduced system for drift-Alfvén wave with uniform $\mathbf{B}_0$, finding effects of FLR and DW coupling on the frequency modification of a SAW (and thus on the ZF generation) much smaller than that of the magnetic field inhomogeneity,
  in the long-wavelength regime $k_\perp^2\rho_f^2 < 1$.

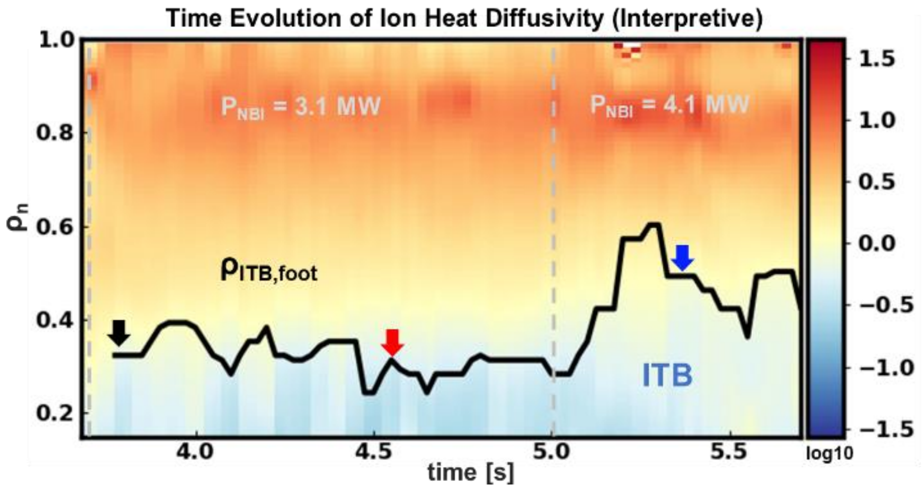
- However, such modifications could be significant with fast ions in the intermediate wavelength regime $k_\perp^2\rho_f^2 > 1 > k_\perp^2\rho_i^2$, even before the kinetic Alfvén wave regime $k_\perp^2\rho_i^2 \sim 1$.
  ⇒ To be addressed in the near future

- Another possible future work would be addressing fast ion effect on the zonal flow generation from electromagnetic drift wave (EM version of our previous work).

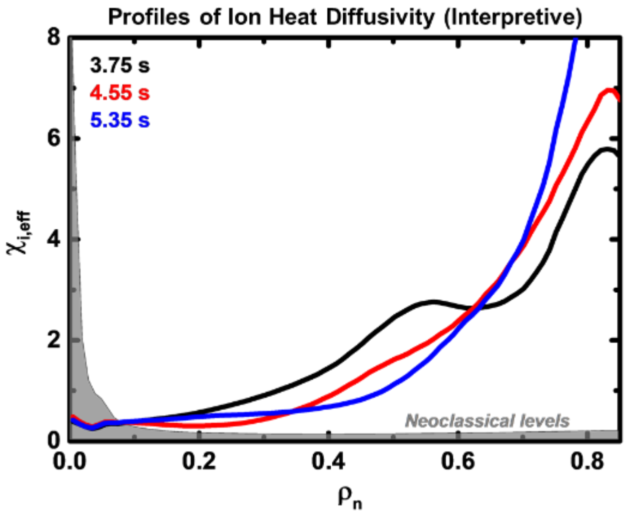
Back-up

ITB characteristics of FIRE mode

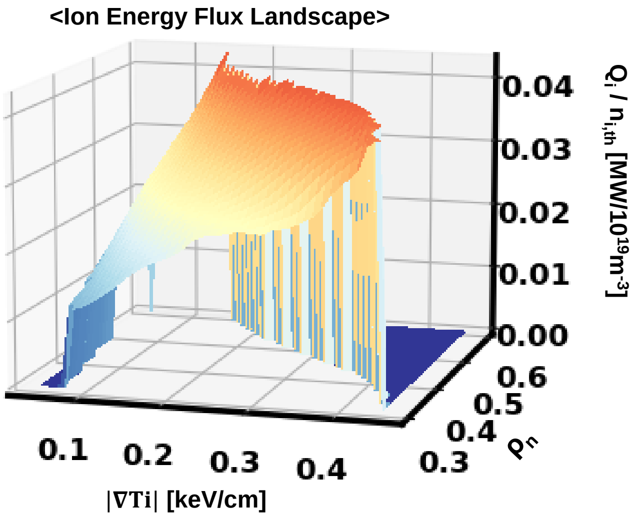
- **Thermal Ion Heat Diffusivity and S-curve**
  - The time evolution of the ion heat diffusivity was calculated from the power balance analysis.
  - The **thermal ion heat diffusivity reduces** in time correlated **with the expansion of ITB** though it is still above the neoclassical level.
  - The relation between the **ion energy flux** and the **ion temperature gradient** shows that there is a “**S-curve**” in the 3-D landscape* [P.H. Diamond *et al*., PRL (1997)].
  - The reduction of the energy flux while the gradient increases implies a transport bifurcation.



Time Evolution of Ion Heat Diffusivity (Interpretive)



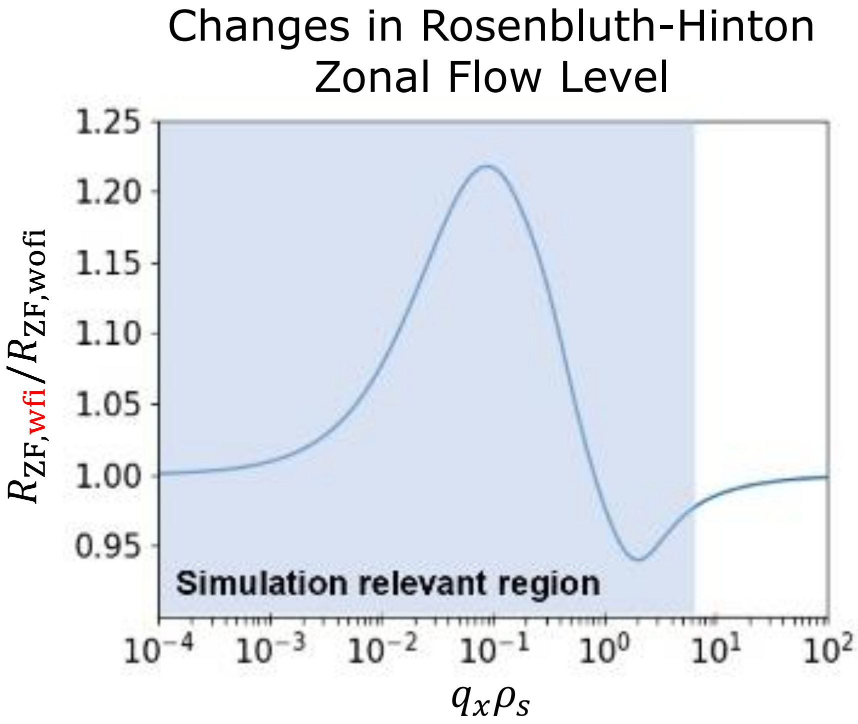
Profiles of Ion Heat Diffusivity (Interpretive)



<Ion Energy Flux Landscape>

Courtesy: Y.-S. Na

Residual ZF level moderately increase with fast ions

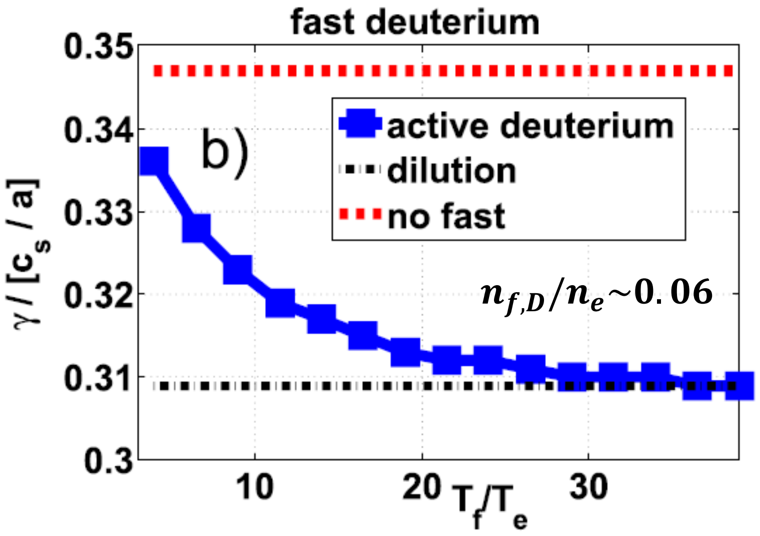


for KSTAR FIRE mode parameters
using an analytic formula from
[Y.W. Cho and T.S. Hahm, NF **59**, 066026 (2019)]

Fast ions' direct contribution to turbulence

- ✓ In finite T_f, fast ions can contribute to electrostatic turbulence near resonance condition. [Di. Siena, NF, '18]
  - Considering simple case : small electrostatic fluctuation neglecting parallel dynamics (or at low field side $\theta = 0$) and trapping terms with $s - \alpha$ geometry,

$$\delta f_f = \frac{1}{\omega_r + i\omega_i - \vec{v}_D \cdot \vec{k}} \left(\frac{k_y \delta\phi}{R_0 B_0}\right) \left[\left(-R_0 \frac{\partial F_f}{\partial r}\right) + v_\parallel \frac{\partial F_f}{\partial v_\parallel}\right]$$



gradient drive of background distribution

$$\widehat{\mathcal{D}}[F_f] \equiv \left(-R_0 \frac{\partial F_f}{\partial r}\right) + v_\parallel \frac{\partial F_f}{\partial v_\parallel}$$

$\widehat{\mathcal{D}}[F_f] > 0$: destabilization (low η_f , NBI-like)

$\widehat{\mathcal{D}}[F_f] < 0$: stabilization (high η_f , ICRH-like)

near a resonance condition $\omega_r \approx \vec{v}_D \cdot \vec{k}$

δf_f : perturbed distribution of fast ions

F_f : background distribution of fast ions

$\omega_r, \ \omega_i$: real and imaginary part of frequency

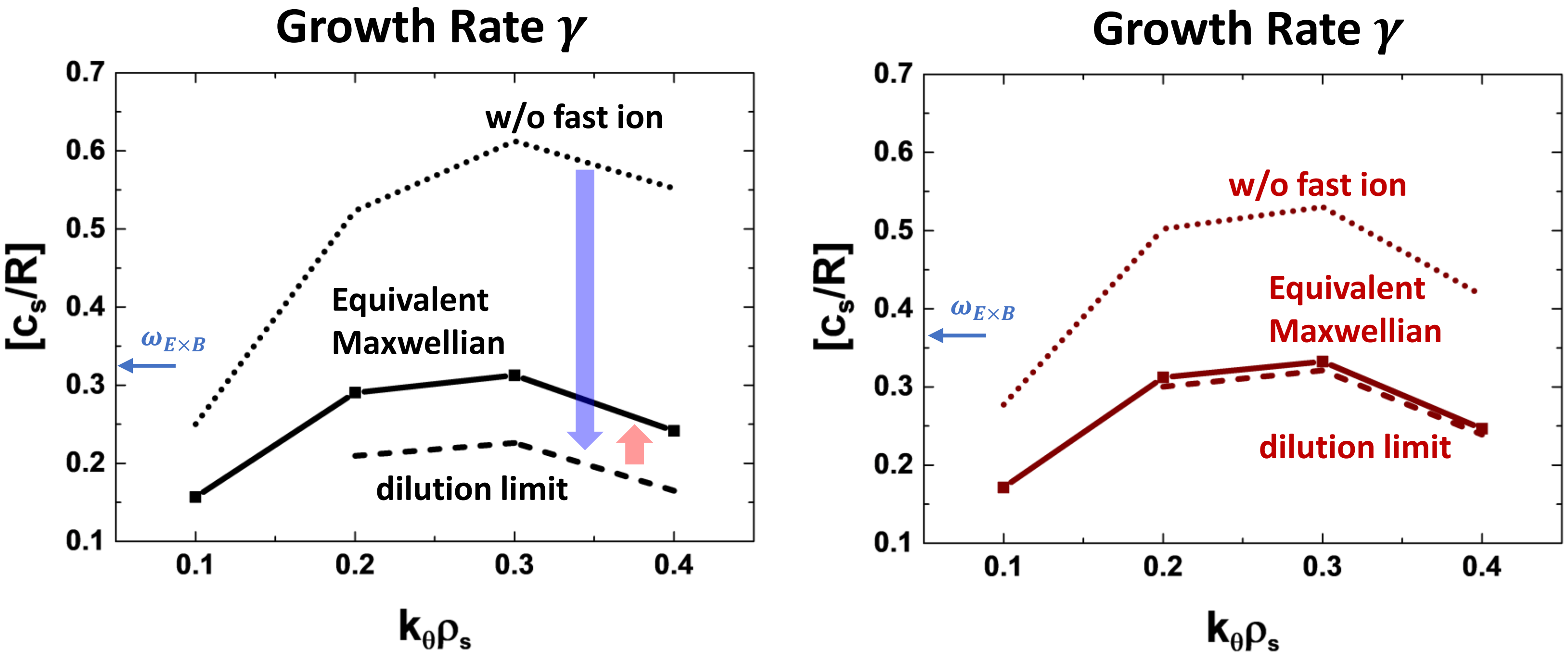
$\vec{v}_D = \frac{m v_\parallel^2 + \mu B}{ZeB^3} \vec{B} \times \nabla \vec{B}$: magnetic drift velocity

→ This is confirmed through GENE code in Ref. [Di. Siena, NF, '18]

Fast ions' direct contribution to turbulence – FIRE mode

- **Local gyrokinetic linear simulation with GKW for the KSTAR FIRE discharges**

#31921 (low performance) vs #30239 (high performance)



- gyrokinetic electrons, thermal ions, fast ions
- electromagnetic fluctuations considered $(\delta A_\parallel, \delta B_\parallel)$
- Collisionless
- Miller geometry
- target radial position : $\rho_{tor,N} = 0.4$

*dilution limit : $T_f/T_e = 100$

✓ Dilution effects are dominant in both discharges.

✓ Fast ions' contribution to ITG turbulence (destabilization, NBI-like) with ExB shearing effects may make a difference between two FIRE mode discharges.

Modulational zonal flow growth

$\tilde{\phi}_{\mathrm{DW}}(\mathbf{k},\omega)$

"Drift Wave"

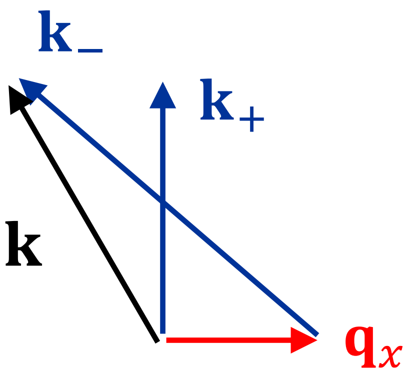
$$\tilde{\phi}_{+}(\mathbf{q}+\mathbf{k},\Omega+\omega_0) \propto \frac{\tilde{\phi}}{\omega_0-\omega_{+}+\Omega}\langle\phi\rangle_{\mathrm{ZF}}$$

from E×B nonlinearity

$$\tilde{\phi}_{-}(\mathbf{q}-\mathbf{k},\Omega-\omega_0) \propto \frac{\tilde{\phi}}{\omega_0+\omega_{-}-\Omega}\langle\phi\rangle_{\mathrm{ZF}}$$

Modulation
via $\bar{\phi}_{\mathrm{ZF}}\,(q_x)$

"Side-bands"
DW produced



$\tilde{\phi}_{+}$, $\tilde{\phi}_{-}$

$$\xrightarrow{\tilde{\phi}(\mathbf{k},\omega)} \frac{\left|\tilde{\phi}_{\mathrm{DW}}\right|^2}{(\omega_0-\omega_{+}+\Omega)(\omega_0+\omega_{-}-\Omega)}\langle\phi\rangle_{\mathrm{ZF}}$$

Source term from
Hasegawa-Mima
nonlinearity

↔ Reynolds stress by Taylor Identity

$$-\left\langle\tilde{\mathbf{v}}_E\cdot\boldsymbol{\nabla}\nabla_{\perp}^2\tilde{\phi}\right\rangle = \nabla_{\perp}^2\left\langle\tilde{v}_{Ey}\tilde{v}_{Ex}\right\rangle$$