A fluid model of non-zonal transitions in electromagnetic, ITG-driven turbulence

Yujia Zhang[1]
T. Adkins[2], M. R. Hardman[3], P. G. Ivanov[1], M. Barnes[1] and A. A. Schekochihin[1]

[1]Rudolf Peierls Centre for Theoretical Physics, U. Oxford
[2]Dept. of Physics, U. Otago
[3]Tokamak Energy Ltd

Fluid model for electromagnetic ITG

- Consider strongly-driven plasma in a z-pinch with cold ions and modest electron pressure:

$$\sqrt{\frac{m_e}{m_i}} \ll \frac{T_i}{T_e} \sim \frac{L_T}{L_B} \ll \beta_e \ll k_\perp \rho_s \sim 1$$

- Include ITG and Alfvénic (fluid) dynamics + critical balance + keep motion along perturbed field line

$$\omega \sim \omega_{*T} \sim k_\parallel v_A \sim k_\perp u_\perp \sim \nu_{ii}\tau$$

- Fields are then of size

$$\frac{\delta n}{n} \sim \frac{e\phi}{T_e} \sim \frac{\delta T_i}{T_e} \sim \frac{A_\parallel}{\rho_s B}\frac{1}{\sqrt{\beta_e}} \sim \frac{\delta B_\parallel}{B}\frac{1}{\beta_e} \sim \frac{\delta u_{\parallel,e}}{v_{\mathrm{th},i}}\sqrt{\frac{\beta_e}{\tau}}$$

Fluid model for electromagnetic ITG

- Consider strongly-driven plasma in a z-pinch with cold ions and modest electron pressure:

$$\sqrt{\frac{m_e}{m_i}} \ll \frac{T_i}{T_e} \sim \frac{L_T}{L_B} \ll \beta_e \ll k_\perp \rho_s \sim 1$$

- Include ITG and Alfvénic (fluid) dynamics + critical balance + keep motion along perturbed field line

$$\omega \sim \omega_{*T} \sim k_\parallel v_A \sim k_\perp u_\perp \sim \nu_{ii} \tau$$

- Fields are then of size

$$n \sim \varphi \sim T \sim A$$

Fluid model for electromagnetic ITG

$$\boxed{\nabla_{\|}(...) \equiv \partial_z(...) - \{A,(...)\}}$$

$$\boxed{\mathrm{d}_t(...) \equiv \partial_t(...) + \{\varphi,(...)\}}$$

$$\boxed{\omega_z \equiv \nabla_{\perp}^2 \varphi}$$

- Induction equation:

$$\partial_t A = \nabla_{\|}\left(n - \varphi\right)$$

- Ion energy equation:

$$\mathrm{d}_t T + \kappa_T \partial_y \varphi = \chi \nabla_{\perp}^2 T$$

- Vorticity equation:

$$\mathrm{d}_t \omega_z + 2\nabla_{\|}\nabla_{\perp}^2 A + \mathbf{I} : \{\nabla_{\perp} T, \nabla_{\perp}\varphi\} + \{T, \omega_z\} + \partial_y(n + T)$$
$$= \kappa_T \partial_y \omega_z + \chi \nabla_{\perp}^4 \left(a\varphi - bT\right)$$

- Continuity equation: $\mathrm{d}_t n + 2\nabla_{\|}\nabla_{\perp}^2 A + \partial_y(n - \varphi) = 0$

The electrostatic limit

- Corresponds to $k_\parallel \equiv \dfrac{k_\parallel^{\mathrm{phys}} L_B}{2\sqrt{\beta_e}} \to \infty$

$\partial_t A = \nabla_\parallel (n - \varphi) \Rightarrow n = \varphi$ (Boltzmann electrons)

- Recovers curvature-ITG and Alfvén waves in appropriate limits:

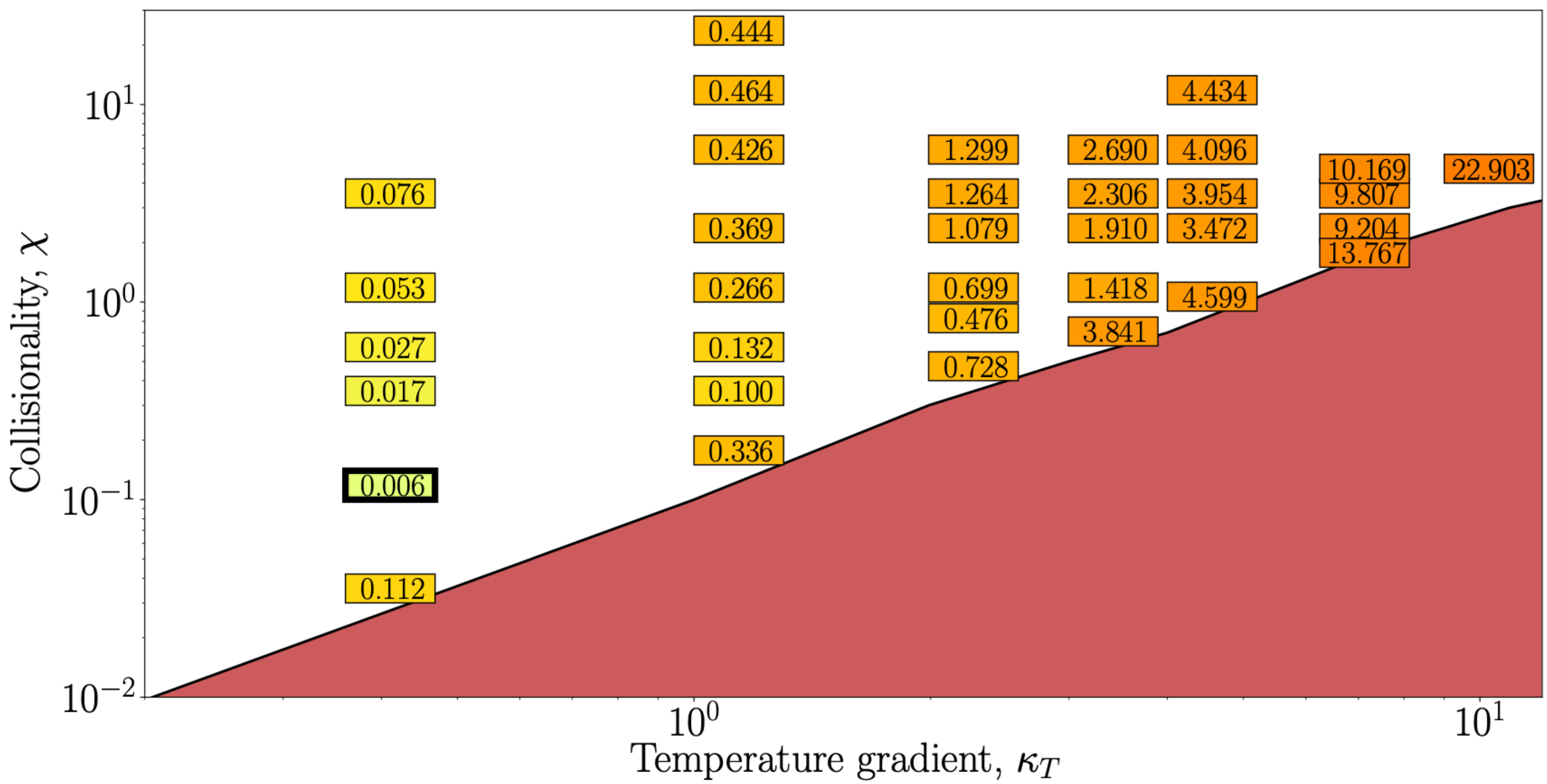
$$\gamma_{\mathrm{ITG}} = k_y \rho_s \frac{v_{\mathrm{th},i}}{\sqrt{L_B L_T}}$$

$$\omega_A = \pm k_\parallel^{\mathrm{phys}} v_A \sqrt{1 + k_\perp^2 \rho_s^2}$$

The electrostatic limit

P. Ivanov et al., JPP **86,** 855860502 (2022)

Heat flux

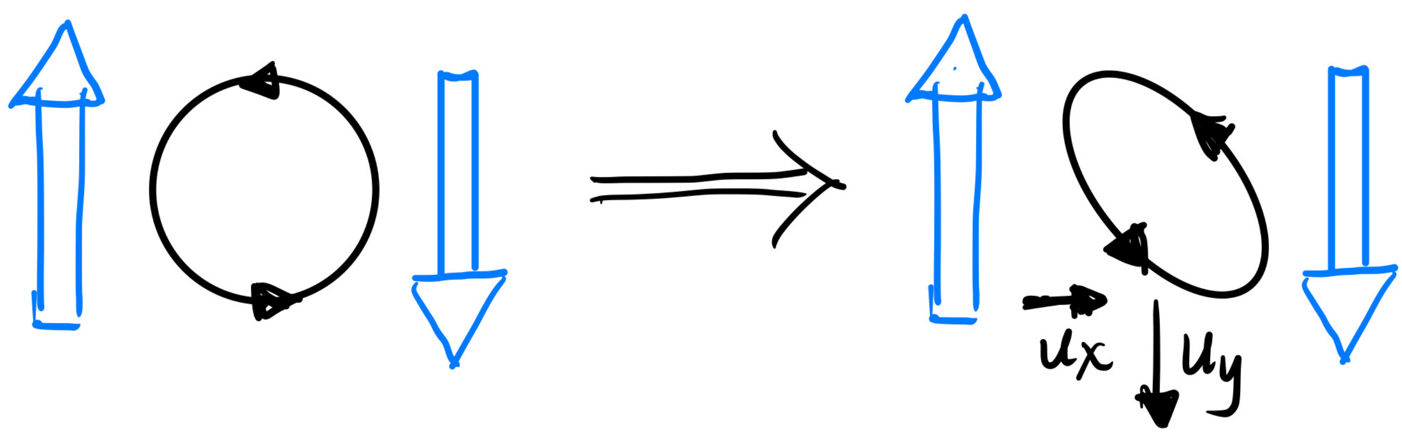


Transition to high transport state beyond critical temperature gradient

The electrostatic limit

P. Ivanov et al., JPP **86,** 855860502 (2022)

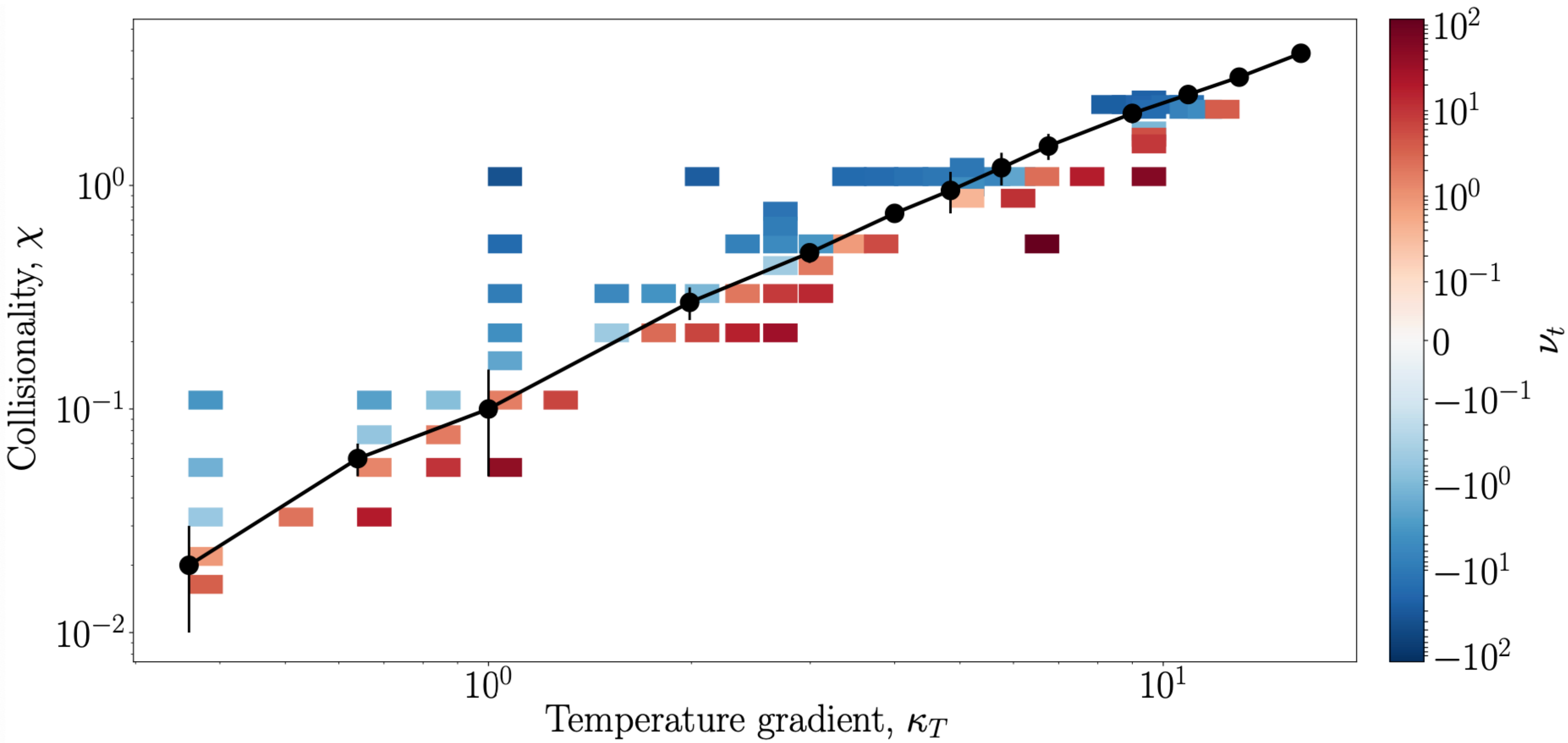
$$\partial_t \overline{\varphi} + \Pi_t + \Pi_d = 0 \qquad \Pi_t = \underbrace{-\overline{(\partial_x \varphi)(\partial_y \varphi)}}_{\Pi_\varphi} \underbrace{- \overline{(\partial_x \varphi)(\partial_y T)}}_{\Pi_T}$$



Reynolds stress reinforces zonal flow

The electrostatic limit

P. Ivanov et al., JPP **86,** 855860502 (2022)



$$\Pi_t = \Pi_\varphi + \Pi_T \qquad \nu_t \equiv -\frac{\left\langle \int \mathrm{d}x \; \Pi_t S \right\rangle_t}{\left\langle \int \mathrm{d}x \; S^2 \right\rangle_t} \qquad S = \partial_x^2 \varphi$$

Linear physics at finite plasma beta

- ‘High’ beta corresponds to $k_{\|} \propto \beta_e^{-1/2} \to 0$

$$\partial_t A = \nabla_{\|} (n - \varphi) \Rightarrow A \propto A_{\|} \beta_e^{-1/2} \to 0$$

- Recovers the interchange mode

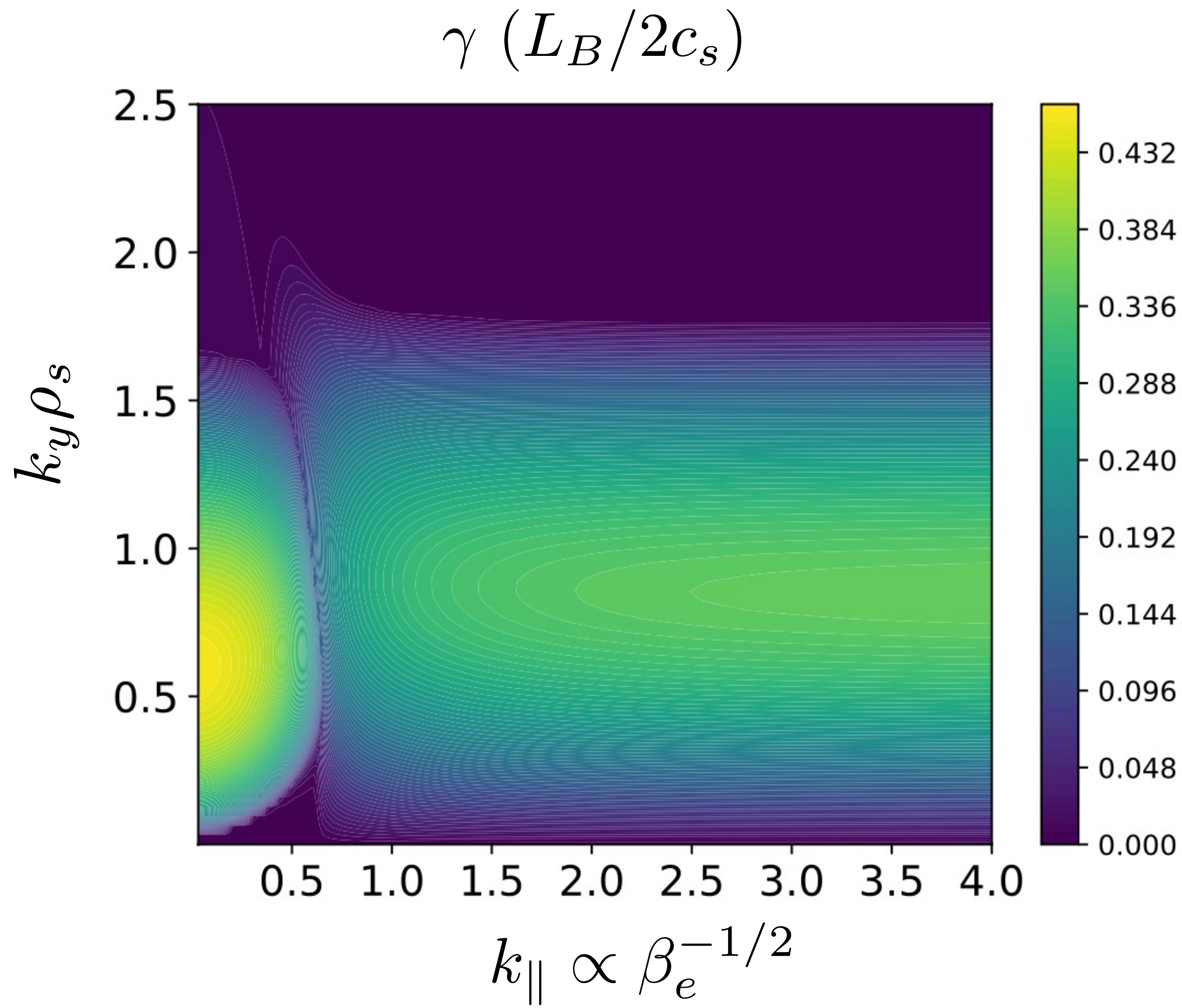
$$\omega_{\leftrightarrow}^2 = -\left(\frac{L_B}{L_T} - 2\frac{T_e}{T_i}\right)\left(\frac{v_{\mathrm{th},i}}{L_B}\right)^2$$

- Complete stabilization of ITG when

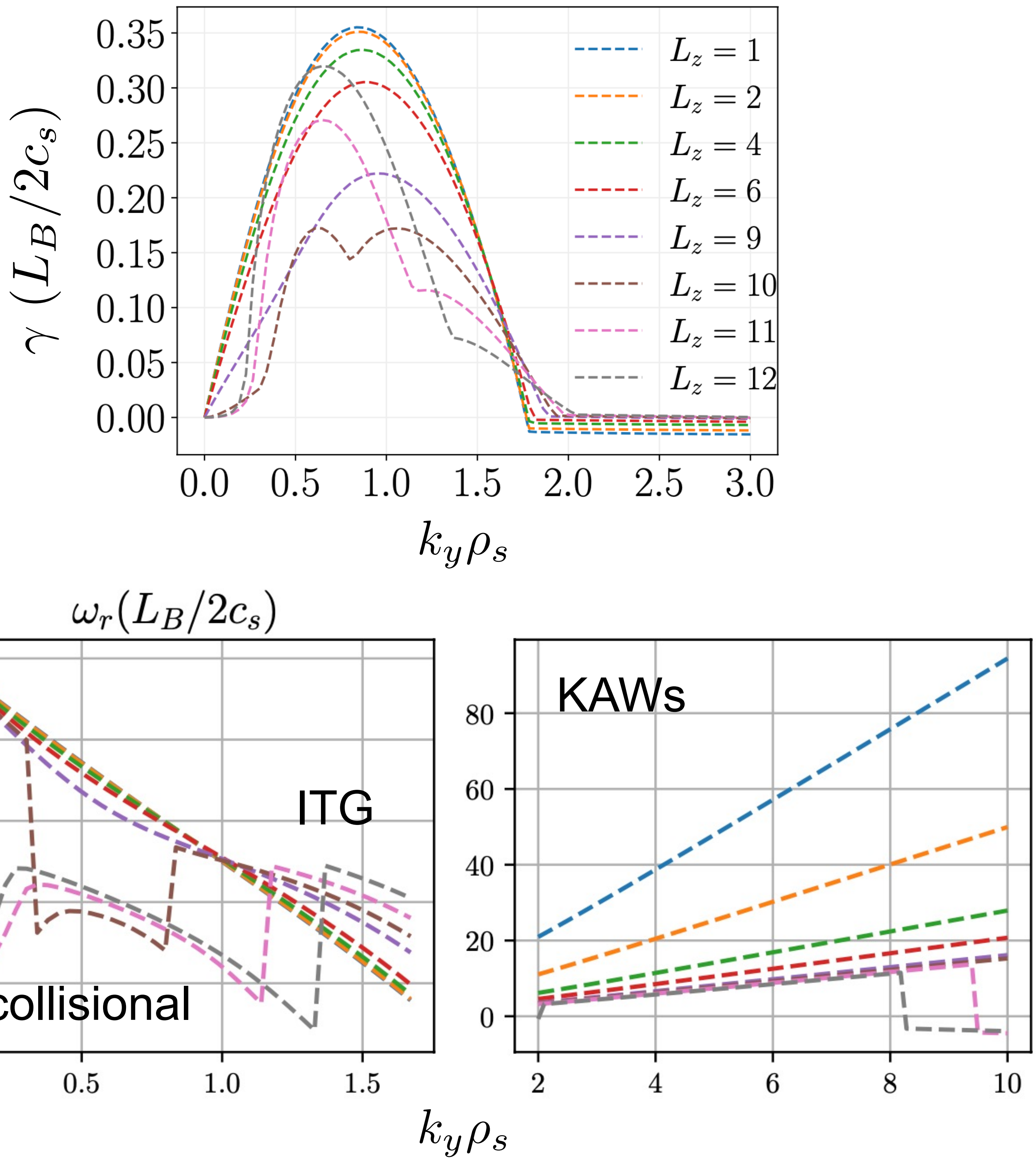
$$\beta_e > \left(k_{\|}^{\mathrm{phys}} L_B\right)^2 \frac{L_T}{L_B}\frac{T_e}{T_i}$$

- Collisional instability introduced at small $k_{\|}$

Linear physics at finite plasma beta



- Low collisionality + below interchange threshold



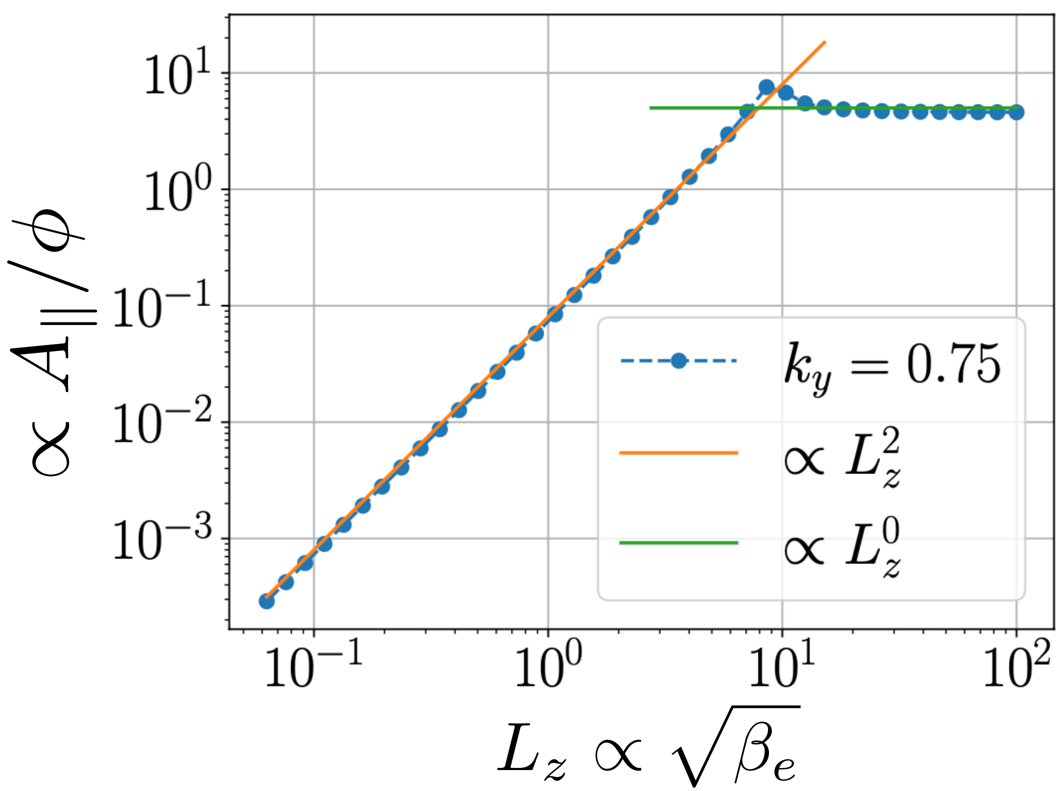
$\gamma\ (L_B/2c_s)$
$L_z = 1$
$L_z = 2$
$L_z = 4$
$L_z = 6$
$L_z = 9$
$L_z = 10$
$L_z = 11$
$L_z = 12$
$k_y \rho_s$
$\omega_r (L_B/2c_s)$
ITG
collisional
KAWs
$k_y \rho_s$

Linear physics at finite plasma beta

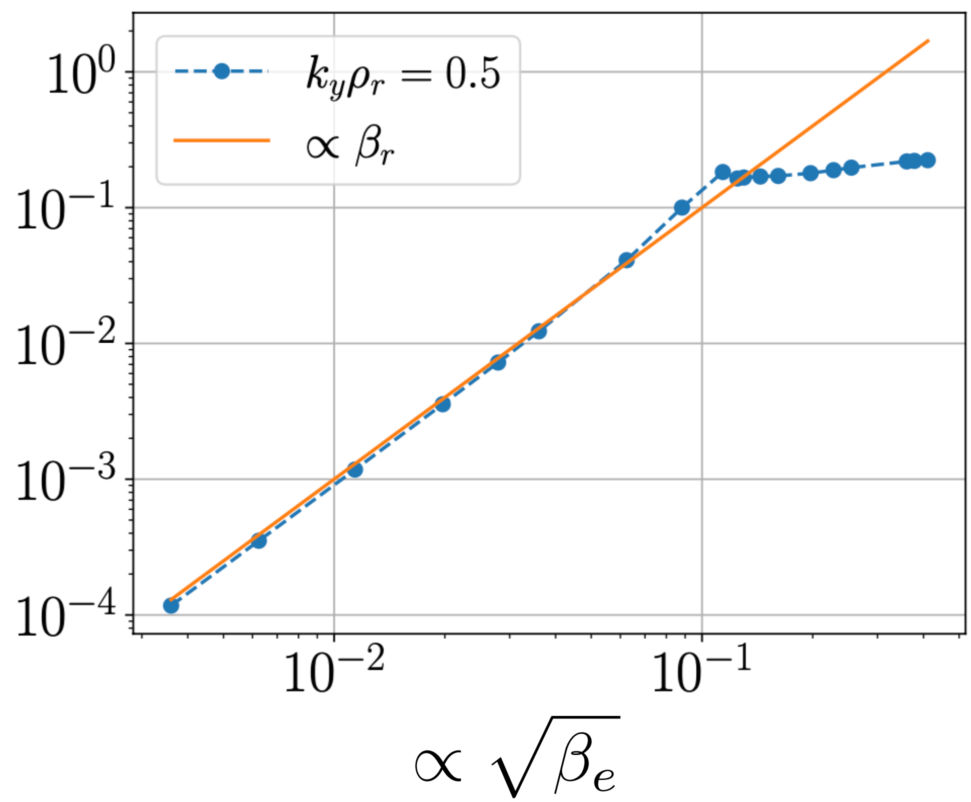
- Considering small beta corrections to electrostatic ITG:

$$A \sim \sqrt{\beta_e \kappa_T}\varphi \Rightarrow A_\parallel \sim \beta_e \sqrt{\kappa_T}\varphi$$

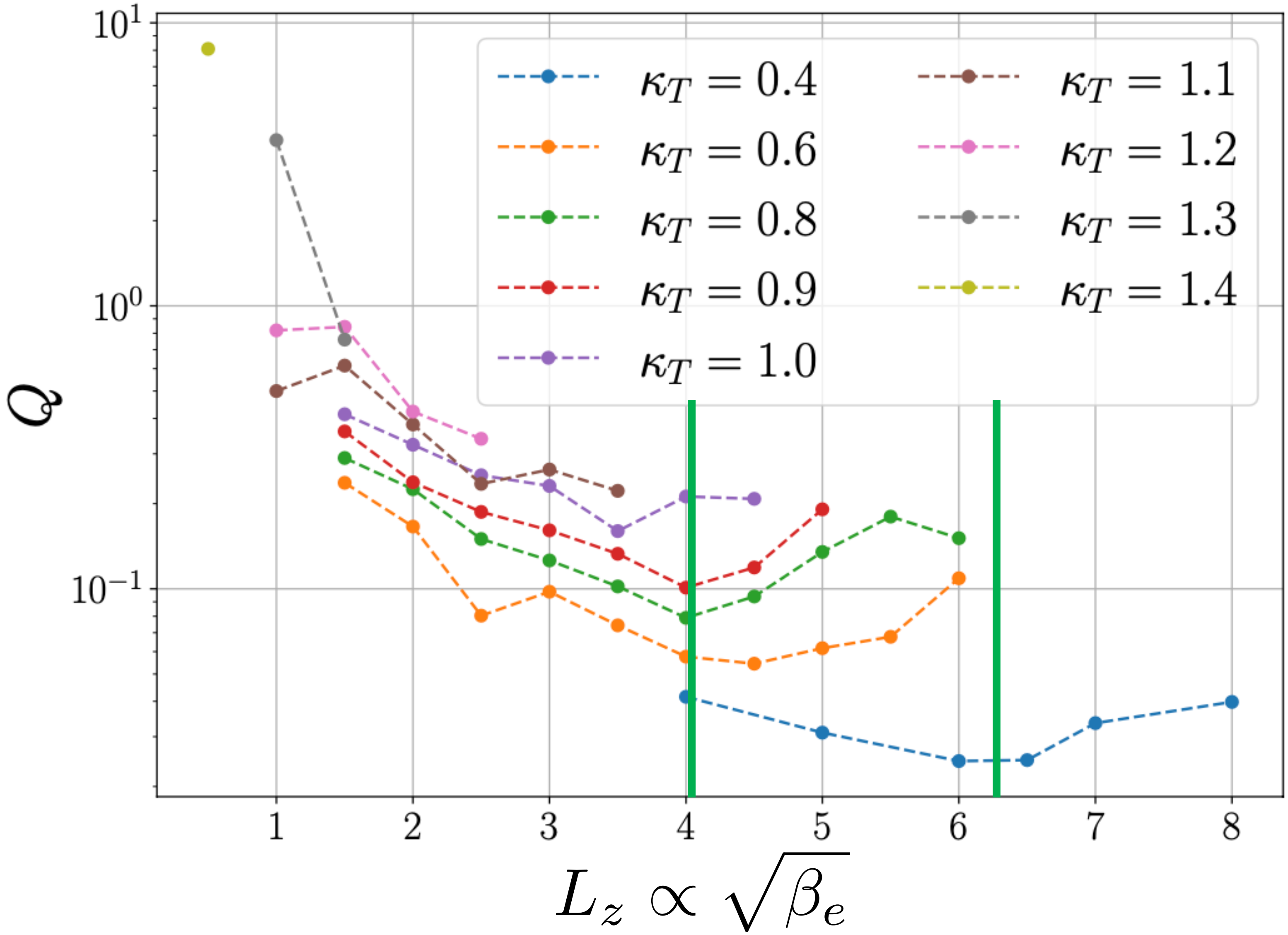
Fluid model

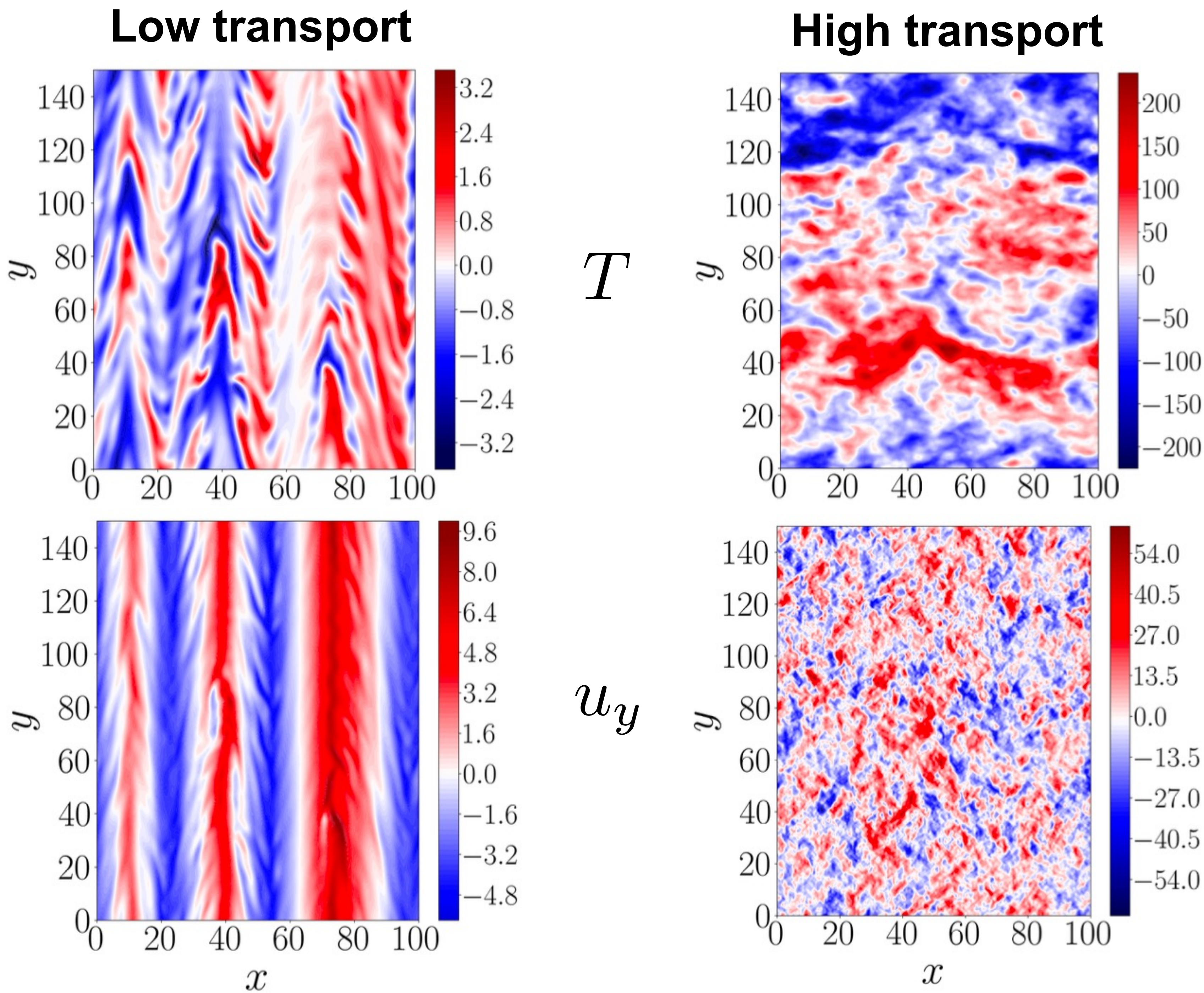


gyrokinetics



Non-zonal transition in fluid model





Low transport
High transport
T
u_y
y
x

Addition of Maxwell stress

$$\Pi_t = \Pi_\varphi + \Pi_T + \Pi_A \qquad \Pi_A \equiv 2\overline{(\partial_x A)(\partial_y A)}$$

Average over region with constant zonal shear:

$$\frac{1}{L_x}\int \mathrm{d}x\ \Pi_A = 2\sum_{\mathbf{k}} k_x k_y \left|A_{\mathbf{k}}\right|^2$$

Assume zonal shear sets x-wavelength:

$$k_x \sim -k_y S \tau_{\mathrm{nl}}$$

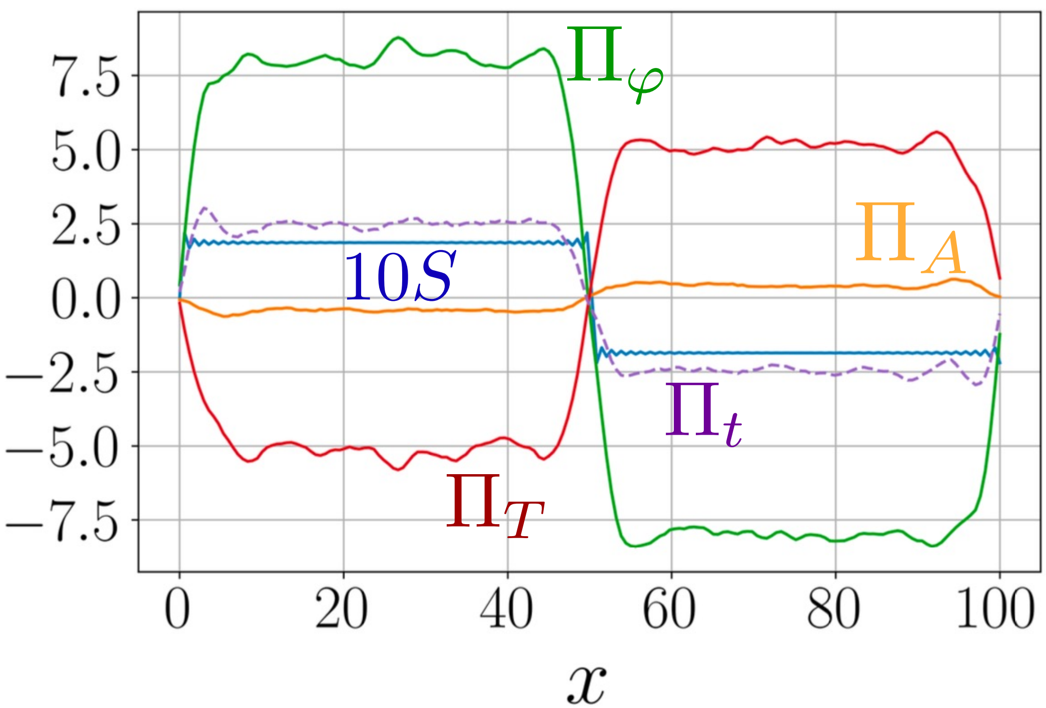
Result is sign-definite stress that opposes zonal shear:

$$\frac{1}{L_x}\int \mathrm{d}x\ \Pi_A \sim -S(k_y^o)^2\left|A^o\right|^2$$

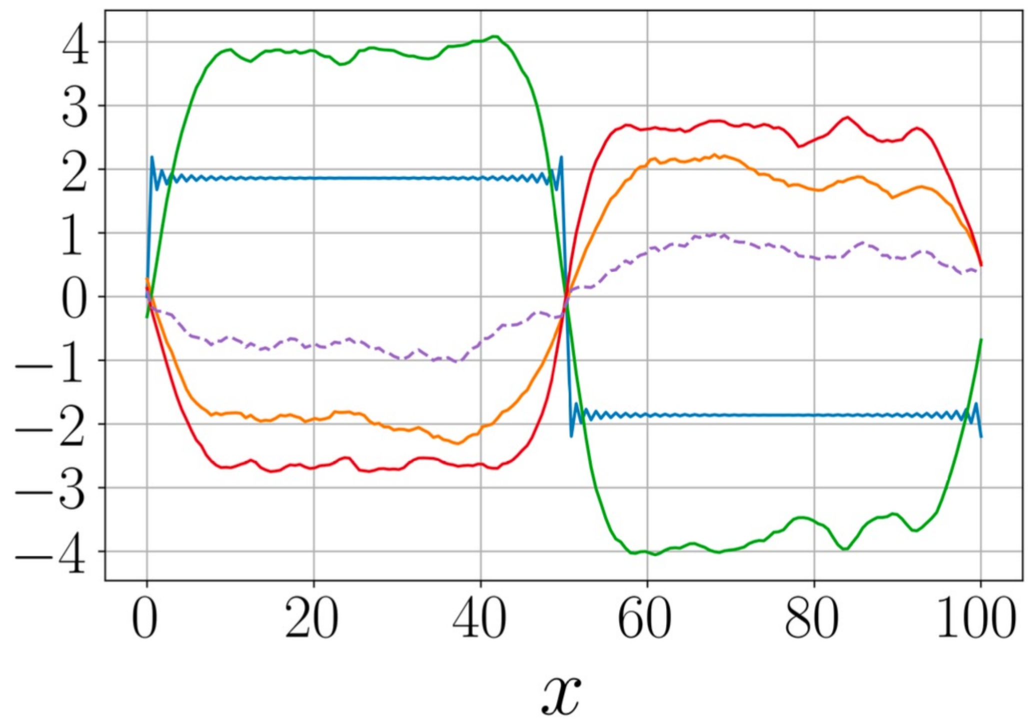
Correlation between turbulent stress and zonal shear

- Fix zonal flow and determine if turbulent stresses tend to support or oppose it:

Low flux state

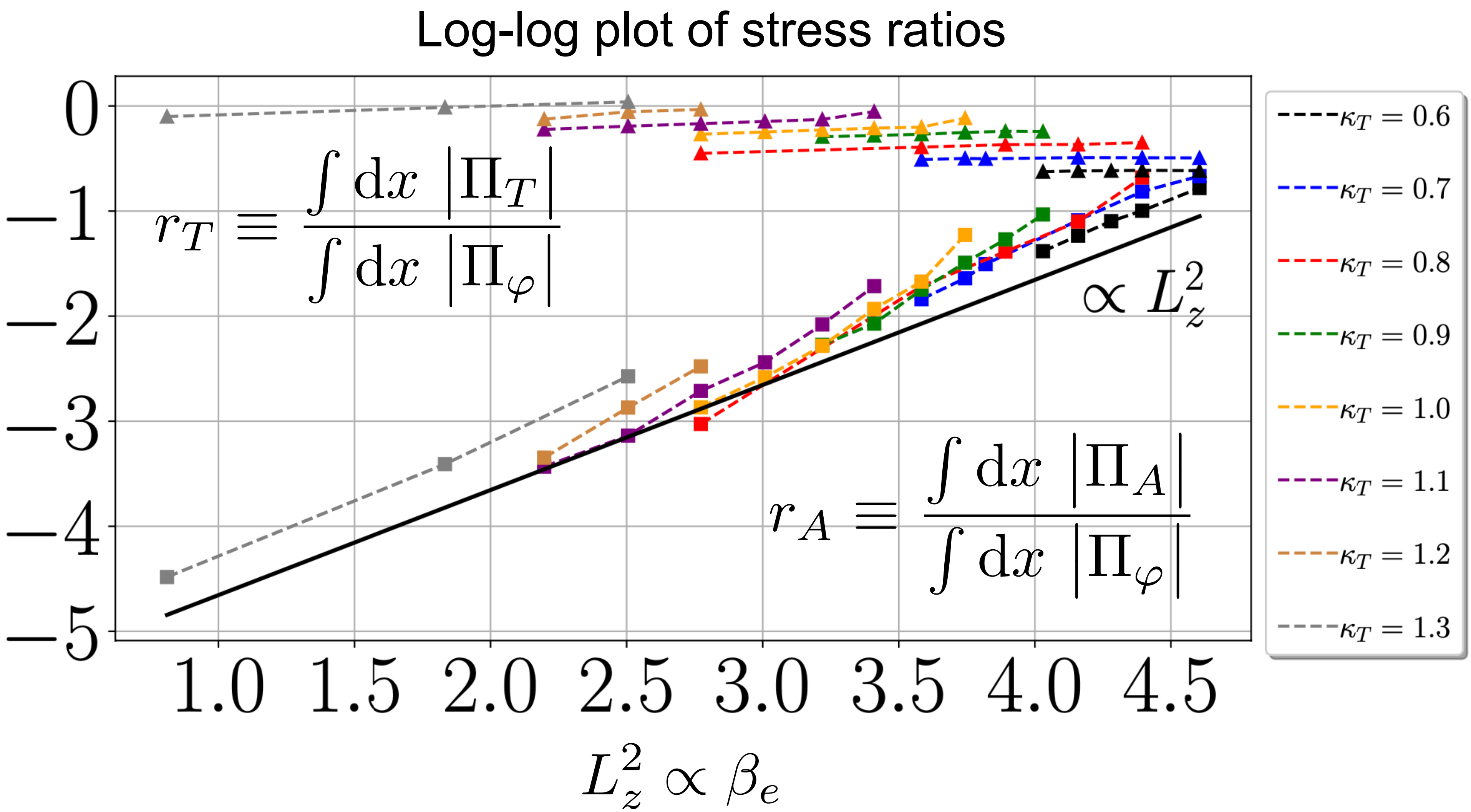


High flux state

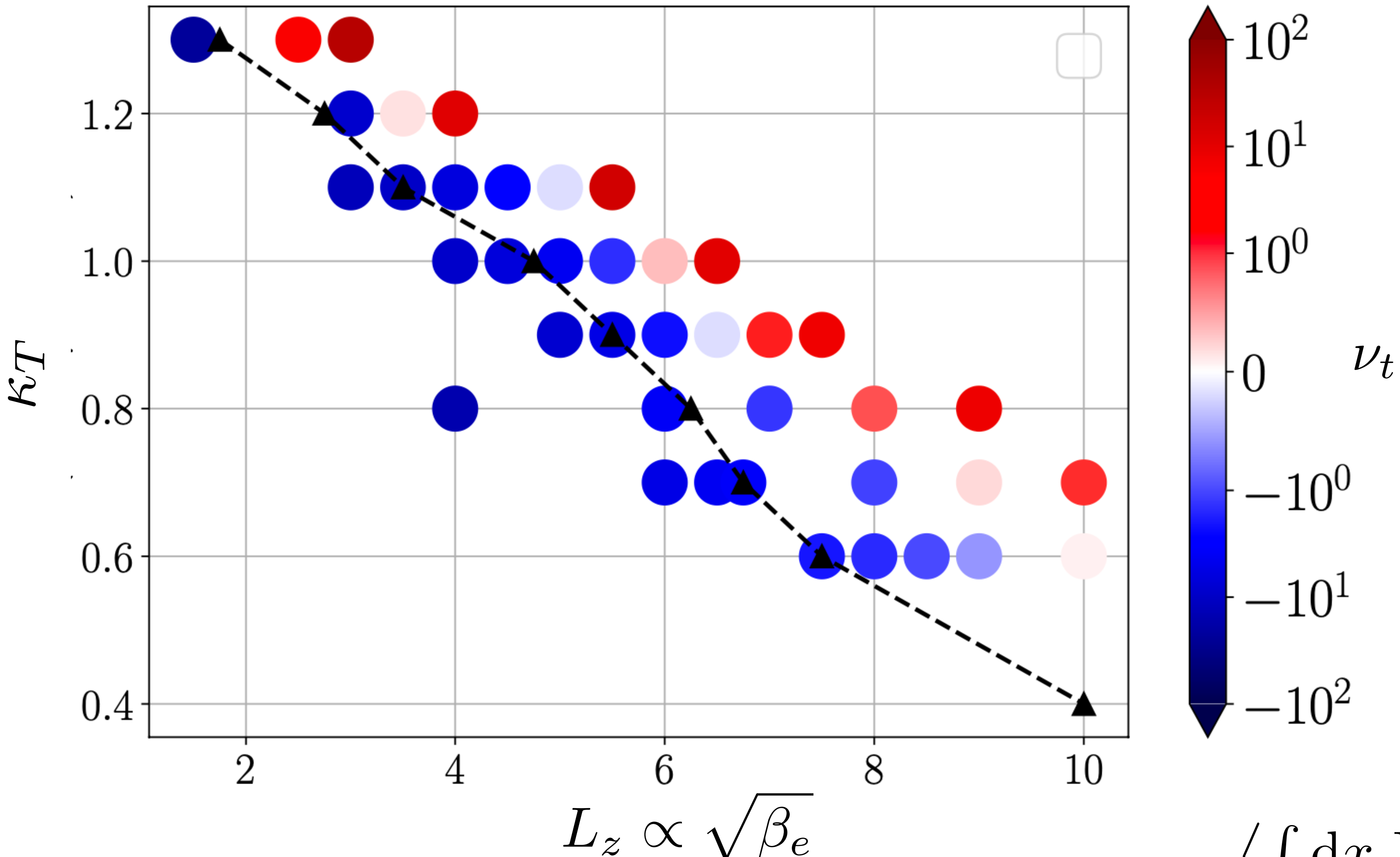


$$\Pi_t = \Pi_\varphi + \Pi_T + \Pi_A \qquad \Pi_A \equiv 2\overline{(\partial_x A)(\partial_y A)}$$

Maxwell-to-Reynolds stress ratio increases with plasma beta

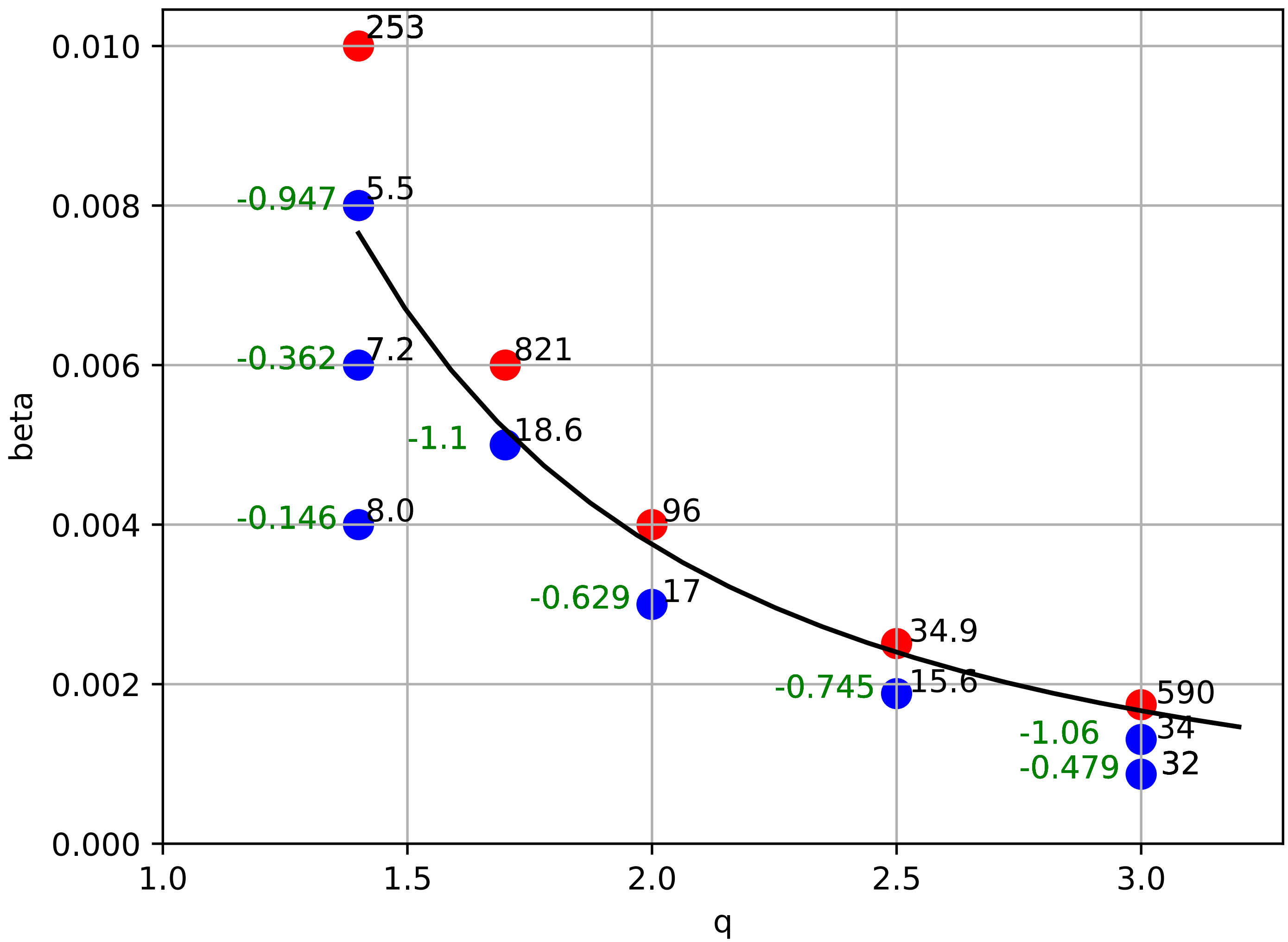


Turbulent viscosity and the non-zonal transition



$$\nu_t \equiv -\frac{\left\langle \int \mathrm{d}x \; \Pi_t S \right\rangle_t}{\left\langle \int \mathrm{d}x \; S^2 \right\rangle_t}$$

Comparison with gyrokinetic data



- Effective plasma beta from fluid model is

$$\beta_{\text{eff}} \equiv \beta_e \left(k_{\parallel}^{\text{phys}} L_B\right)^{-2} \propto q^2 \beta_e$$

Some open questions

- The electrostatic transition to the high-transport state is eliminated by including parallel dynamics – why doesn’t this help in the EM picture?
- Can we do anything useful with this information? E.g., can we predict the critical beta without running a lot of nonlinear gyrokinetic simulations?
- Should we worry about possible ‘gaps’ in the heat flux obtainable in local simulations?