Decaying RMHD turbulence in magnetic flux ropes

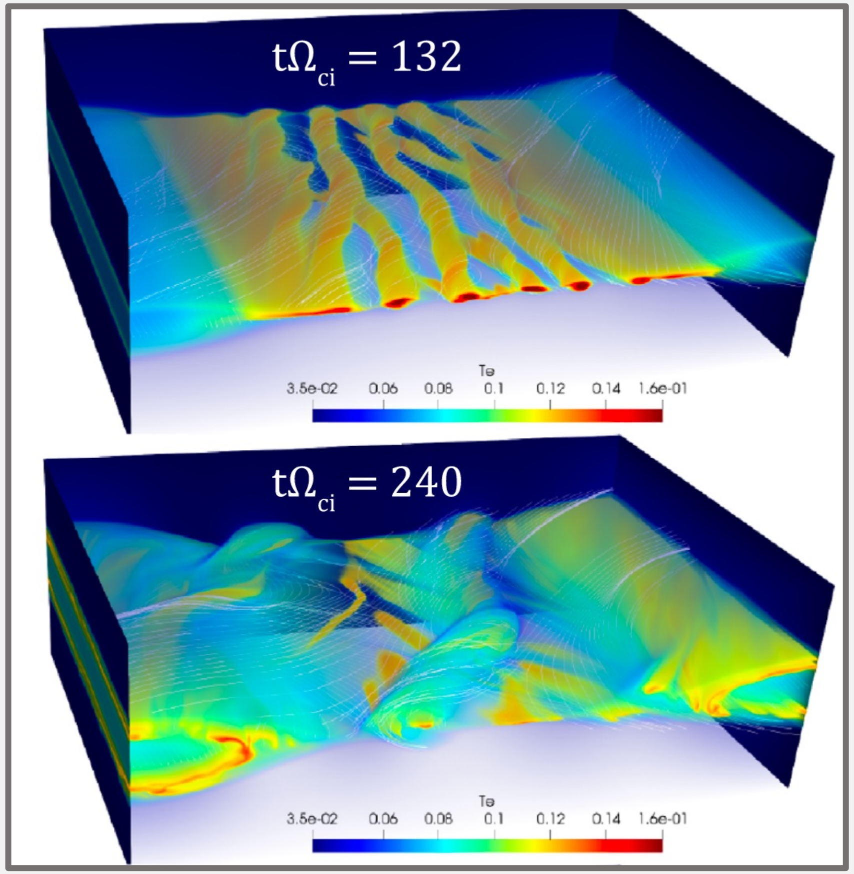
Alex Velberg, Lucas Shoji, Muni Zhou, Nuno F. Loureiro

WPI, Vienna, 7/31/23

PSFC Plasma Science and Fusion Center
Massachusetts Institute of Technology

Prevalence of magnetic flux ropes in plasma dynamics

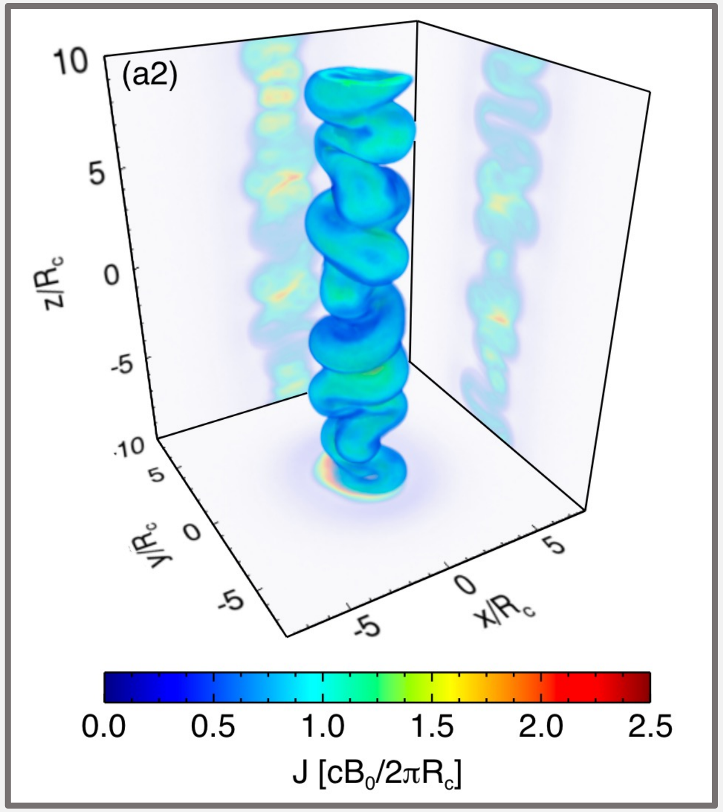
- Magnetic flux ropes (MFR) are an important component of many plasma phenomena, both on their own as an isolated structure and as a part of an interacting system of flux ropes
- Interactions between flux ropes in models of 3D reconnection



Stanier + 2019

Prevalence of magnetic flux ropes in plasma dynamics

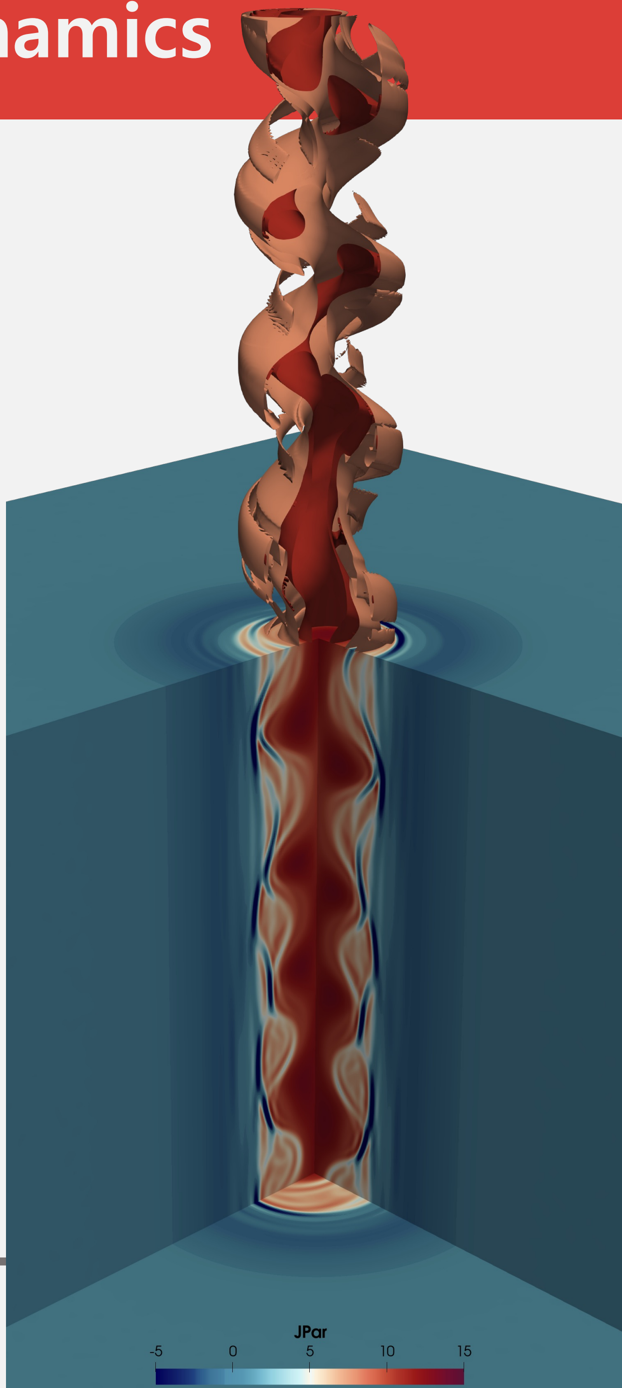
- Magnetic flux ropes (MFR) are an important component of many plasma phenomena, both on their own as an isolated structure and as a part of an interacting system of flux ropes
- Interactions between flux ropes in models of 3D reconnection
- Astrophysical jets modeled as flux ropes



Alves+ 2018

An isolated MFR is subject to its own interesting dynamics

- Magnetic flux ropes have their own rich, complex dynamics!
- Subject to kink-type instabilities
  - Multiple modes can interact, leading to turbulence
- We study this turbulence using reduced-MHD simulations of internal kink-tearing unstable flux ropes
  - Interesting example of decaying RMHD turbulence
  - What are the characteristics of this turbulence and how can we explain them?



(Very brief) review of kink instabilities

- Stability of the flux ropes is determined by the safety factor,

$$q = \frac{2\pi r}{L_z}\frac{\boldsymbol{B}_z}{\boldsymbol{B}_\theta}$$

- Rational surfaces -> $q = \frac{m}{n}$, integer poloidal and longitudinal mode numbers
- Correspond with locations which satisfy $\boldsymbol{k} \cdot \boldsymbol{B} = 0$, so stabilization by field line bending is minimized
- We observe internal kink-tearing modes
  - m=1 only, resistive kink
  - All m, tearing
  - (Ideal stabilized in RMHD)

Reduced MHD equations

- The reduced MHD equations describe the dynamics of Alfvénic MHD turbulence (a la GS95)
- Asymptotic reduction of the MHD equations in the large guide field limit, contains the dynamics of magnetic and velocity field perturbations in the perpendicular plane
- Nice to simulate—two coupled equations for *scalar* stream/flux functions

$$\frac{\partial \psi}{\partial t} + [\phi, \psi] = v_A \frac{\partial \phi}{\partial z} + \eta \nabla_\perp^2 \psi$$

$$\frac{\partial}{\partial t} \nabla_\perp^2 \phi + [\phi, \nabla_\perp^2 \phi] = v_A \frac{\partial}{\partial z} \nabla_\perp^2 \psi + [\psi, \nabla_\perp^2 \psi] + \nu \nabla_\perp^4 \phi$$

Strauss 1976
Kadomtsev + Pogutse 1974
Schekochihin 2009

Flux rope simulations using `Viriato`

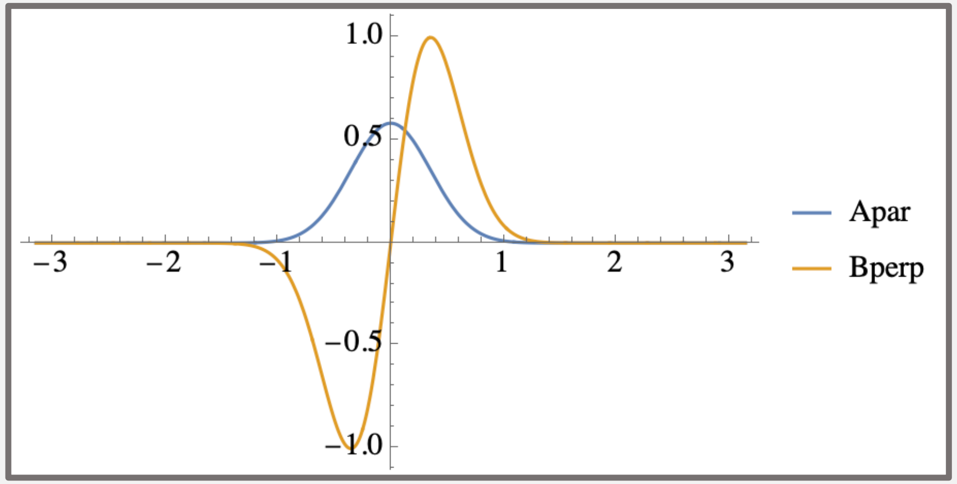
- Equilibrium:

$$A_{\parallel,eq} = A_0 exp\left[-4\left(\frac{2\pi}{L_x}x^2 + \frac{2\pi}{L_y}y^2\right)\right]$$

$$\phi_{eq} = 0$$

- Use triply periodic boundary conditions
- Simulations are resolved with 512^2 grid points in the perpendicular direction
- To get widest possible inertial range, use hyperdissipation:

$$\eta_H \nabla_\perp^6 \psi, \qquad \nu_H \nabla_\perp^6 \phi$$

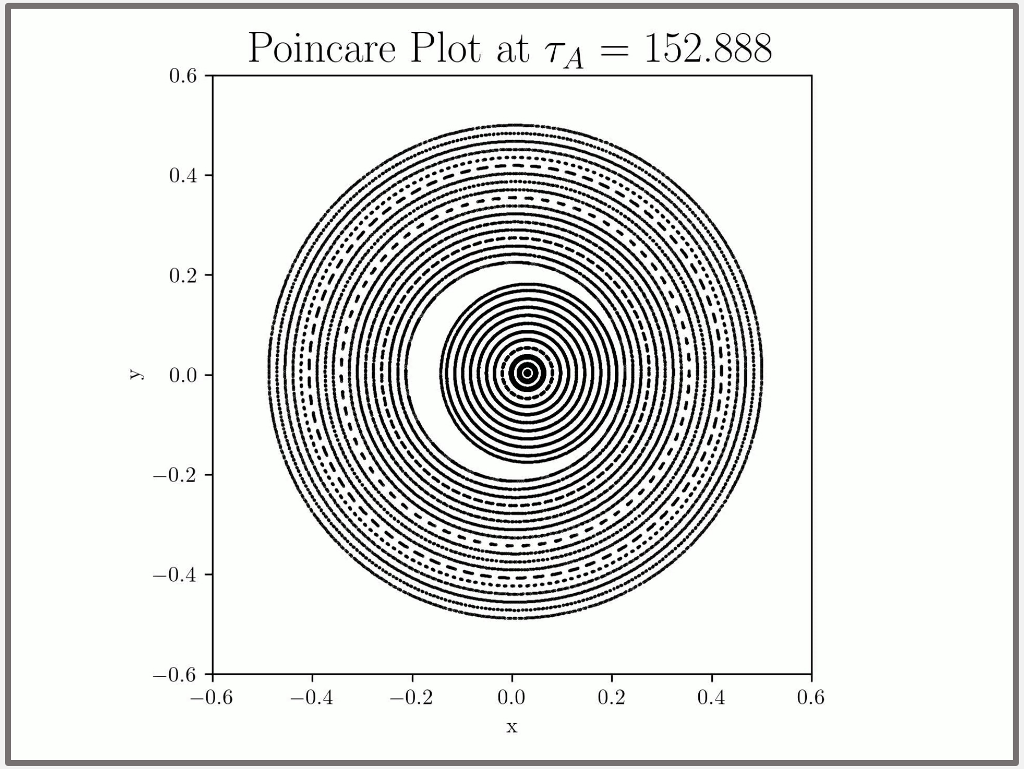


Label	L_z	Unstable parallel mode numbers	z Resolution
I	0.25	1	32
II	0.5	1,2	64
III	1	1-4	128
IV	2	1-9	256
V	4	1-18	512

The development of stochastic magnetic field lines

- There is a convenient equivalence between magnetic field lines and the trajectories of a Hamiltonian system (Morrison 2000, White 2014)
  - This allows an analytic description of how perturbations affect the magnetic field topology:
  - One perturbation -> chain of *m* magnetic islands, exact separatrix
  - Multiple pertubations -> chains of magnetic islands with narrow stochastic bands at separatrices
  - If islands “overlap”, stochasticity fills volume

One mode (simulation I):



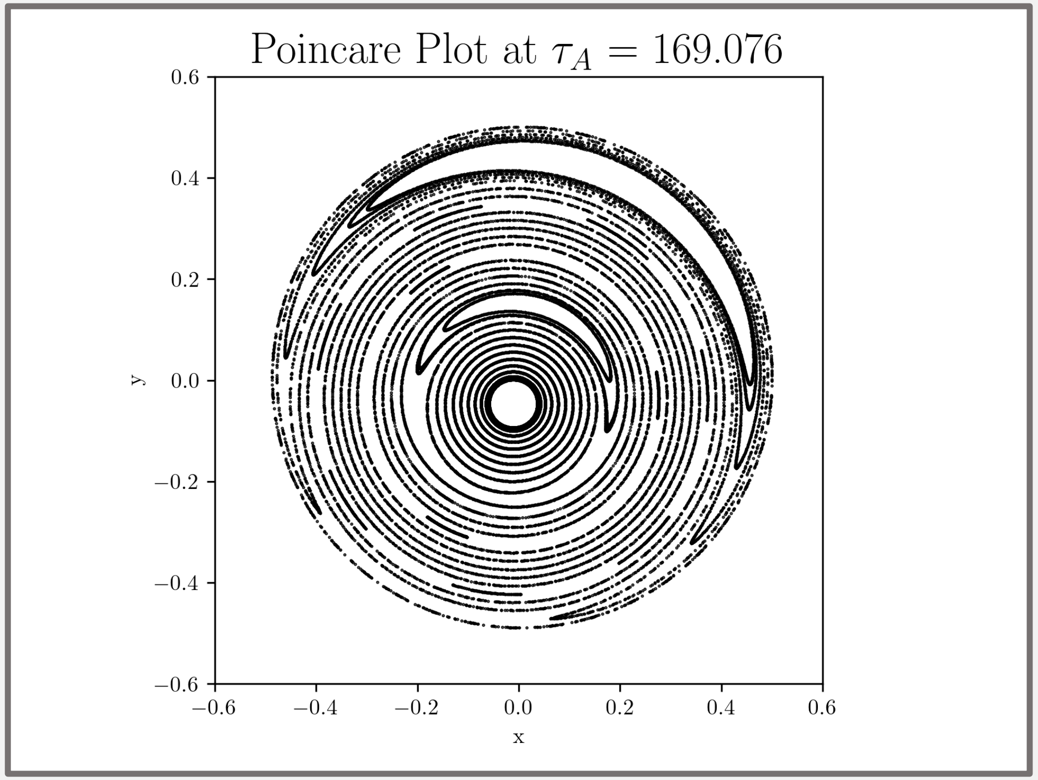
The development of stochastic magnetic field lines

- Create Poincaré maps to study the onset of stochasticity in our system
- Parameterize the magnetic field lines as follows, and integrate over the periodic z direction:

$$\frac{dx}{dz} = -\frac{\partial\psi}{\partial y} = B_x, \qquad \frac{dy}{dz} = \frac{\partial\psi}{\partial x} = B_y$$

Di Giannatale + 2018

Two modes (simulation 2):



*Note that this plot does not show the entire poloidal cross section

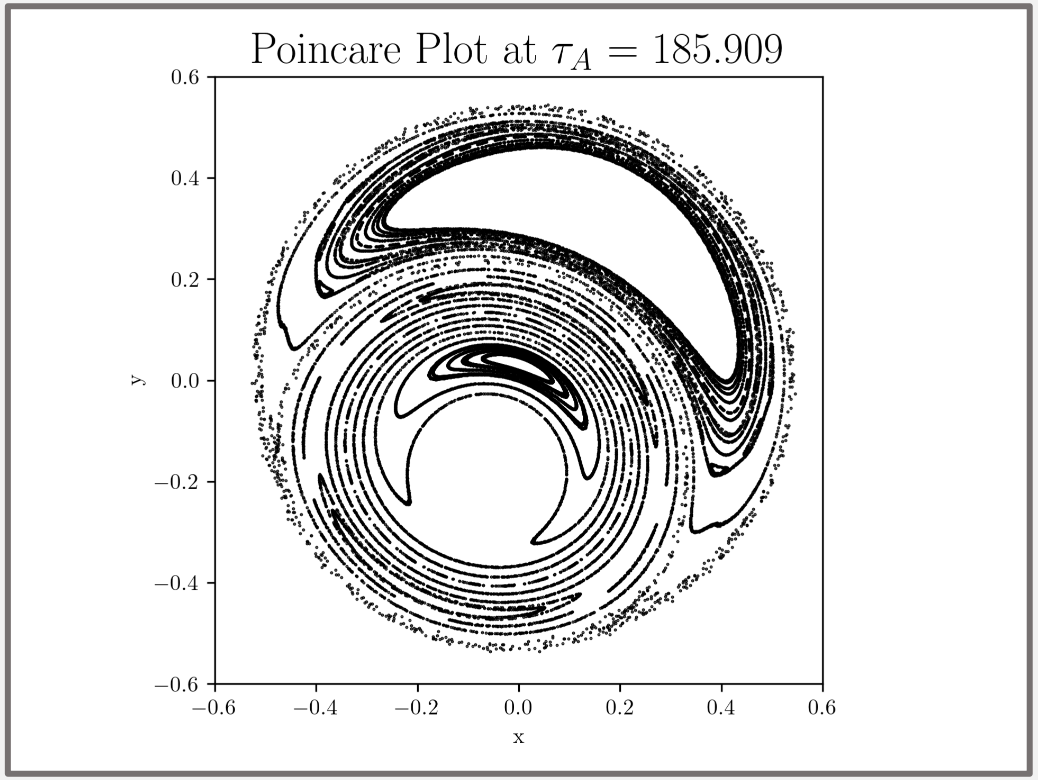
The development of stochastic magnetic field lines

- Create Poincaré maps to study the onset of stochasticity in our system
- Parameterize the magnetic field lines as follows, and integrate over the periodic z direction:

$$\frac{dx}{dz} = -\frac{\partial\psi}{\partial y} = B_x, \qquad \frac{dy}{dz} = \frac{\partial\psi}{\partial x} = B_y$$

Di Giannatale + 2018

Two modes:



*Note that this plot does not show the entire poloidal cross section

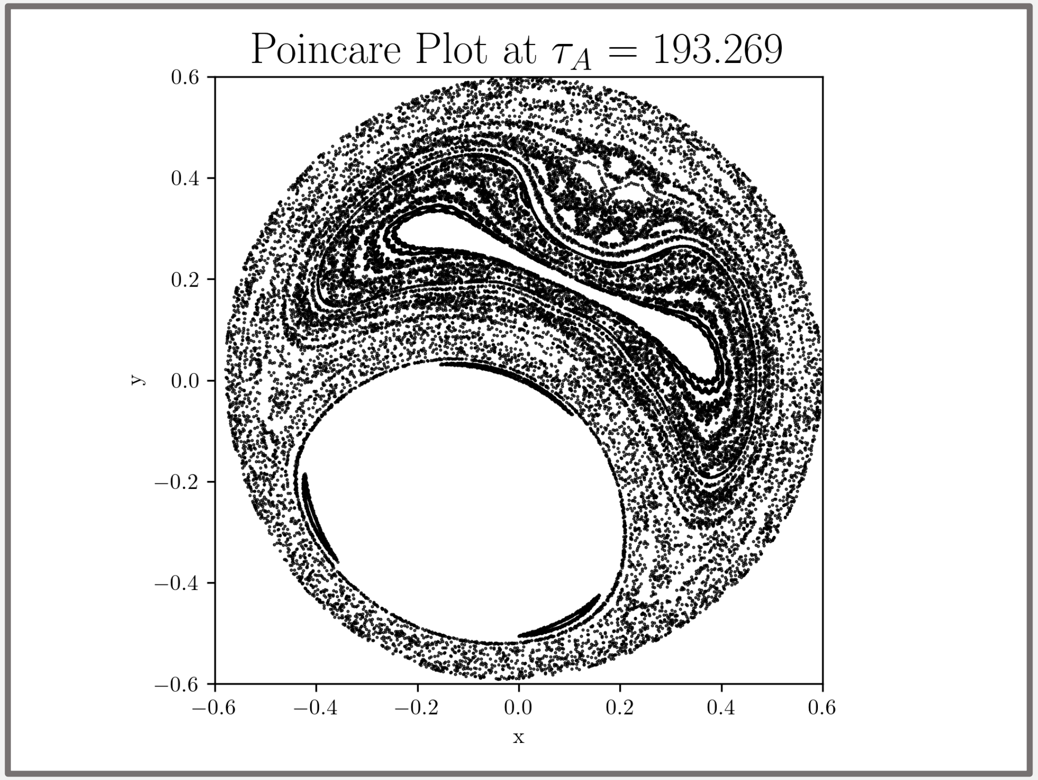
The development of stochastic magnetic field lines

- Create Poincaré maps to study the onset of stochasticity in our system
- Parameterize the magnetic field lines as follows, and integrate over the periodic z direction:

$$\frac{dx}{dz} = -\frac{\partial\psi}{\partial y} = B_x, \qquad \frac{dy}{dz} = \frac{\partial\psi}{\partial x} = B_y$$

Di Giannatale + 2018

Two modes:



*Note that this plot does not show the entire poloidal cross section

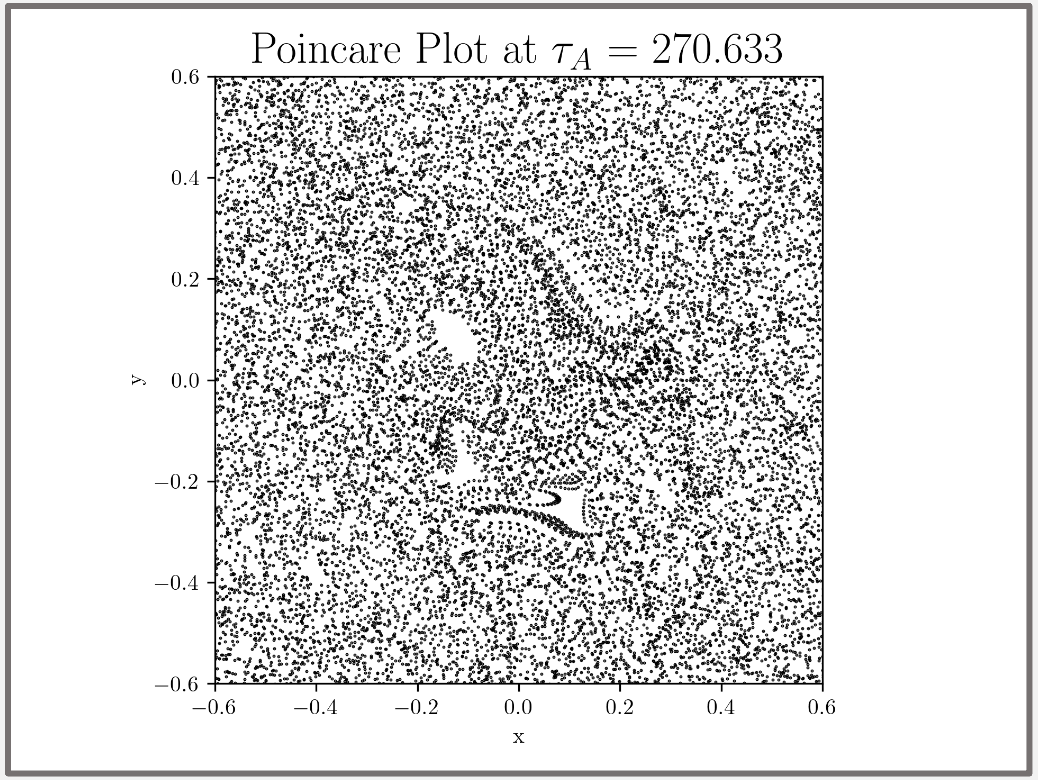
The development of stochastic magnetic field lines

- Create Poincaré maps to study the onset of stochasticity in our system
- Parameterize the magnetic field lines as follows, and integrate over the periodic z direction:

$$\frac{dx}{dz} = -\frac{\partial \psi}{\partial y} = B_x, \quad \frac{dy}{dz} = \frac{\partial \psi}{\partial x} = B_y$$

Di Giannatale + 2018

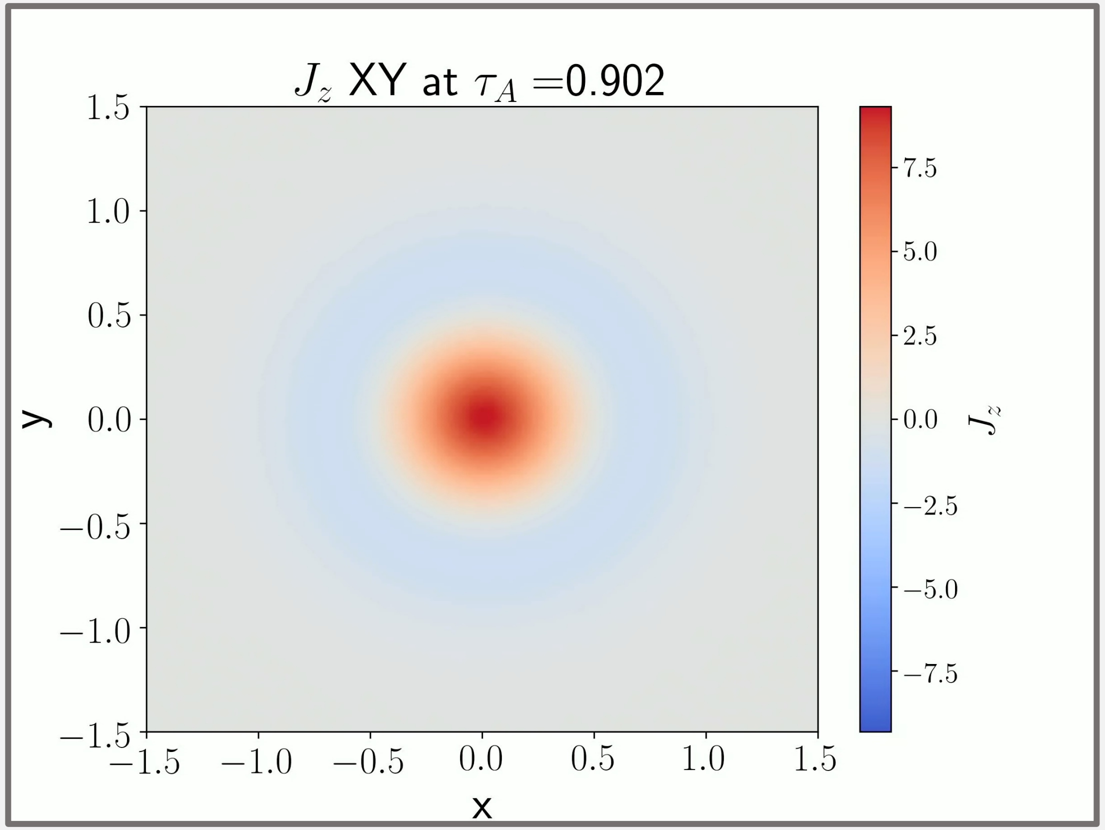
Two modes:



*Note that this plot does not show the entire poloidal cross section

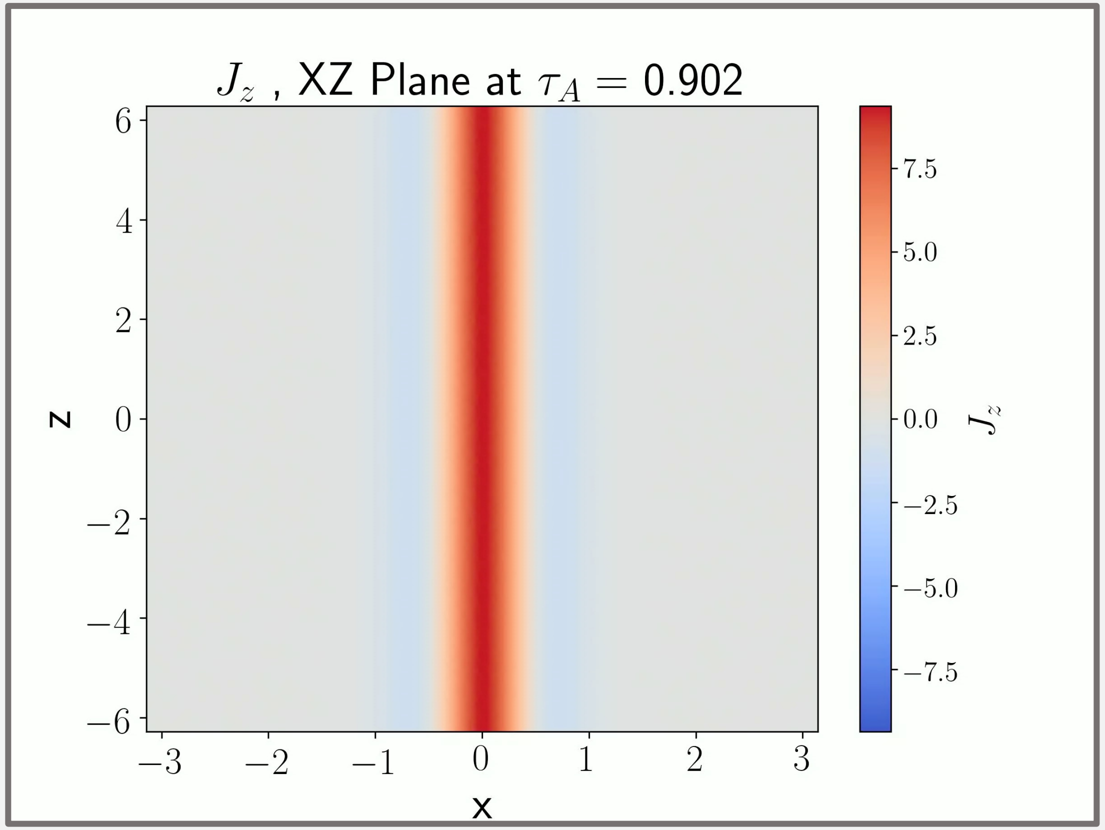
The interaction of fastest growing modes leads to turbulence

- Stochasticity of magnetic field lines leads to random motions of the plasma, generating fluctuations at all scales and leading to turbulence.
- A key feature is the appearance of an intense current sheet which forms at rational surfaces and wraps the modes
  - Envelope current sheet forms
  - Internally, see additional current sheets form
- Importantly, some rational surfaces are preserved in this initial interaction and the associate modes are excited at later times



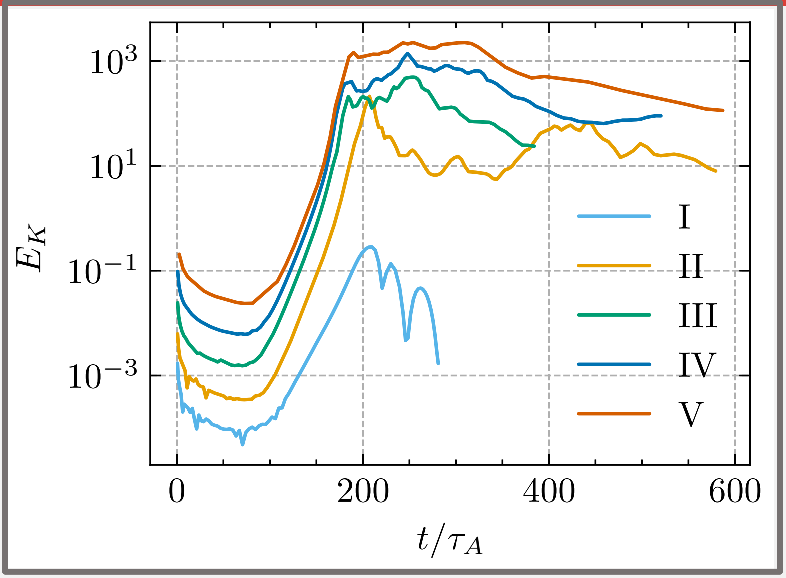
The interaction of fastest growing modes leads to turbulence

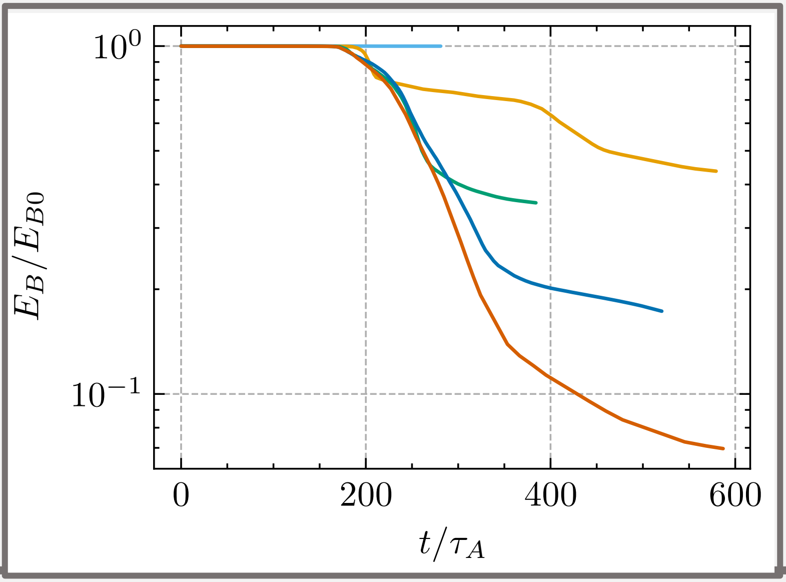
- Stochasticity of magnetic field lines leads to random motions of the plasma, generating fluctuations at all scales and leading to turbulence.
- A key feature is the appearance of an intense current sheet which forms at rational surfaces and wraps the modes
  - Envelope current sheet forms
  - Internally, see additional current sheets form
- Importantly, some rational surfaces are preserved in this initial interaction and the associate modes are excited at later times



Qualitative overview of dynamics

- Kinetic energy
  - Simulation I, oscillatory decay after mode saturates
  - Simulations II-V, nonlinear interaction between modes leads to complex nonlinear dynamics
  - Kinetic energy remains within order of magnitude of maximum value
  - Several bumps in the kinetic energy – correspond to late excitation of modes
  - Call period τ_A ~200-400 the "kinetic energy flattop"
    - Later arriving modes are associated with their own current sheets, which sustain the dynamics

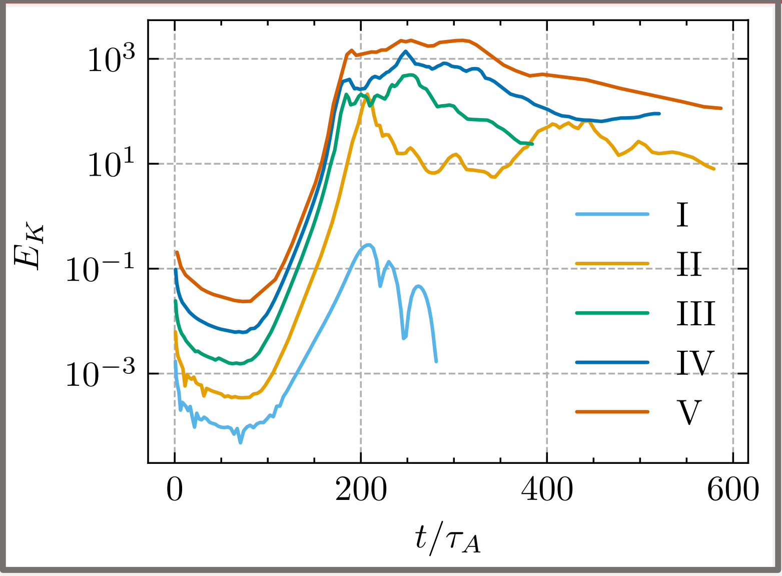


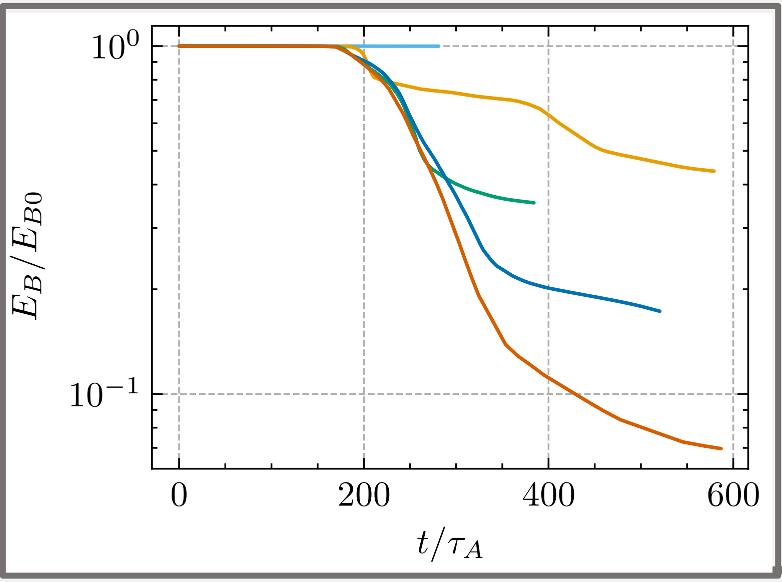


Qualitative overview of dynamics

- Magnetic energy
  - Exponential magnetic energy dissipation during kinetic flattop – resistive dissipation at intense current sheets
  - Longer flux ropes (increasing from I-V) dissipate larger fraction of initial energy
  - Always large compared to kinetic energy (**magnetically dominated** turbulent decay)

All results from simulation IV unless otherwise labeled

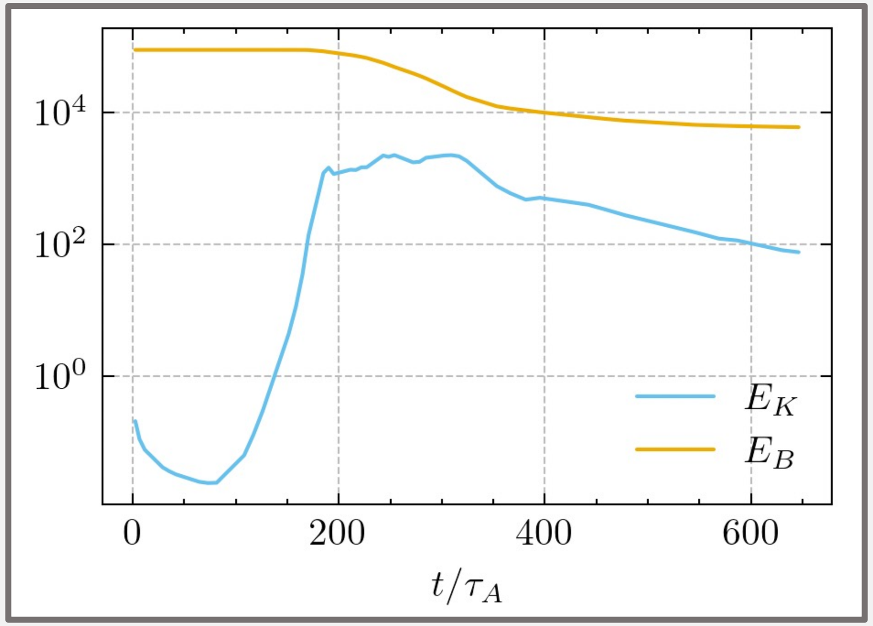




Qualitative overview of dynamics

- Magnetic energy
  - Exponential magnetic energy dissipation during kinetic flattop – resistive dissipation at intense current sheets
  - Longer flux ropes (increasing from I-V) dissipate larger fraction of initial energy
  - Always large compared to kinetic energy (**magnetically dominated** turbulent decay)

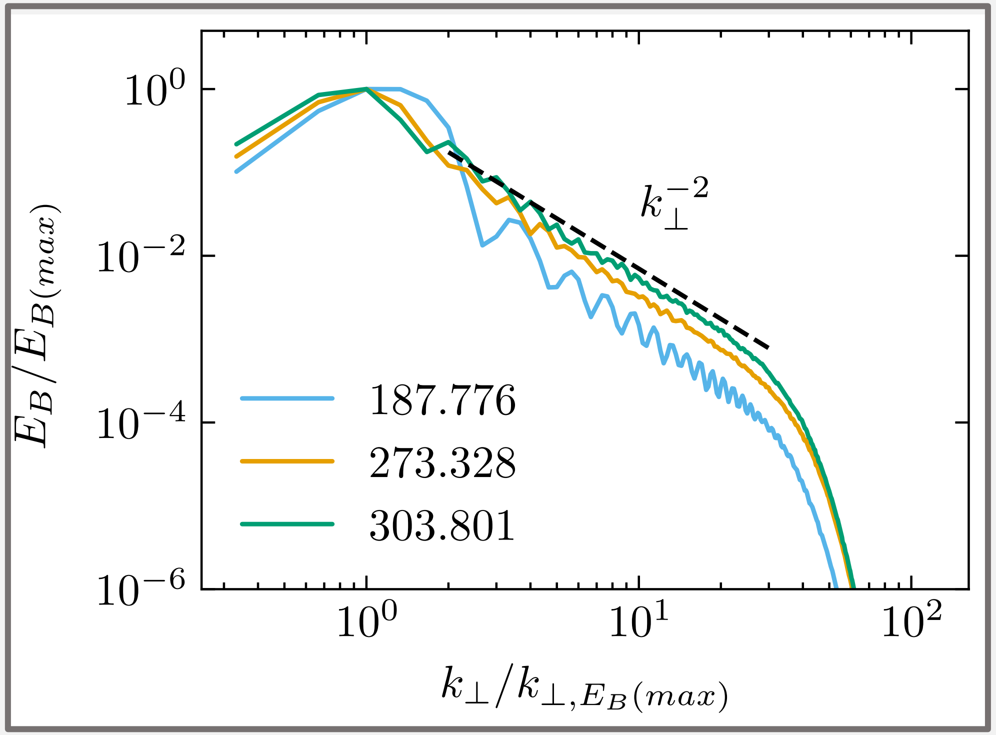
Simulation V



All results from simulation IV unless otherwise labeled

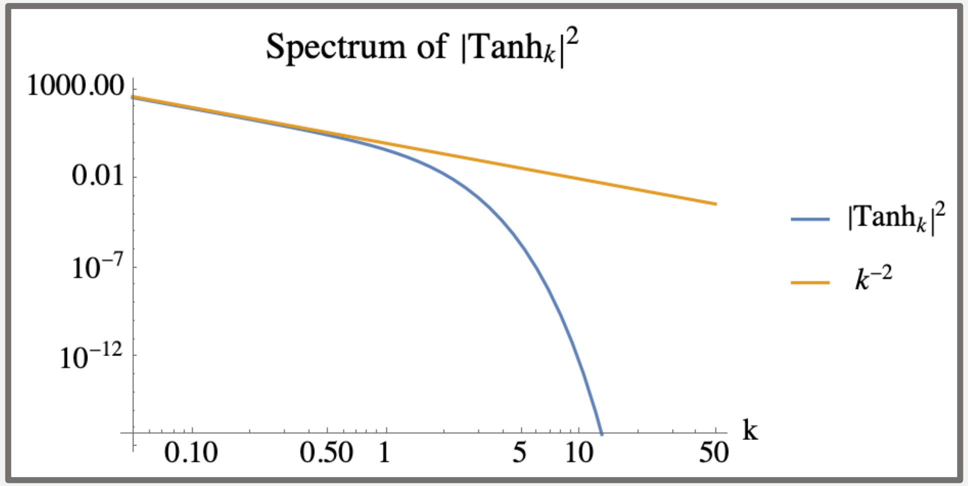
Energy spectra are mediated by strong current sheets

- The current sheets associated with the resistive-kink instabilities are sites of sharp magnetic field reversal
  - Results in a "Burgers" spectrum, $E_B \sim k_\perp^{-2}$
- At early times, only have current sheets, turbulence not developed, and have heavily modulated spectrum
- System is magnetically dominated—turbulent fluctuations aren't able to take over the spectrum



Energy spectra are mediated by strong current sheets

- The current sheets associated with the resistive-kink instabilities are sites of sharp magnetic field reversal
  - Results in a "Burgers" spectrum, $E_B \sim k_\perp^{-2}$
- At early times, only have current sheets, turbulence not developed, and have heavily modulated spectrum
- System is magnetically dominated System is magnetically dominated—turbulent fluctuations aren't able to take over the spectrum



Calculating the energy transfer

- Interested in how energy moves from scale to scale in this system
- Energy transfer equations are readily obtained from the MHD induction and momentum equations
- The "shell-filtered variables", $\boldsymbol{B}_\perp^K$ and $\boldsymbol{u}_\perp^K$ are quantities whose Fourier transform contains only the fields in a shell $k < K \leq k + 1$

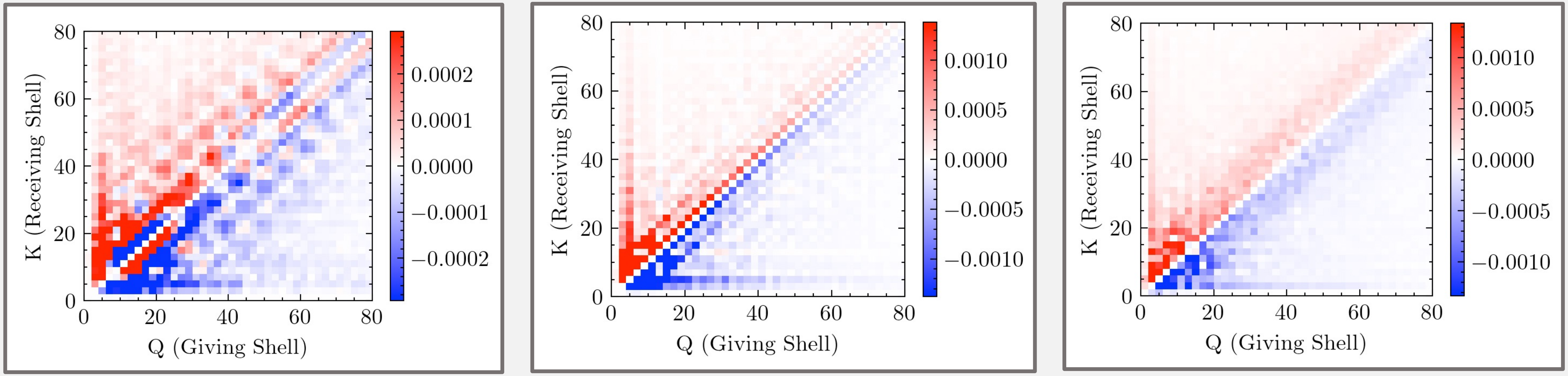
$$T_{bb}(Q, K) \equiv -\left\langle \int d^2\mathbf{x}_\perp^2 \left[\mathbf{B}_\perp^K \cdot (\mathbf{u}_\perp \cdot \nabla_\perp)\mathbf{B}_\perp^Q \right] \right\rangle_z$$

Zhou + 2020, Alexakis + 2005

Signatures of current sheets in the energy transfer

- Magnetic-to-magnetic energy transfer
  - Cascade forms along diagonal
  - Energy transfer from large to small scales evident in bars at low Q,K. This indicates the transfer due to current sheets

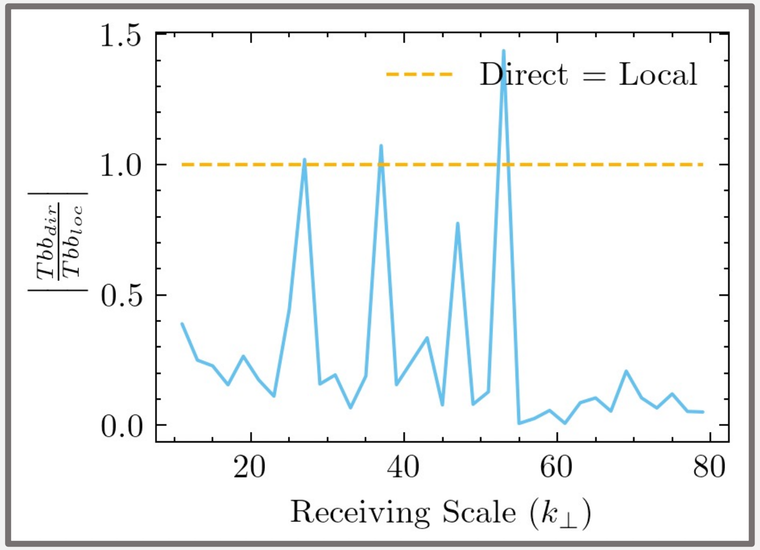
$\tau_A =$ 187 215 273



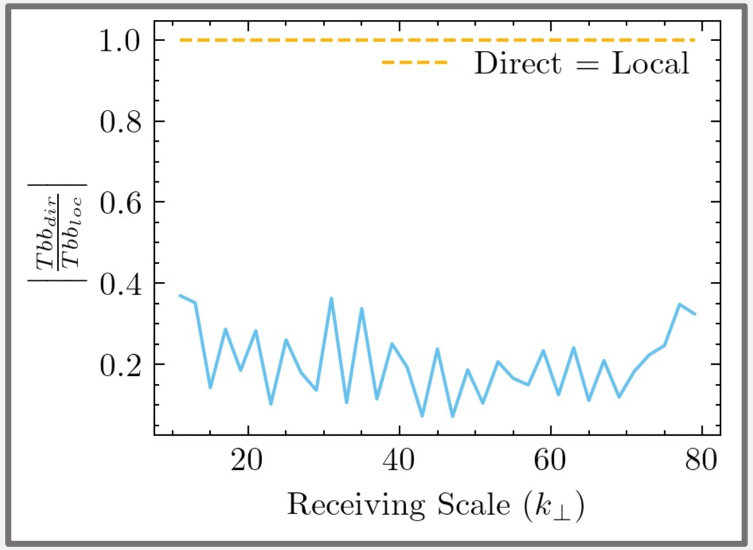
Energy cascade in Burgers turbulence

- Appearance of current sheets in energy transfer is a bit misleading…
- Current sheets may be responsible for the observed spectral indices, but the dynamics of energy transfer are similar to the typical inertial range picture
    - Similar to results for 1D burgers equation in fluids [Kraichnan, Gotoh and Kraichnan]

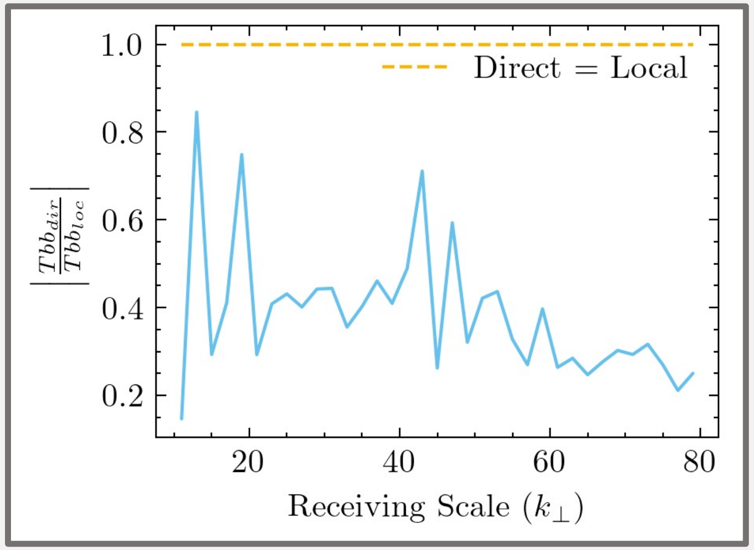
$\tau_A =$ 187



215



273



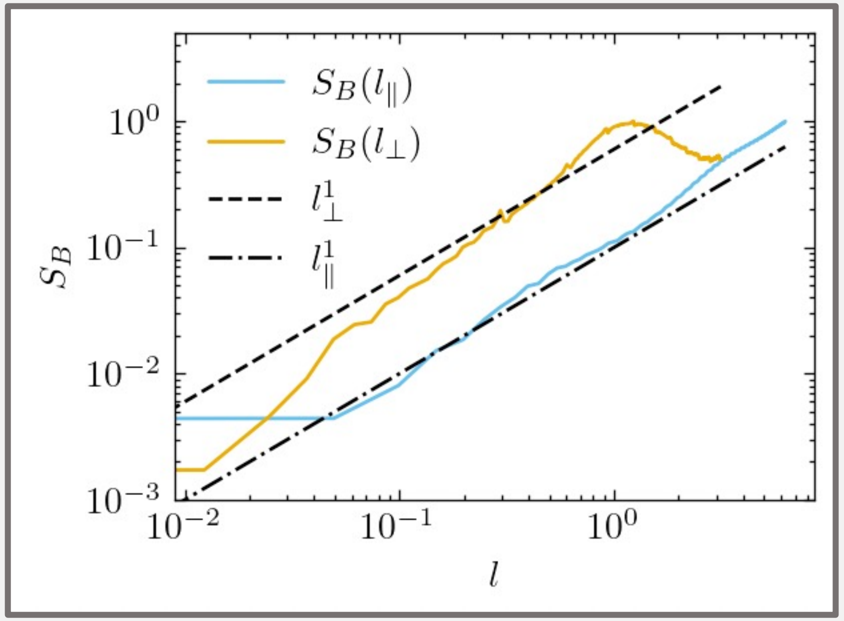
This is not weak turbulence

- $E_B \sim k_\perp^{-2}$ would also be the spectrum of weak turbulence! Let's show that this is definitely not what this is.
- We'd like to show:
  - Parallel cascade
  - Critical balance
- Difficult to obtain parallel spectrum directly from data → use structure function diagnostics
  - $SF \propto l^\alpha \rightarrow E(k) \propto k^{-(\alpha+1)}, l = k^{-1}$
- 5pt, 2nd order structure function
  - Compared to the typical two-point SF, will remove large scale variations, for example those imposed by the remnants of the equilibrium. [Cho+ 2019]

This is not weak turbulence

- $E_B \sim k_\perp^{-2}$ would also be the spectrum of weak turbulence! Let's show that this is definitely not what this is.
- We'd like to show:
  - Parallel cascade
  - Critical balance
- Difficult to obtain parallel spectrum directly from data → use structure function diagnostics
  - $SF \propto l^\alpha \rightarrow E(k) \propto k^{-(\alpha+1)}, l = k^{-1}$
- 5pt, 2nd order structure function
  - Compared to the typical two-point SF, will remove large scale variations, for example those imposed by the remnants of the equilibrium. [Cho+ 2019]

$\tau_A = 273$

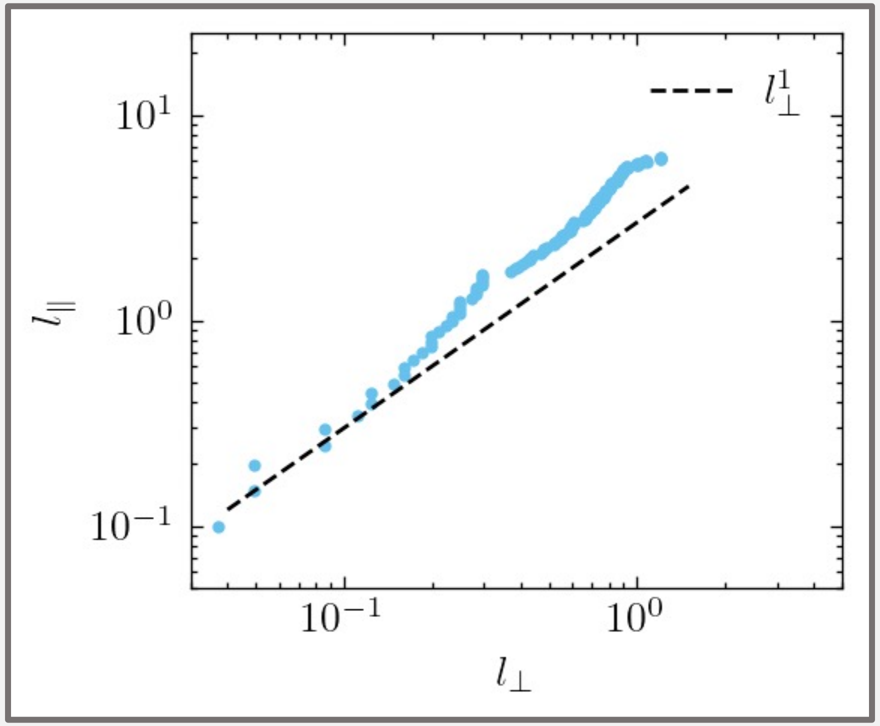


$SF(l_\parallel) \propto l_\parallel^1 \rightarrow E_B(k_\parallel) \propto k_\parallel^{-2}$

Critical balance?

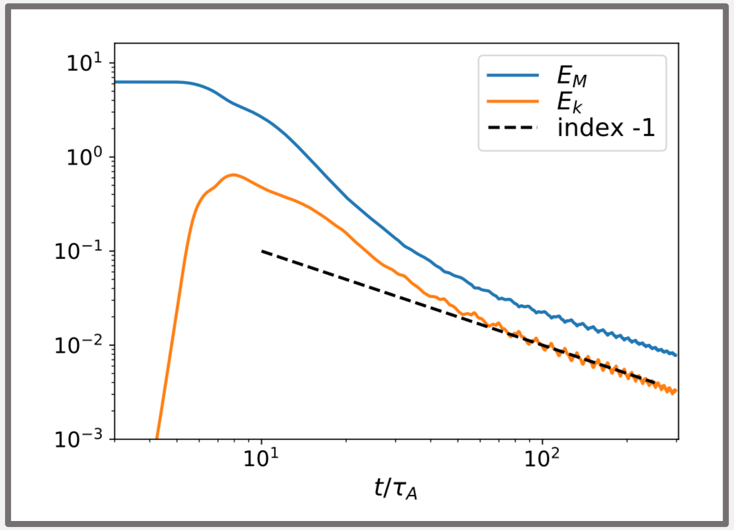
- Structure functions allow us to investigate the scale dependence of turbulent anisotropy
  - For MHD turbulence, would obtain $l_\parallel \sim l_\perp^{2/3}$ by invoking critical balance
- Observe steeper than $l_\parallel \sim l_\perp^1$
- Apparently the typical dimensional analysis doesn't apply here, so how do we approach this?
- Some interesting features here, which we could discuss further…

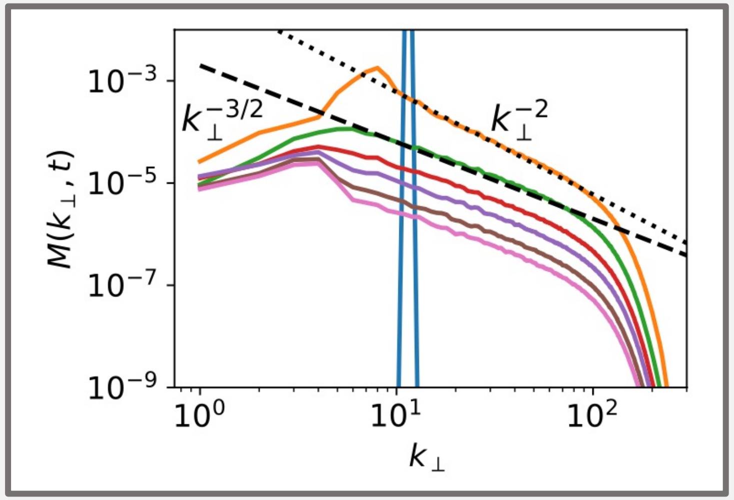
$\tau_A = 273$



Comparison to other decaying RMHD turbulence results

- These results are different from previous studies of decaying RMHD turbulence:
- Zhou et al 2020 investigated turbulence in a system of many merging flux ropes
  - In “transient” state, observe fast magnetic energy dissipation and a Burgers spectrum
  - After decay of magnetic energy so $E_B \sim E_K$, settles into reconnection controlled turbulent decay, mergers of flux ropes to larger and larger scales, as well as conventional MHD turbulence cascade

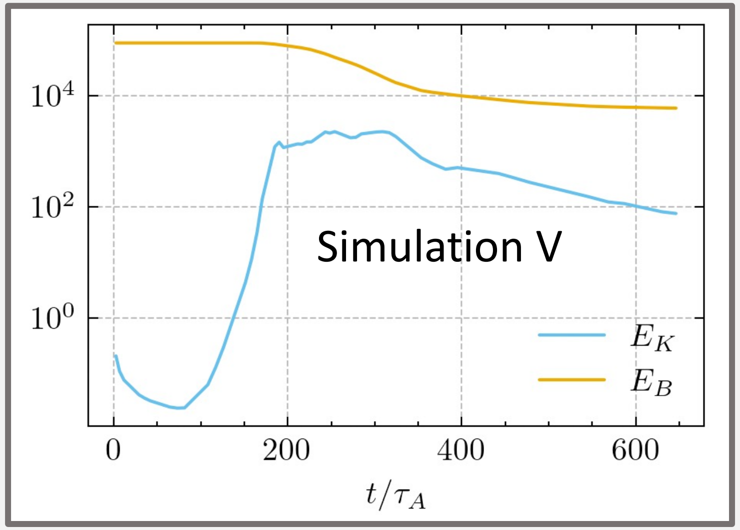


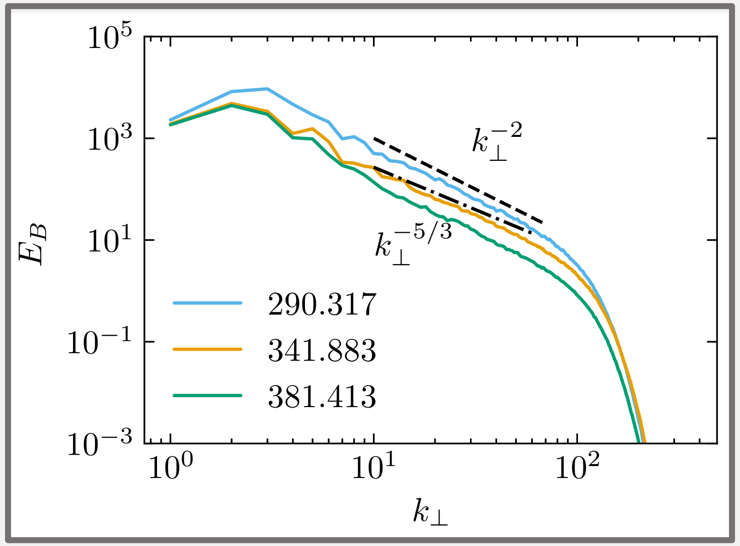


Zhou +, 2020

Comparison to other decaying RMHD turbulence results

- These results are different from previous studies of decaying RMHD turbulence:
- Similar to transient state, but even in run V where the most magnetic energy is dissipated and the E_B/E_K ratio gets the highest, the magnetic energy *still* an order of magnitude larger
- Turbulence is Burgers until end of the kinetic flattop
  - Interestingly, does seem to shallow after this, but only for a short time before diffusion robs us of our inertial range
  - Higher resolution might help to investigate this further

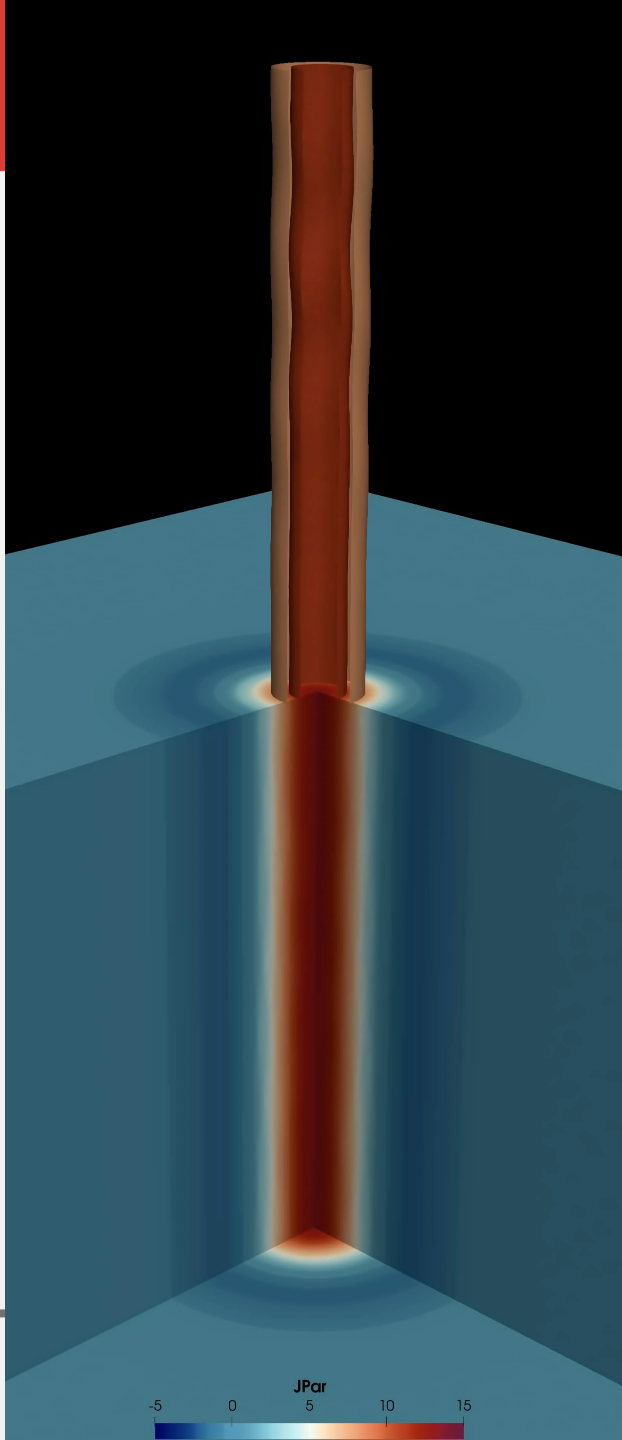




Conclusions

- Study the evolution of magnetic flux ropes under their own dynamics in RMHD
- Modes nonlinearly interact, leading to turbulence
- Current sheets associated with unstable modes allow for exponential dissipation of magnetic energy
- Current sheets mediate the energy spectrum, but an energy cascade appears to be the primary mechanism for energy transfer, as with Burgers turbulence in fluids

- Still would like to figure out critical balance, maybe try at higher resolution!
  - Good news, new GPU code (GX) capable of running these equations is nearly ready!
  - Diagnostics, diagnostics, diagnostics…



References

Stanier et al, “Influence of 3D plasmoid dynamics on the transition from collisional to kinetic reconnection”, Phys. Plasmas , 2019

Stanier et a,l “The role of guide field in magnetic reconnection driven by island coalescence”, Phys. Plasmas , 2017

Alves et al, “Efficient Nonthermal Particle Acceleration by the Kink Instability in Relativistic Jets”, PRL , 2018

Bromberg et al, ”Kink Instability: Evolution and Energy Dissipation in Relativistic Force-free Nonrotating Jets”, ApJ , 2019

Strauss, “Nonlinear, three-dimensional magnetohydrodynamics of noncircular tokamaks”, Phy. Fluids, 1976

Schekochhin et al, “Astrophysical Gryrokinetics: Kinetic and fluid turbulent cascades in magnetized weakly collisional plasmas”, ApJ Supp., 2009

Morrison, “Magnetic field lines, Hamiltonian dynamics, and nontwist systems”, Phys. Plasmas , 2000

White, “Theory of Toroidally Confined Plasmas”, , 2014

Di Giannatale et al, “Coherent transport structures in magnetized plasmas. I. Theory”, Phys. Plasmas, 2018

Loureiro et al, “Plasmoid and Kelvin-Helmholtz instabilities in Sweet-Parker current sheets”, PRE , 2013

Zhou et al, ” Multi-scale dynamics of magnetic flux tubes and inverse magnetic energy transfer”,JPP , 2020

Alexakis, “Shell-to-shell energy transfer in magnetohydrodynamics. I. Steady state turbulence” , PRE, 2005

Kraichnan, “Lagrangian-History Statistical Theory for Burgers' Equation” , Phys. Fluids , 1968

Gotoh et al, “Statistics of decaying Burgers turbulence”, Phys. Fluids , 1993

Cho, “

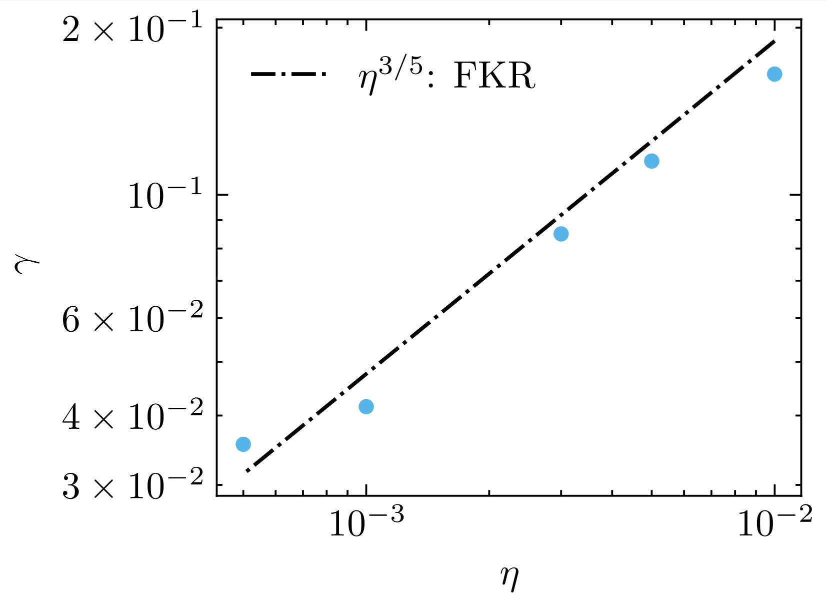
A Technique for Removing Large-scale Variations in Regularly and Irregularly Spaced Data” , ApJ , 2019

BACKUP

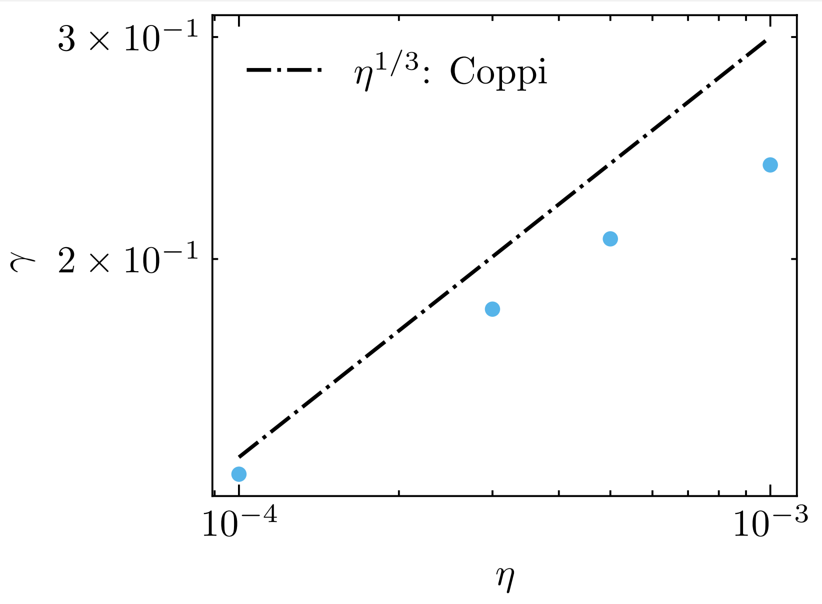
Kink-tearing scalings

- Laplacian resistivity scans for Lz = 0.25 (left) and Lz =0.5 (right).
- 512^2 x (32,64)

$\gamma \sim \eta^{3/5} k^{-2/5}$



$\gamma \sim \eta^{1/3} k^{2/3}$



Dynamics of a quasi-steady nonlinear state

A few interesting features I'd like to explain—

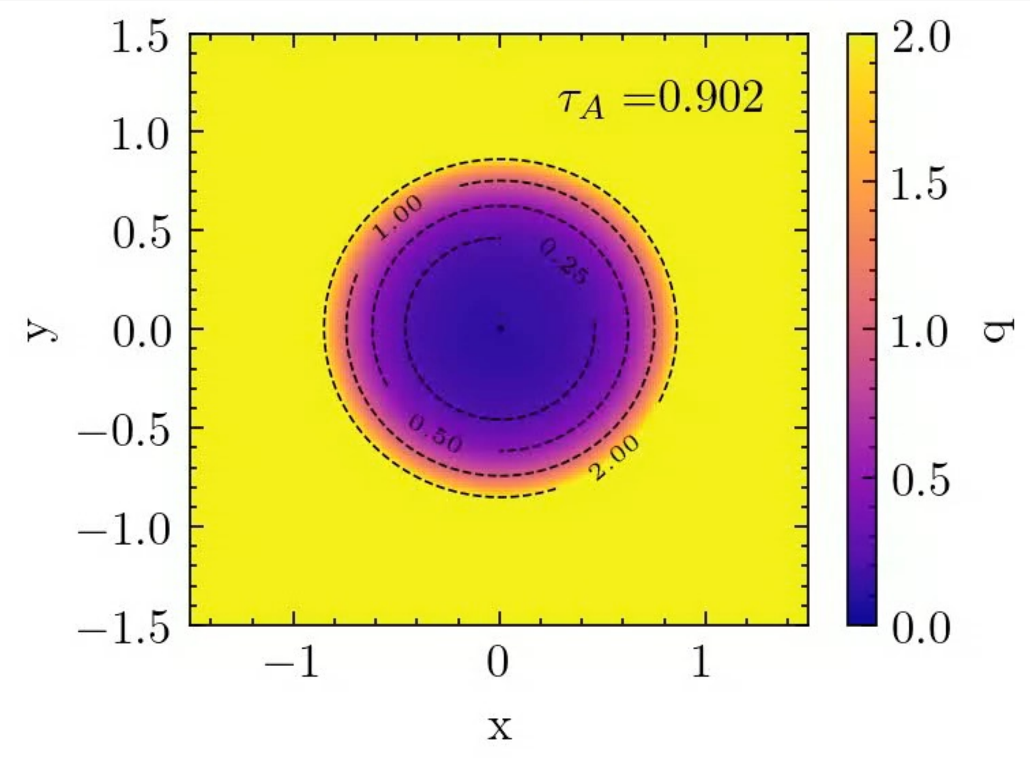
- How is it that the peaks in kinetic energy arise?
- How is it that the rate of magnetic energy dissipation remains exponential?
- How is it that longer flux ropes dissipate a larger fraction of energy

Rational surfaces survive the initial nonlinear interaction

- Peaks in kinetic energy must be associated with some important dynamic event
  - Excitation of instability
  - Are these the initially excited perturbations?
- Calculate safety factor from the magnetic field data:

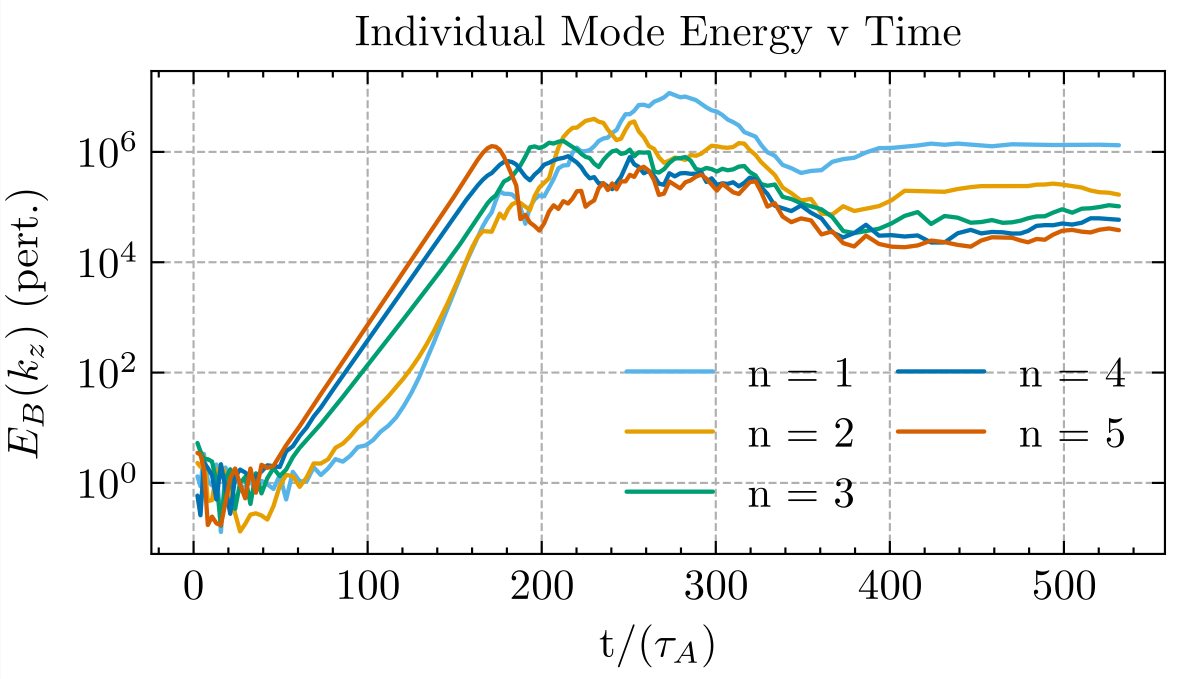
$$\hat{q} = \frac{\hat{x}}{L_z}\frac{1}{\hat{\mathbf{B}}_\perp}$$

- Observe survival of the 1/1 and 1/2 rational surfaces after tau_A ~ 200, where the initial nonlinear interaction occurs
- Rational surfaces are rearranged, so these are not simply the initially growing perturbations but are excited nonlinearly

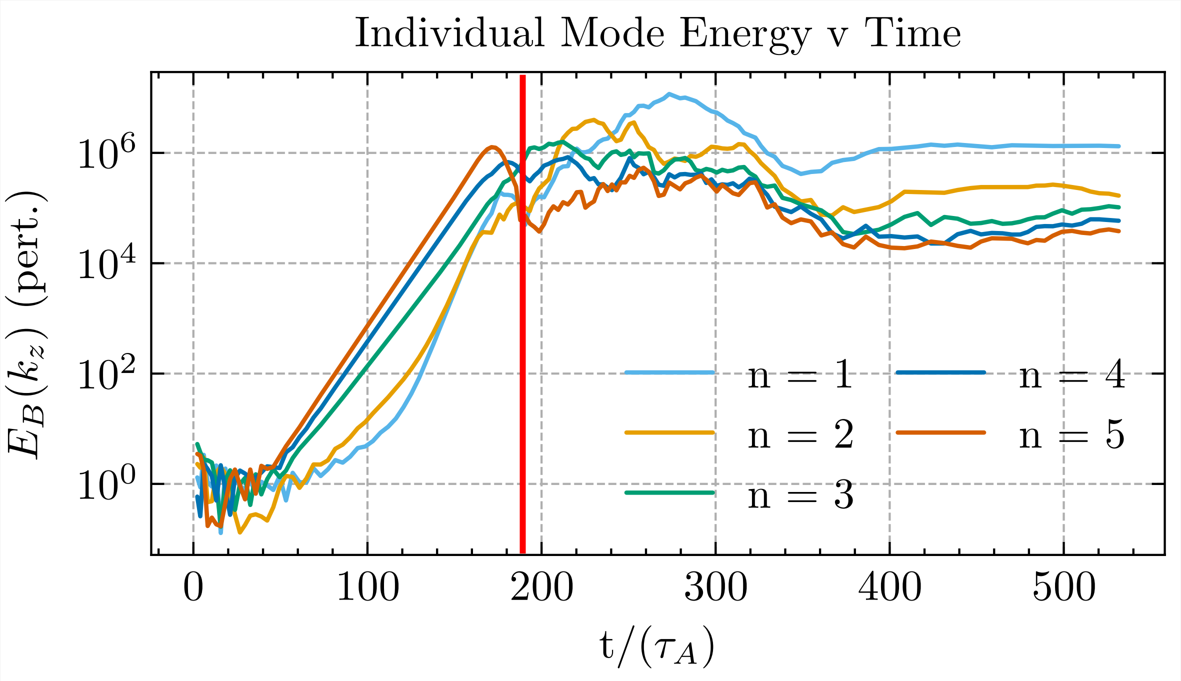


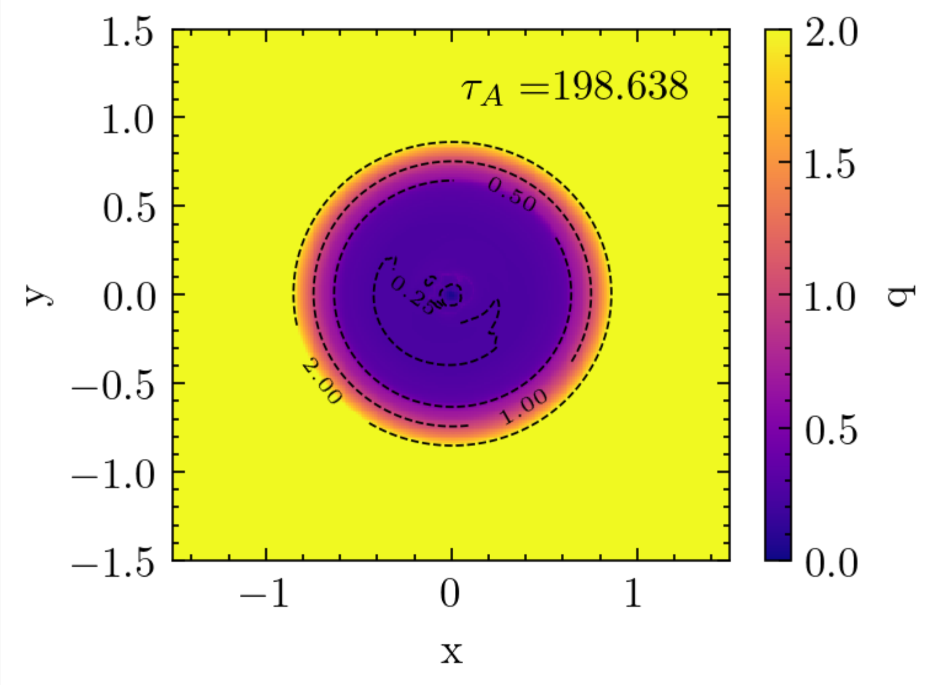
Rational surfaces survive the initial nonlinear interaction

- Calculate the kz spectrum of the perturbed magnetic field
- Indeed, modes experience nonlinear kick at later times
- This allows us to investigate the time evolution of each mode
- See sequential peaks in the mode energies, from n=5 -> n=1

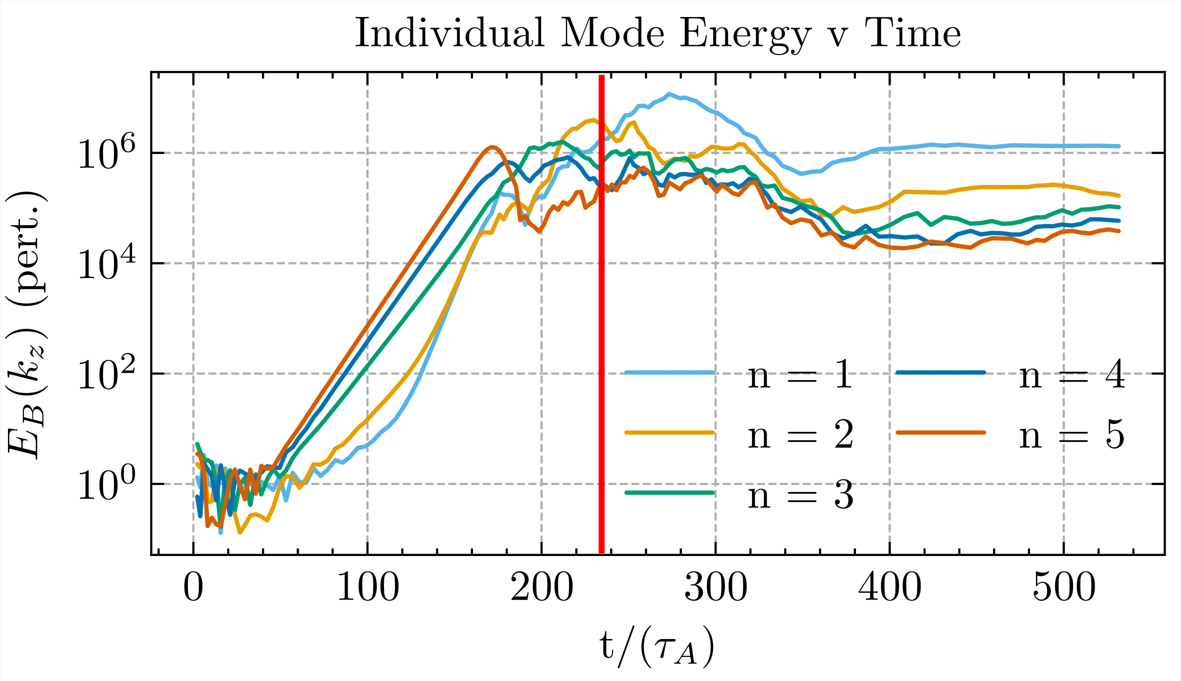


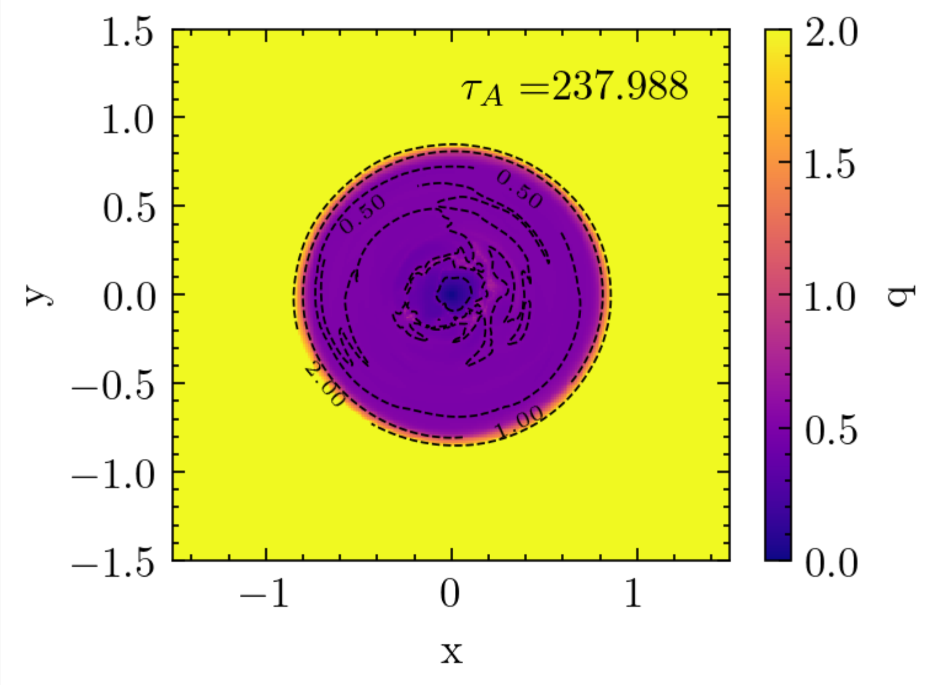
Rational surfaces survive the initial nonlinear interaction



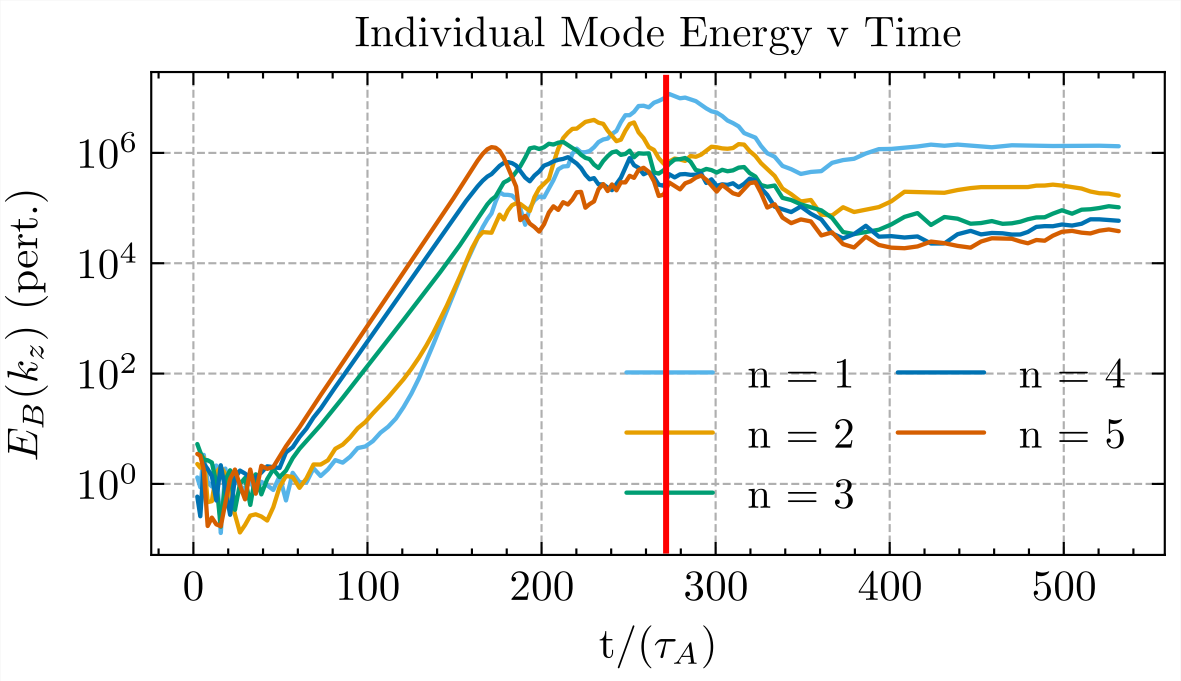


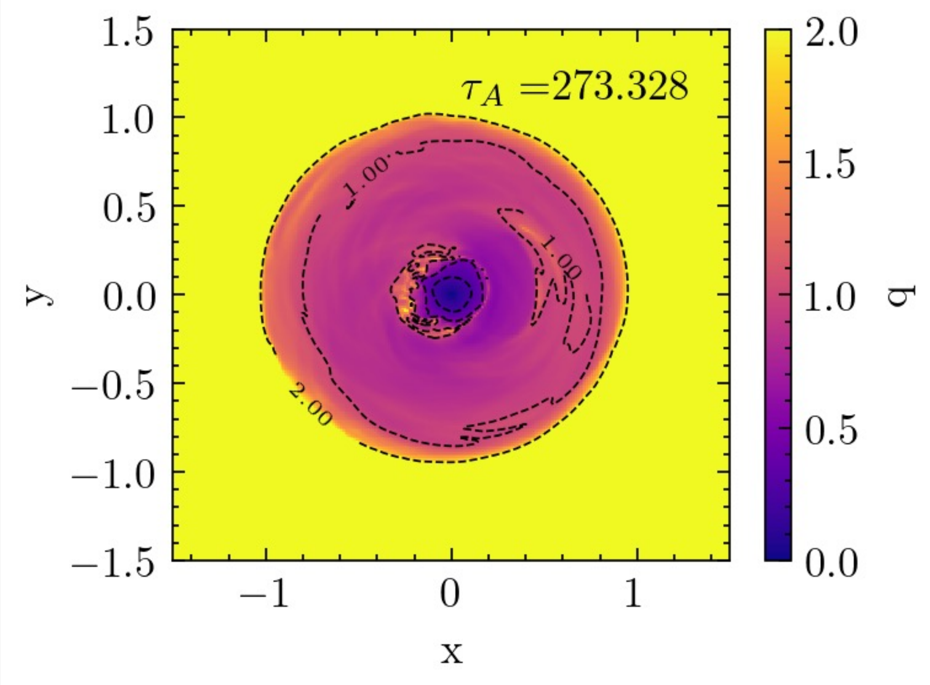
Rational surfaces survive the initial nonlinear interaction





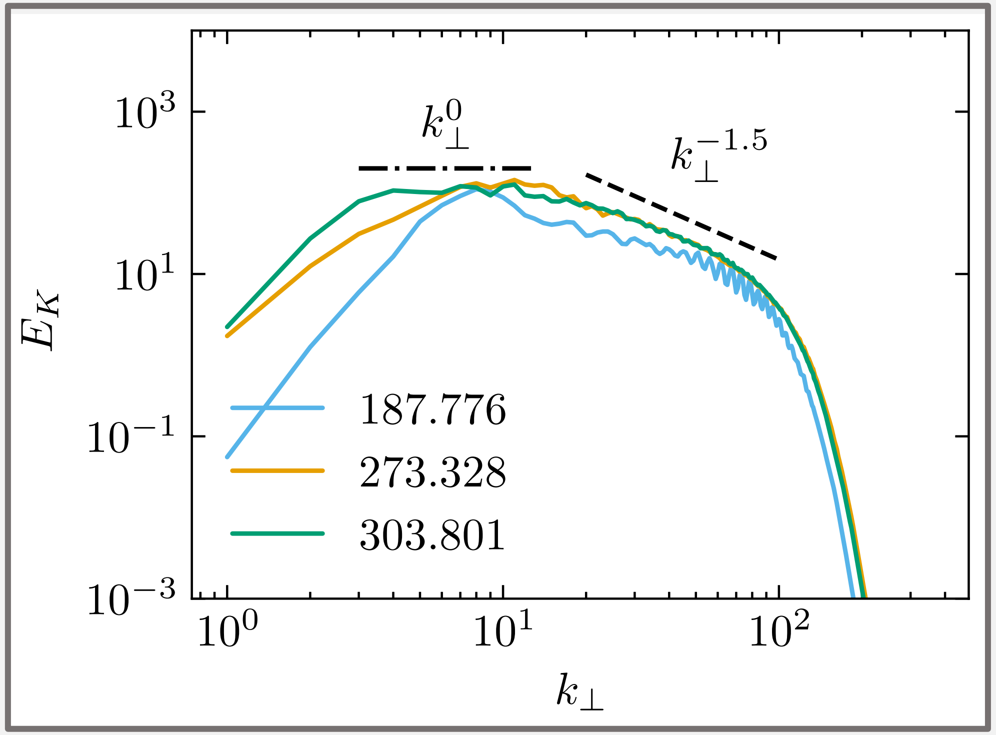
Rational surfaces survive the initial nonlinear interaction





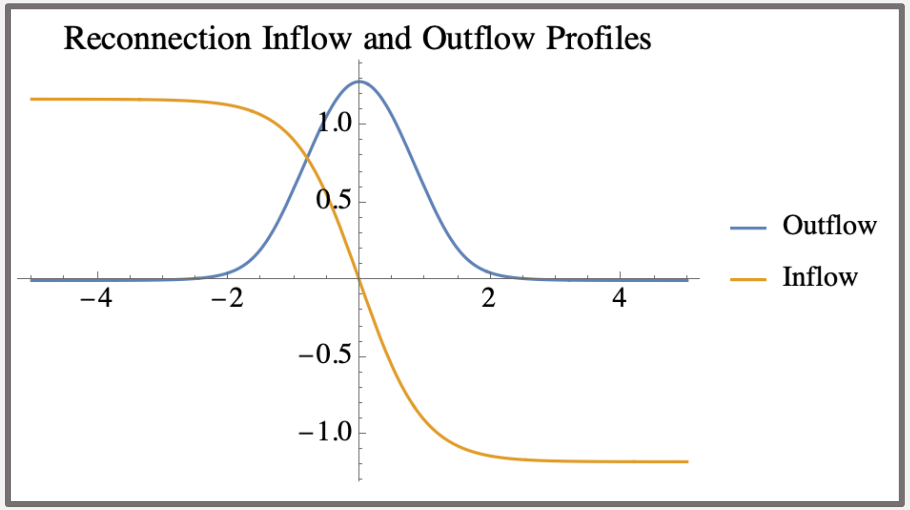
Energy spectra are mediated by strong current sheets

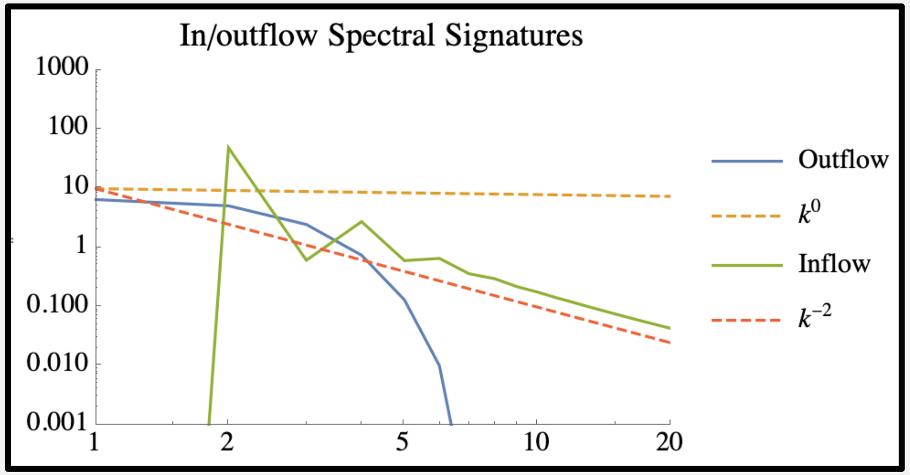
- The kinetic spectrum is also set by features of the magnetic field
- Initially close to $E_K \sim k_\perp^{-1}$. As turbulence develops, see knee at intermediate scales, separating out a flat regime and an inertial range steeper than -1
- Understand this as a combination of the spectral signature of inflow/outflow profiles and the turbulent cascade



Spectral signature of reconnection

- Can obtain analytical expressions for inflow and outflow profiles of an SP-like equilibrium (Loureiro+ 2013)
- Outflows inject energy at all scales—flat spectrum
- Inflows have reversal across cs, so have a $k_\perp^{-2}$ spectrum
- Turbulent cascade steepens the spectrum as well





Calculating the energy transfer

- Interested in how energy moves from scale to scale in this system
- Energy transfer equations are readily obtained from the MHD induction and momentum equations
- The "shell-filtered variables", $\boldsymbol{B}_\perp^K$ and $\boldsymbol{u}_\perp^K$ are quantities whose Fourier transform contains only the fields in a shell $k < K \leq k+1$

$$T_{bb}(Q,K) \equiv -\left\langle \int d^2\mathbf{x}_\perp^2 \left[\mathbf{B}_\perp^K \cdot (\mathbf{u}_\perp \cdot \nabla_\perp)\mathbf{B}_\perp^Q\right] \right\rangle_z$$

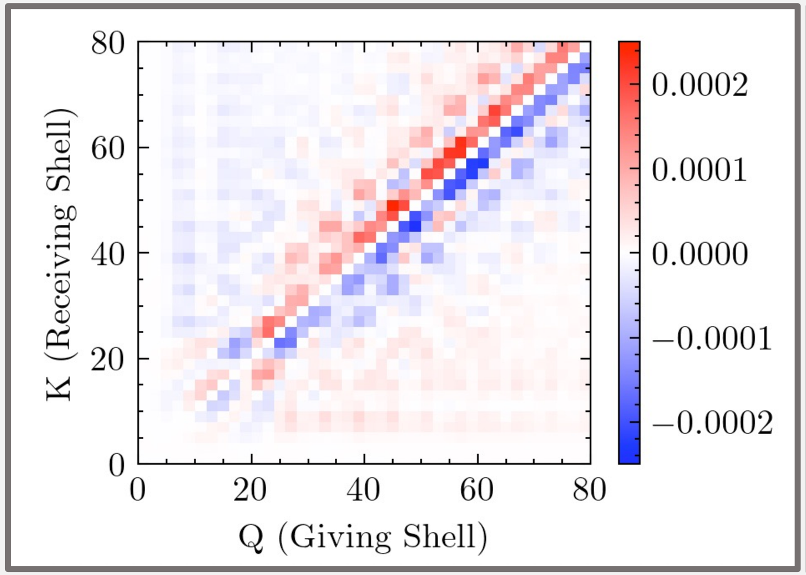
$$T_{uu}(Q,K) \equiv -\left\langle \int d^2\mathbf{x}_\perp^2 \left[\mathbf{u}_\perp^K \cdot (\mathbf{u}_\perp \cdot \nabla_\perp)\mathbf{u}_\perp^Q\right] \right\rangle_z$$

Zhou + 2020, Alexakis + 2005

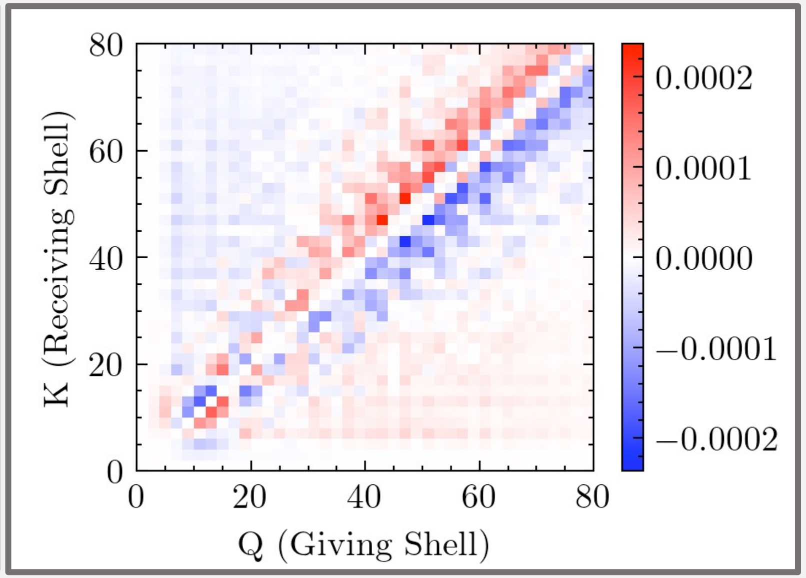
Signatures of current sheets in the energy transfer

- Kinetic-to-kinetic energy transfer
  - Cascade forms along diagonal.
  - Less intense but still significant *inverse* energy transfer from small to large scales indicates the energy injected by reconnection outflows

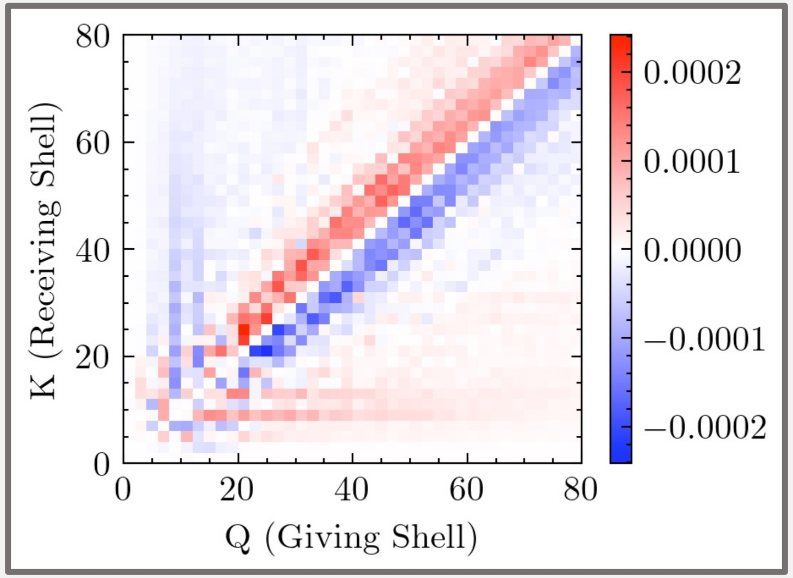
$\tau_A =$ 187



215



273

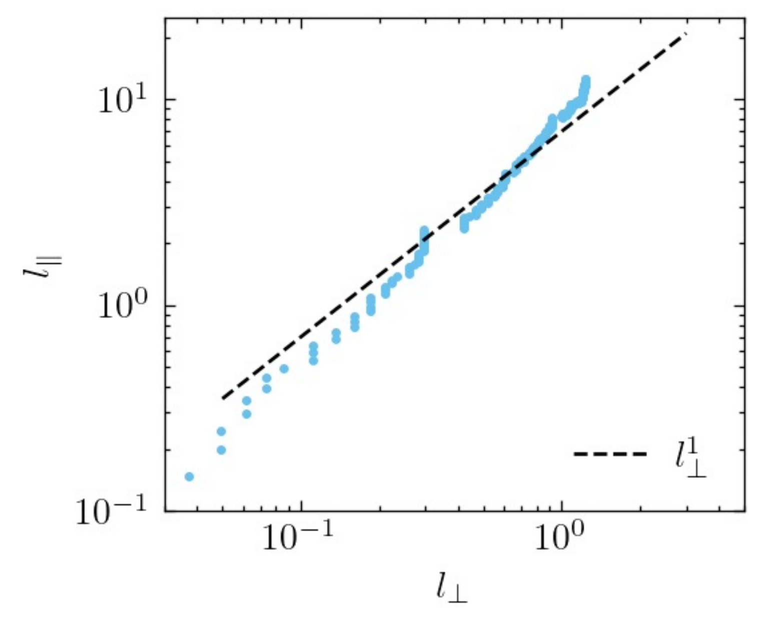


Structure function for Lz4

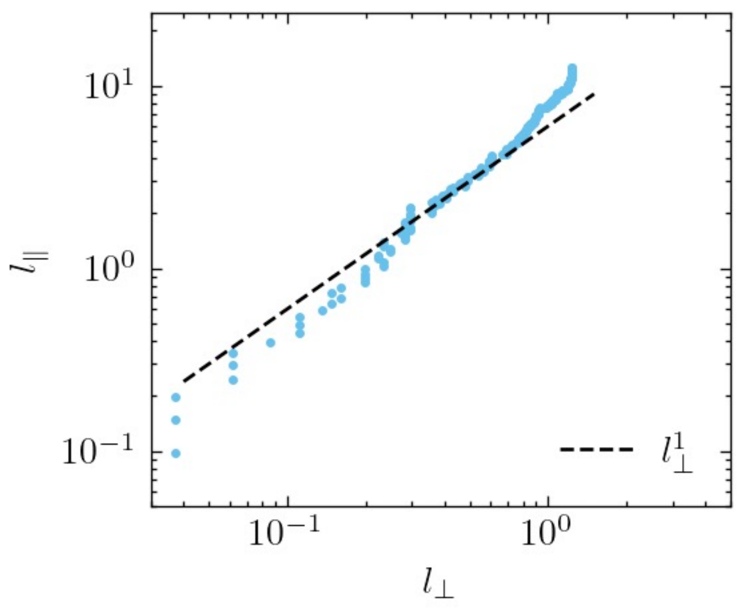
- Lz=4 gives us the best chance of having 3D turbulence
- See that the break in correlation disappears at later times!
- Also becomes perfectly $l_\perp^1$

$$S^{5pt}_{B(2)}(\mathbf{r}) \equiv \frac{1}{35}\langle|\mathbf{B}(\mathbf{x}-2\mathbf{r}) - 4\mathbf{B}(\mathbf{x}-\mathbf{r}) + 6\mathbf{B}(\mathbf{x}) - 4\mathbf{B}(\mathbf{x}+\mathbf{r}) + \mathbf{B}(\mathbf{x}+\mathbf{r})|^2\rangle_x$$

265



284



341