Neoclassical transport in strong gradient regions

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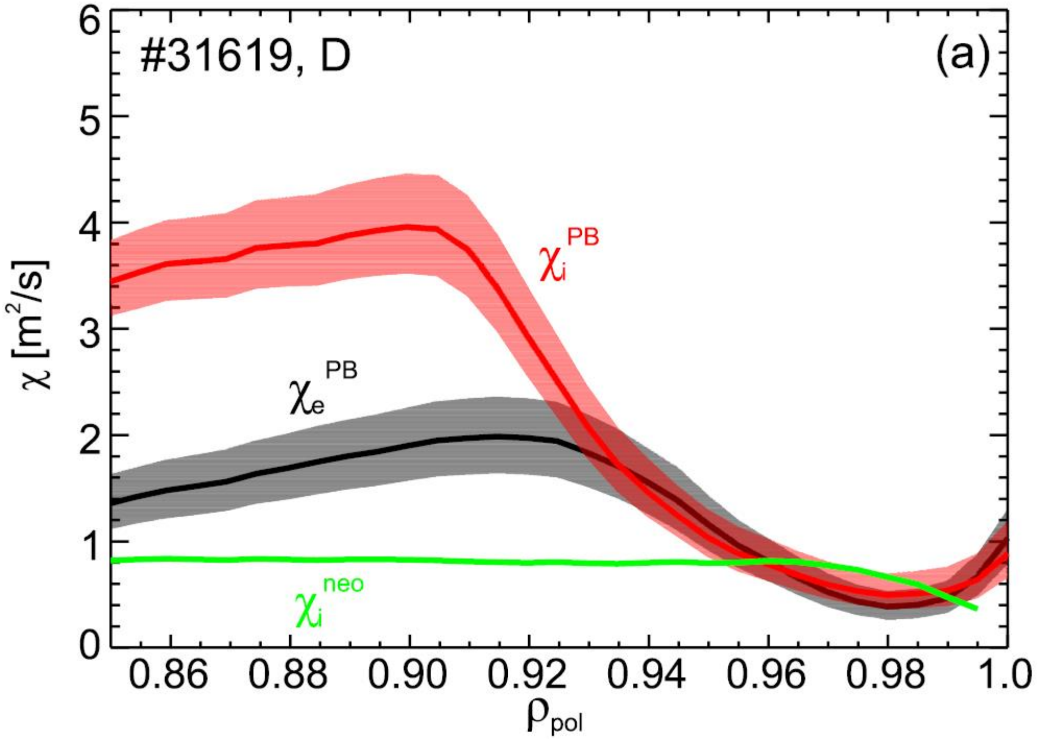
Motivation

- Neoclassical transport relevant in strong gradient regions[1]
- Pedestal width is of the order of ρ_p[1,2]
- Problem: standard neoclassical theory requires weak gradients

$$\frac{\rho_p}{L} \ll 1$$

- Previous work assumed weak temperature gradient[3] or mean parallel flow gradient[4]

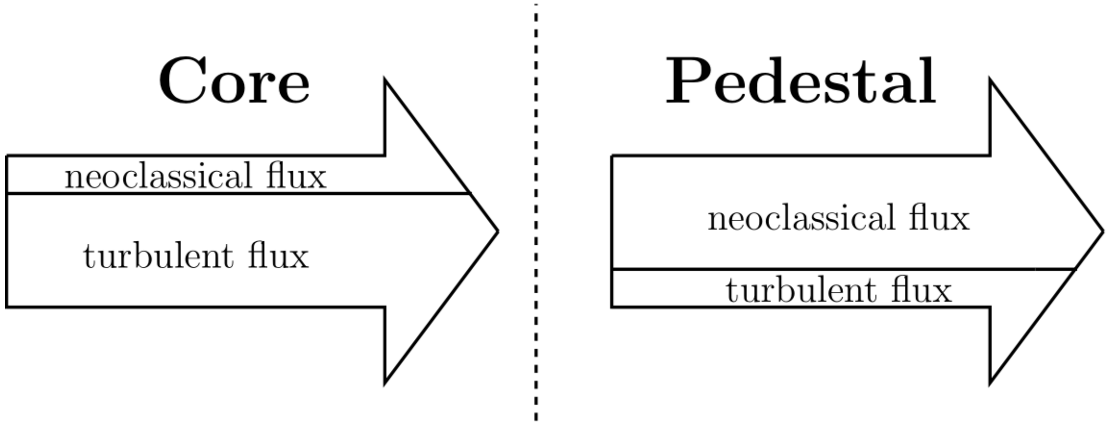


Heat diffusivities[1]

[1] E. Viezzer *et al* 2018 *Nucl. Fusion* **58** 026031
[2] R. M. McDermott et al 2009 *PoP* **16**, 056103
[3] G. Kagan et al 2009 *PoP* **16**, 056105
[4] K. C. Shaing et al 1994 *PoP* **1**

Expectation

Total flux going from core into pedestal is kept (steady state, e.g. I-mode, QH-mode)

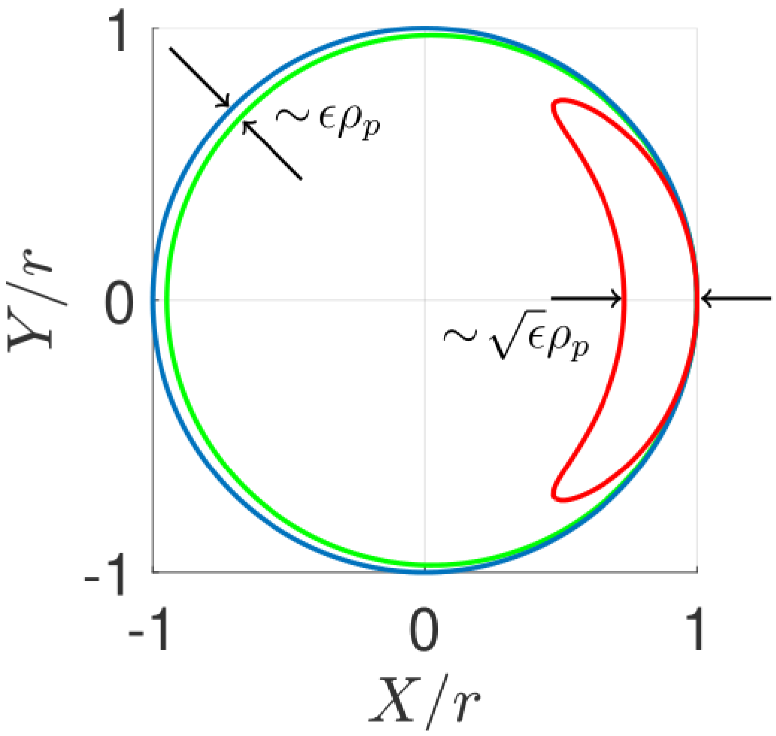


- Turbulent fluxes decrease as turbulence decreases
- Neoclassical fluxes increase as gradients get stronger

Outline

- Assumptions and differences to standard neoclassical theory
- Neoclassical particle and energy flux
- Poloidal variation of potential and density
- Mean parallel flow

Orderings and assumptions



- Large gradients:

$$L_n \sim L_\Phi \sim L_T \sim \rho_p \equiv \rho \frac{B}{B_p} \sim \rho \frac{q}{\epsilon}$$

- Large aspect ratio:

$$\frac{r}{R} \sim \epsilon \ll 1$$

- Banana regime:

$$\frac{qR\nu}{v_t} \ll \epsilon^{3/2}$$

- Circular flux surfaces

Also included: poloidal variation

$$\Phi - \phi(\psi) = \phi_\theta(\theta,\psi) = \phi_c(\psi)\cos\theta \sim \epsilon \frac{T}{e}$$

Trapped and Passing Particles

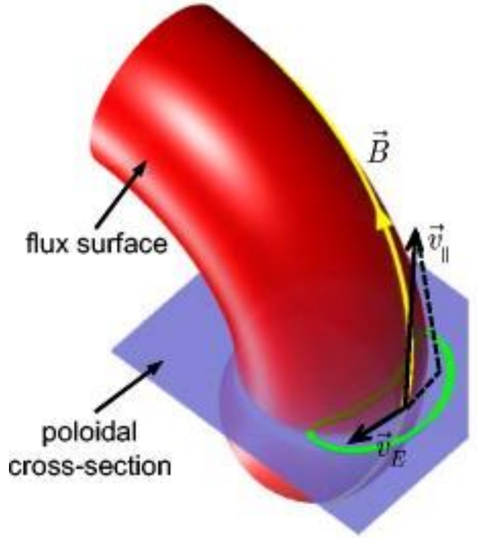
Trapped particles:

Poloidal velocity:

$$\dot{\theta} = (v + v_{E\times B}) \cdot \nabla\theta = \left(v_\parallel + \frac{cI}{B}\frac{\partial\Phi}{\partial\psi}\right)\hat{b}\cdot\nabla\theta \equiv (v_\parallel + u)\hat{b}\cdot\nabla\theta$$

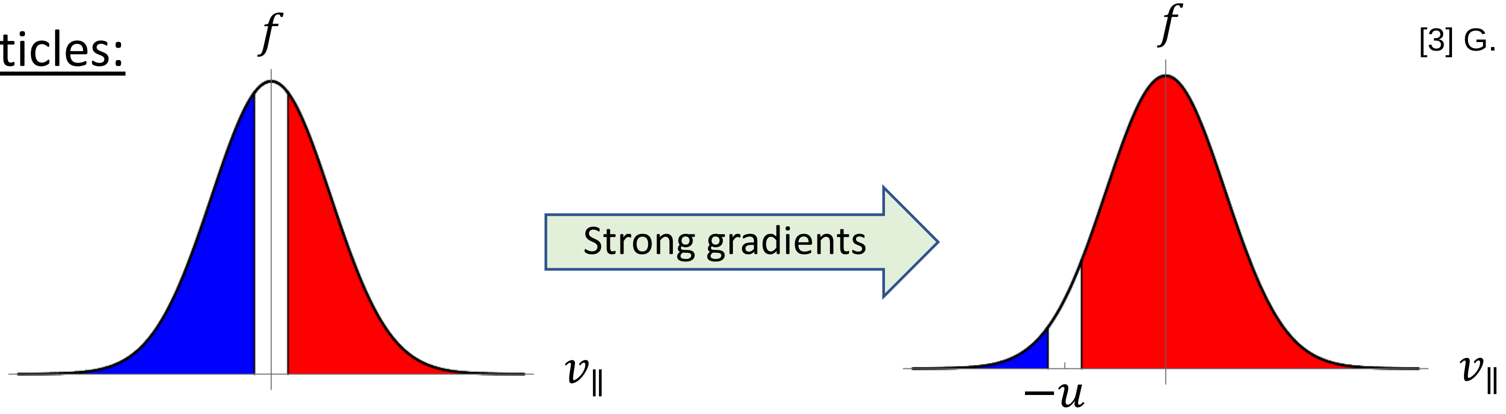
Poloidal components of parallel velocity and $E \times B$ – drift balance[3,5]

$\Rightarrow$ Shift in trapped particle region to $v_\parallel + u \sim \sqrt{\epsilon} v_t$



[3] G. Kagan et al 2009 *PoP* **16**, 056105

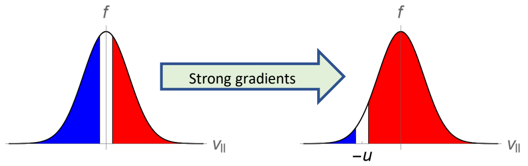
Passing particles:



Shift in trapped particle region causes asymmetry in passing particle number:
more red particles $(v_\parallel + u > 0)$ than blue particles $(v_\parallel + u < 0)$

[3] G. Kagan et al 2009 *PoP* **16**, 056105
[5] K. C. Shaing et al 1992 *Physics of Fluids B: Plasma physics* **4**

Poloidal Variation



Shift in Trapped Particle Region causes asymmetry in passing particle number:
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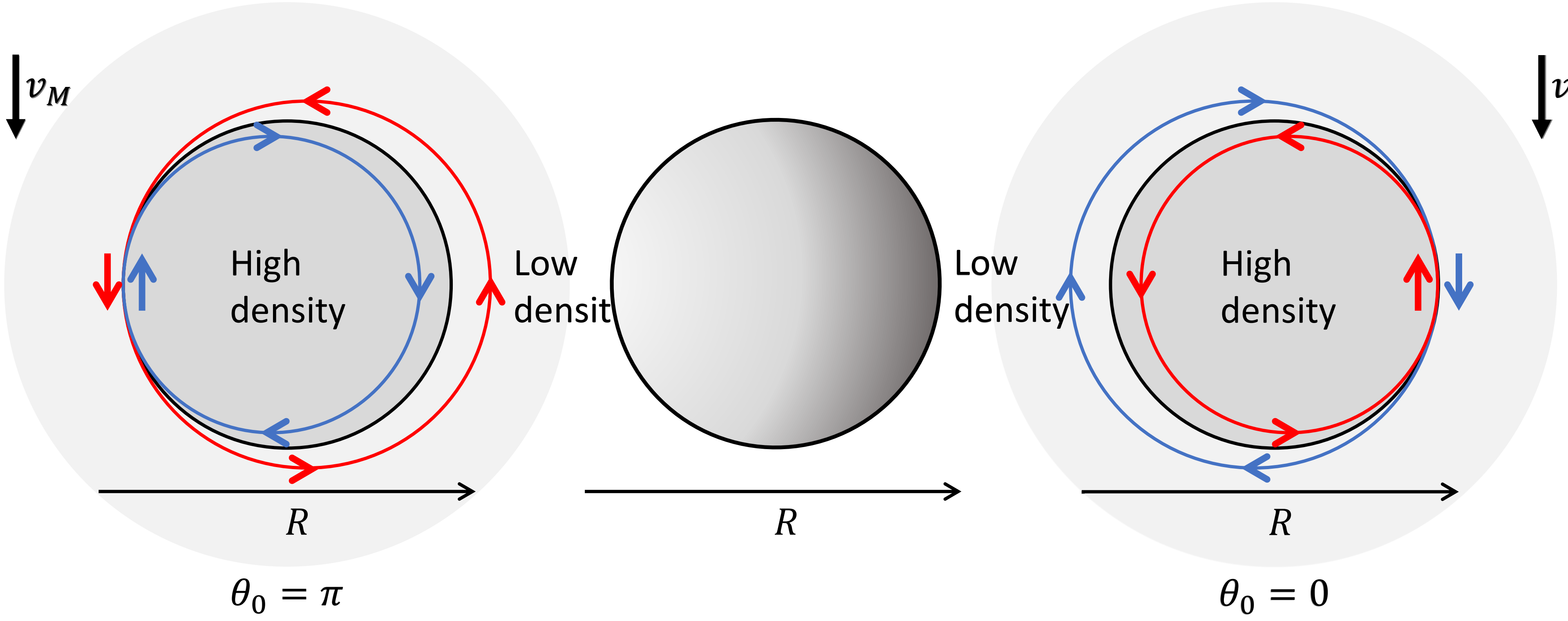


$\Rightarrow$ Poloidal Variation within a flux surface in density, potential, flow, and temperature

Solving the drift kinetic equation

1) Drift kinetic equation

$$\dot{\theta}\frac{\partial f}{\partial \theta} + \dot{\psi}\frac{\partial f}{\partial \psi} + \dot{v}_{\parallel}\frac{\partial f}{\partial v_{\parallel}} = C[f] + \Sigma$$

Change of variables $\{v_{\parallel}, \psi\} \rightarrow \{v_{\parallel f}, \psi_f\}$

$$\dot{\theta}\left.\frac{\partial f}{\partial \theta}\right|_{v_{\parallel f},\psi_f} = C[f] + \Sigma$$

Transit average: $\langle \dots \rangle_\tau = \int \frac{d\theta}{\dot{\theta}}(\dots)/\tau$

$$\langle C[f] \rangle_\tau = -\langle \Sigma \rangle_\tau$$

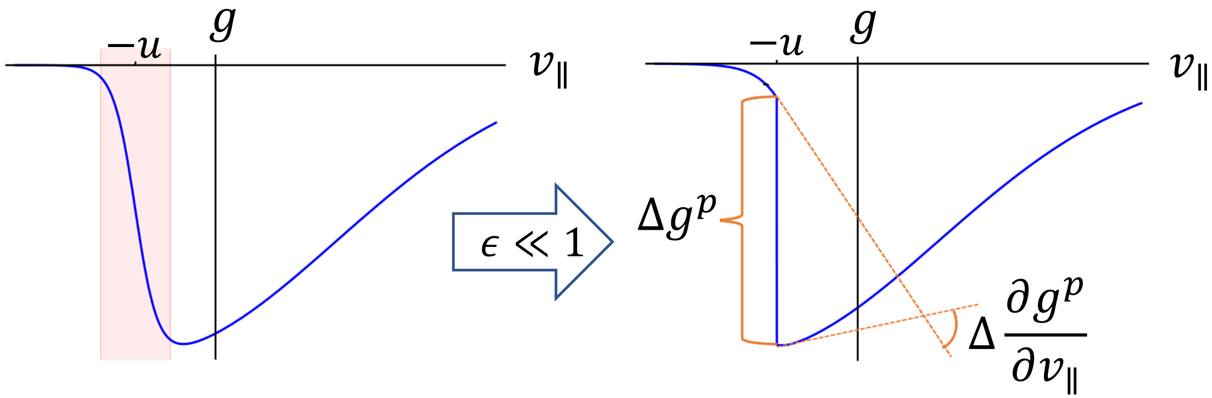
2) Asymptotic matching

$$f = f_M + g$$

The trapped-barely-passing region is small

$$g \simeq g^p$$

Jump in height and derivative across trapped-barely-passing region



3) Moment equations

Take moments of drift kinetic equation

e.g. particle moment

$$\int d^3v \, \langle C[f] \rangle_\tau = -\int d^3v \, \langle \Sigma \rangle_\tau$$

$$\frac{\partial \Gamma}{\partial \psi} = \int d^3v \, \langle \Sigma \rangle_\tau$$

Particle and Energy Flux

$$\underbrace{\Gamma}_{\to 0} = -1.1\sqrt{\frac{r}{R}}\frac{\nu I^2 p}{\underbrace{|S|^{3/2}}_{\to 1} m\Omega^2}\left\{\left[\frac{\partial}{\partial\psi}\ln p - \frac{m(u+V_\parallel)}{T}\left(\underbrace{\frac{\partial V_\parallel}{\partial\psi}}_{\to 0} - \frac{\Omega}{I}\right)\right]\underbrace{G_1(u,V_\parallel,\phi_c)}_{\to 1} - 1.17\frac{\partial}{\partial\psi}\ln T\,\underbrace{G_2(u,V_\parallel,\phi_c)}_{\to 1}\right\}$$

$$Q = \underbrace{\frac{mu^2}{2}}_{\to 0}\Gamma - 1.46\sqrt{\frac{r}{R}}\frac{\nu I^2 pT}{\underbrace{|S|^{3/2}}_{\to 1} m\Omega^2}\left\{\left[\frac{\partial}{\partial\psi}\ln p - \frac{m(u+V_\parallel)}{T}\left(\underbrace{\frac{\partial V_\parallel}{\partial\psi}}_{\to 0} - \frac{\Omega}{I}\right)\right]\underbrace{H_1(u,V_\parallel,\phi_c)}_{\to 1} - 0.25\frac{\partial}{\partial\psi}\ln T\,\underbrace{H_2(u,V_\parallel,\phi_c)}_{\to 1}\right\}$$

Standard neoclassical limit

- Modification of transport coefficient by poloidal dependence of the potential
  - Trapped particles drive transport and particles can be (de-)trapped by ϕ_θ
- Transport driven by gradient of mean parallel flow
- Orbit squeezing[5]

Orbit squeezing:

$$S = 1 + \frac{cI^2}{\Omega B}\frac{\partial^2\Phi}{\partial\psi^2}$$

Trapped particle velocity:

$$u = \frac{cI}{B}\frac{\partial\Phi}{\partial\psi}$$

[5] K. C. Shaing et al 1992 *Physics of Fluids B: Plasma physics* **4**

Particle Flux

Parallel momentum transport:

Flow damping: $\Gamma = -\frac{I}{m\Omega} F_\parallel$

Parallel momentum flux → $\frac{\partial}{\partial \psi}(-mu\Gamma)$

$$\frac{\partial}{\partial \psi}(-mu\Gamma) + F_\parallel = \gamma$$

Parallel momentum source → γ

If there is no parallel momentum source ($\gamma = 0$):

$$\Gamma \propto \exp\left(-\int d\psi \frac{\Omega}{I}\frac{S}{u}\right) \to 0$$

Problem: we were expecting Γ to increase

Parallel momentum source

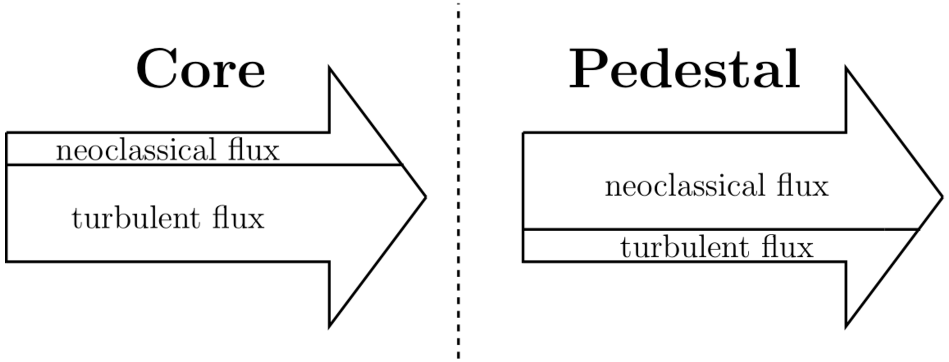
$$\gamma \equiv \int d^3v\, mv_\parallel\, \Sigma$$

Orbit squeezing:

$$S = 1 + \frac{cI^2}{\Omega B}\frac{\partial^2 \Phi}{\partial \psi^2}$$

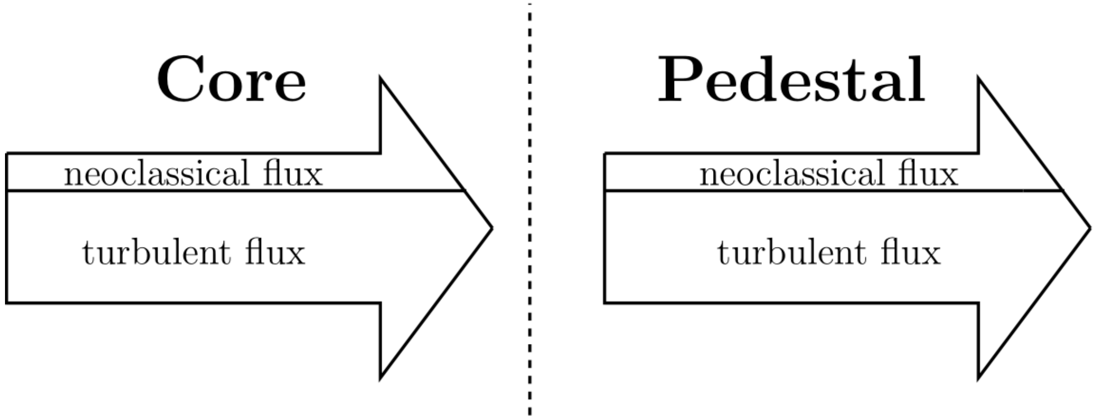
Trapped particle velocity:

$$u = \frac{cI}{B}\frac{\partial \Phi}{\partial \psi}$$

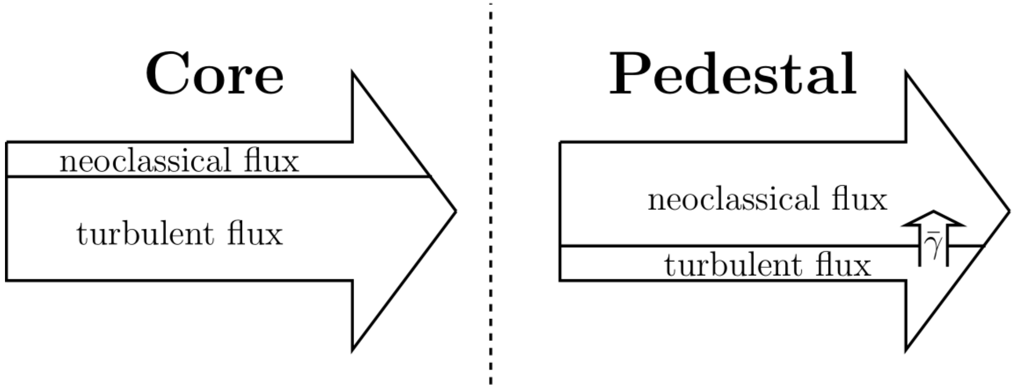


Particle Flux

- Option 1: Particles carried by turbulence
    - Neoclassical ion particle flux remains small
    - No parallel momentum source



- Option 2: Parallel momentum input drives neoclassical particle flux
    - Turbulence or impurities possibly parallel momentum source
    - Neoclassical ion particle flux significant



What about ambipolarity?

Ambipolarity: Radial current must vanish

$$j_r = 0 \Rightarrow \Gamma_{\mathrm{i}} = \Gamma_e$$

Strong gradient fluxes:

$$\Gamma_i^{neo} \sim \sqrt{\epsilon}\nu \text{ and } \Gamma_e^{neo} \sim \sqrt{\frac{m_e}{m_i}}\,\Gamma_i^{neo} \ll \Gamma_i^{neo}$$

?

The **total** fluxes are ambipolar

$$\Gamma_i^{neo} + \Gamma_i^{turb} = \Gamma_e^{neo} + \Gamma_e^{turb}$$

where

$$\Gamma_i^{neo} \sim \Gamma_e^{turb} \sim \sqrt{\epsilon}\nu$$

Example Profiles

- Realistic H-mode density and temperature profiles[6]
- Assumption - radial electric field balances pressure gradient[2]:

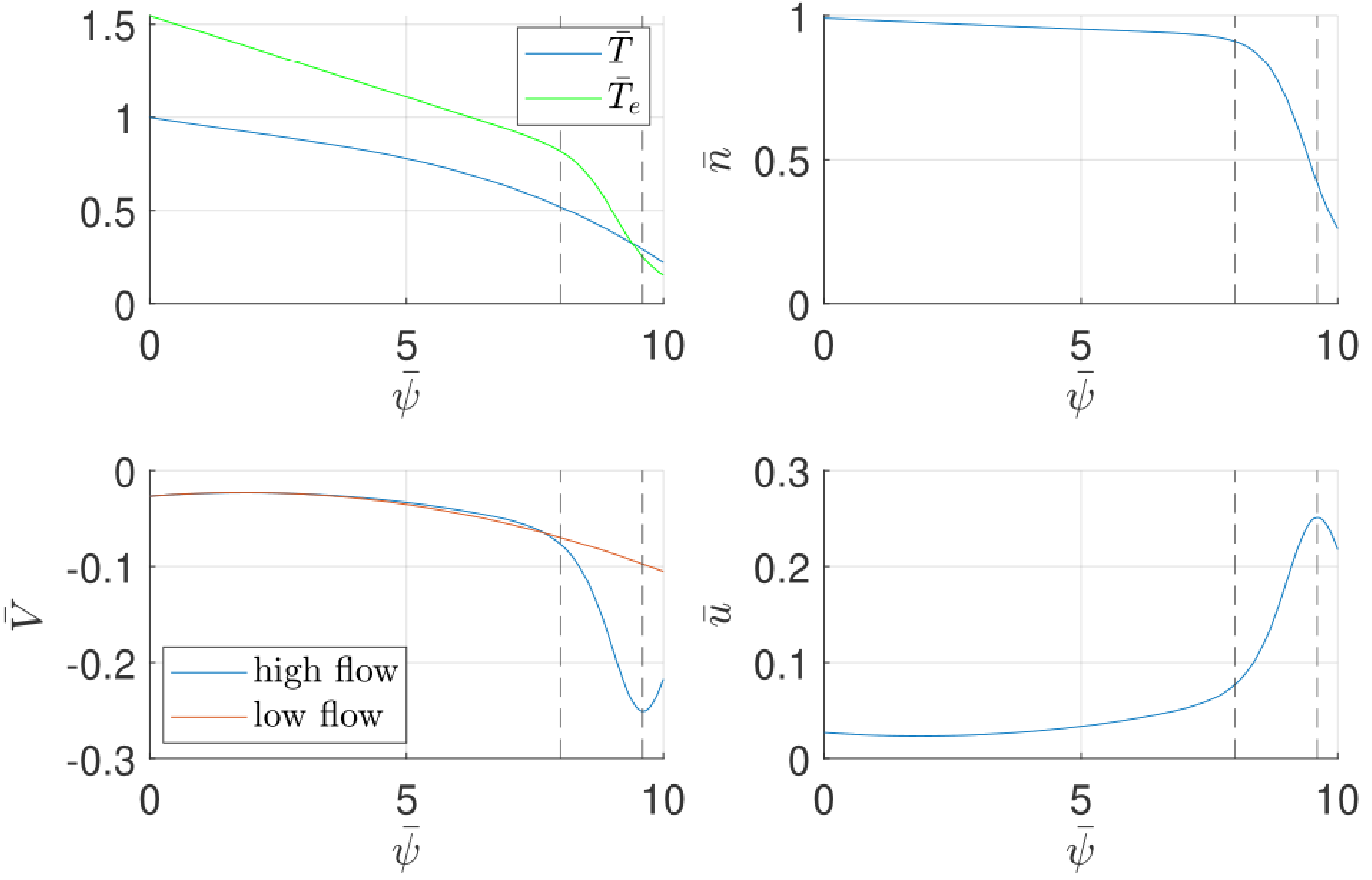
$$Zen\frac{\partial \Phi}{\partial \psi} = \frac{\partial p}{\partial \psi}$$

- Compare two cases:
    - "Low Flow"

$$V_{\parallel} = 1.17\frac{IT}{m\Omega}\frac{\partial}{\partial \psi}\ln T$$

    - "High Flow"

$$V_{\parallel} = -u$$

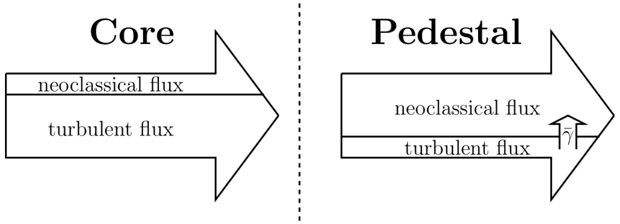


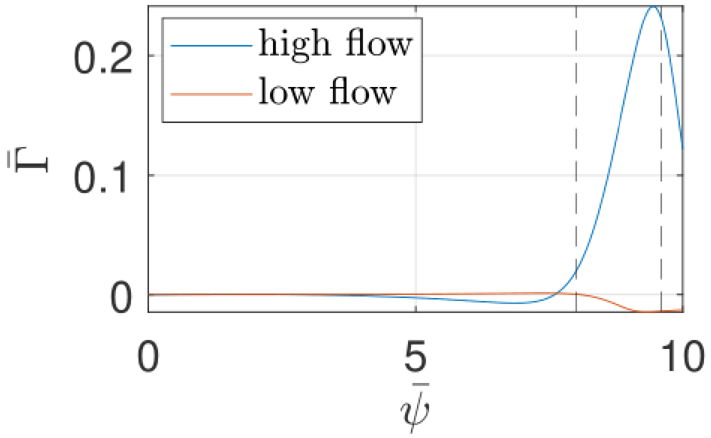
[2] R. M. McDermott et al 2009 *PoP* **16**, 056103

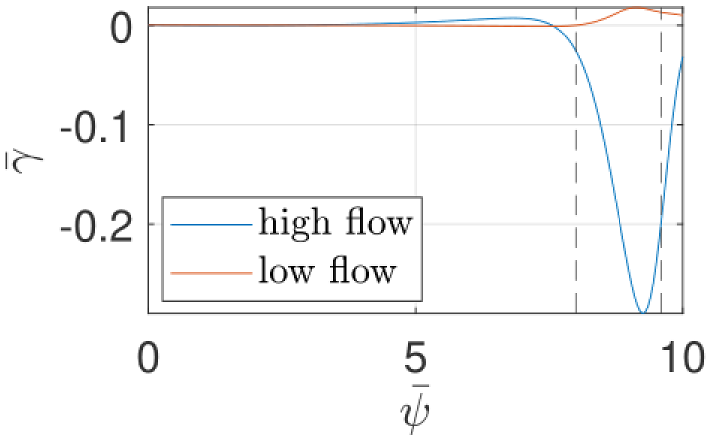
[6] E. Viezzer et al 2017 *Nucl. Fusion* **57**, 022020

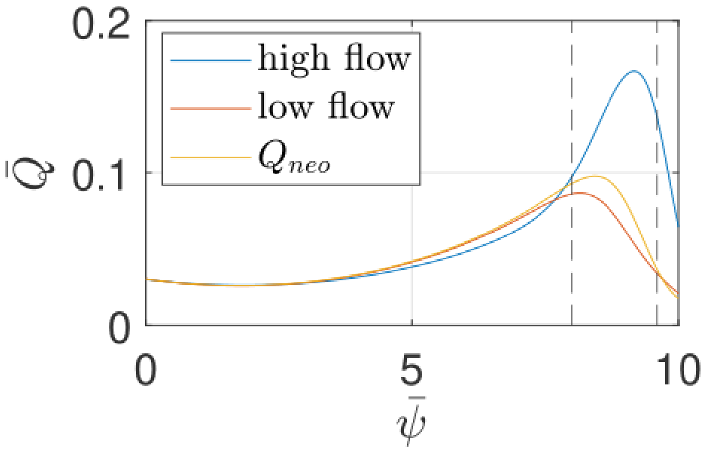
Example Profiles

- Neoclassical particle and energy flux increase in the strong gradient region
- **Parallel momentum input** necessary to sustain neoclassical particle flux







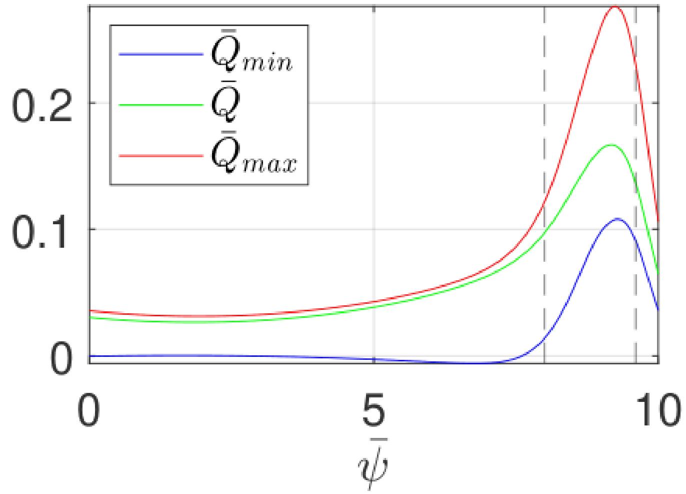


Energy Flux

In the pedestal: $\frac{\partial}{\partial\psi}\ln n < 0$ and $\frac{\partial}{\partial\psi}\ln T < 0$

We can construct bounds for the energy flux (for $\Gamma = 0$):

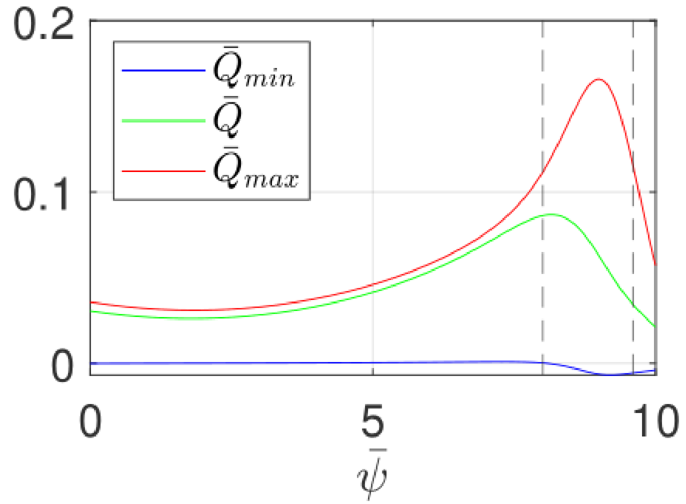
$$0 \leq Q \leq 1.71 \frac{n^2 u}{T^{1/2} S^{3/2}} \Delta Q(u, V_\parallel, \phi_c)$$



"high flow"

In pedestals, usually $L_n < 4L_T$ holds (i.e. $n^2/T^{1/2} \to 0$)

For a growing energy flux, $u\Delta Q/S^{3/2}$ has to grow stronger



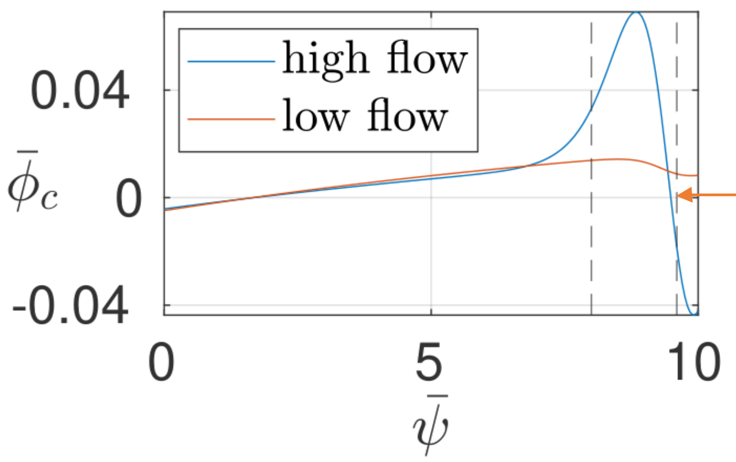
"low flow"

⇒ **Radial electric field** is important for increased neoclassical energy flux

Poloidal Variation

Potential:

Poloidally varying part of potential $\phi_c(\psi)\cos\theta$ can (de-)trap particles



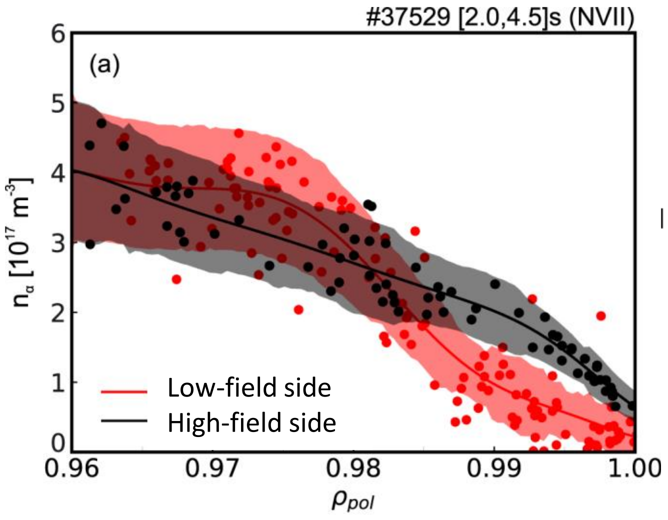
Density:

ASDEX-U[7], Alcator C-Mod[8,9]:

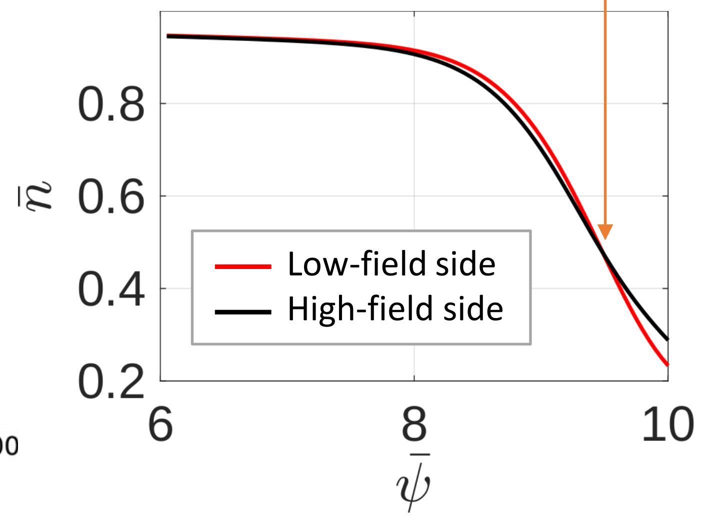
- Accumulation of impurities on high-field side
- In-out asymmetry in temperatures and radial electric field

"High flow" example reproduces high-field side accumulation

"Low flow" example does not



Impurity measurements[7]



Bulk ion model ("high flow")

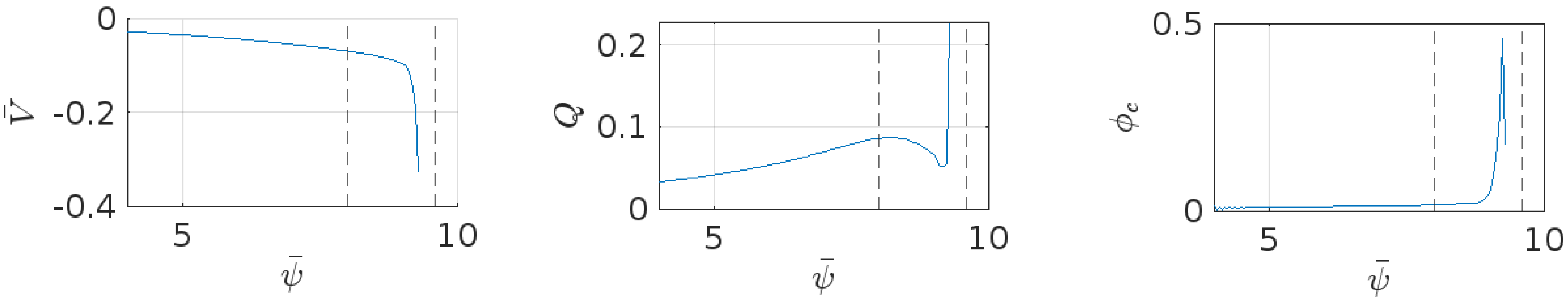
[7] D. J. Cruz-Zabala et al 2022 *Plasma Phys. Control. Fusion* **64** 045021
[8] C. Theiler et al 2014 *Nucl. Fusion* **54** 083017
[9] R. M. Churchill et al 2015 *PoP* **22** 056104

A note on the mean parallel flow

So far: Choose $(n, T, V_\|) \rightarrow (\Gamma, \gamma, Q, u, \phi_c)$

Can we reverse it? Choose $(\Gamma, \gamma, Q) \rightarrow (n, T, V_\|, u, \phi_c)$?

Example: $(n, T, \Gamma = 0) \rightarrow (\gamma, Q, V_\|, u, \phi_c)$:



⇒ Solutions show “shock-like” features

A note on the mean parallel flow

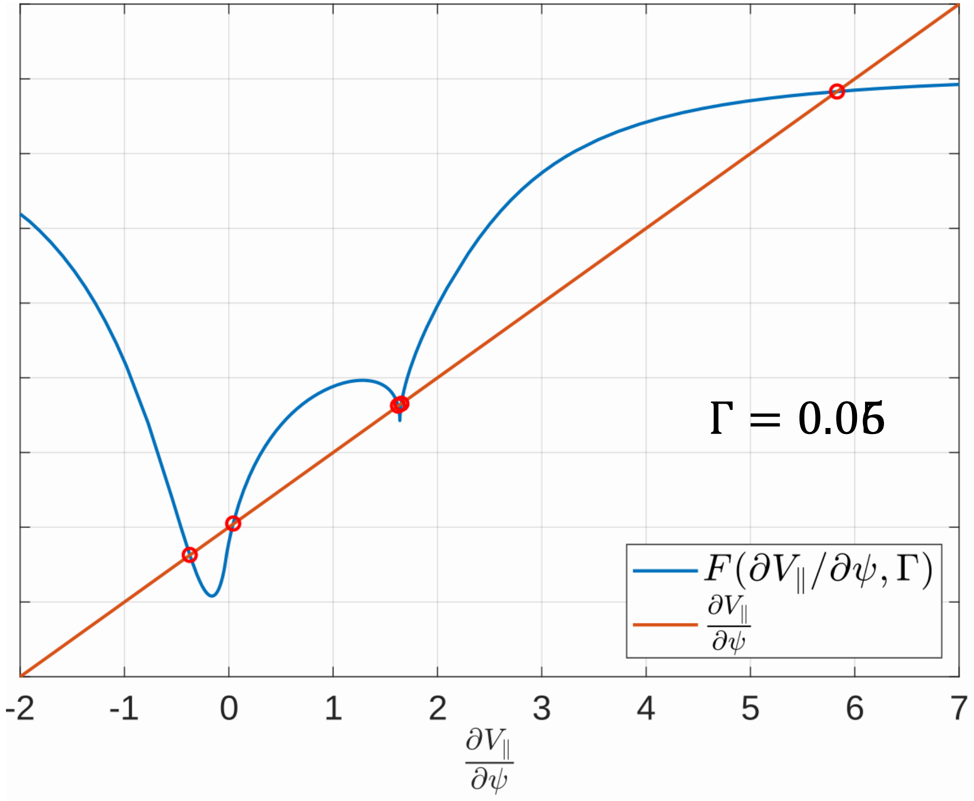
Particle flux equation

$$\Gamma = -1.1\sqrt{\frac{r}{R}}\,\frac{\nu I^2 p}{|S|^{\frac{3}{2}} m\Omega^2}\left\{\left[\frac{\partial}{\partial\psi}\ln p - \frac{m(u+V_\parallel)}{T}\left(\frac{\partial V_\parallel}{\partial\psi} - \frac{\Omega}{I}\right)\right] G_1\left(u, V_\parallel, \phi_\theta\right) - 1.17\,\frac{\partial}{\partial\psi}\ln T\, G_2\left(u, V_\parallel, \phi_\theta\right)\right\}$$

Coupled to quasineutrality equation $\phi_\theta = \phi_\theta\left(\frac{\partial V_\parallel}{\partial\psi}\right)$

$$\Rightarrow \frac{\partial V_\parallel}{\partial\psi} = F\left(\frac{\partial V_\parallel}{\partial\psi}, \Gamma\right)$$

- Highly nonlinear in $\frac{\partial V_\parallel}{\partial\psi}$
- Equations have up to **five** roots for $\frac{\partial V_\parallel}{\partial\psi}$
- Roots very sensitive to sources and boundary conditions



What is next?

Strong Gradient Transport Model

Self-Consistency

Radial electric field: replace assumption

$$Zen\frac{\partial \Phi}{\partial \psi} = \frac{\partial p}{\partial \psi}$$

by higher order calculation

Extensions

Impurity transport
Comparison with codes

Plateau regime
General shapes

Stability Analysis

Self-Consistency

Radial electric field: replace assumption

$$Zen\frac{\partial \Phi}{\partial \psi} = \frac{\partial p}{\partial \psi}$$

by higher order calculation

Purely neoclassical case: $\Gamma = \Gamma^{neo}$

1) Ion particle equation → $O(\sqrt{\epsilon\nu})$: $\Gamma = 0$ → density n
2) Ion parallel momentum equation → $O(\sqrt{\epsilon\nu})$: $\gamma = 0$ →$O(\epsilon\nu)$ → parallel flow $V_\|$
3) Ion energy equation → $O(\sqrt{\epsilon\nu})$: $Q = \cdots$ → temperature T

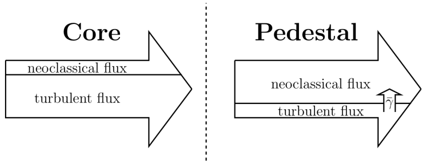
4) Electron particle equation → $O\left(\sqrt{\frac{m_e}{m_i}}\sqrt{\epsilon\nu}\right)$ → density n_e

5) Quasineutrality → $Zn = n_e$ → radial electric field u

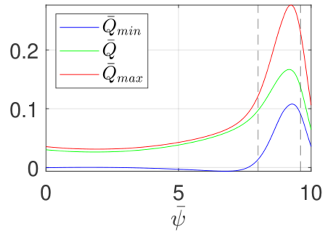
$$\boldsymbol{Sources \rightarrow Profiles}$$

Conclusion

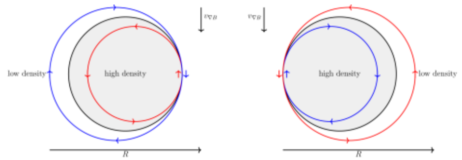
We extended neoclassical transport theory to regions of **strong gradients** $(\rho_p/L \sim 1)$ for large aspect ratio tokamaks



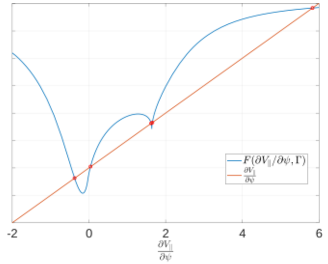
- A **non-zero neoclassical ion particle flux** requires a source of parallel momentum



- Bounds for the energy flux indicate **significance of radial electric field**



- **Poloidal variation** of the electric potential are captured and relevant for transport



- Solutions highly sensitive to **sources**

Our paper: JPP 89, 905890304 (2023)