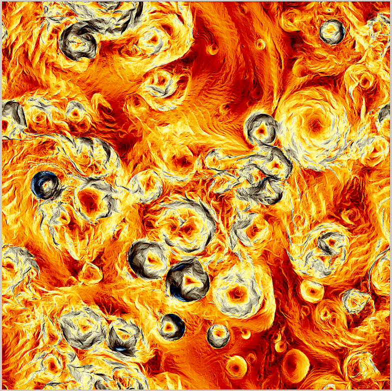
Reflection-driven turbulence in the super-Alfvénic solar wind



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Reflection-driven turbulence

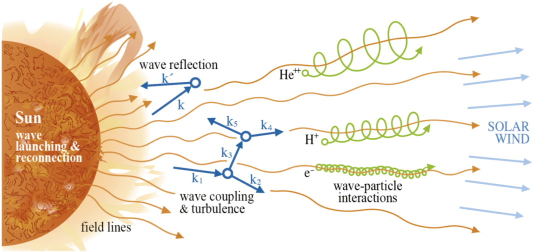


Figure: Image courtesy of B. Chandran

- The Sun launches Alfvén waves, which transport energy outwards
- The waves become turbulent due to wave reflections
- This causes energy to “cascade”, heating the plasma
- This increases the thermal pressure, which then accelerates the solar wind.

Expanding Box Model

We consider the turbulence dynamics in a frame co-moving with a spherically expanding flow.

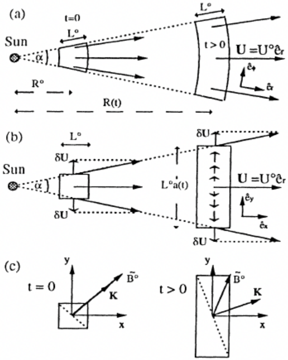


Figure: Grappin et al. 1993

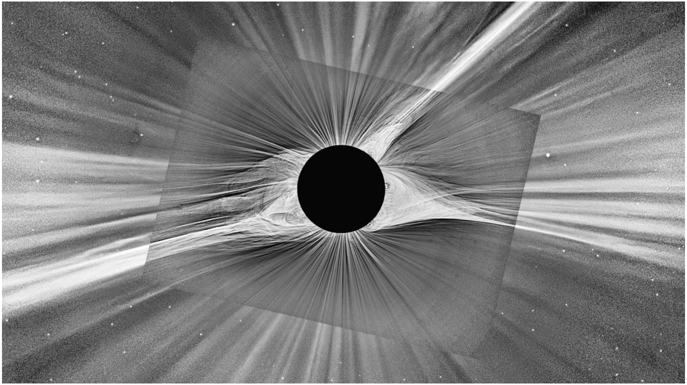
Simplest model imaginable:

- Fluctuations are transverse, non compressible
- Radial background magnetic field
- $k_{\perp}\rho_i \ll 1$
- U is radial, constant and $\gg V_A$
- All fields are 3D periodic

Expanding Box Model

'zeroth-order' questions:

- How fast various types of energy decay?
- How the outer scale evolves?



RMHD Expanding Box Model

$$\dot{a}\frac{\partial \tilde{\mathbf{z}}^{\pm}}{\partial a} \mp \frac{v_{A0}}{a}\frac{\partial \tilde{\mathbf{z}}^{\pm}}{\partial z} + \frac{1}{a^{3/2}}\left(\tilde{\mathbf{z}}^{\mp}\cdot\nabla_{\perp}\tilde{\mathbf{z}}^{\pm} + \frac{\nabla_{\perp}p}{\rho}\right) = -\frac{\dot{a}}{2a}\tilde{\mathbf{z}}^{\mp}$$

$$a(t) = \frac{R(t)}{R_0} = 1 + \dot{a}t, \quad \tilde{\mathbf{z}}^{\pm} \doteq a^{1/2}\boldsymbol{z}_{\perp}^{\pm} \propto \frac{\boldsymbol{z}_{\perp}^{\pm}}{\sqrt{\omega_{\mathrm{A}}}}$$

RMHD equations with two modifications :

- additional linear terms coupling counter-propagating Alfvénic perturbations: $-\frac{\dot{a}}{2a}\tilde{\mathbf{z}}^{\mp}$
- modified expression for the gradients accounting for the increasing lateral stretching of the plasma with distance: $\nabla_{\perp} \to \nabla_{\perp}/a$

Characteristic times scales

$$\dot{a}\frac{\partial \tilde{\mathbf{z}}^{\pm}}{\partial a} \mp \frac{v_{A0}}{a}\frac{\partial \tilde{\mathbf{z}}^{\pm}}{\partial z} + \frac{1}{a^{3/2}}\left(\tilde{\mathbf{z}}^{\mp}\cdot\nabla_{\perp}\tilde{\mathbf{z}}^{\pm} + \frac{\nabla_{\perp}p}{\rho}\right) = -\frac{\dot{a}}{2a}\tilde{\mathbf{z}}^{\mp}$$

- $\chi_{\mathrm{A}} = \dfrac{\text{Alfvénic}}{\text{nonlinear}}$
- $\chi_{\mathrm{exp}} = \dfrac{\text{expansion}}{\text{nonlinear}}$
- $\Delta = \dfrac{\text{expansion}}{\text{Alfvénic}} = \dfrac{\chi_{\mathrm{exp}}}{\chi_{\mathrm{A}}} = \text{const.}$

χ_{A} and χ_{exp} vary during the expansion and their relative values delimit different regimes through which the turbulence evolves during its radial transport.

Basic evolution

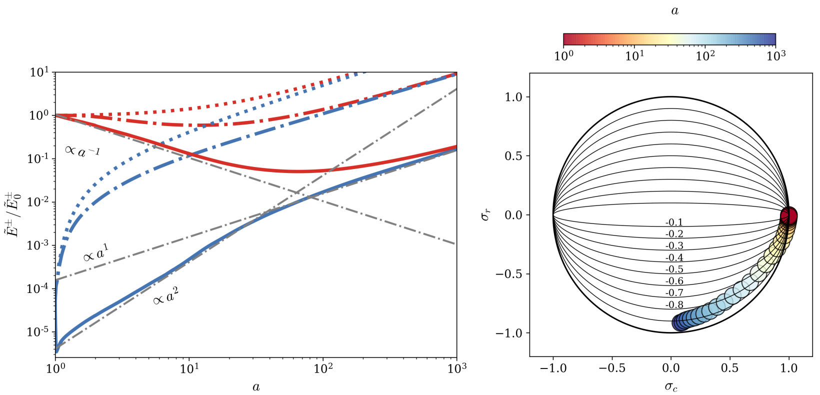
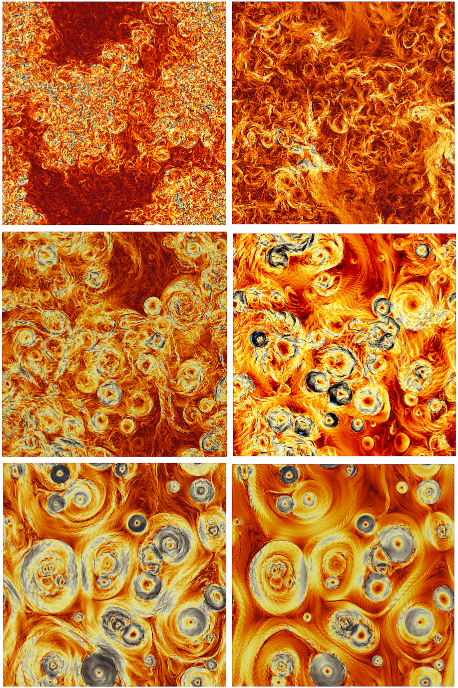


Figure: Left panel: Radial evolution of wave action energies $\tilde{E}^+$ (red lines) and $\tilde{E}^-$ (blue lines) for three simulations with different amplitude initial conditions. Right panel: Parametric representation of σ_r and σ_c during the evolution. The colors (on a logarithmic scale) indicate the normalized radial distance a. Solid lines represent contours of constant σ_θ.



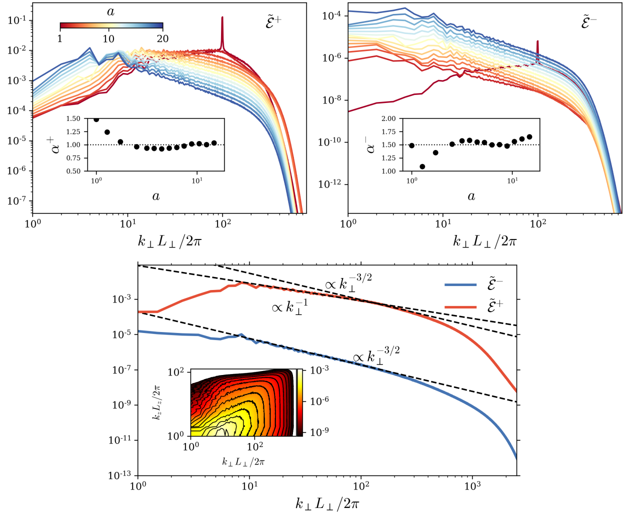


Figure: Wave-action energy spectra $\tilde{\mathcal{E}}^{\pm}(k_\perp)$ during the imbalanced phase of the simulation. The different colors show different time/radii, as indicated by the color bar. In each panel, the inset shows the best-fit power-law spectral slope.

Turbulent decay phenomenology

$$\boxed{\dot{a}\frac{\partial \tilde{E}^{\pm}}{\partial a} \sim -\frac{1}{a^{3/2}}\frac{\tilde{E}^{\pm}}{\tau^{\mp}} - \frac{\dot{a}}{a}\tilde{E}^{r}}$$

- Because $\tilde{E}^{+} \gg \tilde{E}^{r}$ when $\tilde{E}^{+} \gg \tilde{E}^{-}$, $\tilde{z}^{+}$ is decaying , while $\tilde{z}^{-}$ is forced by reflection and damped by turbulence.
- The $\tilde{z}^{-}$ fluctuations remain "anomalously coherent" with the $\tilde{z}^{+}$, because their forcing via reflection is highly coherent ($\propto -\tilde{\mathbf{z}}^{+}$) thus "dragging" $\tilde{z}^{-}$ along with the $\tilde{z}^{+}$ in time.
- The turbulent decay time $\tau^{\pm}$ of $\tilde{z}^{\pm}$ is strong for both field:

$$\tau_{\mp}^{-1} \sim a^{-3/2}\frac{\tilde{z}^{\pm}}{\tilde{\lambda}^{\pm}}$$

Turbulent decay phenomenology

$$\boxed{\dot{a}\frac{\partial \tilde{E}^{\pm}}{\partial a} \sim -\frac{1}{a^{3/2}}\frac{\tilde{E}^{\pm}}{\tau^{\mp}} - \frac{\dot{a}}{a}\tilde{E}^{r}}$$

- $\dot{a}\frac{\partial \tilde{E}^{+}}{\partial a} \sim -\frac{1}{a^{3/2}}\frac{\tilde{z}^{-}}{\tilde{\lambda}^{-}}\tilde{E}^{+}$
- $\frac{1}{a^{3/2}}\frac{\tilde{z}^{+}}{\tilde{\lambda}^{+}}\tilde{E}^{-} \sim \frac{\dot{a}}{a}|\tilde{E}^{r}| \sim \frac{\dot{a}}{a}|\sigma_{\theta}|\tilde{z}^{+}\tilde{z}^{-}, \qquad \tilde{z}^{-} \sim \dot{a}a^{1/2}\tilde{\lambda}^{+}|\sigma_{\theta}|$

$$\sigma_{\theta} \doteq \frac{\langle \tilde{\mathbf{z}}^{+} \cdot \tilde{\mathbf{z}}^{-} \rangle}{\langle |\tilde{\mathbf{z}}^{+}|^{2}\rangle^{1/2}\langle |\tilde{\mathbf{z}}^{-}|^{2}\rangle^{1/2}} = \frac{\sigma_{r}}{\sqrt{1-\sigma_{c}^{2}}},$$

$$\sigma_{c} \doteq \frac{\tilde{E}^{+} - \tilde{E}^{-}}{\tilde{E}^{+} + \tilde{E}^{-}} = \frac{\tilde{E}^{c}}{\tilde{E}}$$

$$\sigma_{r} \doteq \frac{\tilde{E}^{u} - \tilde{E}^{b}}{\tilde{E}^{u} + \tilde{E}^{b}} = \frac{\tilde{E}^{r}}{\tilde{E}}$$

Turbulent decay phenomenology

- $\dot{a}\dfrac{\partial \tilde{E}^+}{\partial a} \sim -\dfrac{1}{a^{3/2}}\dfrac{\tilde{z}^-}{\tilde{\lambda}^-}\tilde{E}^+$
- $\tilde{z}^- \sim \dot{a}a^{1/2}\tilde{\lambda}^+|\sigma_\theta| \Rightarrow \tilde{z}^- \sim \tilde{z}^+/\chi_{\rm exp}$

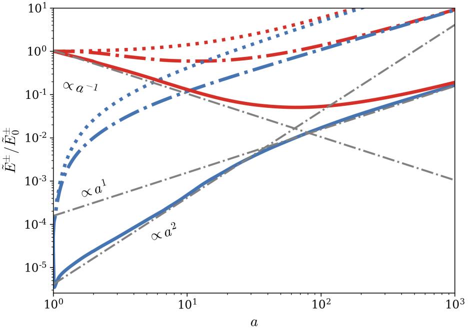
$$\boxed{\frac{\partial \ln \tilde{E}^+}{\partial a} \sim -\frac{1}{a}\frac{\tilde{\lambda}^+}{\tilde{\lambda}^-}\sigma_\theta}$$

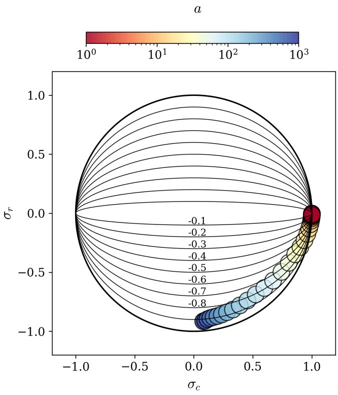
The anomalous coherence will break down once $\tilde{z}^-$ enters the weak regime (in which case $\tilde{z}^-$ can propagate away from the source). The phenomenology thus requires

$$\chi_{\rm A} \doteq \frac{(\tau_-)^{-1}}{v_{\rm A}/\ell_{\|}} \sim \frac{\tilde{z}^+/\tilde{\lambda}^+}{a^{1/2}v_{\rm A0}/\ell_{\|}} \gtrsim 1$$

The phenomenology can only be valid for sufficiently large-amplitude $\tilde{z}^+$ with $\chi_{\rm exp} \gg 1$ irrespective of the fluctuation's parallel scale, and we expect the transition to the balanced regime to occur when $\tilde{E}^+$ decays sufficiently so that $\chi_{\rm exp} \sim 1$.

Basic evolution





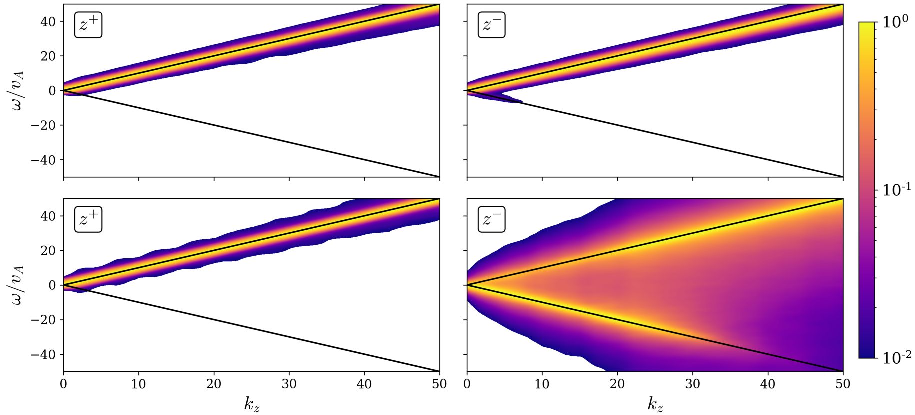
$$\sigma_\theta \tilde{\lambda}^+ / \tilde{\lambda}^- \approx 1 \Rightarrow \tilde{E}^+ \propto a^{-1}$$

$$\sigma_\theta \tilde{\lambda}^+ \propto a^{1/2}$$

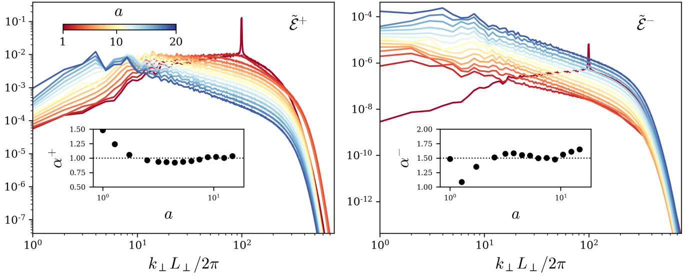
Anomalous coherence

Space-time Fourier spectrum:

$$\tilde{\mathcal{E}}^{\pm}(k_z,\omega) = \frac{1}{2}\left\langle |\hat{\tilde{\boldsymbol{z}}}^{\pm}(k_z,\omega)|^2 \right\rangle_{\perp}$$



The growth of $\tilde{L}_+$



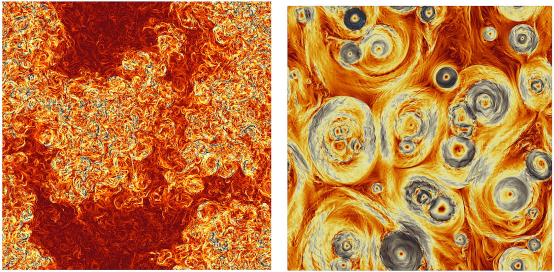
- $\tilde{\mathcal{E}}^+(k_\perp)$ migrate towards large scales during the expansion. As this occurs, $\tilde{\mathcal{E}}^+$ develops a wide $\tilde{\mathcal{E}}^+ \propto k_\perp^{-1}$ range.
- The evolution of $\tilde{\mathcal{E}}^-(k_\perp)$ is quite different, rapidly moving to large scales at very early times.

Anomalous conservation of anastrophy

In the nonlinear regime, the weaker field is born coherent and is short-lived. It doesn't propagate against but with the stronger field. The dynamic is effectively 2D like which suggest that anastrophy $\langle A_z^2 \rangle$ is anomalously conserved.

$$\langle A_z^2 \rangle \sim a^{-1} \tilde{E}^{+} \tilde{L}_{+}^2 \sim \text{const.} \implies \tilde{L}_{+} \propto a$$

The turbulent decay must progress with $\tilde{L}_{+}$ increasing rapidly in time.



Anomalous conservation of anastrophy

We compute the parametric representation,

$$X(a) = -\frac{\partial \ln \tilde{z}^+_{\rm rms}(a)}{\partial \ln a}, \quad Y(a) = \frac{\partial \ln \tilde{L}_+(a)}{\partial \ln a},$$

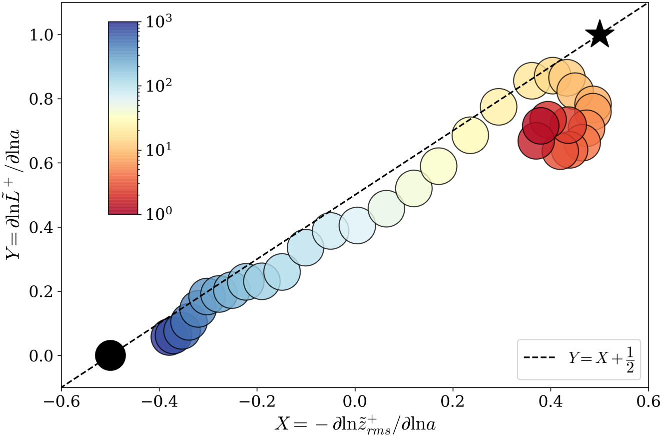
where $\tilde{z}^+_{\rm rms} = \sqrt{2\tilde{E}^+}$ and $\tilde{L}_+$ is computed as

$$\tilde{L}_+ \equiv \int dk_\perp \tilde{\mathcal{E}}^+(k_\perp)/k_\perp.$$

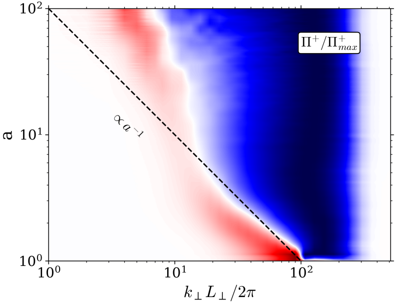
$X(a)$ and $Y(a)$ are the instantaneous scaling exponents of $1/\tilde{z}^+_{\rm rms}$ and $\tilde{L}_+$. If anastrophy is conserved then

$$Y(a) = X(a) + \frac{1}{2}.$$

Anomalous conservation of anastrophy



The split cascade

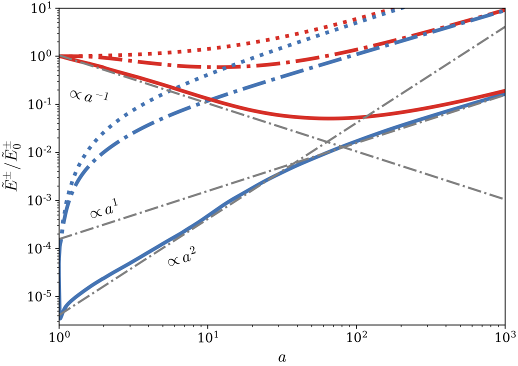


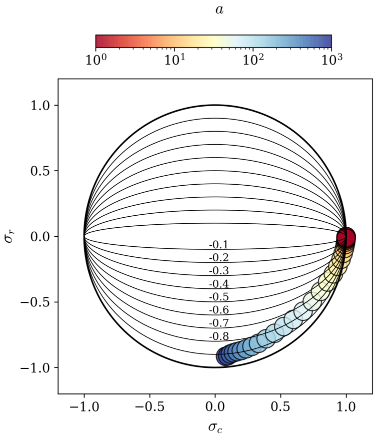
$$\Pi^{\pm}(k_\perp) = a^{-3/2}\frac{2\pi}{L_\perp}\int \frac{d^3\boldsymbol{r}}{V}\left[\tilde{\mathbf{z}}^{\pm}\right]^{<}_{k_\perp}\cdot(\tilde{\mathbf{z}}^{\mp}\cdot\tilde{\nabla}_\perp\tilde{\mathbf{z}}^{\pm}),$$

where the low-pass filter is defined by

$$\left[\tilde{\mathbf{z}}^{\pm}\right]^{<}_{k_\perp} = \sum_{k'_z}\sum_{|\boldsymbol{k}'_\perp|\le k_\perp} e^{i\boldsymbol{k}'\cdot\boldsymbol{r}}\tilde{\mathbf{z}}^{\pm}_{\boldsymbol{k}}.$$

Balanced, magnetically dominated phase





Linear Dynamics

Eigenmodes

$$\xi^{\pm} = \frac{1}{2}\tilde{z}_w^{\pm} \pm i\left(\Delta \mp \omega^{\pm}\right)\tilde{z}_w^{\mp}$$

$$\omega^{\pm} = \pm\sqrt{\Delta^2 - 1/4}$$

- For $\Delta > 1/2$, $\omega^{\pm}$ is real and $\tilde{z}^{\pm}$ oscillates with frequency $\omega^{\pm}$.
- For $\Delta < 1/2$, $\omega^{\pm}$ is imaginary and modes grow exponentially, $\tilde{z}^{\pm} \propto e^{|\omega^{\pm}|\ln a} = a^{|\omega^{\pm}|} = a^{\sqrt{1-4\Delta^2}/2}$.
  The growing expansion-dominated mode, with $\omega = i\sqrt{1/4 - \Delta^2}$, is magnetically dominated with $\tilde{\mathbf{z}}^- \approx -\tilde{\mathbf{z}}^+$ and $|\tilde{\boldsymbol{b}}_\perp| \gg |\tilde{\boldsymbol{u}}_\perp|$

Linear Dynamics

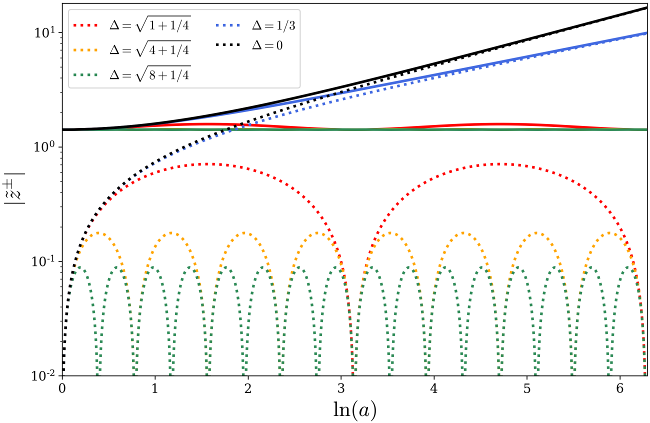


Figure: Solutions of the linearised equations, starting from the initial condition $\tilde{z}^{-}(0) = 0$ and $\tilde{z}^{+}(0) = \sqrt{2}$ with different values of Δ as labelled. Solid lines show $|\tilde{z}^{+}(a)|$; dotted lines show $|\tilde{z}^{-}(a)|$.

Emergence of Alfvén vortices

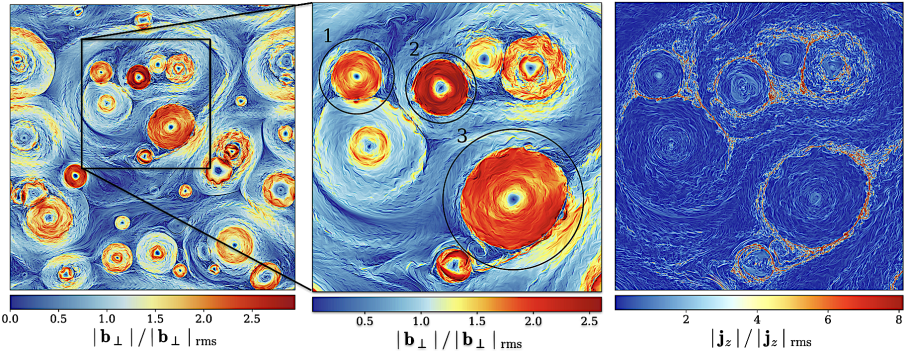


Figure: Left: Snapshot of the magnetic field modulus in a plane perpendicular to B_0 at $a = 250$. Middle: Close-up corresponding to the marked region on the left, illustrating Alfvén vortices colliding and and merging through reconnection. Right: Same region as the middle panel, but showing the out-of-plane current.

Emergence of Alfvén vortices

We start from the variational problem

$$\delta \int d^3\boldsymbol{r} \left(|\tilde{\nabla}_\perp \tilde{A}_z|^2 - \Lambda \tilde{A}_z^2 \right) = 0,$$

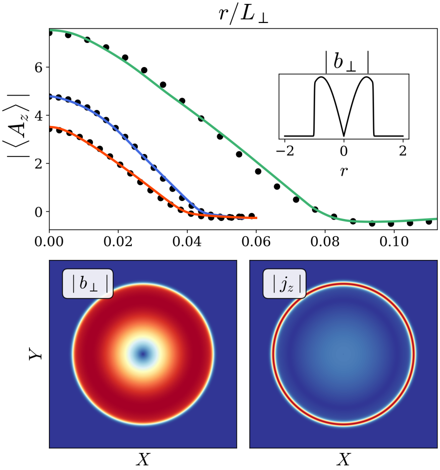
where δ denotes the functional derivative and Λ a Lagrangian multiplier. Identifying Λ with a characteristic scale $K_\perp$ via $\Lambda = -K_\perp^2$, the Euler-Lagrange equation becomes the Helmholtz equation.

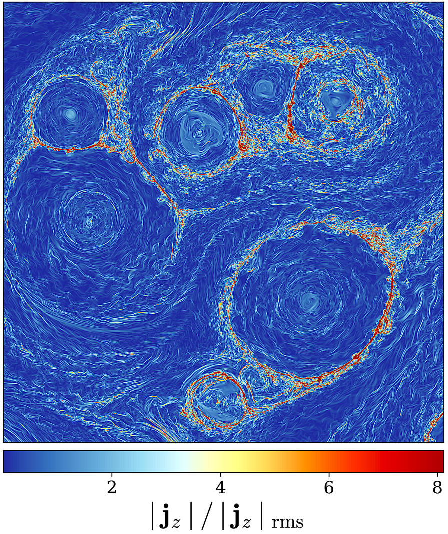
$$\tilde{\nabla}_\perp^2 \tilde{A}_z = -K_\perp^2 \tilde{A}_z.$$

$$\begin{cases} \tilde{A}_z(r) = A_0 J_0(K_\perp r), & r < r_c \\ \tilde{A}_z(r) = A_B, & r \geq r_c \end{cases}$$

The solution corresponds to a particular case of so-called Alfvén vortex solutions, the vortex monopole. The equilibrium is effectively a screw pinch, with its nonlinear equilibrium resulting from the balance between the curvature/tension force and the pressure gradient.

Emergence of Alfvén vortices





Conclusions

In its simplest form, the reflection-driven turbulence may explain essential features of the solar wind:

- Double power law at intermediate and large scales, with power indices -3/2 and -1 respectively
- Bi-directional Elsasser cascades in highly imbalance streams.
- Formation of Alfvén vortices.
- Generation of high negative residual energy states.

Thank you for your attention

Figure: Courtesy of Maxwell Busby