Wave interactions and turbulence in collisionless, high-β plasmas

Stephen Majeski (Advised by M. Kunz, with J. Squire)

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Prerequisites:

- Double adiabats and pressure anisotropy
- Acoustic-Alfvén wave interactions (high β)

Turbulence ($k \ll \rho_i^{-1}$):

- Low-moderate beta
- Magneto-immutability
- High beta (10-100)

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New to this talk

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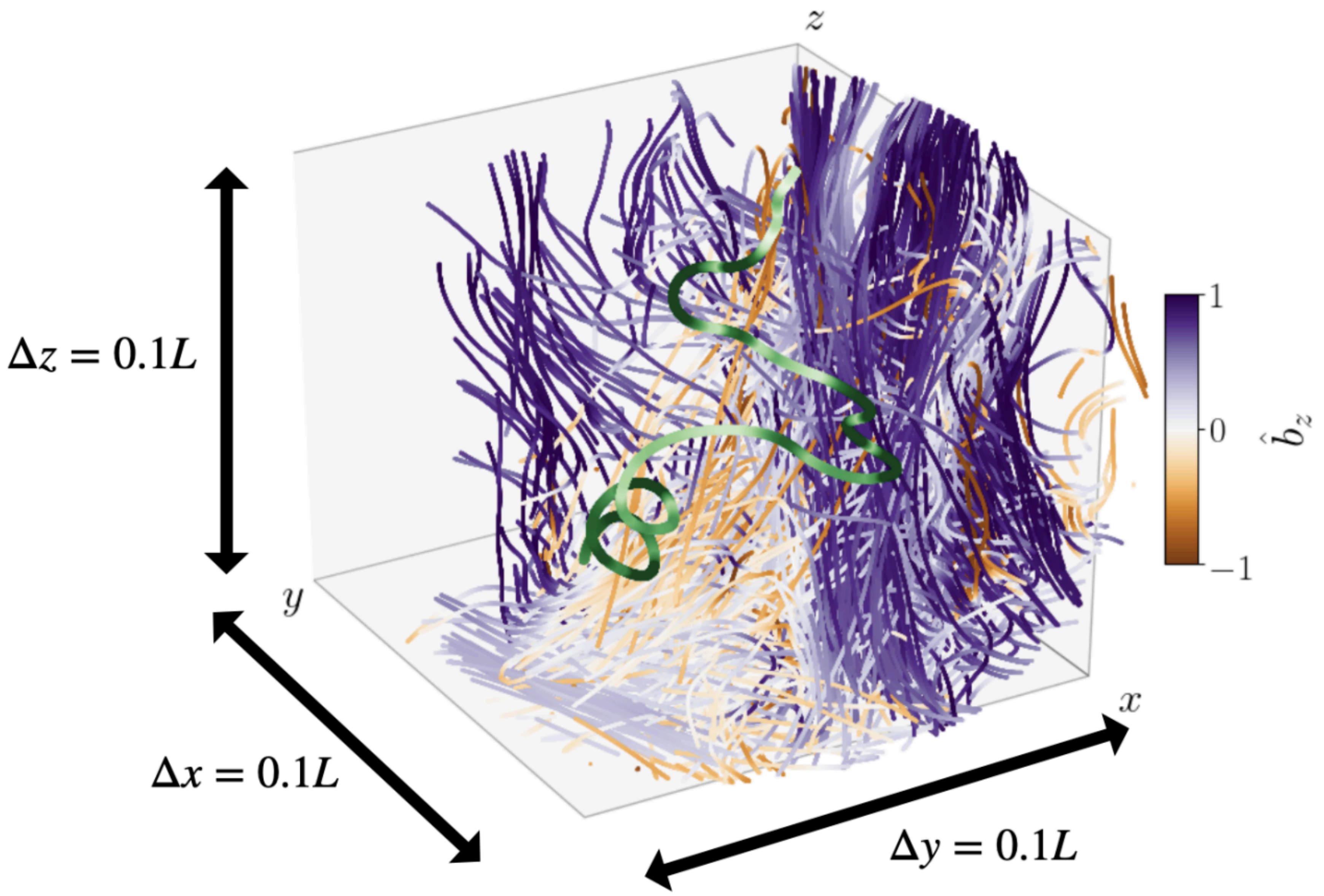
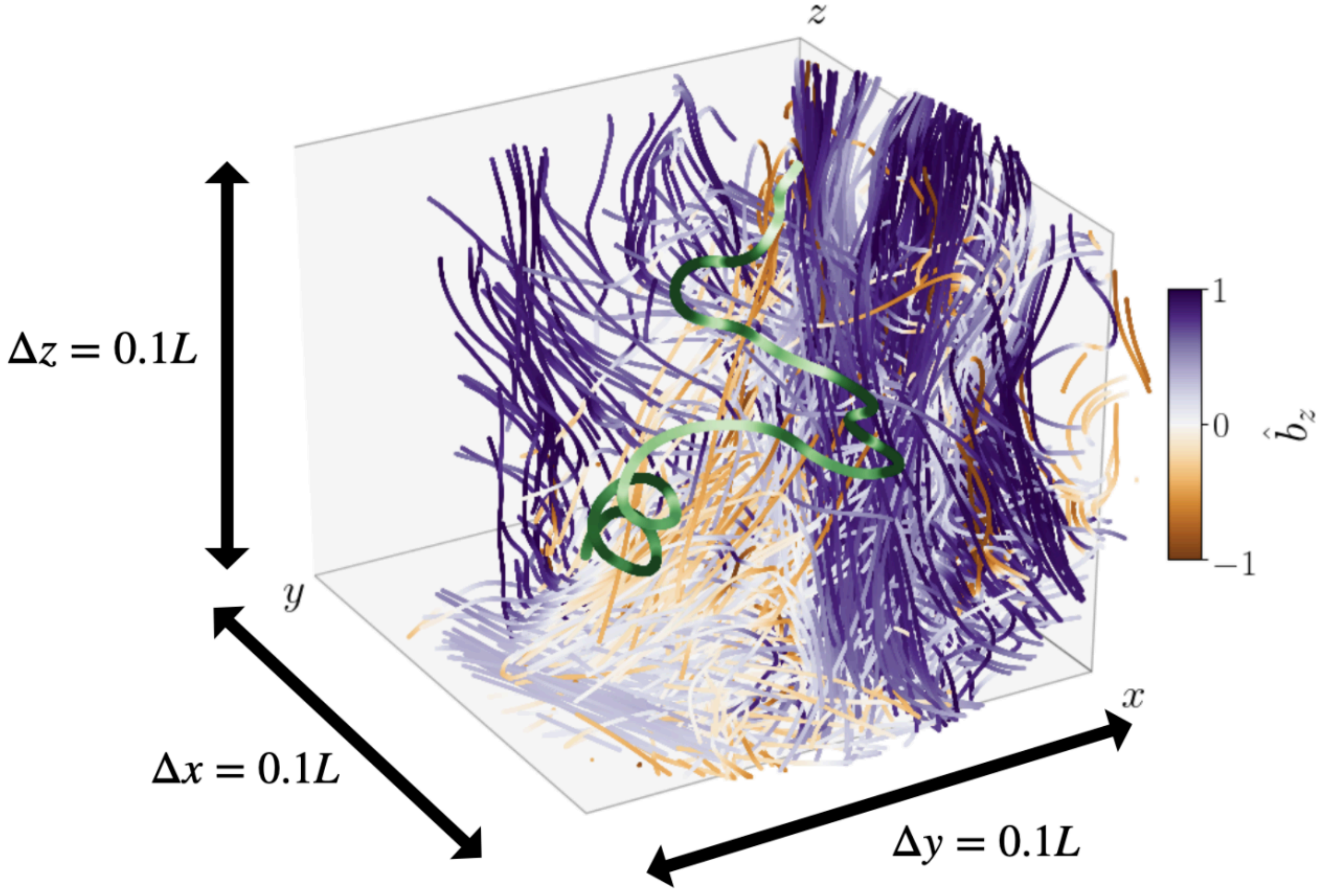
All simulations: $T_e = 0$



Cosmic ray transport by turbulent B fields (Kempskii et al '23)



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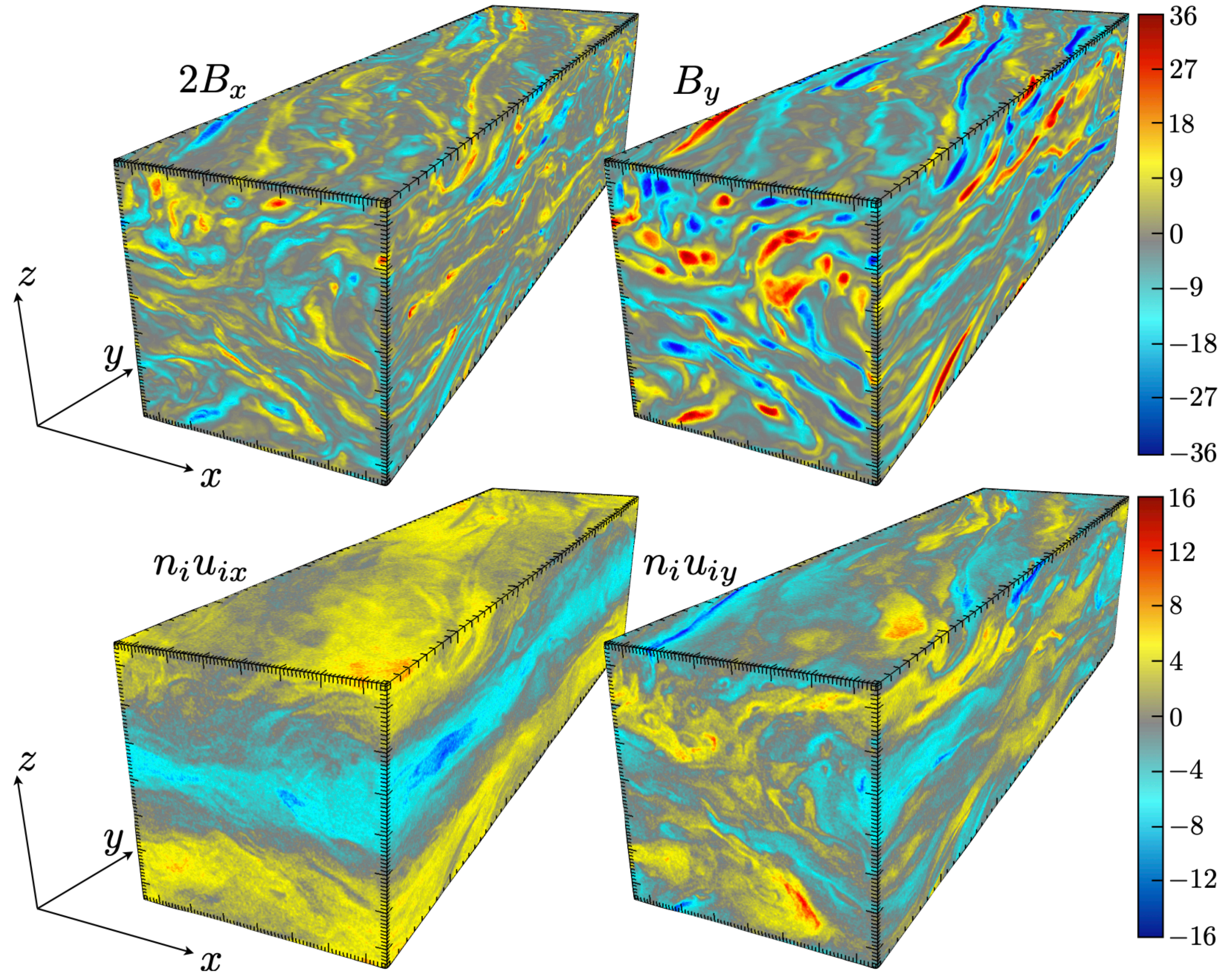

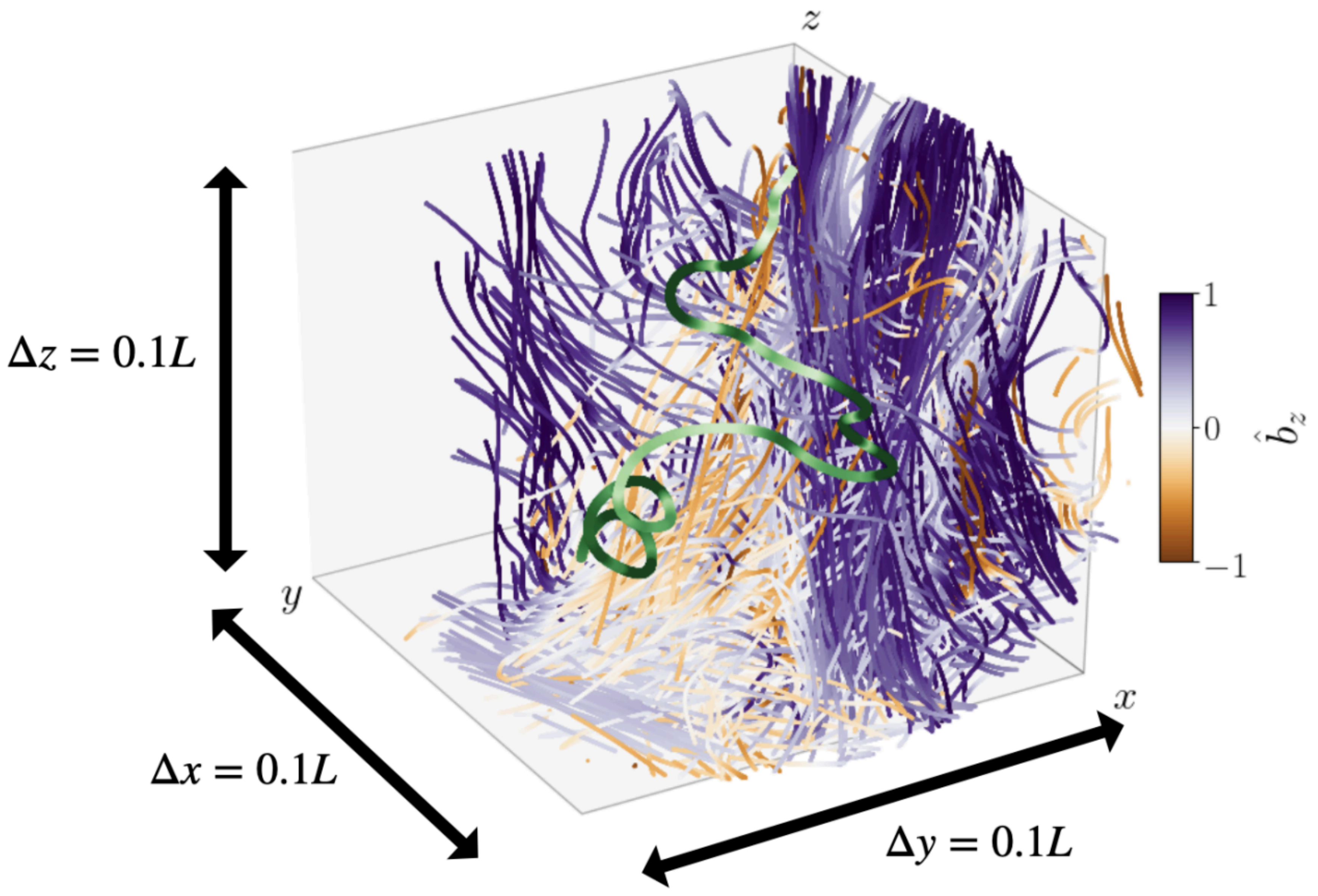

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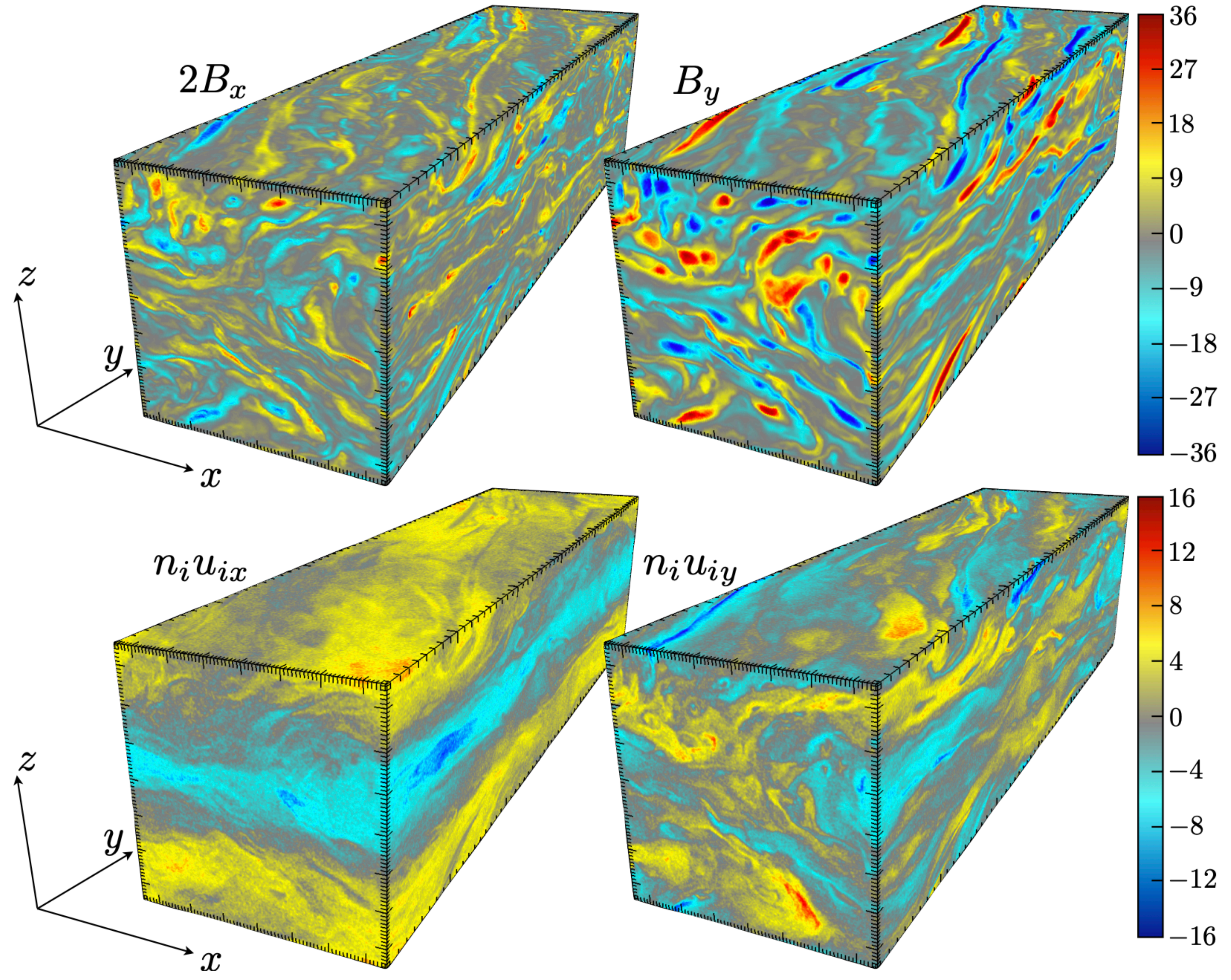
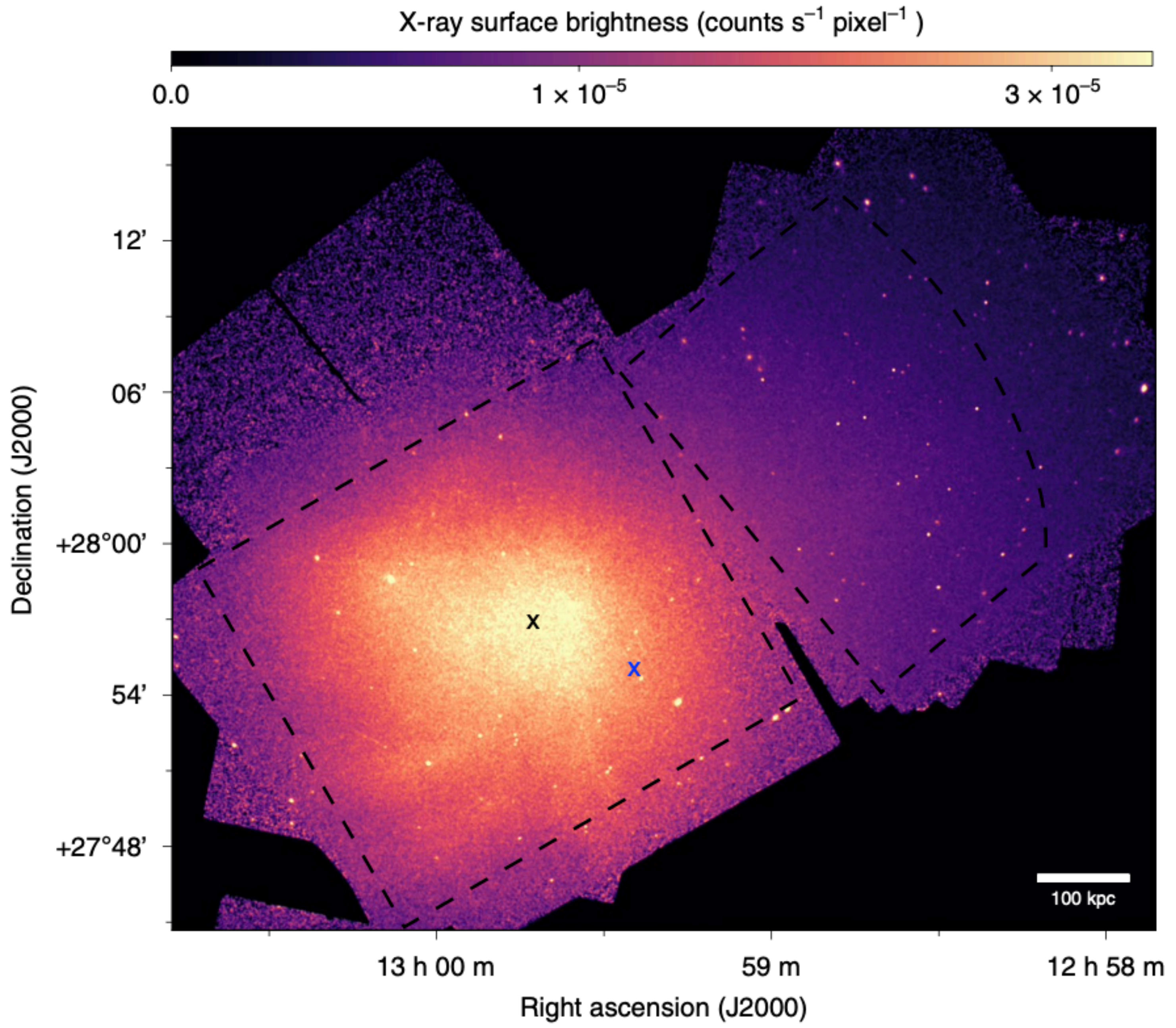




Coma density fluctuations defy viscous scale (Zhuravleva et al ’19)

Prerequisites

Double adiabats:

Individual particles:

$$\mu = \frac{mv_\perp^2}{2B}$$

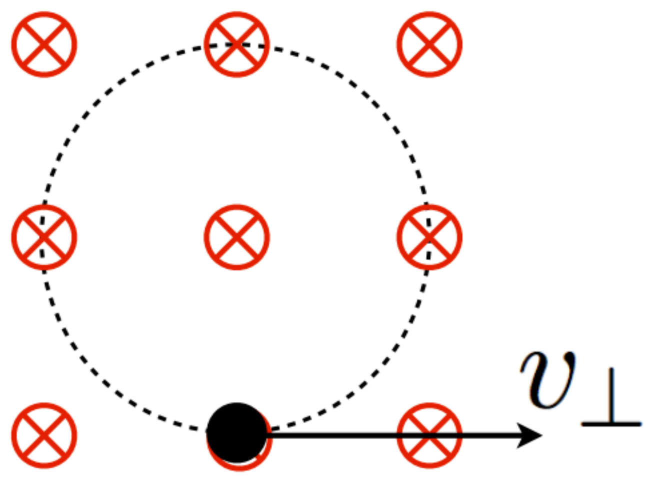
$$\mathscr{J} = \oint v_\| \mathrm{d}s$$

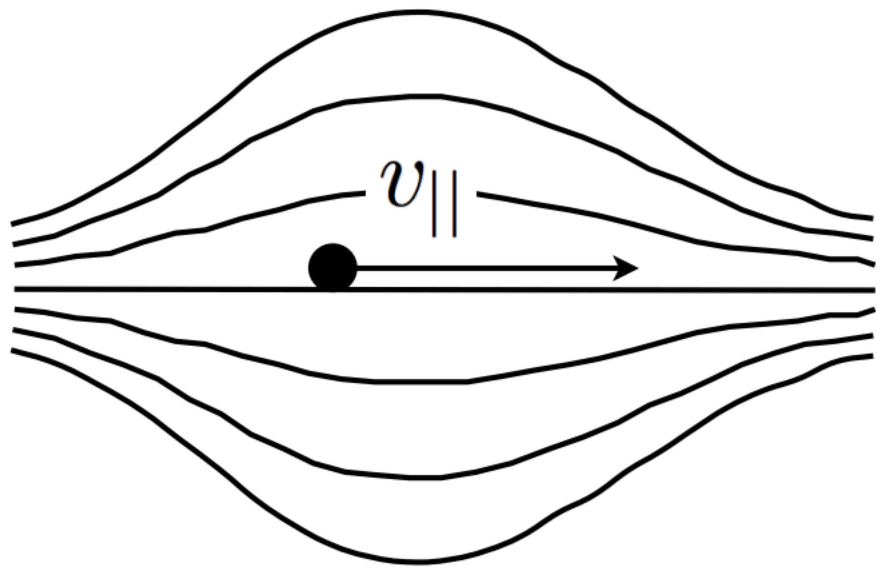
↓ ↓

Bulk plasma:

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{p_\perp}{nB}\right) = 0$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{p_\| B^2}{n^3}\right) = 0$$





(Swiped from Matt Kunz)

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$\searrow$ $\swarrow$

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→ Requires well magnetized, weakly collisional plasmas (with no ρ_i or Ω_i scale variation)

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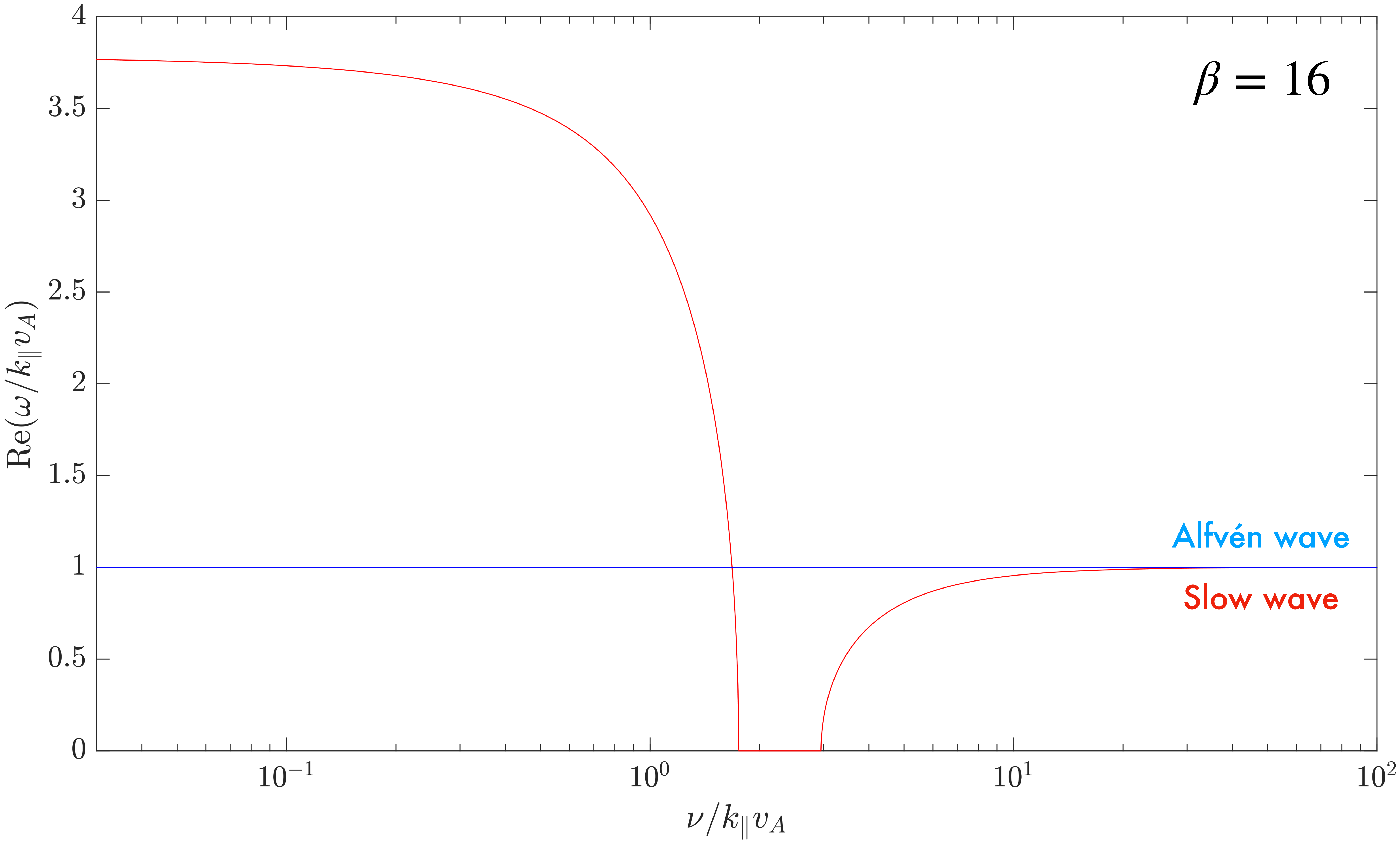
Realistically (heat fluxes, scattering from Coulomb...) → CGL-MHD

MHD


$\beta = 16$
Alfvén wave
Slow wave
$\text{Re}(\omega/k_\parallel v_A)$
$\nu/k_\parallel v_A$
0
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1.5
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3
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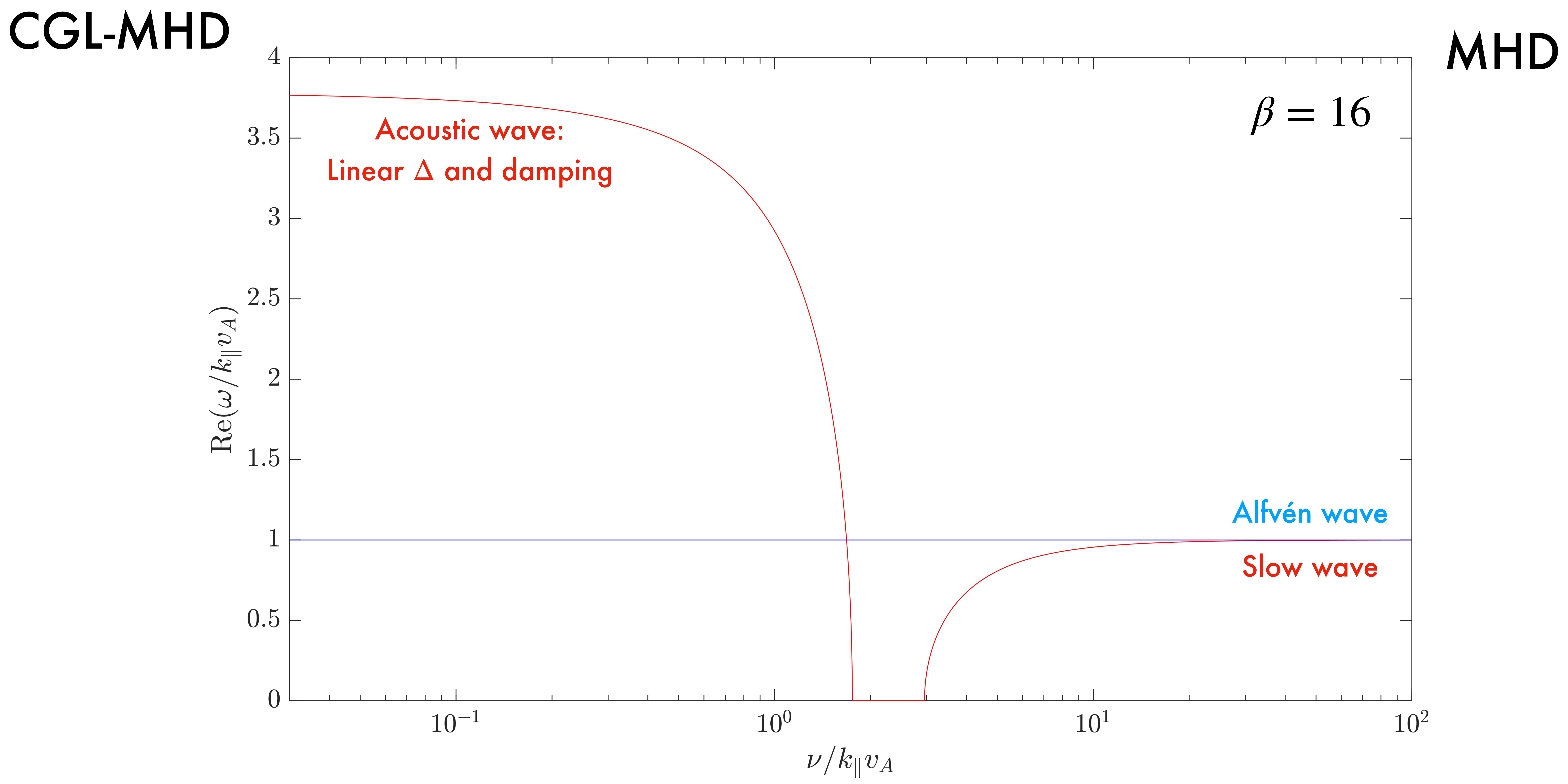
4
10^{-1}
10^0
10^1
10^2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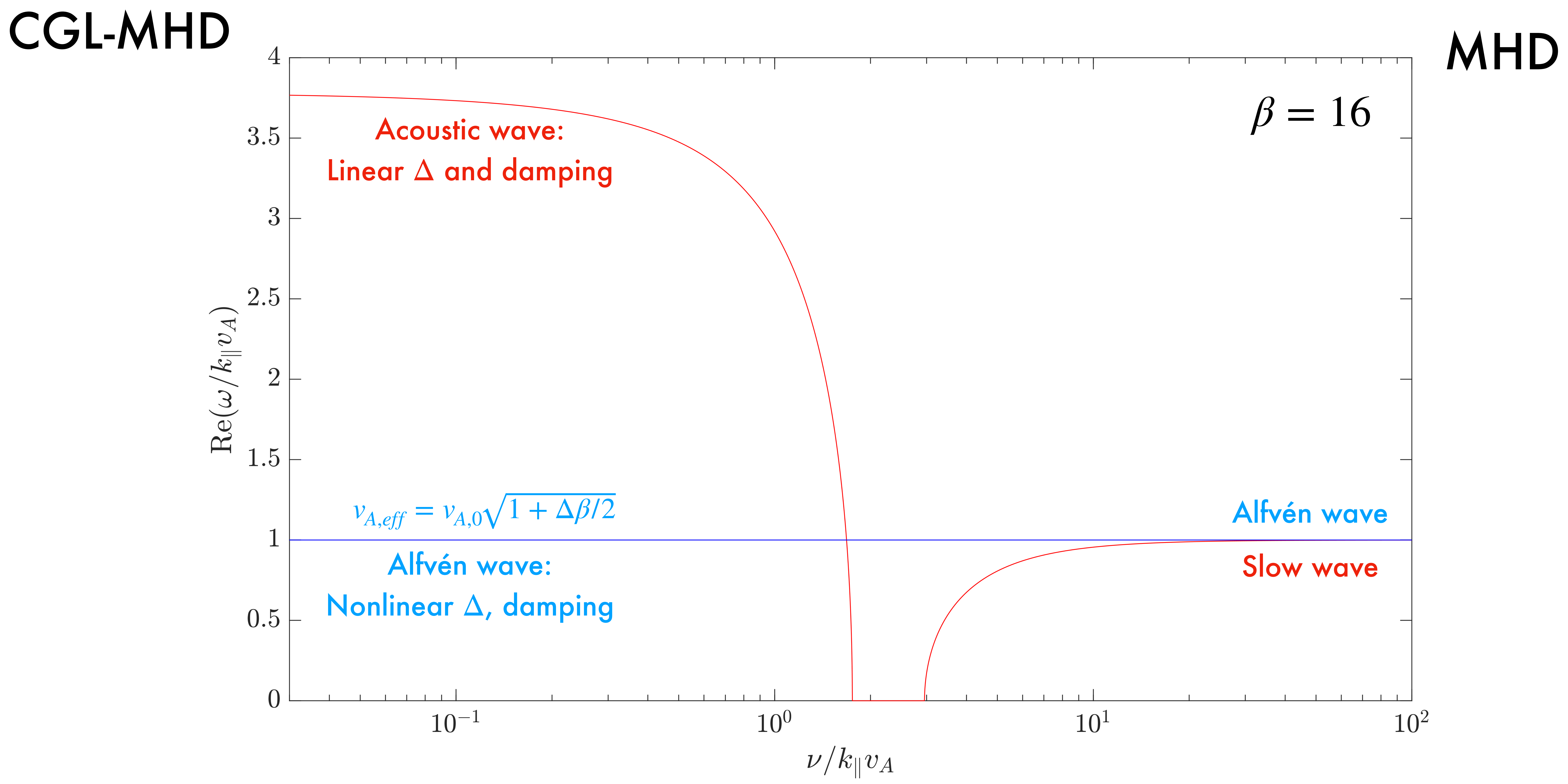
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$$\Delta\beta \sim 1$$

(Sets IA amplitude)

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$$\Delta\beta \sim 1 \quad \longrightarrow \quad \frac{\delta B_{\|}}{B_0} \sim \frac{\delta B_{\perp}}{B_0} \sim \frac{1}{\beta} \sim \Delta \sim \frac{\delta\rho}{\rho_0} \sim \frac{u_{\perp}}{v_A} \sim \frac{u_{\|}}{v_{th}} \sim \epsilon$$

(considering the waves as linear)

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Ordering 1D CGL-MHD... $\downarrow$ $z^\pm = u_\perp \pm \delta B_\perp/\sqrt{\rho_0}$

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$+$ Linear ion acoustic waves

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- AWs do not affect IAs (without going $\beta^{-3/2}$ further down in order)

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Linear ion acoustic waves

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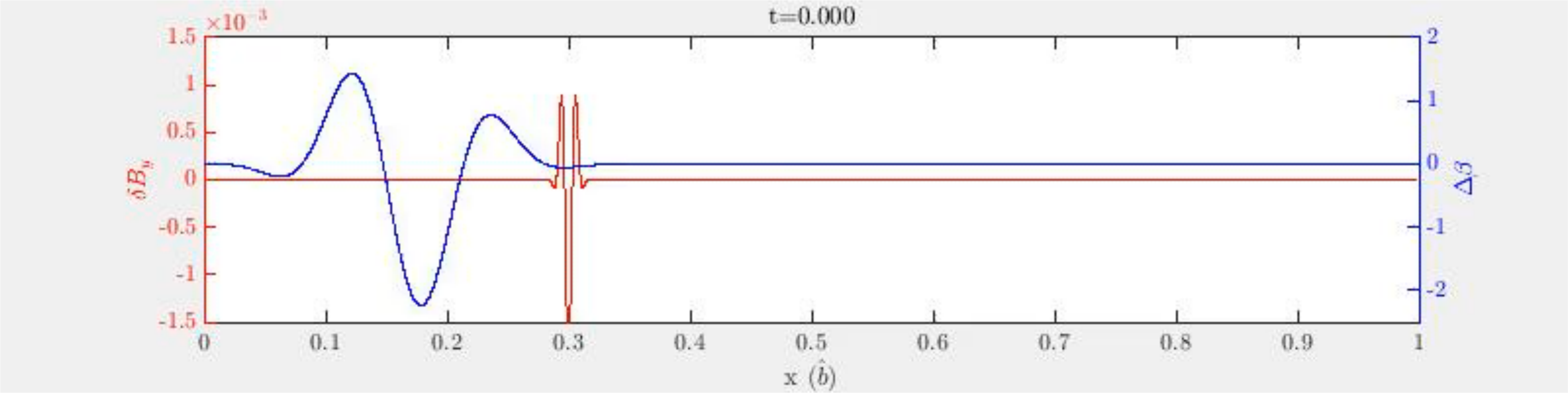
$\beta = 400$



Predicted by solution of a single IA interacting with a single AW.

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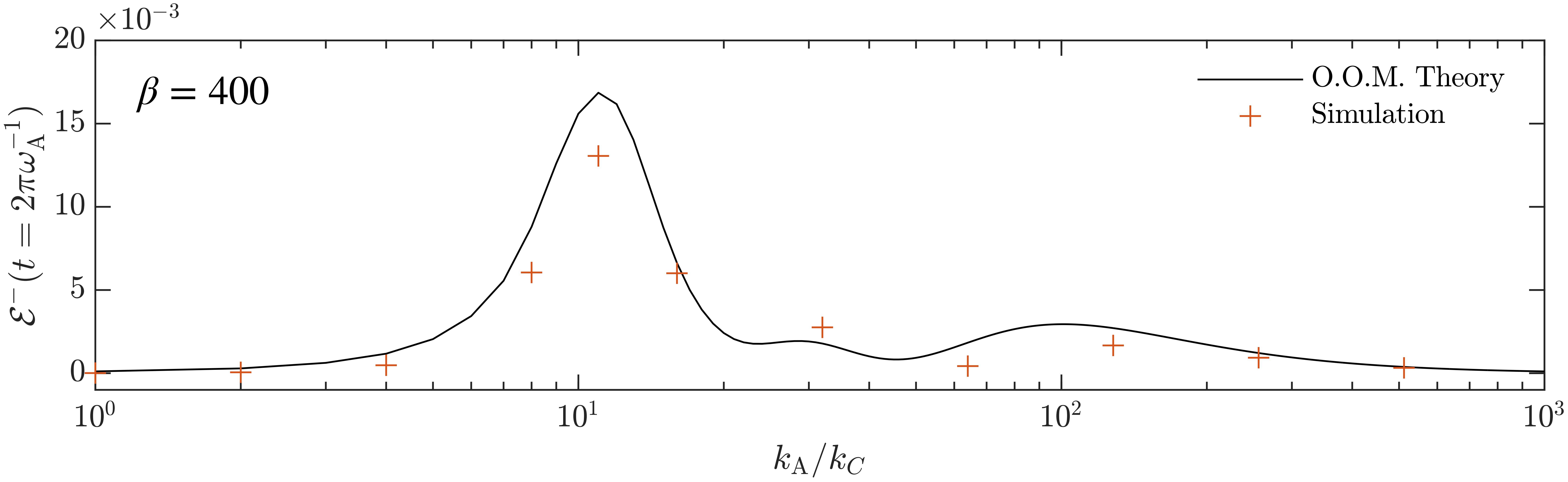

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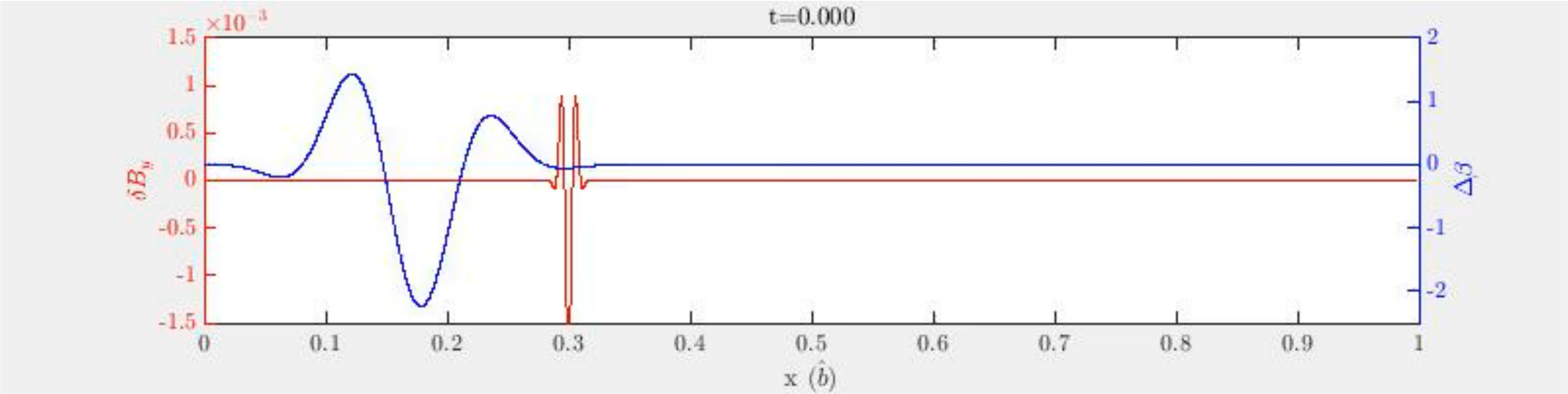

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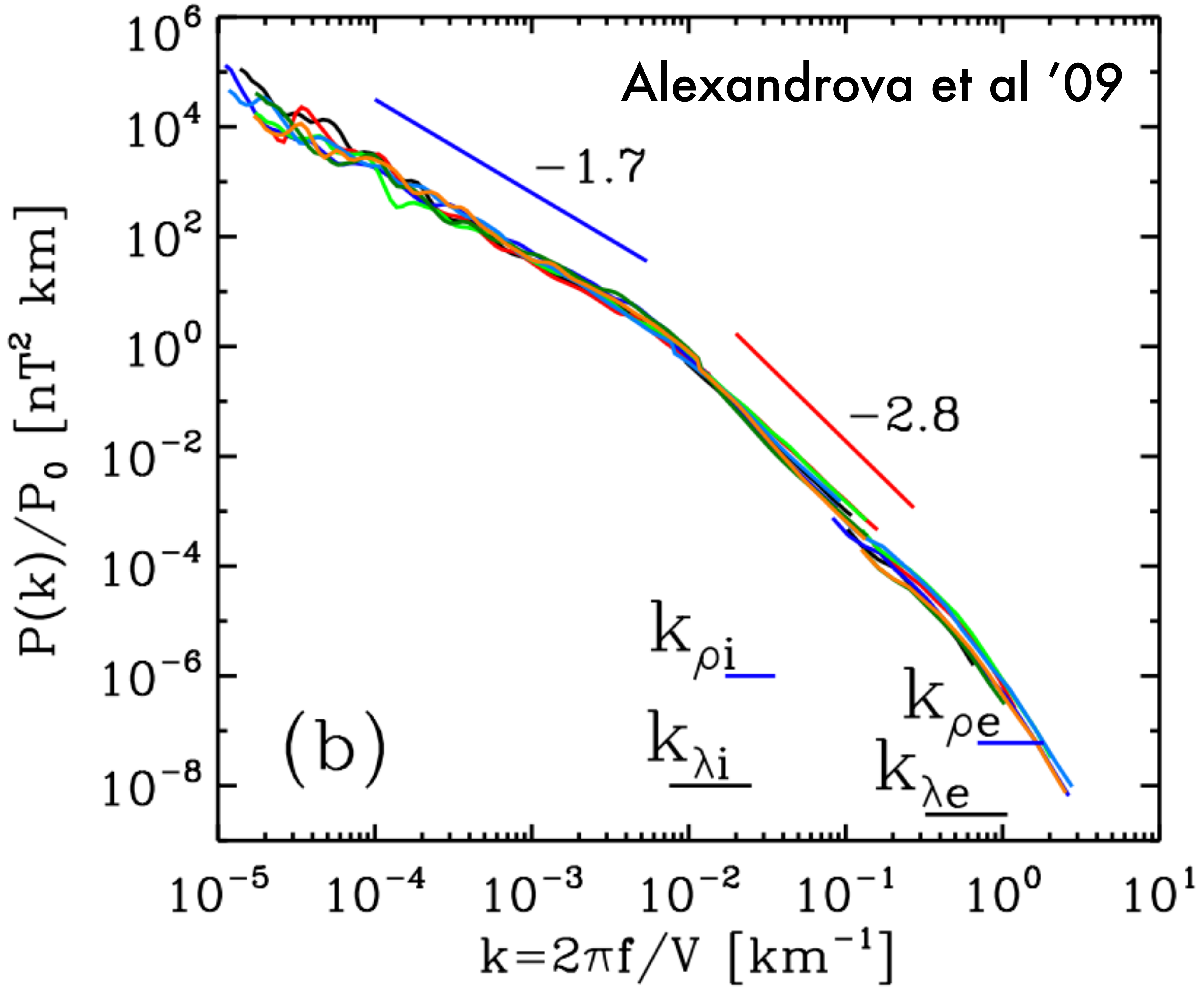
MHD:

AWs are unaffected by slow modes

Slow modes mixed by $\delta B_\perp$ while propagating otherwise linearly

Turbulence

Low-Moderate β, Coll'less:



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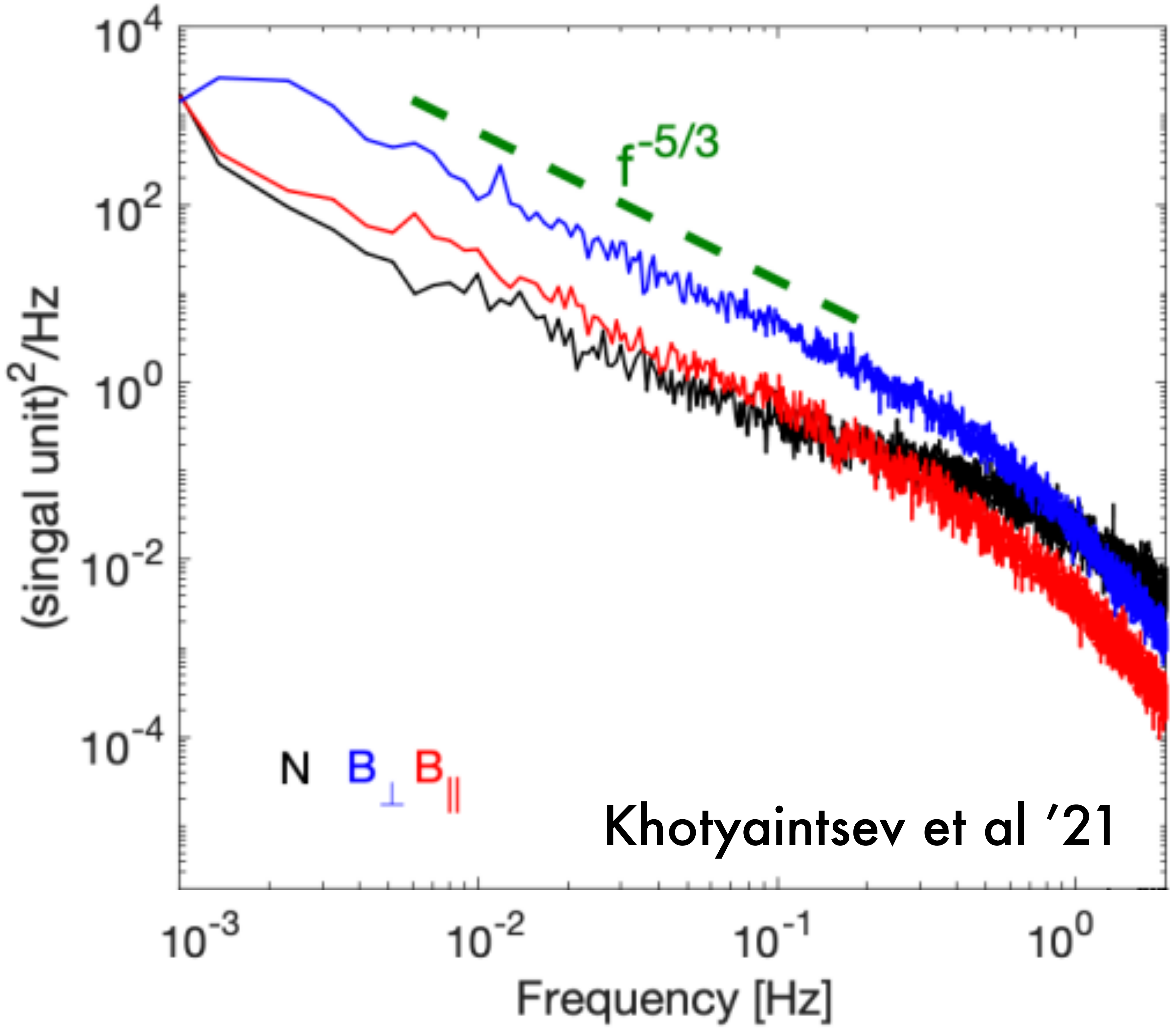

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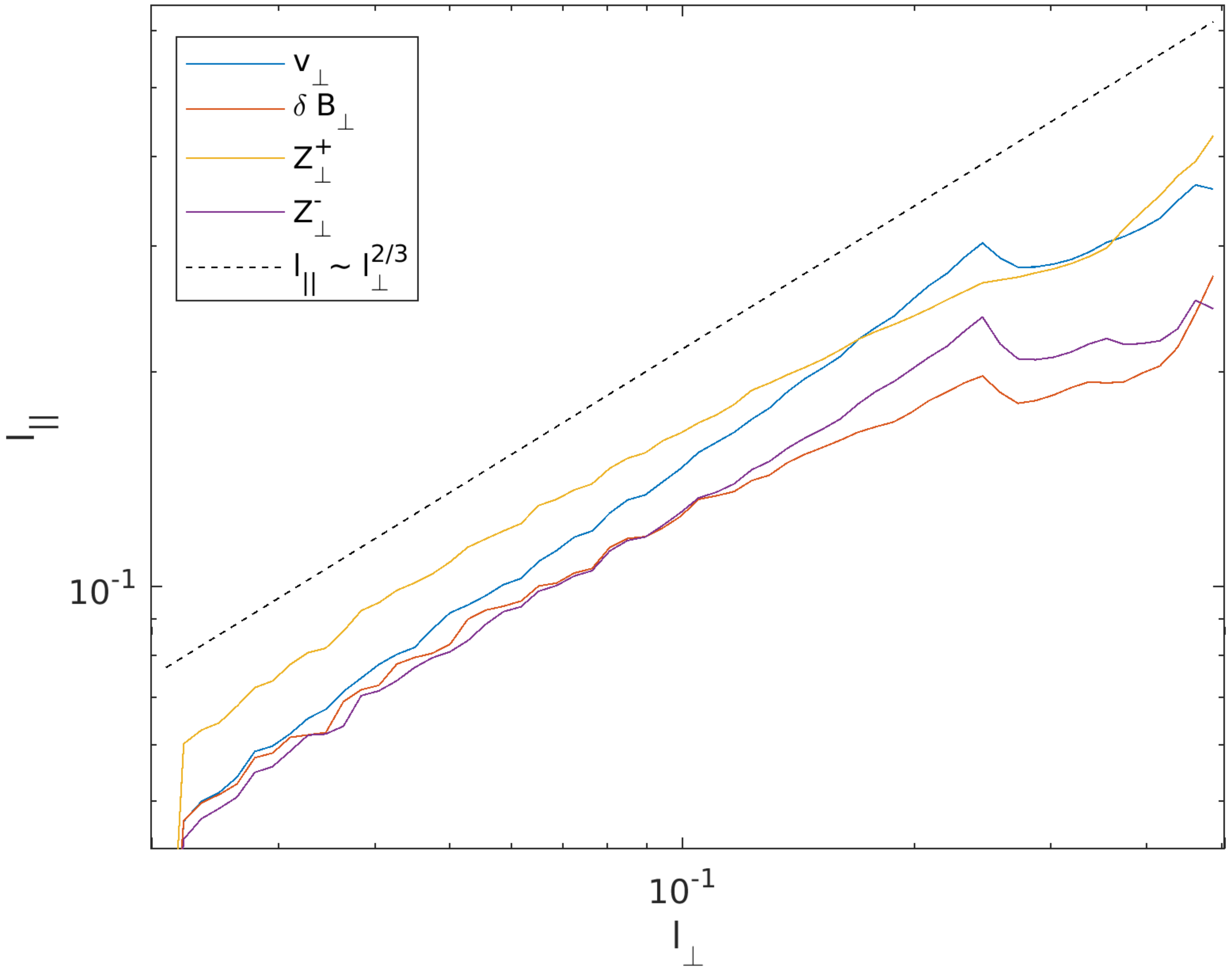

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Low-Moderate β, Coll'less:

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→ Depending on forcing, $\Delta\beta < 1$ so $v_{A,eff} \approx v_{A,0}$

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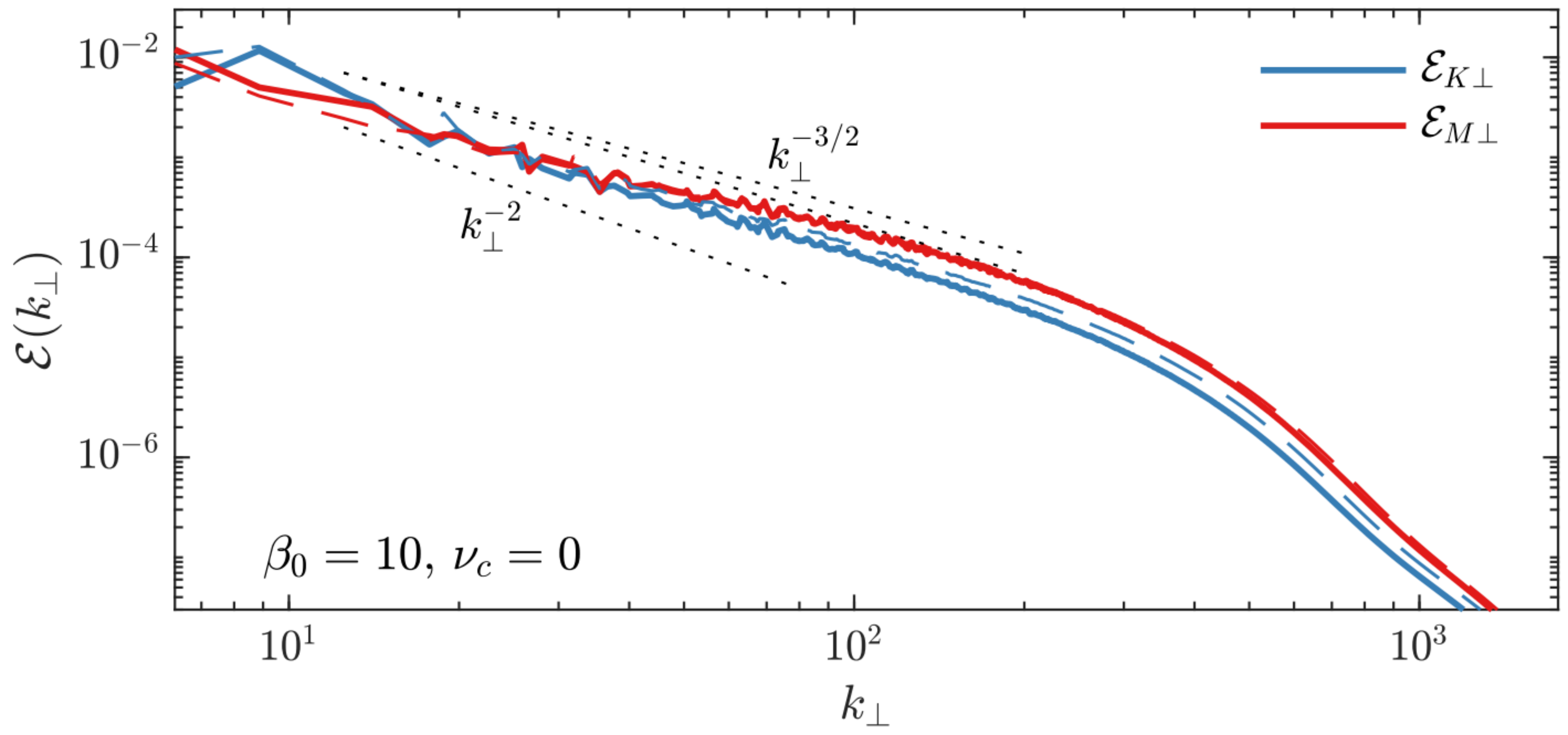

(Squire et al 23)

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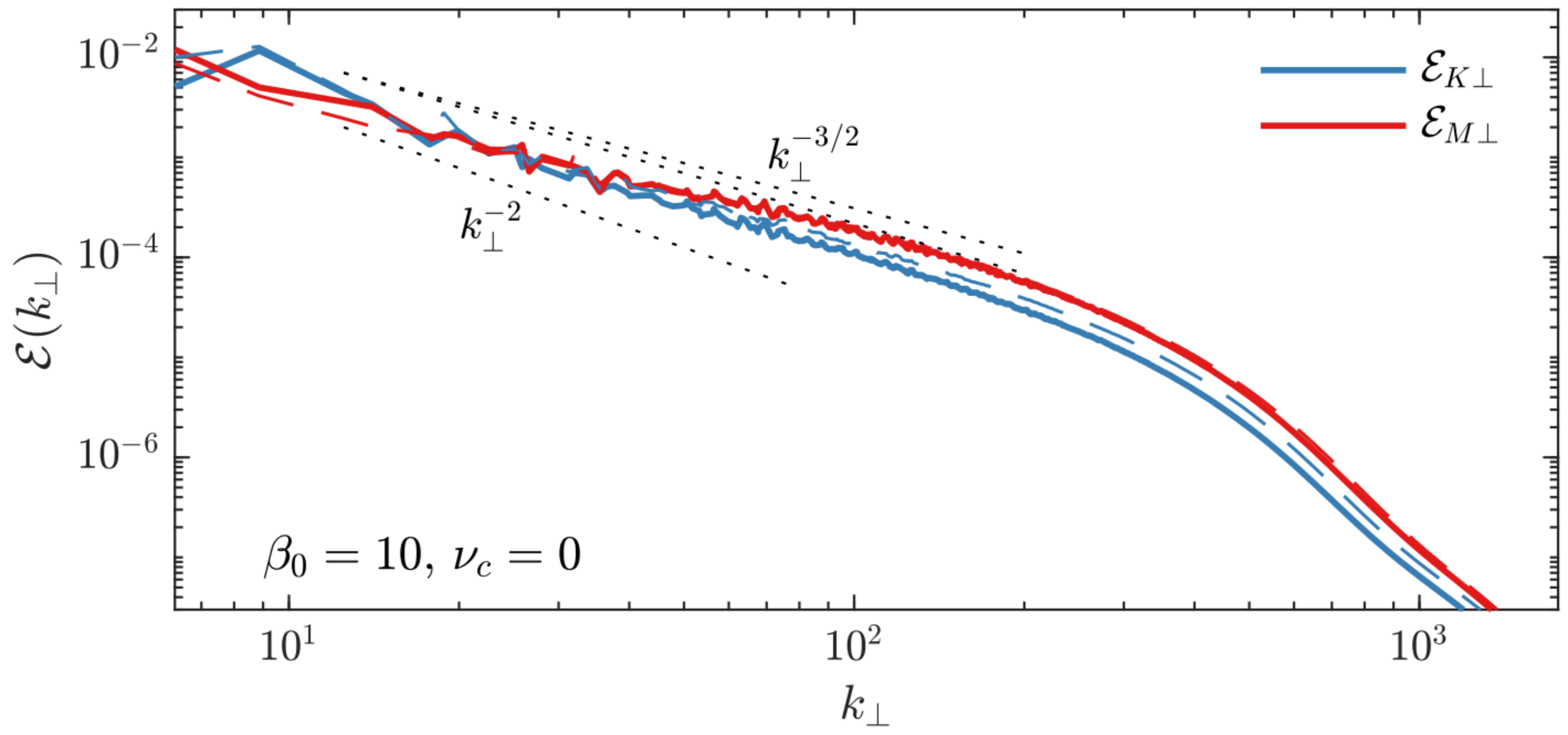

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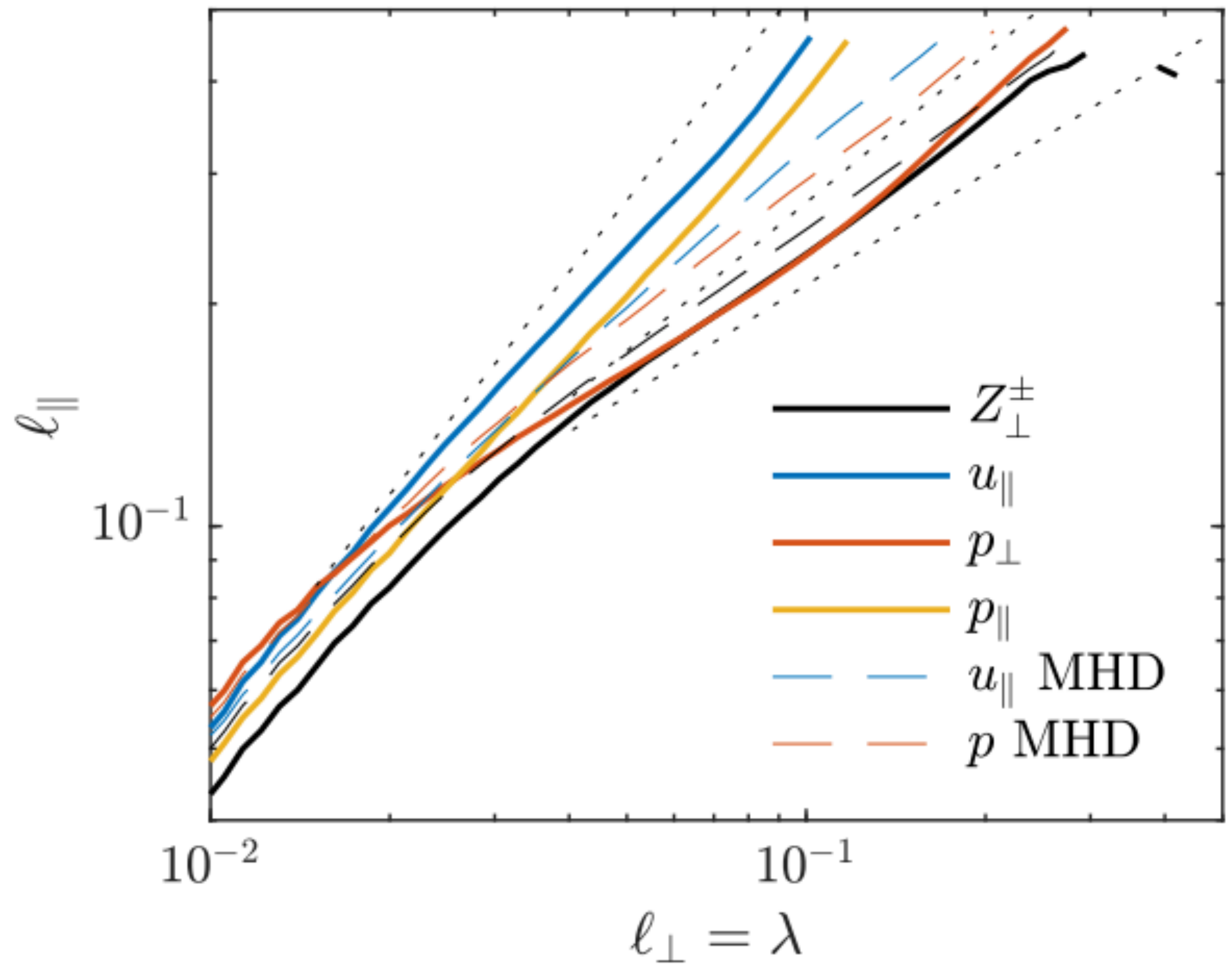

→ Spectra, spatial anisotropy resemble "passive" Δp MHD simulations (even though should have $|\Delta\beta| > 2$ @ outer scale)

Magneto-immutability: self organizing to minimize $\hat{b}\hat{b} : \nabla \vec{u}$, δB

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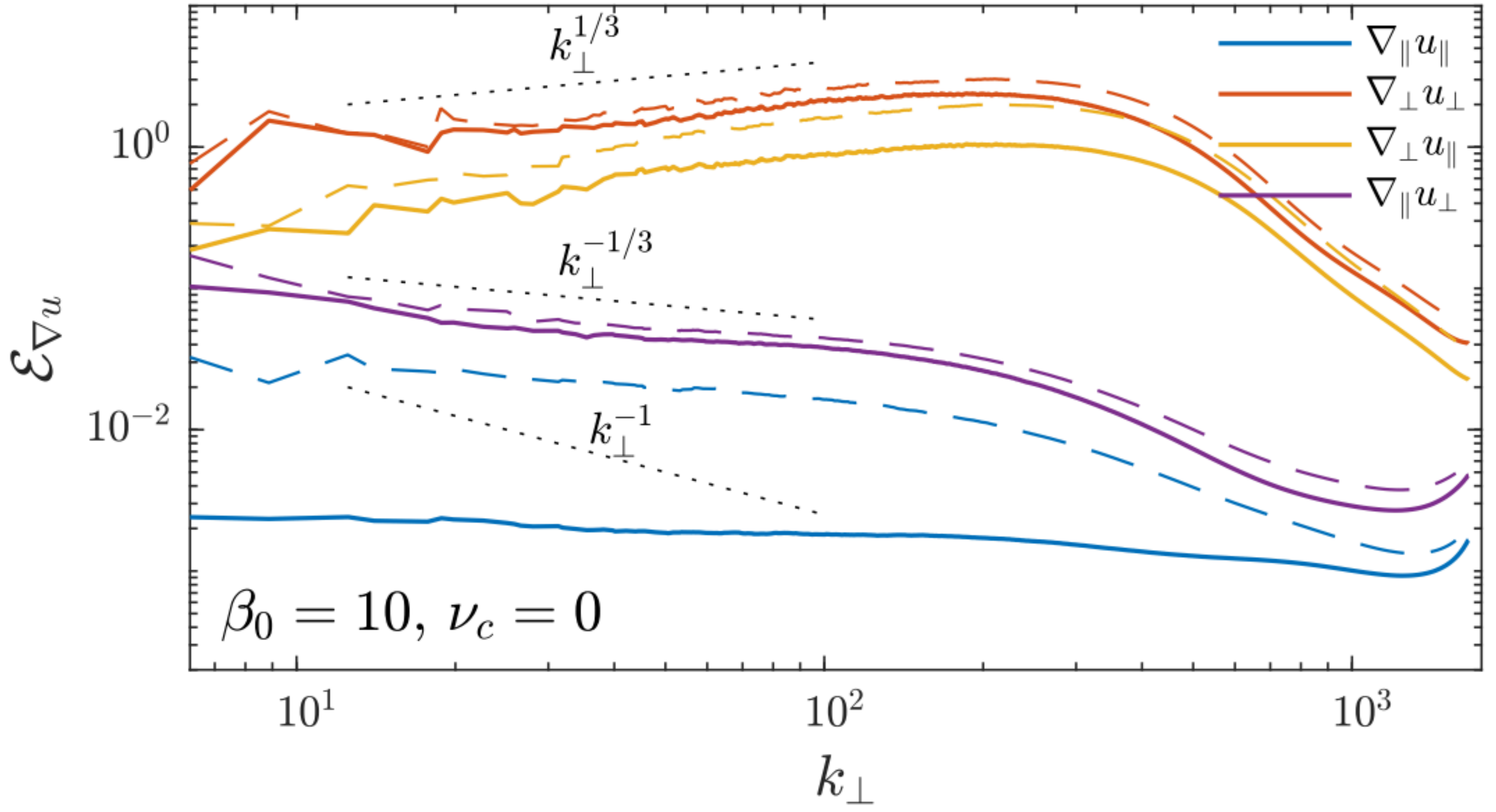
$\nabla \cdot (\hat{b}\hat{b}\Delta p)$ suppresses coll'lessly damped motions

— active, - - passive



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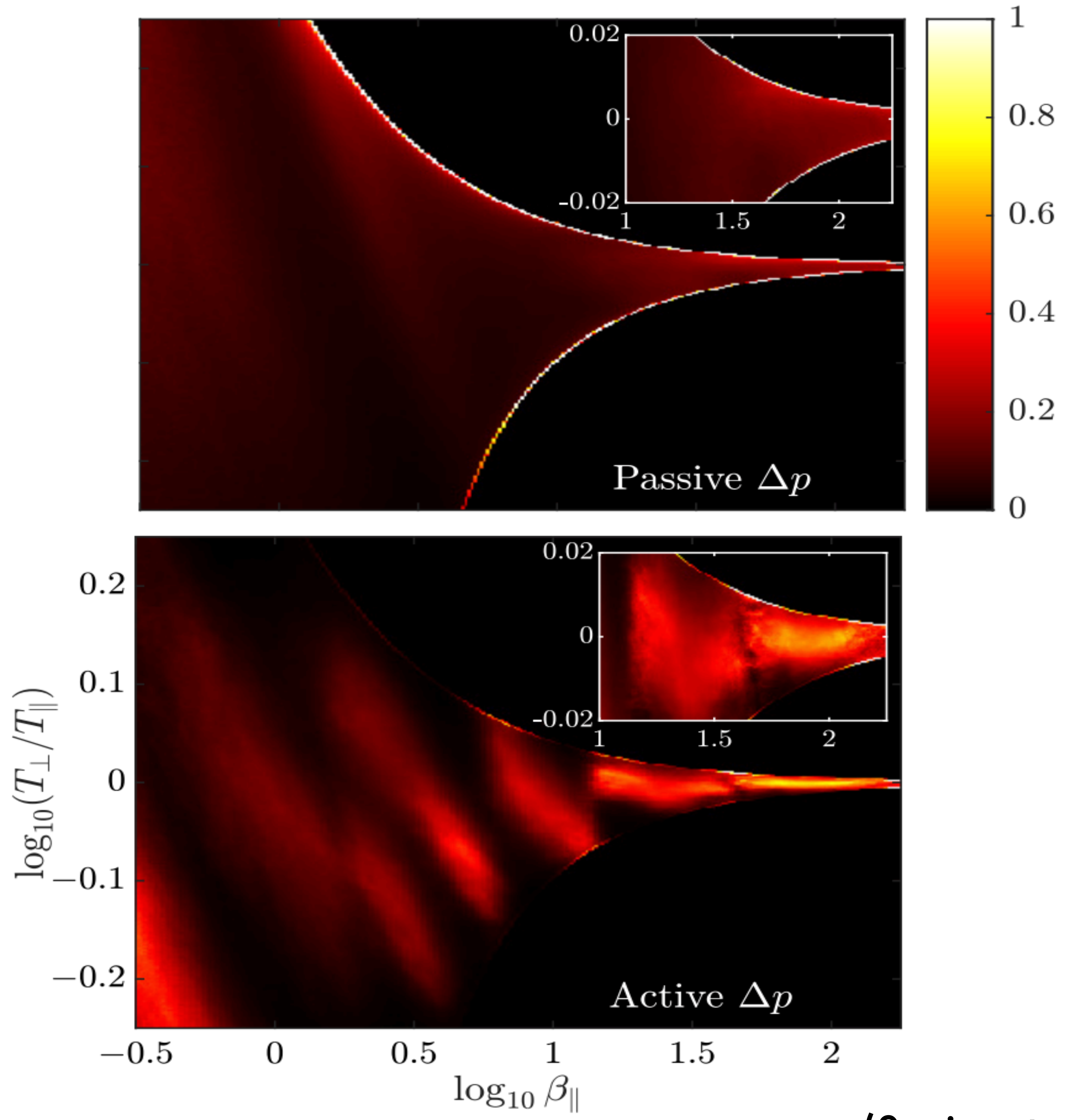
Drives plasma away from instability thresholds



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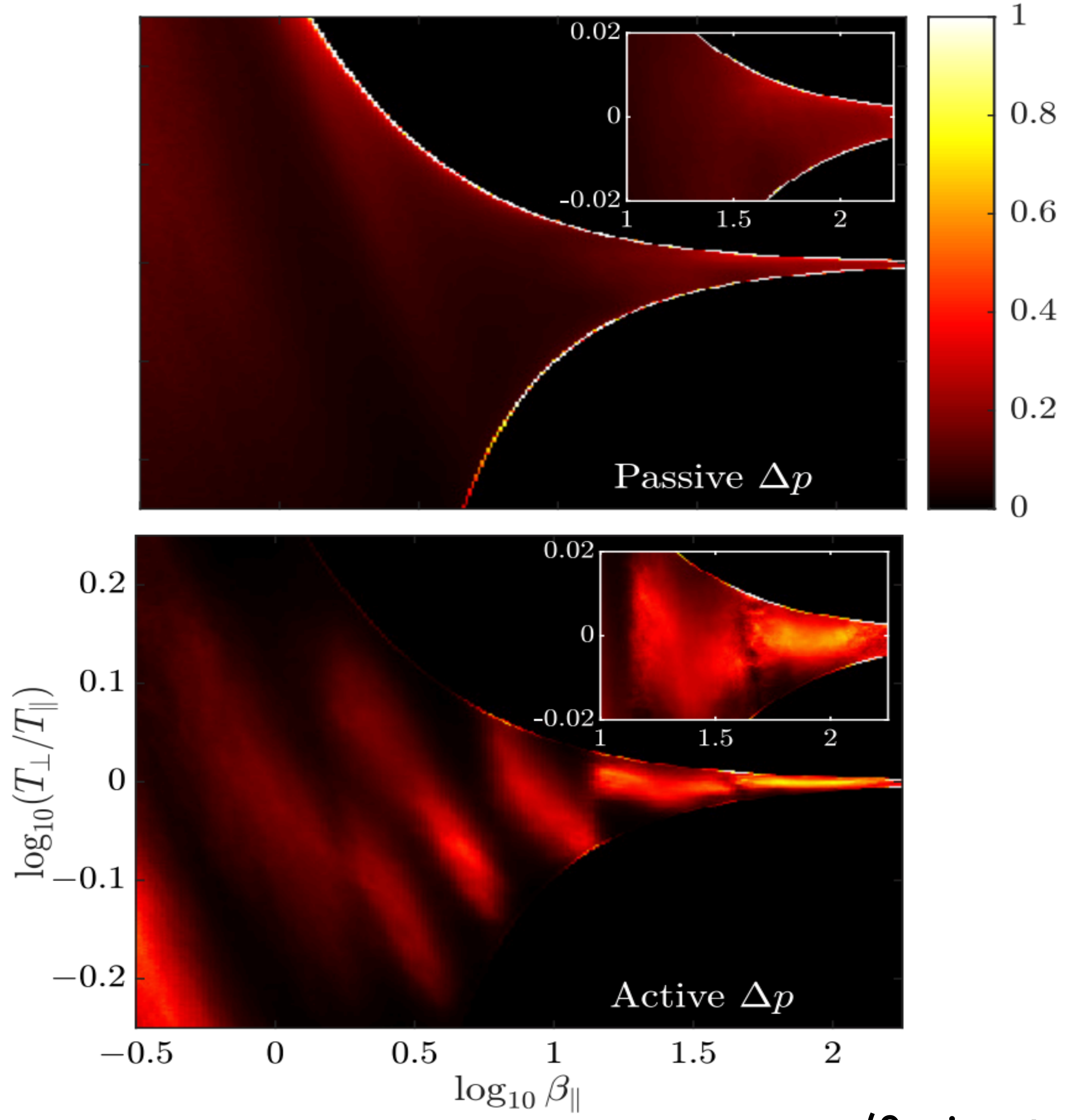

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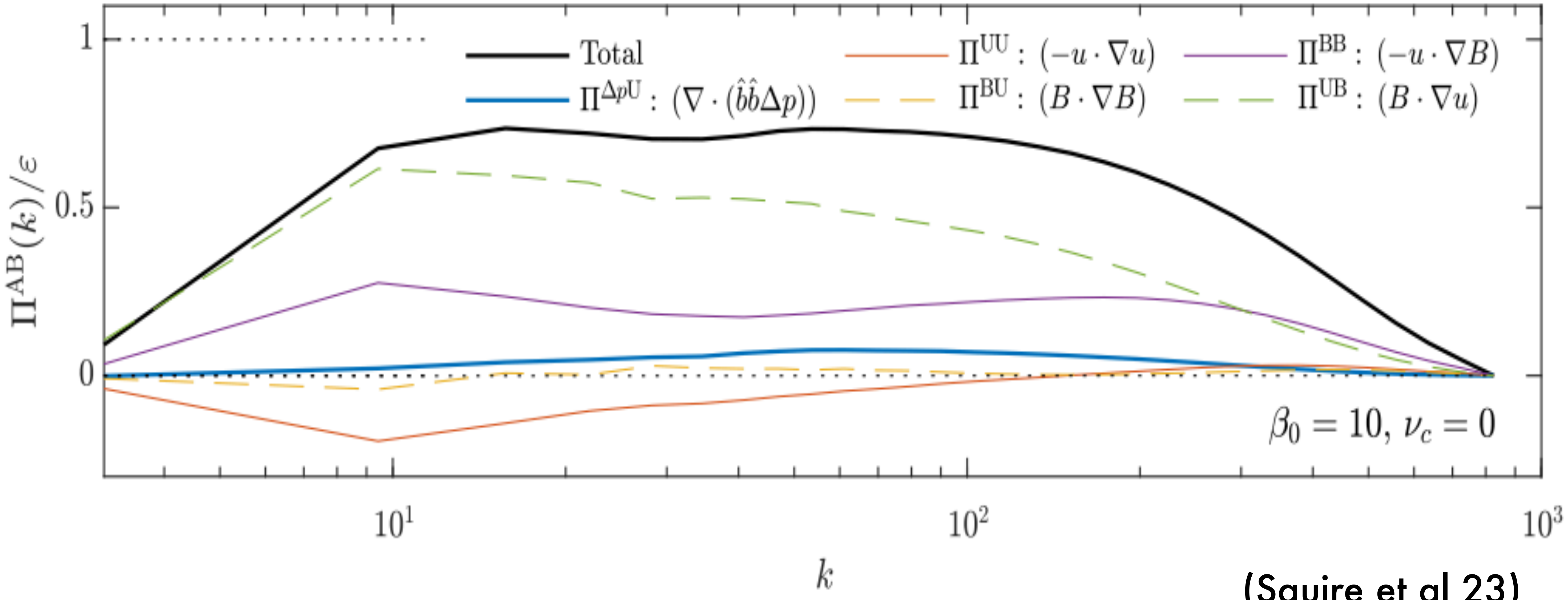
MHD-like conservative cascade



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Could compressive driving be different?

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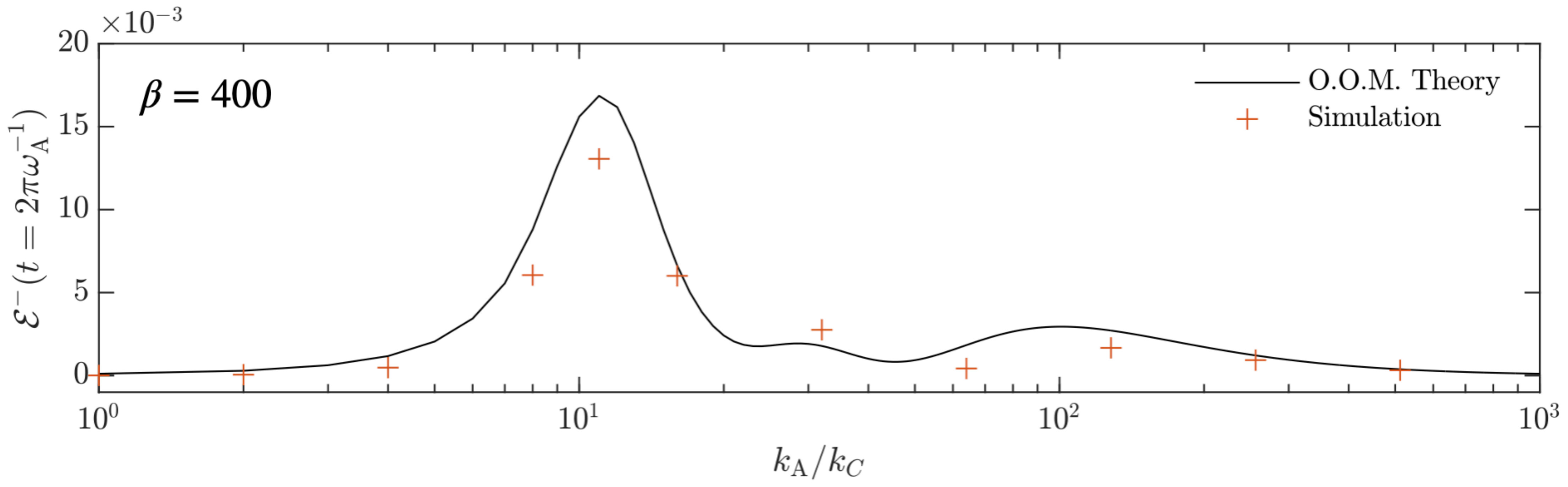

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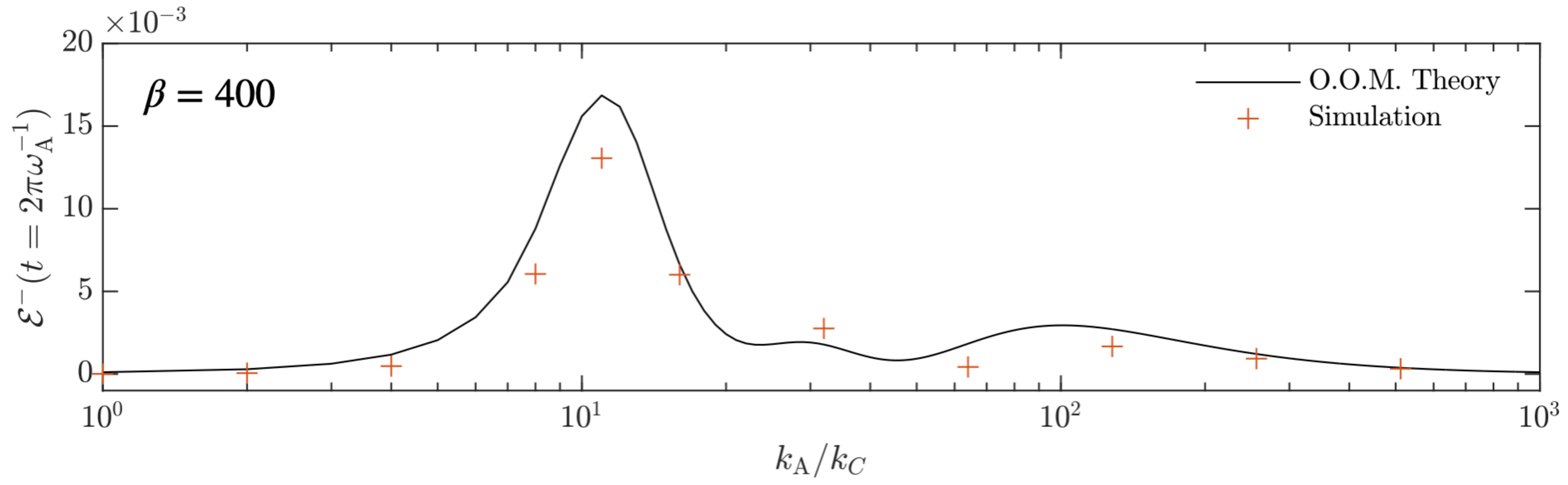

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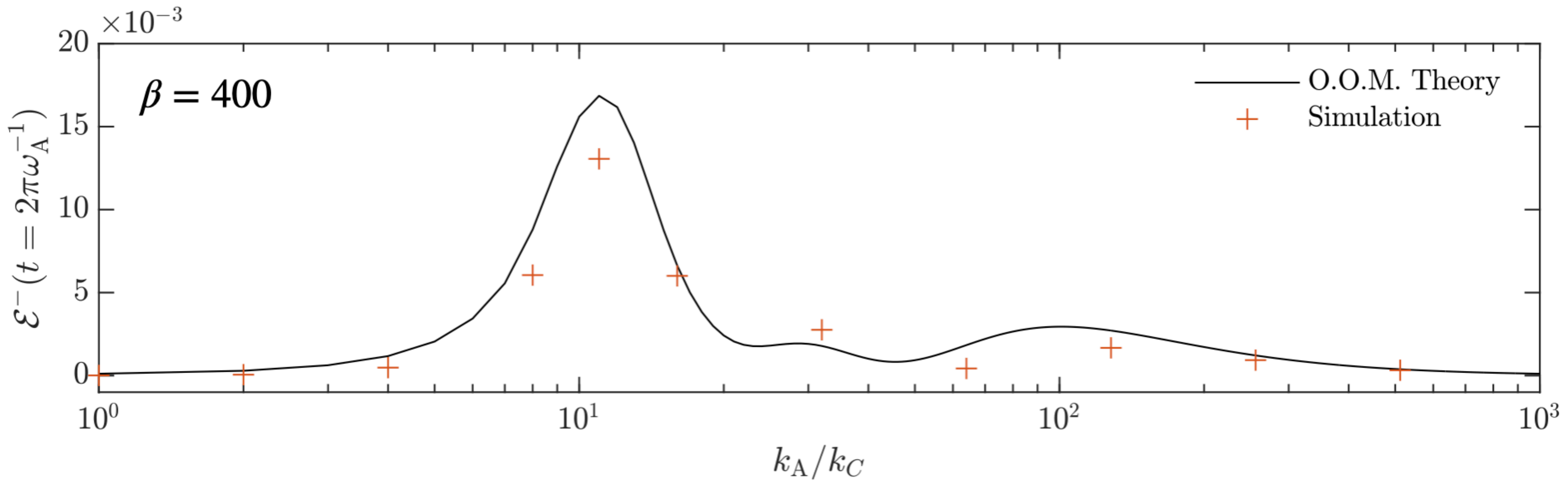

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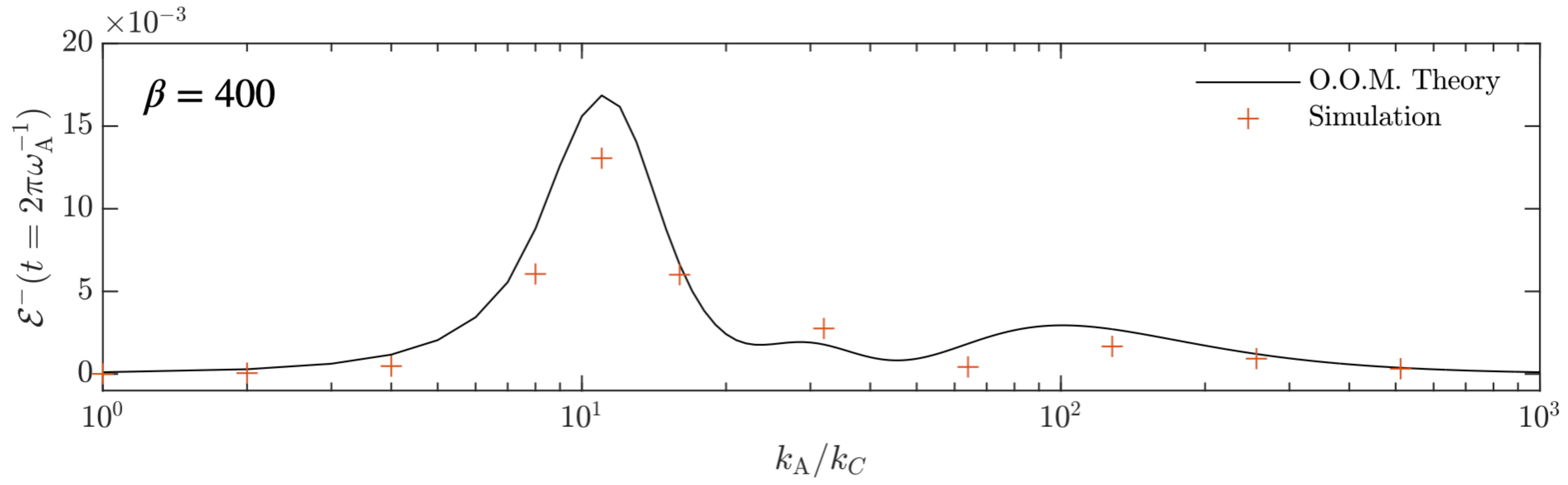

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- AWs may be limited in their ability to mix certain compressive modes
- IAs _are_ _linearly_ collisionlessly damped

Questions to ask:

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Heat fluxes diffuse Δp on $(k_{\|} v_{th})^{-1}$ timescales → can immutability influence the flow before a causal connection is lost?

Do compressive fluctuations, their larger anisotropy, and their own possible immutability, interfere with the evolution and immutability of the Alfvénic cascade?

Simulation parameters: Landau-fluid CGL-MHD

Governing equations

$$\rho\left(\partial_t \boldsymbol{u} + \boldsymbol{u}\cdot\boldsymbol{\nabla}\boldsymbol{u}\right) = -\boldsymbol{\nabla}\left(T_e\rho + p_\perp + \frac{B^2}{8\pi}\right) + \boldsymbol{\nabla}\cdot\left[\hat{\boldsymbol{b}}\hat{\boldsymbol{b}}\left(\Delta p + \frac{B^2}{4\pi}\right)\right],$$

$$\partial_t p_\perp + \boldsymbol{\nabla}\cdot(p_\perp\boldsymbol{u}) + p_\perp\boldsymbol{\nabla}\cdot\boldsymbol{u} + \boldsymbol{\nabla}\cdot(q_\perp\hat{\boldsymbol{b}}) + q_\perp\boldsymbol{\nabla}\cdot\hat{\boldsymbol{b}} = p_\perp\hat{\boldsymbol{b}}\hat{\boldsymbol{b}}\boldsymbol{:}\boldsymbol{\nabla}\boldsymbol{u} - \frac{1}{3}\nu_{\rm c}\Delta p,$$

$$\partial_t p_\| + \boldsymbol{\nabla}\cdot(p_\|\boldsymbol{u}) + \boldsymbol{\nabla}\cdot(q_\|\hat{\boldsymbol{b}}) - 2q_\perp\boldsymbol{\nabla}\cdot\hat{\boldsymbol{b}} = -2p_\|\hat{\boldsymbol{b}}\hat{\boldsymbol{b}}\boldsymbol{:}\boldsymbol{\nabla}\boldsymbol{u} + \frac{2}{3}\nu_{\rm c}\Delta p.$$

Heat fluxes

$$-\boldsymbol{\nabla}\cdot(q_\perp\hat{\boldsymbol{b}}) \approx -\hat{\boldsymbol{b}}\cdot\boldsymbol{\nabla}q_\perp \approx \sqrt{\frac{2}{\pi}}\nabla_\|\left[\frac{c_{{\rm s}\|}^2}{c_{{\rm s}\|}|\nabla_\|| + a_\perp\nu_{\rm c}}\nabla_\| p_\perp\right],$$

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(Squire et al 23)

- Riemann solver based on Athena MHD code (Squire et al ’23)

Simulation parameters: Landau-fluid CGL-MHD

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(Squire et al 23)

- Riemann solver based on Athena MHD code (Squire et al '23)
- Landau fluid $k_L = |\nabla_\parallel|$ is set to parallel box length
- When $\Delta\beta \geq 1$ or $\Delta\beta \leq -2$, $\nu = 1e10$ locally

Simulation parameters: Landau-fluid CGL-MHD

β	**1**	**10**	**100**
Grid	384 x 192^2	768 x 384^2	768 x 384^2
dE/dt ($v_A^3/L_\perp$)	0.16	0.16	0.16
No flux?	√	√	X
Passive?	X	√	√

Each setup has been run with compressive (completely random) forcing and Alfvénic (incompressible, $\perp \overrightarrow{B}_0$)

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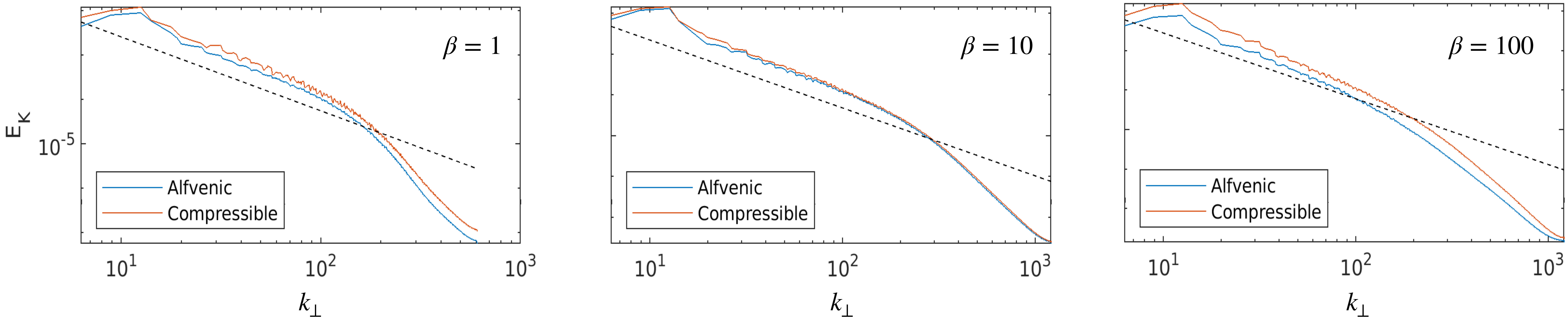
Quite fresh! What is shown represents patterns identified over the last ~week

Alfvénic vs Compressive driving: u^2 Spectra



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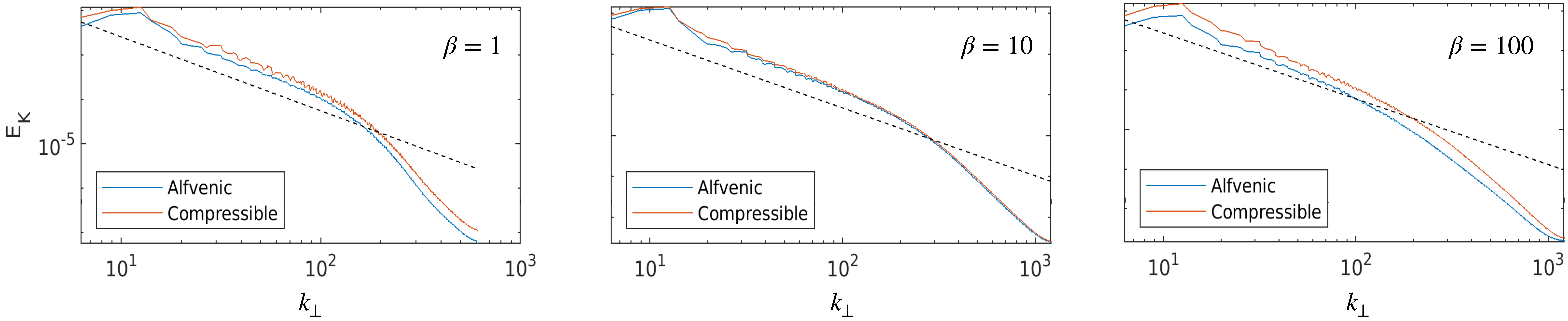
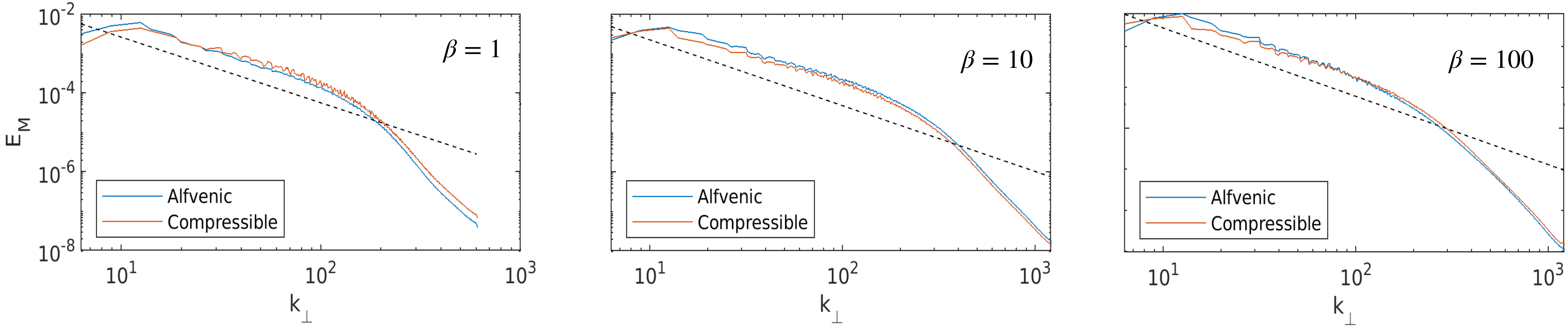


→ Largely similar, with spectrum growing steeper as $\beta \uparrow$

→ For $\beta = 1 - 10$, $E_K \sim k_\perp^{-5/3}$, while $\beta = 100$ exhibits $E_K \sim k_\perp^{-2}$

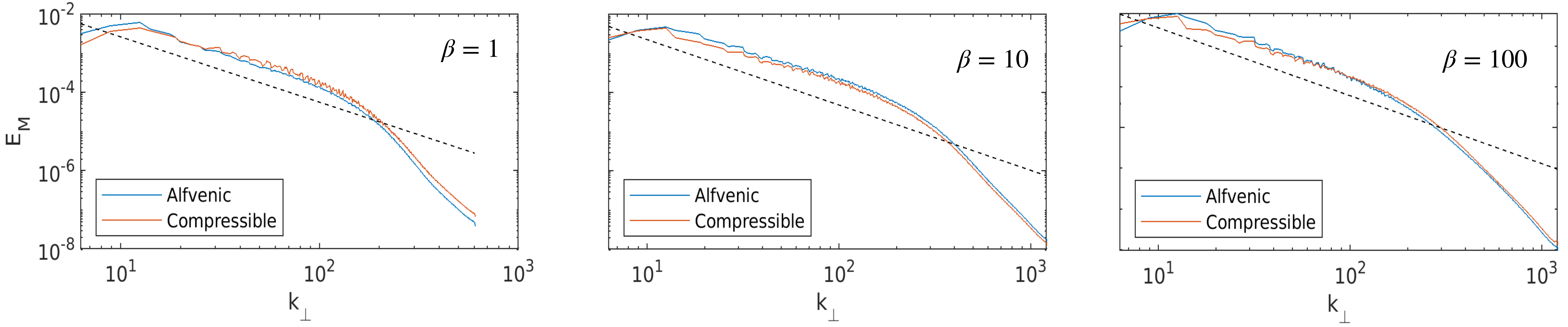
Alfvénic vs Compressive driving: B^2 Spectra



Alfvénic vs Compressive driving: B^2 Spectra

→ Compressive spectrum *appears* to be less steep ($\sim k_\perp^{-3/2}$) than Alfvenic ($\sim k_\perp^{-5/3}$)

→ Relatively consistent with increasing β



Alfvénic vs Compressive driving: Immutability

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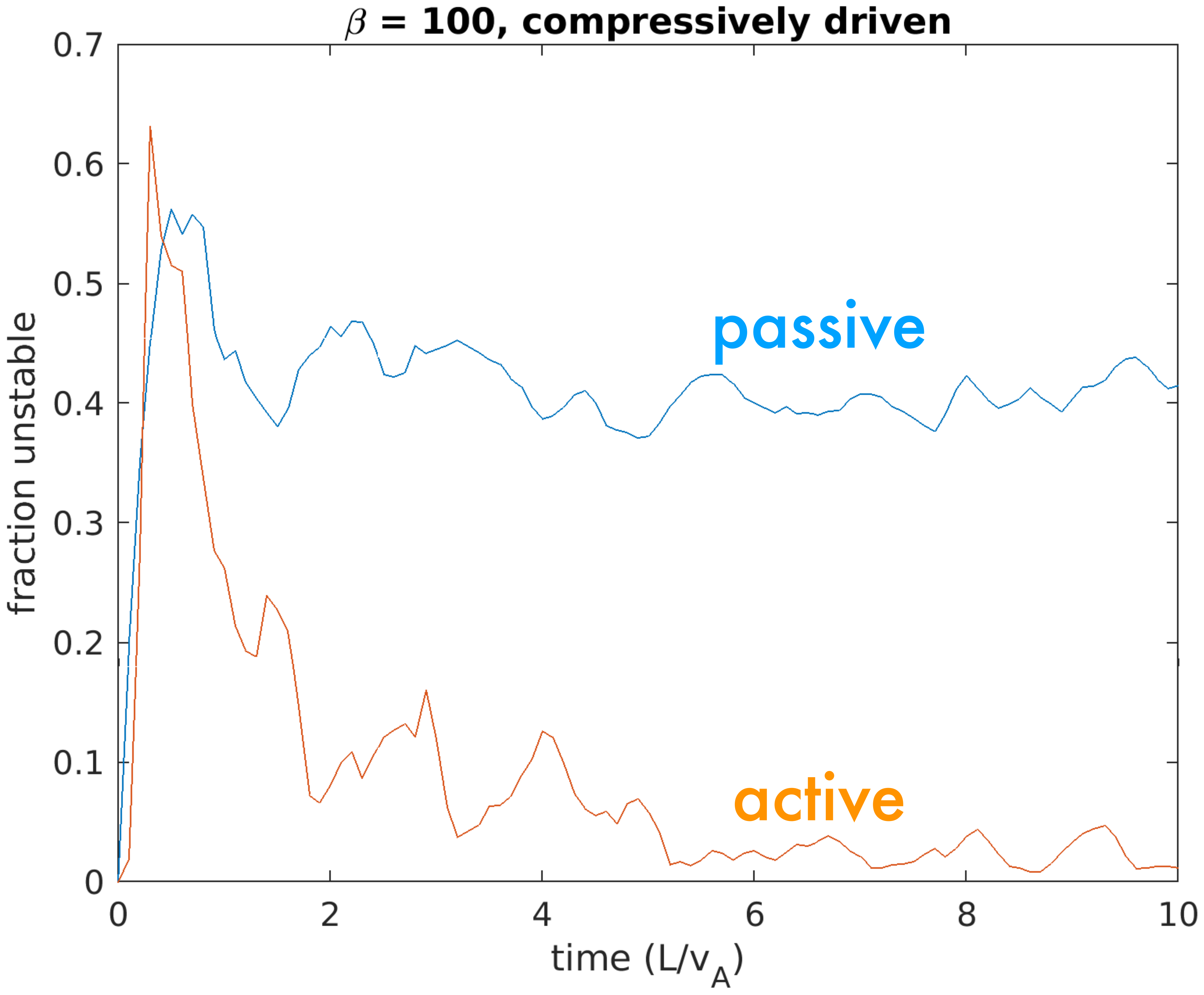

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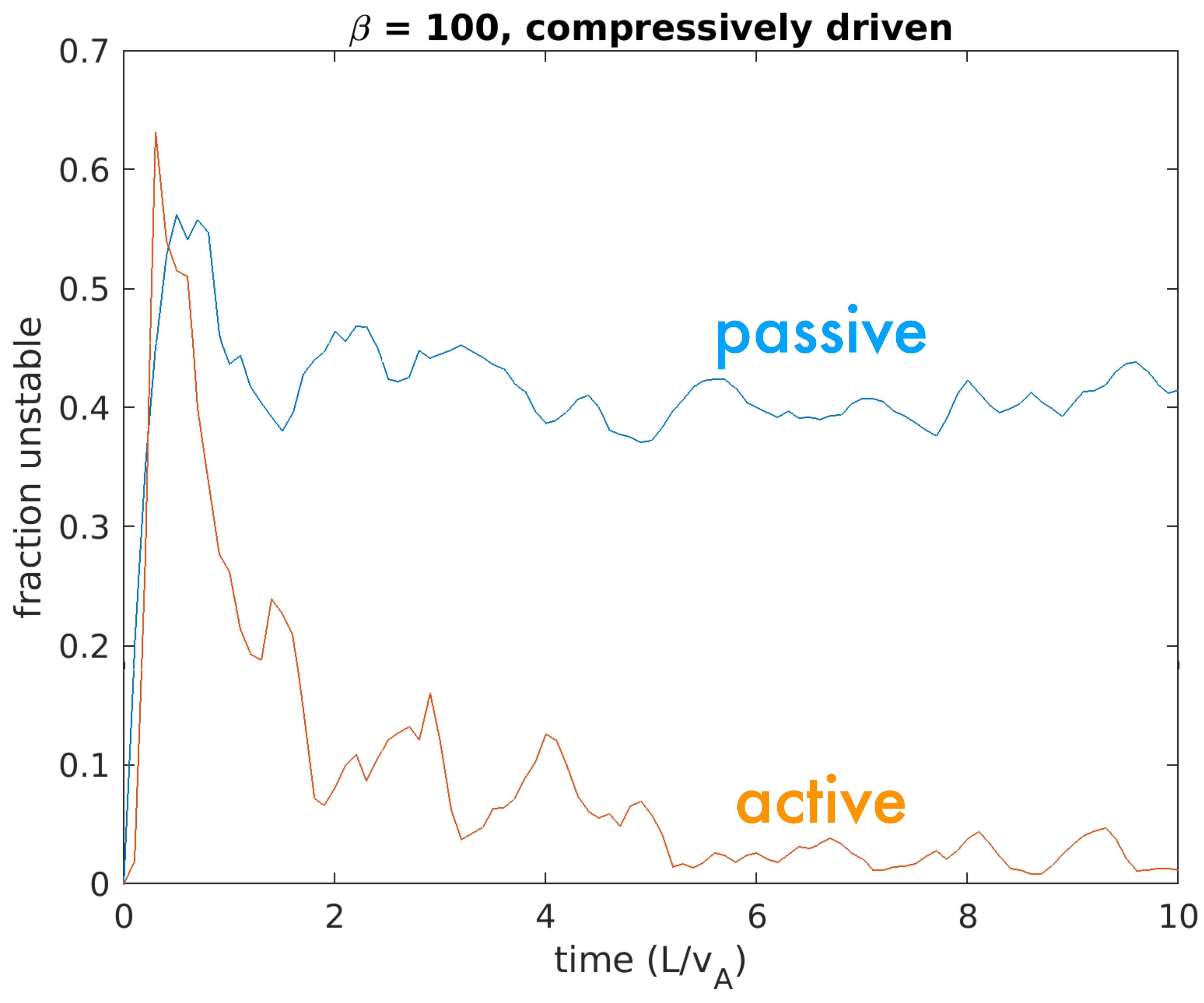
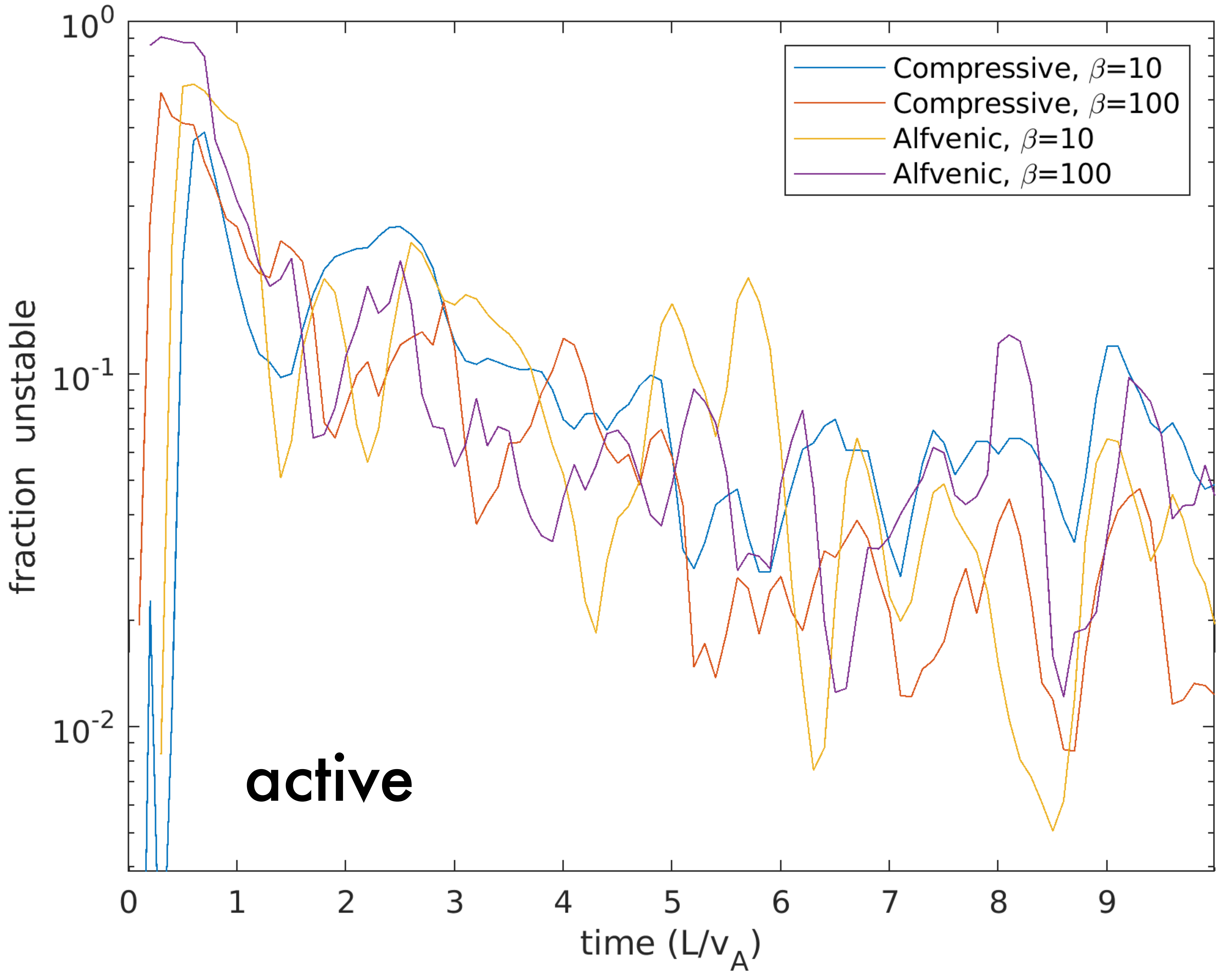
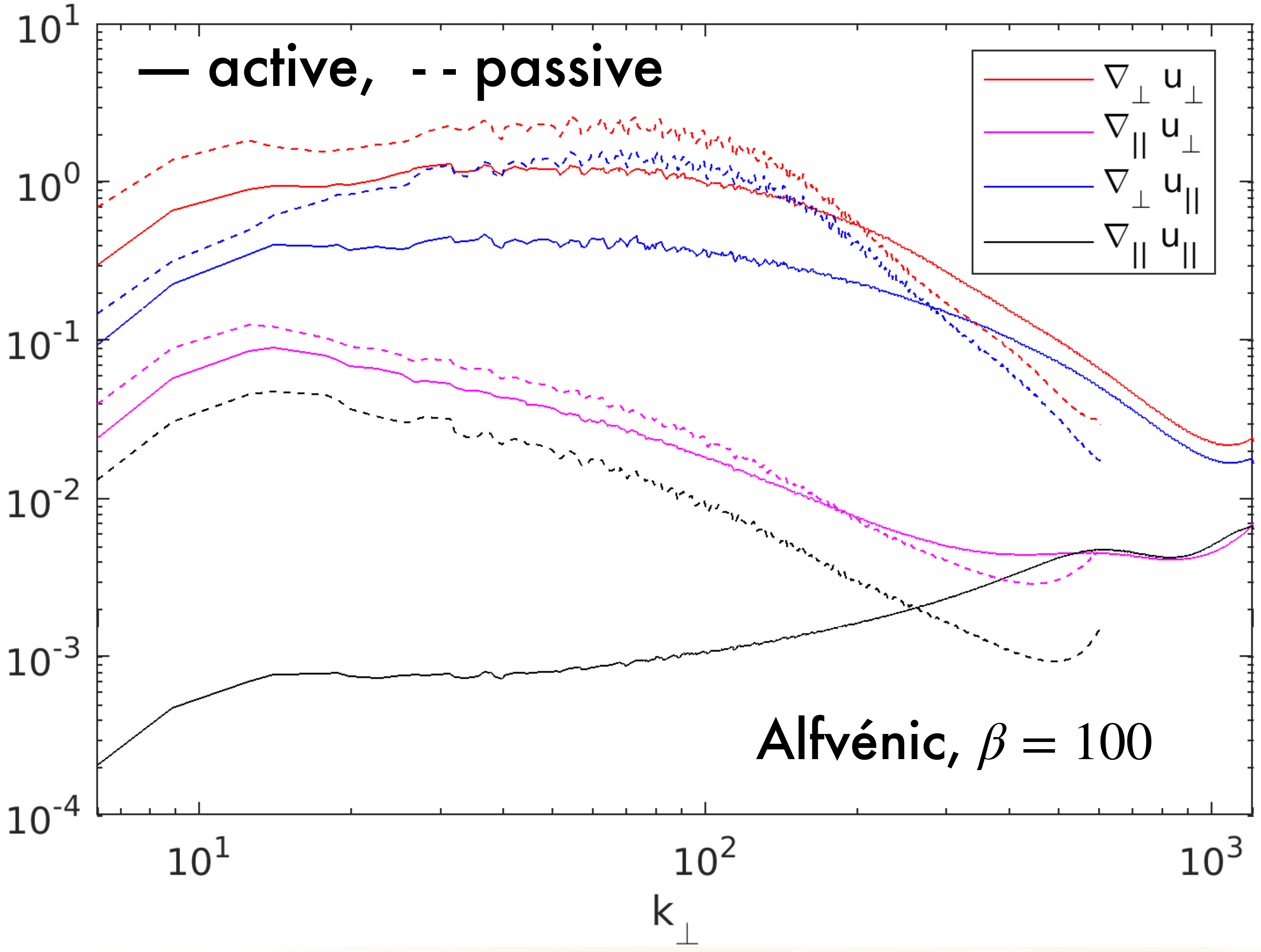




→ Quite collisionless!

Alfvénic vs Compressive driving: Immutability (rates of strain)

→ $\nabla_{\|} u_{\|}$ suppressed, but $\nabla u_{\|}$ spectra changing



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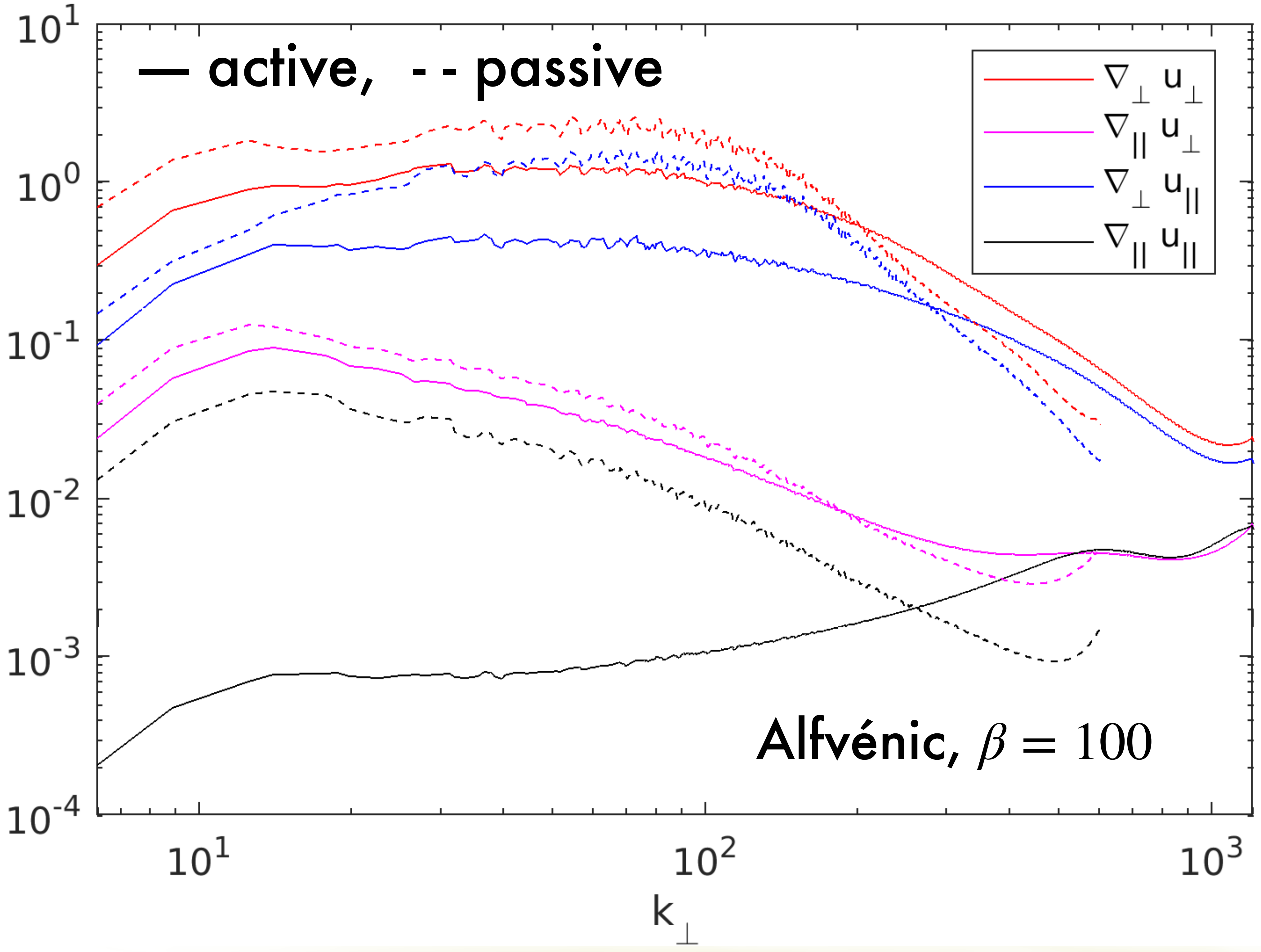
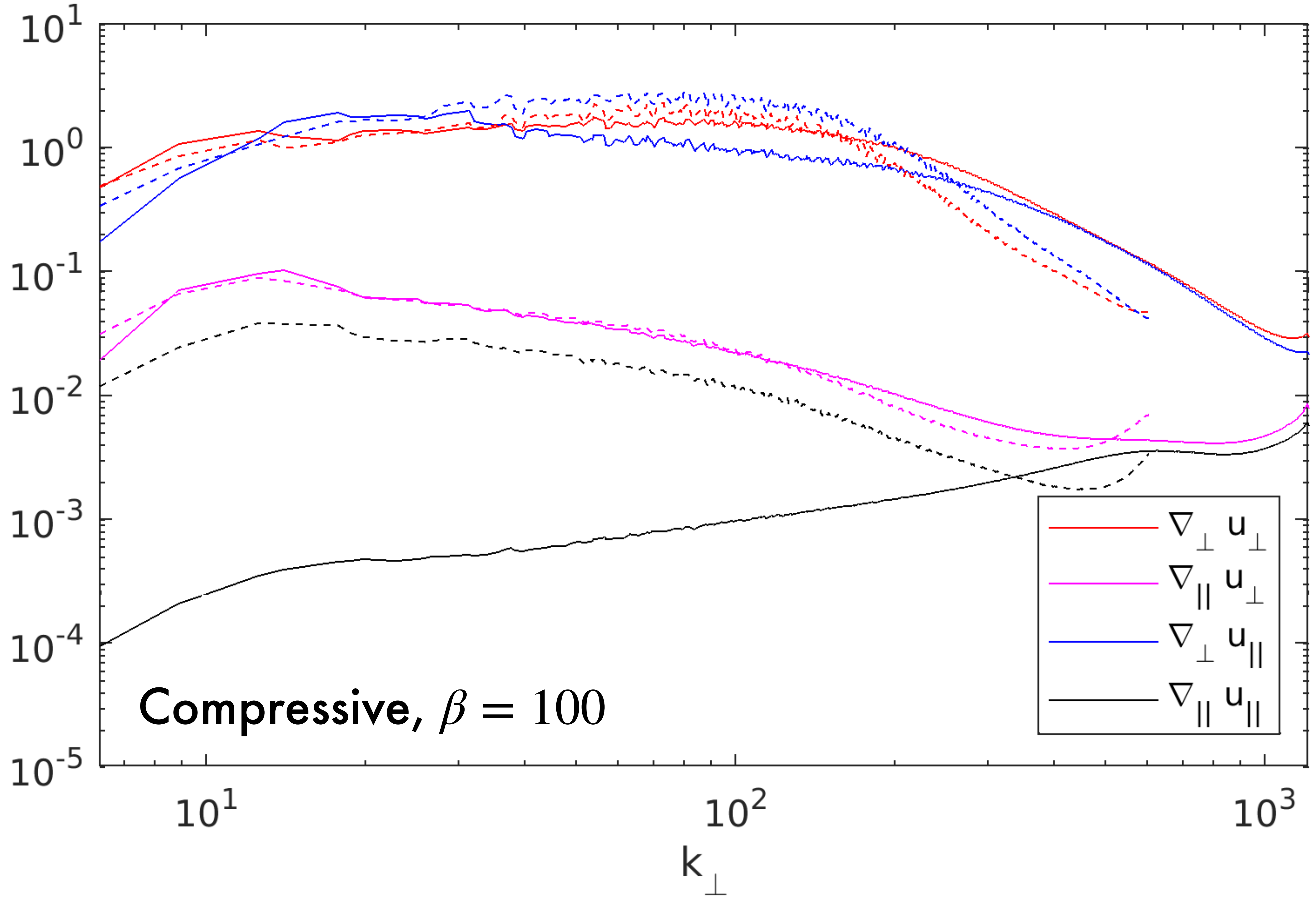
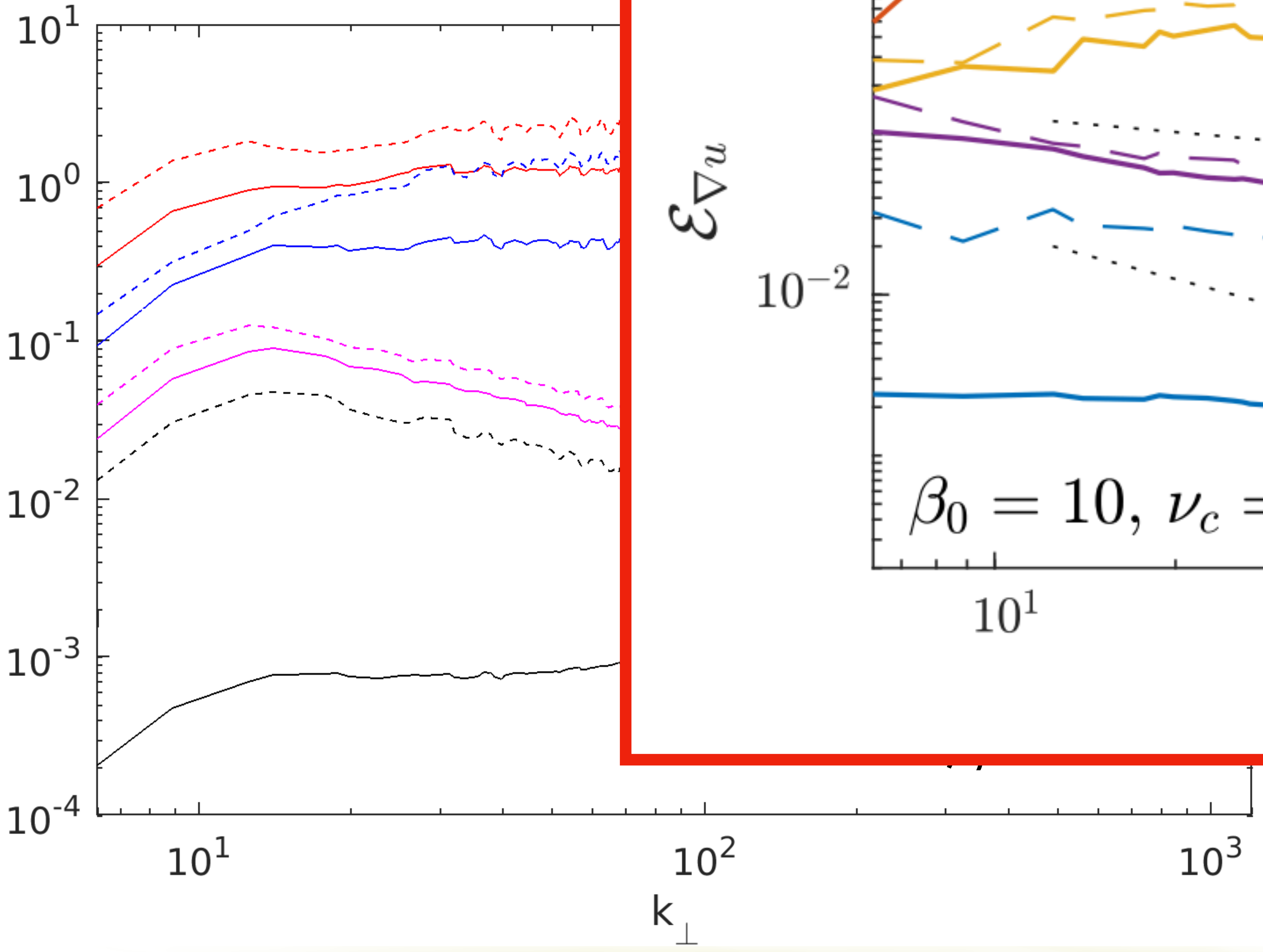


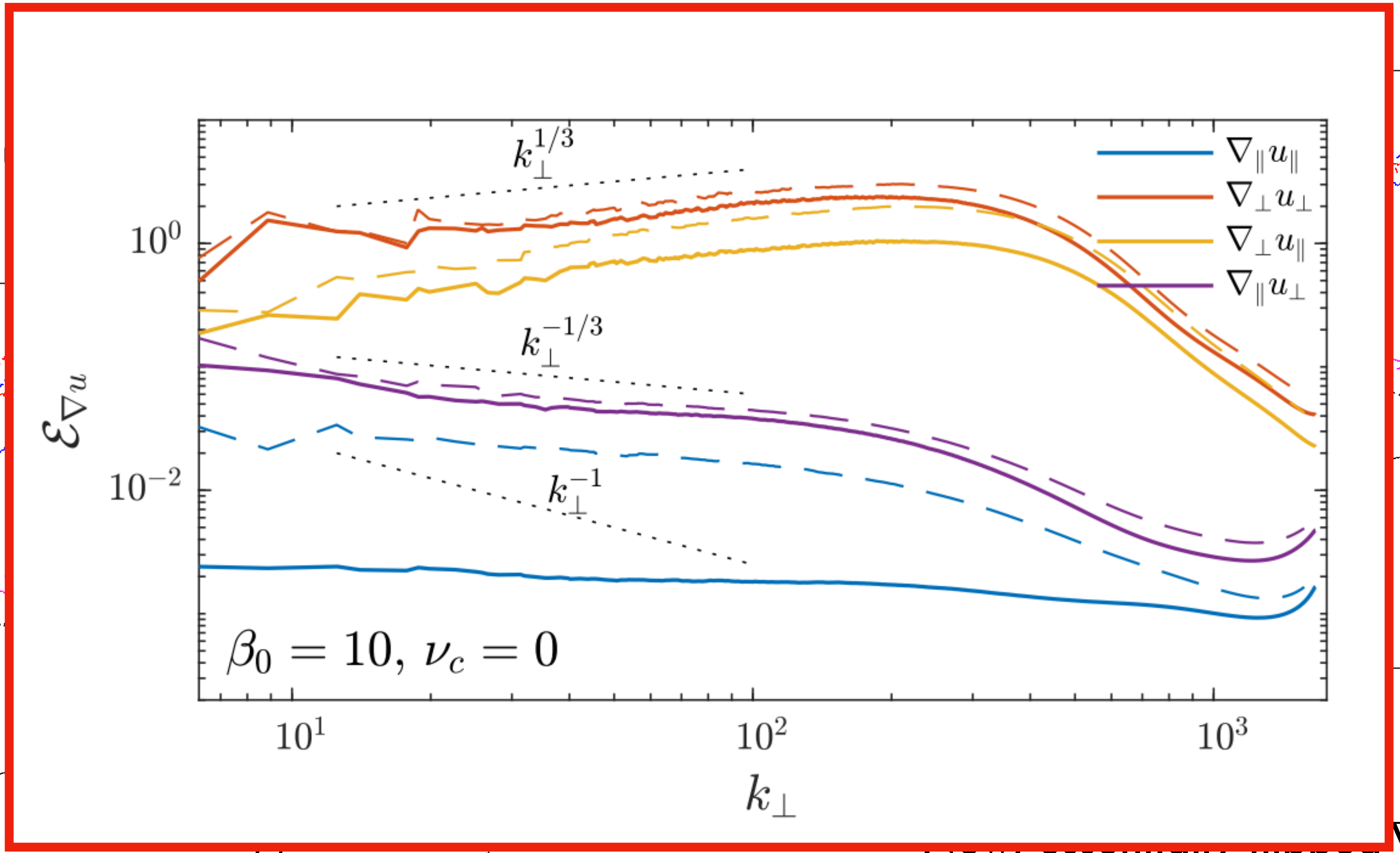


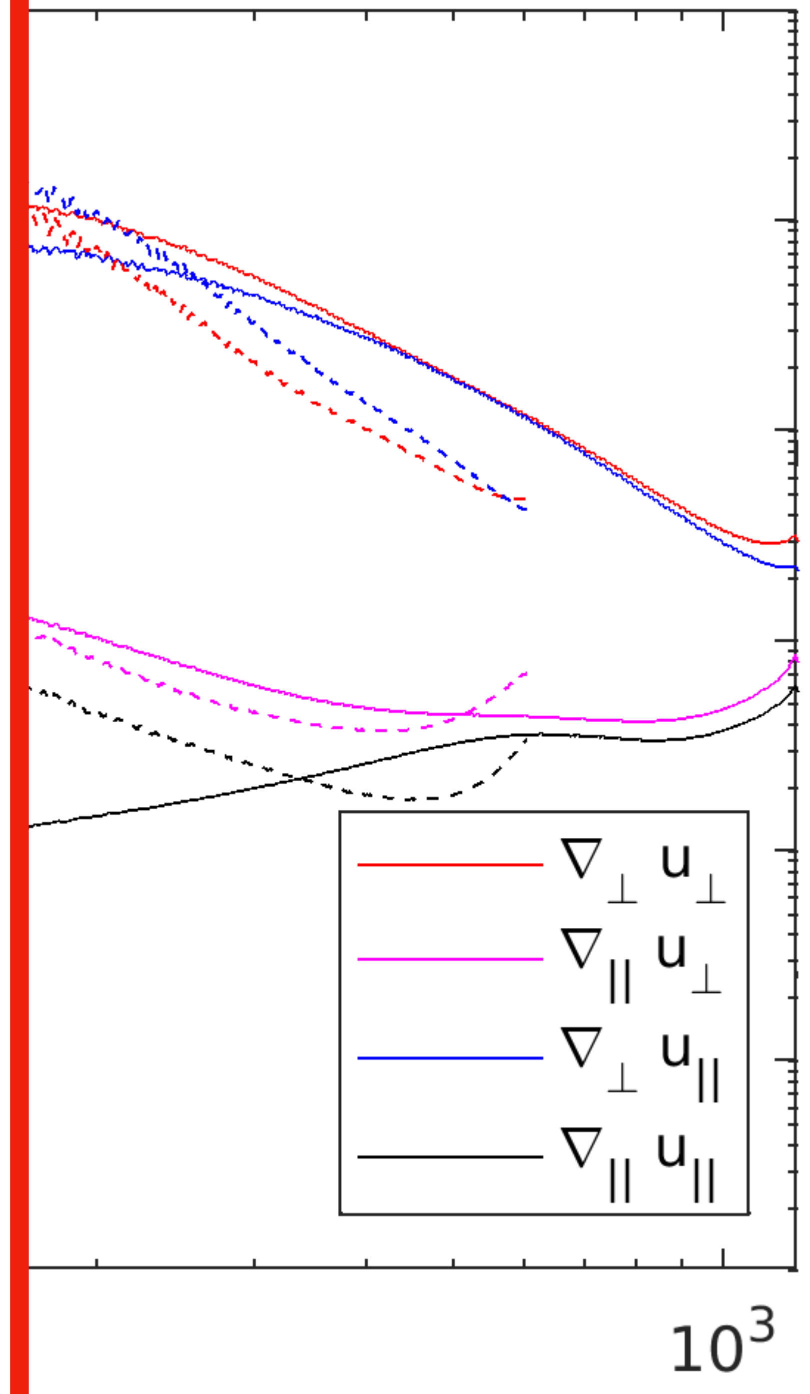
→ New, essentially flipped $\nabla u_{\|}$ scalings support more dissipation

Alfvénic vs Compressive driving: Immutability (rates of strain)

→ $\nabla_{\|} u_{\|}$ suppressed, bu



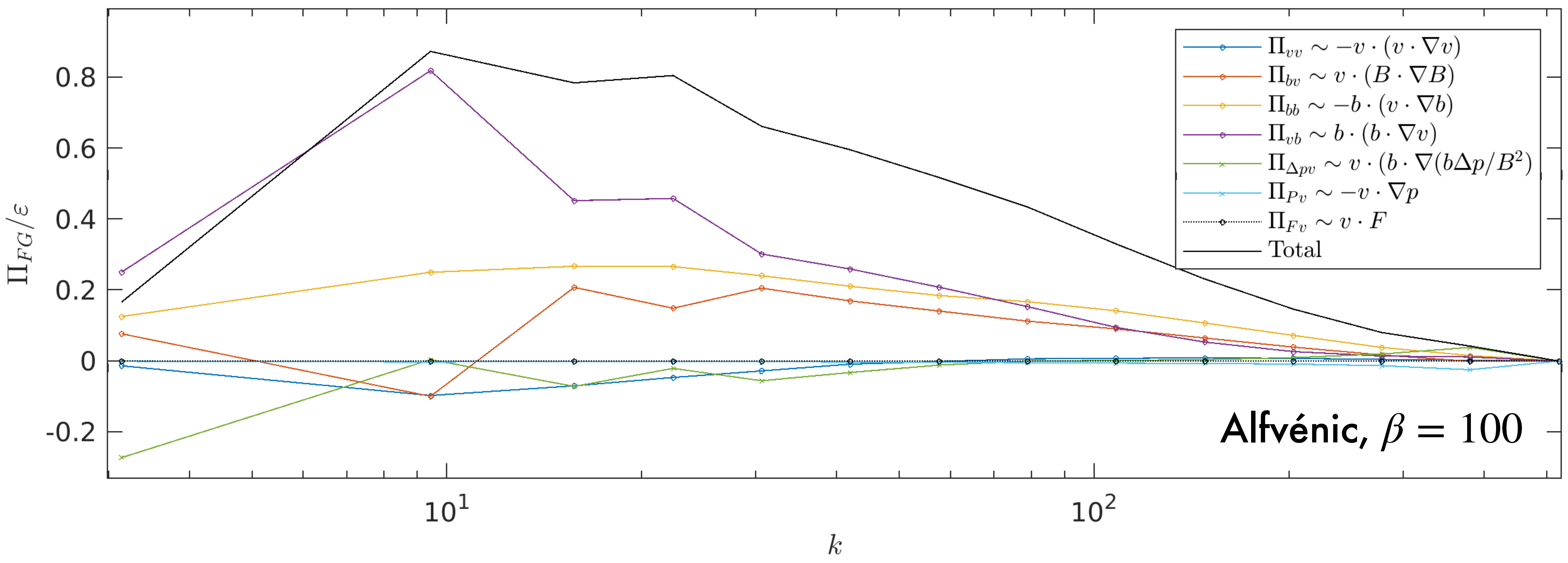


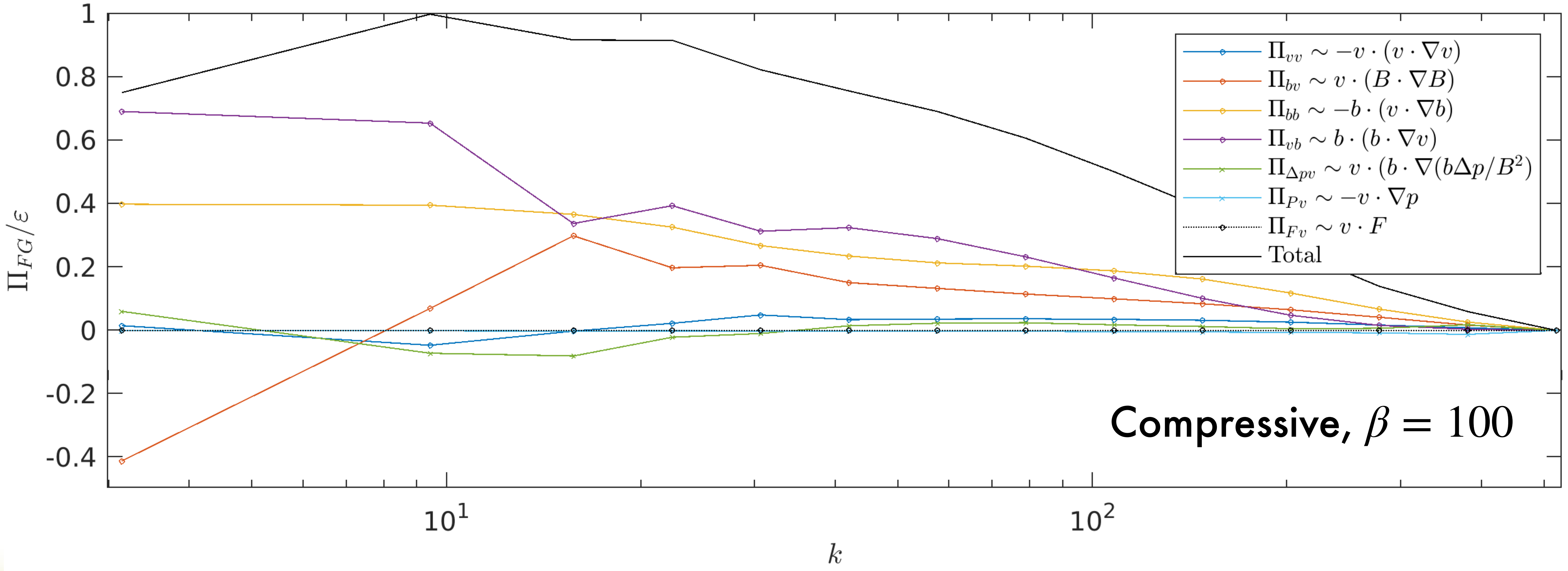


$\nabla u_{\|}$ scalings

support more dissipation

Alfvénic vs Compressive driving: Fluxes





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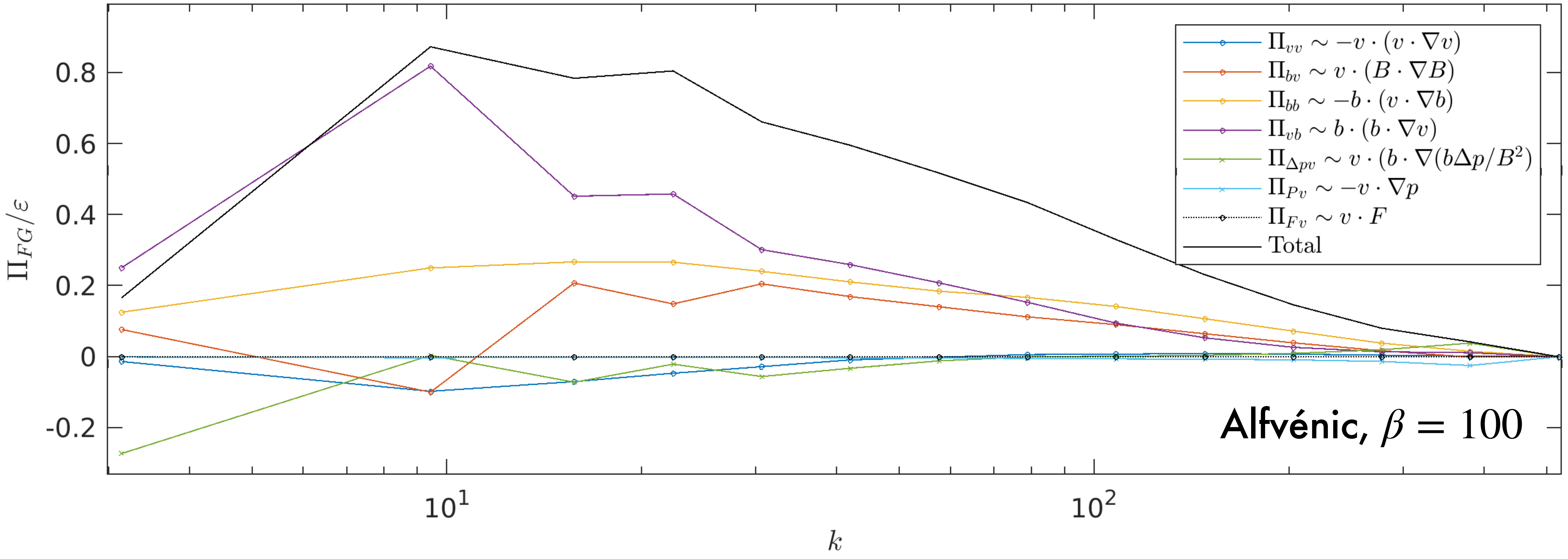
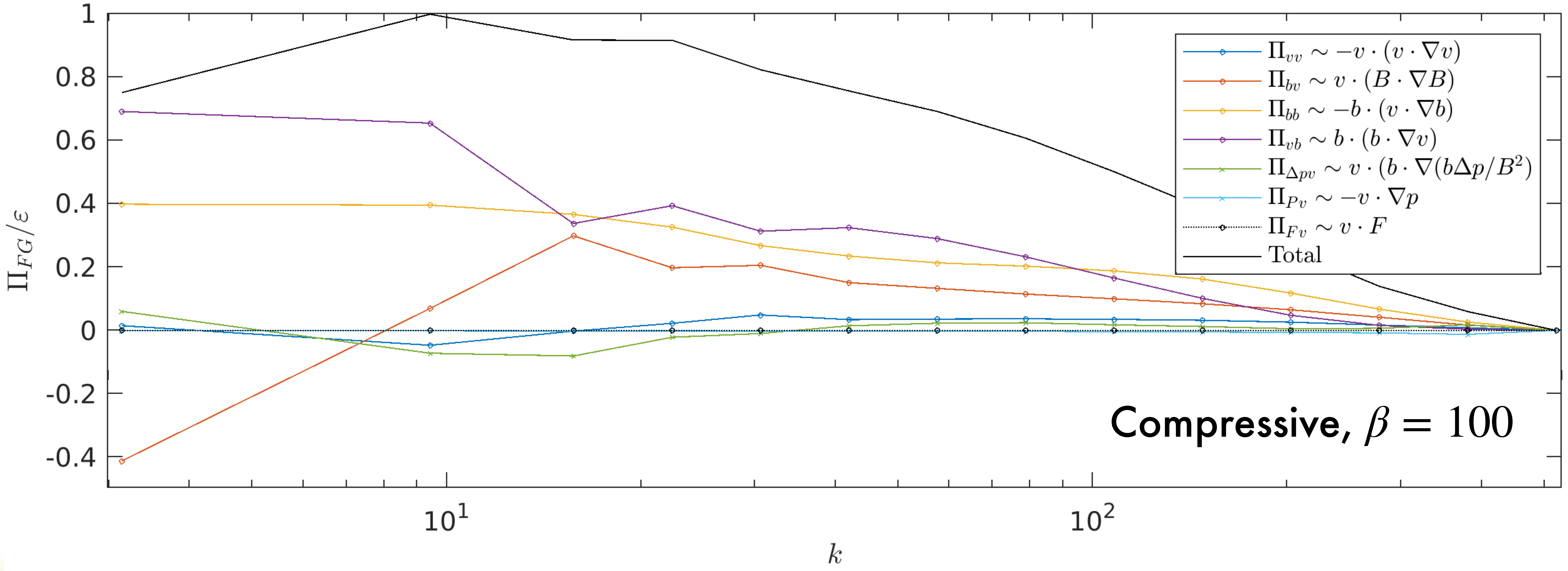


Dominant mechanisms for cascade still appear to be Alfvénic



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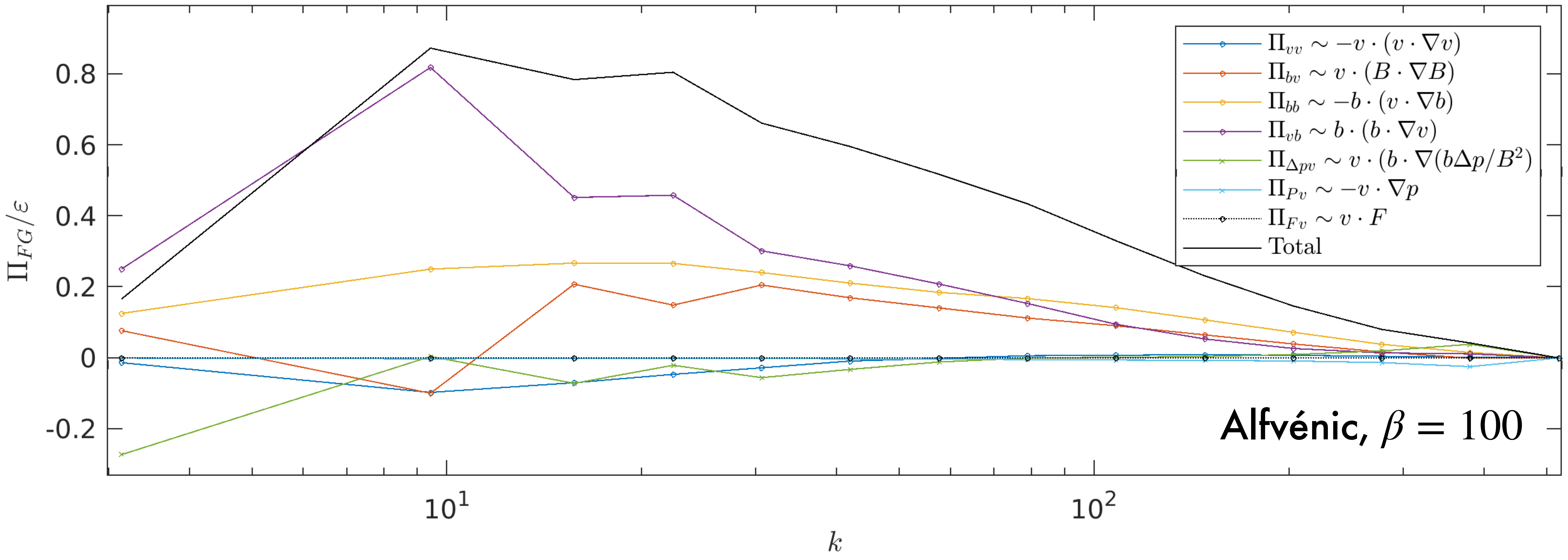

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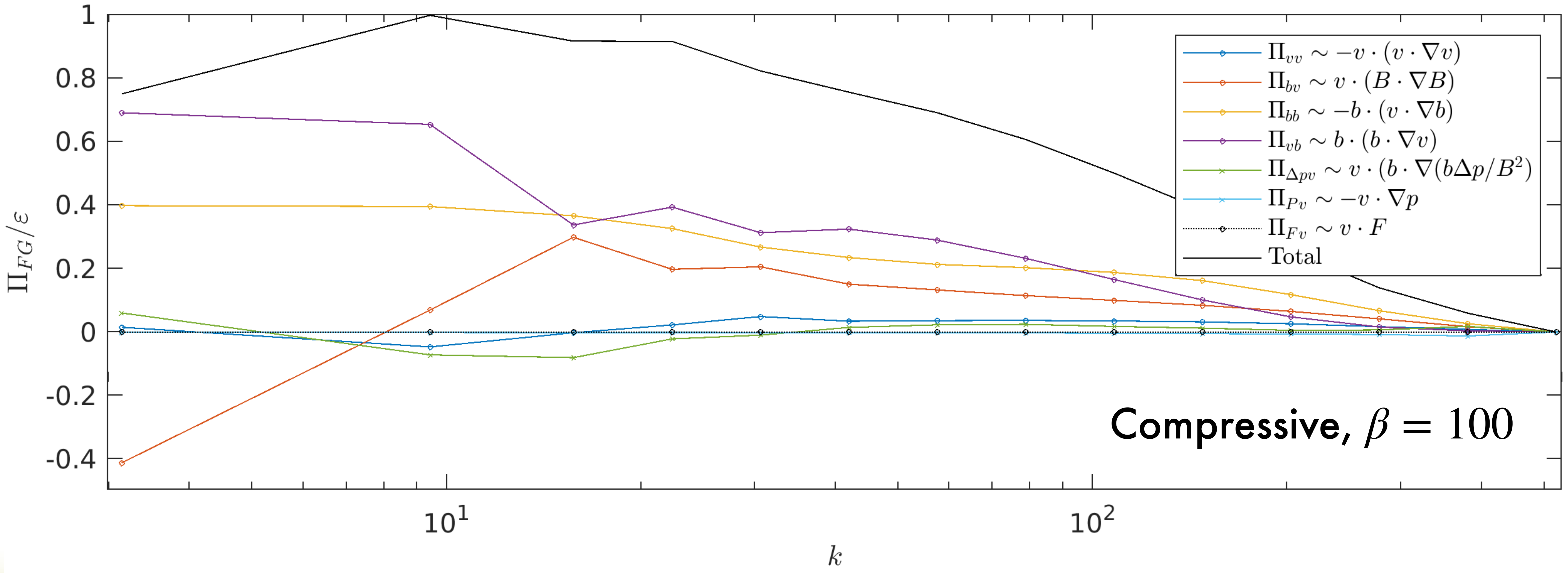
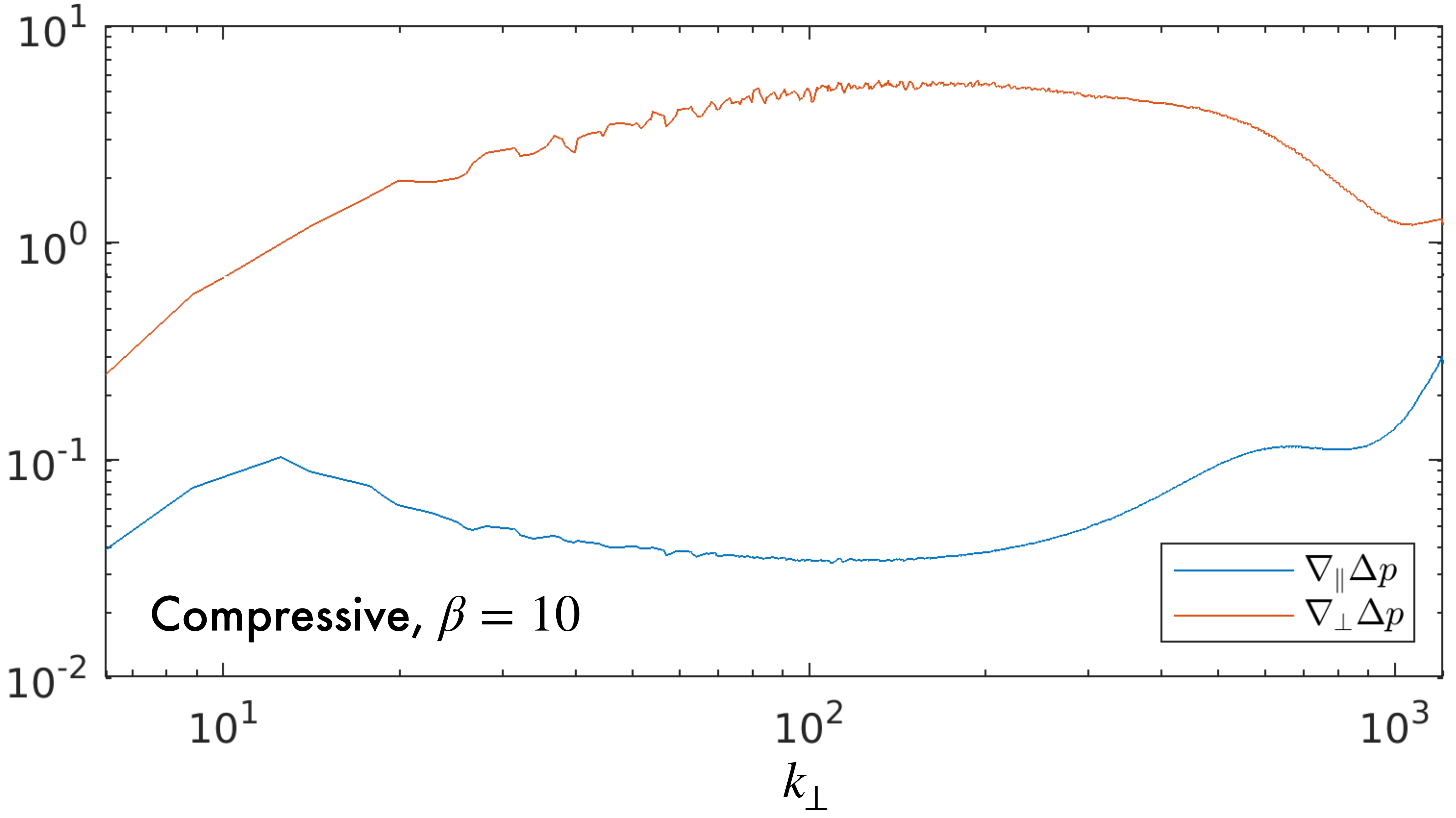


No obvious energy sink among terms considered

→ Heat fluxes possibly responsible

Compressive driving: Scale of Δp



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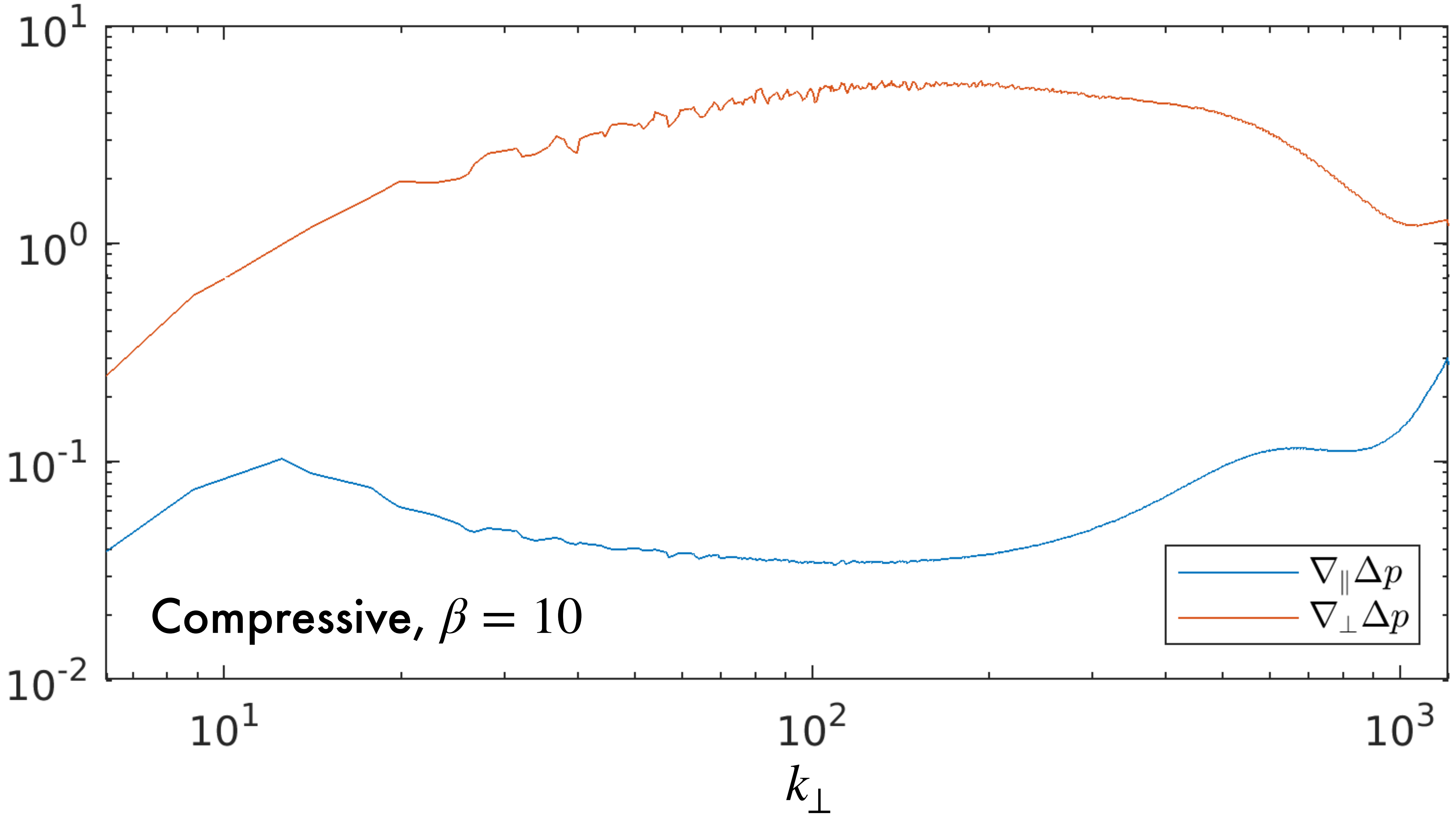

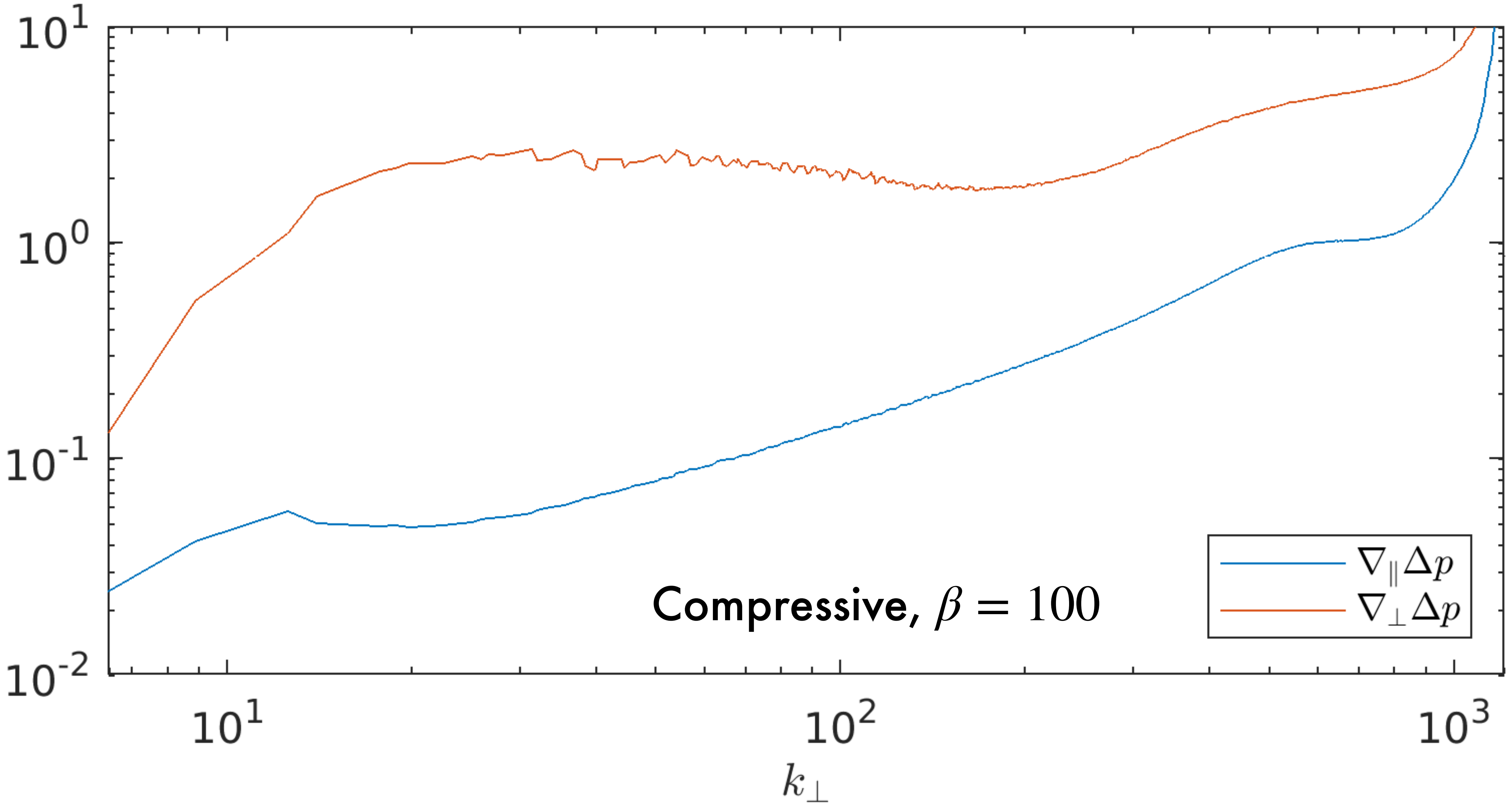

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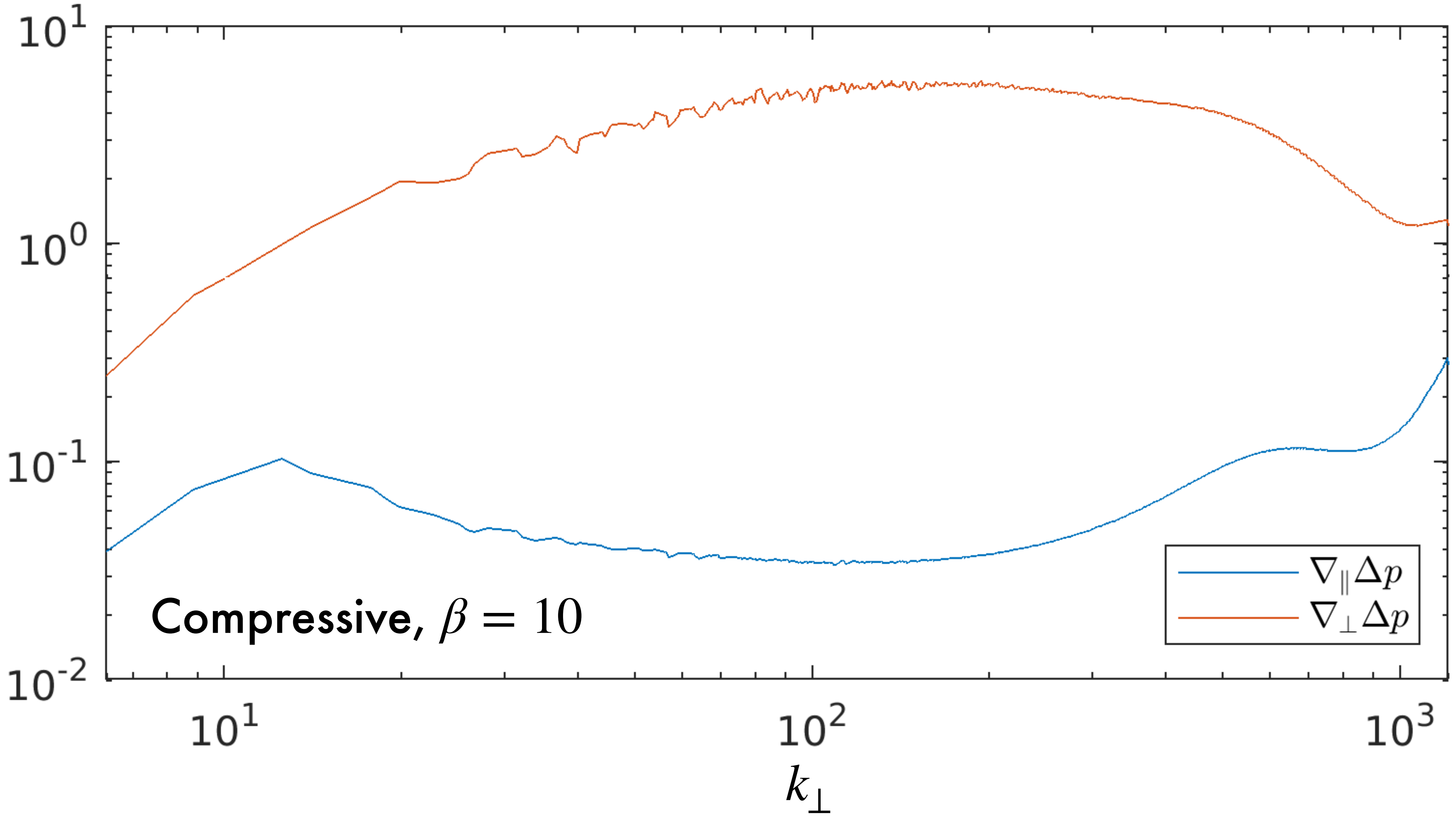
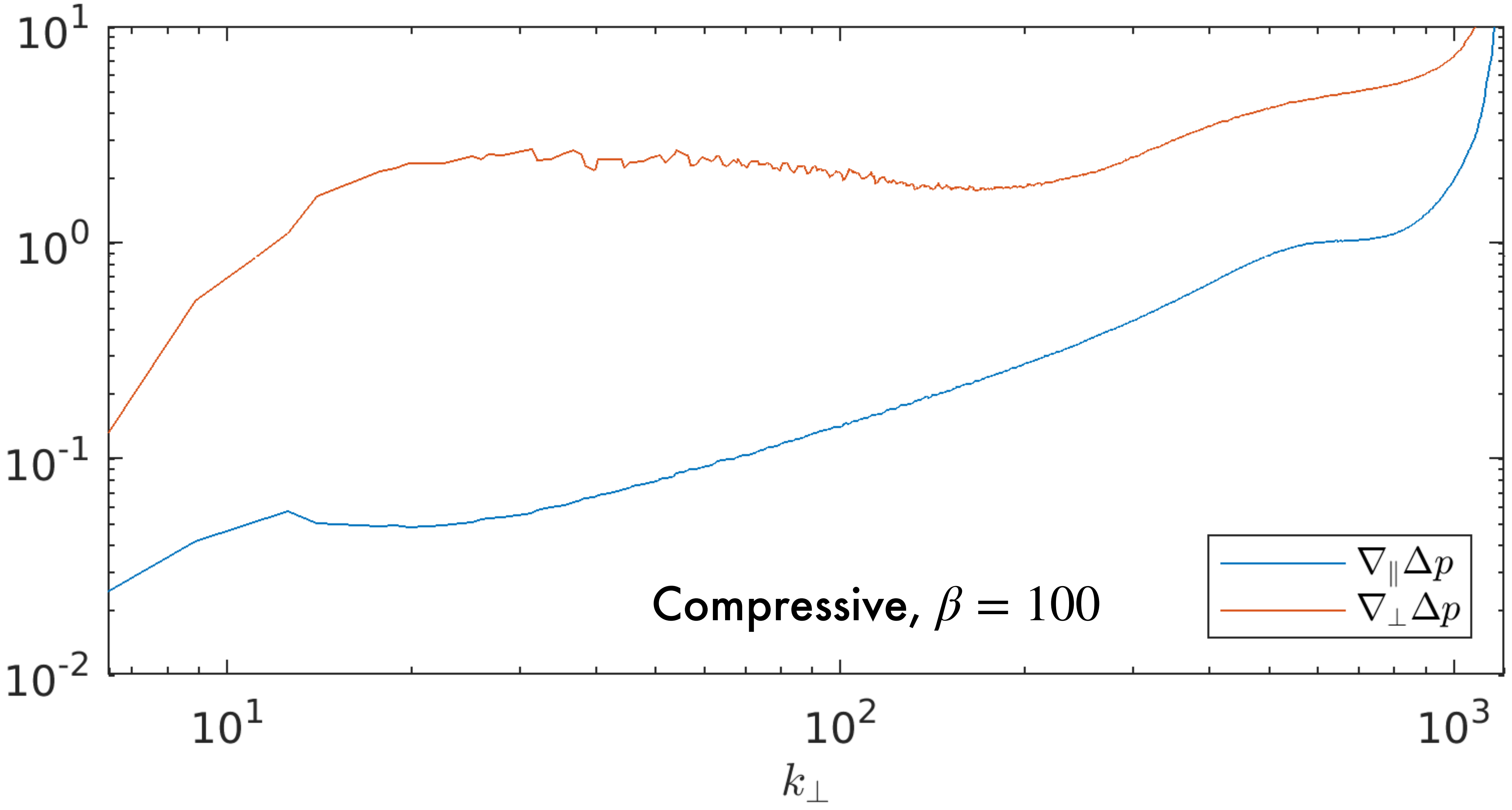


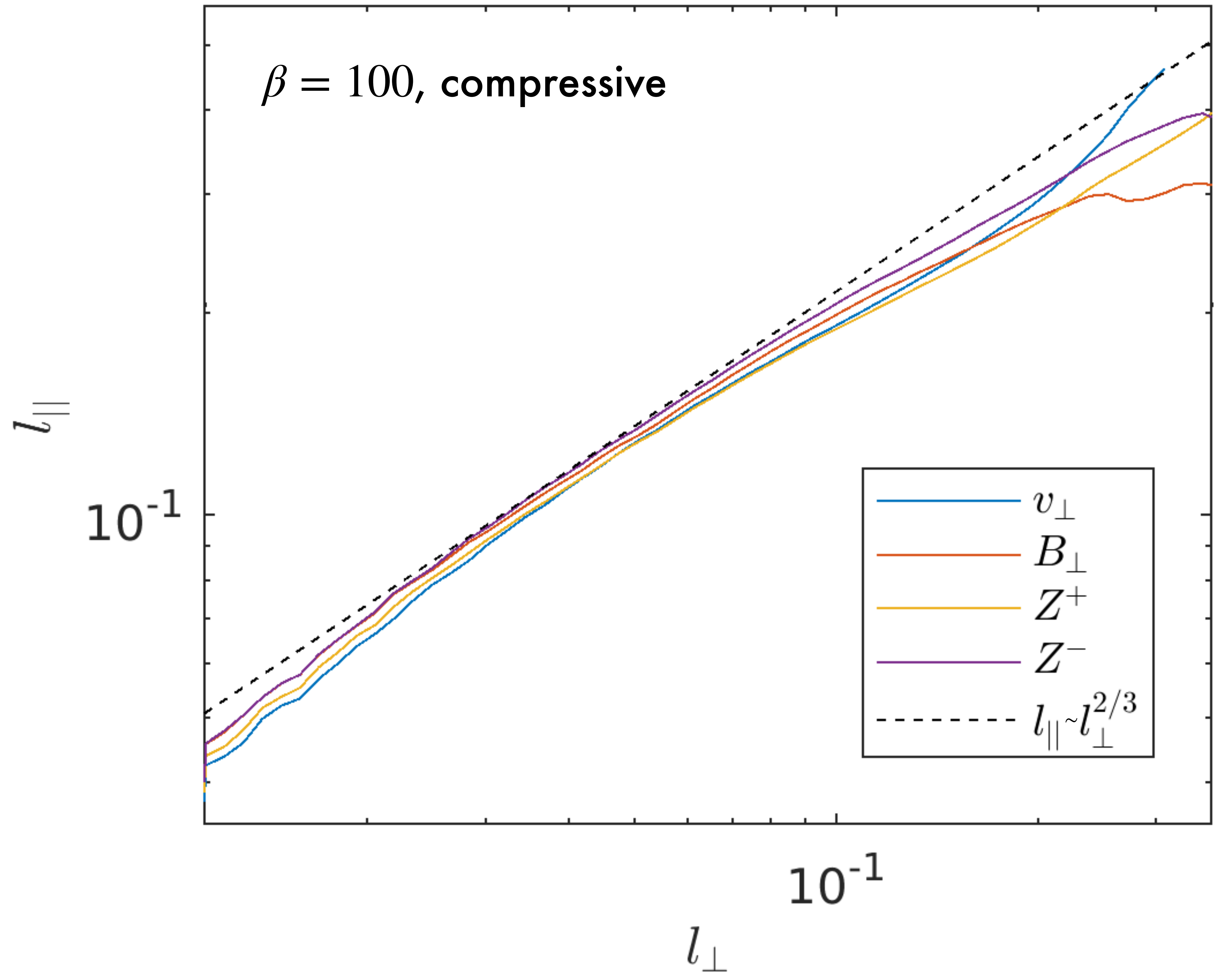
→ AWs cannot maintain large $\nabla_\parallel \Delta p$ against heat fluxes



→ Suggests compressive source

Compressive driving: Spatial anisotropy

→ AWs obey critically balanced $l_{\|} \sim l_{\perp}^{2/3}$ scaling very well



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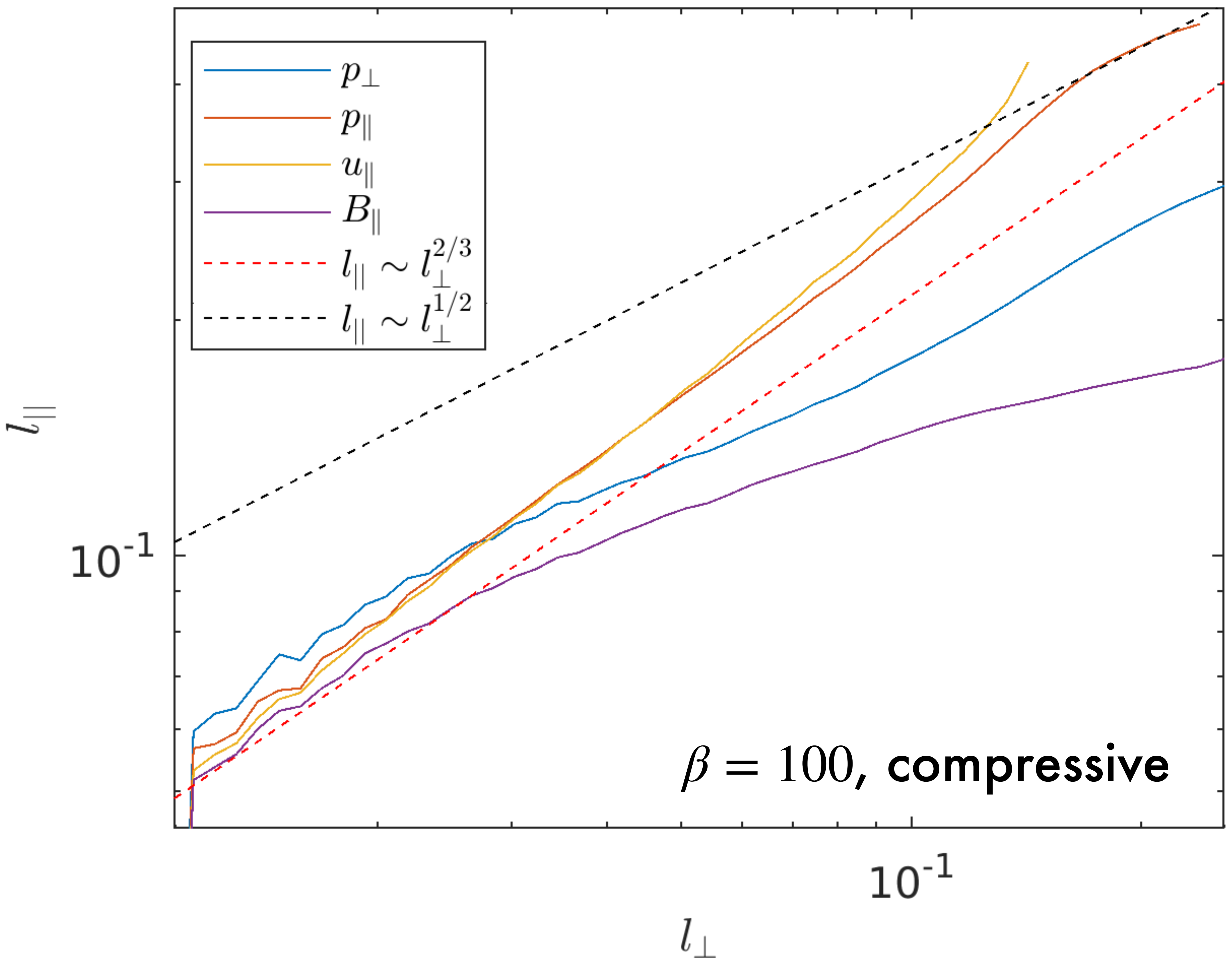
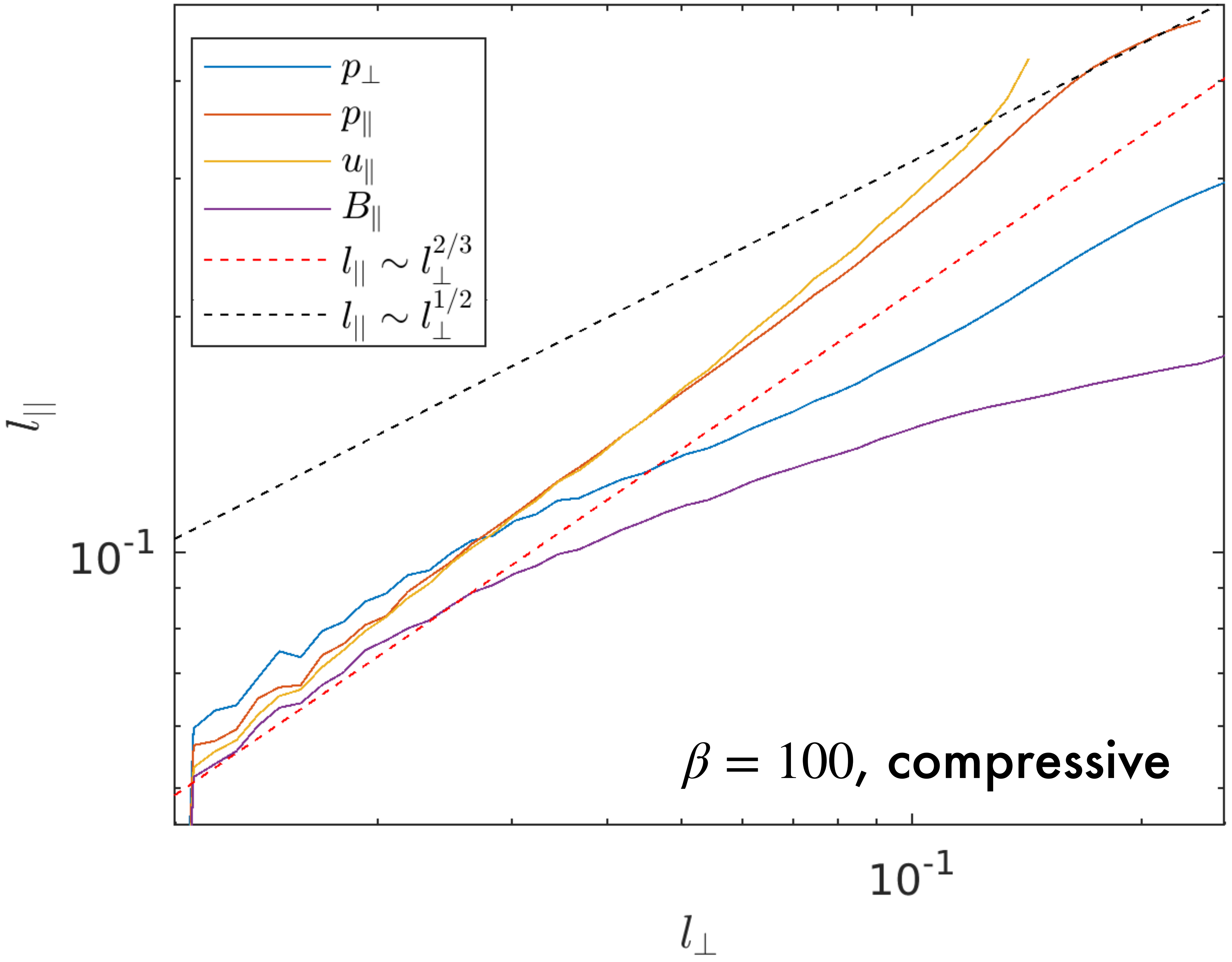


→ $p_\parallel$ and $u_\parallel$ structures (associated w/ IAs) appear to also follow AW CB scaling very well

Compressive driving: Spatial anisotropy



→ $p_\|$ and $u_\|$ structures (associated w/ IAs) appear to also follow AW CB scaling very well

→ $p_\perp$ and $B_\|$ spectra follow each other, but don't quite agree with any Alfvénic scalings

Summary/To-Do

Both compressively driven and high β turbulence are well regulated by magneto-immutability, suggesting immutability can compete with heat fluxes.

Immutability does not appear to interfere with heat flux driven damping.

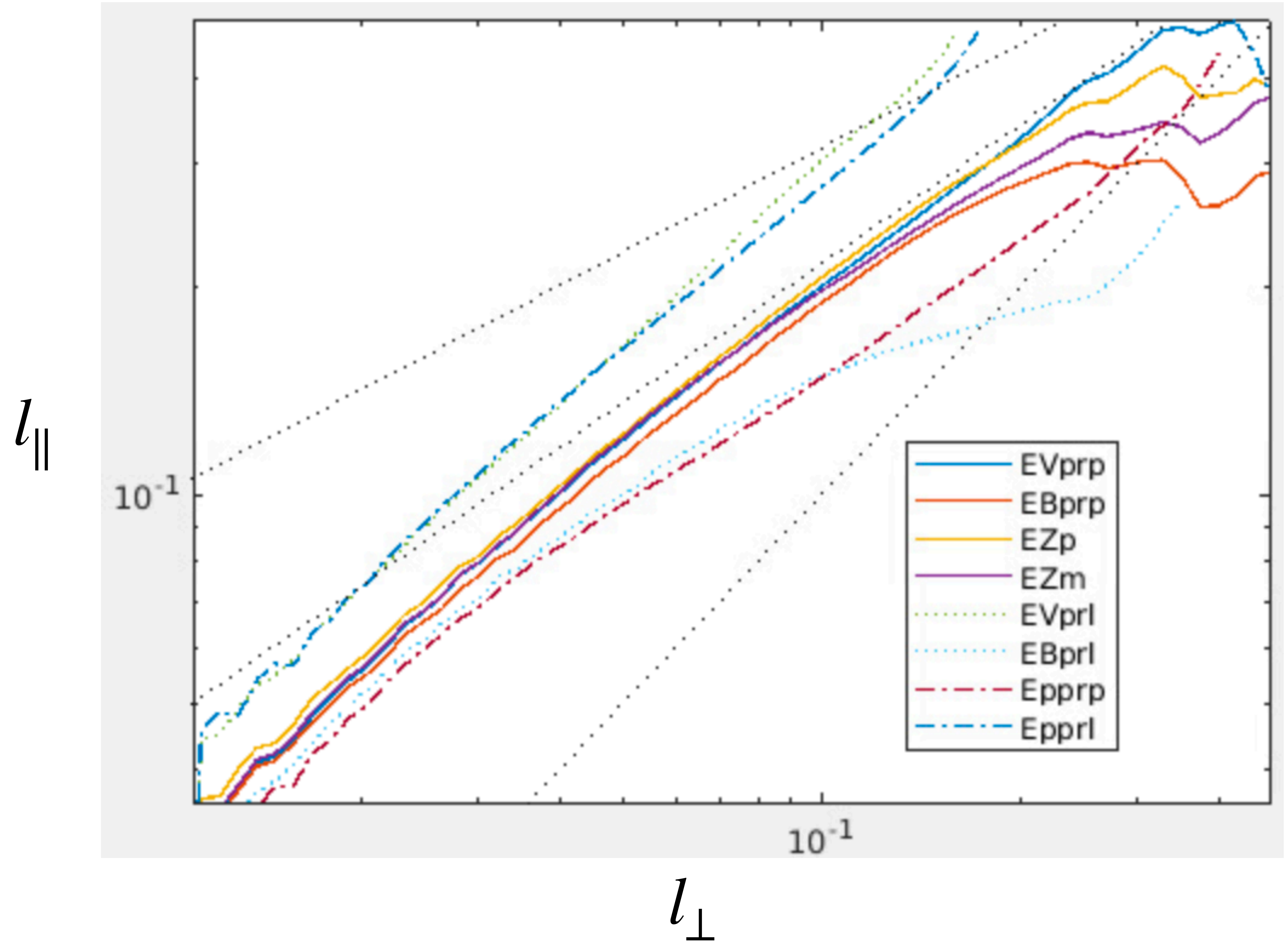
Surprisingly, IA-dominated quantities follow $l_\perp$, $l_\|$ scalings of critical balance.

Source of resilience of the magnetic spectrum not yet clear. (Fast, NP modes?)

→ Eigenmode decomposition: understand which modes appear to be dominating the energy partition at each β, forcing

→ Investigate role of heat fluxes by comparing contribution with other transfer functions.

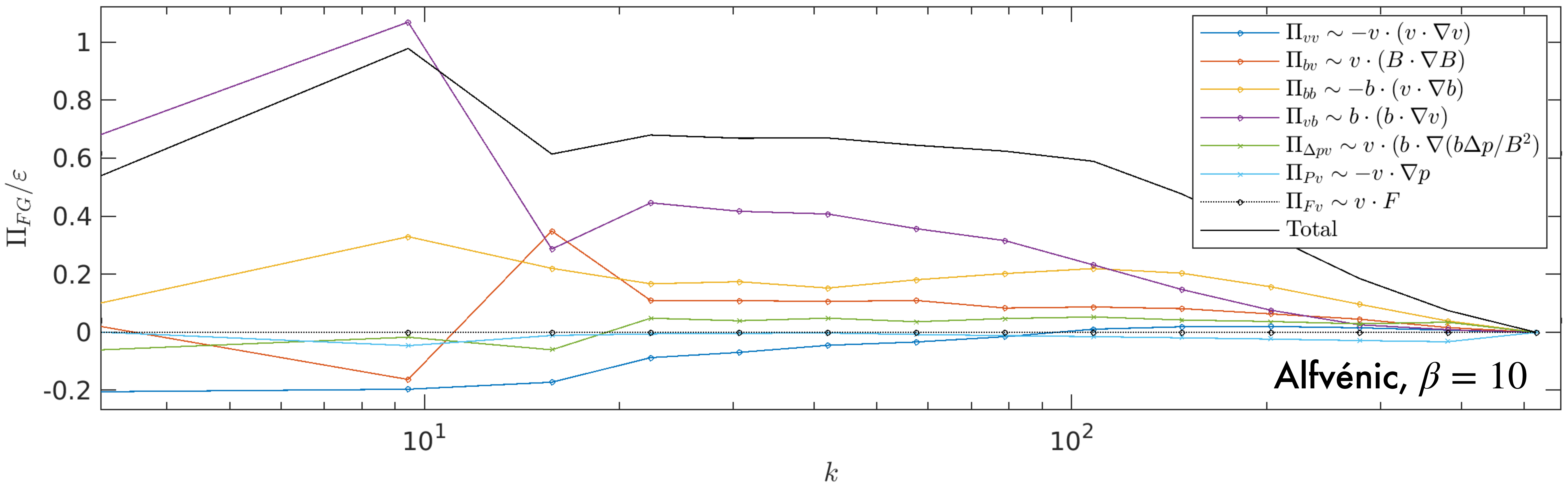
Alfvénic driving: Spatial anisotropy (bonus)



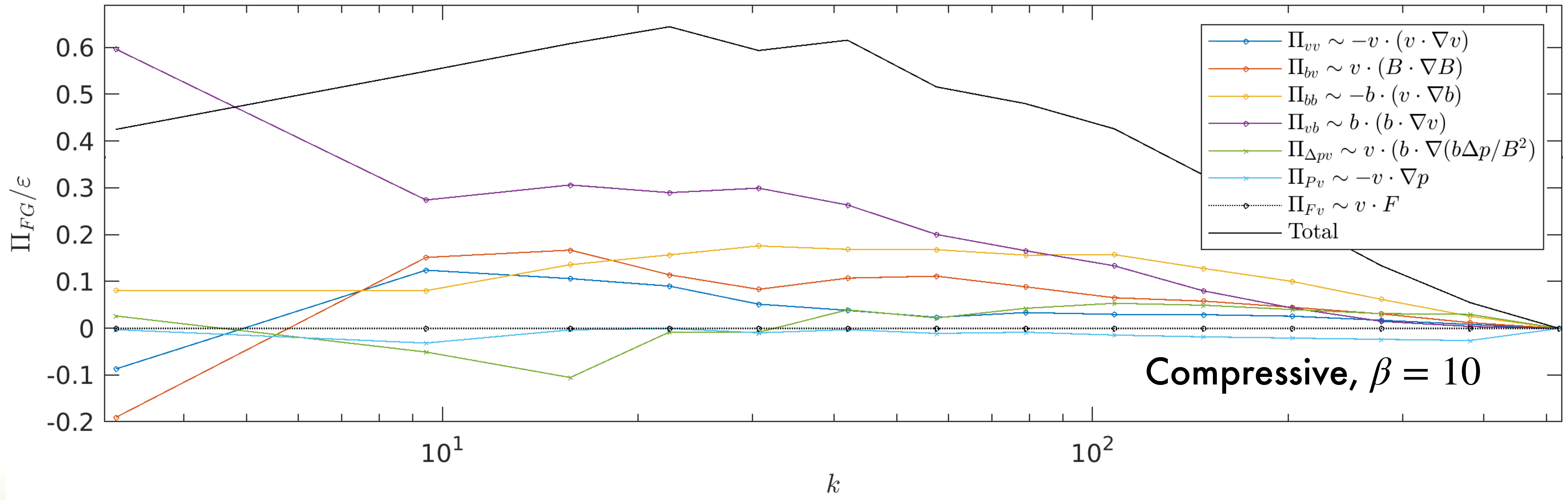
→ $p_{\perp}$ and $B_{\parallel}$ spectra approach CB scaling, with $p_{\parallel}$ and $u_{\parallel}$ only slightly differing from compressive driving

→ Spatial anisotropy does not appear to be very sensitive to beta (explains magnetic spectrum?)

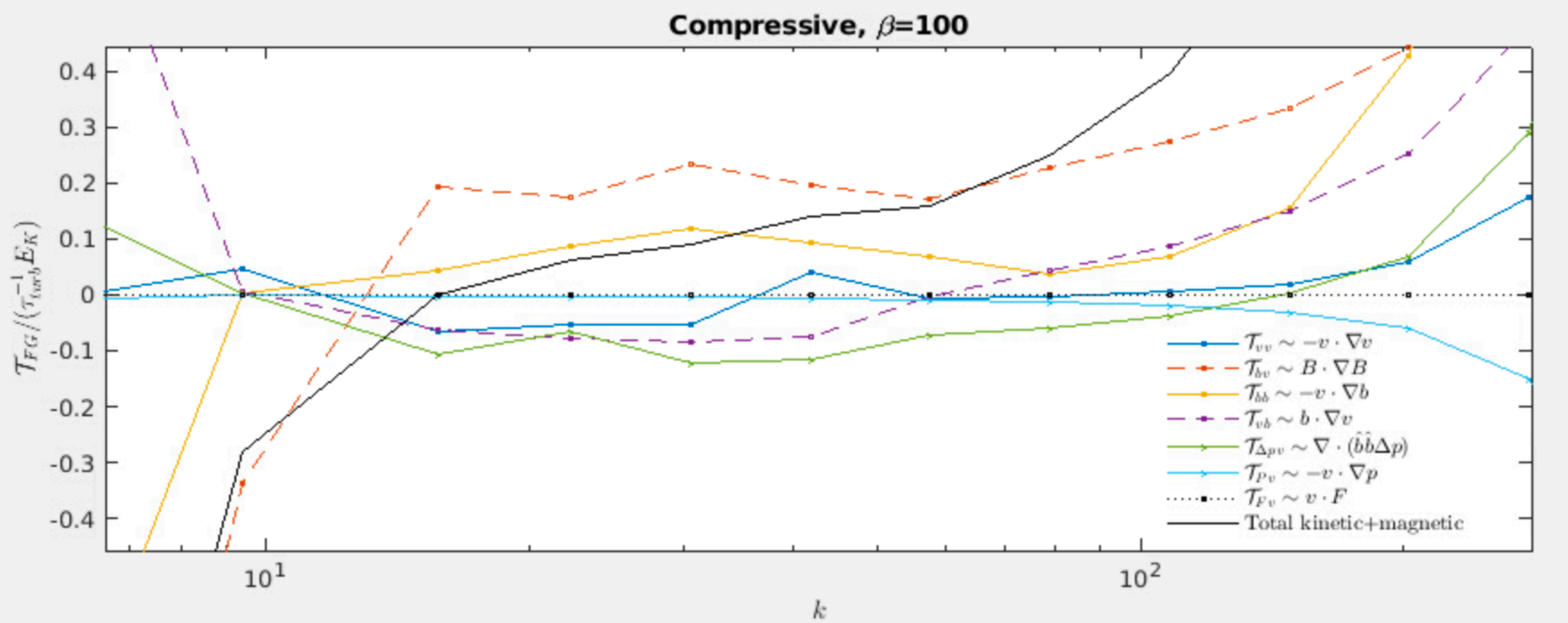
Alfvénic vs Compressive driving: Fluxes (bonus)



Hint of dissipation in compressive $\beta = 10$ run?

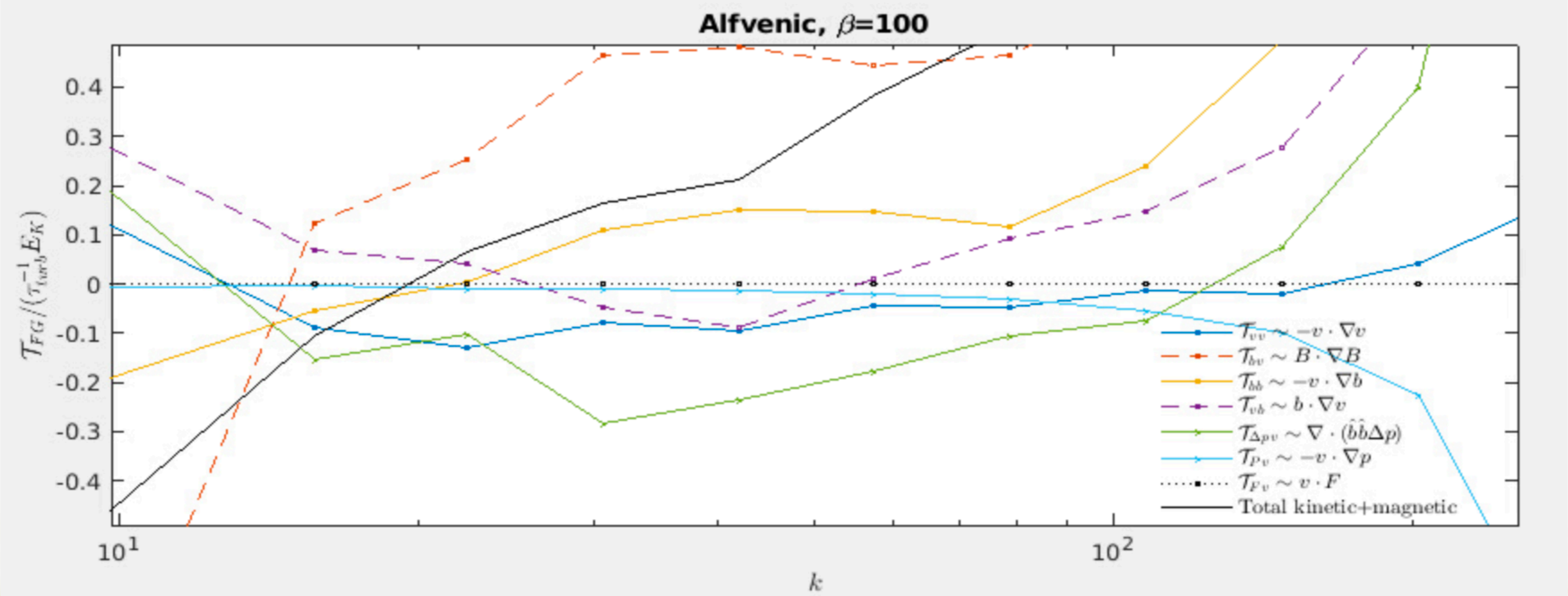


Alfvénic vs Compressive driving: Transfer functions (bonus)



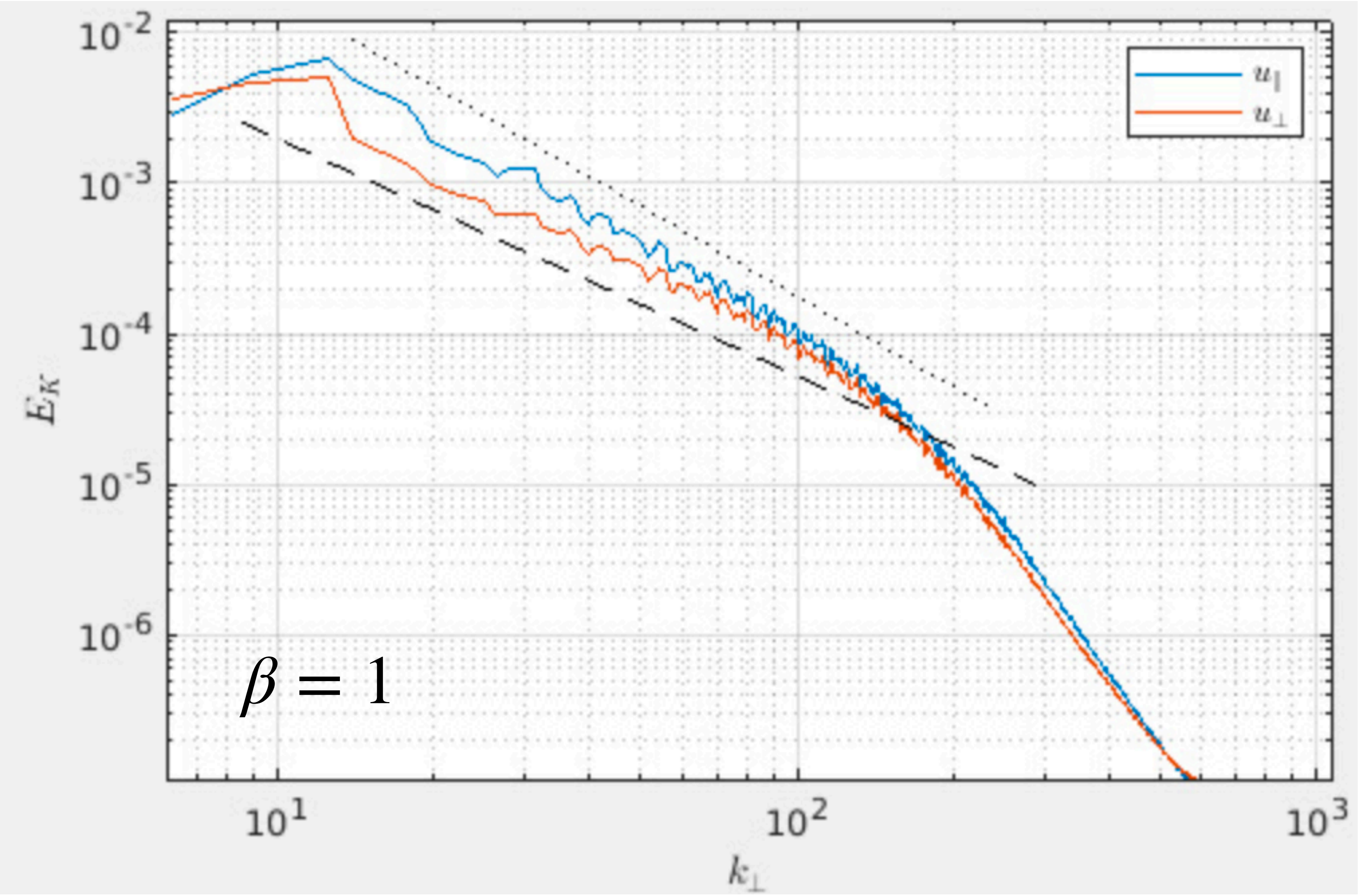
Seems like u^2 spectrum is not very sensitive to viscous dissipation yet.

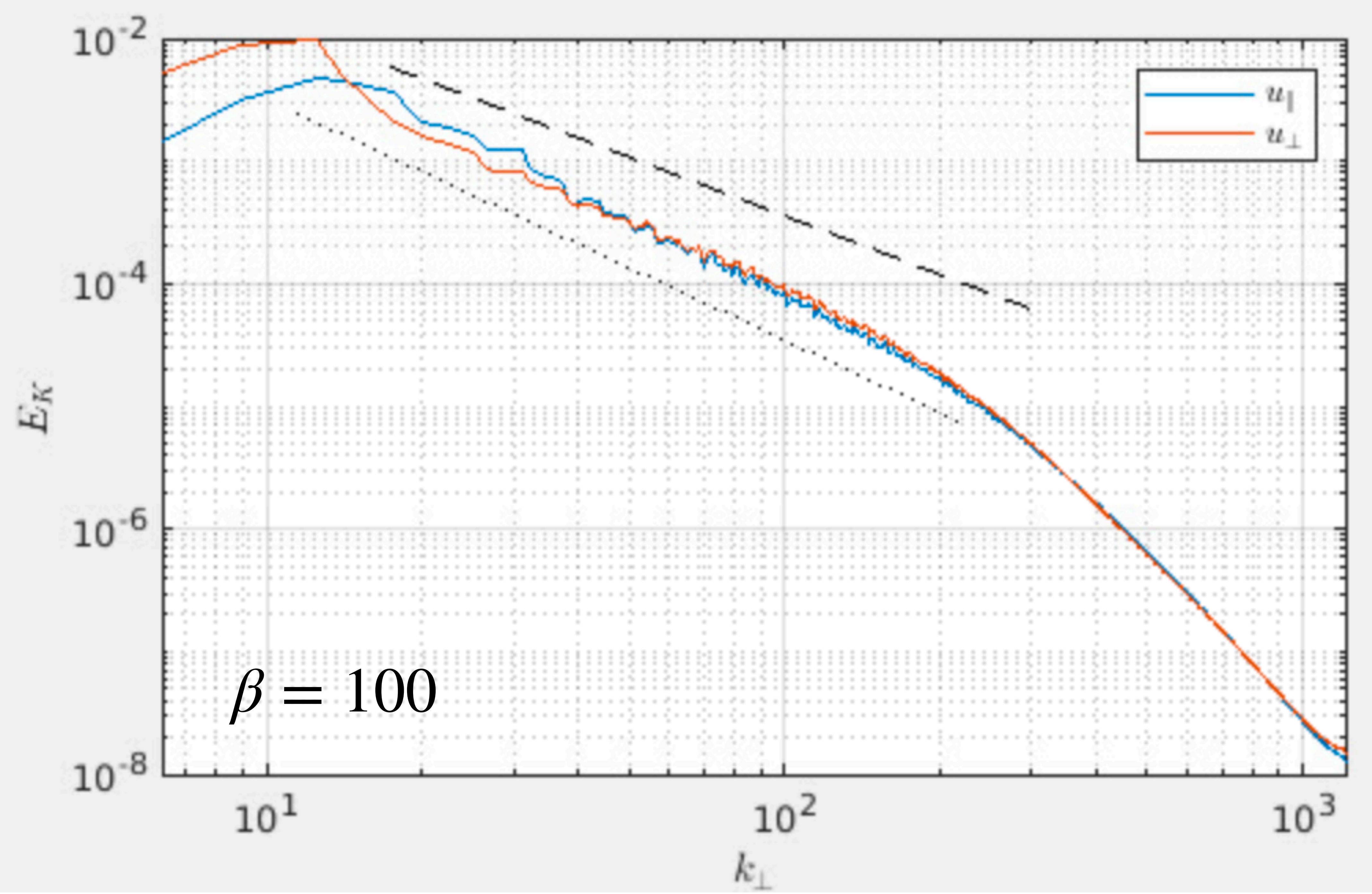
$\mathscr{T}_{\Delta pu}$ for compressive $\beta = 10$ run is $\sim 75\,\%$ of $\beta = 100$ run.



Compressive driving: $u_{\|}$ vs $u_{\perp}$ spectra (bonus)

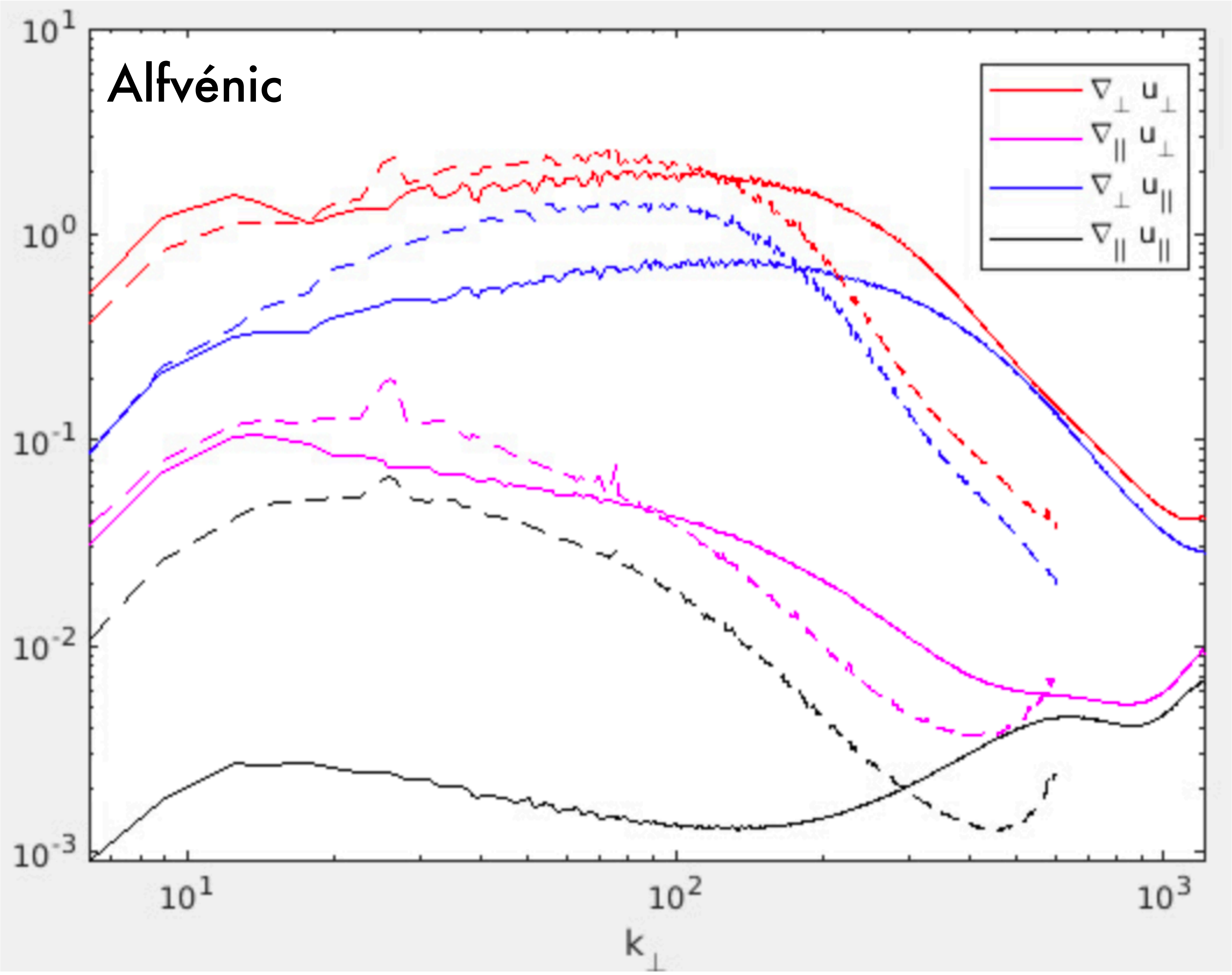
→ Spectrum of $u_{\|}$ is steeper than $u_{\perp}$

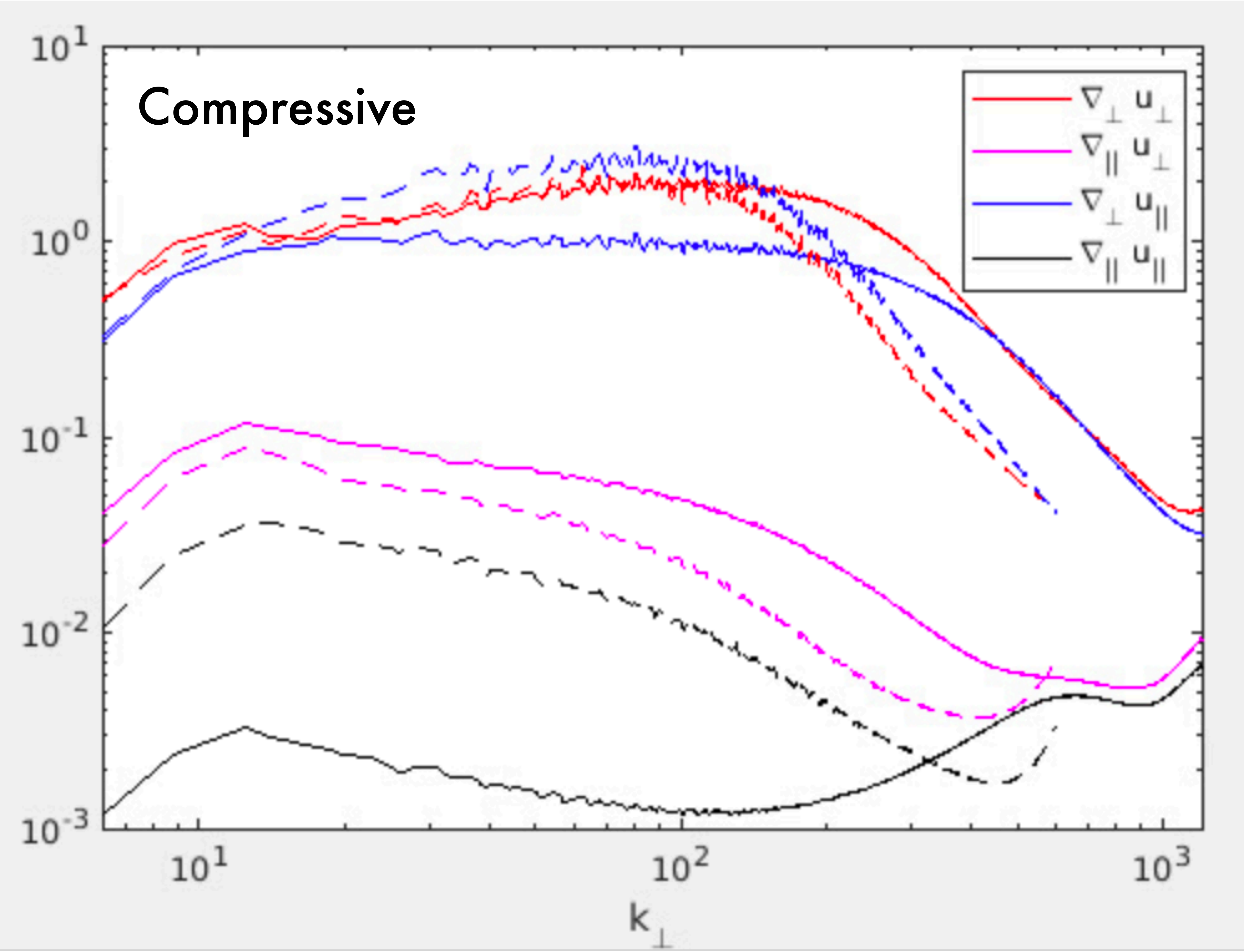




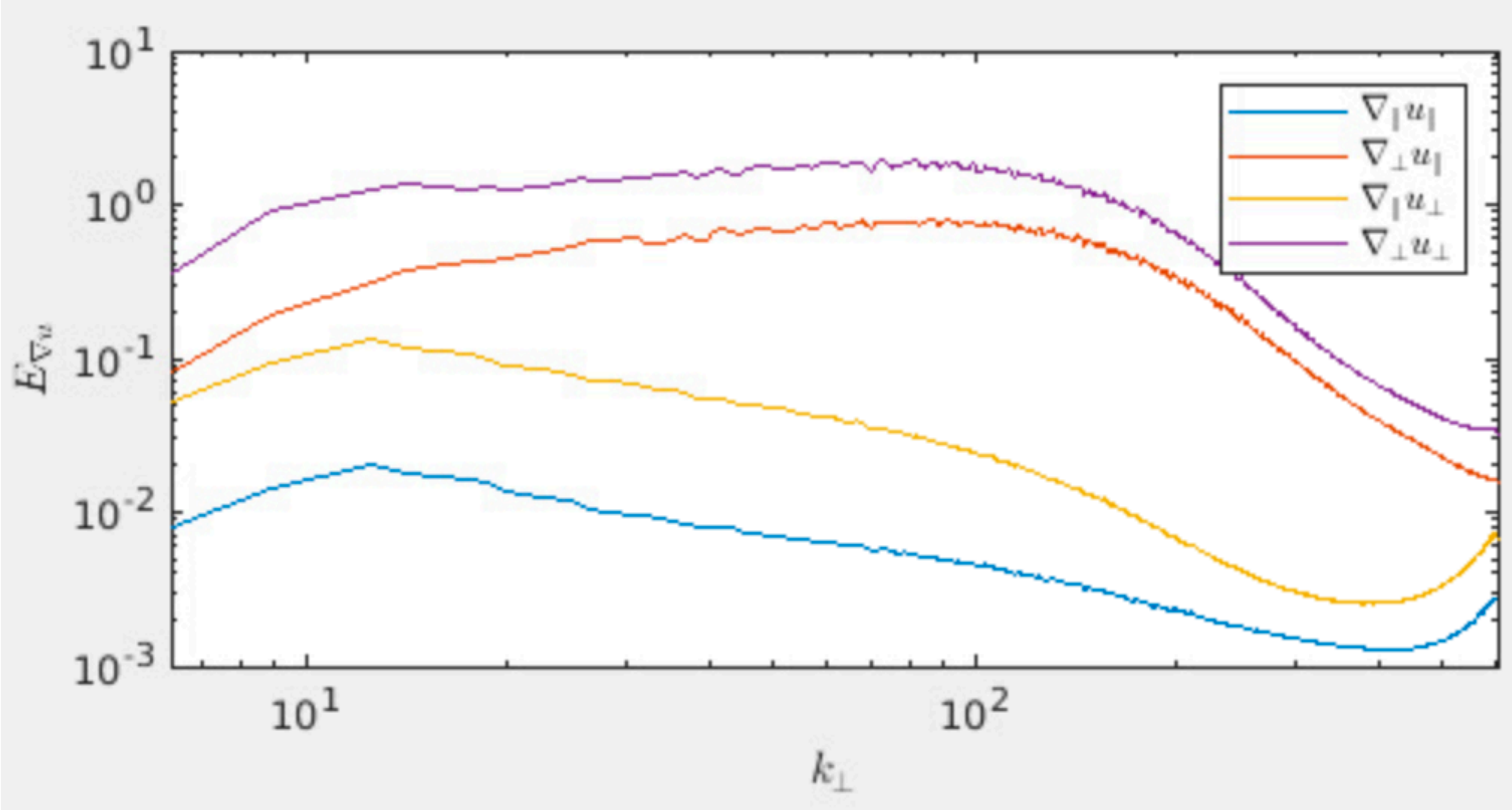
→ Difference between spectra appears insensitive to β

Compressive driving: Rate of Strain $\beta = 10$ (bonus)



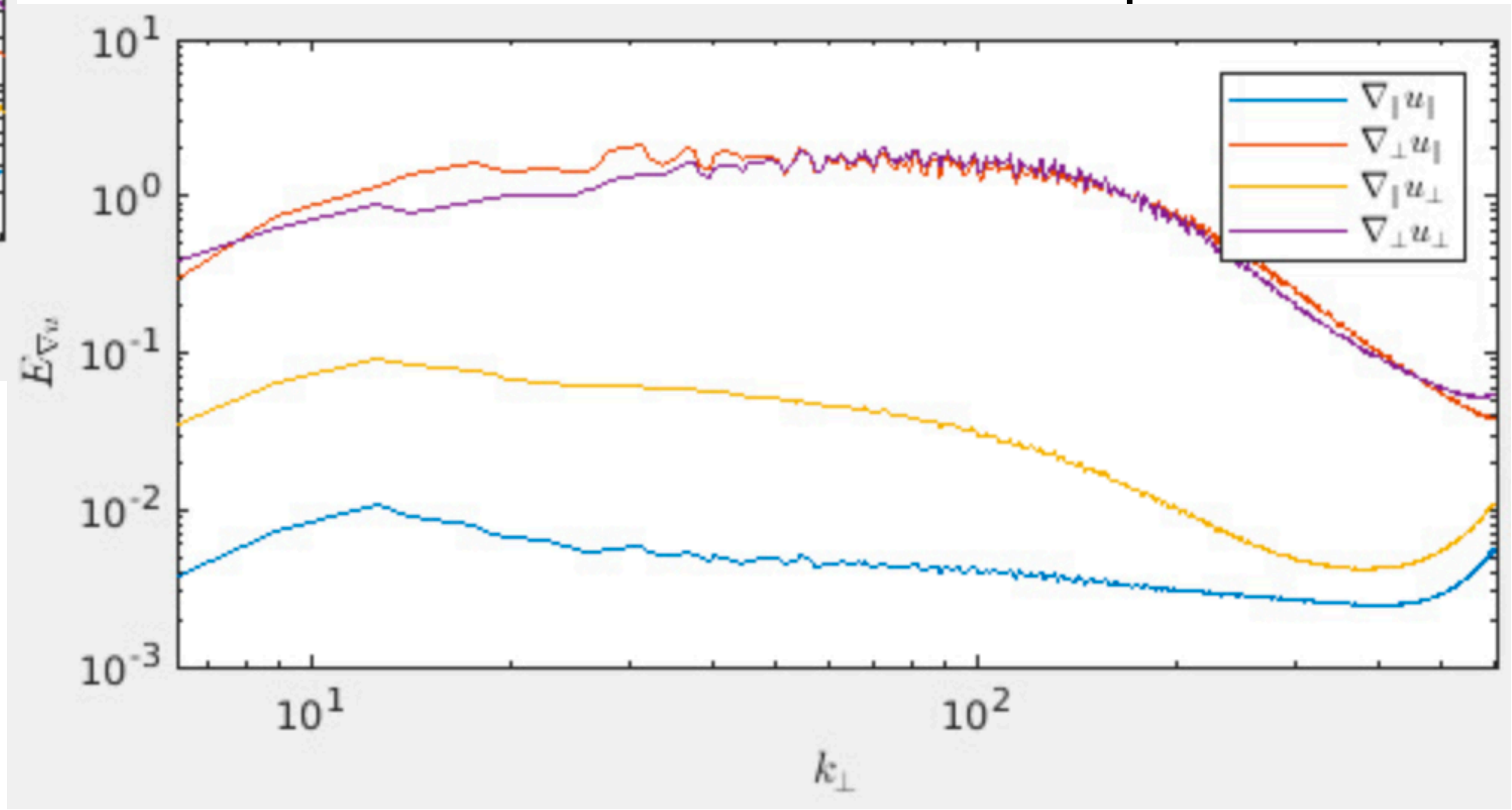


Compressive driving: Rate of Strain $\beta = 1$ (bonus)

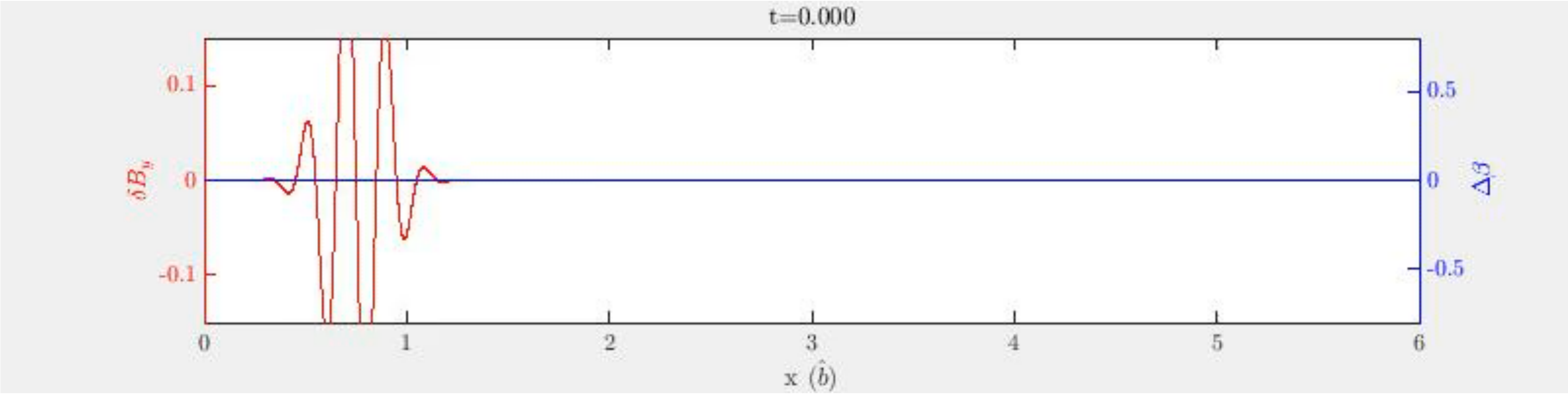


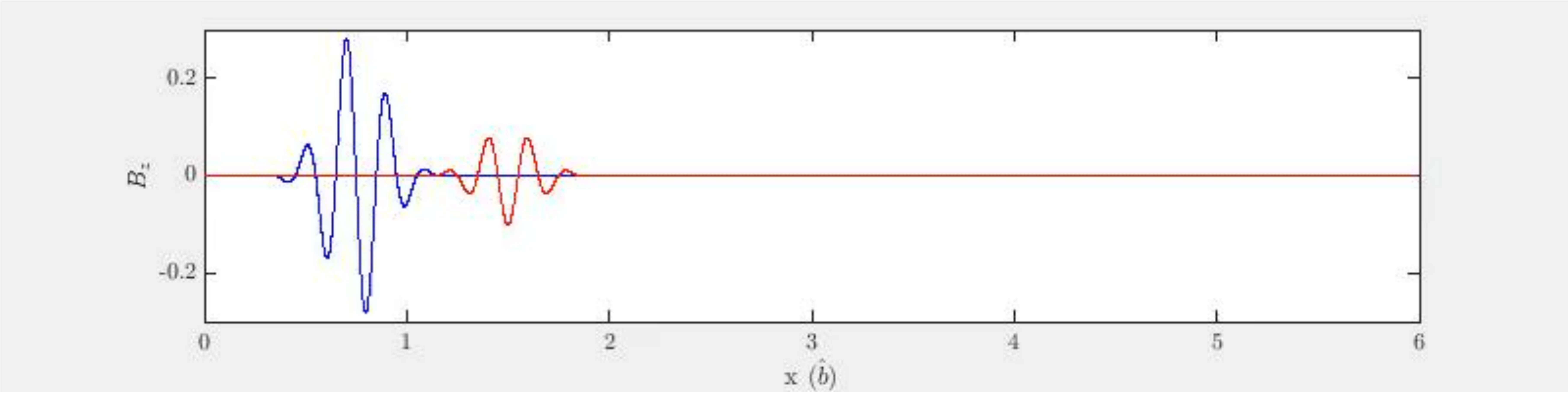
Alfvénic

Compressive



AW interaction: Pressure anisotropy (bonus)





AW interaction: Kinetic + magnetic energy (bonus)