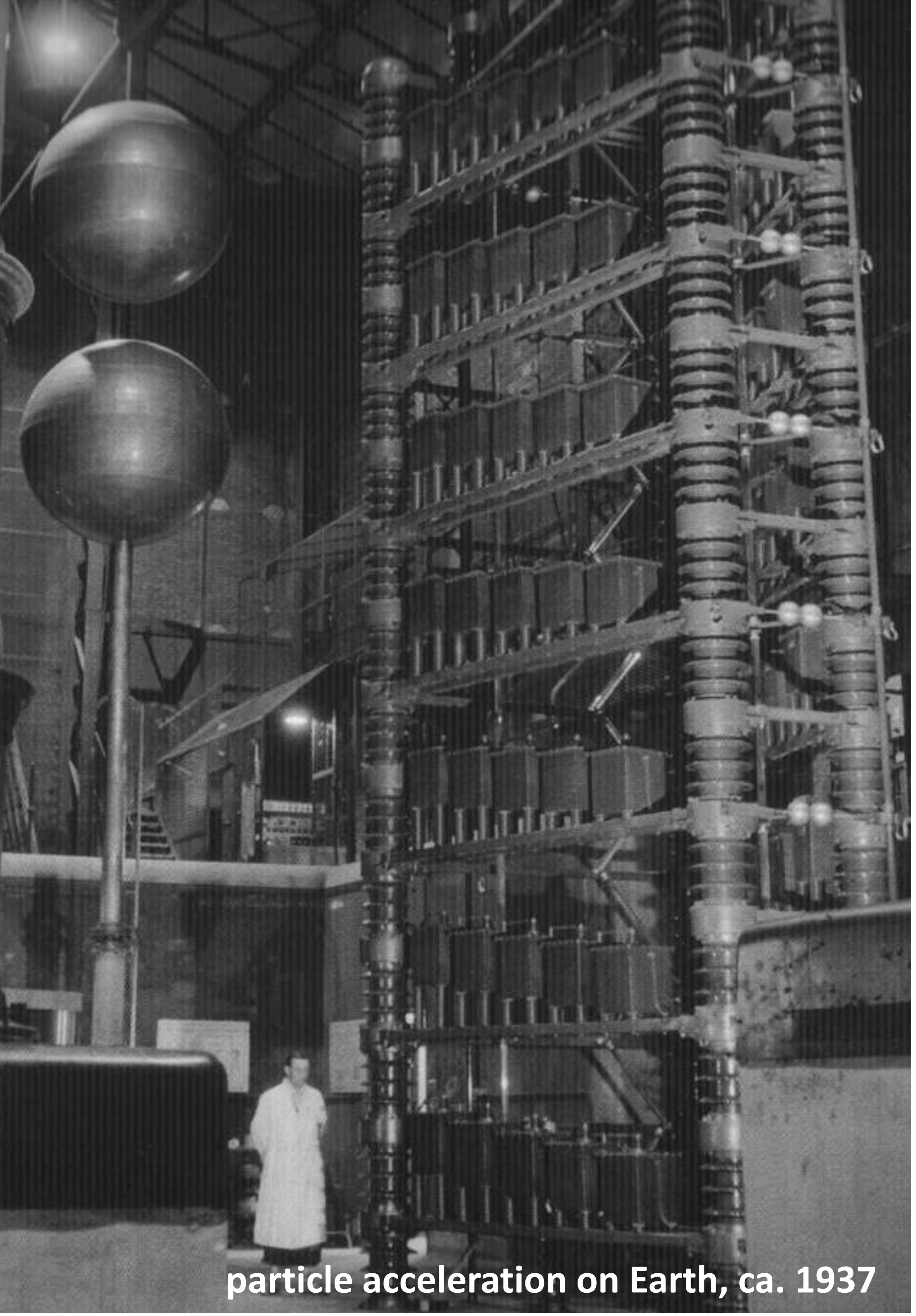
particle acceleration on Earth, ca. 1937

Particle acceleration in astrophysical, magnetized turbulent plasmas

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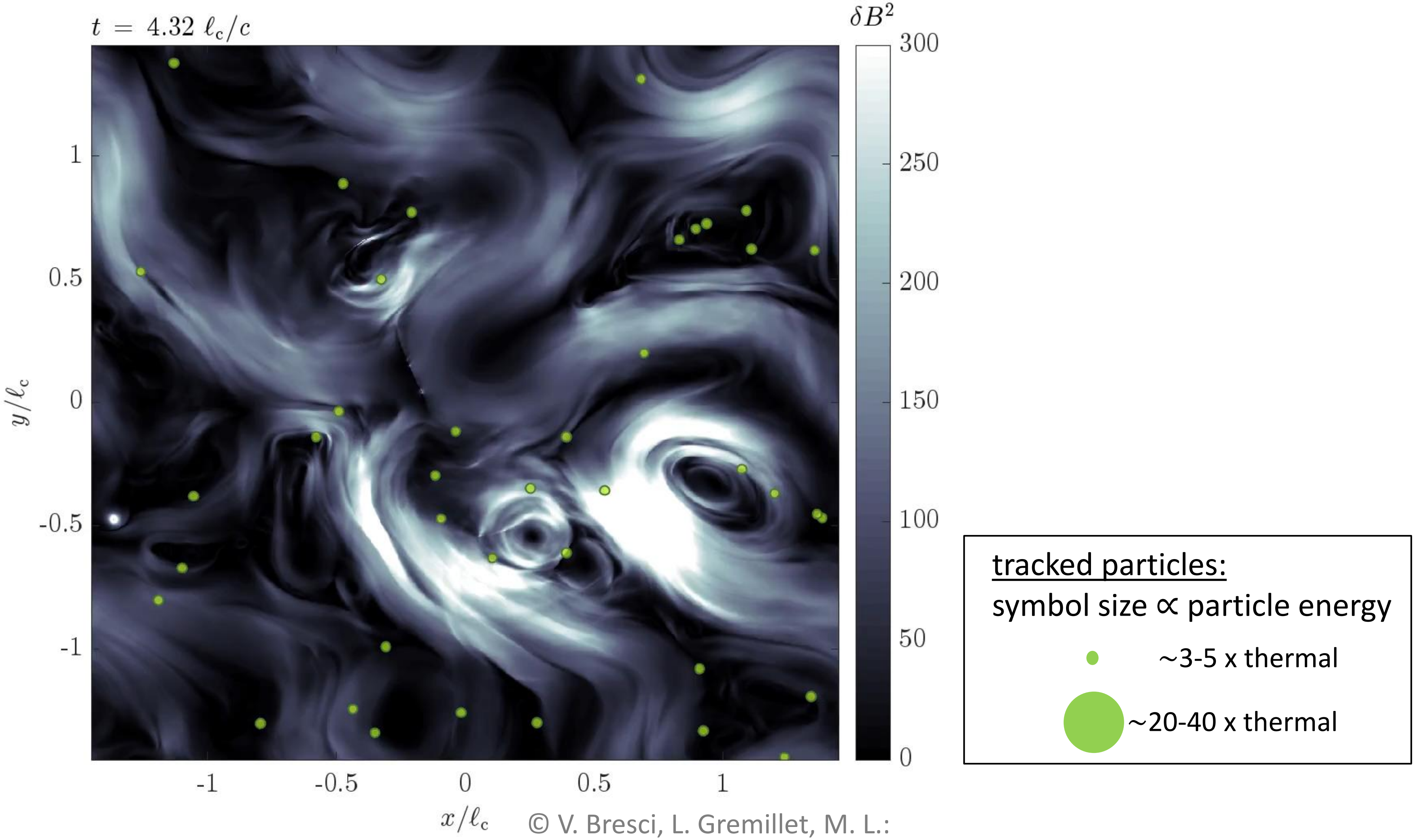
L. Comisso & L. Sironi (Columbia)

(Ideal) magnetized, turbulent astrophysical plasmas as particle accelerators

→ the Fermi picture[1]: perfectly conducting magnetized plasma composed of moving scattering centers…

⇒ particle acceleration on (ideal) motional electric fields $\boldsymbol{E} = -\boldsymbol{v_E} \times \boldsymbol{B}/c$



© V. Bresci, L. Gremillet, M. L.:
2D PIC, driven turb., e^+e^-, 10 000^2, $\delta B/B \sim 3$, $\sigma \sim 1$

Refs: 1. Fermi 49, 54

Probing turbulent acceleration with kinetic numerical simulations

→ recent breakthrough particle-in-cell simulations[1] of particle acceleration in magnetized, collisionless turbulence:

… in semi- (Alfvén $v_A \gtrsim 0.1\ c$) and fully-relativistic turbulence, at large amplitude $\delta B/B \sim \mathcal{O}(1)$

… relativistic eddy velocity ↔ magnetization parameter $\sigma \equiv \dfrac{B^2/4\pi}{n\,mc^2} = (u_A/c)^2 \sim 1$

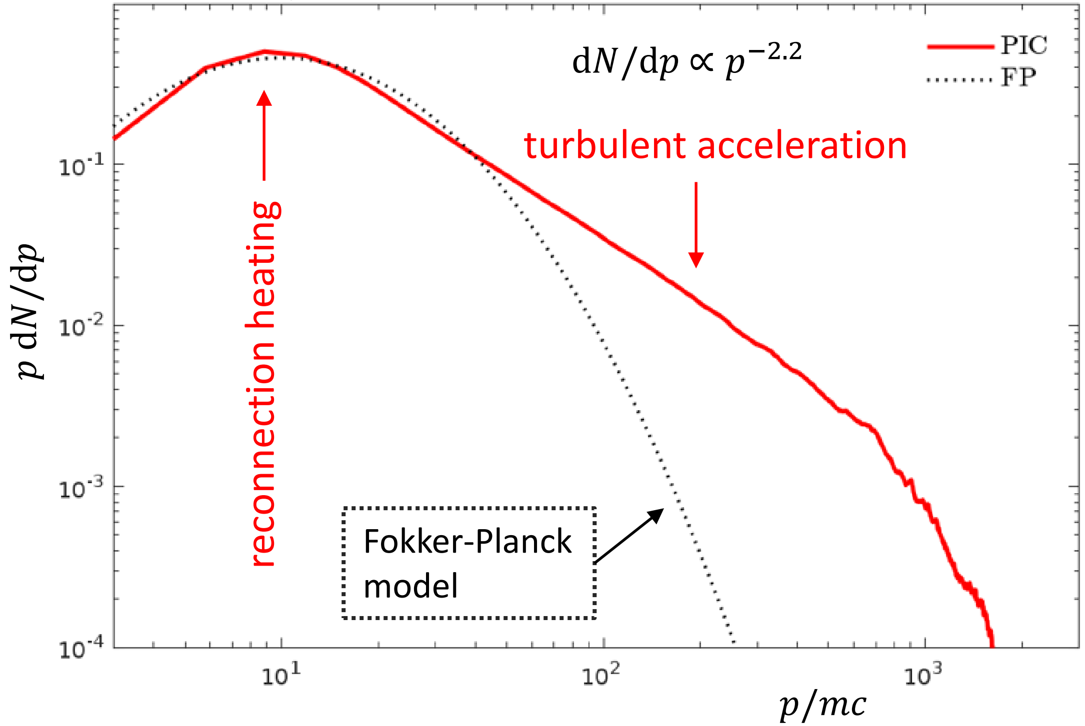
[note: acceleration timescale $t_{acc} \sim (\delta B/B)^{-2}\ \ell_c/\sigma c$ ⇒ rapid acceleration in large amplitude, fast turbulence]

→ findings:

1. two-stage particle acceleration:
   … injection by reconnection up to $p \sim \sigma\, mc$ …
   … then, particle acceleration in "ideal" fields

2. unexpected[2] emergence of powerlaws, spectrum $dN/dp \propto p^{-s}$ with $s \sim 2\ \dots\ 4$

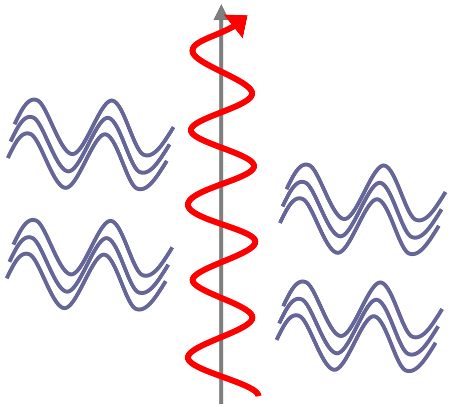
… signature of a rich phenomenology!



Refs: 1. Zhdankin+17,18,20,… Wong+ 19, Comisso+Sironi 18, 19, Nättilä + Beloborodov 20, Vega+20, … Bresci+22
(+ many MHD/hybrid sims: Dmitruk+03, Arzner+06, …, Isliker+17, Trotta+20, Pezzi+22)
2. discussion in M.L. + Malkov 20

The standard model for particle acceleration in turbulence: quasilinear calculations

→ Quasi-linear picture: particles gyrate around background magnetic field and collect the influence of random linear waves…
⇒ energization through resonant wave-particle interactions[1]



resonant interactions:
$k\, r_g \sim 1$

… momentum diffusion coefficient: $D_{pp} \sim \int \mathrm{d}\tau \ldots \langle \delta \boldsymbol{E}\left[\boldsymbol{x}(t+\tau),\, t+\tau\right]\, \delta \boldsymbol{E}\left[\boldsymbol{x}(t),\, t\right]\rangle$

$$\sim \int \mathrm{d}\boldsymbol{k} \ldots \delta\left(k_\| v_\| - \omega + n\, c/r_\mathrm{g}\right) \langle \delta B_{\boldsymbol{k}}^2 \rangle \propto \frac{\langle \delta B^2\rangle}{B^2} \frac{v_\mathrm{A}^2}{c^2}\, p^q$$

… phenomenological applications: solve Fokker-Planck with D_{pp} as input

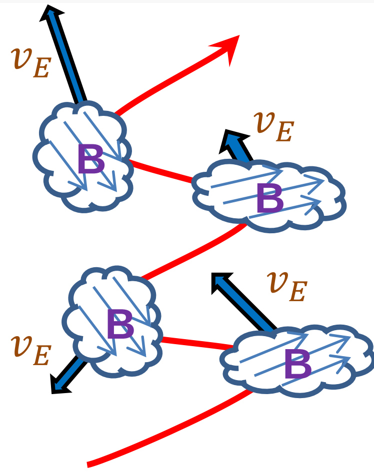
→ however:

… fails to reproduce: scaling with σ, scaling with momentum, and energy distributions seen in PIC[2]

… in anisotropic turbulence, resonances are washed out[3]… acceleration becomes non-resonant?
… +validity of random phase approximation[4]: random phase approximation ⇒ no structures

Refs:
1. e.g. Kennel + Engelmann 66, …, R. Schlickeiser 02 + refs;
2. e.g. Zhdankin+17,18,19, Comisso+Sironi19,20, …, Bresci+22
   see however: Wong+19, Zhdankin20
3. Chandran 00, Yan+Lazarian 02, …, Xu+Lazarian 20, …
4. Maron+Goldreich 01, Grošelj+19, Gan+22, Fu+23,

Particle acceleration à la Fermi, implementation in a random velocity flow



→ **original Fermi model:** discrete interactions with moving magnetic scattering centers (structures)

… elastic interactions in scattering center frame where $\boldsymbol{E} = 0$

→ **Fermi-type acceleration in a large-scale flow:**

… many models[1], a direct connection to structures on various scales…

… important: acceleration controlled by the shear of the velocity flow $\partial_\alpha {v_E}^\beta$, with $\boldsymbol{v_E} = c\, \boldsymbol{E} \times \boldsymbol{B}/B^2$

… in ideal MHD conditions $\boldsymbol{E}$ vanishes in (comoving) frame moving at $\boldsymbol{v_E} \Rightarrow$ no acceleration in absence of shear…

→ **implementation in turbulence[2]:**

… generalize Fermi approach : follow particle momentum in the (non-inertial) frame where $\boldsymbol{E} = 0$

… in that frame, energy variation ∝ non-inertial forces characterized by velocity shear of $\boldsymbol{v_E}$

$$\frac{\mathrm{d}\gamma'}{\mathrm{d}\tau} = -\Gamma^0_{\ a\,b}\,\frac{p'^a\,p'^b}{m^2c^2}$$

… **inertial forces:** $\Gamma^0_{\ a\,b} \propto \partial_a v_{E\,b}$

… γ' comoving particle Lorentz factor

connection to intermittency: gradients become non-Gaussian on small scales

Refs: 1. Fermi 49,…, Bykov+Toptygin 83, Ptuskin 88, Chandran+Maron 04, Cho+Lazarian 06, Ohira 13, …

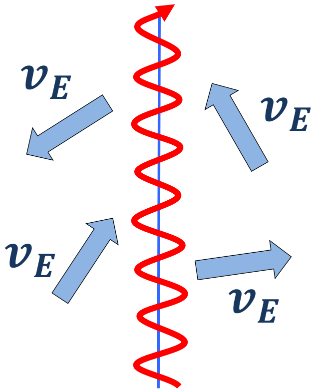
2. M.L. 19 [PRD 99, 083006 (2019)], 21 [PRD 104, 063020 (2021)]; see also previous works by Webb 85, 89

The crucial element in Fermi-type particle acceleration: particle transport

→ in Fermi-type scenarios: $\boldsymbol{E} = -\boldsymbol{v}_E \times \boldsymbol{B}/c \Longrightarrow$ particles must perform cross-field transport to gain energy…

… generic solution: include turbulence to sustain diffusive transport
… other solutions: include forces (external or drift-type, e.g. curvature, gradient etc.)

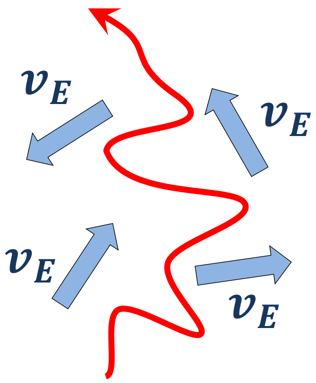
→ approximate description of particle transport: some generic cases,



gyration around a strong background magnetic field

e.g.: weak coupling to turbulence, perturbative approximation…

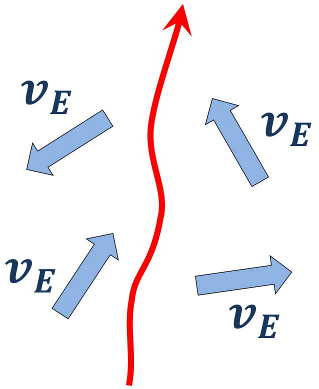
example: quasilinear theory in turbulent accceleration



diffusive transport in turbulence

example: diffusive shock acceleration, shear acceleration

models: simplified description as rectilinear transport between two scattering events, or distinction between parallel and perpendicular diffusion…



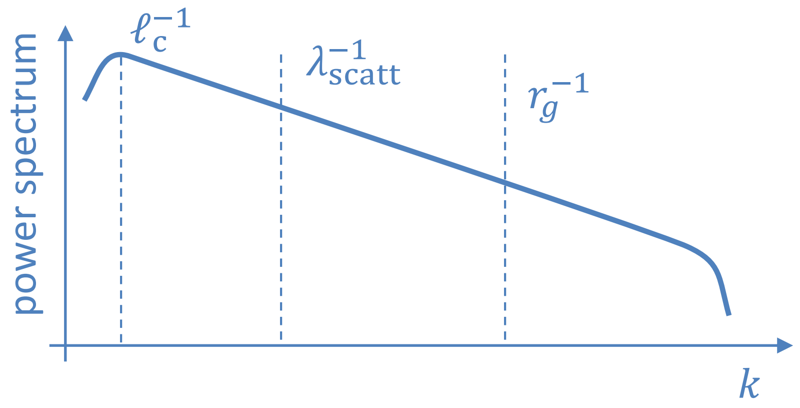
near-ballistic regime

e.g.: pitch-angle deflection w/o spatial diffusion, transport in small-scale turbulence, in absence of mean field…

example: relativistic shock acceleration…

Generalized Fermi acceleration: interaction with large scale modes

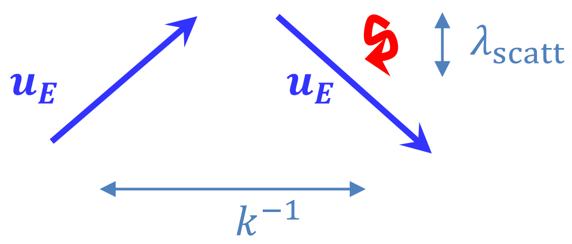
→ different scales act differently:



… e.g.: for $r_g/\ell_c \to 0$ adiabatic limit (MHD)
for $r_g/\ell_c \to \infty$ decoupling from turbulence

… for large scale modes: k (mode wavenumber) $\ll \lambda_{\rm scatt}^{-1}$ (scattering m.f.p.) $\ll r_g^{-1}$ (gyroradius)

⇒ momentum diffusion coefficient[1]: $$D_{pp} \sim \underbrace{\frac{u_E^2}{c^2}\frac{p^2}{\lambda_{\rm scatt}/c}}_{\text{Fermi scaling}} \underbrace{\left(\frac{\lambda_{\rm scatt}}{k^{-1}}\right)^2}_{\ll 1}$$



… u_E gradient weak on scale $\lambda_{\rm scatt}$
⇒ inefficient acceleration in comoving frame…

⇒ “shielding” from large scale modes $k \ll \lambda_{\rm scatt}^{-1}$

Refs: 1. Bykov+Toptygin 83, Ptuskin 88, …, M.L. 19, Rieger 20 + refs, Demidem+20

Fermi acceleration in magnetized turbulence: interaction with modes on scales $\gtrsim r_g$

→ model for acceleration[1]:

... dominant contribution: modes on scales $l \gtrsim r_g$ ⇒ coarse-grained view on scale $\sim r_g$
... transport: gyration around local mean magnetic field lines ⇒ local gyro-average
↔ particles collect effect of structures while traveling along mean field defined on scale $\gtrsim r_g$

→ formally:

$$\frac{\mathrm{d}\gamma'}{\mathrm{d}\tau} = -\gamma' u'_\parallel \, \boldsymbol{a_E} \cdot \boldsymbol{b} - {u'_\parallel}^2 \, \Theta_\parallel - \frac{1}{2} {u'_\perp}^2 \Theta_\perp$$

energy change in local comoving frame

$u'_\parallel = \boldsymbol{p'} \cdot \boldsymbol{b}/mc$

$u'_\perp = \left[u'^2 - {u'_\parallel}^2\right]^{1/2}$

effective gravity along field line

$\boldsymbol{a_E} = u_E^\alpha \, \partial_\alpha \, \boldsymbol{u_E}$

velocity shear along field line

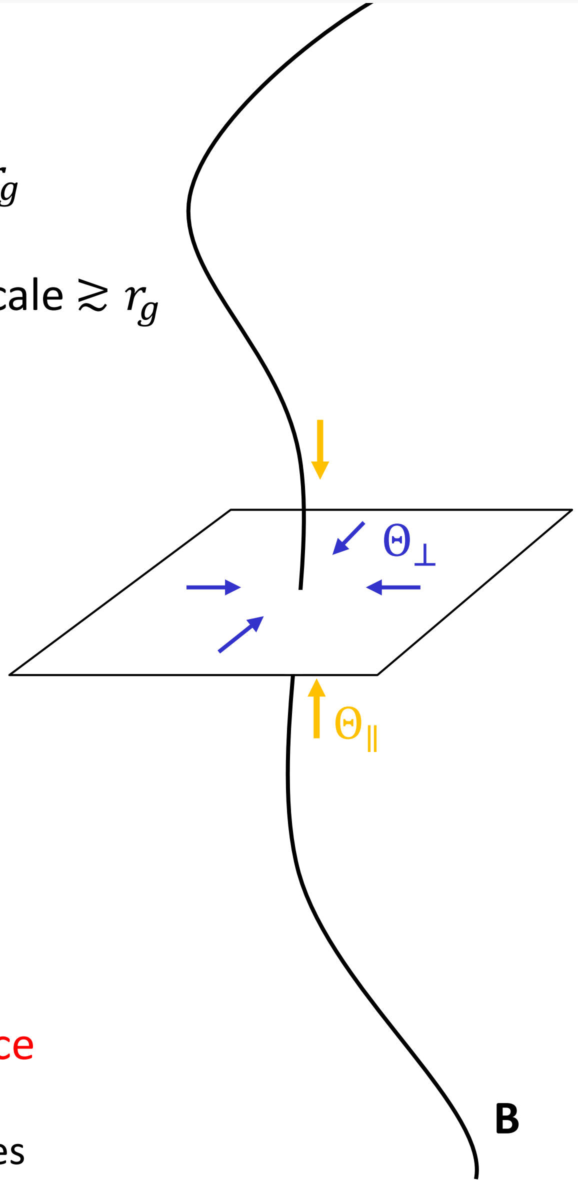
$\Theta_\parallel = b^\alpha b^\beta \partial_\alpha u_{E\,\beta}$

[Fermi type-B]
[field line curvature]

compression transverse to field line

$\Theta_\perp = (\eta^{\alpha\beta} - b^\alpha b^\beta) \partial_\alpha u_{E\,\beta}$

[Fermi type-A]
[magnetic mirrors]



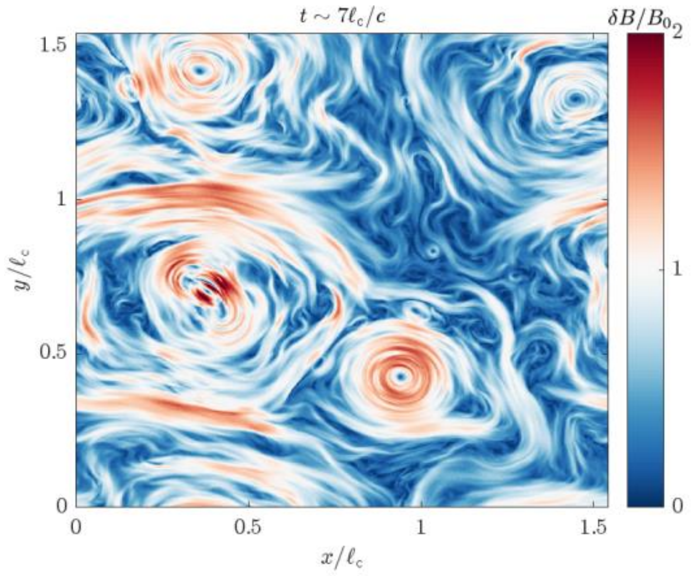
... terms $a_E \cdot B$, $\Theta_\parallel$ and $\Theta_\perp$ are random forces: ⇒ random walk in momentum space

... average over gyro-orbit: ~drift-kinetic theory in magnetic field coarse-grained on r_g scales

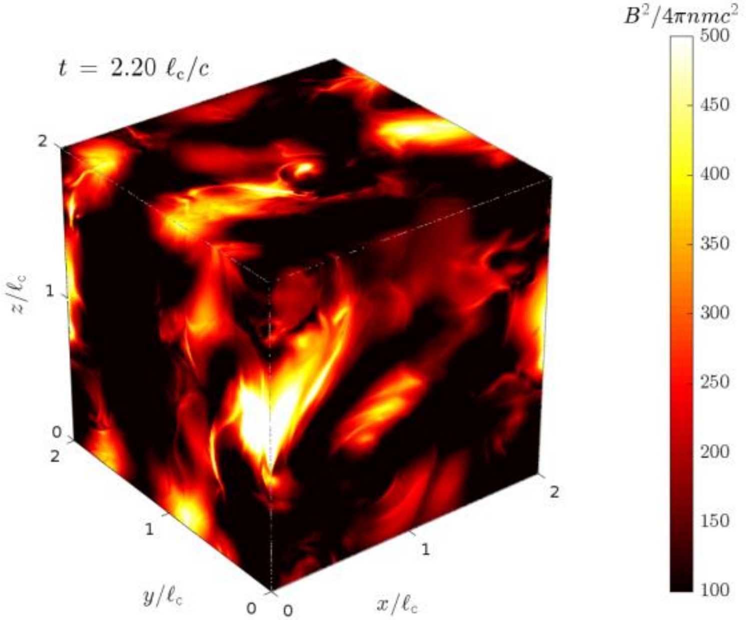
Refs: 1. M.L. 19 [PRD 99, 083006 (2019)], 21 [PRD 104, 063020 (2021)]

Fermi acceleration in magnetized turbulence: comparison to numerical experiments

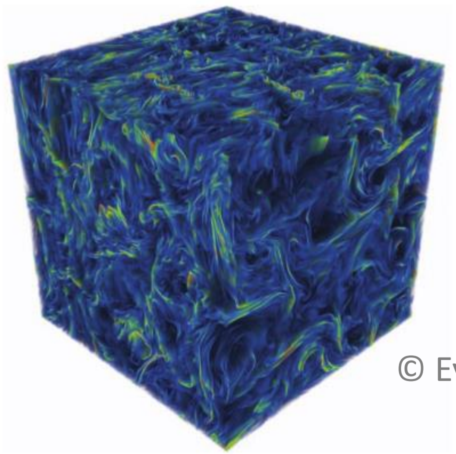
2D PIC simulation[1]: forced and decaying, $10\,000^2$, e^-e^+, $\delta B/B \sim 3$, $\sigma \sim 1$



3D PIC simulation[1]: forced, $1\,080^3$, e^-e^+, $\delta B/B \sim 3$, $\sigma \sim 1$



3D MHD simulation[2]: forced, $1\,024^3 \times 1\,024$, $< B >= 0, v_A/c = 0.4$



© Eyink+13

\+ synthetic turbulence[3] : sum of plane waves (Alfvén or fast magnetosonic)

Note: magnetization parameter $\sigma = \dfrac{\text{mag. energy dens.}}{\text{plasma energy dens.}}$

Refs: 1. V. Bresci, ML, L. Gremillet, L. Comisso, L. Sironi, C. Demidem 22 2. JHU database 3. Demidem+ 20

Non-resonant Fermi-type acceleration: comparison to numerical experiments

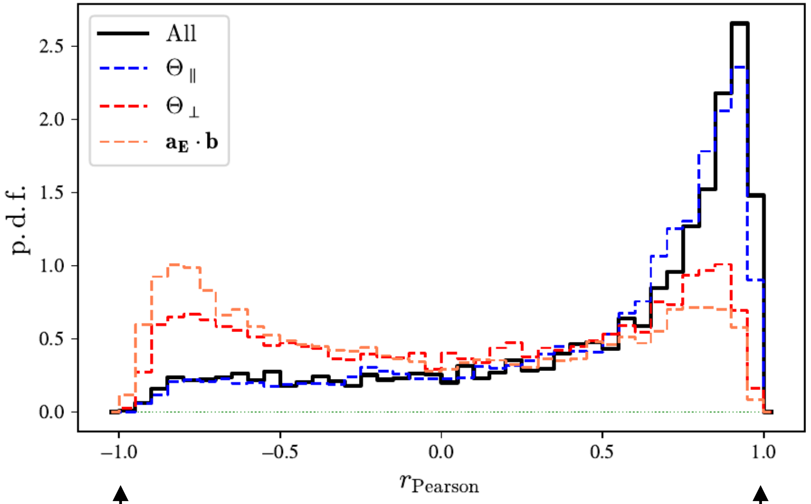
→ model:

$$\frac{d\gamma'}{d\tau} = -\gamma' u'_\parallel \, \boldsymbol{a_E}\cdot\boldsymbol{b} - {u'_\parallel}^2\,\Theta_\parallel - \frac{1}{2}{u'_\perp}^2\Theta_\perp$$

$$u'_\parallel = \boldsymbol{p'}\cdot\boldsymbol{b}/mc$$
$$u'_\perp = \left[u'^2 - {u'_\parallel}^2\right]^{1/2}$$

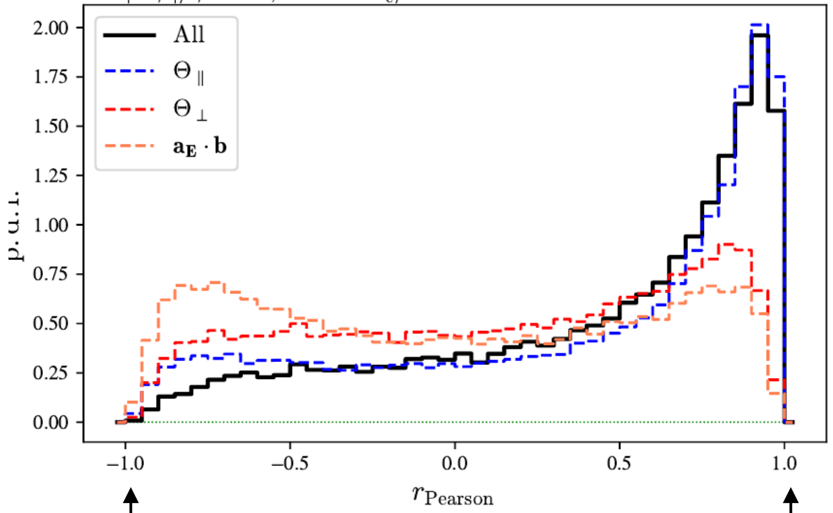
→ test[1]: for each particle history in a simulation, reconstruct $\gamma'(t)$ using above model and velocity gradients measured in the simulation at $\boldsymbol{x}, t$…
… then measure degree of correlation $r_{\rm Pearson}$ between the observed and reconstructed $\gamma'(t)$

PIC simulation: 3D, 1 080^3, e^-e^+, $\delta B/B \sim 3$, $\sigma \sim 1$



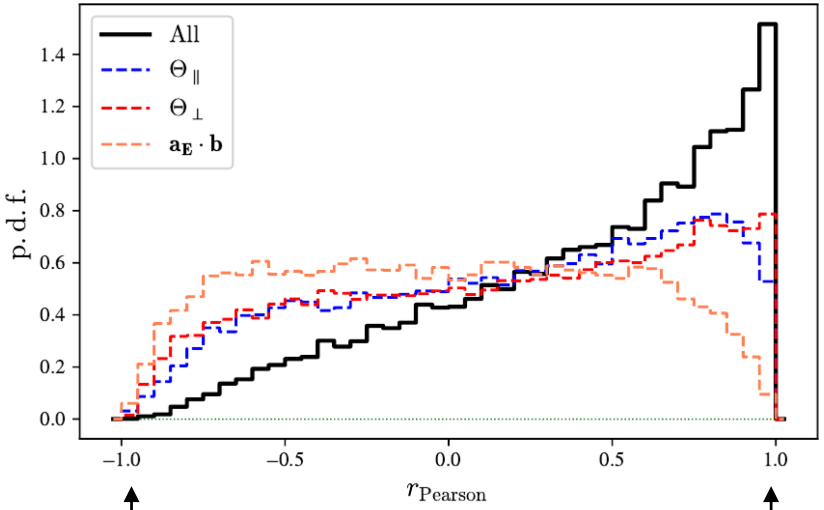
100% anti-correlation | 100% correlation

PIC simulation: 2D, 10 000^2, e^-e^+, $\delta B/B \sim 3$, $\sigma \sim 1$



100% anti-correlation | 100% correlation

incompressible MHD, 3D, 1024^3, $v_A = 0.4c$



100% anti-correlation | 100% correlation

⇒ model captures the dominant contribution to particle energization

+ note: in wave turbulence w/ resonant wave-particle interactions, no apparent correlation seen (as expected)

Refs: 1. V. Bresci+ 22

Connection to intermittency: statistics of the random force

→ a simplified model[1]: $\dot{p} = \Gamma_l \, p$

... with $\Gamma_l \sim$ random forces $(a_E \cdot b, \Theta_\parallel, \Theta_\perp)$ coarse-grained on $l \sim 2\pi \, r_g$
⇒ momentum p jumps on timescale $\sim l/c$ by

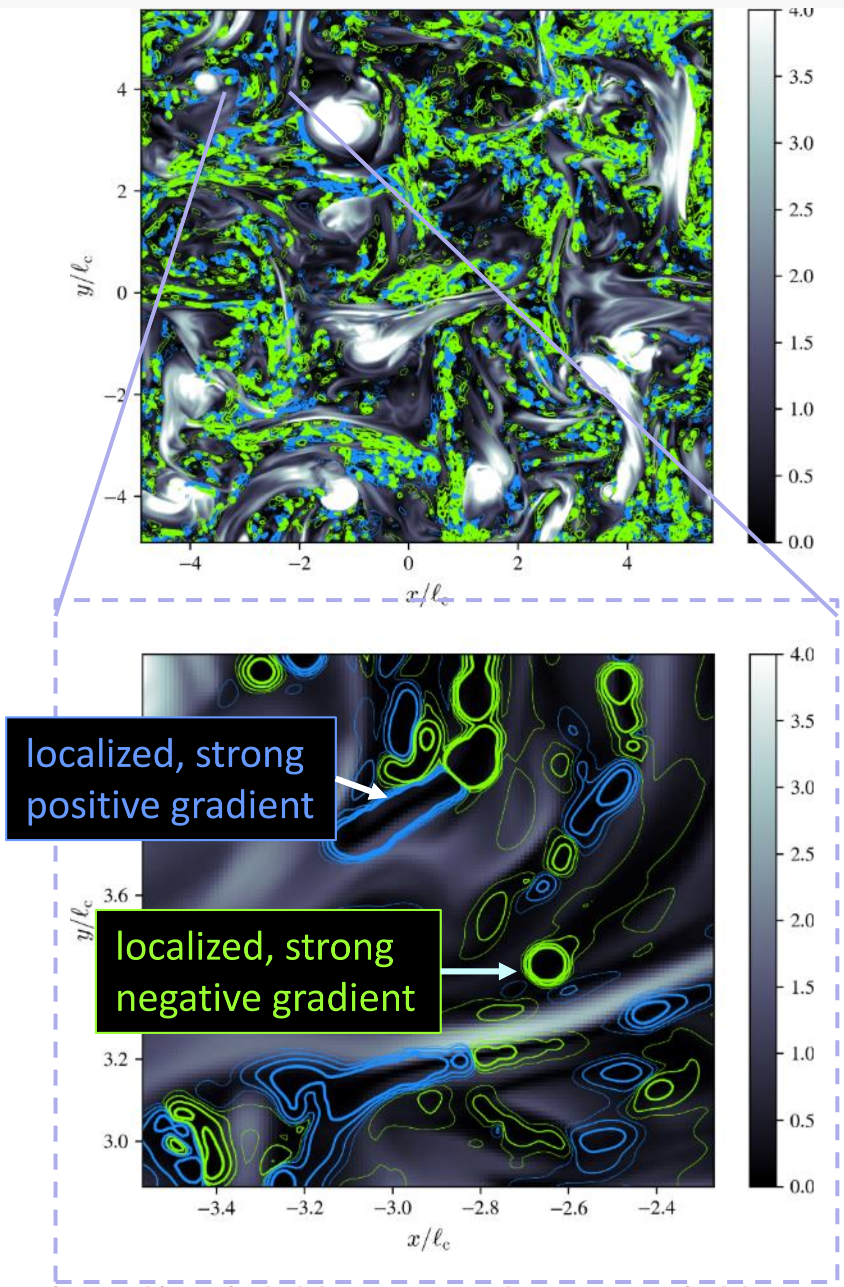
$$\Delta \ln p \sim \Gamma_l \; \Delta t \;\Rightarrow\; \mathrm{Prob.}\,(\Delta \ln p) \sim \mathrm{Prob.}\,(\Gamma_l)$$

→ statistics of Γ_l: neither Gaussian, nor white noise...

... gradients increasingly non-Gaussian at small scales (↔ small r_g)...
... hope: capture statistics through intermittency model to derive statistics of $\Delta \ln p$

→ phenomenology:

... some particles interact frequently with strong scattering centers, some not at all, even over long timescales ⇒ not Fokker-Planck!
⇒ anomalous transport[3] + powerlaws in momentum[4]



Refs.: 1. ML 22 [PRL 129, 215101 (2022)]
3. Trotta+20, ML + Malkov 20, Maiti+21, Pezzi+22
4. e.g. Zhdankin+17,18,19, Comisso+Sironi19,20, Wong+19, ..., Bresci+22

A transport model accounting for the p.d.f. of random forces

→ a simplified model[1]: $\dot{p} = \Gamma_l\, p$

... with $\Gamma_l \sim$ random forces $(a_E \cdot b, \Theta_\parallel, \Theta_\perp)$ coarse-grained on $l \sim 2\pi\, r_g$
⇒ momentum p jumps on timescale $\sim l/c$ by

$$\Delta \ln p \sim \Gamma_l\ \Delta t \ \Rightarrow\ \text{Prob.}\,(\Delta \ln p) \sim \text{Prob.}\,(\Gamma_l)$$

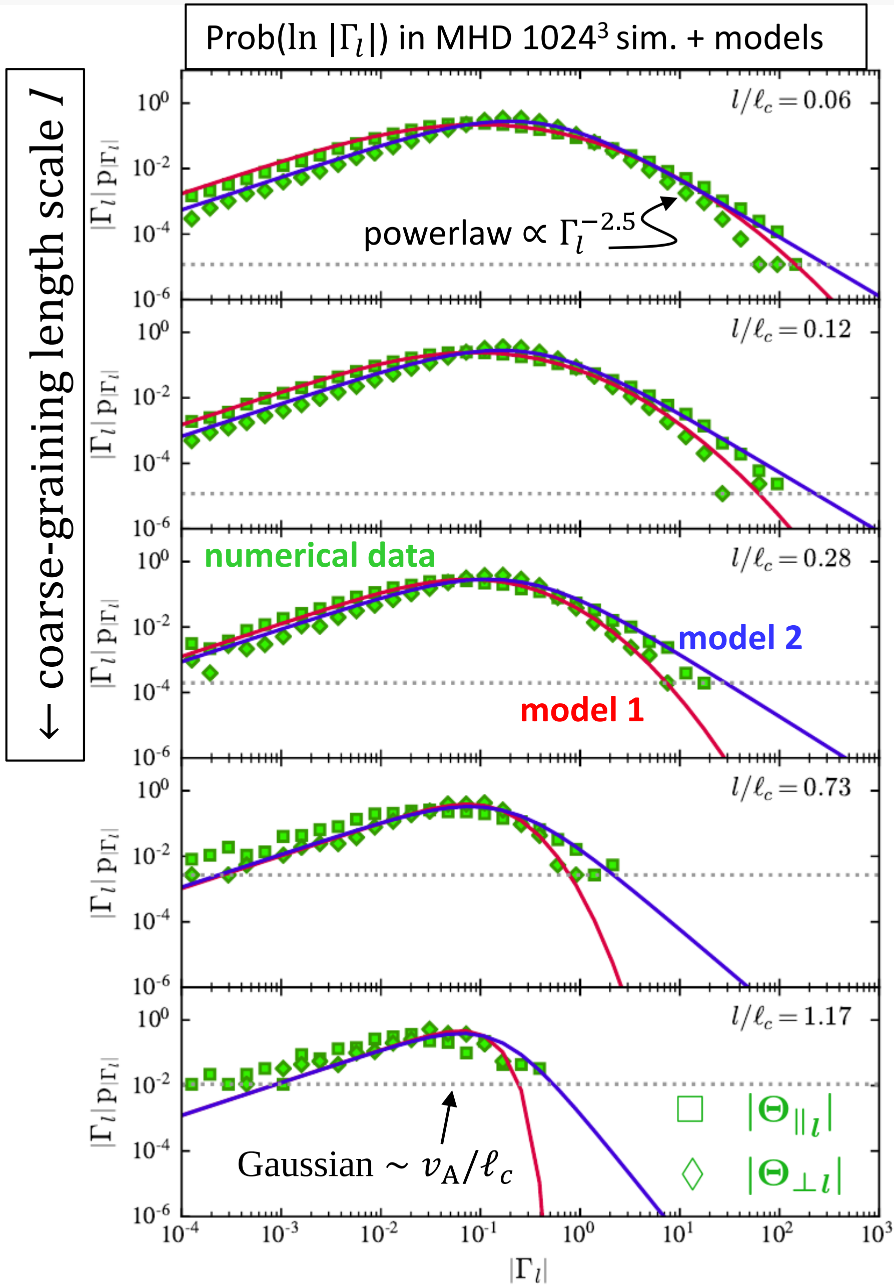
→ transport equation: $$\partial_t\, n_p = \int_0^{+\infty} \mathrm{d}p' \left[\frac{\varphi\,(p|p')}{t_{p'}} n_{p'}(t) - \frac{\varphi\,(p'|p)}{t_p} n_p(t)\right]$$

with: $n_p = \dfrac{\mathrm{d}N}{\mathrm{d}p}\,, \quad t_p \sim l/c \sim 2\pi r_g/c$

→ model: statistics of Γ_l vs l measured in MHD simulation

... **model 1:** multifractal model[2], function[v_A, ℓ_c + ~3 free parameters]

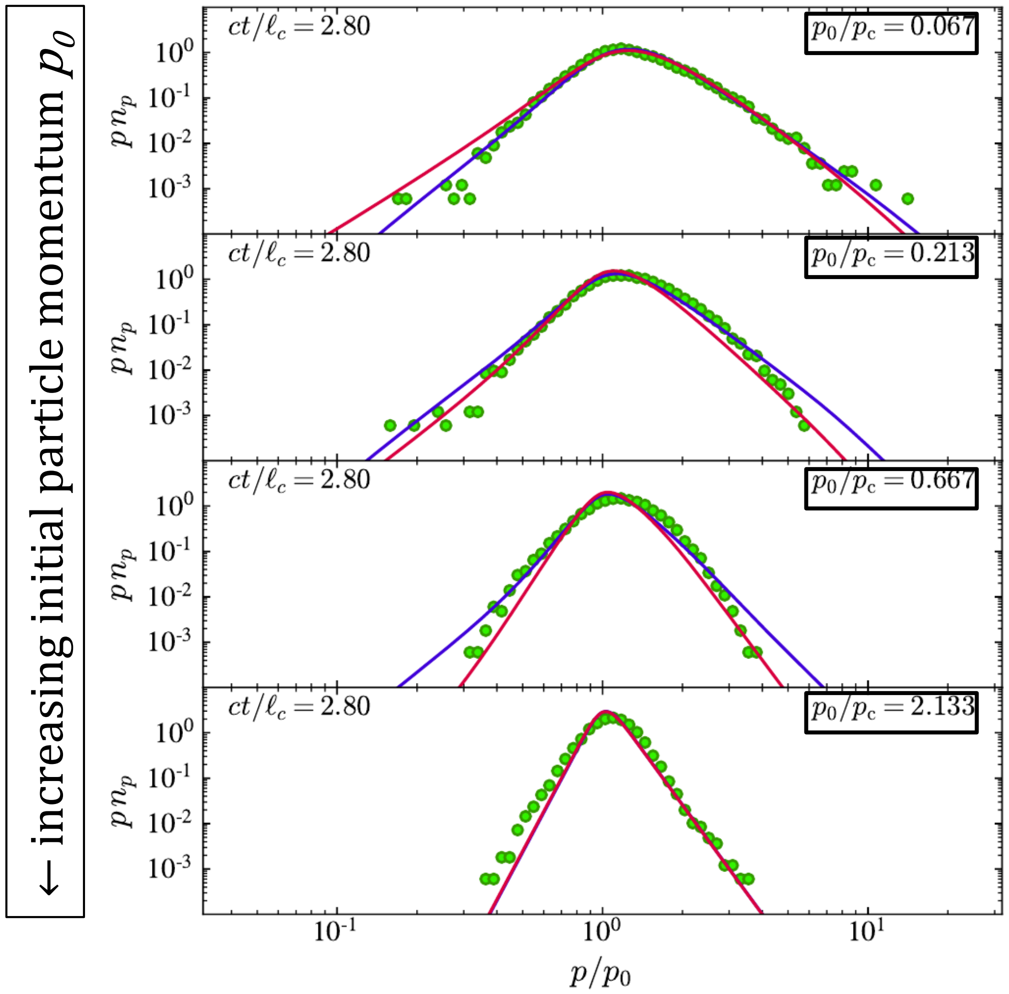
... **model 2**: ad-hoc broken powerlaw, function[v_A, ℓ_c + ~3 free param.]



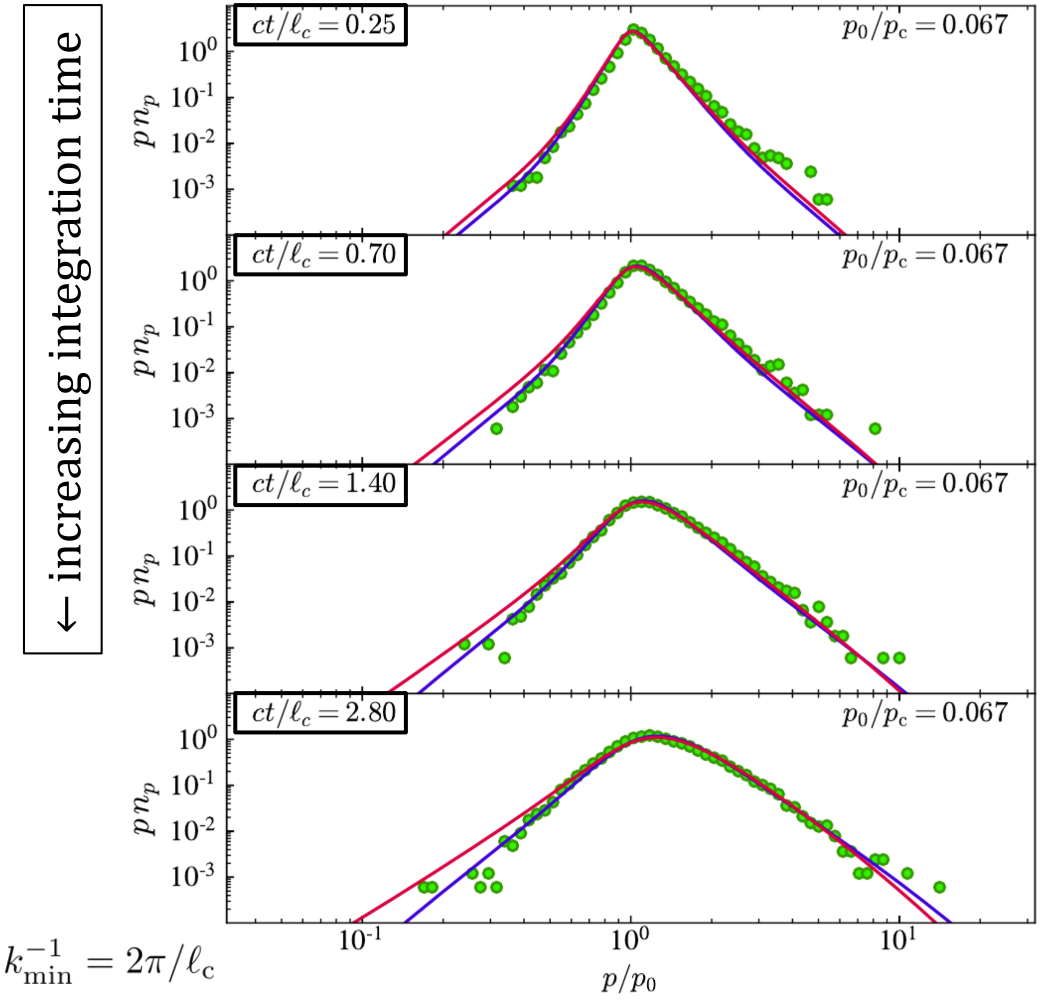
Refs.: 1. ML 22 [PRL 129, 215101 (2022)] 2. Parisi+Frisch 85, ..., She+Levêque 94, Dubrulle 94, ... Biskamp+Müller 00, ... , Chandran+15, ...

Comparison of model spectra to particle tracking in MHD simulation

→ comparison to numerical data: integrate kinetic equation and compare solution (Green function) to distribution measured in MHD 1024^3 simulation by time-dependent particle tracking…



$$p_c : \quad r_g(p_c) = k_{\min}^{-1} = 2\pi/\ell_c$$



⇒ model can reproduce time- and energy- dependent Green functions… + explain origin of powerlaw spectra

The (dominant?) role of the field line curvature

→ energization through curvature drift:

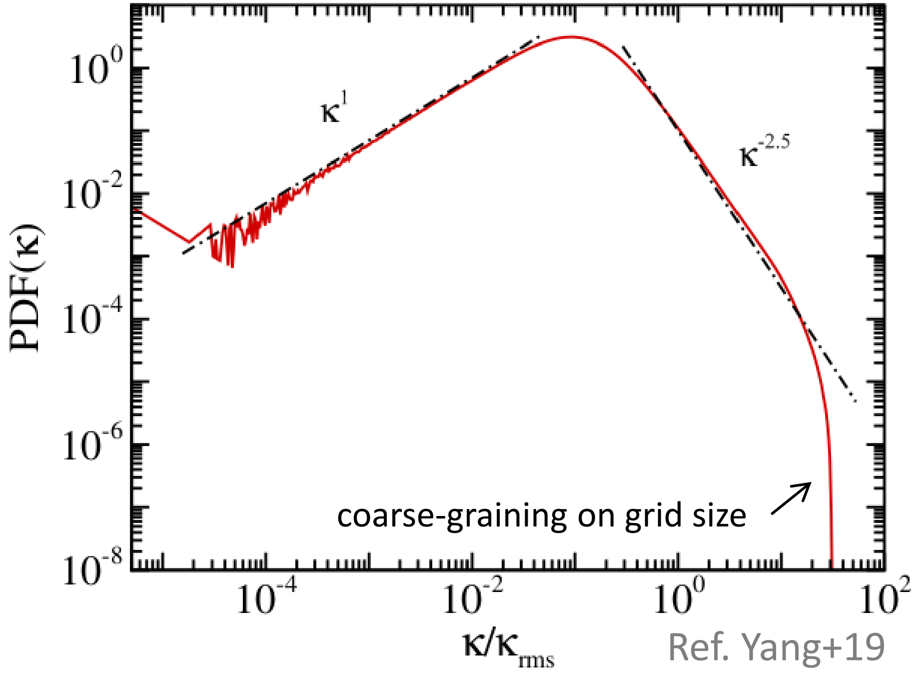
… a dominant process in reconnection physics[1]

… field line curvature: $\boldsymbol{\kappa} = \boldsymbol{b}\cdot\nabla\boldsymbol{b} \qquad (\boldsymbol{b} \equiv \boldsymbol{B}/|\boldsymbol{B}|)$

… curvature drift: $\boldsymbol{v}_{\mathrm{d}} \propto u_{\parallel}^2\, \boldsymbol{b}\times\boldsymbol{\kappa} \Rightarrow \boldsymbol{v}_{\mathrm{d}}\cdot\boldsymbol{E} \propto -\Theta_{\parallel} \propto u_{\parallel}^2\, \boldsymbol{v}_{\boldsymbol{E}}\cdot\boldsymbol{\kappa}$

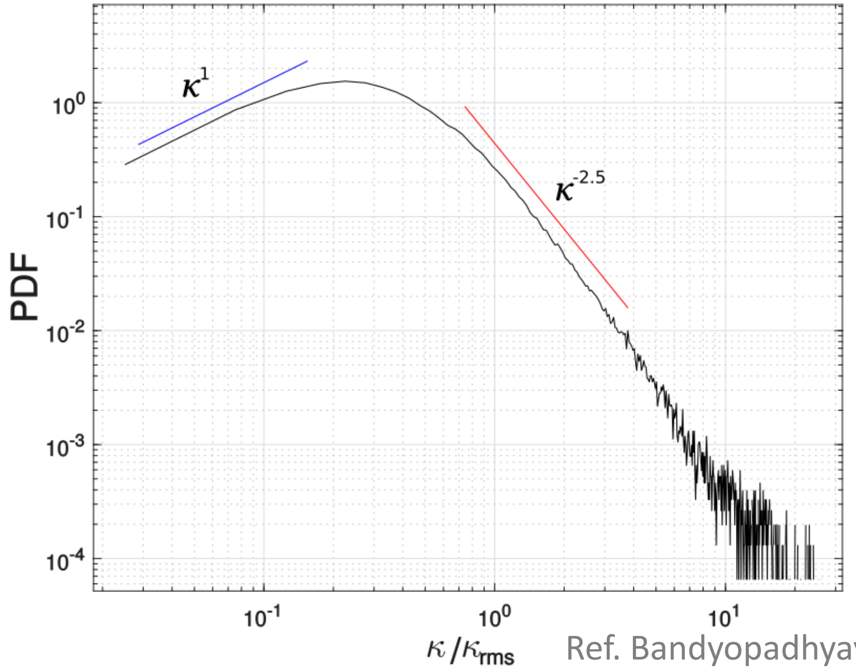
… statistics of κ: a powerlaw a large values[2], p.d.f.(κ) ∝ $\kappa^{-2.5}$ ⇒ origin, connection to statistics of random force?

3D MHD simulation[2]



Ref. Yang+19

Solar wind data[3]



Ref. Bandyopadhyay+20

Refs.: 1. e.g. Drake+06, …, Dahlin+14, … 2. Schekochihin+01, Yang+19, Yuen+Lazarian20 3. Bandyopadhyay+20, Huang+20

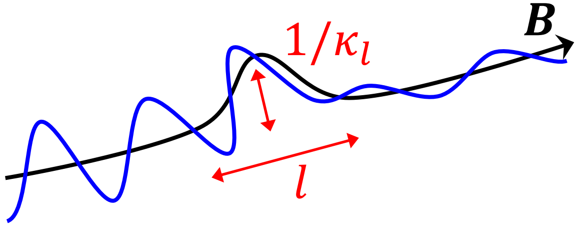
Field line curvature and spatial transport in magnetostatic turbulence?

→ impact of field line curvature κ[1]:

as a particle crosses a region where $\kappa r_g \gtrsim 1 \Rightarrow$ strong, non-adiabatic scattering

… ``magnetic field line curvature scattering'' in magnetospheric physics…

… seen in a numerical model of a tokamak[2] …



→ in MHD turbulence:

statistics of κ_l highly non-Gaussian[3] at $l \ll \ell_c$: $\mathrm{p.d.f.}(\kappa) \propto \kappa^{-2.5}$

… also seen in solar wind[4]

→ consequences for transport[5]:

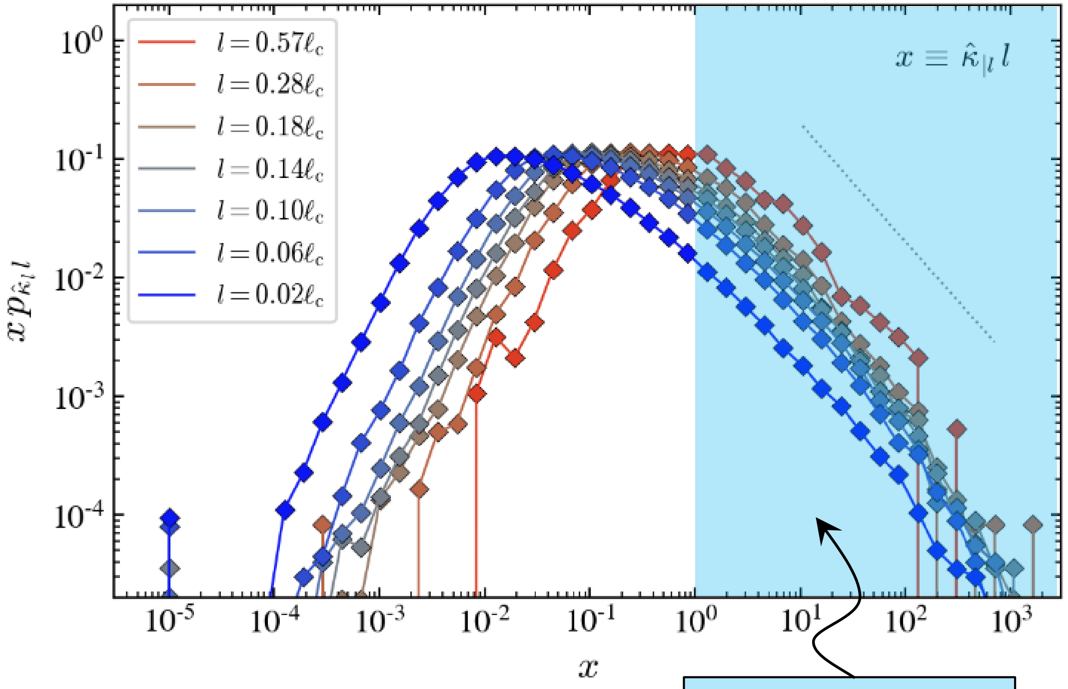
… filling fraction of scattering regions:

size $l \sim r_g$ with $\kappa_l\, l \gtrsim 1$

… at small r_g, powerlaw tail guarantees sufficiently many regions exist to sustain scattering…

… mean free path comparable to QLT prediction

statistics of $\kappa_l l$ from MHD simulation



Refs.:

1. Chen+Palmadesso 86, Büchner+Zelenyi 89, …, Artemyev+13,…
2. Escande + Sattin 21
3. Schekochihin+01, Yang+19, Yuen+Lazarian 20
4. Bandyopadhyay+20, Huang+20
5. M.L.23 [arXiv:2304.03023], Kempski+23 [arXiv:2304.12335]

Summary + perspectives

→ Recent numerical investigations on particle acceleration in strong, semi-relativistic turbulence:

… efficient particle acceleration in ``ideal'' electric fields

… powerlaw spectra are generic, index $s \sim -4 \quad \to \quad -2$

→ The Fermi picture is well alive:

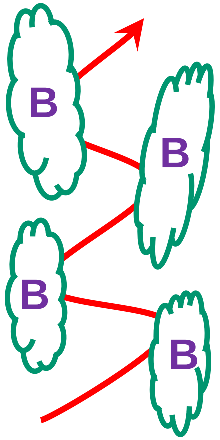
… generalized Fermi model in turbulence: supported by numerical simulations

… particle acceleration in velocity gradients: intermittency rules…

… multi-fractal model of gradient statistics → a transport equation…

→ Some limitations, paths for future explorations:

1. *extrapolation to small spatial length scales, e.g. role of turbulence anisotropy ?*

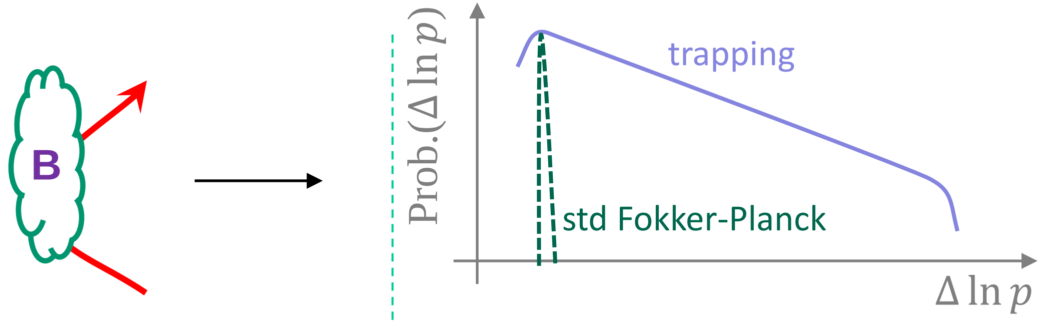


$$k_\parallel \sim k_\perp^{2/3} l_c^{-1/3} \;\Rightarrow\; k_\parallel \ll k_\perp$$

… particles can interact with sharp perpendicular gradients:

$$k_\perp^{-1} < r_g < k_\parallel^{-1}$$

2. *particle trapping inside structures[1]?*



… main effect: jump propagator ∼ powerlaw (energy gain inside structure vs. escape)

Refs.: 1. e.g. M.L. + Malkov 20, Vega+20

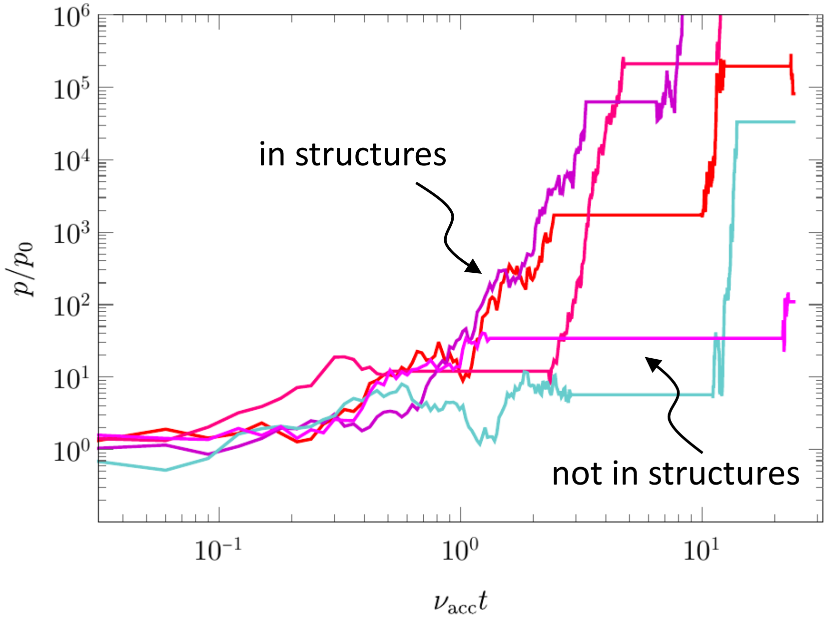
Fokker-Planck model with intermittent structures

→ a two-population model[1]: distribution $f_0(p,t)$ = particles not undergoing acceleration; $f_1(p,t)$ = in structures

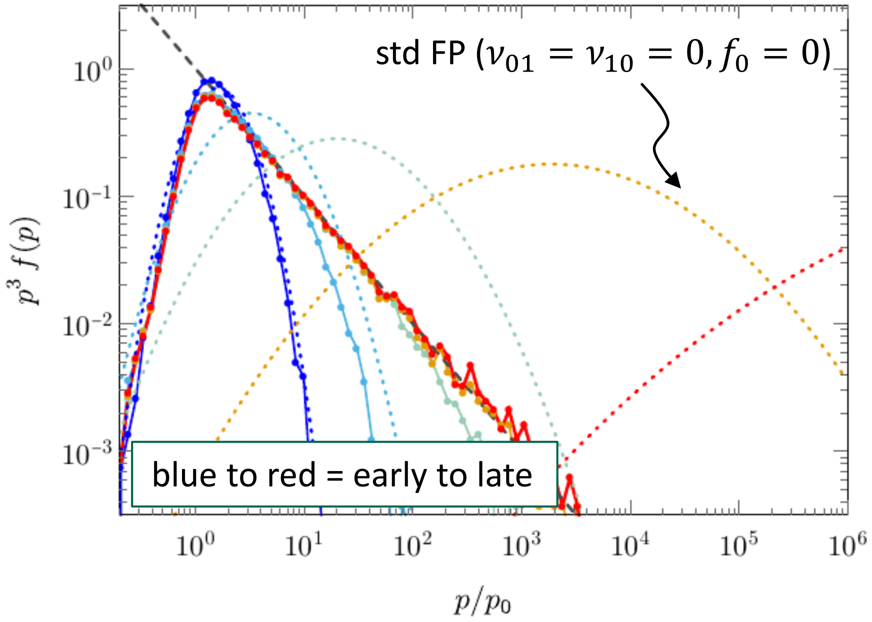
$$\partial_t f_0(p,\,t) \;=\; -\nu_{01} f_0(p,\,t) + \nu_{10} f_1(p,\,t),$$
$$\partial_t f_1(p,\,t) \;=\; +\nu_{01} f_0(p,\,t) - \nu_{10} f_1(p,\,t) \;+\; p^{-2}\partial_p \left\{ D_{pp}\, p^2 \partial_p f_1(p,\,t) \right\}\;.$$

entry rate into structures (ν_{01}); escape rate from structures (ν_{10})

<u>examples of particle histories</u>



<u>momentum distributions</u> ($f_0 + f_1$)



→ **Note:** full Fokker-Planck involves all phase space… integrating out variables ($\boldsymbol{x}, \mu$ …) can lead to deviations from simple Fokker-Planck form for $f(p,t)$

Refs.: 1. e.g. M.L. + Malkov 20