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Firehose-induced collisionality

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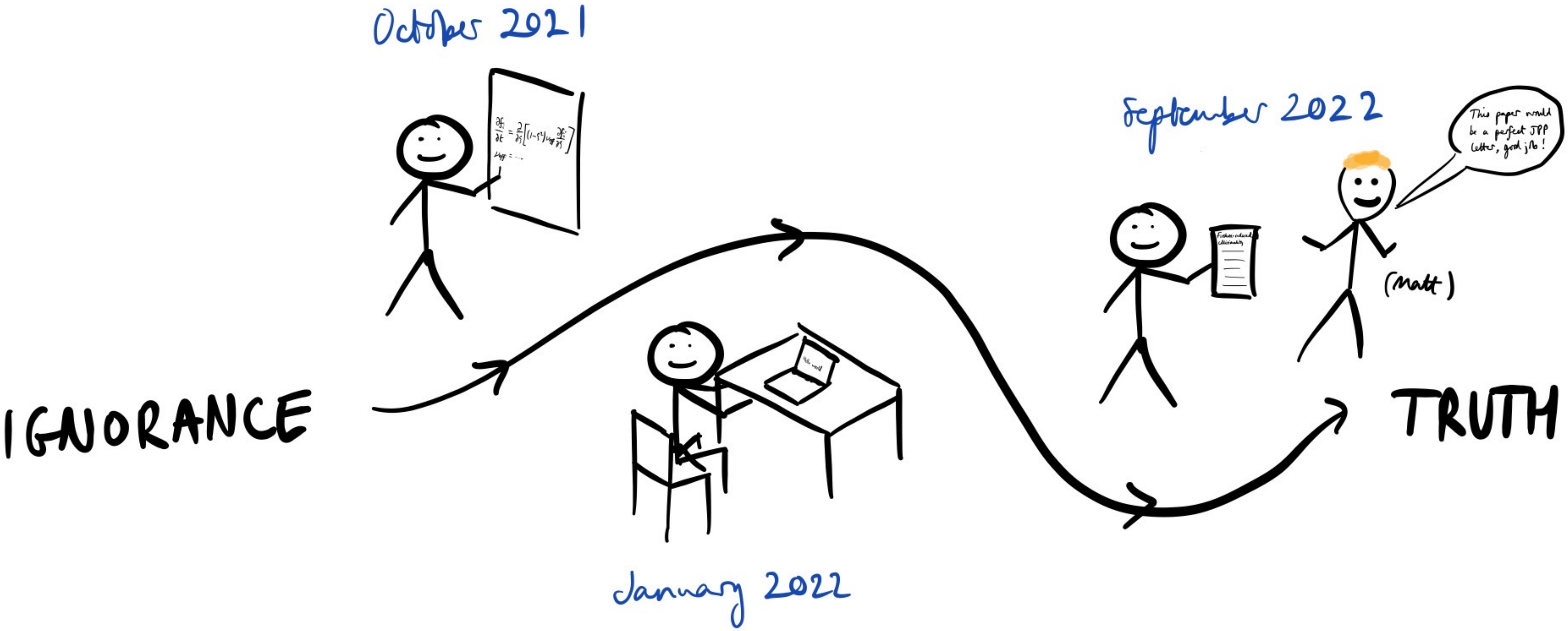
A visual disclaimer *(inspiration: Kate Hammett)*

What I envisaged doing this project would be like



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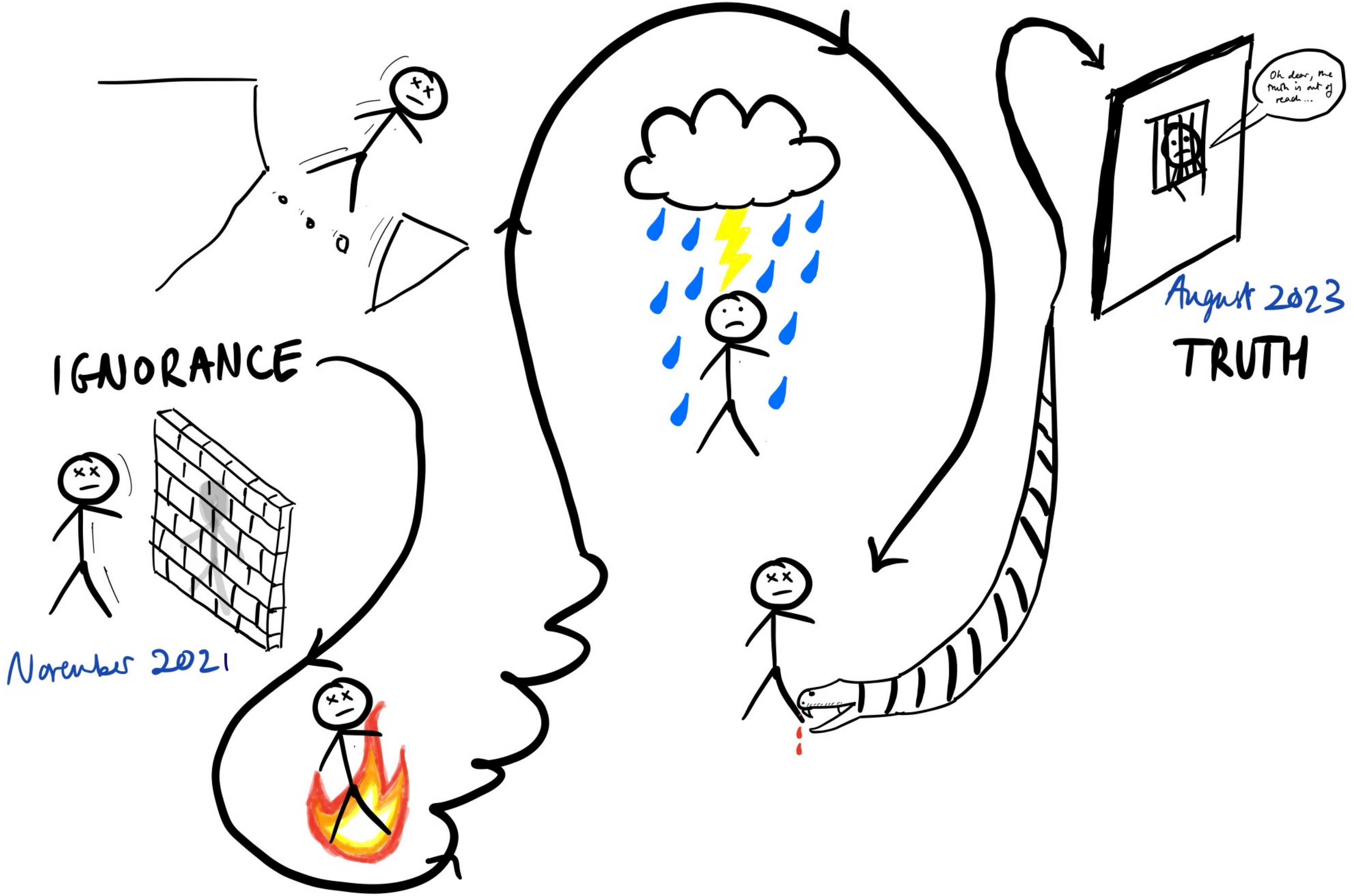
What I actually felt like while doing this project



Background

High-β, magnetised collisionless plasmas have a complicated interplay between macro/microscales

Macroscopic evolution

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{p_\perp}{nB}\right) = 0, \quad \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{p_\parallel B^2}{n^3}\right) = 0$$

Pressure anisotropy

$\Delta = p_\perp / p_\parallel - 1$

Kinetic instabilities

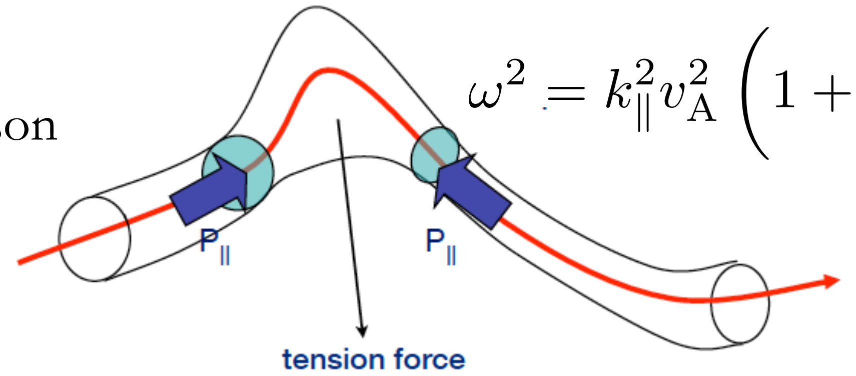
$|\Delta| \gtrsim 1/\beta = B^2/8\pi p$

Anomalous thermodynamics, wave dynamics, collisionality

Simple example: the (ion) firehose instability

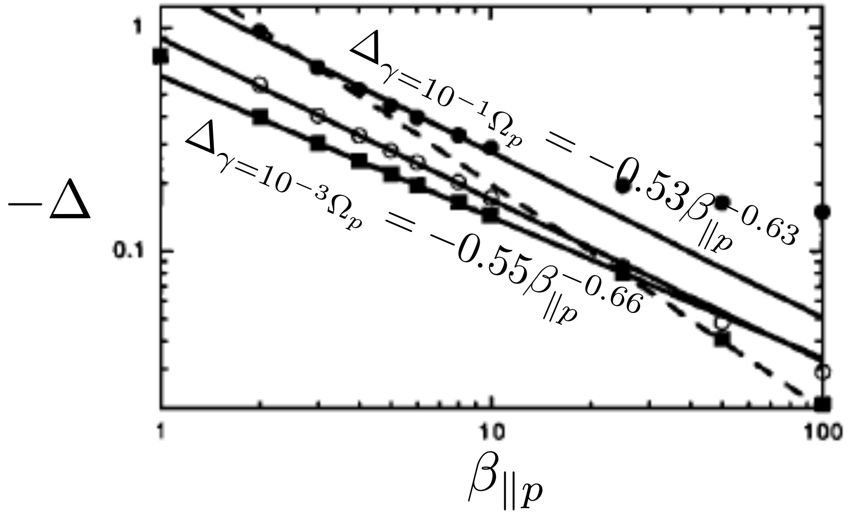
Rosenbluth (1956); Chandrasekhar, Kaufman & Watson (1958); Parker (1958); Vedenov & Sagdeev (1958):
Alfvén waves linearly unstable if $\Delta < -2/\beta_{\|}$.

$$\omega^2 = k_{\|}^2 v_{\rm A}^2 \left(1 + \frac{\Delta\beta_{\|}}{2}\right) \equiv k_{\|}^2 v_{\rm A,eff}^2$$

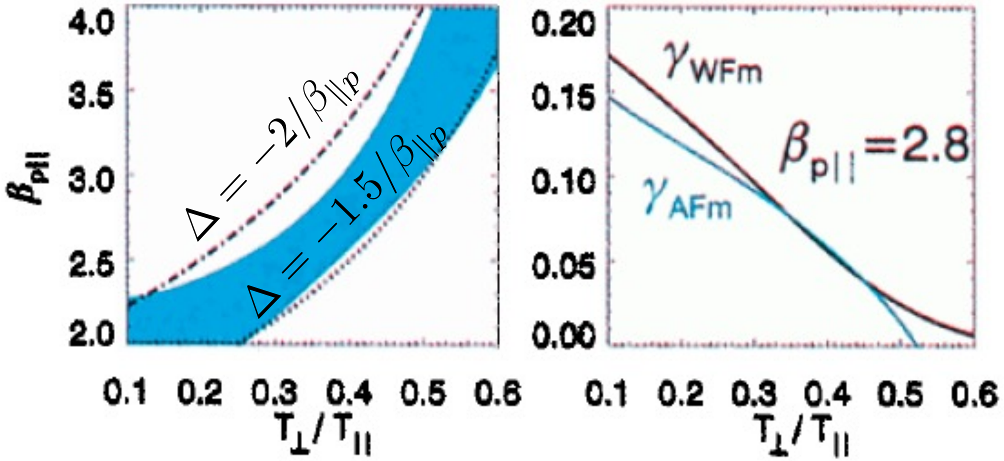


'Kinetic' variants of the firehose instability ($k\rho_i \sim 1$) have the fastest growth rates and least stringent thresholds:

Gary *et al* (1998): *resonant parallel firehose instability*



Hellinger & Matsumoto (2000): *resonant oblique firehose instability*



Nonlinear evolution and saturation of firehose

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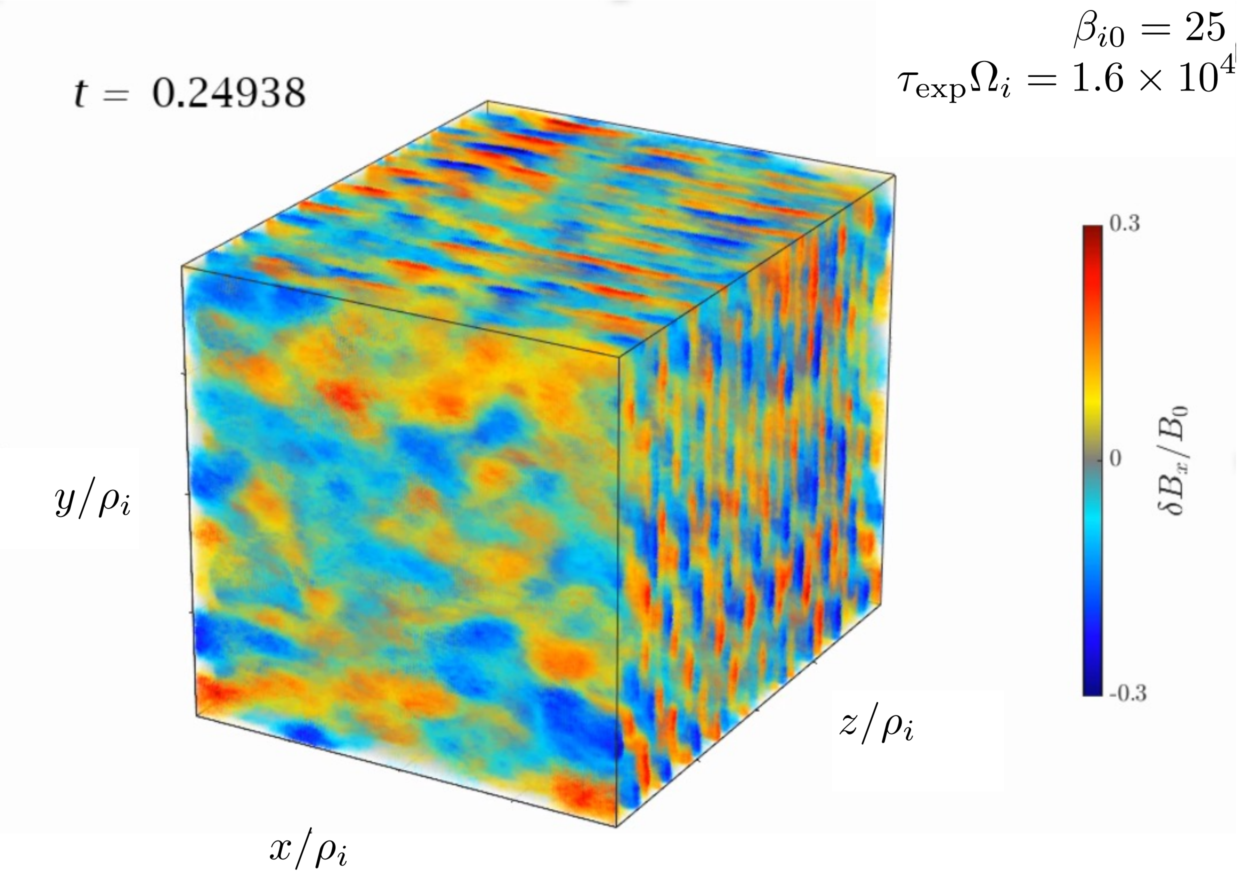
<u>Q:</u> how does the firehose instability evolve nonlinear/saturate in response to a plasma's macroscopic evolution?

→ Addressed primarily using hybrid-PIC expanding-box (e.g. Hellinger & Travnicek 2008) or shearing-box simulations (e.g. Kunz, Schekochihin & Stone 2014); in special cases, quasilinear theory (e.g. Rosin *et al* 2011)

$\beta_{i0} = 25$

$\tau_{\rm exp}\Omega_i = 1.6 \times 10^4$



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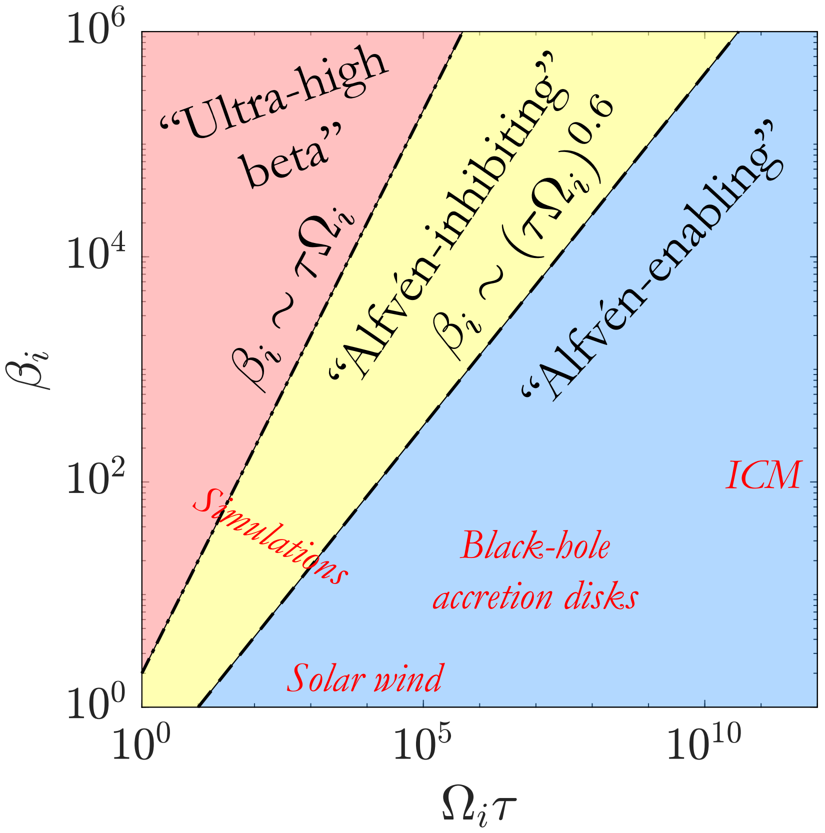
A: Depends on two parameters: β_i and $\tau\Omega_i$. Key common feature: anomalous collisionality/viscosity!



	Ultra-high beta	Alfvén-inhibiting	Alfvén-enabling
$\Delta_{\rm sat}$	$\ll -2/\beta_i$	$\simeq -2/\beta_i$	$\simeq -1.6/\beta_i$
$\langle \delta B_\perp^2/B_0^2\rangle$	~ 1	$\gtrsim \beta_i/\tau\Omega_i$?	$\sim (\tau\Omega_i)^{-1/2}$?
$k\rho_i \ll 1$?	Yes?	Yes	No
$\nu_{\rm eff}$	$\lesssim \Omega_i$?	$\simeq 2\beta_i/\tau$	$\simeq 1.6\beta_i/\tau$
$\mu_{\rm B}$	$\gtrsim p_i\Omega_i^{-1}$?	$\simeq \tau B^2/4\pi$	$\simeq 0.8\tau B^2/4\pi$

Why find a firehose collision operator?

A1: simple argument for estimating $\delta B_{\mathrm{f}}^2/B_0^2$ based on considering scattering of `average' particles fail:

Marginal stability $\Longrightarrow$ $\left|\frac{\partial f_i}{\partial \xi}\right| \sim |\Delta_i| f_{\mathrm{M}i} \sim \frac{1}{\beta_i} f_{\mathrm{M}i}$

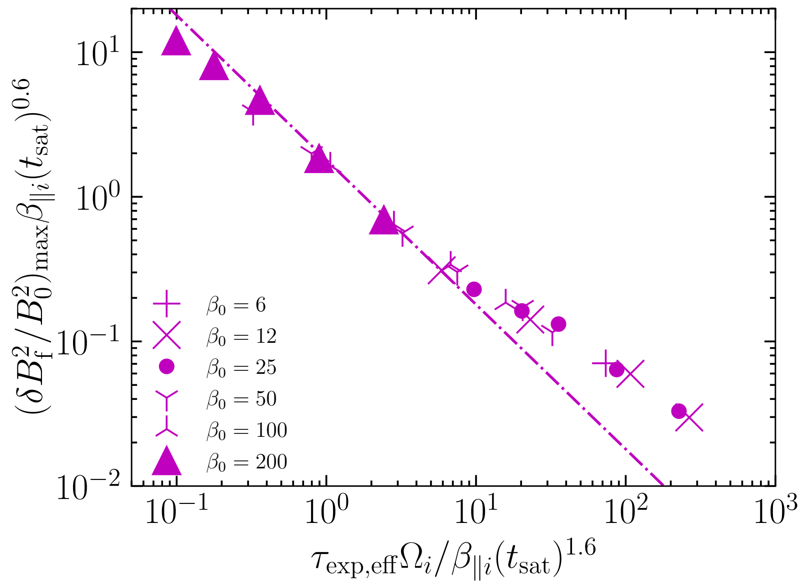
Quasi-steady state $\Longrightarrow$ $\nu_{\mathrm{eff}} \frac{\partial f_i}{\partial \xi} \sim \frac{1}{\tau} f_{\mathrm{M}i}$

Pitch-angle diffusion $\Longrightarrow$ $\nu_{\mathrm{eff}} \sim \Omega_i \frac{\delta B_{\mathrm{f}}^2}{B_0^2}$

$$\Longrightarrow \quad \frac{\delta B_{\mathrm{f}}^2}{B_0^2} \sim \frac{\beta_i}{\tau \Omega_i}$$

To understand physics of firehose saturation, need collision operator!

From my Vienna talk last year…



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To understand physics of firehose saturation, need collision operator!

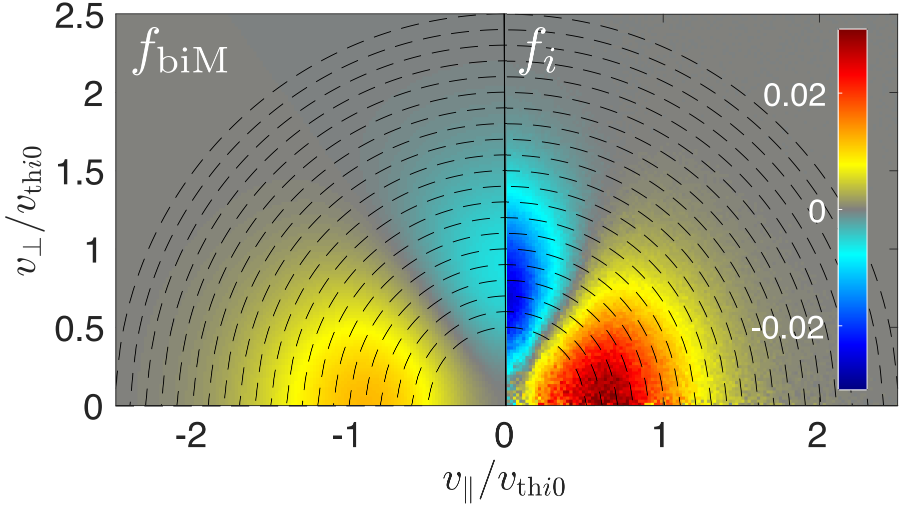
A2: characterise distribution-function anisotropy in saturated state
→ Explain why $(\Delta_i)_{\rm sat} \simeq -1.6/\beta_i < \Delta_{\rm c,biM}$ in AE state

A3: modelling of high-β plasmas with reduced kinetic models e.g. pressure-anisotropic gyrokinetics, drift kinetics

A4: modelling of cosmic-ray transport

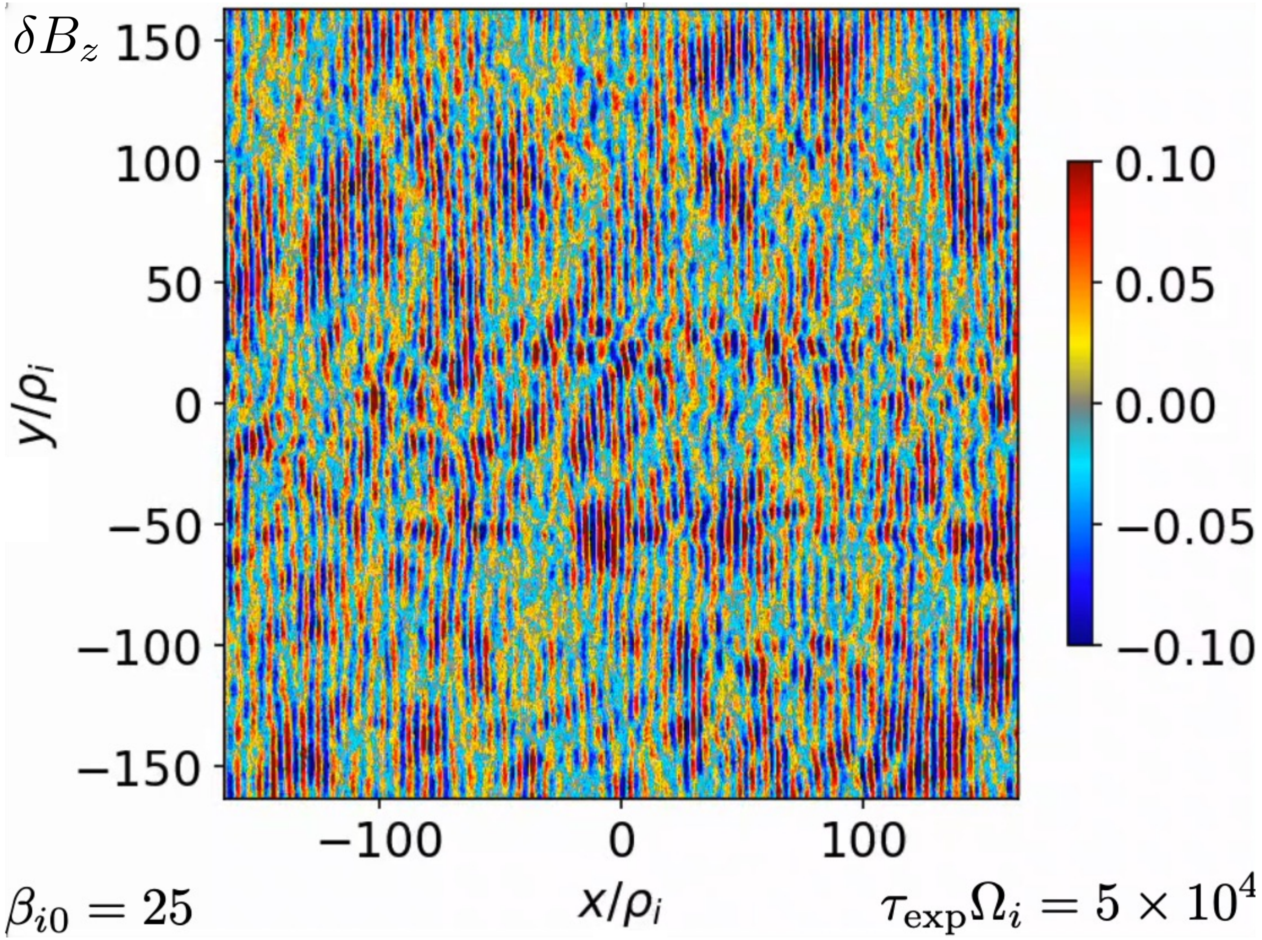
For astrophysical applications, study Alfvén-enabling state

From my Vienna talk last year…



Characterising the Alfvén-enabling states

To motivate form of collision operator, review emergence of firehose modes on path to AE state:



1. Oblique firehose modes first…

$$\delta\boldsymbol{B} = \delta B_z\hat{\boldsymbol{z}},\ |\delta\boldsymbol{E}| \sim \frac{1}{\beta_i}\frac{v_{\rm thi}}{c}|\delta\boldsymbol{B}|$$

$$k_\parallel\rho_i \sim k_\perp\rho_i \sim 0.5,\ \ \Delta k_\parallel \sim \rho_i^{-1},$$

$$\varpi_{\rm ob} = 0,\ \ \gamma_{\rm ob} \simeq 2.9\left|\Delta_i - \frac{1.35}{\beta_i}\right|\Omega_i.$$

2. … then secondary parallel firehose modes:

$$\delta\boldsymbol{B} = \delta B_y(\hat{\boldsymbol{y}} + {\rm i}\hat{\boldsymbol{z}})\,,\ |\delta\boldsymbol{E}_\perp| \sim \frac{1}{\beta_i}\frac{v_{\rm thi}}{c}|\delta\boldsymbol{B}|$$

$$k_\perp\rho_i \ll k_\parallel\rho_i \sim 1,\ \ \Delta k_\parallel \lesssim \rho_i^{-1}$$

$$\varpi_{\rm pl} \sim \frac{\Omega_i}{\beta_i},\ \gamma_{\rm pl} \lesssim \frac{\Omega_i^{1/2}}{\tau_{\rm exp}^{1/2}}.$$

Both types of modes persist in steady state, with $\delta B_{\rm ob} \ll \delta B_{\rm pl} \sim \beta_i^{0.25}/(\tau_{\rm exp}\Omega_i)^{1/2} \lesssim \beta_i^{-0.55}B_0 \ll B_0$

Assumptions for scattering model

Goal: construct simplest possible collision operator that captures dominant physics → **simplifying assumptions**

1. Broad *k*-spectra of small-amplitude firehose modes → ***quasilinear diffusion operator***

2. For both oblique and secondary parallel firehose modes, $|\boldsymbol{F}_{\rm E}| \ll |\boldsymbol{F}_{\rm B}|$ → ***neglect electric fields***
→ ***Resonant pitch-angle scattering operator*** that isotropises f_i in the wave frame (Kulsrud & Pearce 1969):

$$\mathfrak{C}_{\rm f}[f_i] = \frac{1}{2}\frac{\partial}{\partial \xi'}\left[(1-\xi'^2)\nu_{\rm eff}(v_\parallel', v_\perp)\frac{\partial f_i}{\partial \xi'}\right], \quad \text{where} \quad \nu_{\rm eff}(v_\parallel', v_\perp) = \pi \frac{\Omega_i^2}{v_\parallel'}\frac{\tilde{\mathcal{E}}_B(\Omega_i/v_\parallel')}{B_0^2/8\pi},$$

$$\tilde{\mathcal{E}}_B(k_\parallel) \equiv \sum_{n\neq 0} n^2 \int {\rm d}^2\boldsymbol{k}_\perp\, E_B(nk_\parallel, k_\perp)\frac{J_n^2(k_\perp v_\perp/\Omega_i)}{k_\perp^2 v_\perp^2/\Omega_i^2}, \quad v_\parallel' \equiv v_\parallel - v_{\rm wv}, \quad \xi' = \xi - v_{\rm wv}(1-\xi^2)/v.$$

3. Assume ***small anisotropy*** and ***parallel phase velocities*** $[(\partial f_i/\partial \xi)/f_{\rm M} \sim \tilde{v}_{\rm wv} \sim 1/\beta_i \ll 1\,]$.

$$\mathfrak{C}_{\rm f}[f] = \frac{1}{2}\frac{\partial}{\partial \xi}\left\{(1-\xi^2)\nu_{\rm eff}(v_\parallel, v_\perp)\left[\frac{\partial f_i}{\partial \xi} - \frac{w v_{\rm wv}(v_\parallel)}{v_{{\rm th}i}^2} f_{{\rm M}i}\right]\right\}.$$

4. Adopt ***quasi-parallel*** approximation ($v_\perp^2 \ll \Omega_i^2/k_\perp^2$) for all modes

The simplified collision operator

Our final operator is

$$\mathfrak{C}_{\mathrm{f}}[f] = \frac{1}{2}\frac{\partial}{\partial \xi}\left\{(1-\xi^2)\nu_{\mathrm{eff,pl}}(v\xi)\left[\frac{\partial f_i}{\partial \xi} - \frac{2v v_{\mathrm{wv,pl}}(v\xi)}{v_{\mathrm{th}i}^2} f_{\mathrm{M}}\right] + (1-\xi^2)\nu_{\mathrm{eff,ob}}(v\xi)\frac{\partial f_i}{\partial \xi}\right\},$$

where the pitch-angle scattering rates $\nu_{\mathrm{eff,pl}}(v_\|)$ and $\nu_{\mathrm{eff,ob}}(v_\|)$ and phase velocity $v_{\mathrm{wv,pl}}(v_\|)$ are given by

$$\nu_{\mathrm{eff,pl}}(v_\|) \simeq \frac{\pi}{2}\frac{\Omega_i^2}{|v_\||}\frac{E_{B,\mathrm{pl}}(\Omega_i/v_\|)}{B_0^2/8\pi}, \quad \nu_{\mathrm{eff,ob}}(v_\|) \simeq \frac{\pi}{2}\frac{\Omega_i^2}{|v_\||}\frac{E_{B,\mathrm{ob}}(\Omega_i/v_\|)}{B_0^2/8\pi}, \quad v_{\mathrm{wv,pl}}(v_\|) \simeq \varpi(\Omega_i/v_\|)v_\|,$$

and $E_{B,\mathrm{pl}}(k_\|)$ and $E_{B,\mathrm{ob}}(k_\|)$ are the 1D magnetic-energy spectra of parallel and oblique firehose modes.

Initial thoughts:

1. This simplified collision operator is, in fact, quite complicated.

2. A Maxwellian isn't an equilibrium solution, but maybe that's okay.

Q: can we test this with hybrid-PIC simulations of the firehose instability?

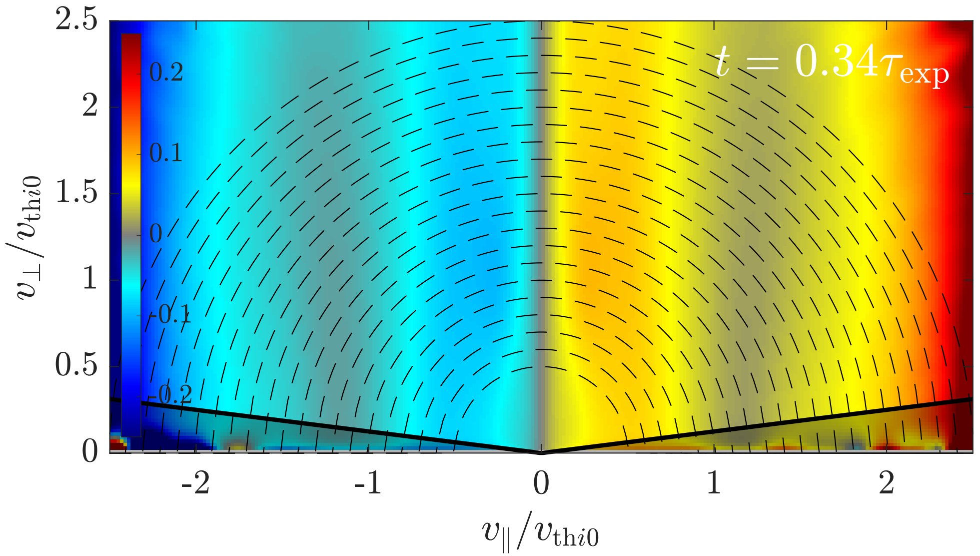
Distribution-function anisotropy

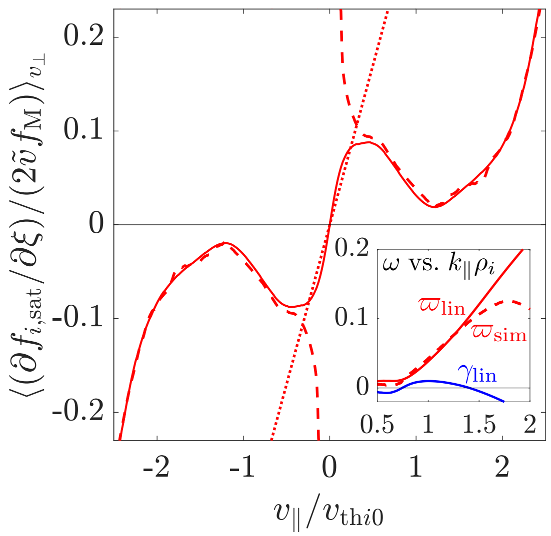
Test 1: relate $\partial f_i/\partial\xi$ to $\nu_{\mathrm{eff,pl}}(v_\parallel)$, $\nu_{\mathrm{eff,ob}}(v_\parallel)$ and $v_{\mathrm{wv,pl}}(v_\parallel)$ via Chapman-Enskog-style expansion:

$$\mathfrak{C}_{\mathrm{f}}[f_{1i}] = \left\{\left(\hat{\boldsymbol{b}}\hat{\boldsymbol{b}} - \frac{1}{3}\mathbf{I}\right) : \mathbf{W}_i\right\}\tilde{v}^2\frac{3\xi^2-1}{2}f_{\mathrm{M}i} = -\frac{2}{3\tau_{\mathrm{exp}}}\tilde{v}^2\frac{3\xi^2-1}{2}f_{\mathrm{M}i},$$

$$\Longrightarrow \boxed{\frac{1}{2\tilde{v}}\frac{\partial f_i}{\partial\xi} \approx f_{\mathrm{M}i}(v)\left\{\frac{\nu_{\mathrm{eff,pl}}(\tilde{v}_\parallel)\tilde{v}_{\mathrm{wv}}(\tilde{v}_\parallel)}{\nu_{\mathrm{eff,pl}}(\tilde{v}_\parallel)+\nu_{\mathrm{eff,ob}}(\tilde{v}_\parallel)} + \frac{\tilde{v}_\parallel}{3\tau_{\mathrm{exp}}[\nu_{\mathrm{eff,pl}}(\tilde{v}_\parallel)+\nu_{\mathrm{eff,ob}}(\tilde{v}_\parallel)]}\right\}}.$$

Then, calculate RHS using collision operator, compare with numerically evaluated LHS (time-averaged):





- **Model agrees well with data, save for particles with $|v_\parallel| \ll v_{\mathrm{th}i}$ (which remain double-adiabatic)**

Direct measurement of collision operator

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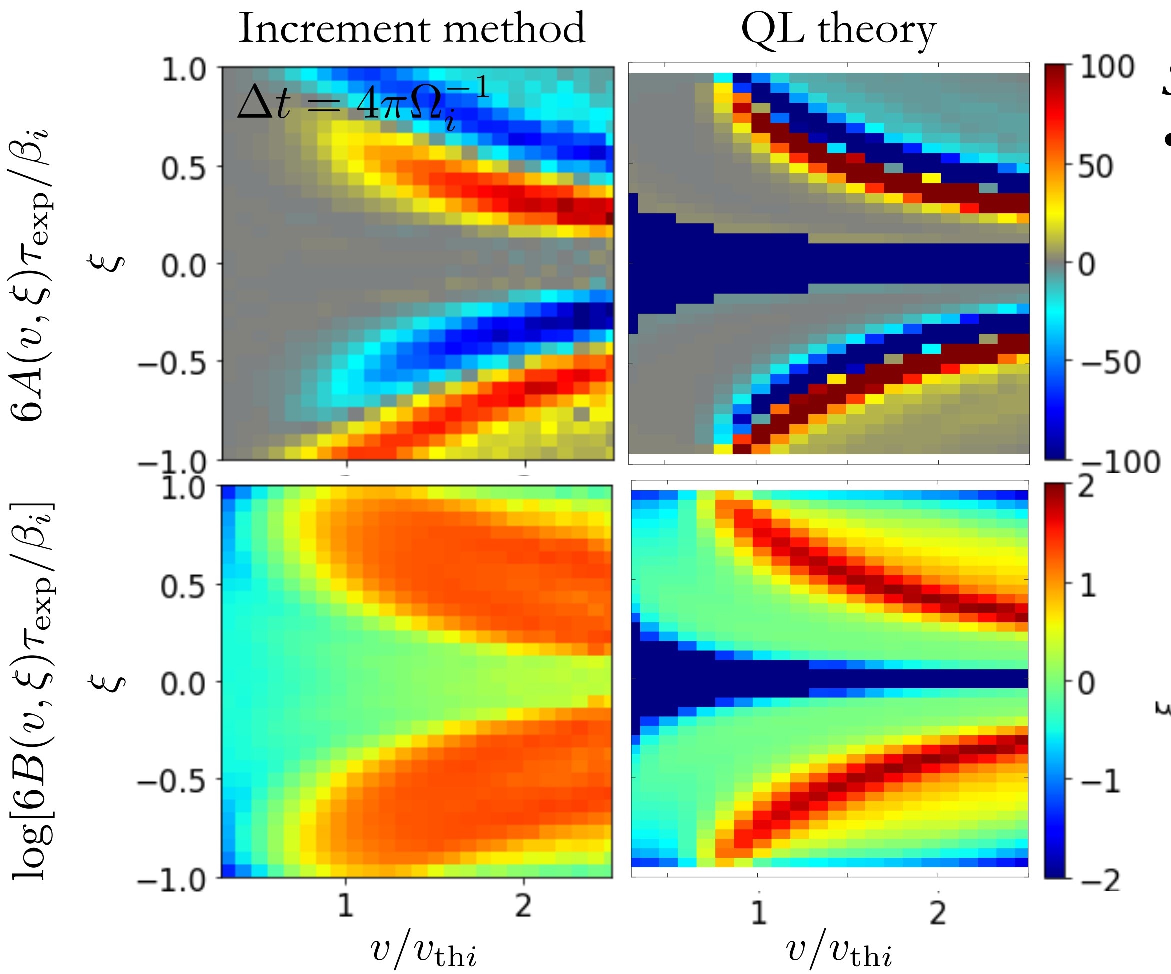
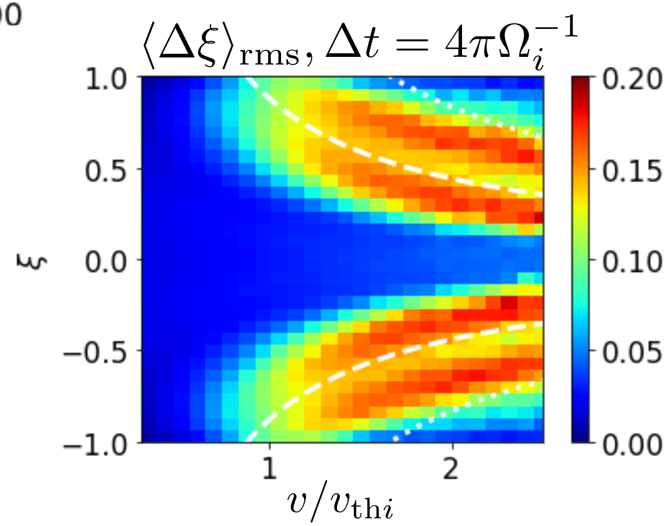
<u>Test 2:</u> measure Fokker-Planck coefficients of pitch-angle collision operator directly using increment method:

$$\mathfrak{C}[f] = -\frac{\partial}{\partial \xi}(Af) + \frac{1}{2}\frac{\partial^2}{\partial \xi^2}(Bf) \implies A \approx A_{\rm inc} \equiv \lim_{\Delta t \to \text{“}0\text{”}} \frac{\langle \Delta \xi \rangle}{\Delta t}, B \approx B_{\rm inc} = \lim_{\Delta t \to \text{”}0\text{”}} \frac{\langle \Delta \xi^2 \rangle - \langle \Delta \xi \rangle^2}{\Delta t}$$



Similar to model, but with broadened resonant features

- *Why?* Fluctuations in ξ of ions with $v_\parallel \sim v_{\rm thi}$ on timescales $\sim \Omega_i^{-1}$ due to interaction with magnetic fluctuations with $k_\parallel \rho_i < 1$
  - ➢ Particles starting at given ξ sample range of pitch angles ($\delta\xi \sim 0.15$).



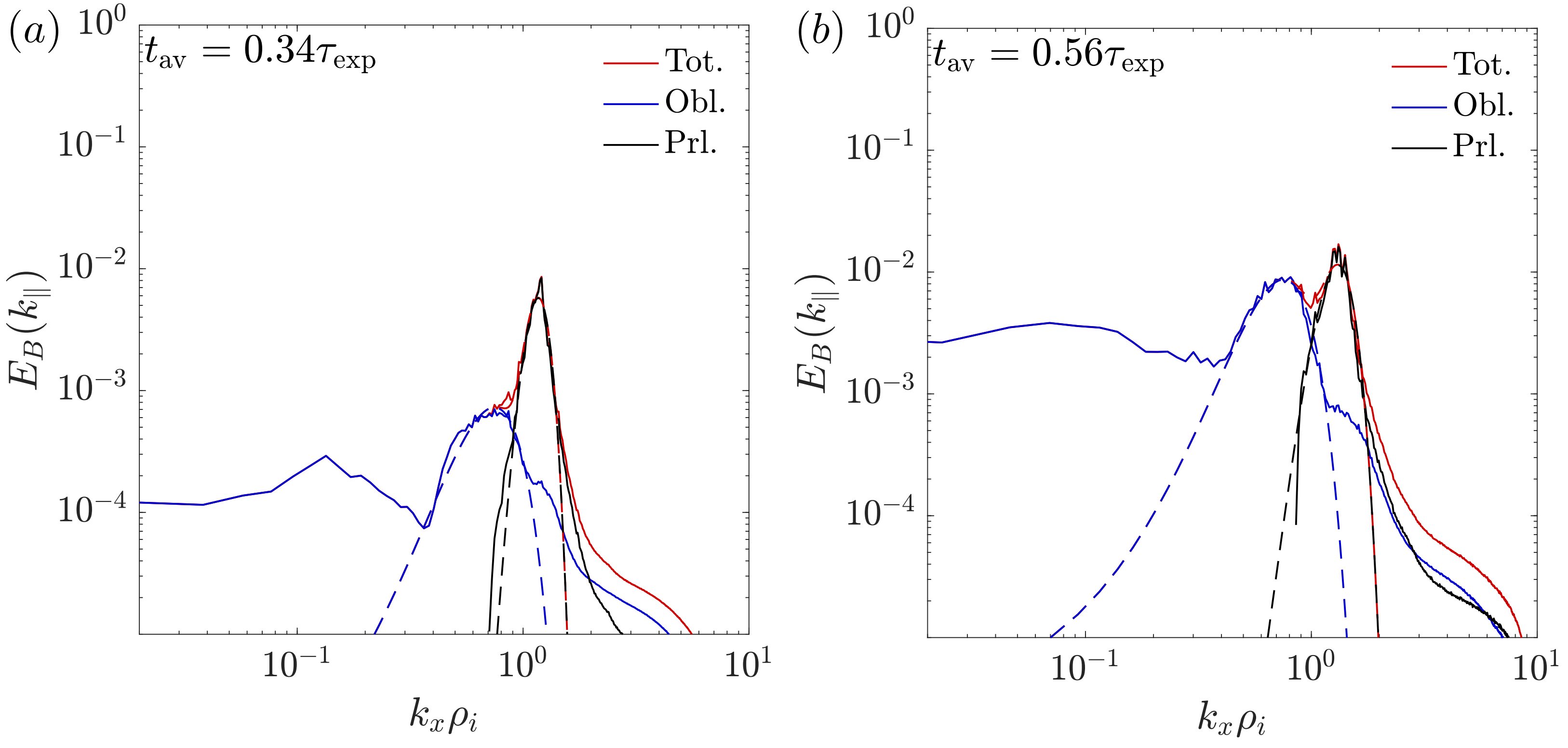
Already for this short increment, the average resonant particle has experienced a significant change in pitch angle!

An explicit form for the collision operator

Q: can we simple functional forms for the functions $\nu_{\text{eff,pl}}(v_\parallel)$, $\nu_{\text{eff,ob}}(v_\parallel)$ and $v_{\text{wv,pl}}(v_\parallel)$?

Unsophisticated approach: fit pre-specified functions to $E_{B,\text{pl}}(k_\parallel)$, $E_{B,\text{ob}}(k_\parallel)$, and $\varpi(k_\parallel \rho_i)$:

$$E_{B,\text{pl}}(k_\parallel) \simeq \frac{B_0^2}{8\pi} \frac{\bar{E}_{B,\text{pl}}}{\sqrt{\pi}\Delta k_{\parallel,\text{pl}}\rho_i} \exp\left[-\frac{(k_\parallel \rho_i - k_{\parallel,\text{pl}}\rho_i)^2}{(\Delta k_{\parallel,\text{pl}}\rho_i)^2}\right], \quad E_{B,\text{ob}}(k_\parallel) \simeq \frac{B_0^2}{8\pi} \frac{\bar{E}_{B,\text{ob}}}{\sqrt{\pi}\Delta k_{\parallel,\text{ob}}\rho_i} \exp\left[-\frac{(k_\parallel \rho_i - k_{\parallel,\text{ob}}\rho_i)^2}{(\Delta k_{\parallel,\text{ob}}\rho_i)^2}\right]$$



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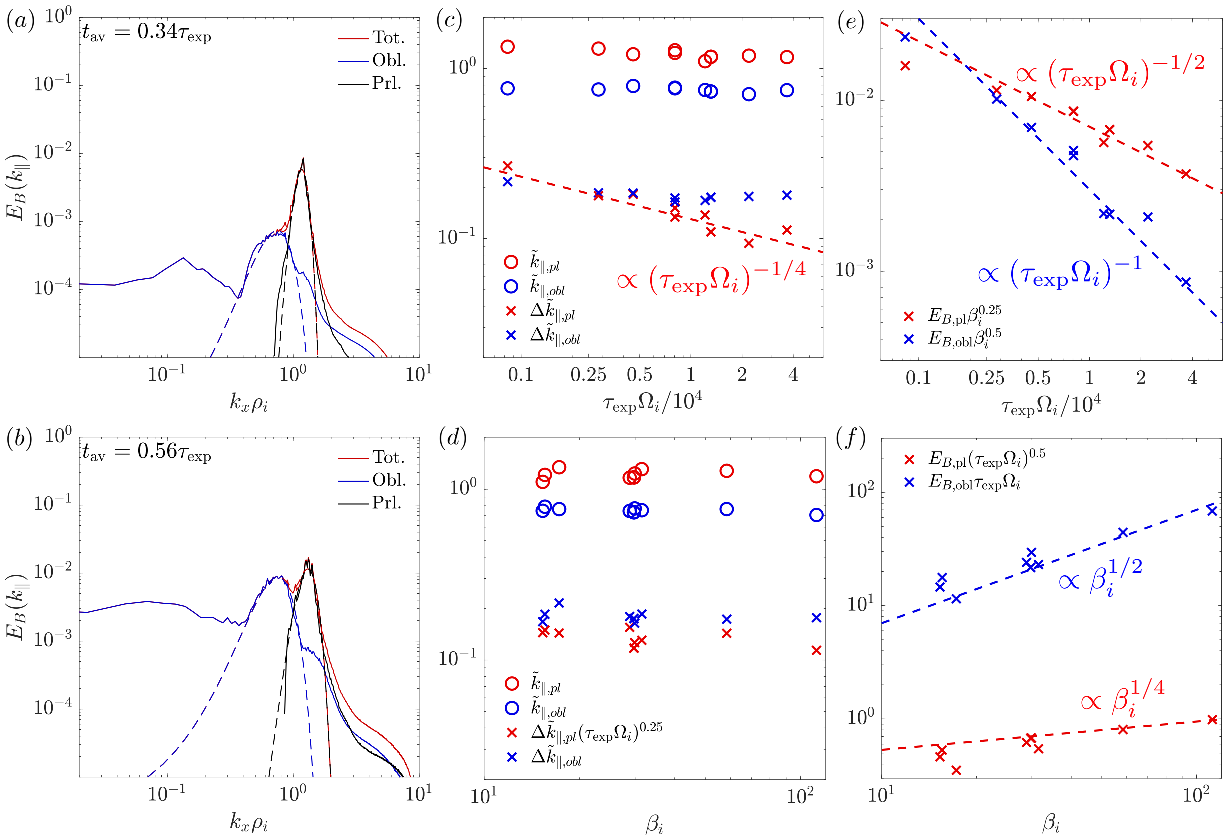

$$k_{\parallel,\rm pl}\rho_i \approx 1.2, \quad k_{\parallel,\rm ob}\rho_i \approx 0.75,$$

$$\Delta k_{\parallel,\rm pl}\rho_i \approx \frac{1.8}{(\tau_{\rm exp}\Omega_i)^{1/4}}, \quad \Delta k_{\parallel,\rm ob}\rho_i \approx 0.28,$$

$$\bar{E}_{B,\rm pl} \approx 0.3\frac{\beta_i^{1/4}}{(\tau_{\rm exp}\Omega_i)^{1/2}}, \quad \bar{E}_{B,\rm ob} \approx 0.7\frac{\beta_i}{\tau_{\rm exp}}.$$

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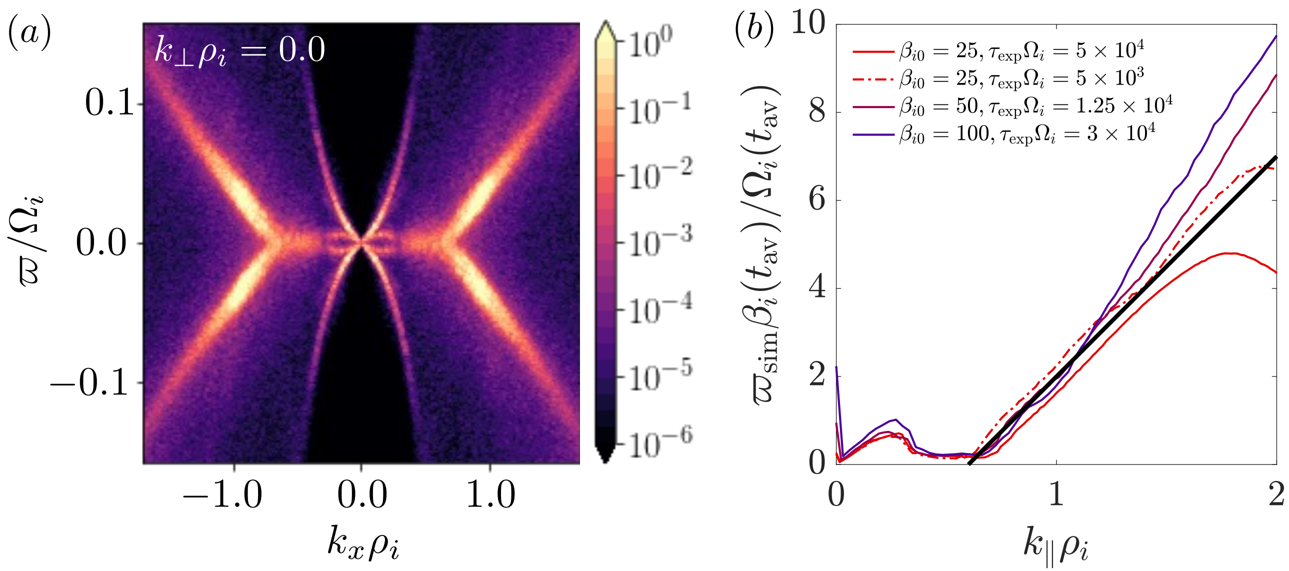
$$k_{\parallel,\mathrm{pl}}\rho_i \approx 1.2, \; k_{\parallel,\mathrm{ob}}\rho_i \approx 0.75, \; \Delta k_{\parallel,\mathrm{pl}}\rho_i \approx \frac{1.8}{(\tau_{\mathrm{exp}}\Omega_i)^{1/4}}, \; \Delta k_{\parallel,\mathrm{ob}}\rho_i \approx 0.28, \; \bar{E}_{B,\mathrm{pl}} \approx 0.3\frac{\beta_i^{1/4}}{(\tau_{\mathrm{exp}}\Omega_i)^{1/2}}, \; \bar{E}_{B,\mathrm{ob}} \approx 0.7\frac{\beta_i}{\tau_{\mathrm{exp}}}.$$



$$\langle\varpi\rangle(k_\parallel) \equiv \frac{\int_0^{k_{\perp,\max}} \mathrm{d}k_\perp \int_0^{\varpi_{\max}} \mathrm{d}\varpi\, \varpi E_B(k_\parallel, k_\perp, \varpi)}{\int_0^{k_{\perp,\max}} \mathrm{d}k_\perp \int_0^{\varpi_{\max}} \mathrm{d}\varpi\, E_B(k_\parallel, k_\perp, \varpi)},$$

$$\approx \frac{\Omega_i}{\beta_i}(4.9 k_\parallel \rho_i - 2.9).$$

This scheme works fine, but it is physically disgusting (Bruno Despres, Vienna 2023)

An explicit form for the collision operator II

Q: can we do anything more sophisticated (e.g. determine $\nu_{\mathrm{eff,pl}}(v_\parallel)$, $\nu_{\mathrm{eff,ob}}(v_\parallel)$ and $v_{\mathrm{wv,pl}}(v_\parallel)$ analytically)?

A: some progress, but not fully solved (solvable?)… Recall distribution function anisotropy:

$$\frac{1}{2\tilde{v}}\frac{\partial f_i}{\partial \xi} \approx f_{\mathrm{M}i}(v)\left\{\frac{\nu_{\mathrm{eff,pl}}(\tilde{v}_\parallel)\tilde{v}_{\mathrm{wv}}(\tilde{v}_\parallel)}{\nu_{\mathrm{eff,pl}}(\tilde{v}_\parallel)+\nu_{\mathrm{eff,ob}}(\tilde{v}_\parallel)} + \frac{\tilde{v}_\parallel}{3\tau_{\mathrm{exp}}[\nu_{\mathrm{eff,pl}}(\tilde{v}_\parallel)+\nu_{\mathrm{eff,ob}}(\tilde{v}_\parallel)]}\right\}.$$

Oblique firehose marginality $\Longrightarrow \dfrac{1}{3\sqrt{\pi}}\mathcal{P}\displaystyle\int_{-\infty}^{\infty}\frac{\tilde{v}_{\parallel i}^2 \mathrm{d}\tilde{v}_{\parallel i}}{1-k_\parallel^2\tilde{\rho}_i^2\tilde{v}_{\parallel i}^2}\frac{1}{\nu_{\mathrm{eff,ob}}(v_{\parallel i})}\exp\left(-\tilde{v}_{\parallel i}^2\right)\approx\frac{\tau_{\mathrm{exp}}}{\beta_{\parallel i}},\ k_\perp\rho_i \ll |k_\parallel\rho_i| < 1$

→ Follows that $k_{\parallel,\mathrm{ob}}\rho_i \sim \Delta k_{\parallel,\mathrm{ob}} \sim 1,\ \bar{E}_{B,\mathrm{ob}} \sim \beta_i/\tau_{\mathrm{exp}}$.

Secondary parallel firehose marginality: for $k_\parallel\rho_i \sim 1$,

$$\frac{\gamma}{\Omega_i}\approx\sqrt{\pi}\left[-\frac{\nu_{\mathrm{eff,ob}}\left(1/k_\parallel\rho_i\right)\tilde{v}_{\mathrm{wv}}(1/k_\parallel\rho_i)}{\nu_{\mathrm{eff,pl}}(1/k_\parallel\rho_i)}+\frac{1}{3k_\parallel\rho_i\tau_{\mathrm{exp}}\nu_{\mathrm{eff,pl}}(1/k_\parallel\rho_i)}\right]\exp\left(-\frac{1}{k_\parallel^2\rho_i^2}\right)\left[1+\frac{1}{k_\parallel\rho_i}\mathrm{Re}\,Z\left(\frac{1}{k_\parallel\rho_i}\right)\right]^{-1}$$

$$\Longrightarrow\ \tilde{v}_{\mathrm{wv}}(0.93)\approx\frac{0.31}{\tau_{\mathrm{exp}}\nu_{\mathrm{eff,ob}}\left(0.93\right)}\approx\frac{0.84}{\beta_i},\ \tilde{v}_{\mathrm{wv}}(k_\parallel\rho_i)\simeq\frac{k_\parallel\rho_i}{\beta_i}\quad \text{for } k_\parallel\rho_i \gg 1.$$

An explicit form for the collision operator III

Q: can we do anything more sophisticated (e.g. determine $\nu_{\mathrm{eff,pl}}(v_{\|})$, $\nu_{\mathrm{eff,ob}}(v_{\|})$ and $v_{\mathrm{wv,pl}}(v_{\|})$ analytically)?

A: some progress, but not fully solved (solvable?)… Recall distribution function anisotropy:

$$\frac{1}{2\tilde{v}}\frac{\partial f_i}{\partial \xi} \approx f_{\mathrm{M}i}(v)\left\{\frac{\nu_{\mathrm{eff,pl}}(\tilde{v}_{\|})\tilde{v}_{\mathrm{wv}}(\tilde{v}_{\|})}{\nu_{\mathrm{eff,pl}}(\tilde{v}_{\|})+\nu_{\mathrm{eff,ob}}(\tilde{v}_{\|})} + \frac{\tilde{v}_{\|}}{3\tau_{\mathrm{exp}}[\nu_{\mathrm{eff,pl}}(\tilde{v}_{\|})+\nu_{\mathrm{eff,ob}}(\tilde{v}_{\|})]}\right\}.$$

Secondary parallel firehose marginality cont.: can also estimate $\delta B_{\mathrm{pl}}^2/B_0^2$ using expression for growth rate:

$$\gamma_{\mathrm{pl}}\frac{\delta B_{\mathrm{pl}}^2}{B_0^2} \sim \frac{1}{\tau_{\mathrm{exp}}},\ \gamma_{\mathrm{pl}} \sim \Delta t_{\mathrm{c}}^{-1} \sim \frac{\Omega_i^{1/2}}{\tau_{\mathrm{exp}}^{1/2}} \implies \frac{\delta B_{\mathrm{pl}}^2}{B_0^2} \sim \frac{1}{(\tau_{\mathrm{exp}}\Omega_i)^{1/2}}.$$

Unanswered questions:

1. Why does our numerical measurement of $\delta B_{\mathrm{pl}}^2/B_0^2$ depend on β_i?

2. Why is $\nu_{\mathrm{eff,pl}}(v_{\|})$ sharply peaked? Could this lead to quasilinear theory breaking asymptotically?

3. What happens to particles with small $v_{\|}$?

Conclusions

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1. Weakly collisional and collisionless magnetised $\beta \gtrsim 1$ plasmas often become susceptible to the firehose instability in the course of their macroscopic evolution, altering their basic physical properties, and generating anomalous collisionality.

2. We have characterised the effective collision operator associated with firehoses in Alfvén-enabling statse (the astrophysically relevant regime) by a simple quasilinear pitch-angle scattering operator
   - The box-averaged collision frequency is $\nu_{\rm eff} \sim \beta_i/\tau$, in agreement with previous results
   - Certain sub-populations of particles experience collisions at a much greater (or smaller) rate depending on their velocity in the direction parallel to the magnetic field.

$$\mathfrak{C}_{\rm f}[f] = \frac{1}{2}\frac{\partial}{\partial \xi}\left\{(1-\xi^2)\nu_{\rm eff,pl}(w\xi)\left[\frac{\partial f_i}{\partial \xi} - 2\tilde{w}\tilde{v}_{\rm wv,pl}(w\xi) f_{{\rm M}i}\right] + (1-\xi^2)\nu_{\rm eff,ob}(w\xi)\frac{\partial f_i}{\partial \xi}\right\},$$

$$\nu_{\rm eff,pl}(v_\|) = \frac{0.15 v_{{\rm th}i}}{|v_\||}\frac{\beta_i^{1/4}\Omega_i^{3/4}}{\tau_{\rm exp}^{1/4}}\exp\left[-0.31(\tau_{\rm exp}\Omega_i)^{1/2}\left(\frac{v_{{\rm th}i}}{|v_\||} - 1.2\right)^2\right],$$

$$v_{\rm wv,pl}(v_\|) = {\rm sgn}(v_\|)\frac{v_{{\rm th}i}}{\beta_i}\left(5.0 - 3.0\frac{|v_\||}{v_{{\rm th}i}}\right), \quad \nu_{\rm eff,ob}(v_\|) = \frac{1.4 v_{{\rm th}i}}{|v_\||}\frac{\beta_i}{\tau_{\rm exp}}\exp\left[-13\left(\frac{v_{{\rm th}i}}{|v_\||} - 0.75\right)^2\right].$$