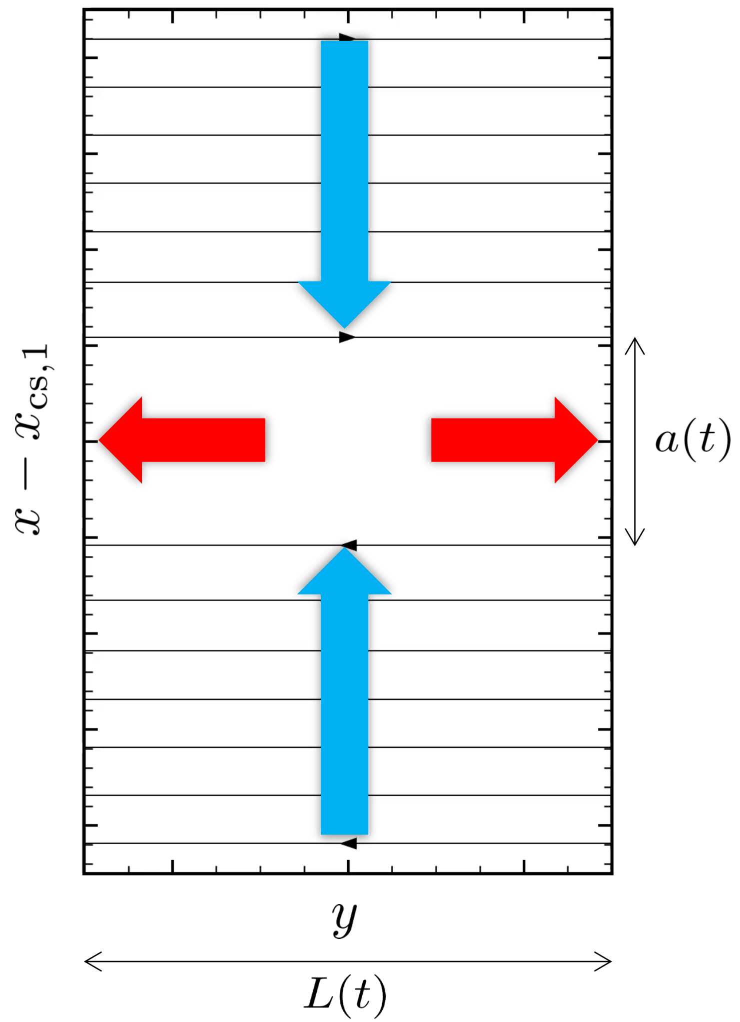
Tearing and Mirror Instability

Himawan Winarto and Matthew Kunz
Princeton University

13th Plasma Kinetics Working Meeting
Wolfgang Pauli Institute, Vienna, 25 July 2022

Current Sheet (CS) Formation



Reconnection does not immediately start in nature

Consider field reversal with Harris equilibrium

$$\boldsymbol{B}_{\mathrm{r}}(t,x) = B_{\mathrm{r}}(t)\tanh\left[\frac{x}{a(t)}\right]\hat{\boldsymbol{y}}$$

Impose background incompressible fluid flow:

$$\boldsymbol{u}(t,x,y) = \frac{1}{\Gamma(t)\tau_{\mathrm{cs}}}(-x\hat{\boldsymbol{x}} + y\hat{\boldsymbol{y}})$$

$$\Gamma(t) \doteq 1 + \frac{t}{\tau_{\mathrm{cs}}}$$

Hyper-resistivity Tearing Instability

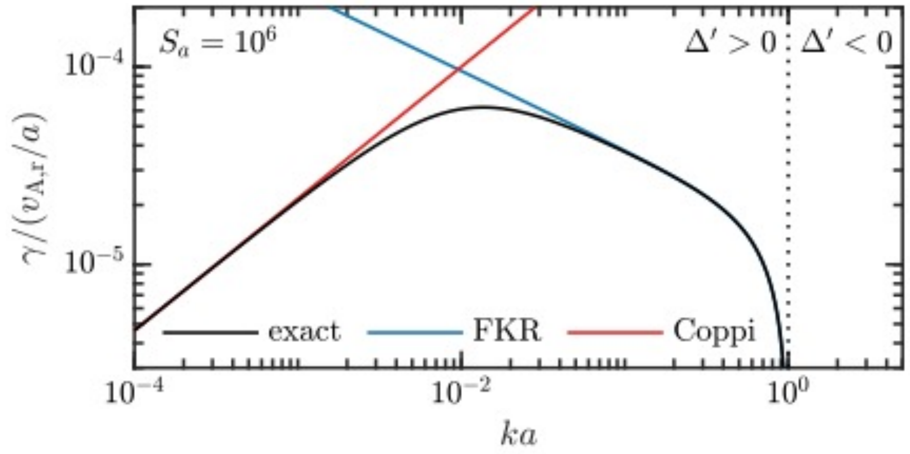
→ Reconnection needs CS to go tearing unstable, for Harris sheet:

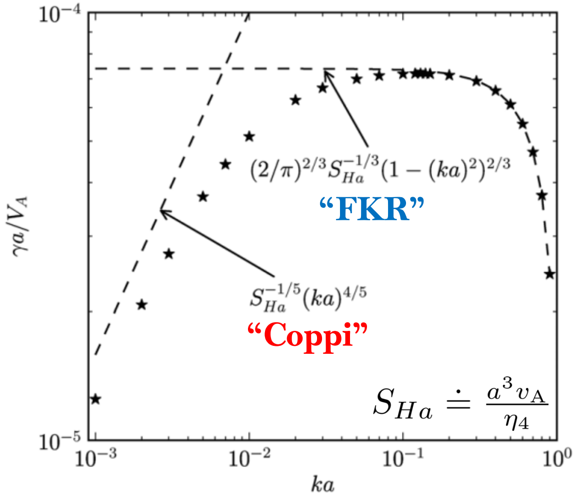
$$\Delta'(t,N) = \frac{2N}{L(t)}\left[\frac{1}{N^2}\frac{L^2(t)}{a^2(t)} - 1\right]$$

→ In Ohmic tearing, there are two asymptotic regimes:

- Coppi : Multiple magnetic islands
- FKR : Single magnetic island

→ In hyper-resistive tearing has flat "FKR" regime, thus tearing mode with single island is always dominant.





Huang+(2013)

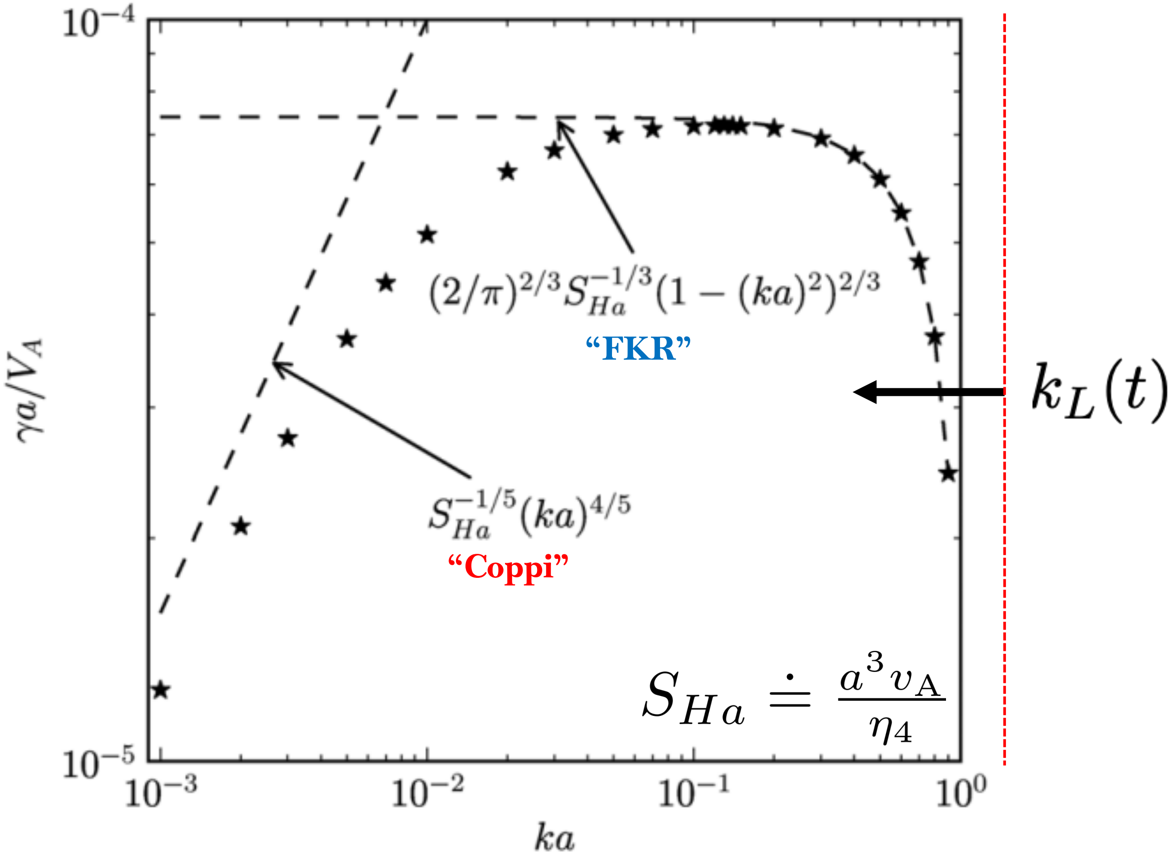
Hyper-resistivity Tearing Instability

→ As CS evolves, aspect ratio changes and longer tearing wavelengths become available.

→ Estimate of tearing onset is from the first unstable mode of $N \sim 1$.

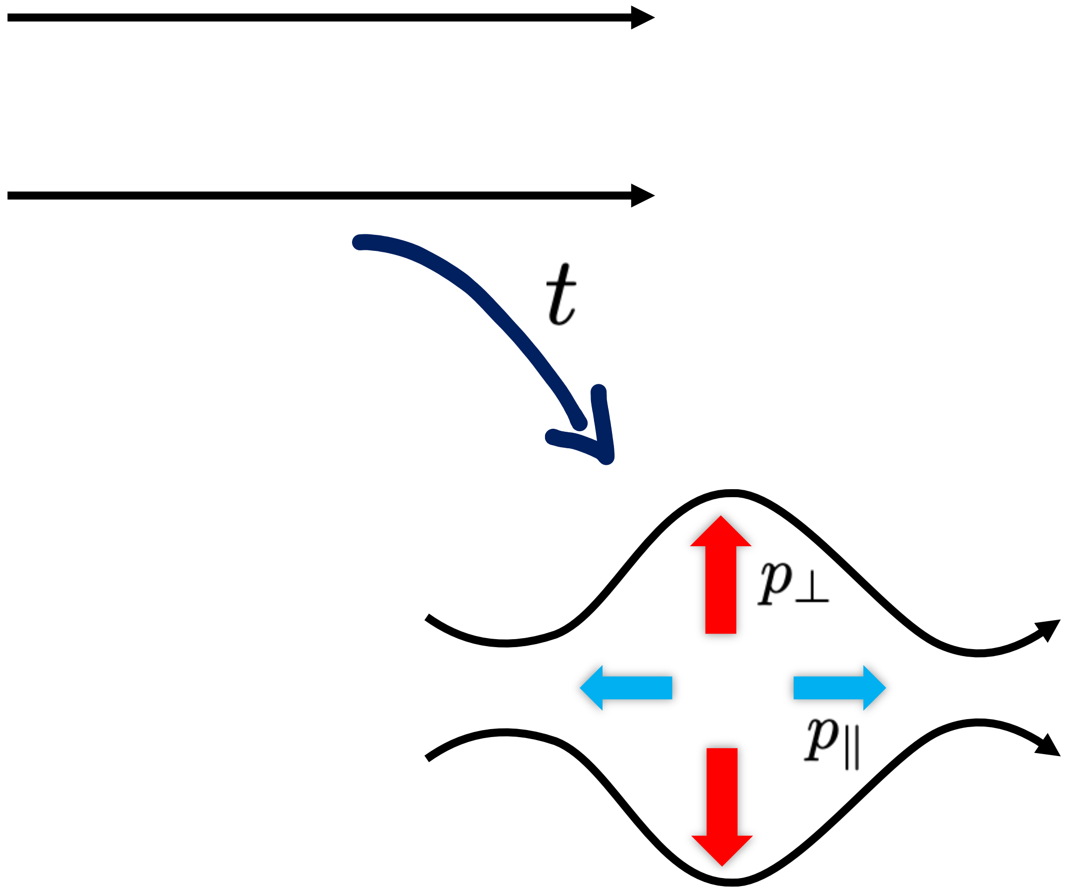
→ Growth rate must satisfy $\gamma_{\mathrm{t}}\tau_{\mathrm{cs}} \gtrsim 1$ and $k_{\mathrm{t}}a \ll 1$ giving

$$\left(1+\frac{t_{\mathrm{cr}}}{\tau_{\mathrm{cs}}}\right)^{2/3}\frac{a_0}{L_0} \ll S_{a0}^{1/3} M_{\mathrm{A}0} \lesssim \left(1+\frac{t_{\mathrm{cr}}}{\tau_{\mathrm{cs}}}\right)^{8/3}.$$



Huang+(2013)

Magnetic Accumulation Causes Mirror Instability



→ CS compression cause magnetic amplification

$$B_\mathrm{r}(t) = B_\mathrm{r0}\Gamma(\mathrm{t})$$

→ In collisionless system, the double adiabatic conservations gives pressure anisotropy

$$\Delta_p(t,\xi(t)) = [\Gamma(\mathrm{t})]^3 - 1 \approx \frac{3\mathrm{t}}{\tau_\mathrm{cs}}$$

→ Large pressure anisotropy can be unstable to mirror instability given that

$$\Lambda_\mathrm{m} \doteq \Delta_p - \frac{1}{\beta_\perp} > 0$$

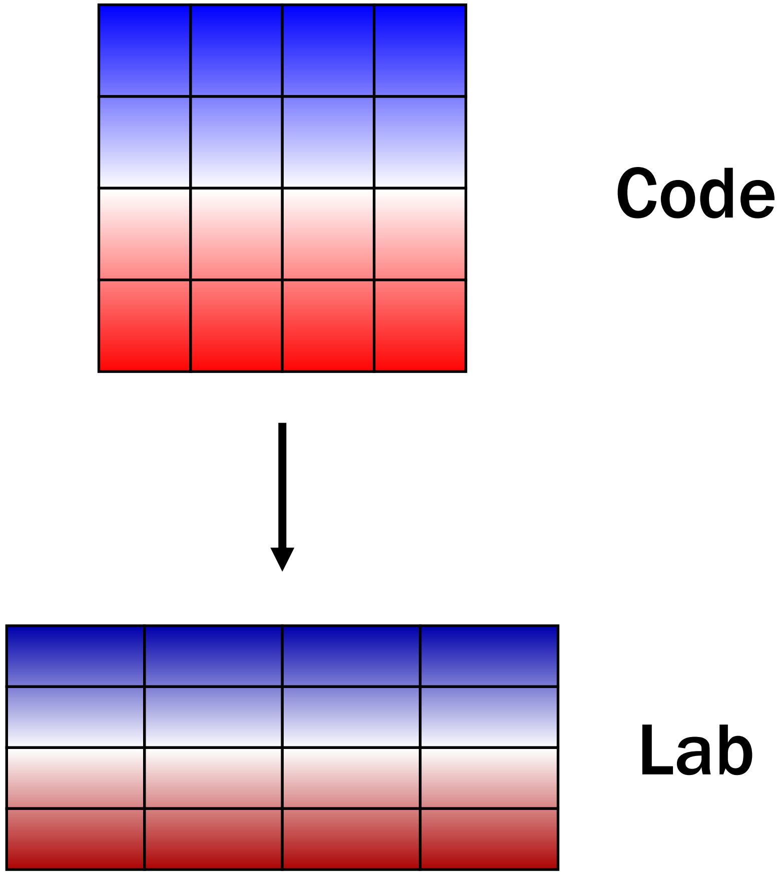
→ Bulk region with stronger initial field will be unstable first with mirror time satisfying

$$\frac{t_\mathrm{m}}{\tau_\mathrm{cs}} \approx \frac{1}{3\beta_\perp(0,\xi_0)} \ll 1$$

→ The pressure anisotropy will then be regulated at mirror regulation time

$$\frac{t_\mathrm{m,reg} - t_\mathrm{m}}{\tau_\mathrm{cs}} \approx \frac{C_\mathrm{m}}{3}(\Omega_i\tau_\mathrm{cs})^{-1/2}$$

Numerical Framework



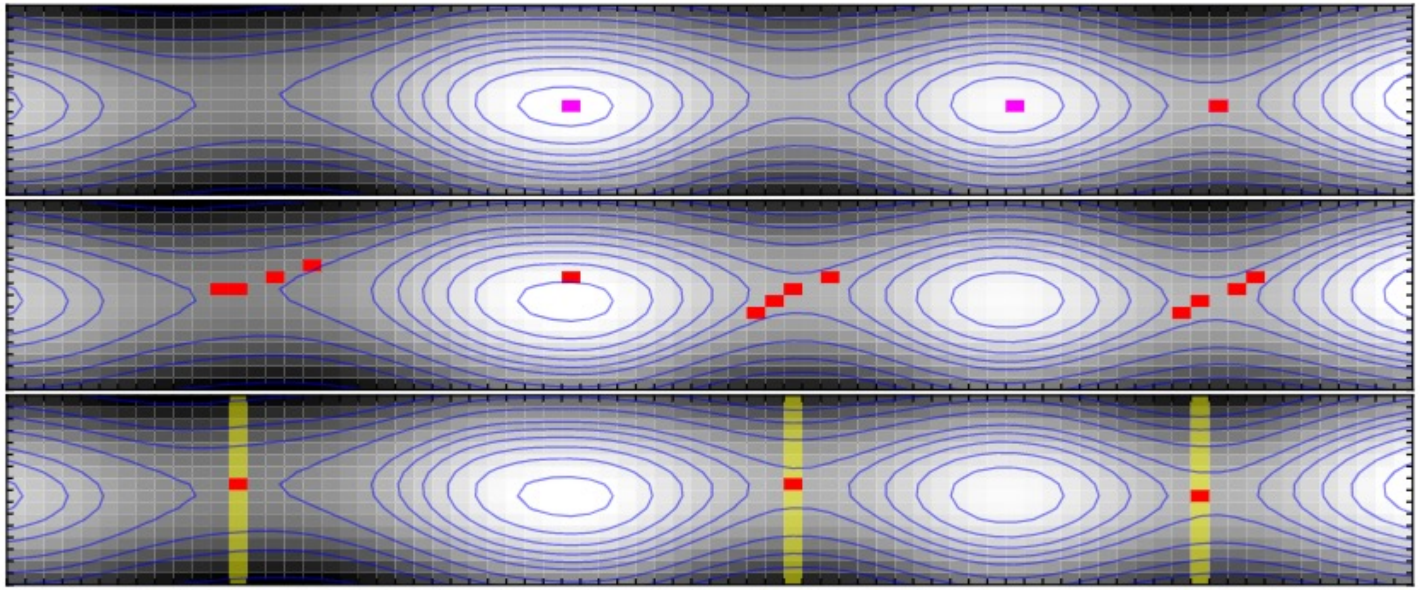
- → Hybrid-kinetic code **`Pegasus++`** with kinetic ions and neutralizing electron background fluid.
  - Hyper-resistivity acts as the electron inertia
- → Compression is modeled using continuous frame transformation

$$\begin{aligned}\Lambda(\mathrm{t}) &= \Gamma^{-1}(\mathrm{t})\hat{\boldsymbol{x}}\hat{\boldsymbol{x}} + \Gamma(\mathrm{t})\hat{\boldsymbol{y}}\hat{\boldsymbol{y}} \\ \Gamma(\mathrm{t}) &\doteq 1 + \frac{t}{\tau_{\mathrm{cs}}}\end{aligned} \quad \longrightarrow \quad \begin{aligned}\boldsymbol{E'} &= \Lambda \boldsymbol{E} \\ \boldsymbol{B'} &= \lambda \Lambda^{-1} \boldsymbol{B} \\ n' &= \lambda n \\ \boldsymbol{u'} &= \Lambda^{-1} \boldsymbol{u}\end{aligned}$$

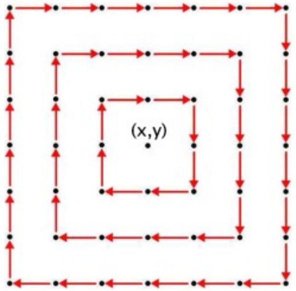
- → Initialize with double Harris-sheet with periodic boundary conditions

$$\boldsymbol{B}_{\mathrm{r}}(t=0) = B_{\mathrm{r}0}\left[\tanh\left(\frac{x-x_{\mathrm{cs},1}}{a_0}\right) - \tanh\left(\frac{x-x_{\mathrm{cs},2}}{a_0}\right) - 1\right]\hat{\boldsymbol{y}}$$

Watershed Segmentation to Identify X-Points



→ Simple saddle-point algorithm

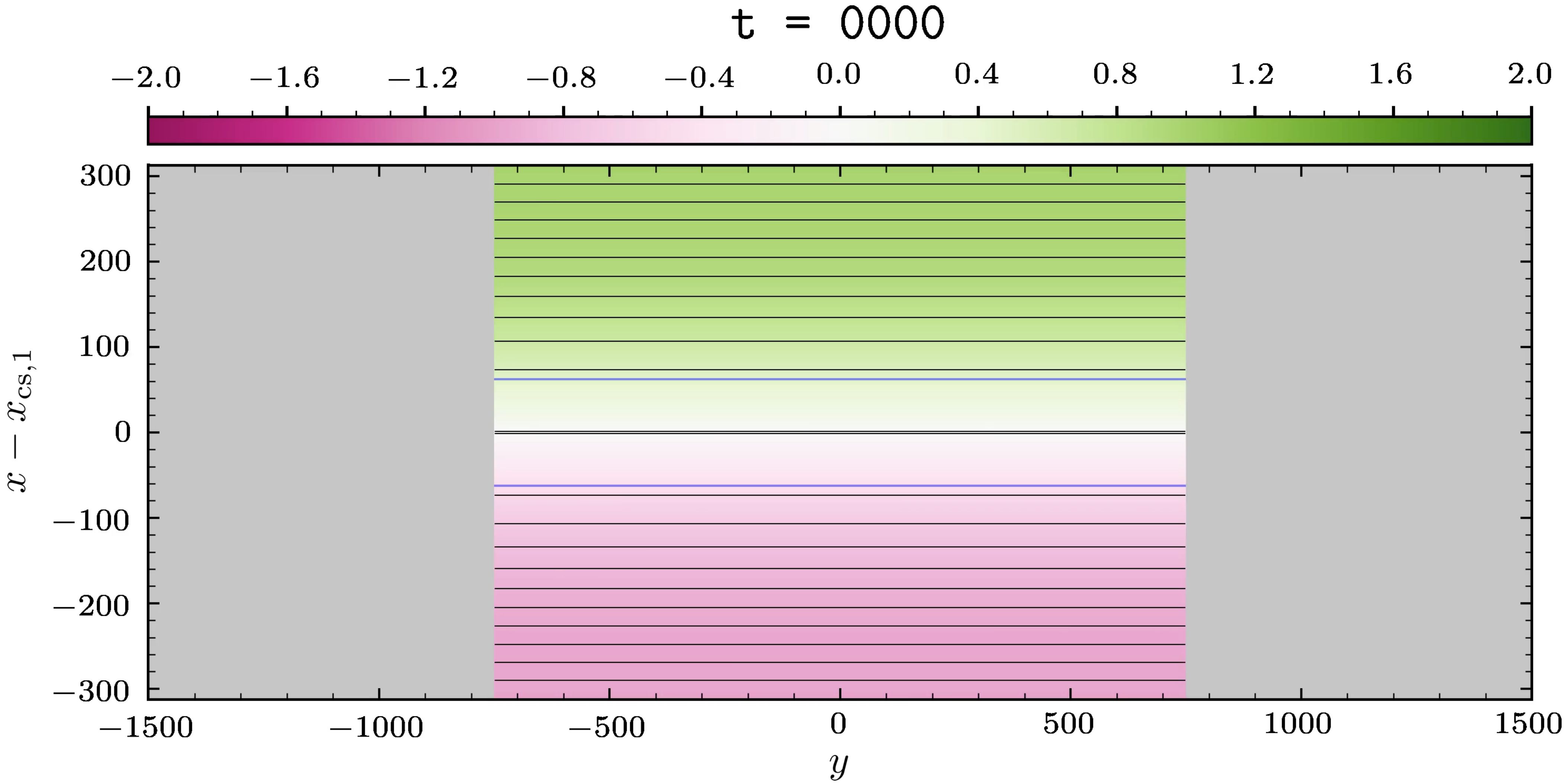


→ Zhdankin+(2013)

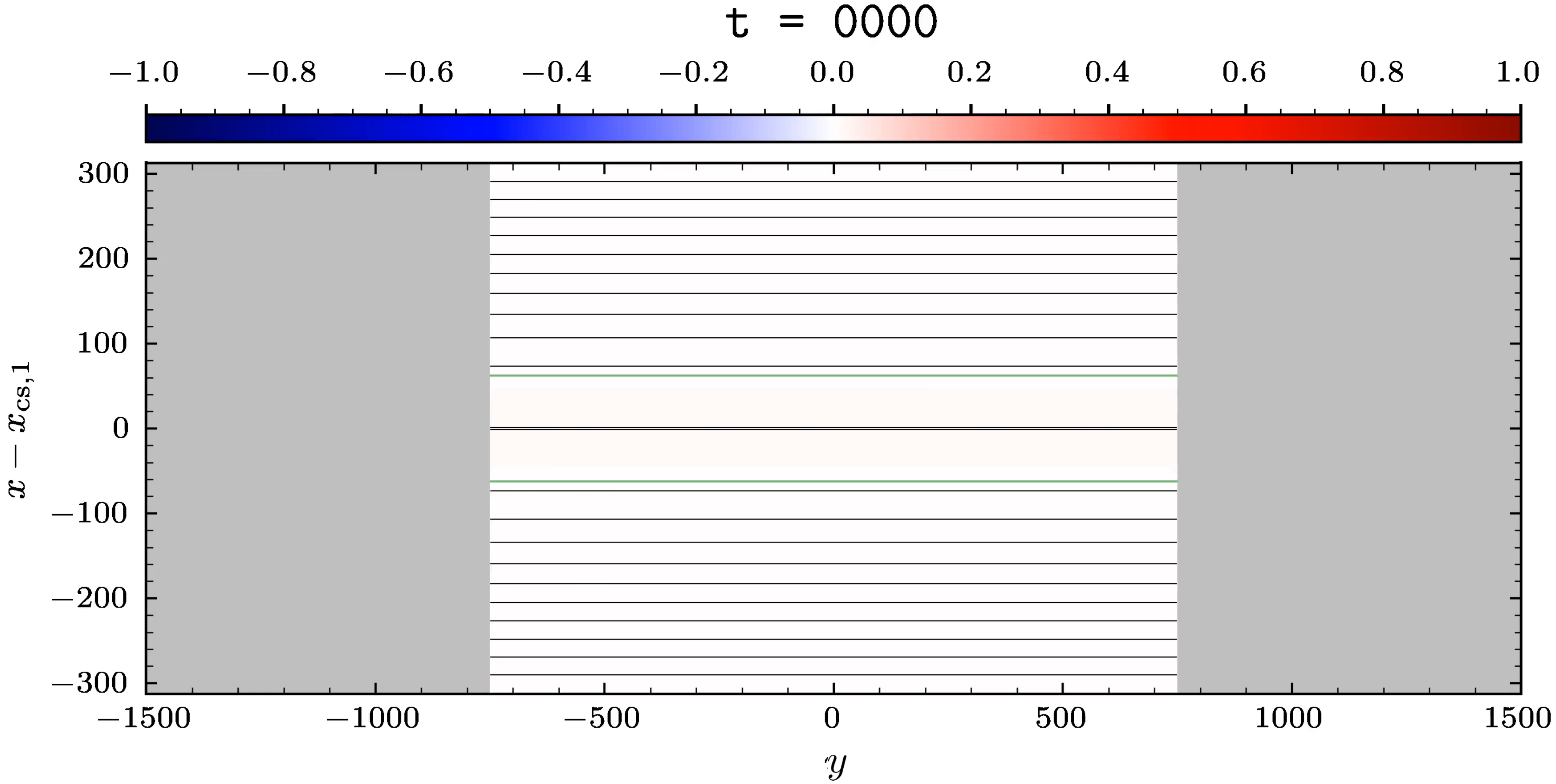
→ Watershed segmentation

Magnetic island acts as flood basins with X-points as the local minimum point along the boundaries

Fiducial Evolution $\tau_{cs} = 1000, a_0 = 125$



Fiducial Evolution $\tau_{cs} = 1000, a_0 = 125$



Fiducial Evolution $\tau_{cs} = 1000, a_0 = 125$

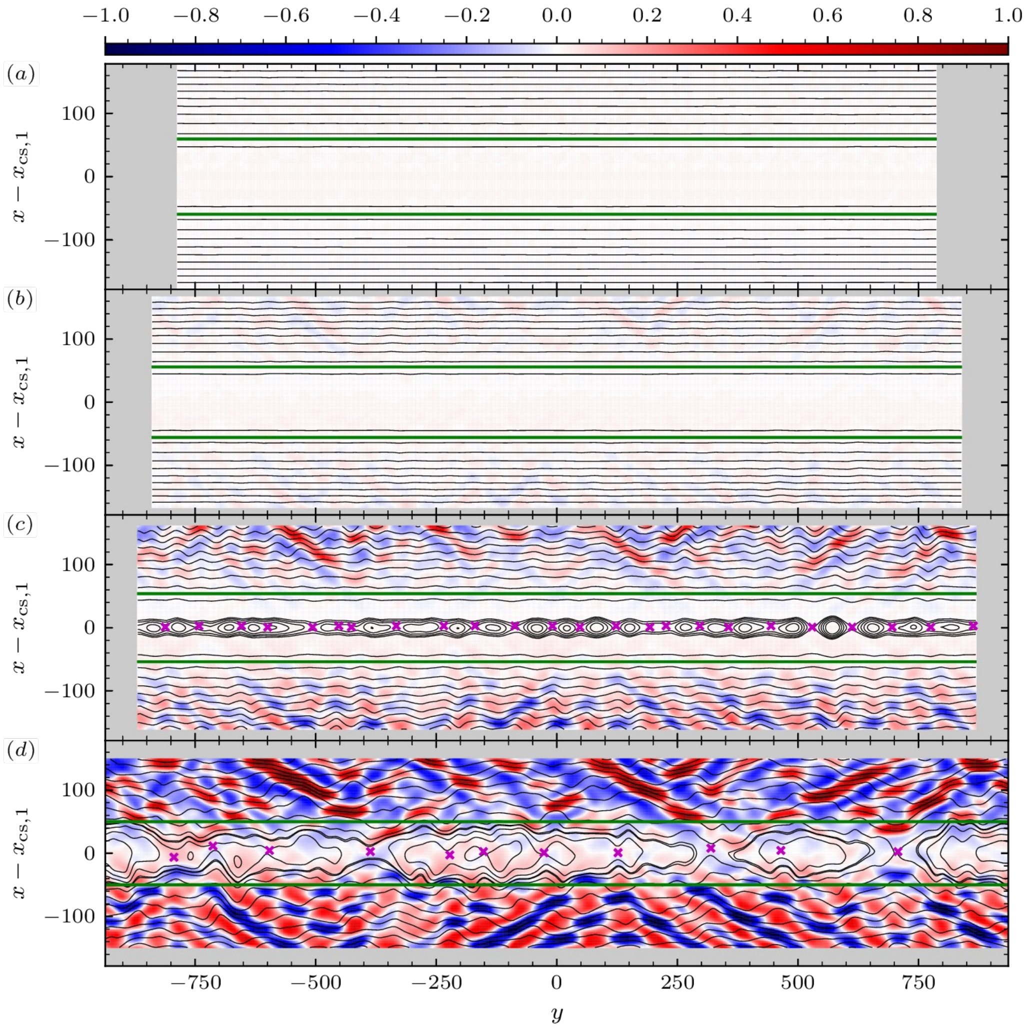
→ Focus at the four stages of evolution:

- (a) : $t = 50$
- (b) : $t = 120$
- (c) : $t = 160$
- (d) : $t = 250$

→ Higher field strength in `Bulk` region starts the mirror instabilities early as shown in figure (b).

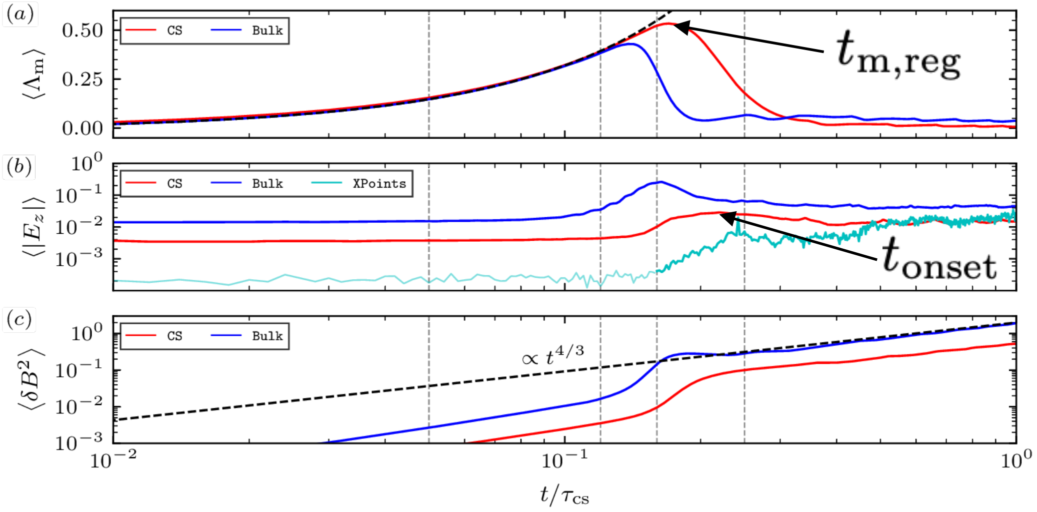
→ In figure (c), the CS should still be **tearing stable**, but we start seeing magnetic mirror.

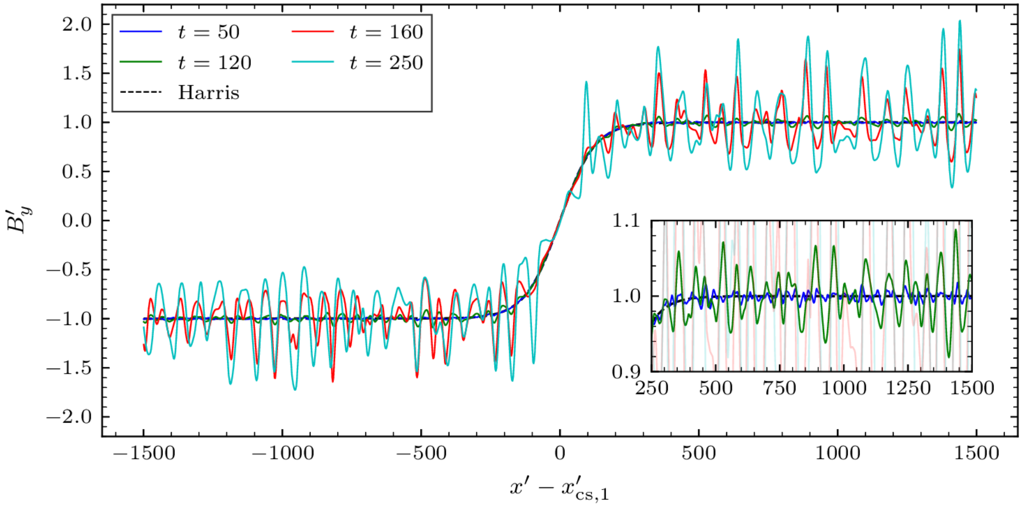
→ In figure (d), the reconnection is well underway with non-linear tearing as indicated by **merging magnetic islands**.



Fiducial Evolution $\tau_{cs} = 1000, a_0 = 125$

- → Average quantities based on the `Bulk` vs `CS` region as well as the averaged values over the location of X-points as `XPoints`.
- → Confirmed secular growth of magnetic fluctuations due to mirror instabilities in the `Bulk`.
- → Mirror fluctuations get progressively closer to the neutral line as time progress.
- → The value of average E_z in `CS` region decrease slowly as the magnetic islands grow.
- → Measured average E_z over the `XPoints` matches the previously reported value in prior hyper-resistive reconnection publications (e.g., Huang, 2013).





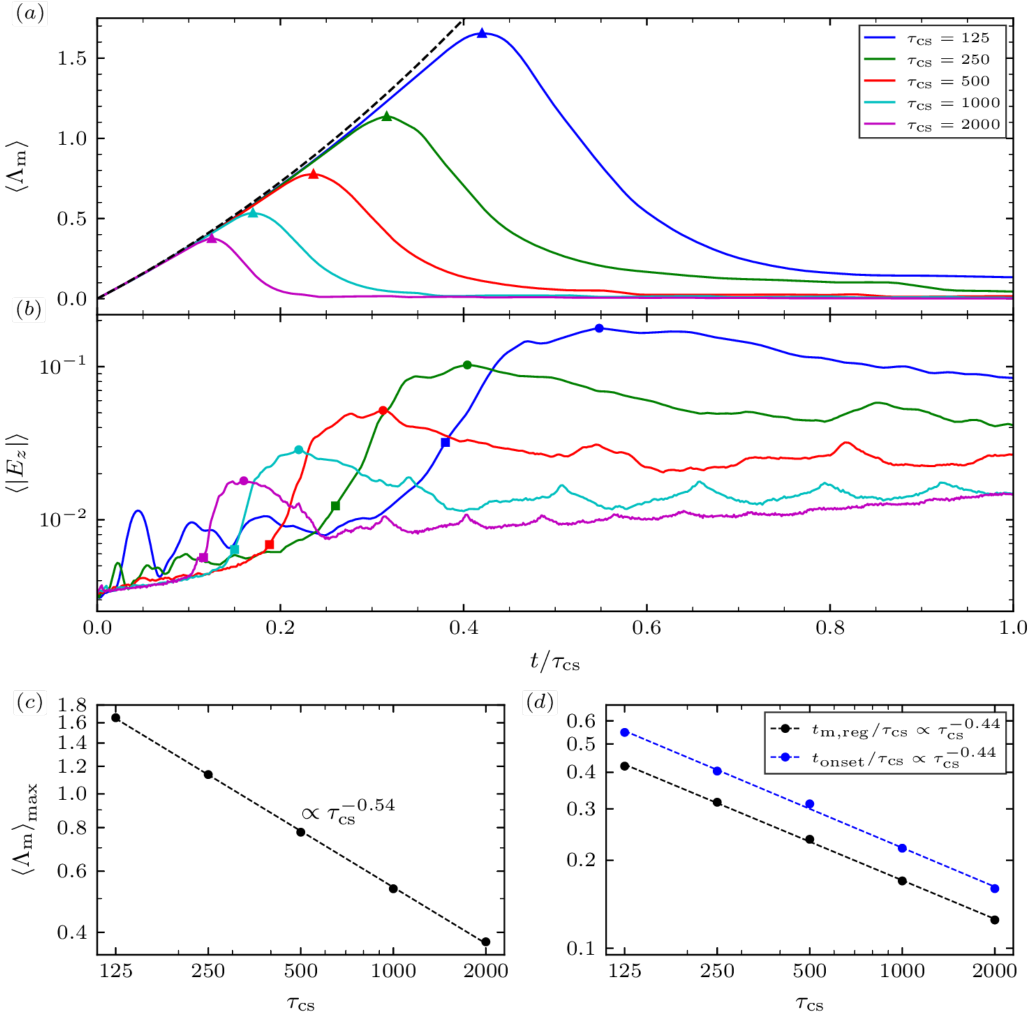
Variation of τ_{cs}

→ Initial mirror growth in CS follows double adiabatic condition, with peak values of Λ_{m} satisfies theoretical prediction

$$\Lambda_{\mathrm{m,max}} = C_{\mathrm{m}}(\Omega_i \tau_{\mathrm{cs}})^{-1/2}$$

$$\frac{t_{\mathrm{m,reg}} - t_{\mathrm{m}}}{\tau_{\mathrm{cs}}} \approx \frac{C_{\mathrm{m}}}{3}(\Omega_i \tau_{\mathrm{cs}})^{-1/2}$$

→ The dominant linear tearing modes can be obtained from time averaging between the peak of mirror instability parameters and the second E_z peak, i.e. between $t_{\mathrm{m,reg}}$ and t_{onset}.



Variation of τ_{cs}

→ Initial mirror growth in CS follows double adiabatic condition, with peak values of $\Lambda_{\rm m}$ satisfies theoretical prediction

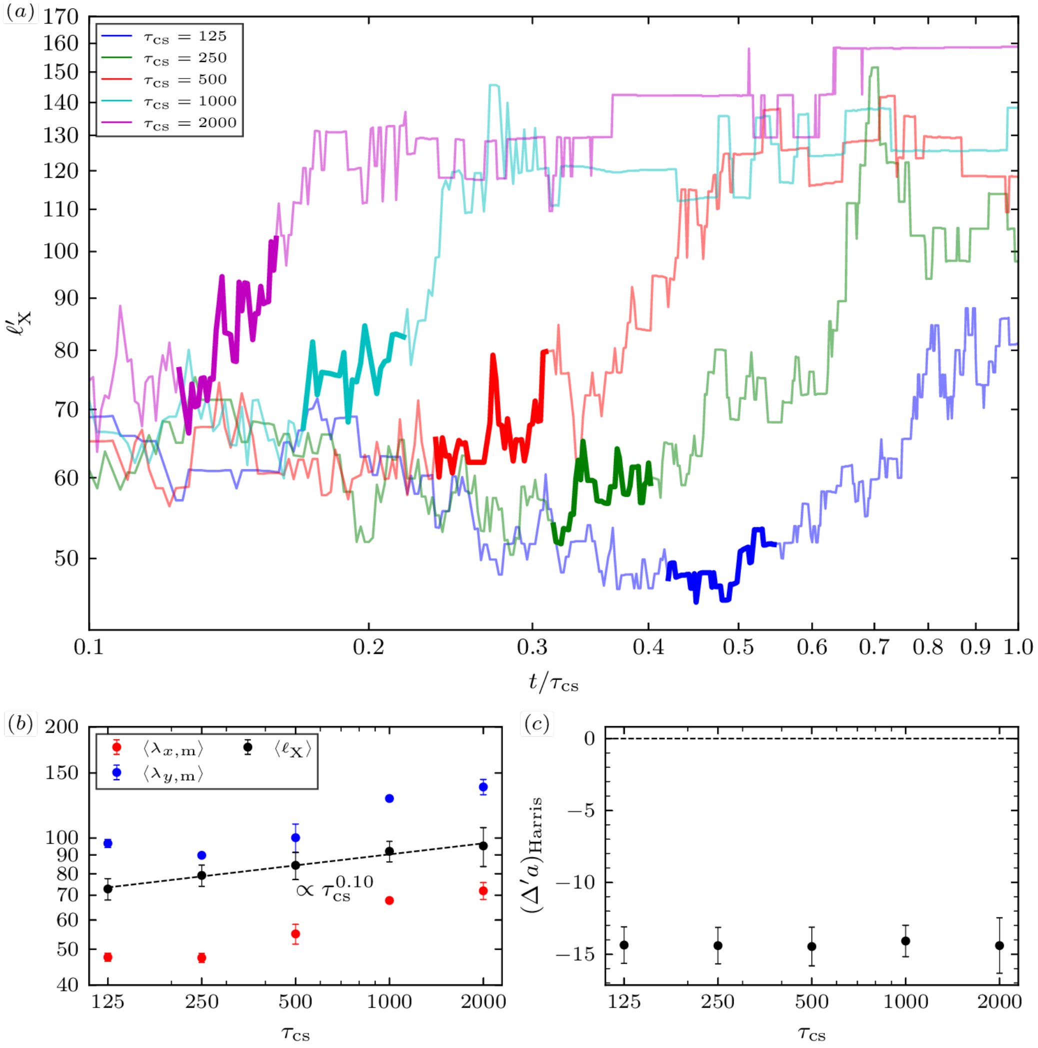
$$\Lambda_{\rm m,max} = C_{\rm m}(\Omega_i \tau_{\rm cs})^{-1/2}$$

$$\frac{t_{\rm m,reg} - t_{\rm m}}{\tau_{\rm cs}} \approx \frac{C_{\rm m}}{3}(\Omega_i \tau_{\rm cs})^{-1/2}$$

→ The dominant linear tearing modes can be obtained from time averaging between the peak of mirror instability parameters and the second E_z peak, i.e. between $t_{\rm m,reg}$ and $t_{\rm onset}$.

→ The most dominant mode scale similar to mirror modes indicating strong interaction between the two.

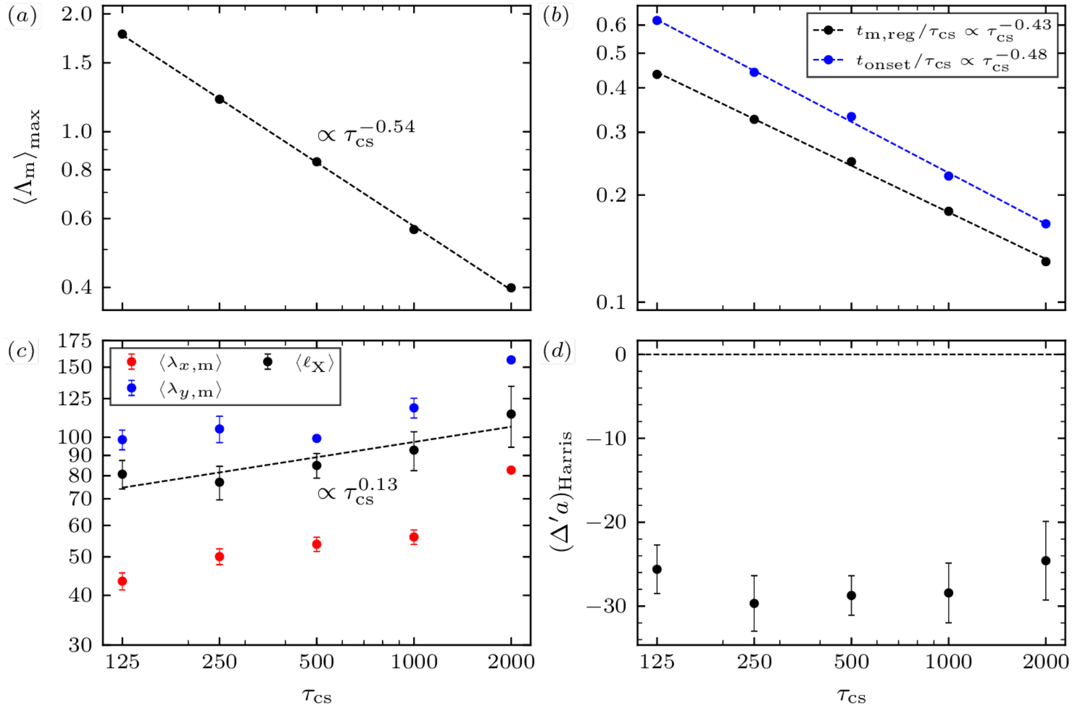
- Classically these tearing modes should still be stable.



Wide Sheet with Variation of τ_{CS}

→ The reconnection onset time does not differ significantly from the thinner sheet, while the wider initial sheet should still be much more stable to tearing instability.

→ Consistent scaling for CS with double initial thickness confirms that the mirror-instability is seeding the tearing modes.



Summary

→ During CS formation in high-beta collisionless plasma, the pressure anisotropy build up can trigger **mirror instability** disrupting the sheet with ion Larmor scale perturbations.

→ The rapid growth of the **mirror fluctuations stimulates tearing modes** by wrinkling the current sheet, effectively reducing CS thickness and changing the value of Δ'.

→ The resulting tearing instability **onsets at earlier time** and **on smaller scales** than it would have without the mirrors.

- Onset time scales at approximately $\propto \tau_{\mathrm{cs}}^{1/2}$.
- Tearing modes are at intermediate scales between the parallel and perpendicular wavelengths of the mirror modes.

→ In turbulence context, this result puts the importance of **kinetic instabilities** that can disturb magnetic folds, which locally can be seen as CS.

* **Watershed segmentation** can be used to robustly determine locations of X-points.