13^{th} Plasma kinetics Working meeting

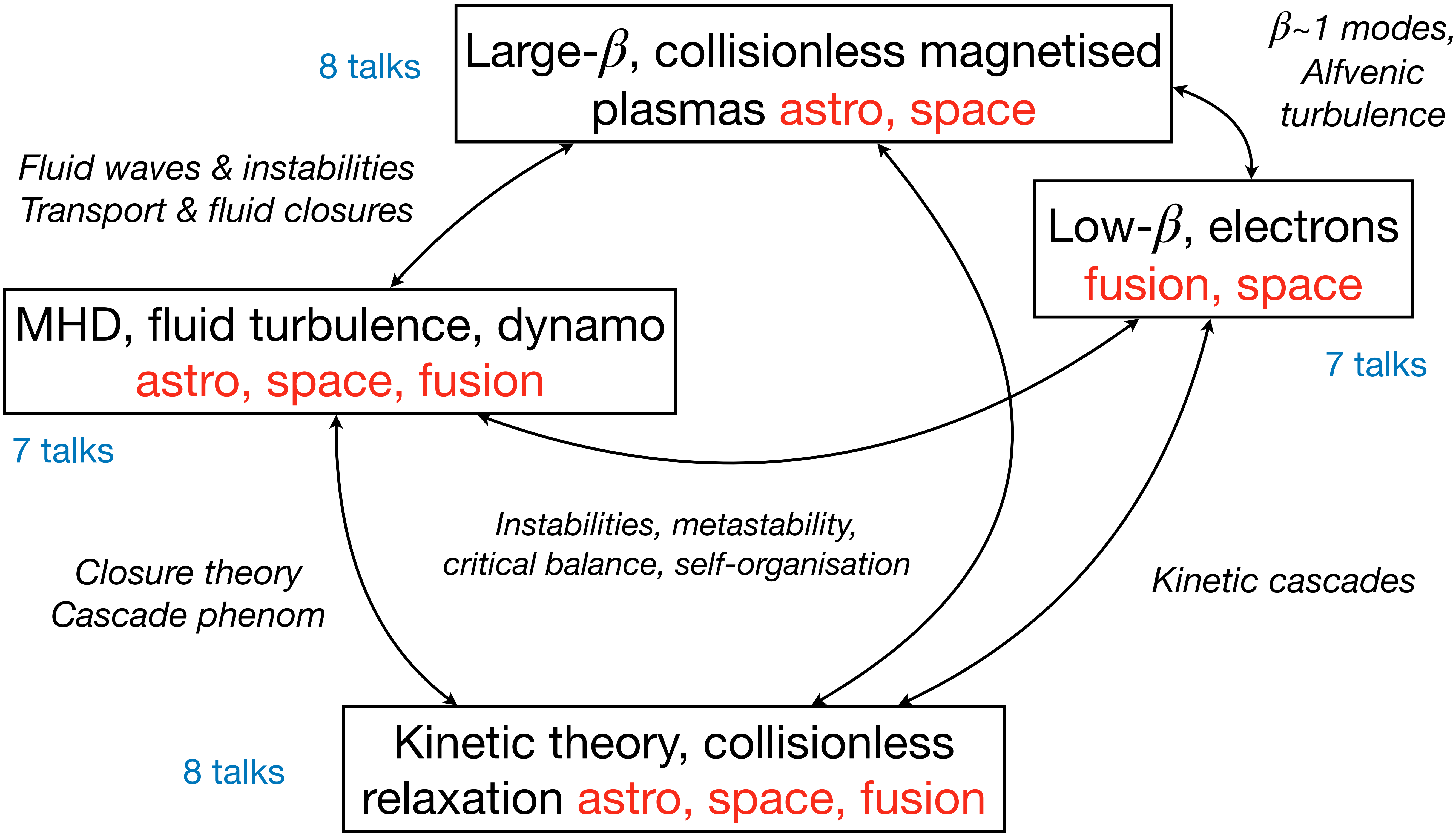
Wolfgang Pauli Institute
Vienna

Summary and discussion

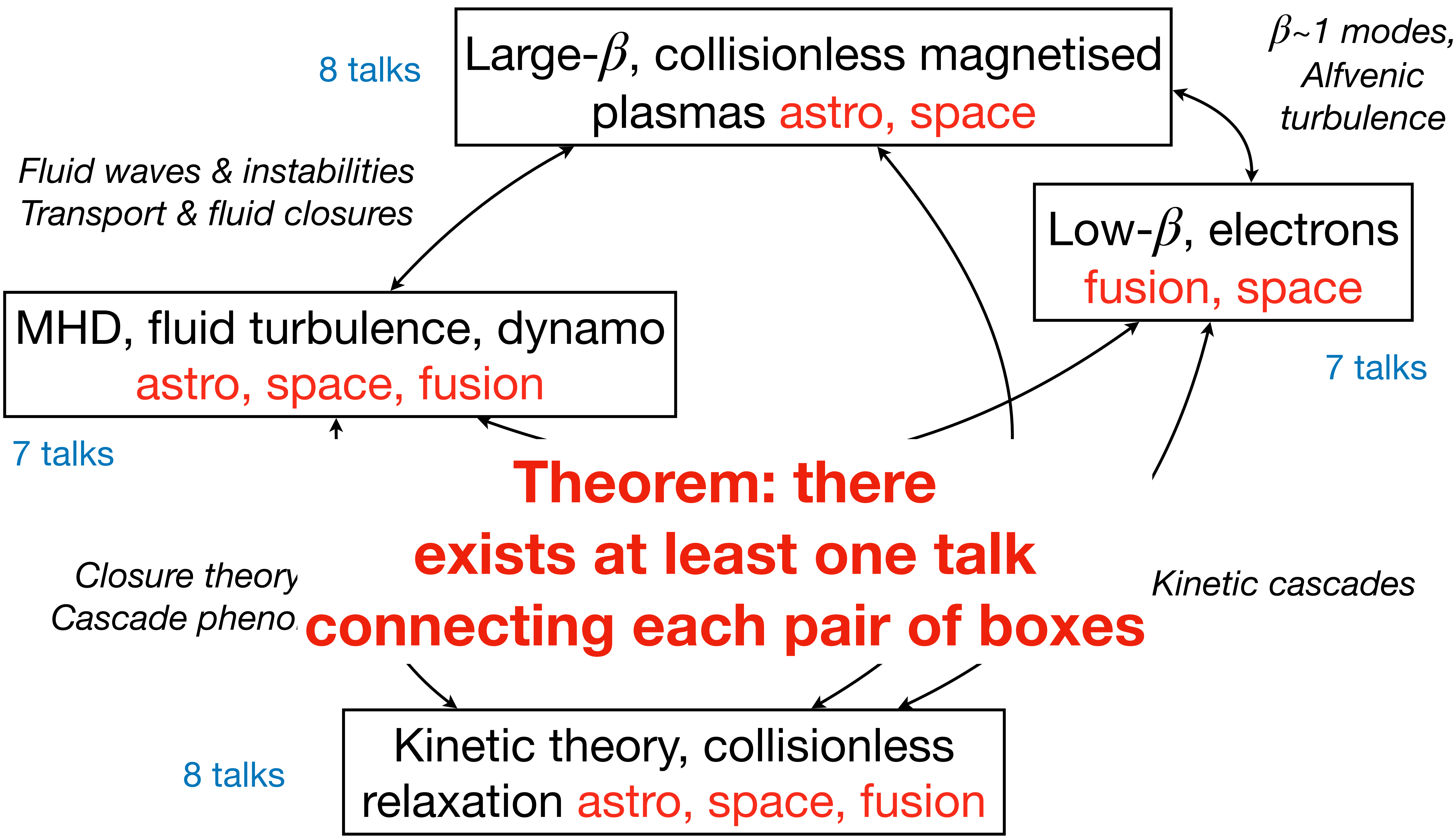
A pat on the back

- Great **organization**: thank you Norbert, WPI staff and Alex
- 13th edition: a **long-term collective effort** starting to pay off
  - Lots of progress on **many problems** only **loosely defined 15 years ago**
  - **Technical maturity** (theory formalism, numerics) —> **fast conceptual progress**
  - New exciting **emerging directions** & **connections**
- Stimulating environment for students, postdocs, researchers
- Top students & postdocs: bright future !

Thematic landscape



Thematic landscape



Methodology landscape

2.4 The basic stochastic theorem

From the above derivation one deduces

Theorem 1 *Let* $\{E^\varepsilon(t,x;\omega)\}_{\omega\in\Omega} = \{-\nabla\Phi(t,x;\omega)\}_{\omega\in\Omega}$ *be a family of stochastic (with respect to the random variable* $\omega \in \Omega$*) gradient vector fields. Assume that such vector fields satisfy the* ε*-independent local in time regularity hypothesis*

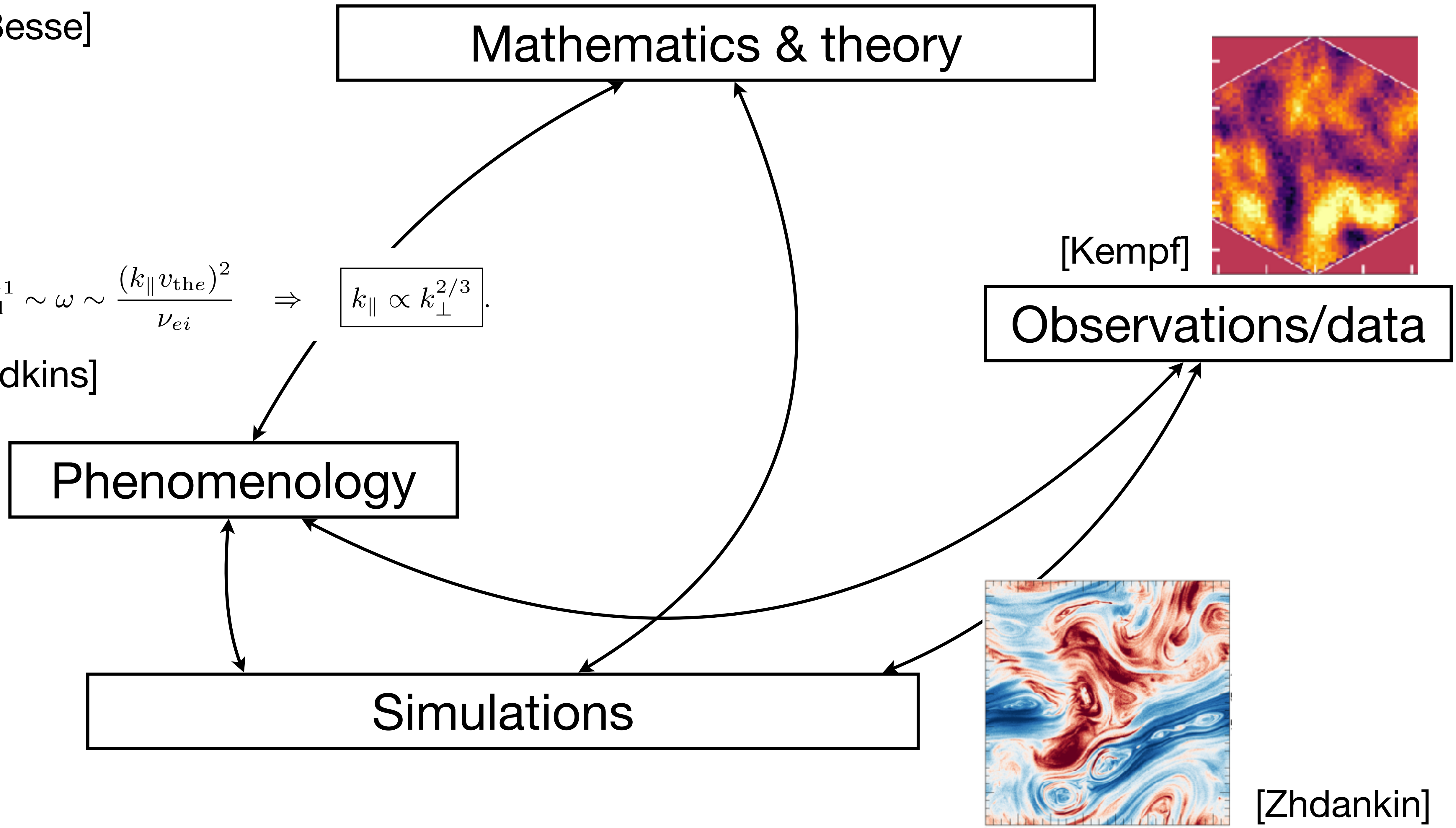
[Besse]

$$\frac{\partial F(v)}{\partial t} = 2\pi^3\mu^2 \frac{\partial}{\partial v}\left[\sum_{k_1,k_2} \frac{k_2^2}{k_1^2(k_1+k_2)}\mathcal{U}(k_1,k_2)\,\mathcal{P}\!\int \frac{\mathrm{d}v_1}{(v-v_1)^4} \times \int \mathrm{d}v_2\,\delta_{\mathrm{D}}[\mathbf{k}\cdot\mathbf{v}]\left(\mathbf{k}\cdot\frac{\partial}{\partial \mathbf{v}}\right)F_3(\mathbf{v})\right], \quad (4)$$

[Fouvry]

$$t_{\mathrm{nl}}^{-1} \sim \omega \sim \frac{(k_\parallel v_{\mathrm{the}})^2}{\nu_{ei}} \quad \Rightarrow \quad \boxed{k_\parallel \propto k_\perp^{2/3}}.$$

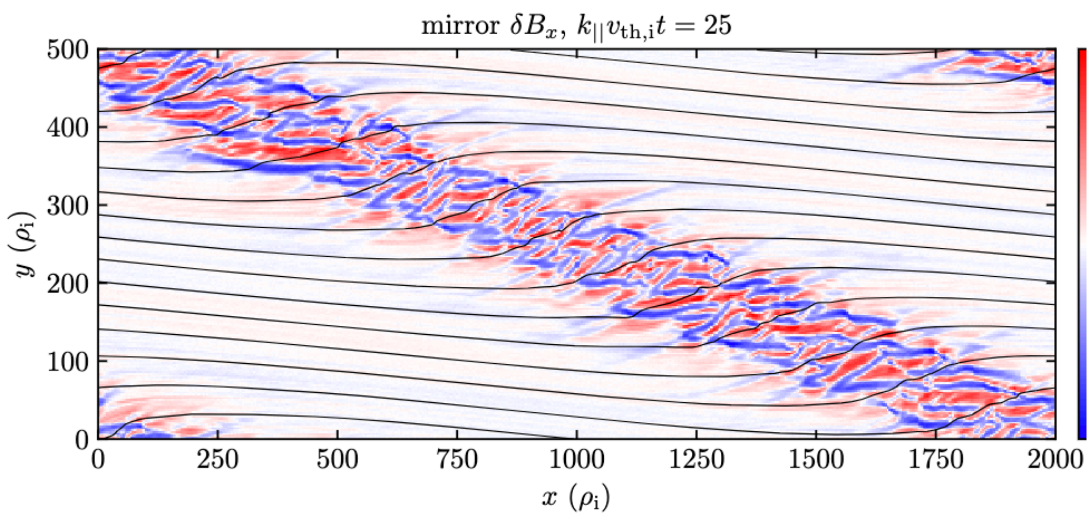
[Adkins]

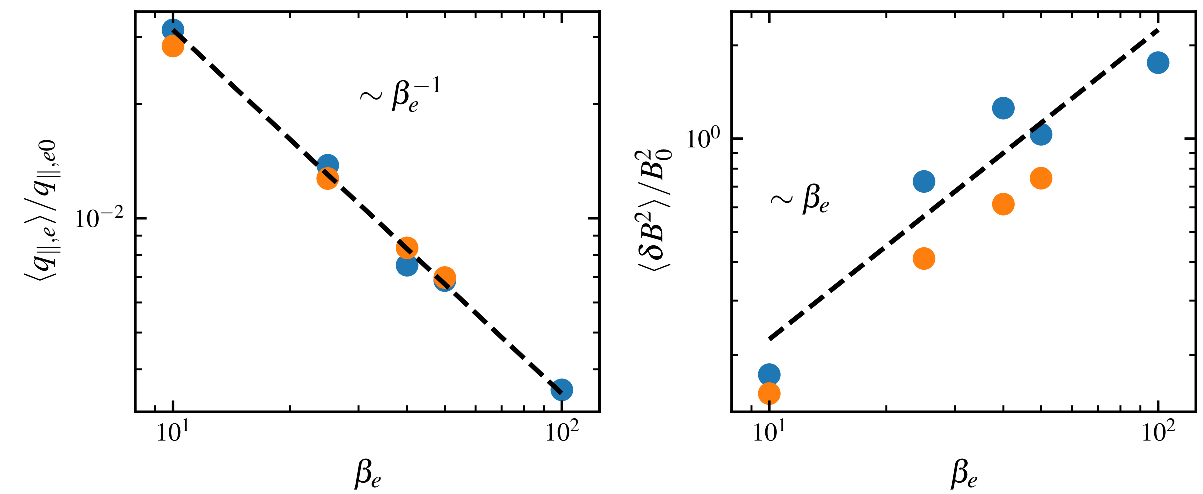


Highlights

Large-β, Kunzology

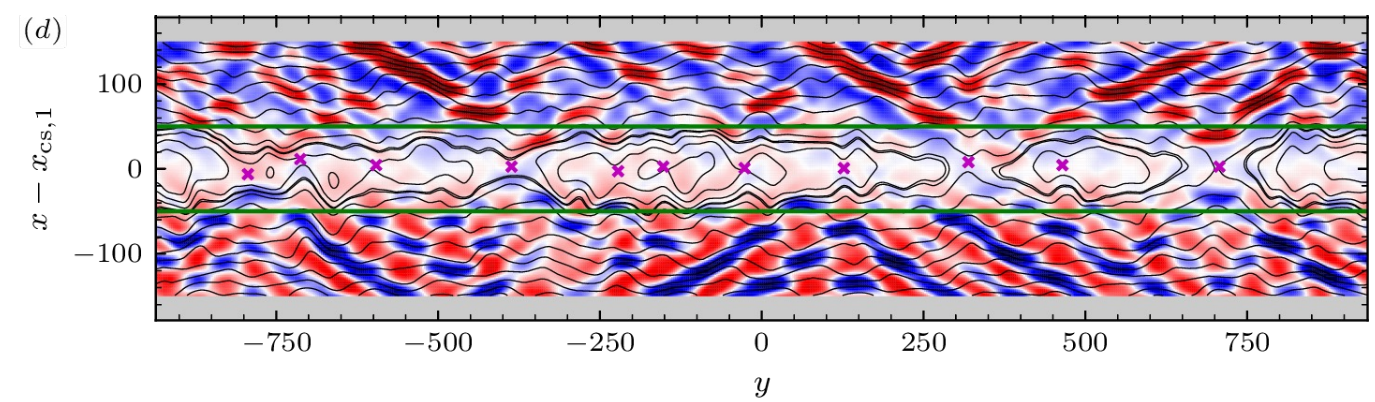
Majeski (magnetosonic modes)

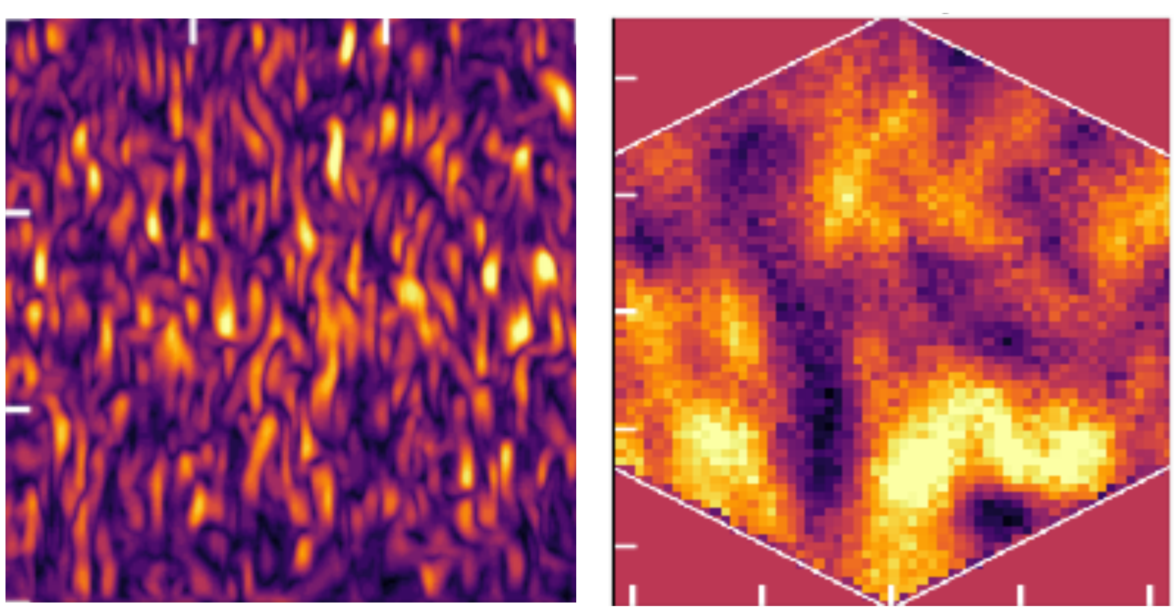




Yerger (whistler thermal conduction)

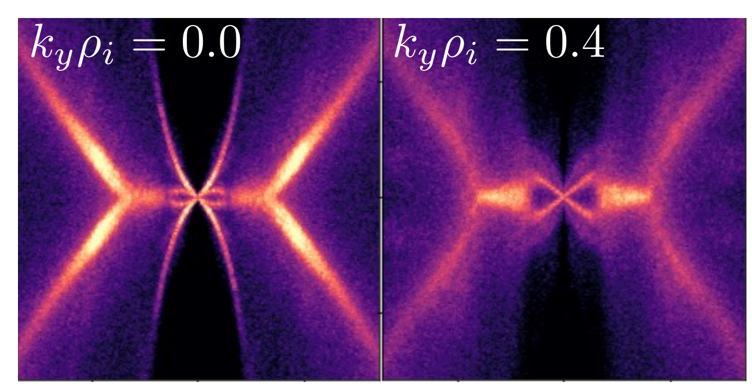
Winarto (reconnection)

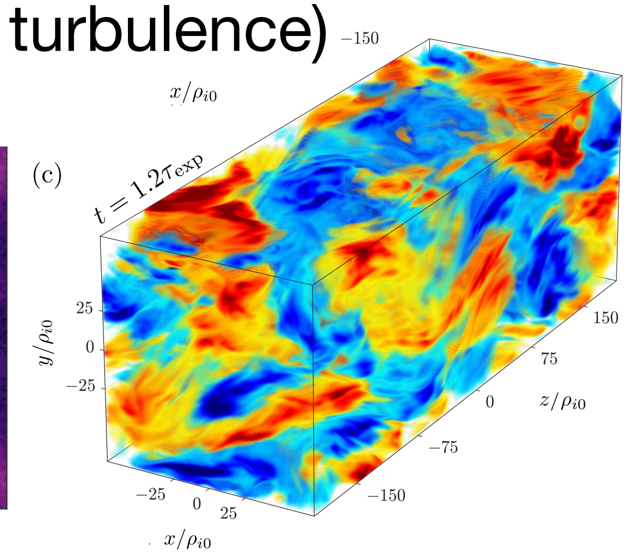




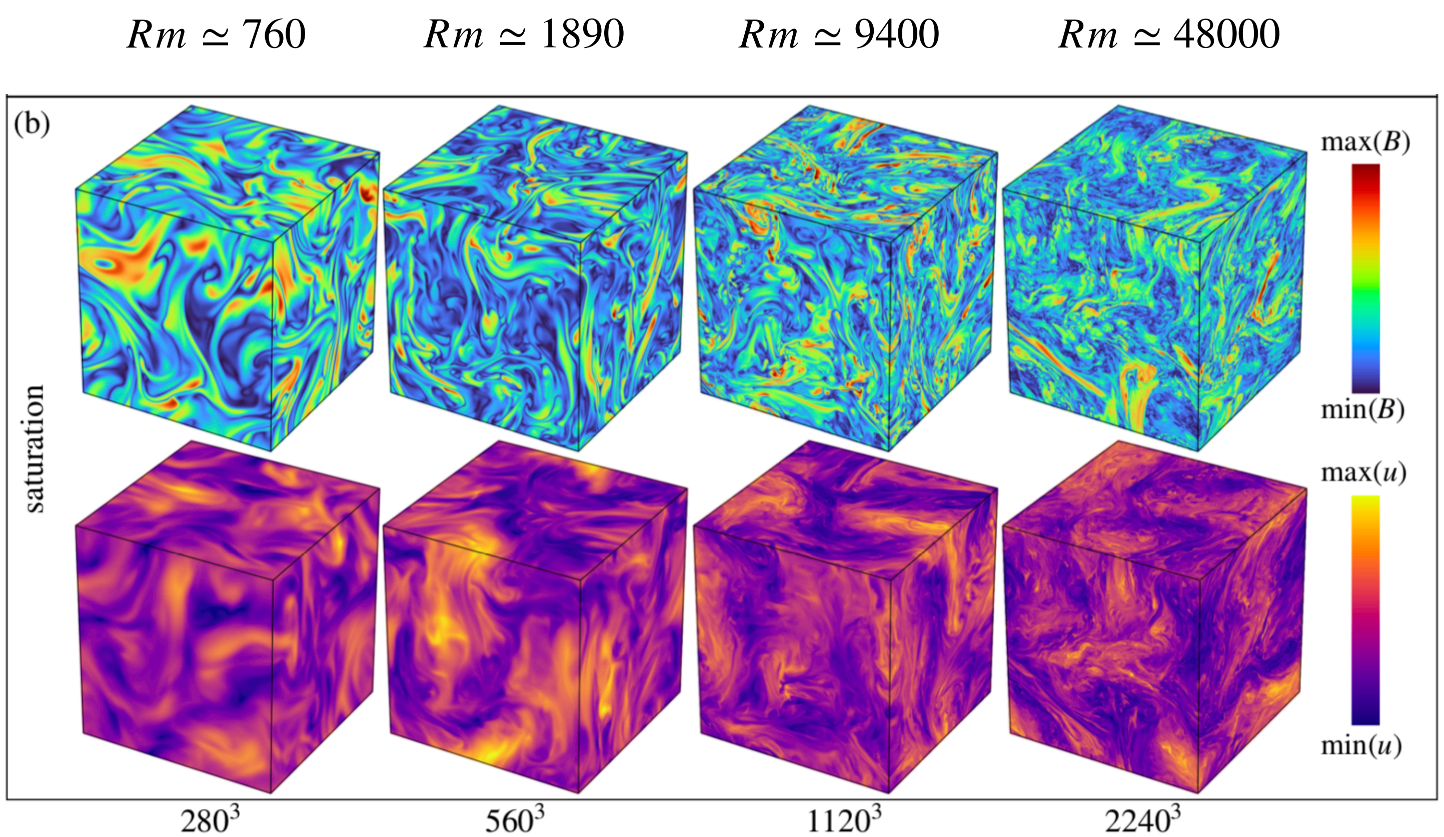
Kempf (MTI in ICM)

Bott (firehose+ turbulence)

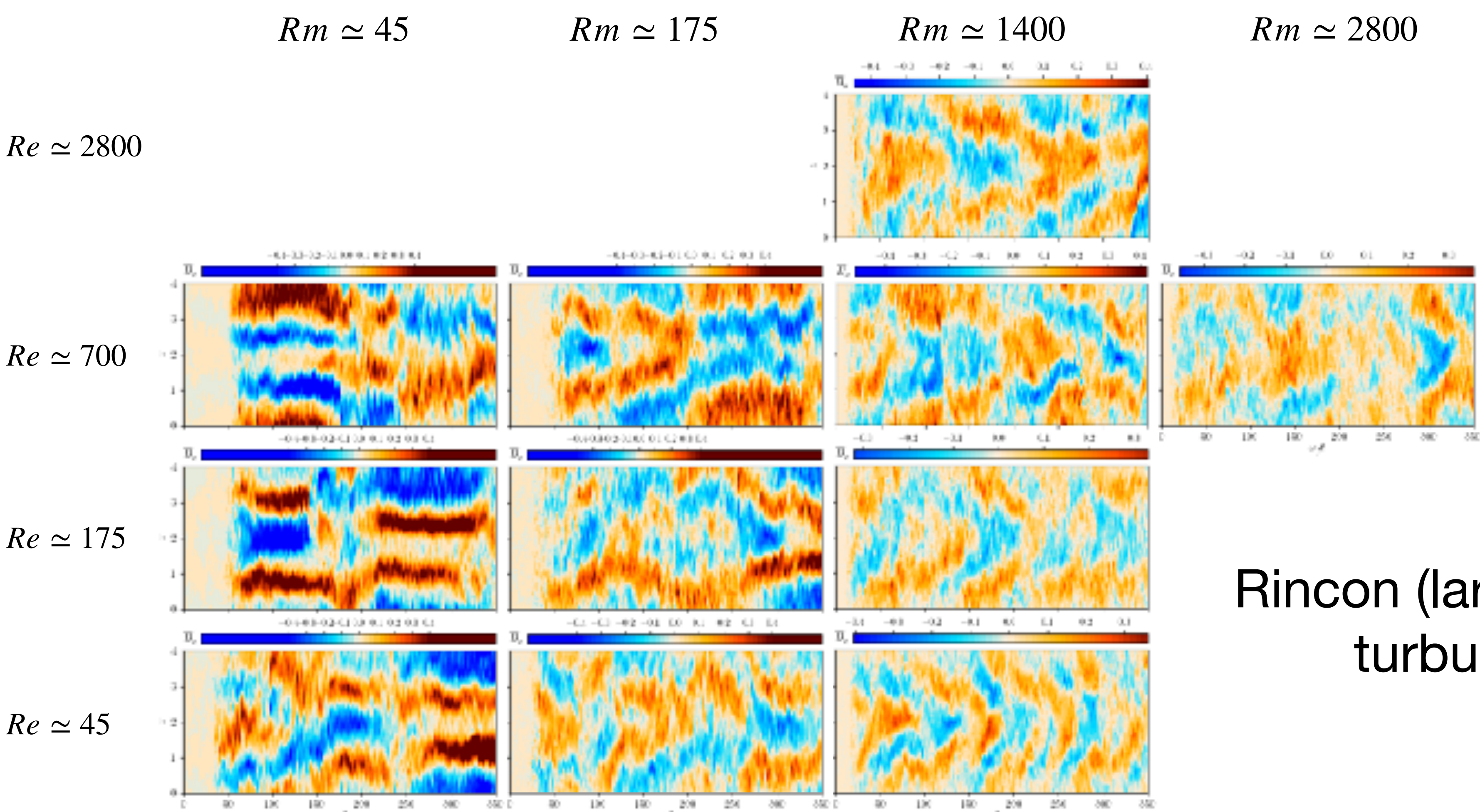




Large-Rm dynamos



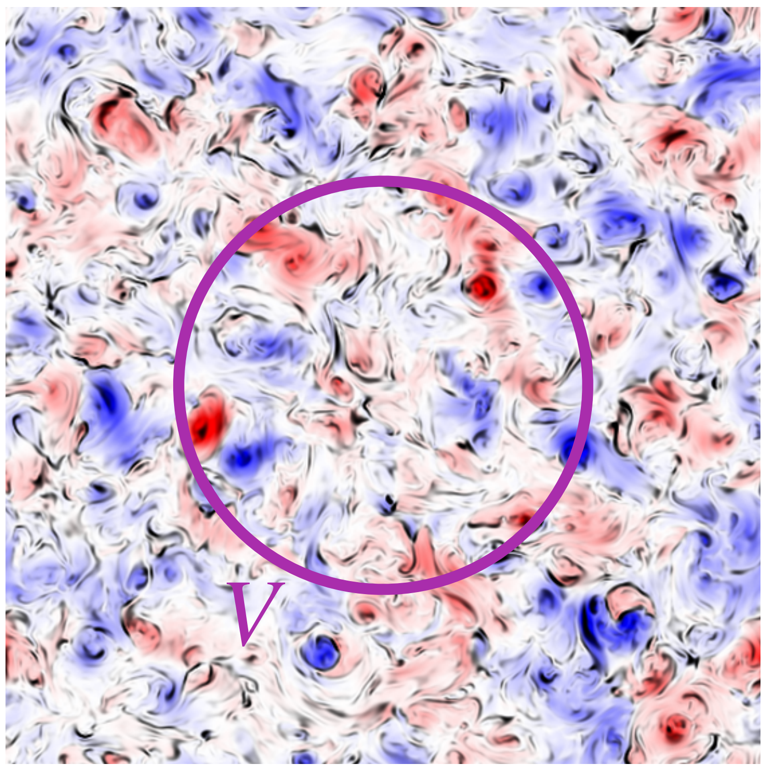
Galishnikova-Kunz (small-scale reconnecting turbulent dynamo)



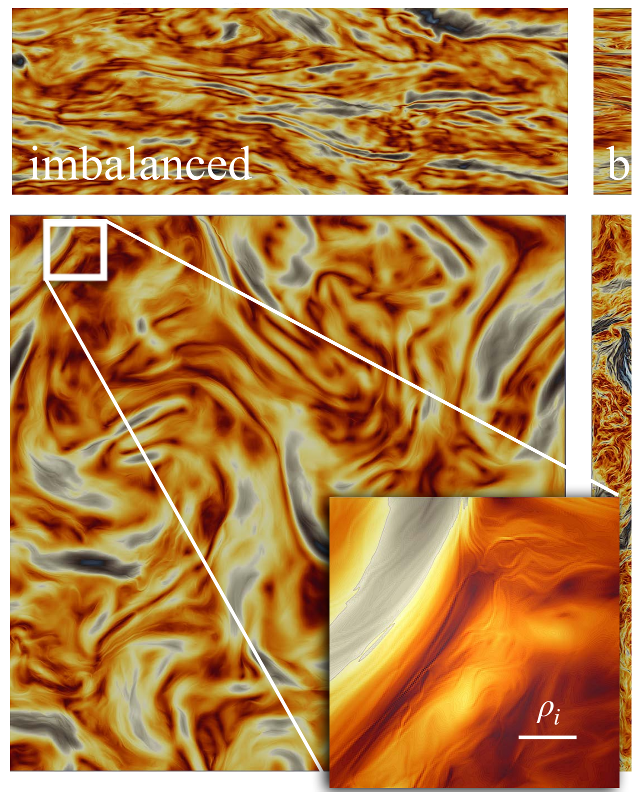
Rincon (large-scale inhomogeneous turbulent helical dynamo)

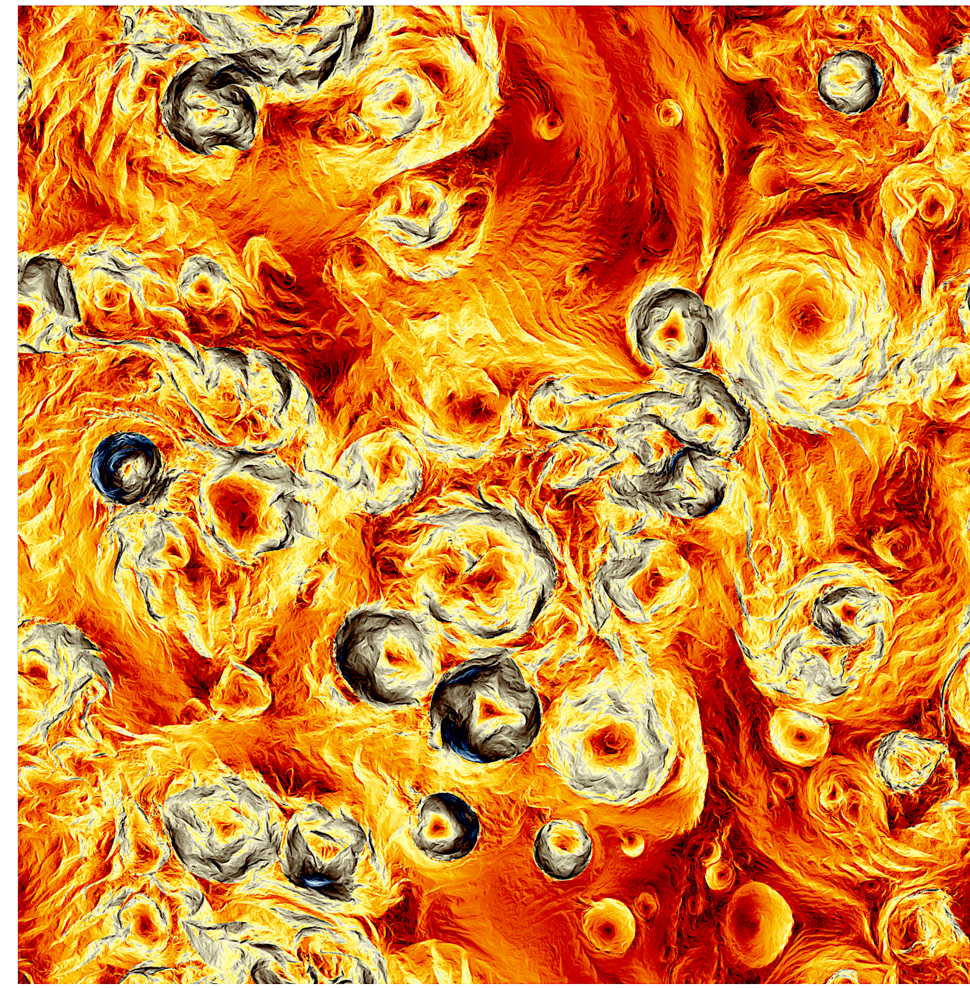
Nonlinear MHD & turbulence

Hosking's invariants



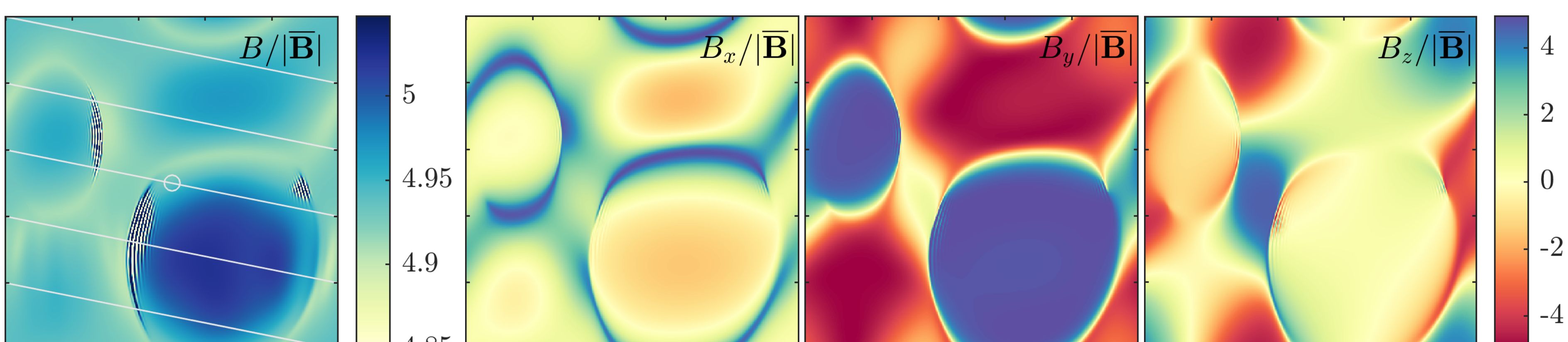
Squire-Meyrand
(imbalanced FLR-MHD)





Meyranistic art (reflection-driven MHD turbulence)

Squiritons



Disruption, reconnection

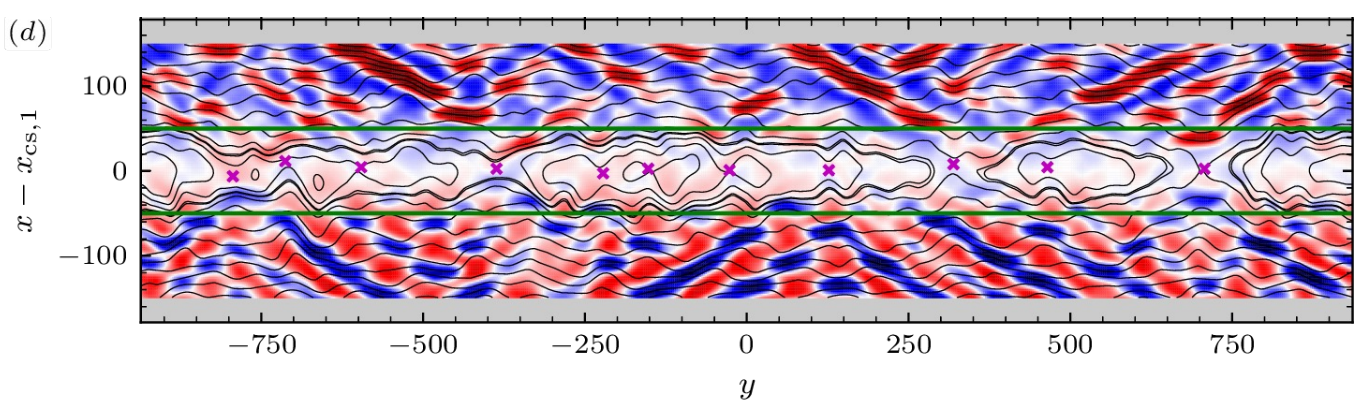
Loureiro (reconnection + Ion-Acoustic)

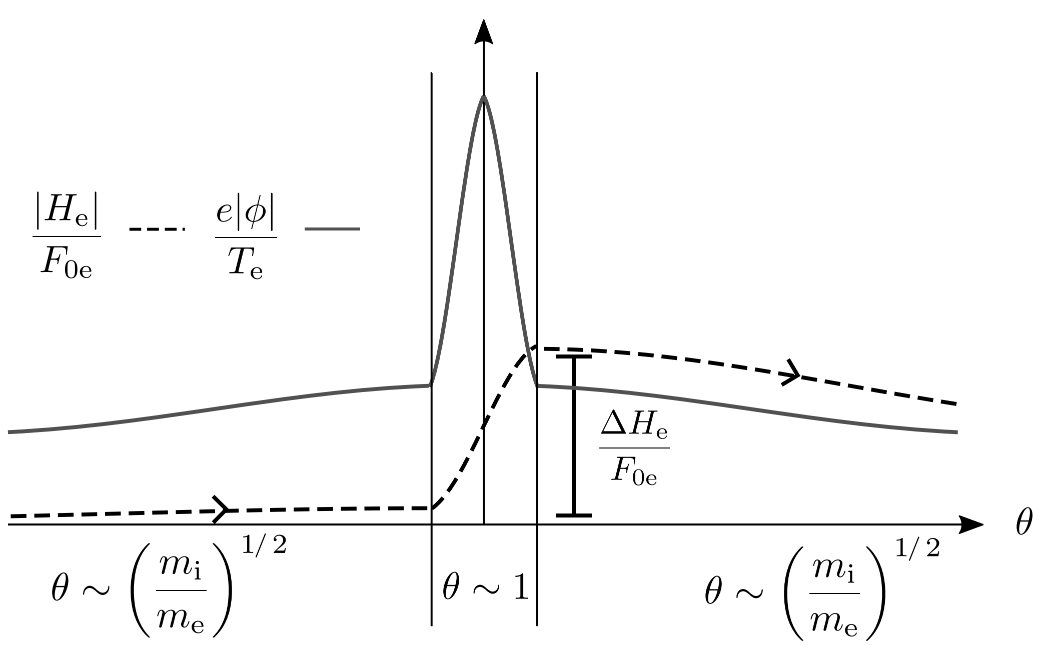
Cowley's explosive instabilities & metastability

- Current sheet instability implies that very large aspect ratio, super-critical, current sheets cannot form in the first place.

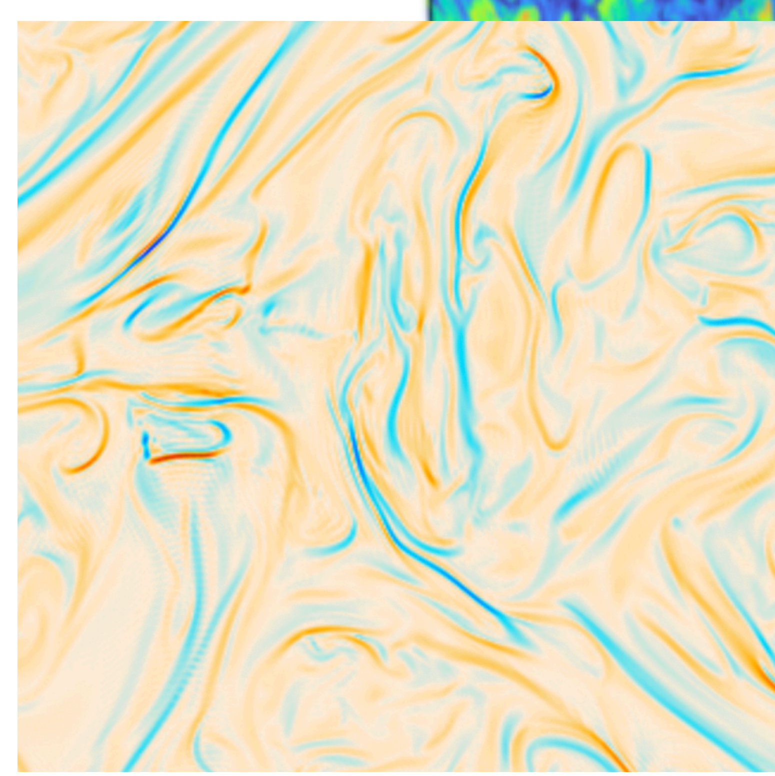
Kunz+Rincon (dynamo+reconnection)

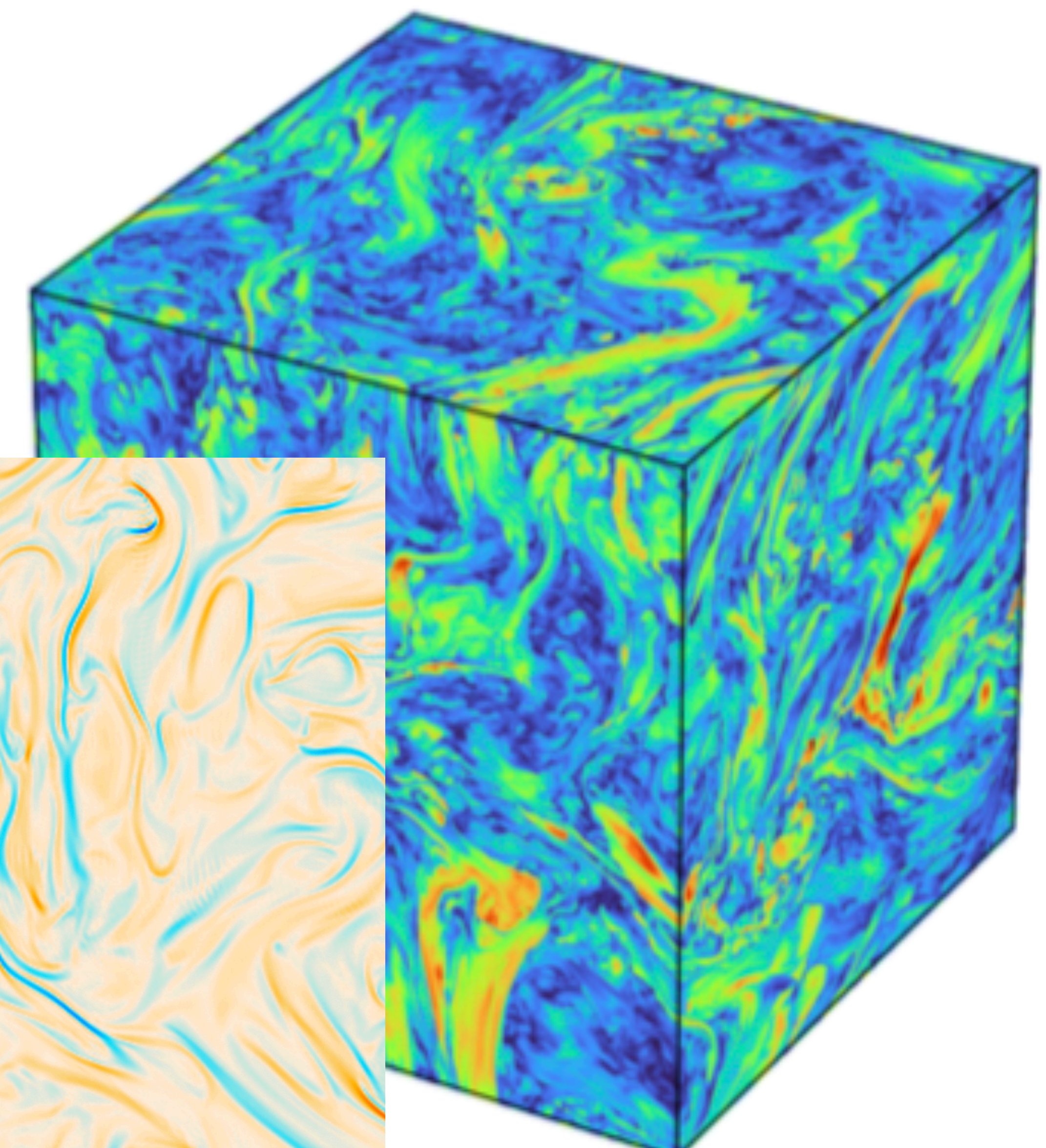
Winarto (reconnection + mirror)





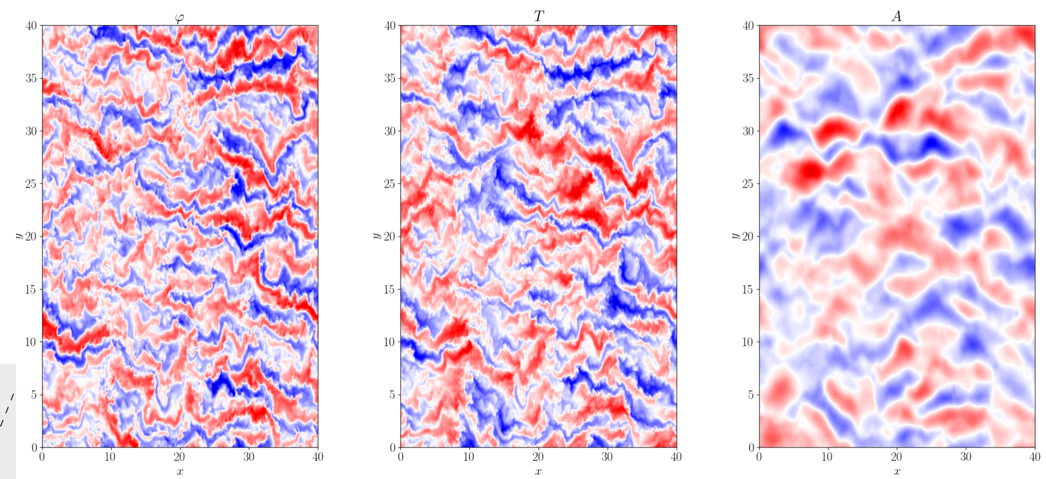
Hardman, Chandran (MTM)

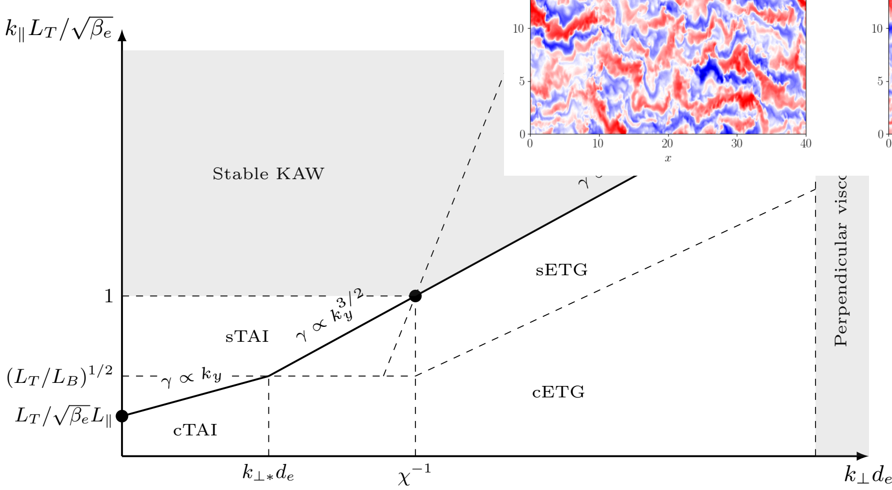




Low-β stability & turbulence

Adkins (ETG, TAI)





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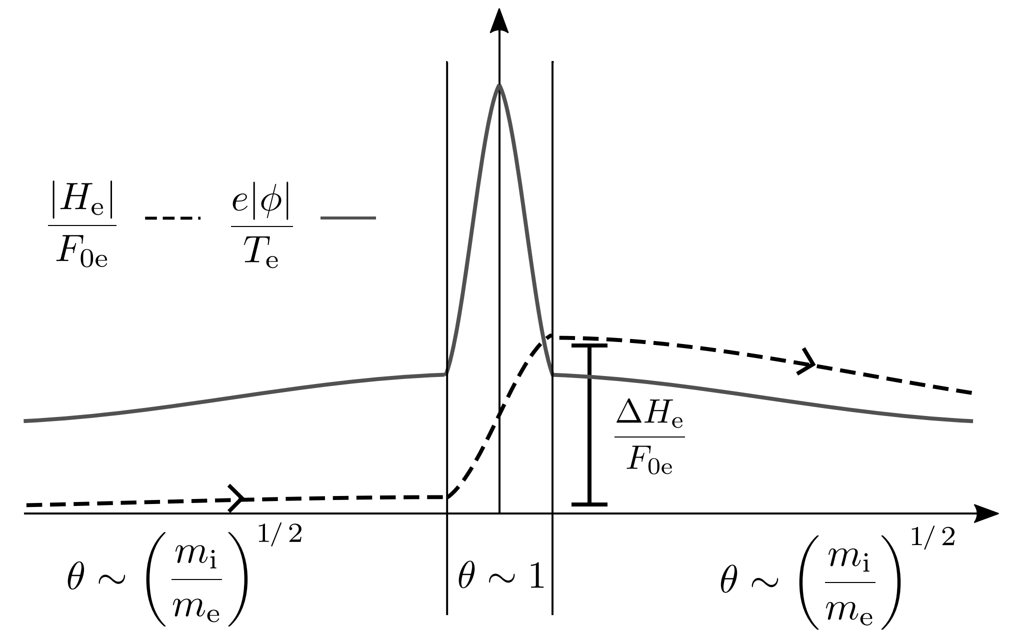
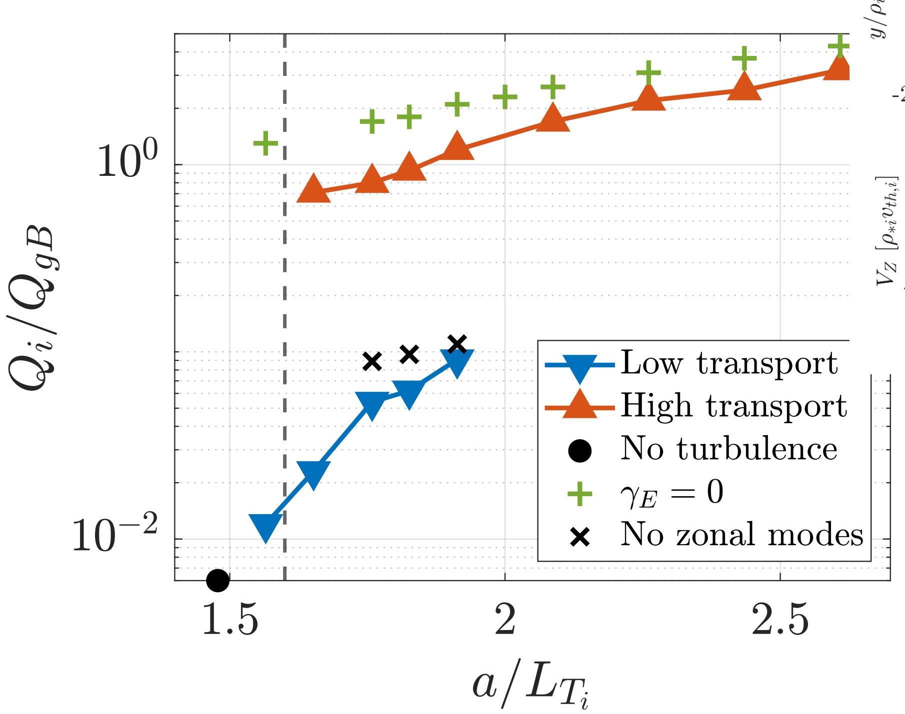
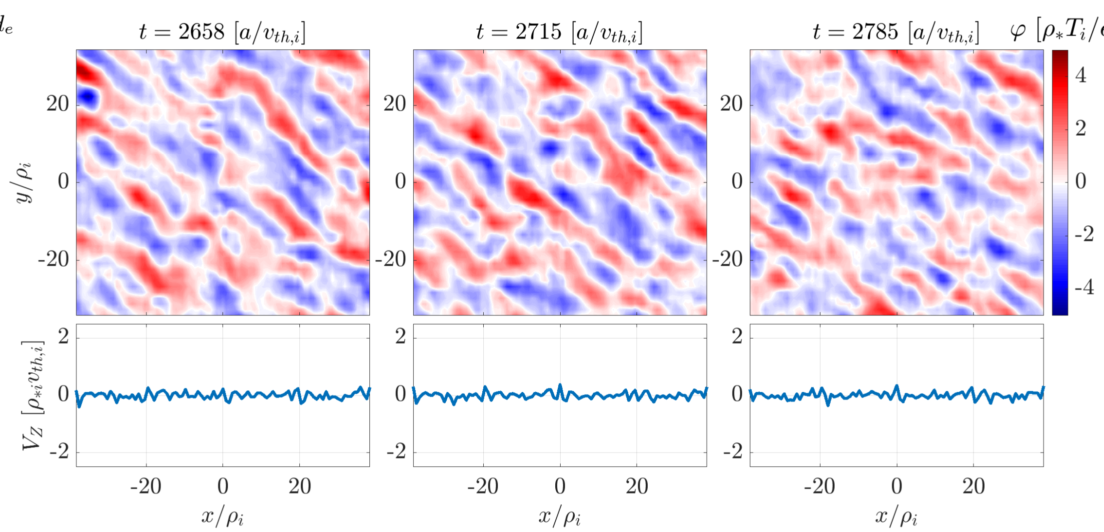


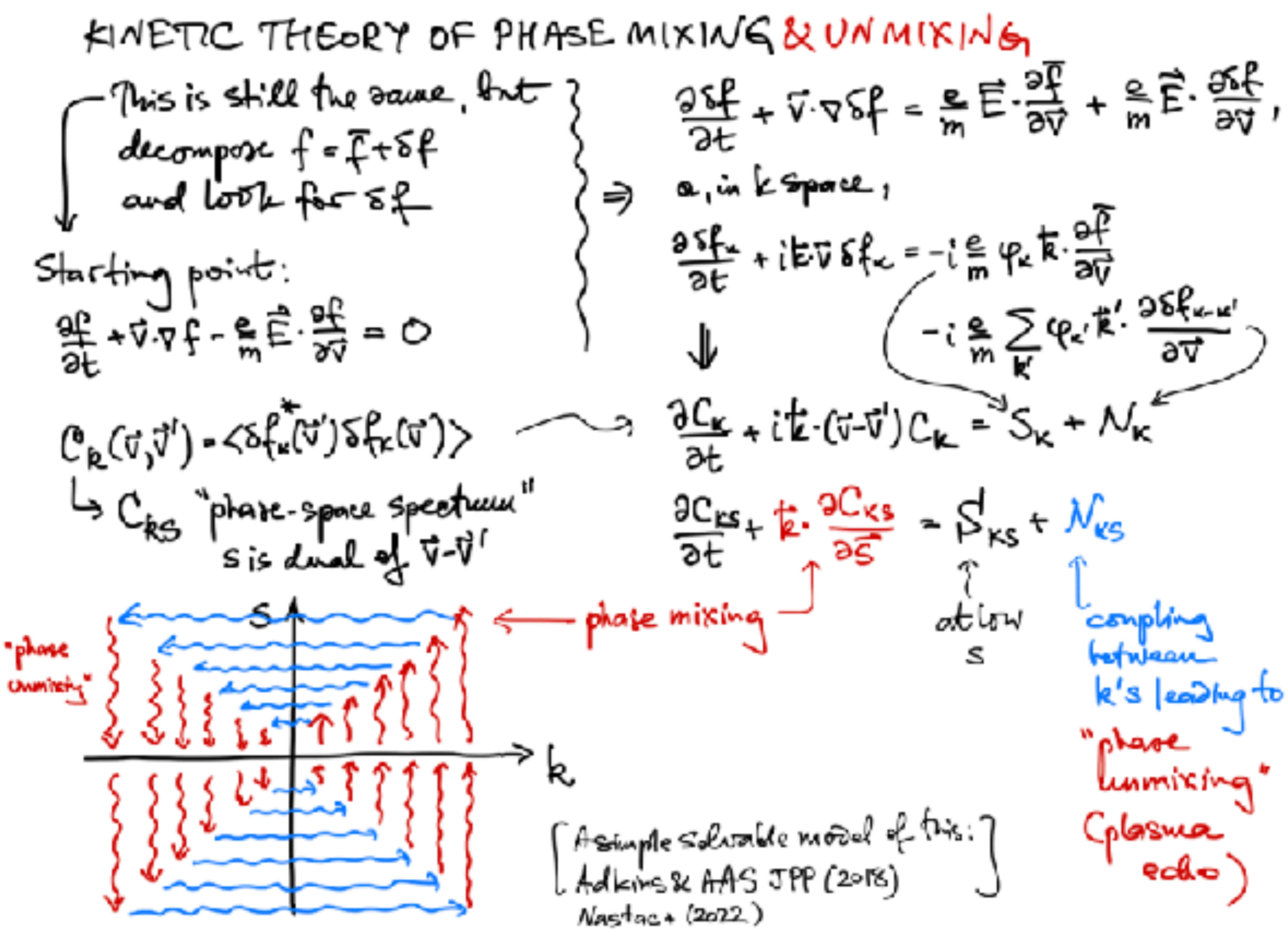
Barnes (ITG bistability)

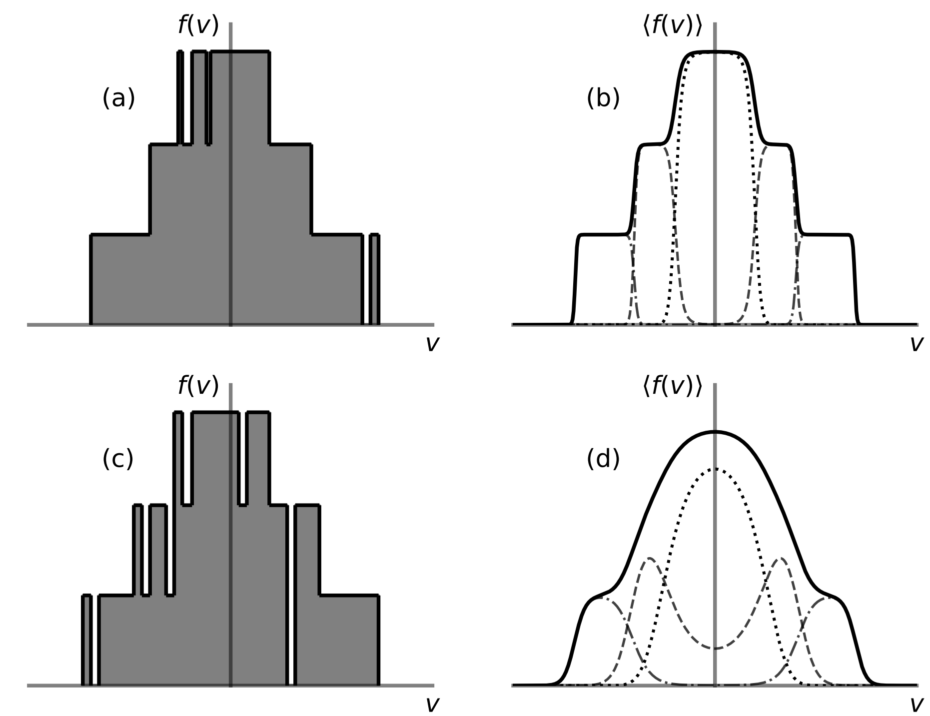




Ivanov (Drift-kinetic linear theory with complex-plane acrobatics)

Schekochinamics & Hyperobbinetics





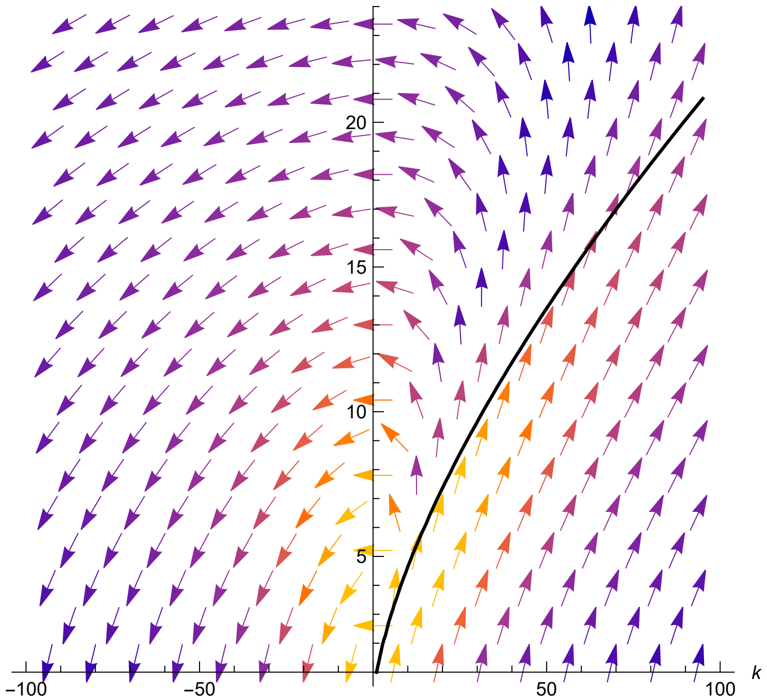
Ewart, Nastac, Adkins

$$\frac{\mathrm{d}S}{\mathrm{d}t} = \sum_{\alpha\alpha''} \frac{8\pi^3 q_\alpha^2 q_{\alpha''}^2}{\Delta\Gamma_\alpha \Delta\Gamma_{\alpha''}} \iint \mathrm{d}\boldsymbol{v}\,\mathrm{d}\boldsymbol{v}'' \sum_{\boldsymbol{k}} \frac{\delta(\boldsymbol{k}\cdot(\boldsymbol{v}-\boldsymbol{v}''))}{k^4 |\epsilon_{\boldsymbol{k},\boldsymbol{k}\cdot\boldsymbol{v}}|^2}$$

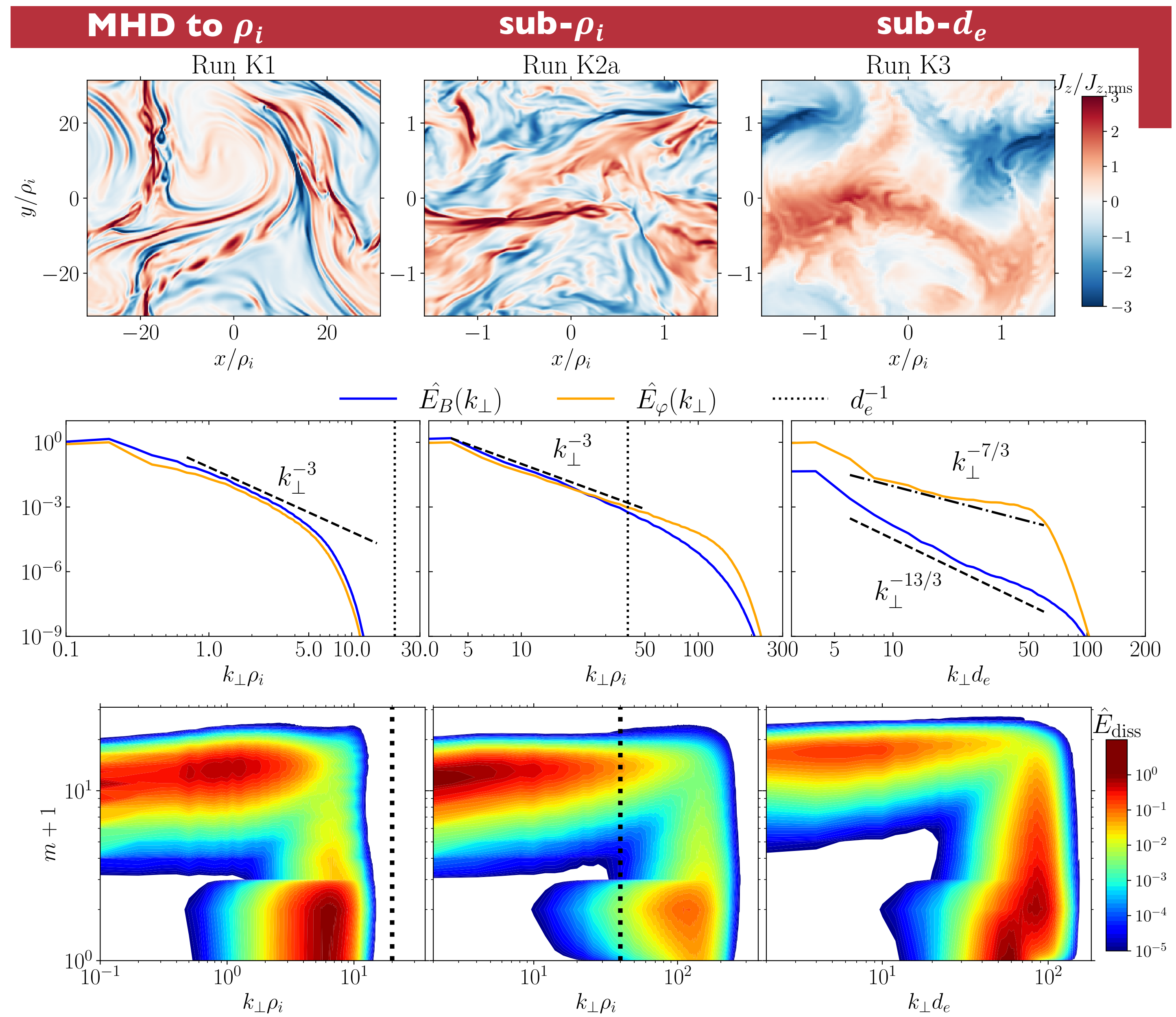
$$\iint \mathrm{d}\eta\,\mathrm{d}\eta'' \left\{ \frac{\Delta\Gamma_{\alpha''}}{m_\alpha}\left[\eta'' - f_{0\alpha''}(\boldsymbol{v}'')\right]\sqrt{\frac{P_{0\alpha''}(\boldsymbol{v}'',\eta'')}{P_{0\alpha}(\boldsymbol{v},\eta)}}\,\boldsymbol{k}\cdot\left.\frac{\partial P_{0\alpha}}{\partial \boldsymbol{v}}\right|_{\eta} - \frac{\Delta\Gamma_{\alpha}}{m_{\alpha''}}\left[\eta - f_{0\alpha}(\boldsymbol{v})\right]\sqrt{\frac{P_{0\alpha}(\boldsymbol{v},\eta)}{P_{0\alpha''}(\boldsymbol{v}'',\eta'')}}\,\boldsymbol{k}\cdot\left.\frac{\partial P_{0\alpha''}}{\partial \boldsymbol{v}''}\right|_{\eta''} \right\}^2 \geqslant 0.$$

Plasma entropy cascades, heating, particle acceleration

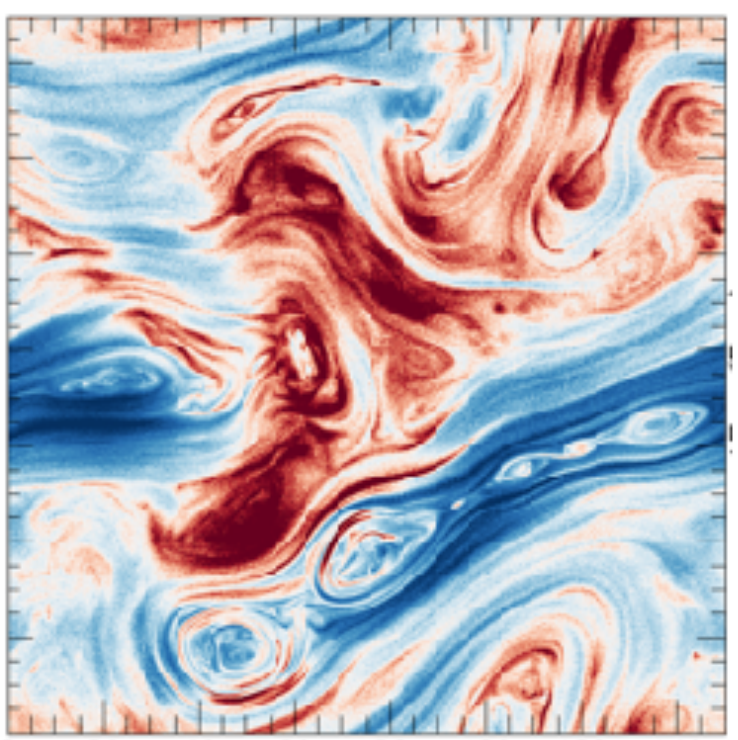
Nastac (Vlasov-Kraichnan)



Zhou (electron heating)



Zhdankin (cascade+acceleration)



+Kanekar, Parker & Meyrand's older work

Gravitational Fouvrynetics

Kinetic blockings and 1/N² kinetic theory

Simplifying the collision operator

Interaction potential

$$U(\mathbf{w},\mathbf{w}') = \sum_k U_k[J,J']\,\mathrm{e}^{\mathrm{i}k(\theta-\theta')}$$

Monotonic frequency profile

$$J \mapsto \Omega(J)$$

Large time limit

$$\lim_{t\to+\infty}\int_0^t \mathrm{d}t'\,\mathrm{e}^{\mathrm{i}(t-t')\omega_\mathrm{R}} = \pi\delta_\mathrm{D}(\omega_\mathrm{R}) + \mathrm{i}\,\mathscr{P}\left(\frac{1}{\omega_\mathrm{R}}\right)$$

Number of terms keeps growing: ~10,000 terms

Do NOT perform these calculations by hand!

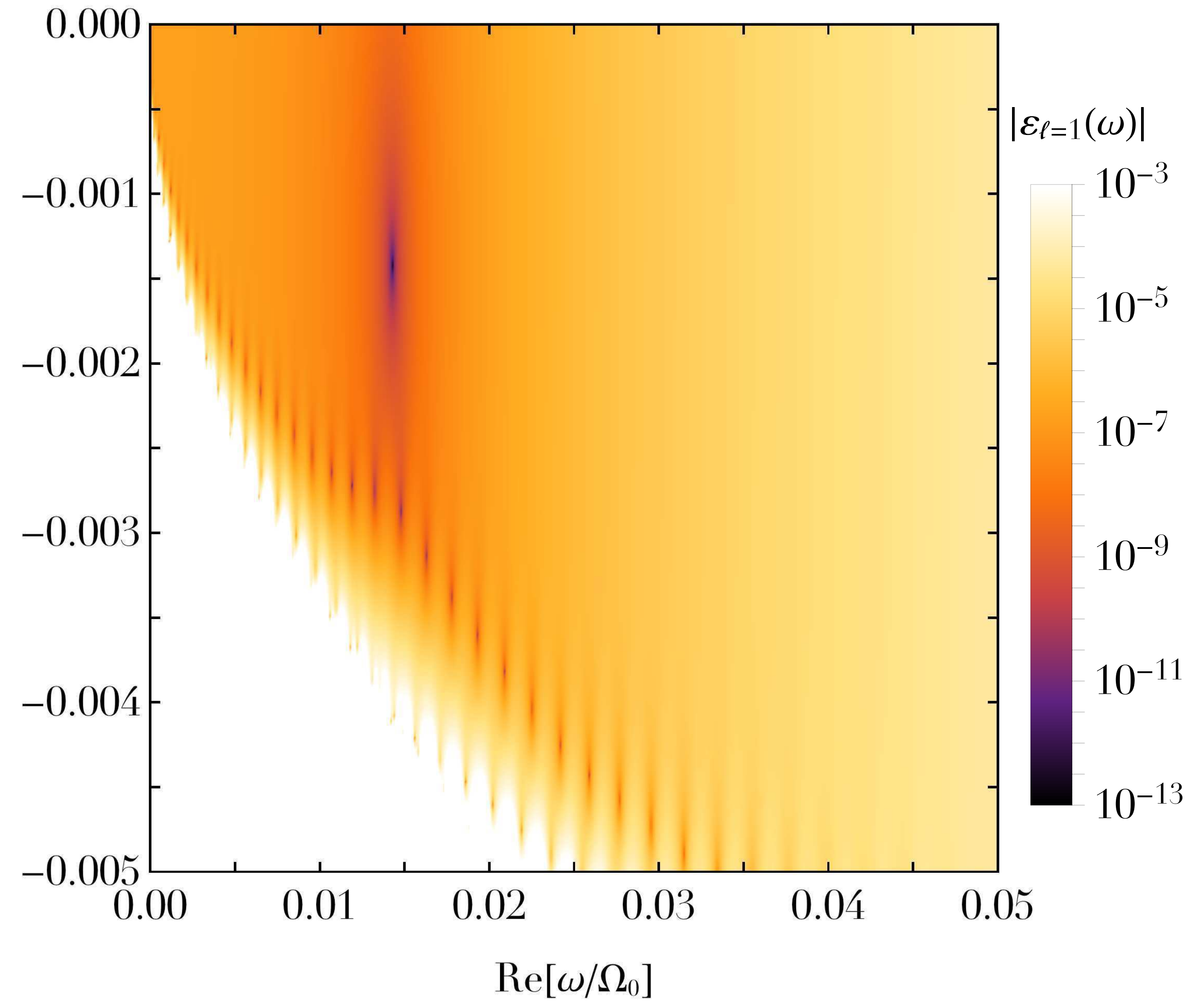
Using a custom grammar in **Mathematica**

Kinetic blocking &

1/N² kinetic theory

Damped modes of stellar clusters

$\det\left[\boldsymbol{\varepsilon}_\ell(\omega)\right]$



Quasi-linear theory

« À Vienne, il était très fort. »

Besse & Bardos

Necesseary condition for non-degenerate diffusion 1/2

Theorem (A)

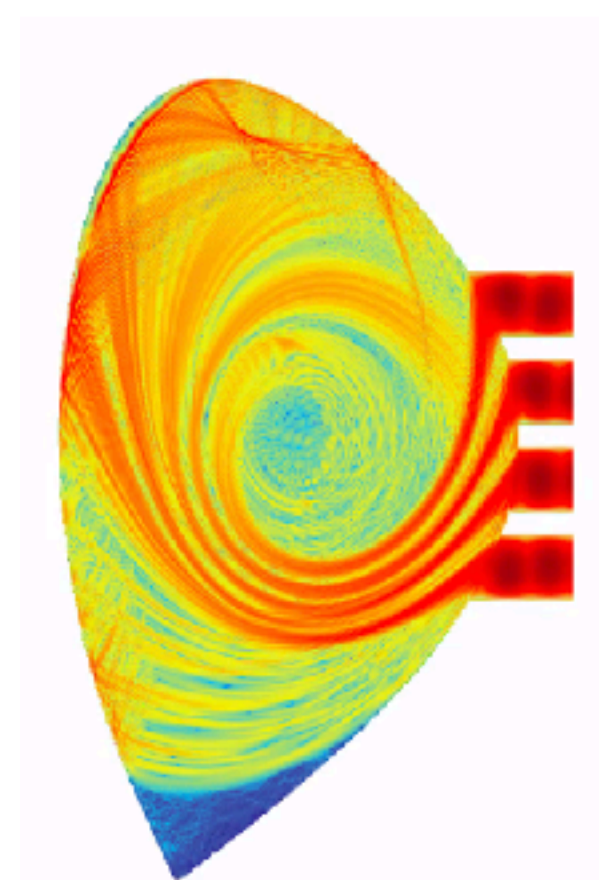
Let $\{f_0^\varepsilon\}_{\varepsilon>0}$ *be a sequence of non-negative initial data and* C_0 *be a positive constant such that*

$$\|f_0^\varepsilon\|_{L^1(Q)} + \|f_0^\varepsilon\|_{L^\infty(Q)} \le C_0, \quad \int_Q f_0^\varepsilon |v|^2 \, dxdv \le C_0, \quad \left\| E_0^\varepsilon := \nabla\Delta^{-1}\Big(\int_{\mathbb{R}^d} f_0^\varepsilon \, dv - 1\Big)\right\|_{L^2(\mathbb{T}^d)} \le C_0.$$

Dodin

- **Result:** QL theory is corrected and derived from first principles as a *local theory*.
  Wigner tensors vs. global-mode decomposition
- **Take-home message:** $\mathcal{O}(\bar{\partial}_t, \bar{\partial}_x)$ is non-negligible on $t \gg \omega^{-1}$ and $\ell \gg k^{-1}$. Calculations ignoring this are unreliable. Weyl calculus is *the* way to get things right.

$$\widetilde{f}_k = -\frac{i(e/m)\widetilde{E}_k}{\omega_k - kv}\frac{\partial \overline{f}}{\partial v} + \boldsymbol{\mathcal{O}(\partial_t \overline{f}, \partial_x \overline{f})}, \qquad F - \overline{f} = \frac{\partial}{\partial \boldsymbol{p}} \cdot \left(\boldsymbol{\Theta}\,\frac{\partial \overline{f}}{\partial \boldsymbol{p}}\right) = \boldsymbol{\mathcal{O}(\widetilde{E}^2)}$$



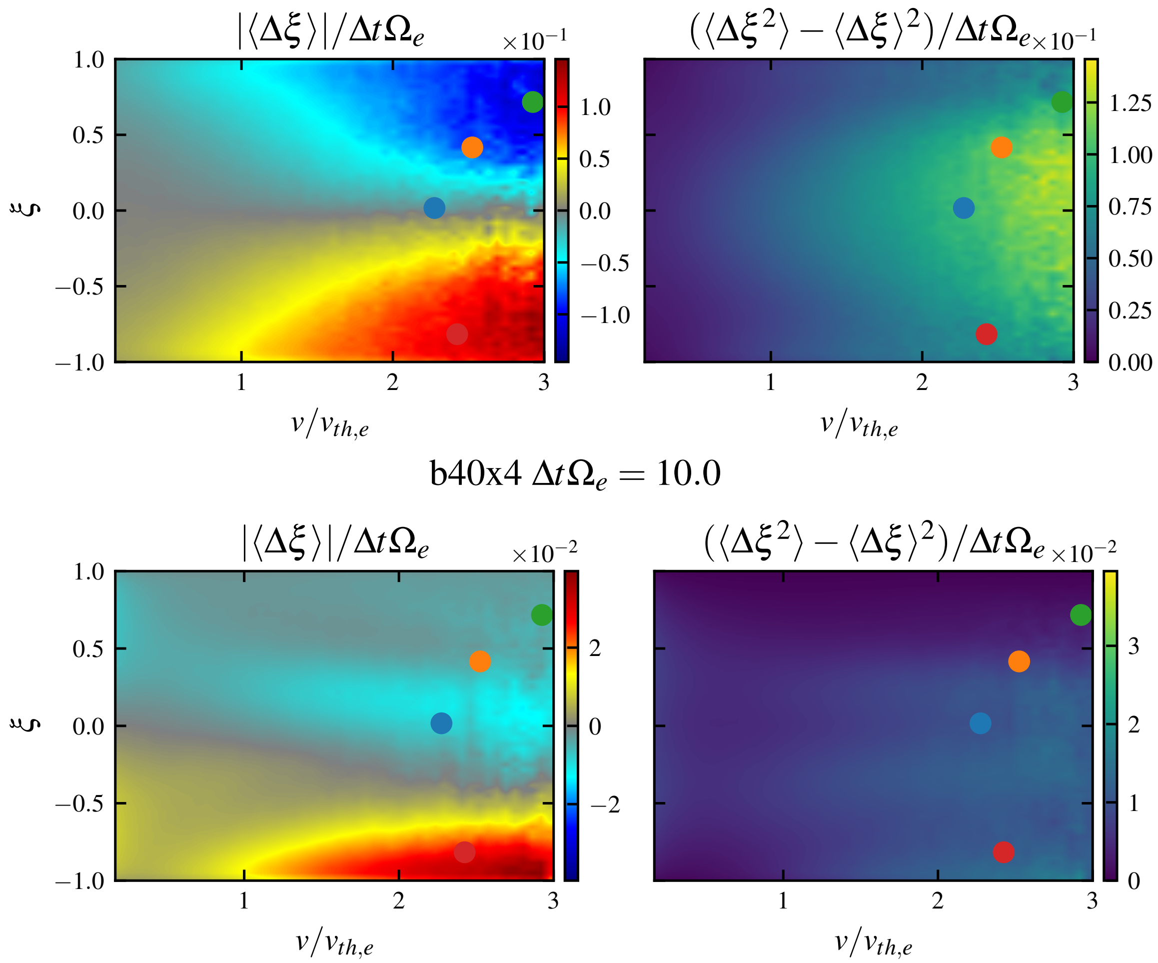
Catto

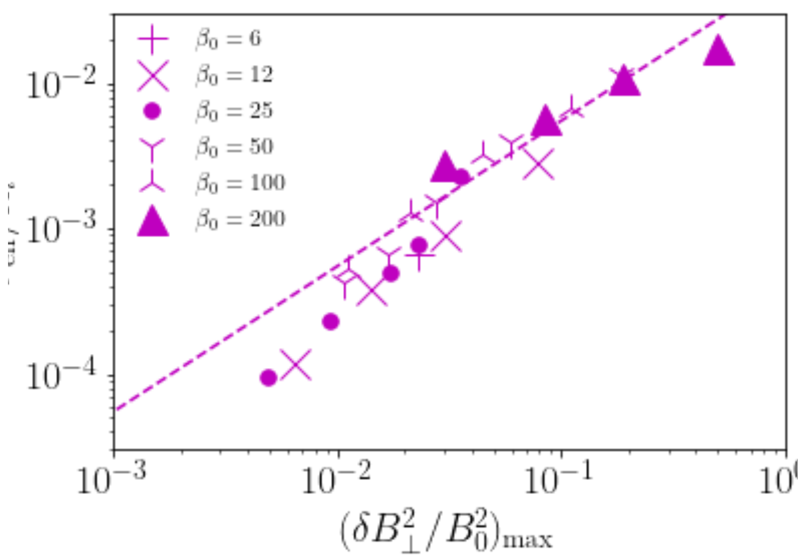
$$Q\{f_0\} = \frac{v_\parallel}{v}\frac{\partial}{\partial v}\left(D\,\frac{v}{v_\parallel}\frac{\partial f_0}{\partial v}\right),$$

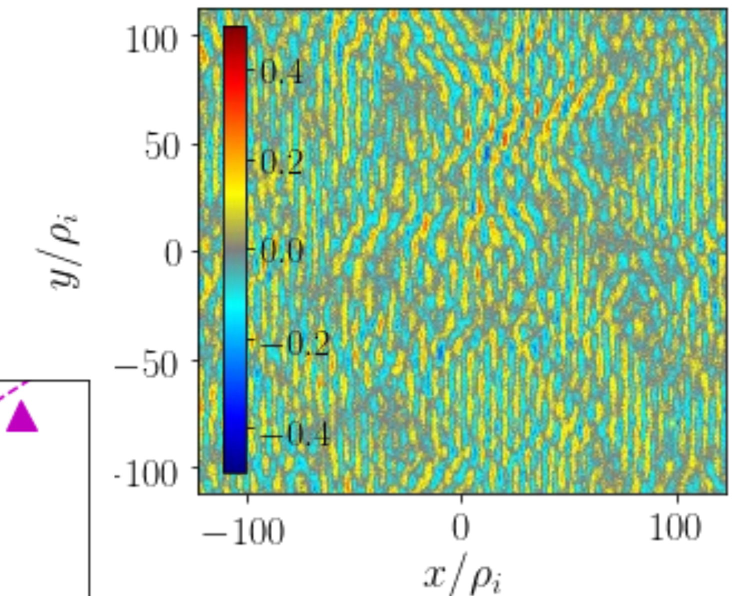
$$D = \frac{\pi e^2}{2m^2v^2}\sum_k \delta(\omega - k_\parallel v_\parallel)\left|\vec{e}_k \cdot [\vec{z} v_\parallel J_0(\eta) + i\vec{z}\times\vec{k}k_\perp^{-1} v_\perp \partial J_0/\partial\eta]\right|^2$$

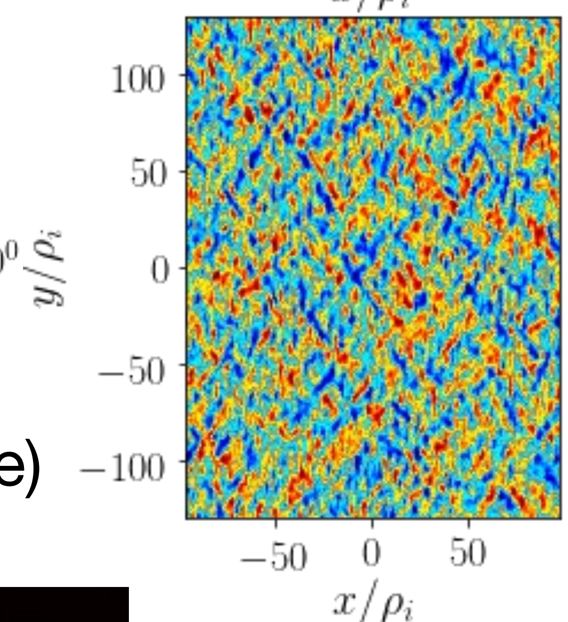
More closure

Yerger (thermal conduction)

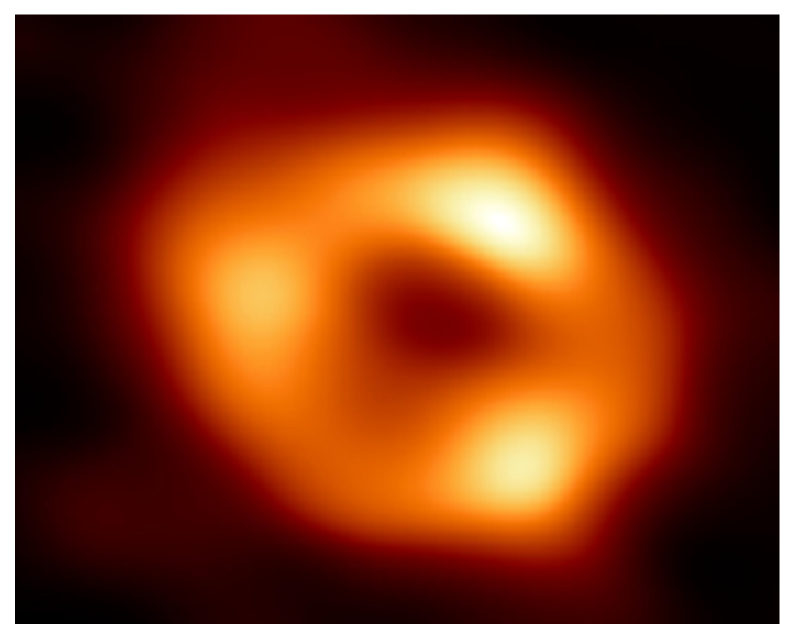








Bott (firehose turbulence)



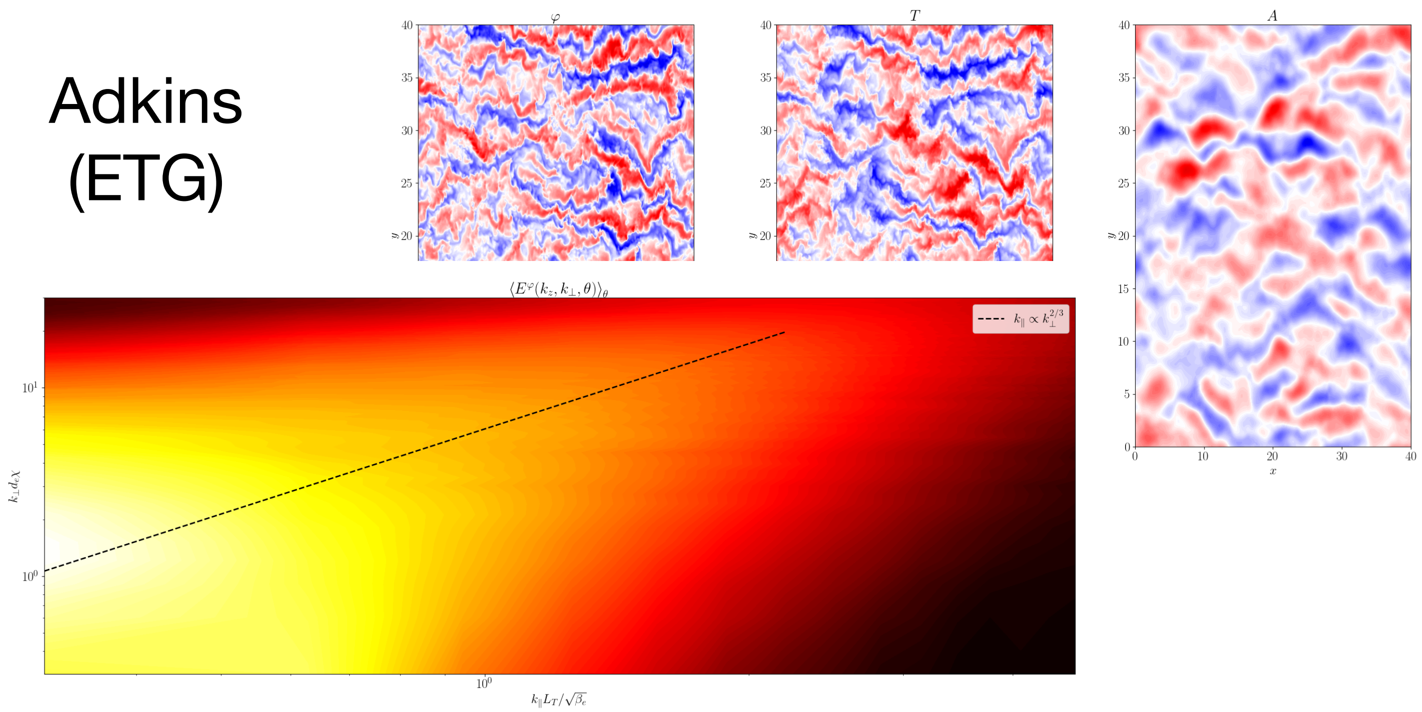
Uzdensky

3. What is the message here?

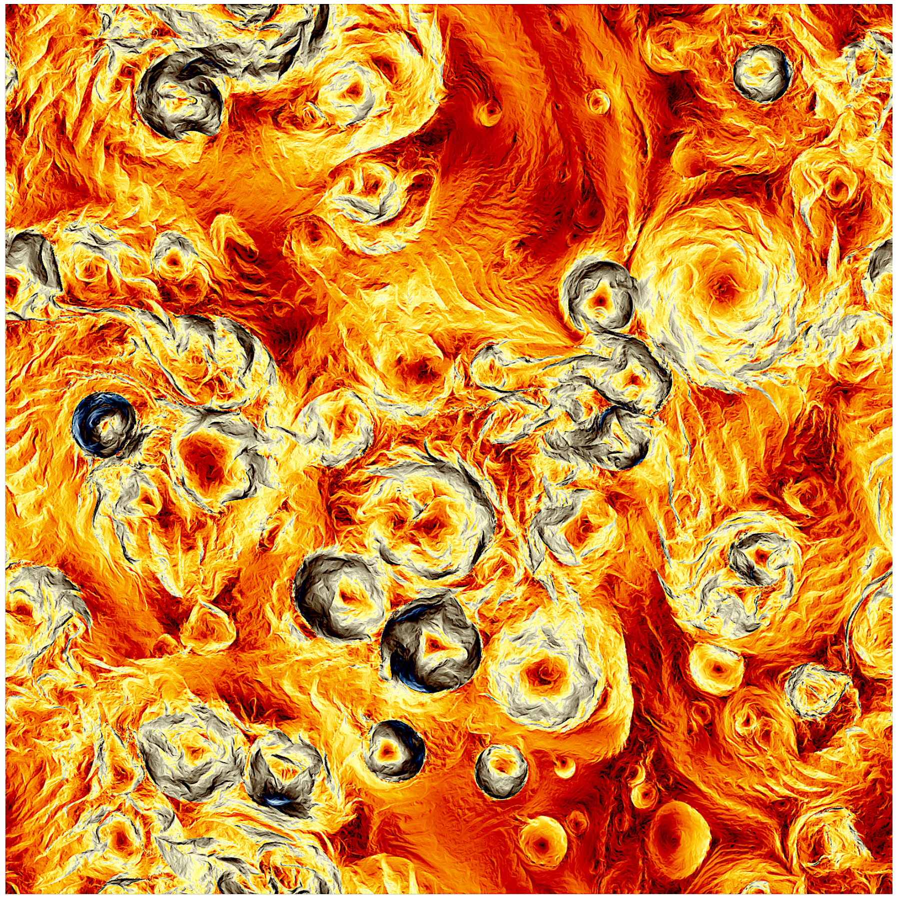
A few lessons learned

Nonlinear self-organisation is a thing

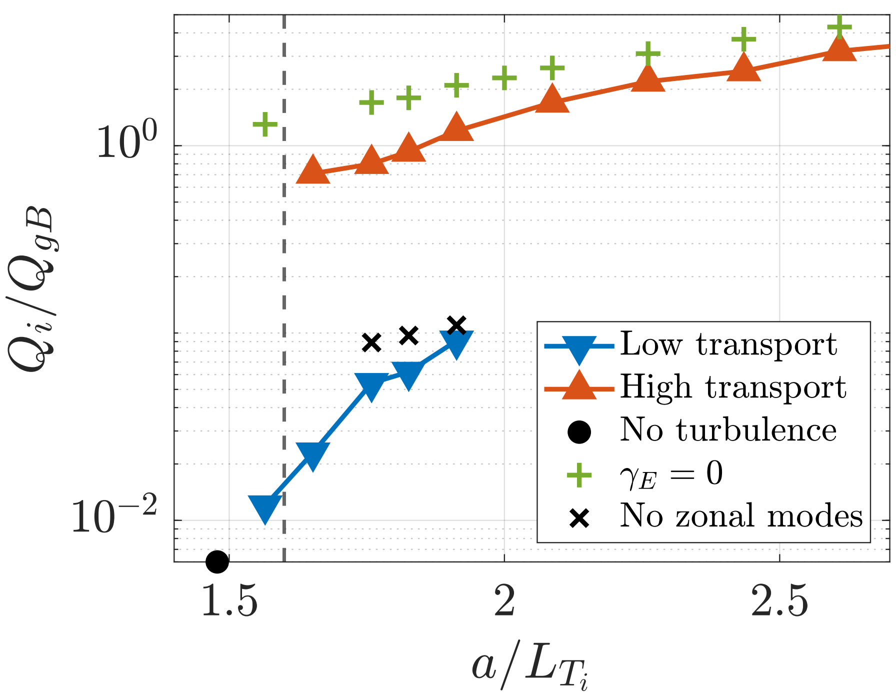
Adkins (ETG)

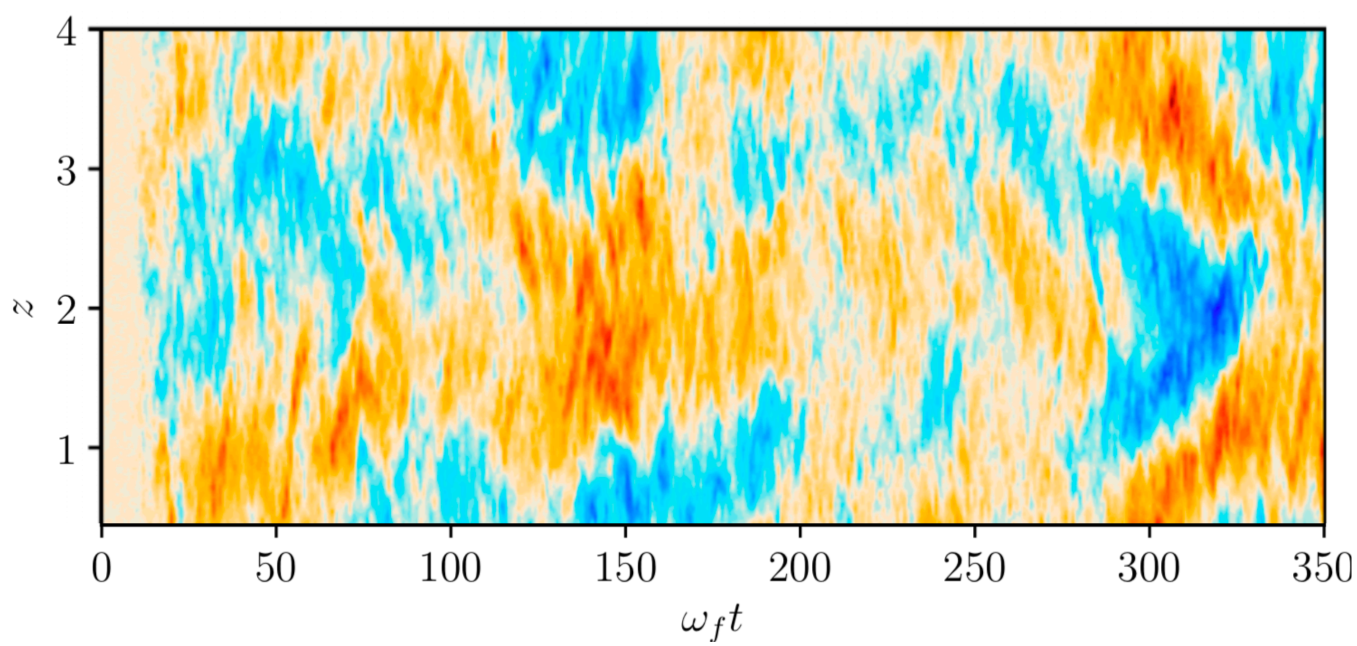


Meyrand (MHD turbulence)



Barnes (ITG bistability)

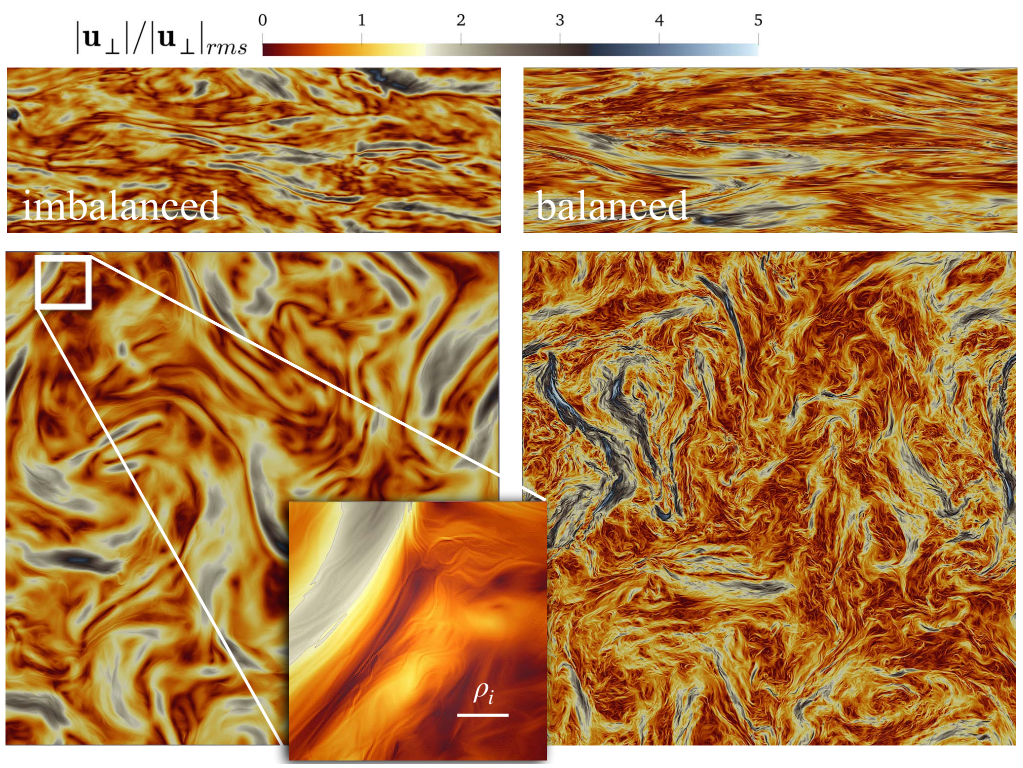


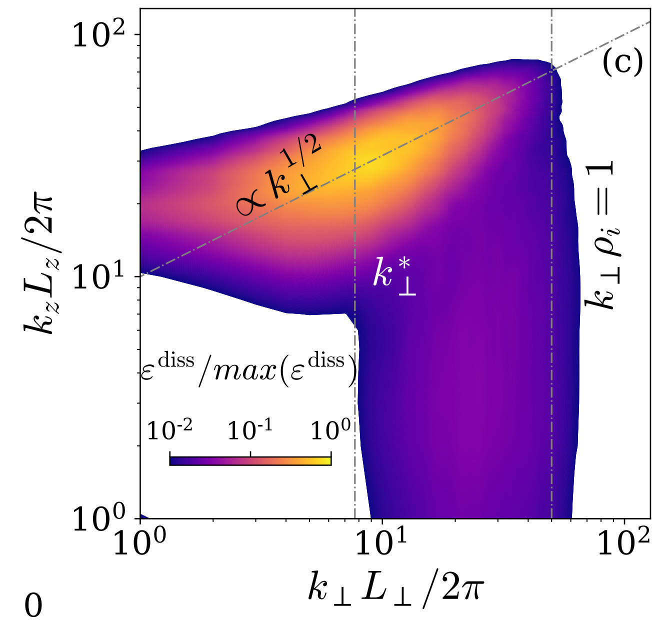


Rincon (dynamo)

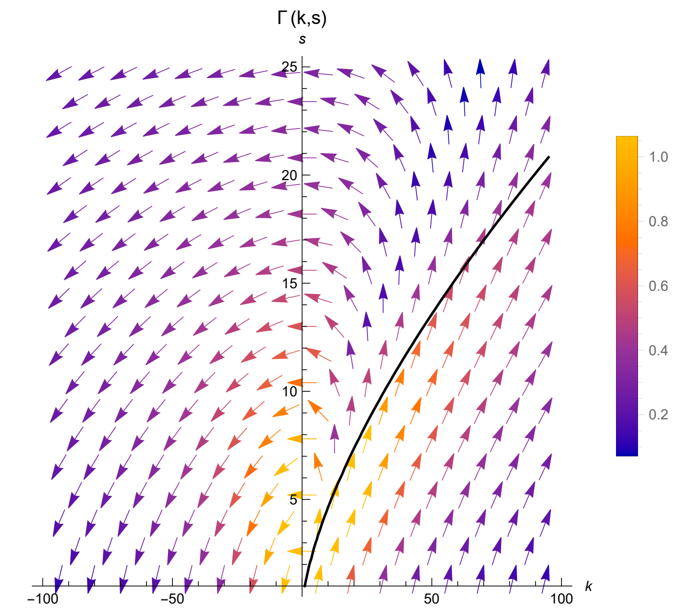
Critical balance is going strong

Squire-Meyrand (imbalanced FLR-MHD turbulence)

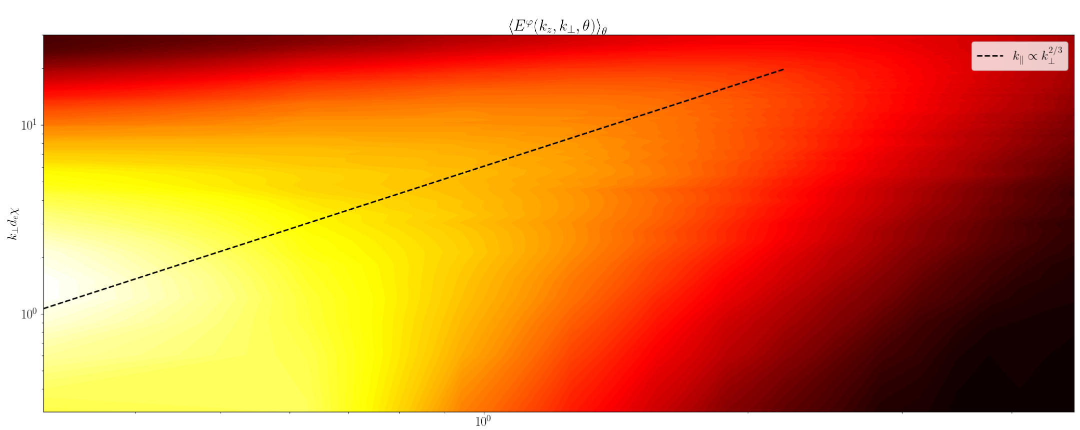


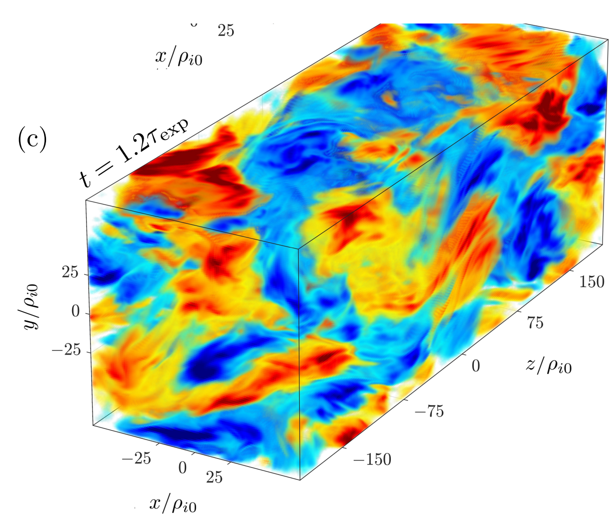


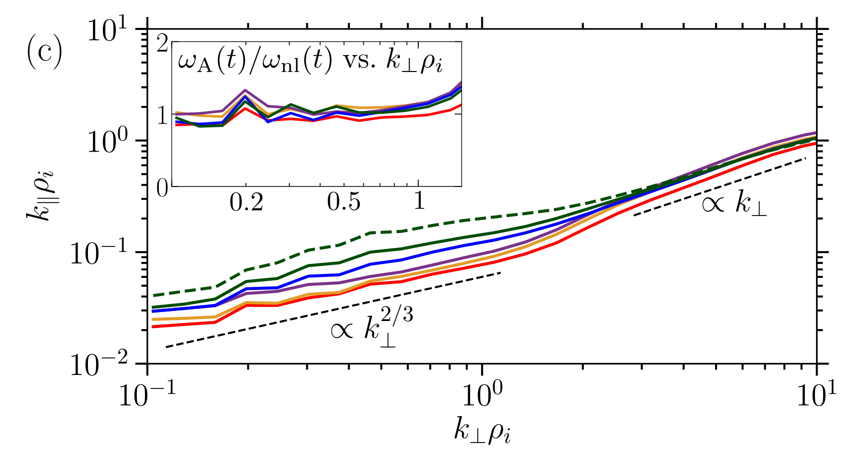
Nastac (Vlasov-Kraichnan entropy cascade)



Adkins (fluid ETG)





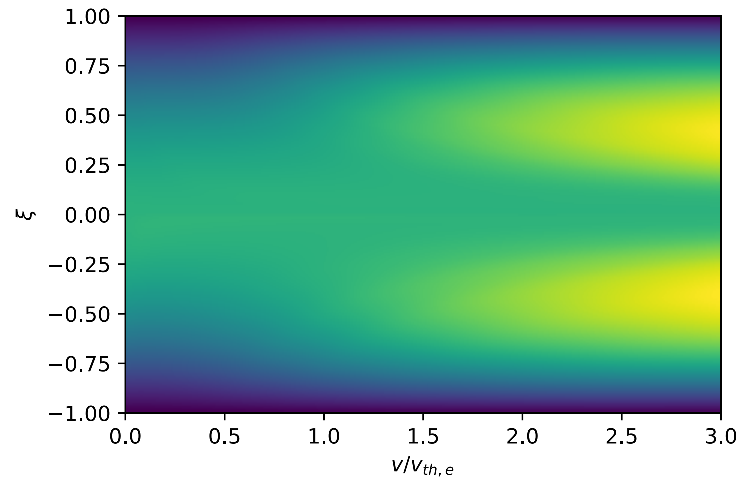


Bott (collisionless, high-β expanding turbulence)

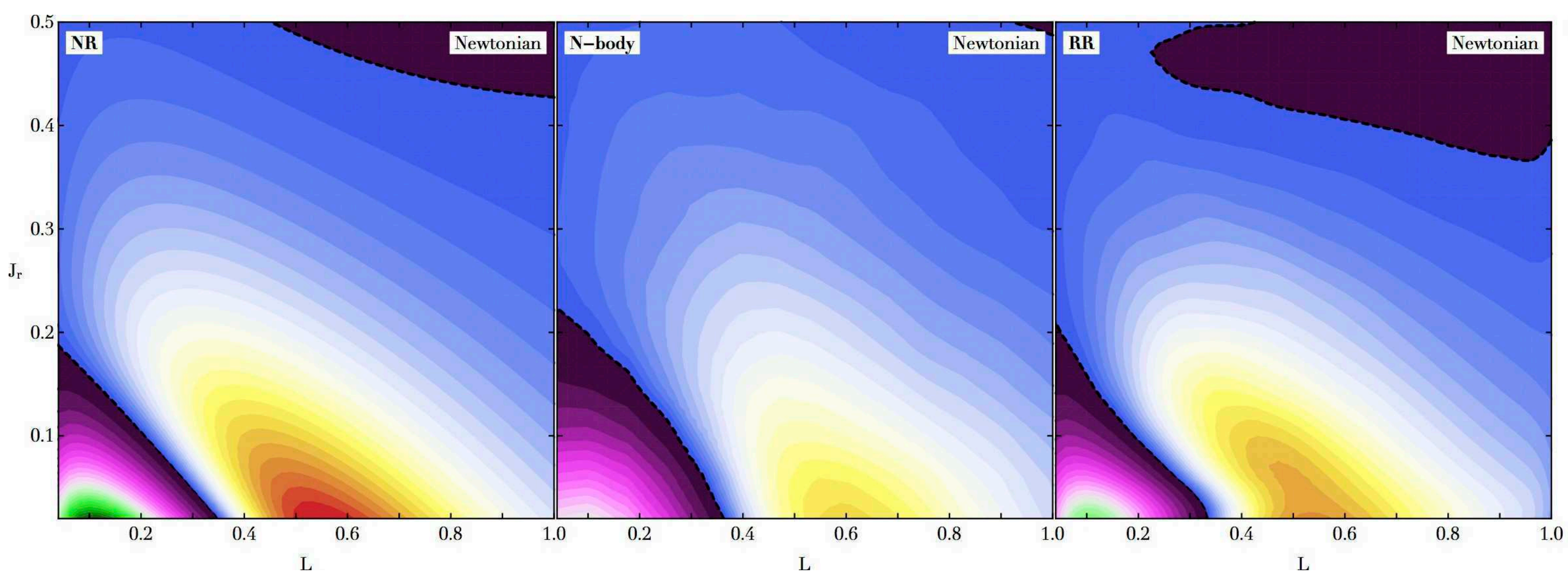
Closure limitations

Yerger

1. We didn't have enough scale separation.
   - We tried to go larger L_T (lower $\delta B/B_0$), but PIC noise suppressed the instability.
   - $\delta B/B_0 \ll 1$ for Gaussian statistics
   - Presumably there are real collisionless systems where $\delta B/B_0 \geq 1/10$ where this is important

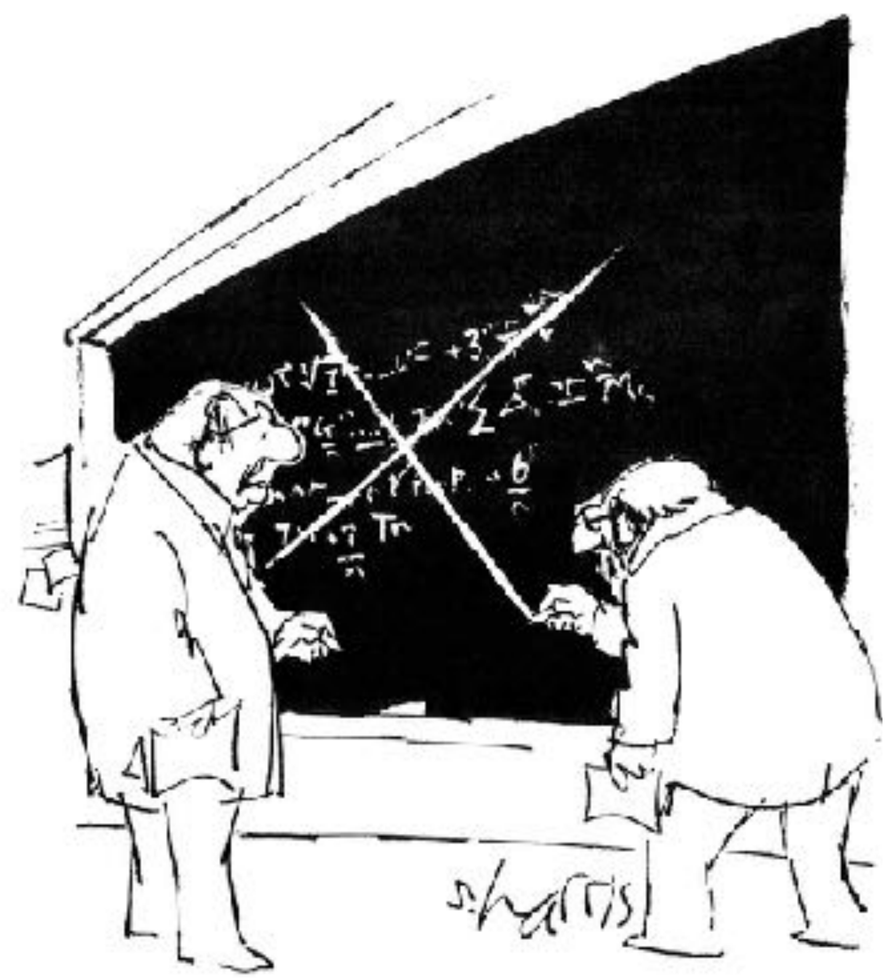


Fouvry



Orbit-averaged homogeneous Landau | *Direct N-body* | *Inhomogeneous Balescu-Lenard*

Why don't we find any trace of QL in numerical simulations?



Taking a step back

- **Things that look encouraging / much better than 15 years ago**
    - **Deeper understanding of phenomenology**
      (high-β kinetic effects, MHD turbulence, entropy cascades, i/eTG turbulence)
    - **Emerging theory/ heavy numerics connections** to solve key closure & relaxation problems
    - **"Semi-asymptotic"** high-resolution **simulations**
      (reconnection, dynamo, high- and low β turbulence…)
    - **Connection** between **theory/numerics** and data in **space plasmas**

- **Significant pockets of resistance**
    - Collisionless **relaxation**
    - particle **acceleration** (origin of power tails, velocity-cascade)
    - **electron dynamics** (ETG, TAI / MTM, resistivity, thermal conduction)
    - **electron vs ion heating & transport** (disks, ICM, fusion…)
    - Self-accelerating/**explosive dynamo** & plasma "batteries"

Three discussion threads

- What can **physicists learn from mathematics** (& conversely) ?
    - Can **physically relevant** (albeit mathematically “hard”) processes be **mathematically constrained** ?
    - What **mathematical developments** can be of **physical interest** ?
- What can we **learn** & what should we **hunt** for **on theory front** ?
    - Even **linear theory** remains **challenging**
    - **Invariants**: a lot of untapped potential ?
    - **Kinetic relaxation** & **thermodynamics …** work still very much in progress
    - **Closures: anomalous diffusion, non-perturbative** musings **?**
- How do we **make theoretical & numerical knowledge practically relevant** ?
    - Is **meta/bistability/subcriticality & phenomenological turbulence** understanding **actionable**?
    - **Reliable closures** for transport theory/large-scale structure: **“One MHD to rule them all” ?**
    - Bridges with **observation** and **data** — physics **constraints**

Thank you