Thermodynamics and collisionality in firehose-susceptible high-β plasmas

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13th Plasma Kinetics Working Meeting, Wolfgang Pauli Institute, Vienna – July 27th 2022

‘The Firehose Instability Resurrections’

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‘The Firehose Instability’ – Rosenbluth (`56); Chandrasekhar, Kaufman & Watson (`58); Parker (`58); Vedenov & Sagdeev (`58)…
‘The Firehose Instability Reloaded’ – Yoon (`93); Gary et al (`97, `98…); Hellinger et al (`00, `01…)
‘The Firehose Instability Revolutions’ – Schekochihin et al (`05, `10); Rosin et al (`11); Kunz, Schekochihin & Stone (`14)…

"…falls short compared to the original, but doesn't skimp on the action or cool visual effects." (Rotten Tomatoes)

Background/motivation

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- Collisionless/weakly collisional magnetised plasmas naturally develop ***pressure anisotropy*** $\Delta = p_\perp/p_\parallel - 1$
  E.g. collisionless magnetised plasma with no heat fluxes → *Chew-Goldberger-Low (CGL 1956) equations:*

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{p_\perp}{nB}\right) = 0, \quad \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{p_\parallel B^2}{n^3}\right) = 0$$

- Magnetised plasmas with 'sufficiently large' pressure anisotropies unstable to 'zoo' of ***kinetic instabilities***
  Generic condition for instability: $|\Delta| \gtrsim 1/\beta = B^2/8\pi p \implies$ if $\beta \gg 1$, 'small' pressure anisotropies drive instabilities

- Evolution and saturation of pressure-anisotropy-driven kinetic instabilities determine fundamental properties of magnetised $\beta \gtrsim 1$ plasmas:

Microphysics $\Longrightarrow$
- Anomalous collisionality (e.g. Kunz, Schekochihin & Stone 2014)
- Anomalous (trapped, accelerated) particle populations (e.g. Riquelme *et al* 2015)

Thermodynamics $\Longrightarrow$
- Pressure-anisotropy regulation & discrepancies from CGL (e.g. Camporeale & Burgess 2008)
- Anomalous heating (e.g. Sironi & Narayan 2015)

Wave dynamics $\Longrightarrow$
- Interruption of large-amplitude shear-Alfvén waves (e.g. Squire *et al* 2017)
- Propagation of large-amplitude sound waves (Kunz *et al* 2020)

$$m_i n_i \frac{\mathrm{d}\boldsymbol{u}}{\mathrm{d}t} = -\nabla\left(\frac{B^2}{8\pi} + p_\perp\right) + \nabla\cdot\left[\hat{\boldsymbol{b}}\hat{\boldsymbol{b}}\left(p_\perp - p_\parallel + \frac{B^2}{4\pi}\right)\right]$$

This talk: the (ion) firehose instability

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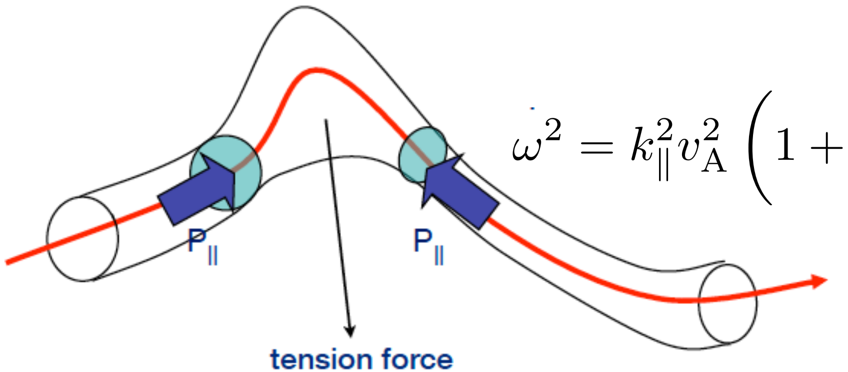
Rosenbluth (1956); Chandrasekhar, Kaufman & Watson (1958); Parker (1958); Vedenov & Sagdeev (1958):
Alfvén waves linearly unstable if $\Delta < -2/\beta_{\|}$.

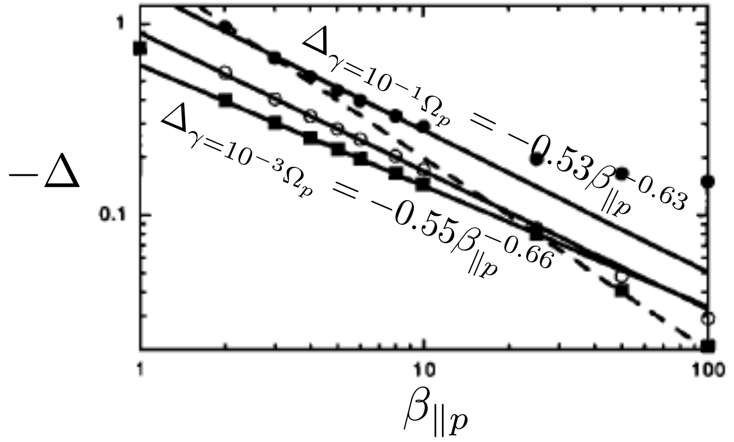


$$\omega^2 = k_{\|}^2 v_{\rm A}^2 \left(1 + \frac{\Delta\beta_{\|}}{2}\right) \equiv k_{\|}^2 v_{\rm A,eff}^2$$

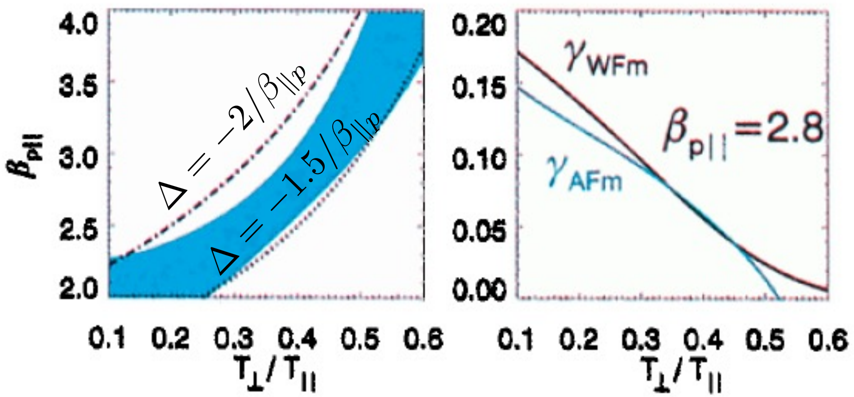
Subsequent developments in linear theory:

1. Dispersive FLR effects determine fastest growing firehose mode (Shapiro & Shevchenko \`64; Kennel & Sagdeev \`67)
2. Kinetic-scale variants of the firehose instability exist at $\beta_{\|} \gtrsim 1$ → \`less stringent' thresholds, faster growth rates:

Gary *et al* (1998): *resonant parallel firehose instability*



Hellinger & Matsumoto (2000): *resonant oblique firehose instability*



Nonlinear evolution of the firehose instability

Q: how does firehose instability evolve nonlinearly (and then saturate) in response to a plasma's macroscopic evolution?

→ Long-wavelength parallel firehose instability: quasilinear theory (Rosin *et al* 2011)…

→ General case: hybrid (HEB) expanding-box simulations (e.g. Hellinger & Travnicek 2008) at $\beta \sim 1$, and shearing-box simulations at $\beta \gg 1$ (Kunz, Schekochihin & Stone 2014):

Example: linearly, transversely expanding plasma: $L_{\|} = \text{const.},\ L_{\perp}(t) = 1 + t/\tau_{\exp}$.

Conservation of mass and magnetic flux

$$n \propto B \propto \left(1 + \frac{t}{\tau_{\exp}}\right)^{-2}$$

\+ Conservation of μ- and bounce invariants in magnetised plasma

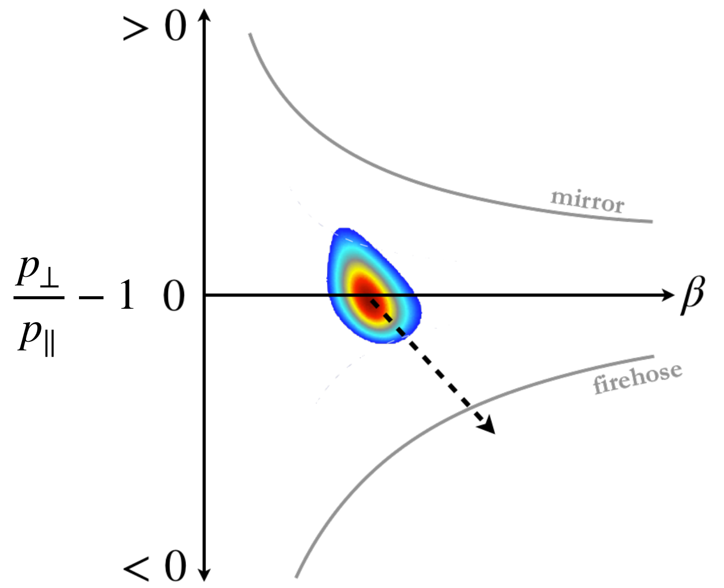
$$p_{\|} \propto n^3/B^2 \propto \left(1 + \frac{t}{\tau_{\exp}}\right)^{-2}$$

$$p_{\perp} \propto nB \propto \left(1 + \frac{t}{\tau_{\exp}}\right)^{-4}$$

$\Longrightarrow$ Decreasing pressure anisotropy & increasing plasma beta

$$\Delta = \left(1 + \frac{t}{\tau_{\exp}}\right)^{-2} - 1$$

$$\beta_{\|} \propto \left(1 + \frac{t}{\tau_{\exp}}\right)^{2}$$



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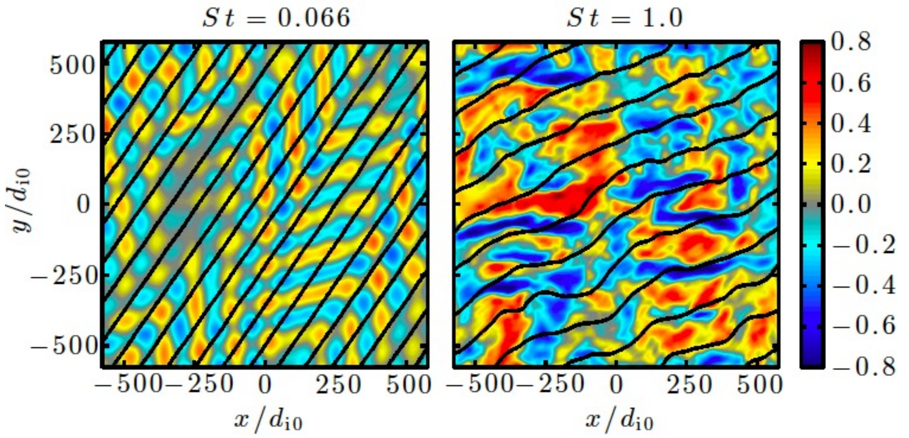
A: 1) Three-stage evolution: linear phase, secular phase, then saturation…

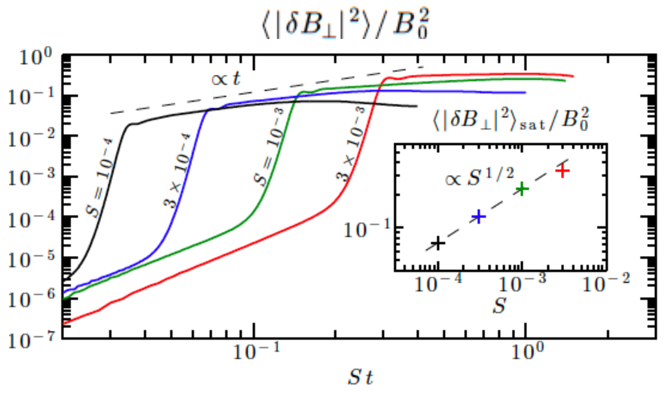
2) with $\delta B_\perp^2/B_0^2 \sim (S\beta_i/\Omega_i)^{1/2}$ (cf. Melville 2016), …

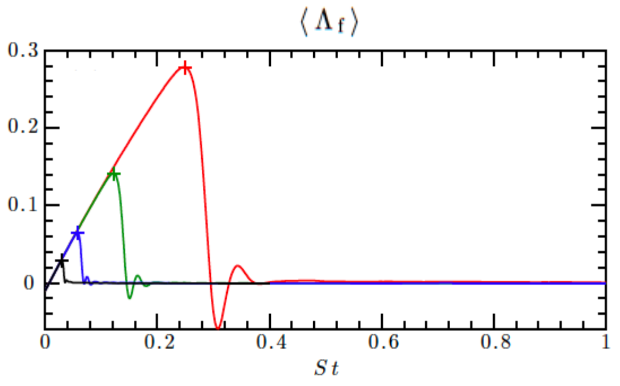
3) …regulated pressure anisotropy…

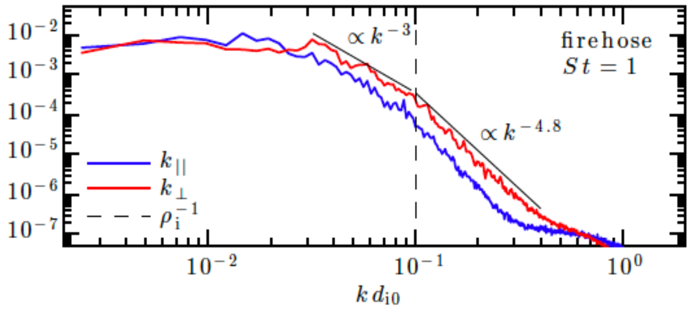
4) …extended magnetic-energy spectrum, …

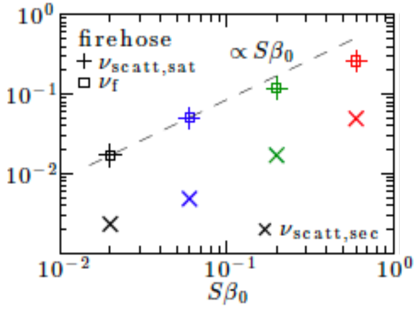
5) …and anomalous collisionality $\nu_{\rm eff} \sim S\beta_i$







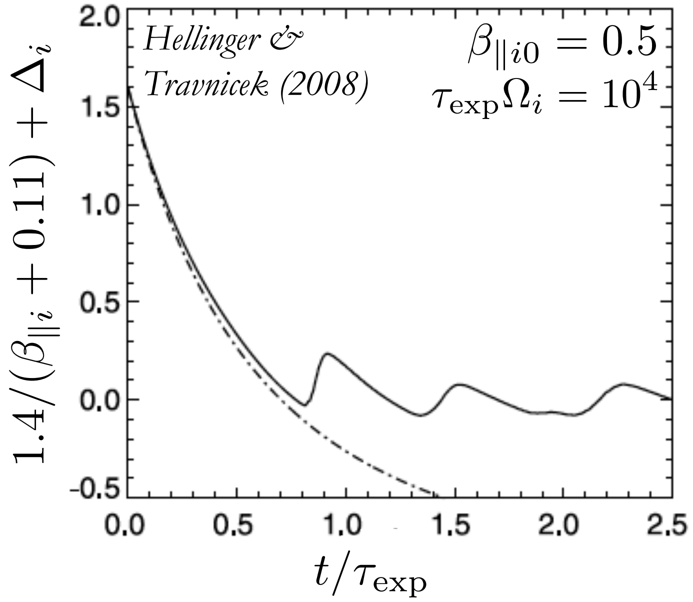




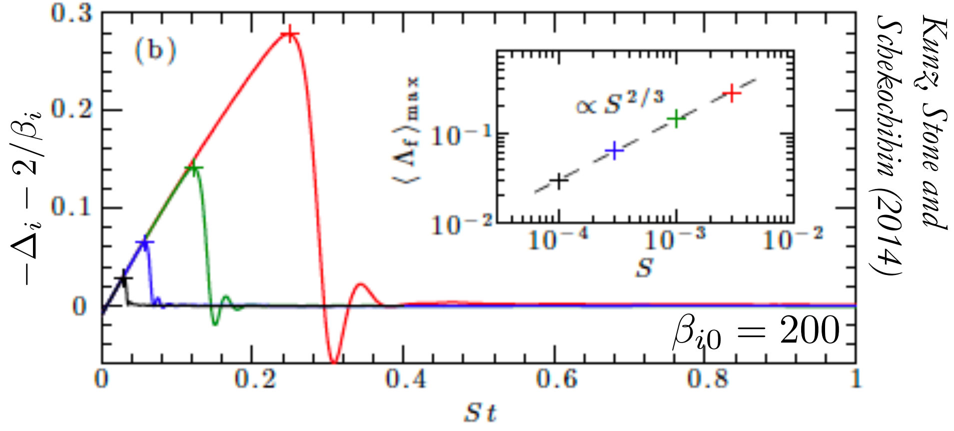
Problem solved? *Not quite – genuinely good reasons for revisiting problem…*

Thermodynamics at $\beta \gtrsim 1$ vs. $\beta \gg 1$

HEB simulations with $\beta \gtrsim 1$, $\tau_{\rm exp}\Omega_i \gg 1$:
in saturation, $\Delta_i \approx -1.4/\beta_{\parallel i}$.



Shearing-box simulations
with $\beta \gg 1, S \ll \Omega_i$:
in saturation, $\Delta_i \approx -2/\beta_i$.



Why does this numerical factor matter?

If $\Delta_i \approx -1.4/\beta_{\parallel i}$, $v_{\rm A,eff} \approx 0.5 v_{\rm A} > 0$

If $\Delta_i \approx -2/\beta_i$, $v_{\rm A,eff} \approx 0$

In latter case, Alfvénic restoring force vanishes; anomalous plasma dynamics!

A consistent explanatory framework?

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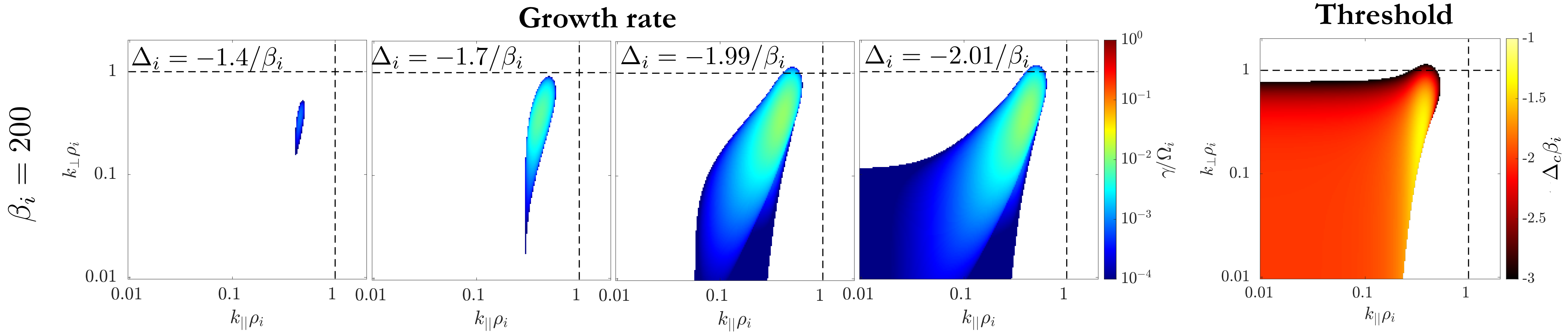
Q: how do these two results relate to each other?

A: for a given β_i, the steady-state value of Δ_i depends on macroscopic evolution rate τ

If $\tau \gg \tau_\mathrm{c}(\beta,\Omega_i) \implies (\Delta_i)_\mathrm{sat} \approx -2/\beta_i \; (v_\mathrm{A,eff} \approx 0 v_\mathrm{A})$; if $\tau \ll \tau_\mathrm{c}(\beta,\Omega_i) \implies (\Delta_i)_\mathrm{sat} > -2/\beta_i \; (v_\mathrm{A,eff} \approx 0.4 v_\mathrm{A})$.

Alfvén-inhibiting state — **Alfvén-enabling state**

Why? J. Squire (2016): 'Kinetic oblique firehose has less stringent threshold than long-wavelength firehose…'



If $\gamma_\mathrm{f}(t_\mathrm{sec} - t_\mathrm{c}) \gtrsim 1$ before $\Delta_i > -2/\beta_i$, then kinetic oblique firehose back-reacts before `fluid' firehose destabilised!

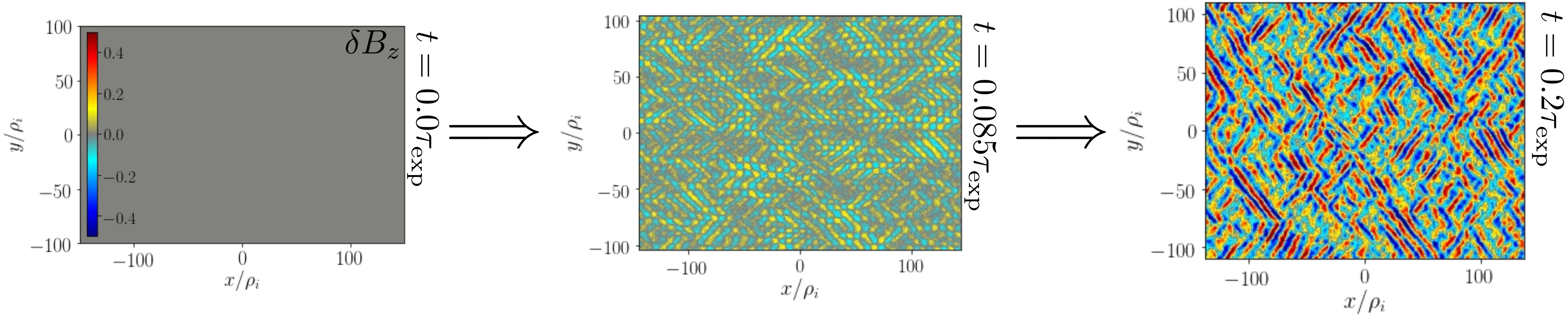
Q: what is τ_c?

$$(t_\mathrm{sec} - t_\mathrm{c}) \sim \gamma_\mathrm{f}^{-1} \sim (|t_\mathrm{sec}/\tau - \Delta_\mathrm{c}|)^{-1/2}\Omega_i^{-1} \implies (t_\mathrm{sec} - t_\mathrm{c})\Omega_i \sim (\tau\Omega_i)^{1/3} \implies (\Delta_i)_\mathrm{min} - \Delta_\mathrm{c} \sim (\tau\Omega_i)^{-2/3}$$

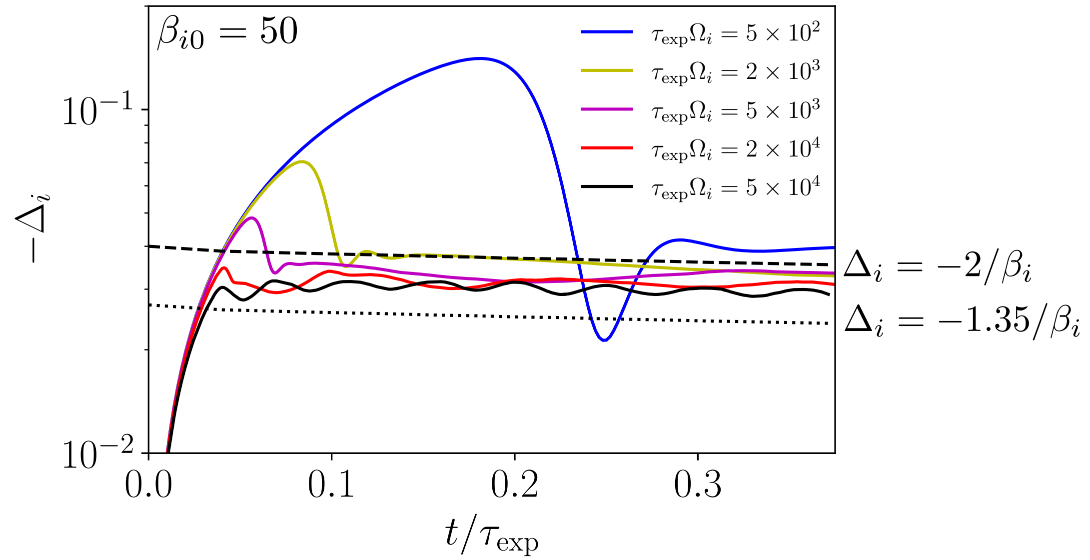
For $(\Delta_i)_\mathrm{min} \sim -2/\beta_i$, deduce $\tau_\mathrm{c}\Omega_i \sim \beta_i^{3/2}$

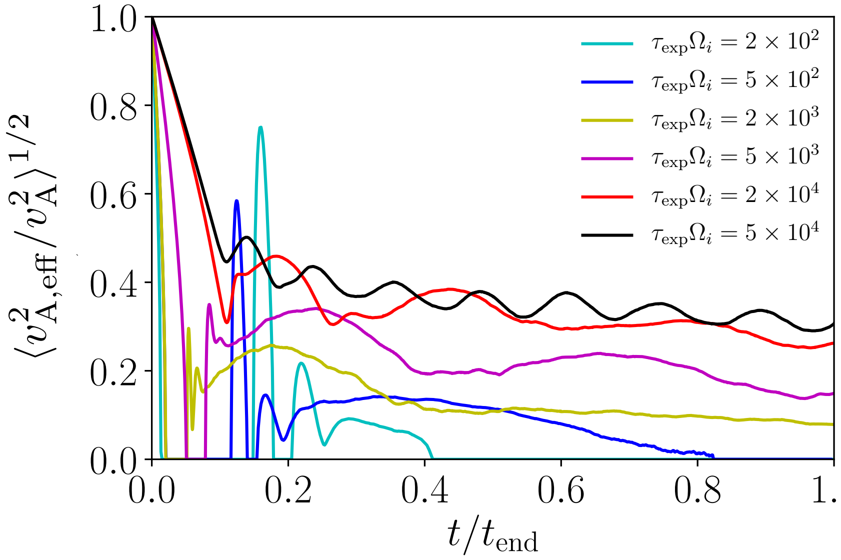
Parameter study results

- Confirm by running HEB simulations over $(\tau_{\rm exp}, \beta_i)$ parameter space! E.g. $L_y = 1 + t/\tau_{\rm exp}\,,\ L_x = L_z = {\rm cnst.}$



Example: fix β_i, vary expansion time…





Scaling verification I

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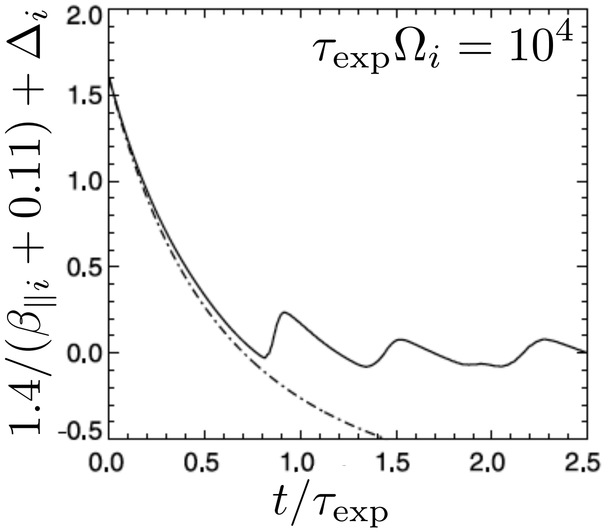
Consistency check: is scaling estimate $(\tau\Omega_i)_c \sim 30\beta_i^{3/2}$ consistent with previous expanding-box/shearing box simulations?

$\beta_i \sim 1$ plasma: $(\tau\Omega_i)_c \sim 30$

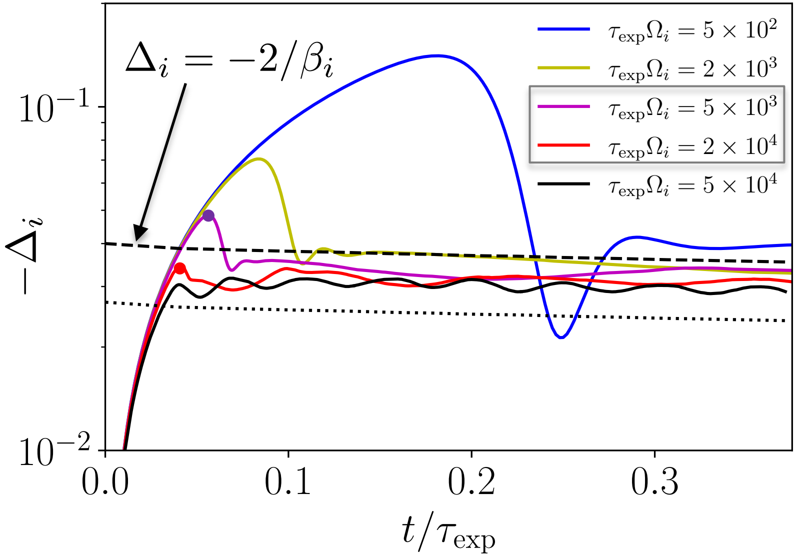
Much faster than typical expansion times (e.g. Hellinger & Travnicek 2008)

$\implies v_{A,\mathrm{eff}} > 0$



$\beta_i \approx 50$ plasma: $(\tau\Omega_i)_c \sim 10^4$

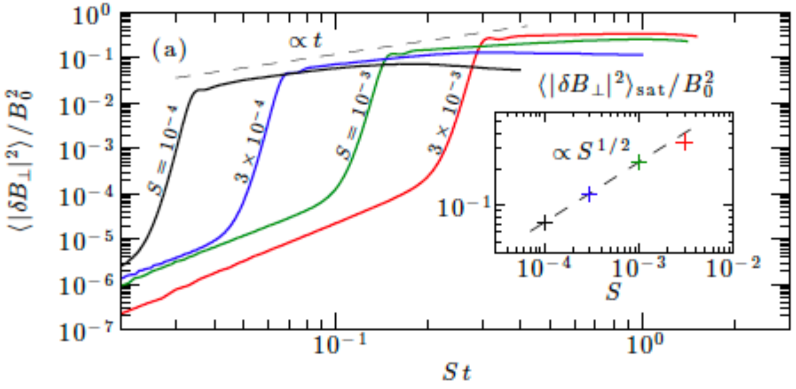
Consistent with observed numerical behaviour



$\beta_i \sim 200$ plasma: $(\tau\Omega_i)_c \sim 10^5$

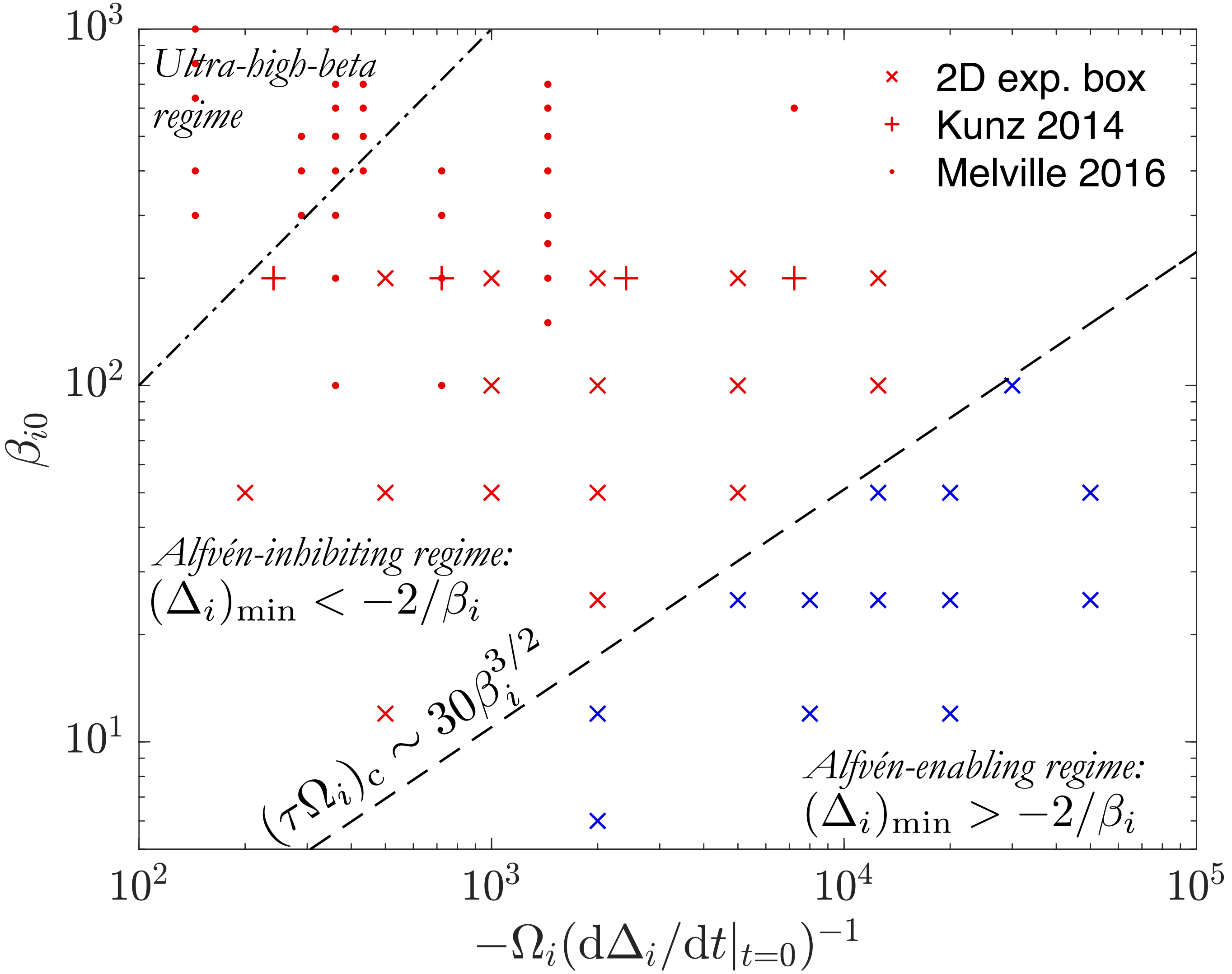
Much slower than shear rates previously investigated (e.g. Kunz et al. 2014, Melville et al. 2016)

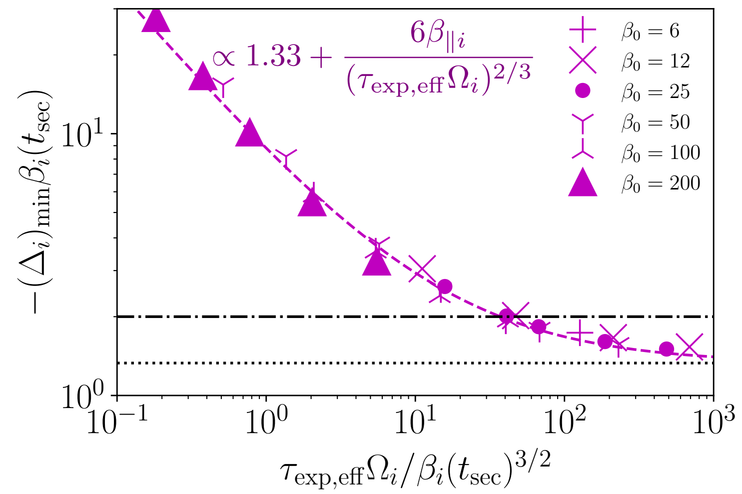
$\implies v_{A,\mathrm{eff}} \approx 0$

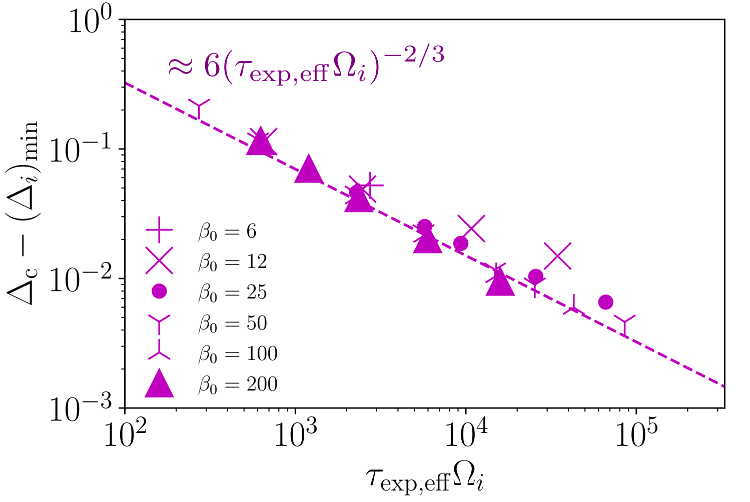


Scaling verification II

Is scaling estimate $(\tau\Omega_i)_c \sim 30\beta_i^{3/2}$ consistent with larger set of 2D HEB simulations?







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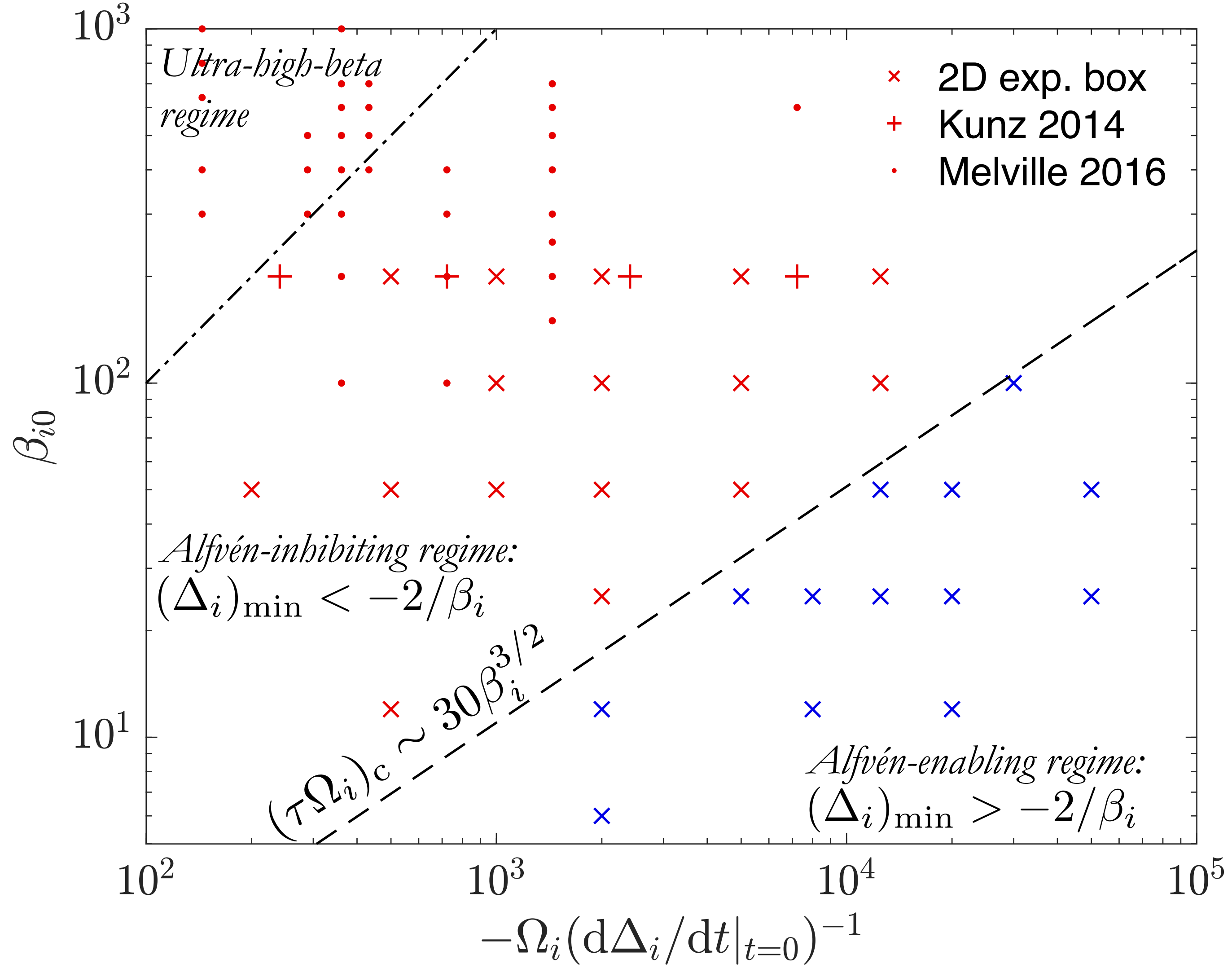

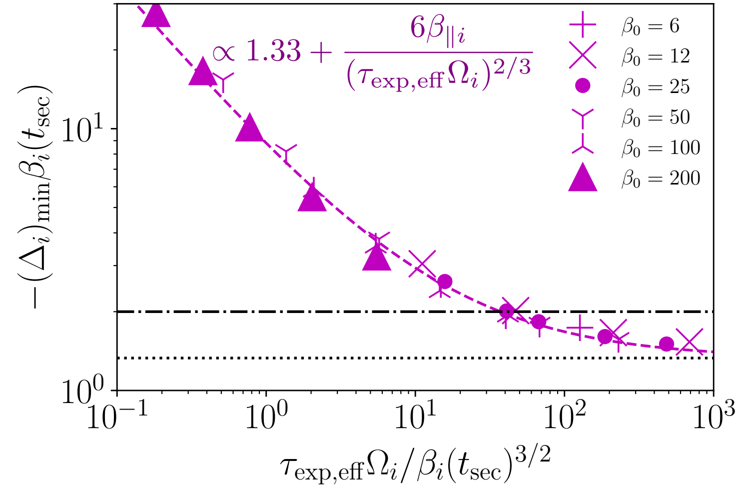

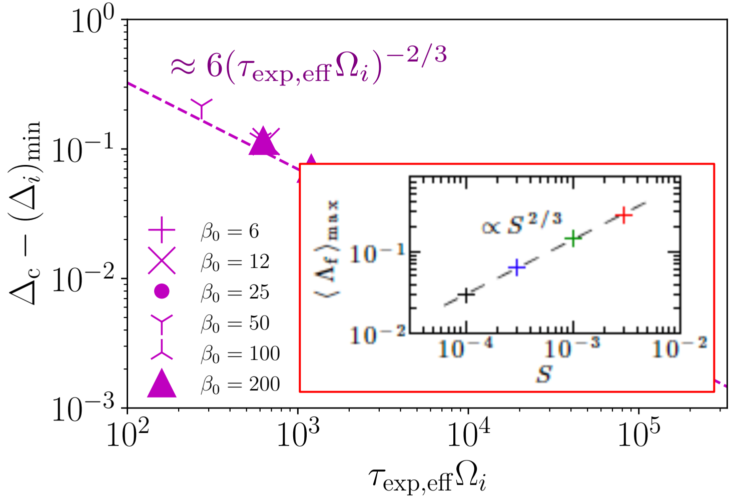

[Agrees with Kunz et al. (2014)]

Saturation in different states

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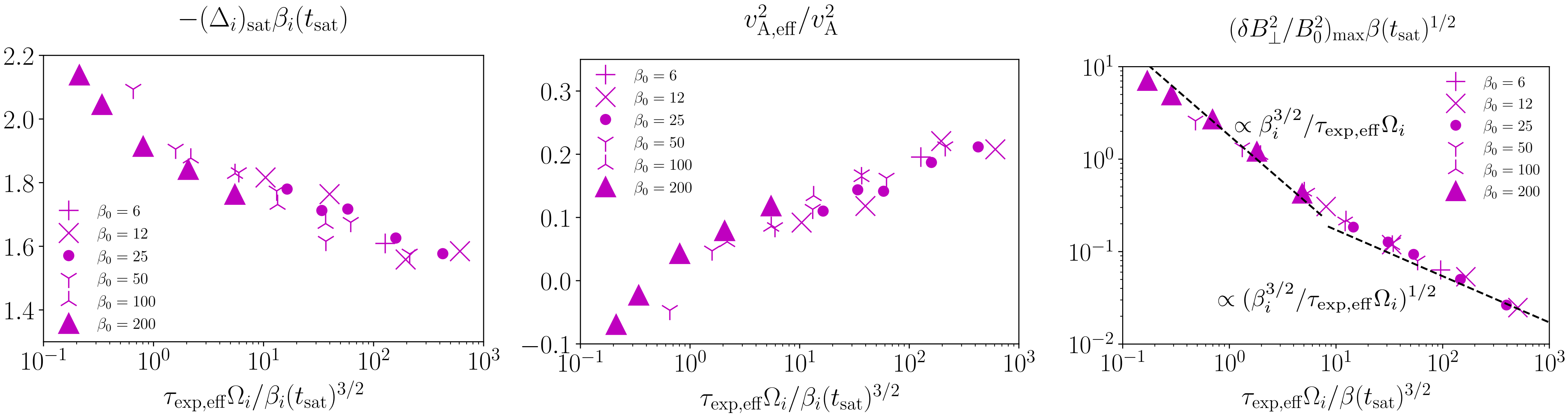
Q: what characteristics does the plasma have in the saturated state of the firehose instability?



Gradual transition from state in which $(\Delta_i)_{\rm sat} \lesssim -2/\beta_{\parallel i}$ to state in which $(\Delta_i)_{\rm sat} > -2/\beta_i$

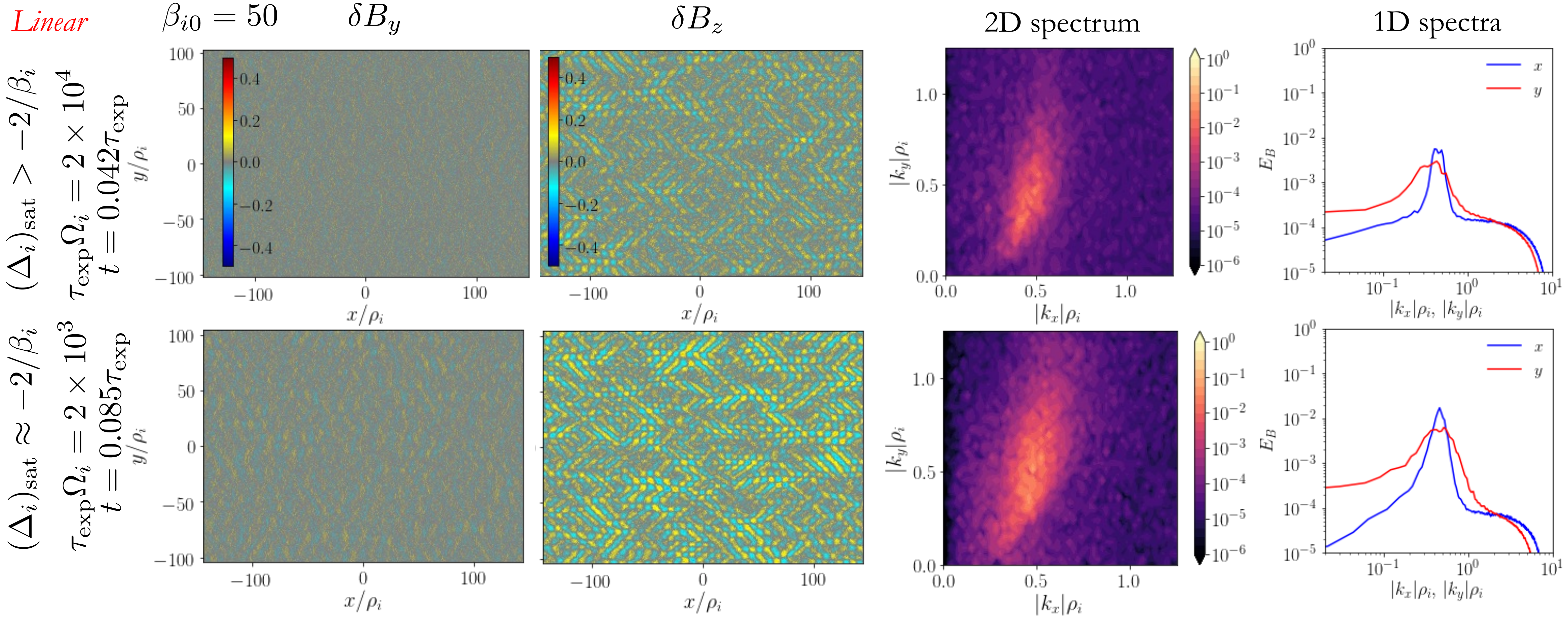
Gradual transition from Alfvén-inhibiting to Alfvén-enabling state

Unexpected magnetic-field strength scaling in Alfvén-enabling state…

$$\nu_{\rm eff} \sim \frac{\delta B_\perp^2}{B_0^2}\Omega_i \sim \frac{\beta_i}{\tau_{\rm exp}}$$

Magnetic-field structure

Firehose magnetic-field perturbations depend qualitatively on whether $(\Delta_i)_{\rm sat} \approx -2/\beta_i$ ***or*** $(\Delta_i)_{\rm sat} > -2/\beta_i$



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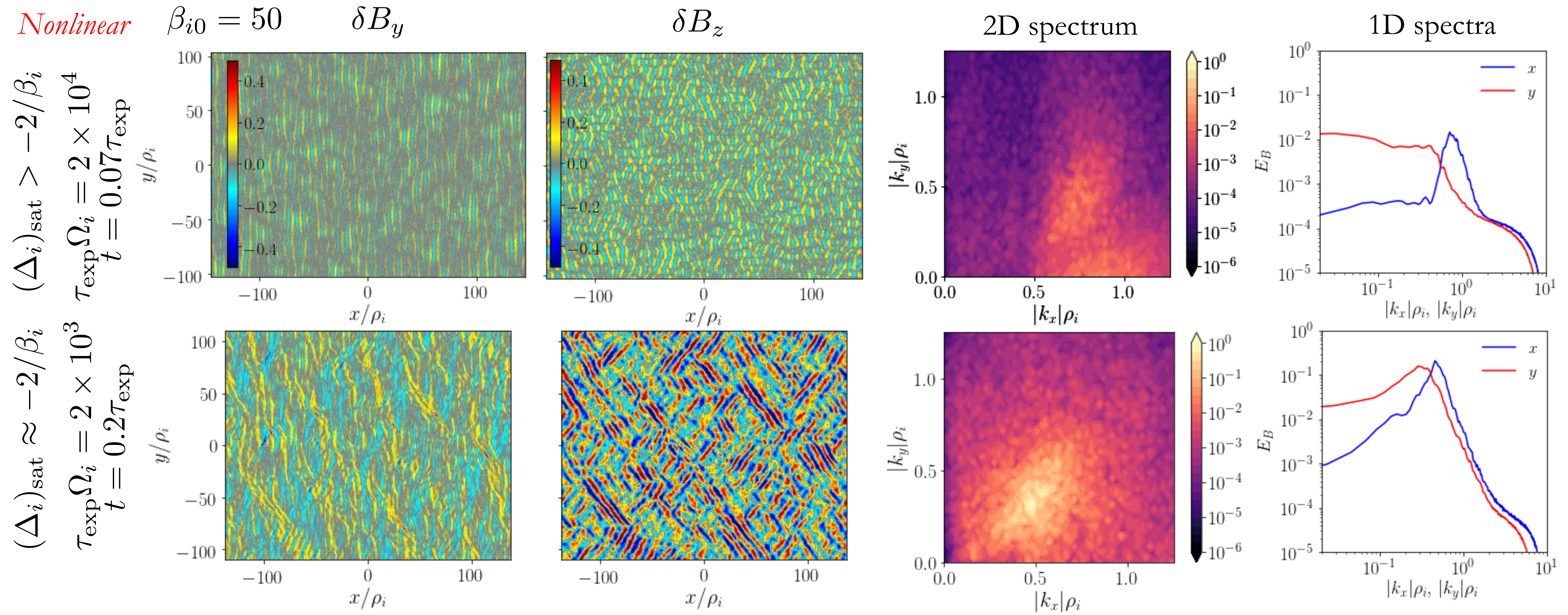

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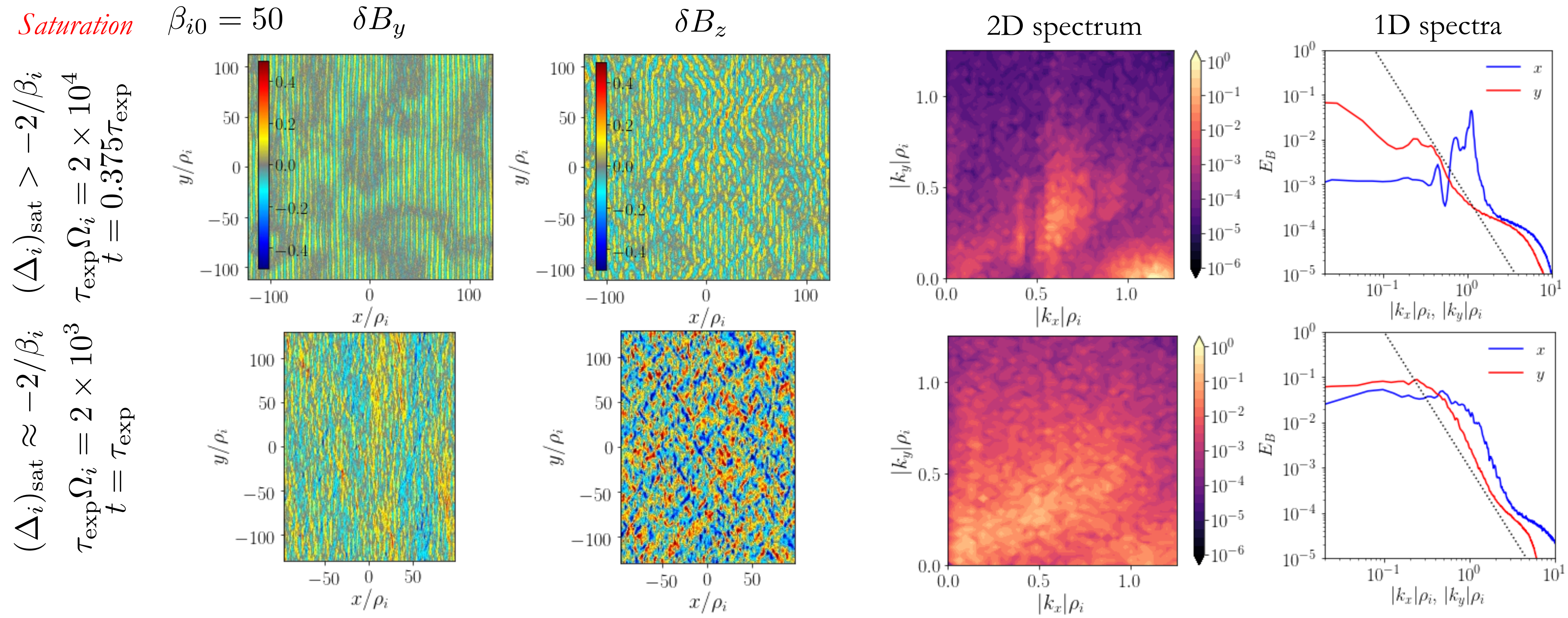
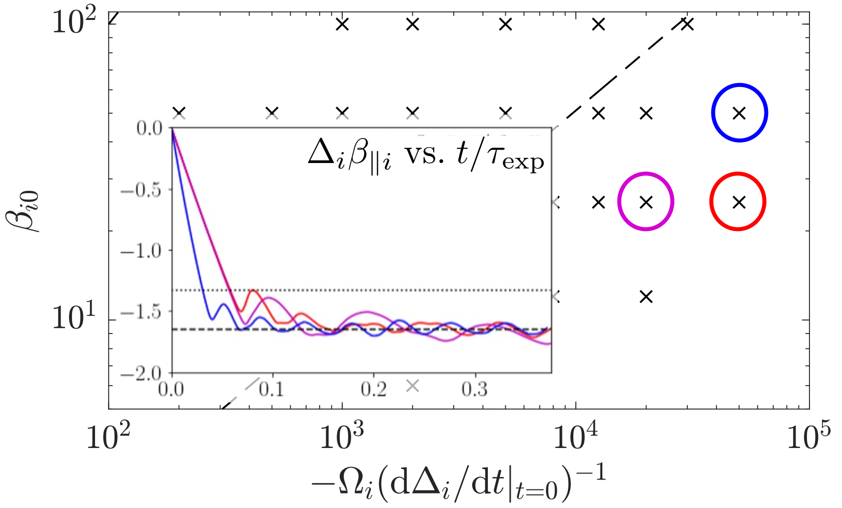


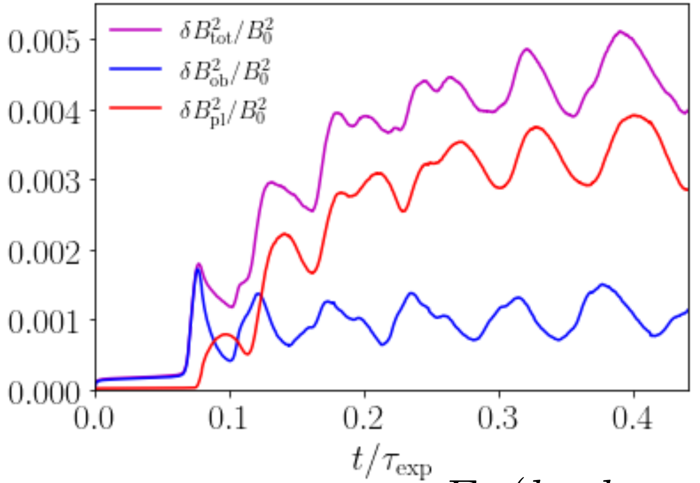
- In Alfvén-enabling state, quasi-periodic spectrum of oblique and parallel modes on ion-Larmor scales
- In Alfvén-inhibiting regime, broad quasi-static spectrum of modes including above ion-Larmor scales

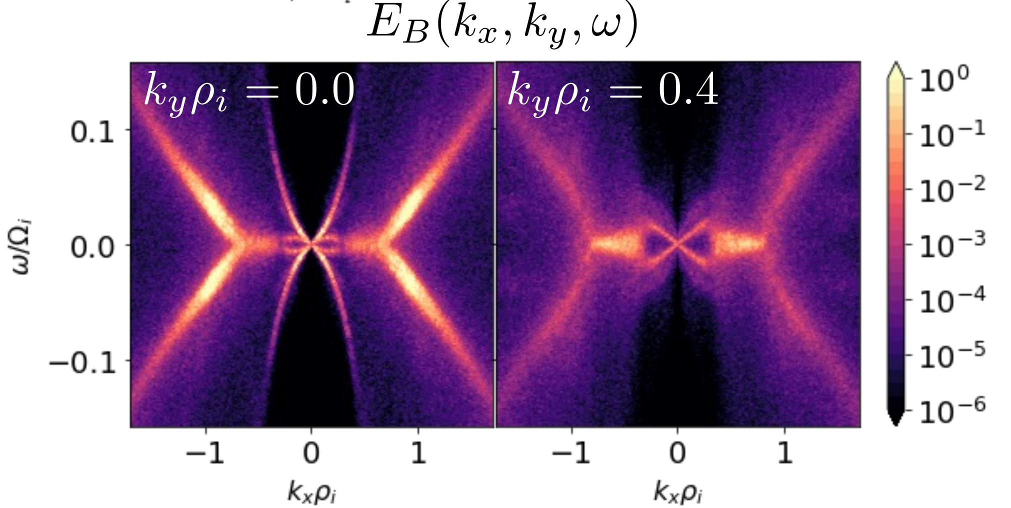
Firehoses in Alfvén-enabling state

Anomalously large $(\delta B_\perp^2/B_0^2)_{\max}$ ***in Alfvén-enabling state due to parallel ion-Larmor-scale modes***

For `asymptotic' Alfvén-enabling state… → …parallel modes eventually grow larger than oblique ones







Q: what excites parallel modes? Linearly stable for bi-Maxwellian…

A: evolution has two initial phases before reaching quasi-periodic state:

1. Bi-Maxwellian with $T_\parallel > T_\perp$ drives (non-propagating) ion-Larmor-scale oblique firehoses unstable (with $k_\parallel\rho_i \approx 0.5$)
2. Back-reaction isotropizes distribution function near $v_\parallel \approx 2v_{\rm th i}$, destabilising propagating right-handed parallel modes

Distinct wave populations clearly seen in Fourier analysis!

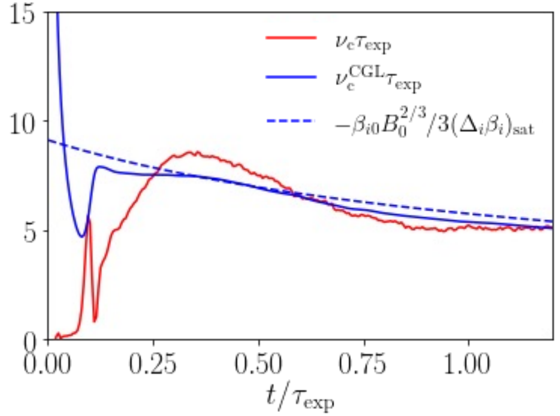
Box-averaged collisionality

Anomalous collisionality model with $\nu_c \sim \beta_i/\tau_{\rm exp}$ agrees with box-averaged numerical estimates

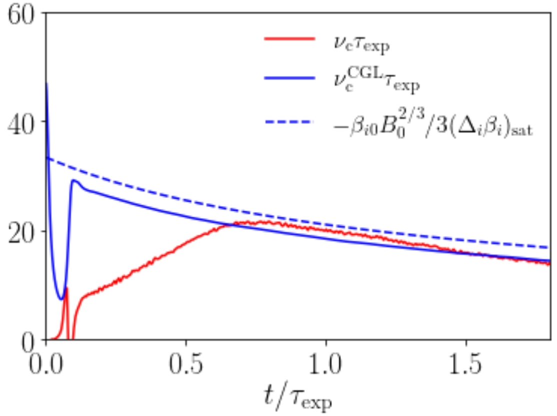
$$\nu_{\rm c} = \dot{\bar{\mu}}/\overline{(T_{\parallel i} - T_{\perp i})/B} \quad \text{vs.} \quad \nu_c^{\rm CGL} = \frac{1}{3\Delta_i}\frac{\rm d}{{\rm d}t}\log B$$

Alfvén-inhibiting states

$\tau_{\rm exp}\Omega_i = 2000, \beta_{i0} = 50$

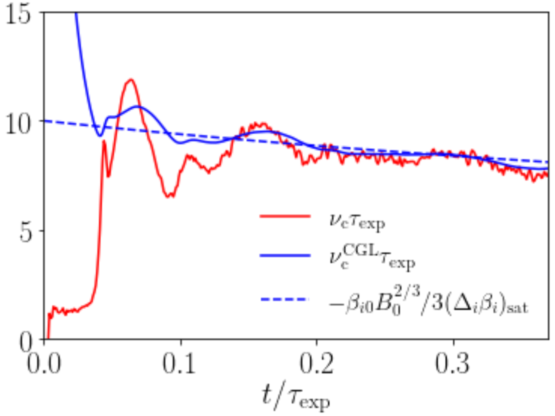


$\tau_{\rm exp}\Omega_i = 2000, \beta_{i0} = 200$

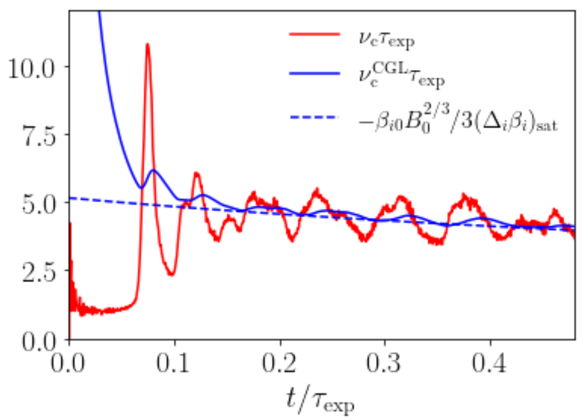


Alfvén-enabling states

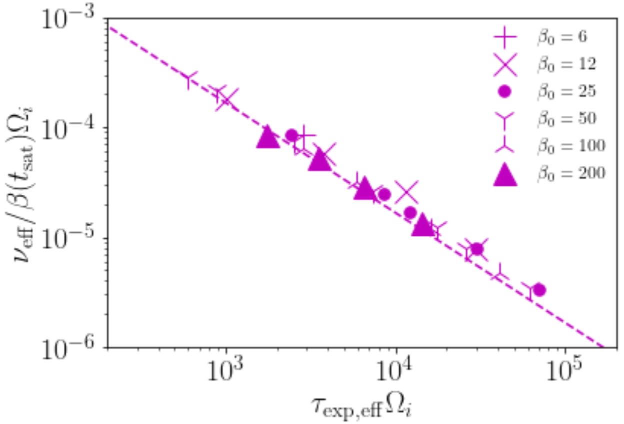
$\tau_{\rm exp}\Omega_i = 20000, \beta_{i0} = 50$



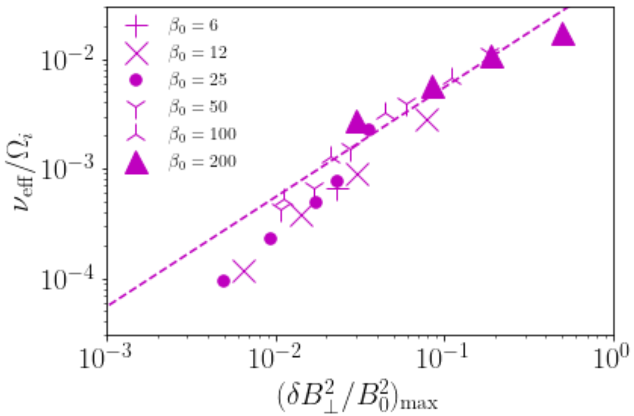
$\tau_{\rm exp}\Omega_i = 50000, \beta_{i0} = 25$



Box-averaged collisionality:
$\nu_{\rm eff} \sim \beta_i/\tau_{\rm exp}$ …



… but $\nu_{\rm eff}/\Omega_i$ **not** proportional to $\delta B_\perp^2/B_0^2$.
(Why? Answer depends on velocity-dependent collisionality…)



Discussion/astrophysical implications

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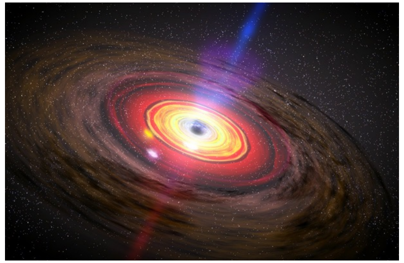
Q: which state is most relevant to astrophysical systems of interest?

A: large-scale separations – ***Alfvén-enabling state generally most relevant***

ICM at $R_{\rm cool}$

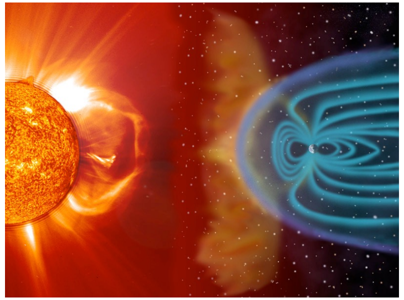
$\tau\Omega_i \sim 10^{12} \gg (\tau\Omega_i)_{\rm c} \sim 10^4$
$\beta_i \sim 100$

Sgr A* at $R_{\rm Bondi}$



$\tau\Omega_i \sim 10^{7} \gg (\tau\Omega_i)_{\rm c} \sim 10^3$
$\beta_i \sim 10$

Solar wind at 1 au



$\tau\Omega_i \sim 10^{4} \gg (\tau\Omega_i)_{\rm c} \sim 30$
$\beta_i \sim 1$

Implications

1. In astrophysical plasma, $(\Delta_i)_{\rm sat} > -2/\beta_i$
   → MHD plasma models with $v_A \to v_{\rm A,eff} > 0$ generally valid
   → No wave interruption…?
2. Firehose fluctuations concentrated at ion-Larmor scales

N.B. Alfvén-inhibiting state could still be relevant if $\beta_i \gg 1$:

- Reionization epoch (weak magnetisation)
- Local regions of ICM plasma in which tangled stochastic field is reversing sign
- Systems with much smaller scale separations… like experiments!

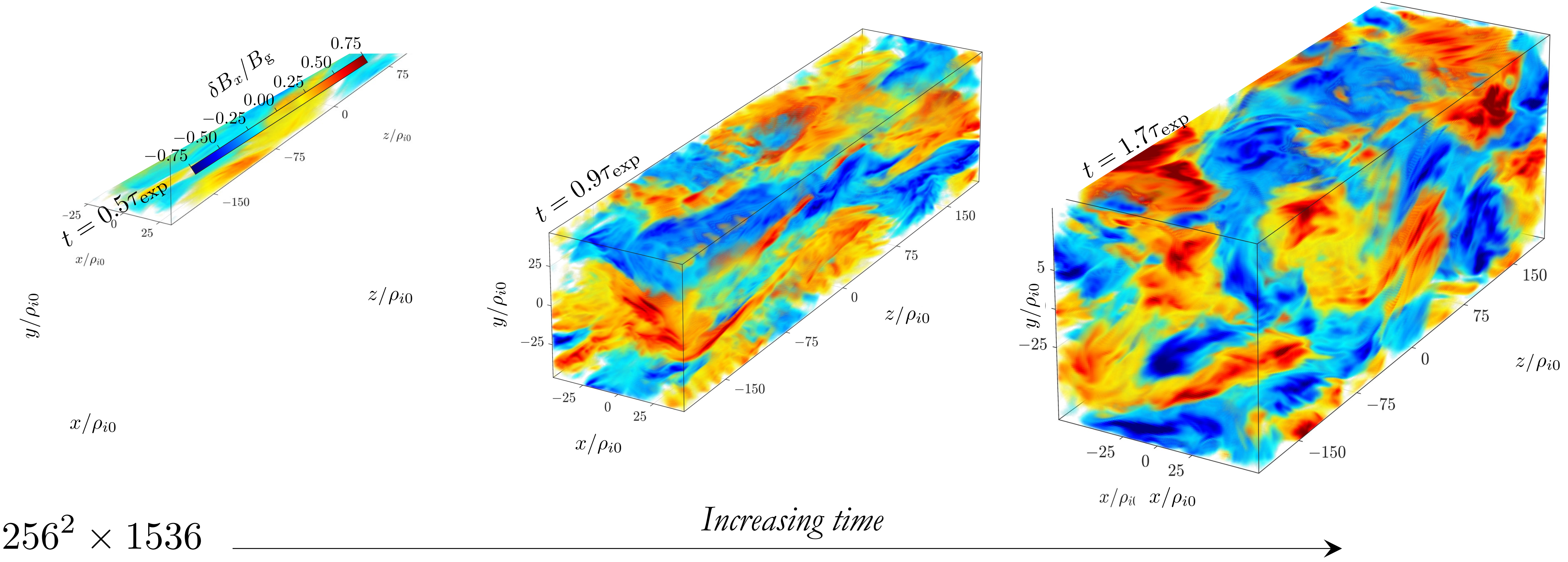
Application: turbulence in expanding plasma

Consider strong Alfvénic turbulence in an expanding ($L_\parallel = \text{const.},\ L_\perp(t) = 1 + t/\tau_{\text{exp}}$), collisionless plasma (Bott *et al* 2021):

$L_\parallel / L_\perp(0) = 6$
$\beta_{\parallel i}(0) = 2$

$L_\parallel / v_A(0) \approx 551 \Omega_i^{-1}$
$\tau_{\text{exp}} = 10 L_\parallel / v_A(0)$

$L_\parallel / L_\perp(\tau_{\text{exp}}) \approx 3$
$\beta_{\parallel i}(\tau_{\text{exp}}) \approx 4$



$256^2 \times 1536$

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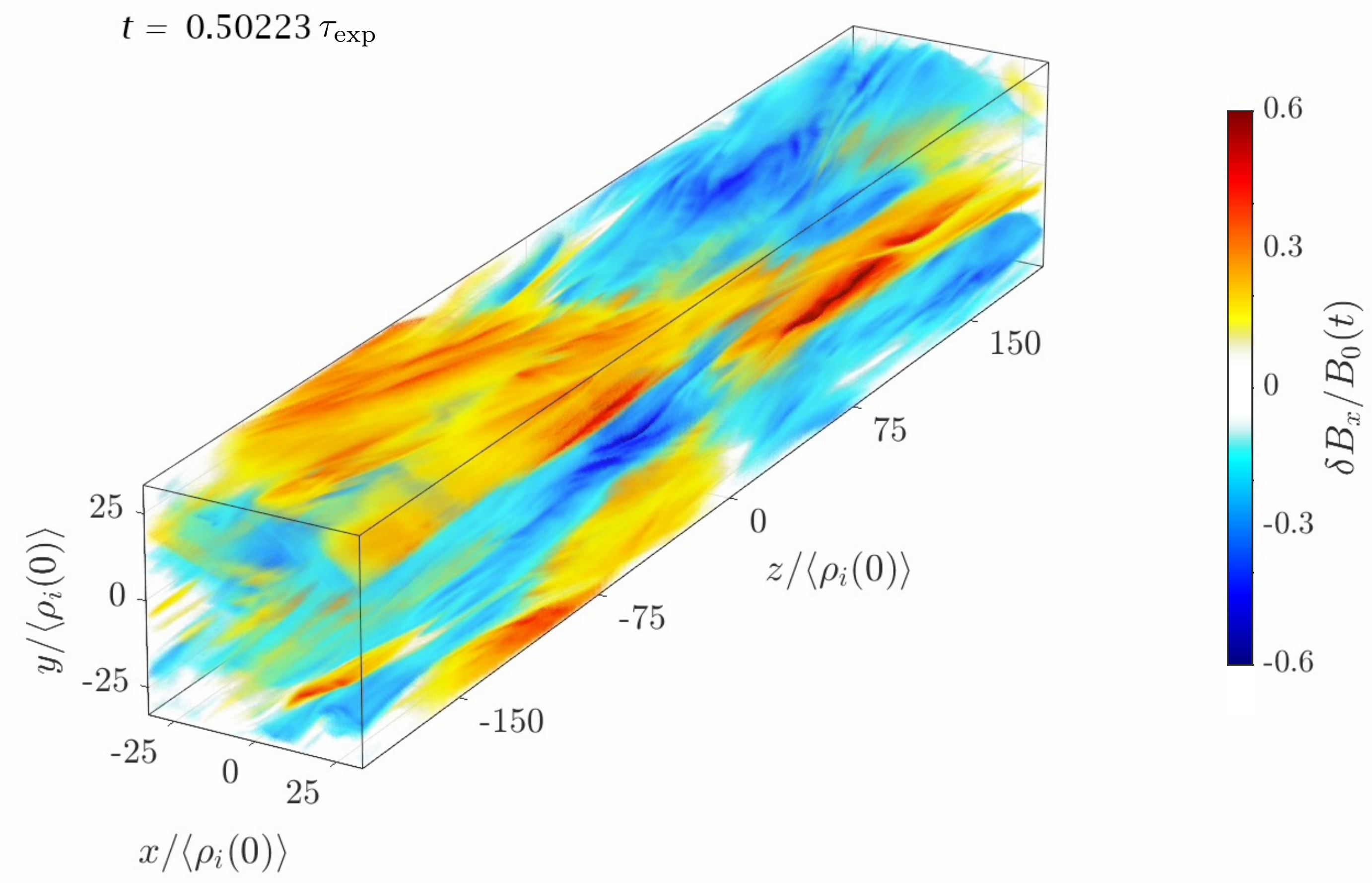

Key findings

- Fluctuations oscillate slower and $\delta B/B_0(t)$ gets larger as effective Alfvén speed drops:

$$v_{A,\rm eff}(t) \equiv \frac{B(t)}{\sqrt{4\pi\rho(t)}}\sqrt{1 + \frac{p_\perp - p_\parallel}{B^2/4\pi}(t)}$$

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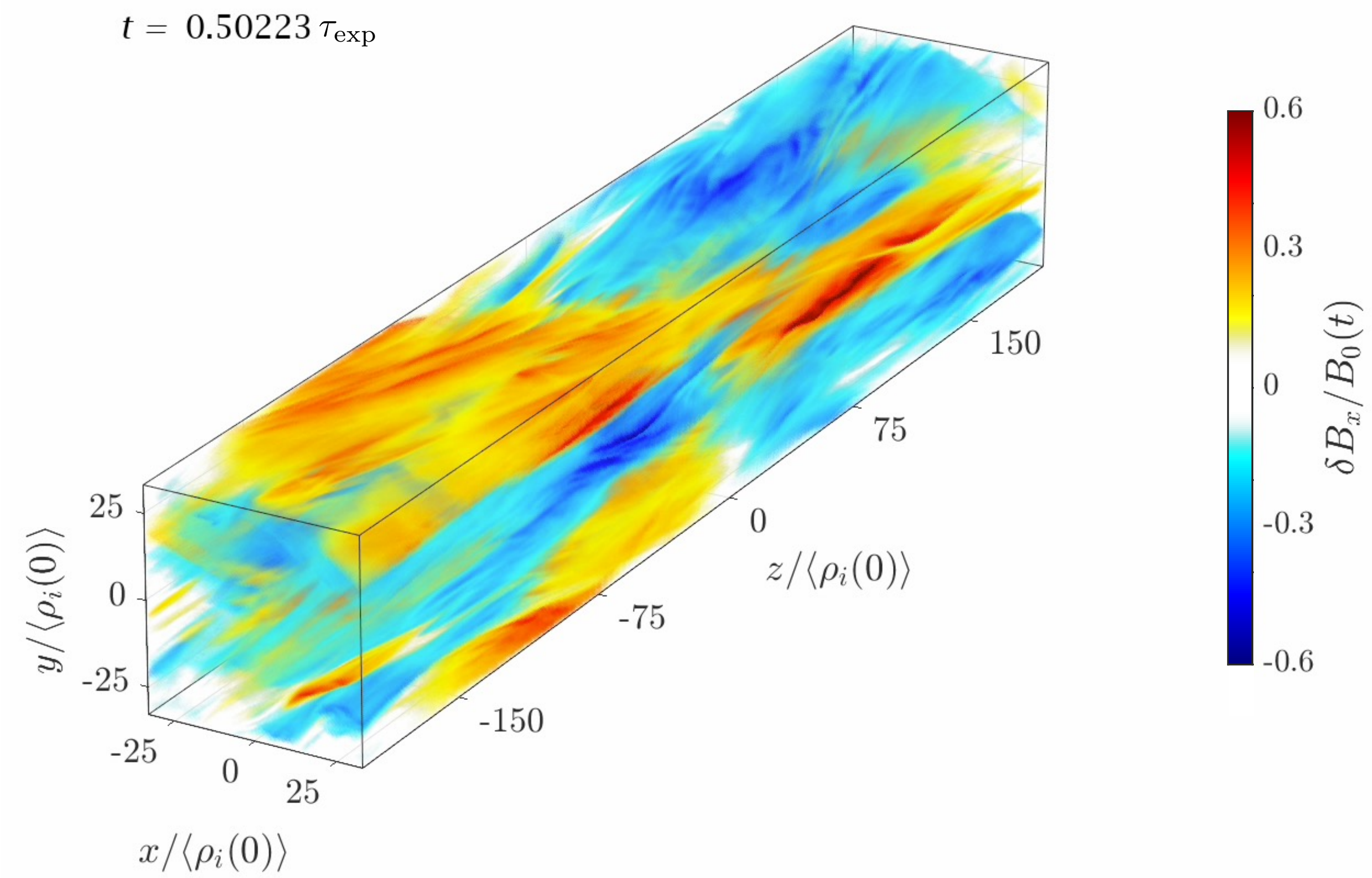

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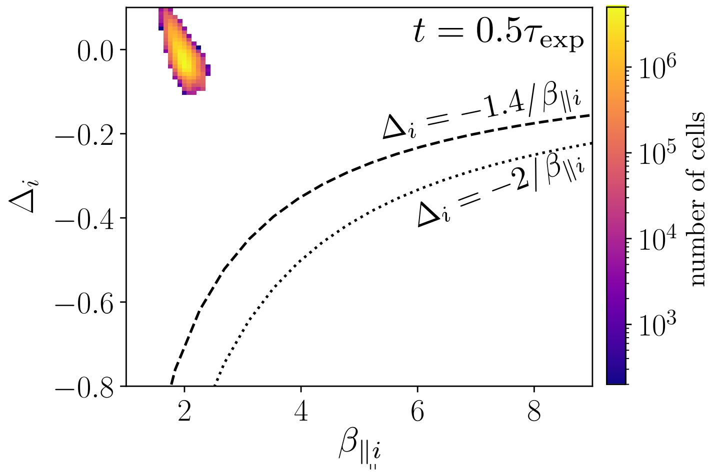

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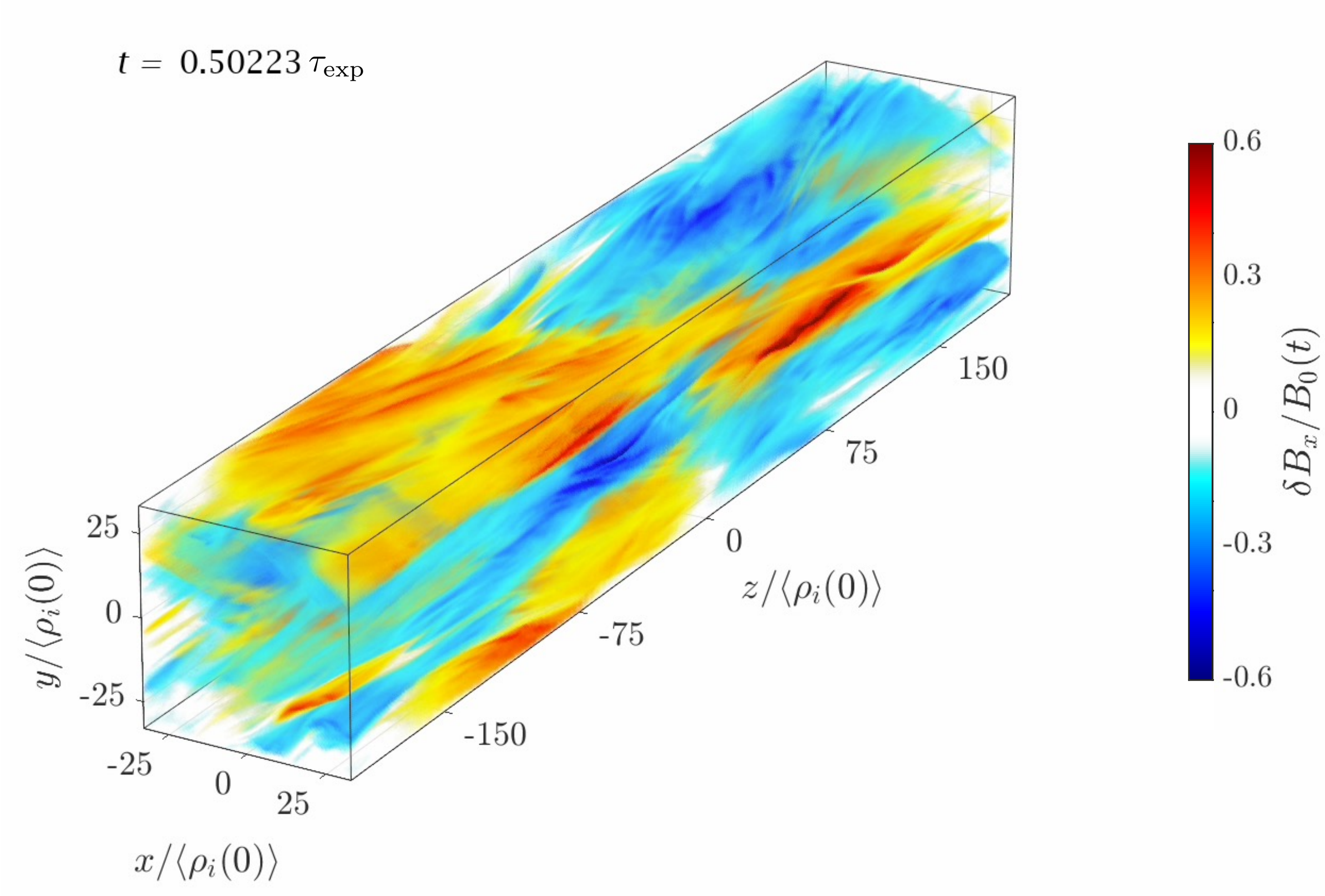

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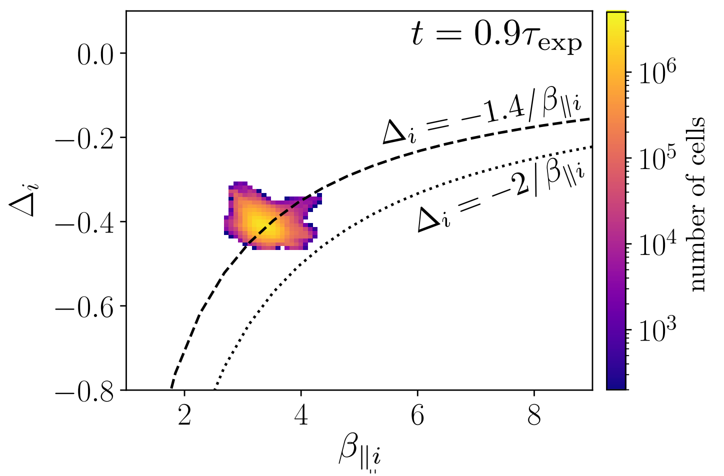

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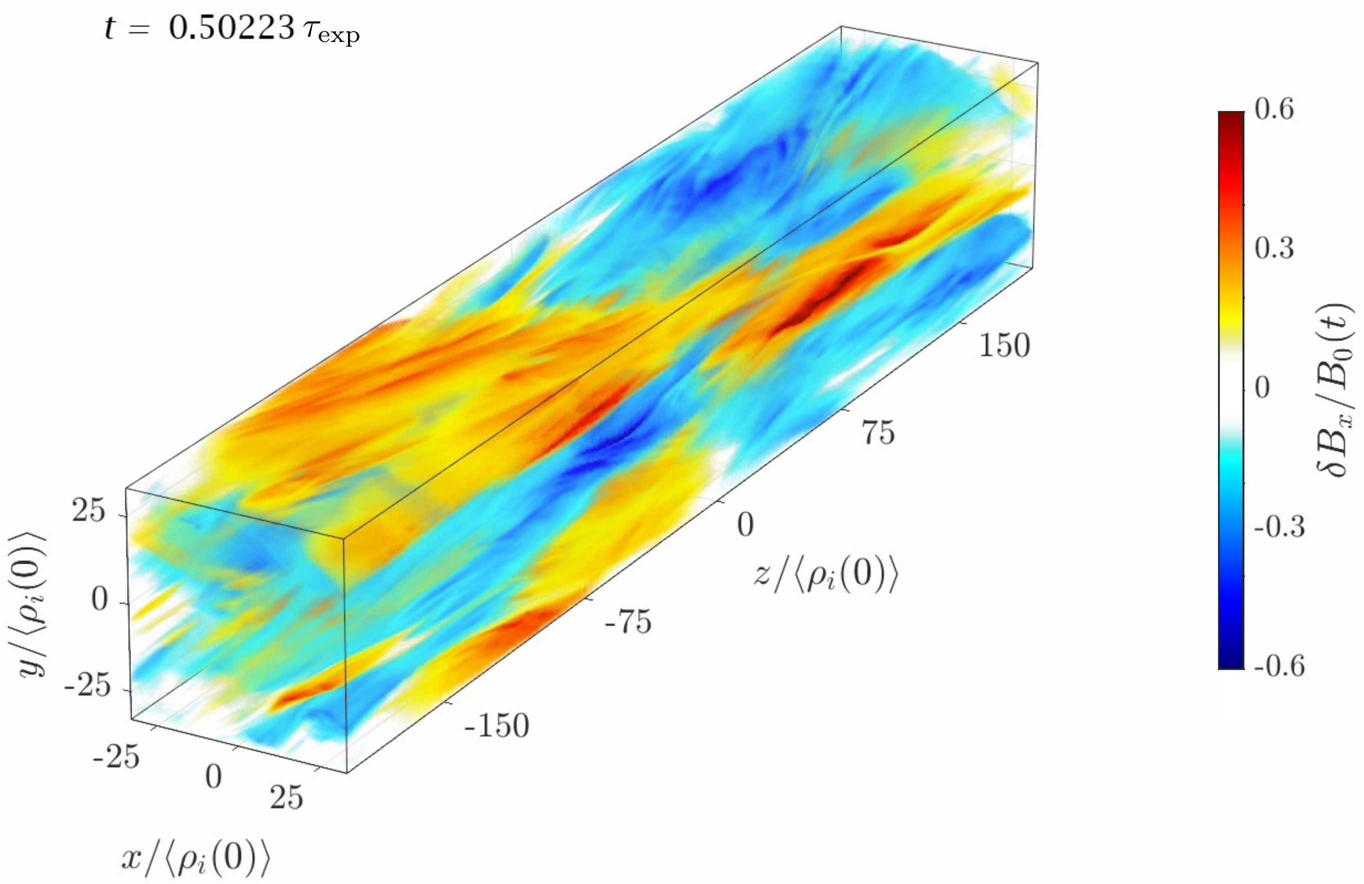

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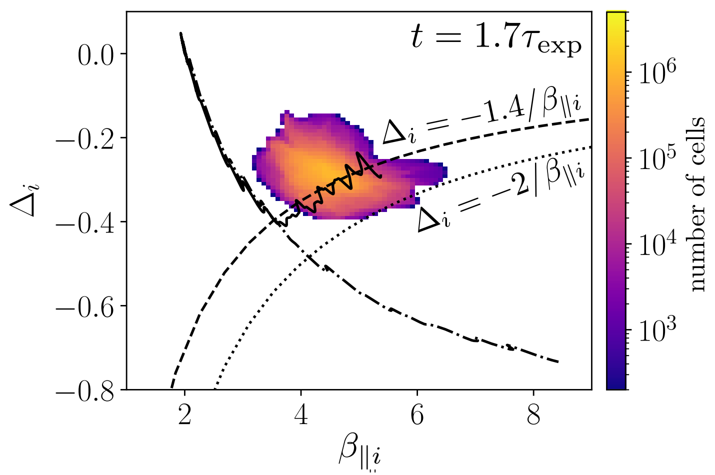

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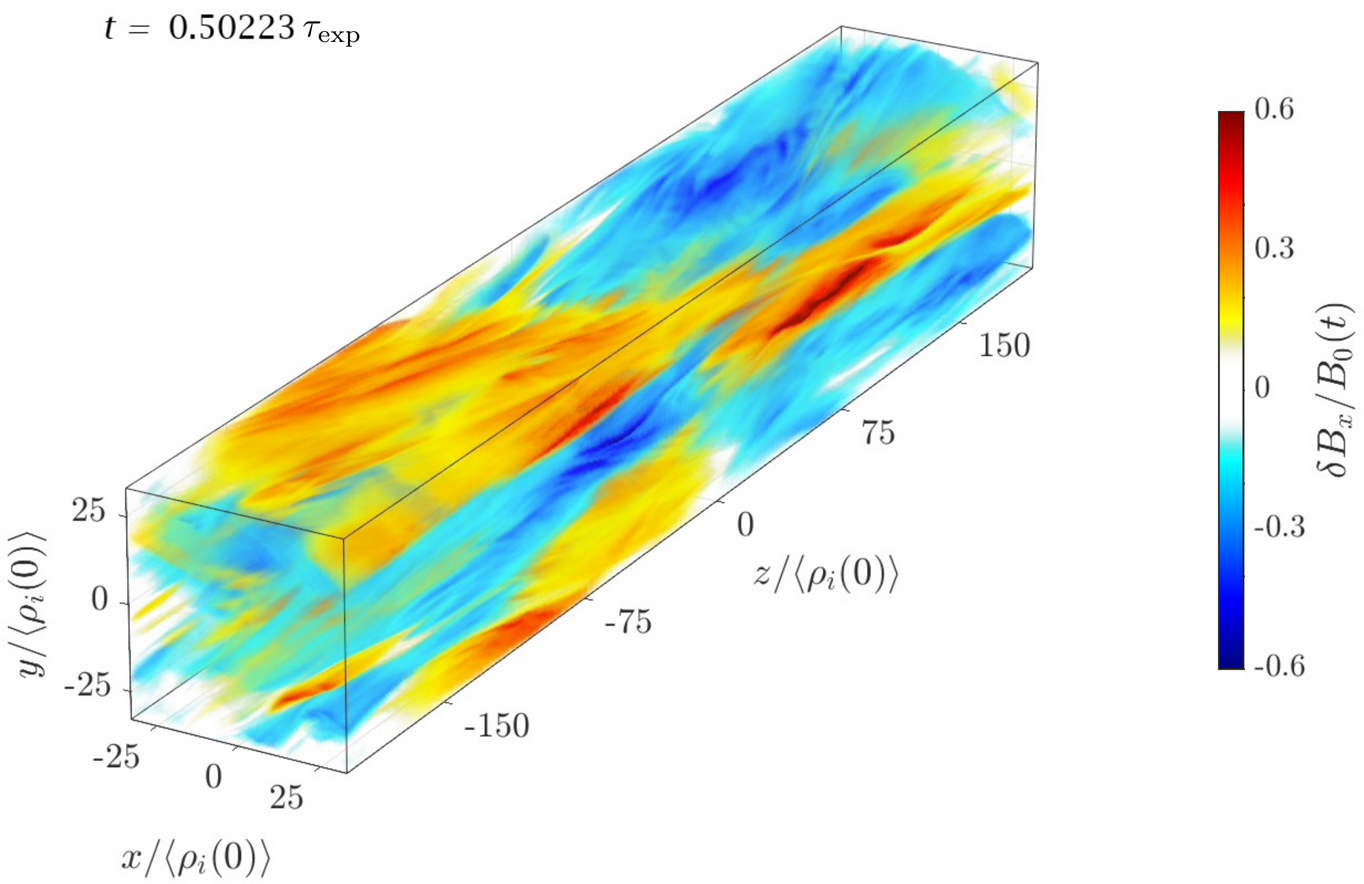

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- Alfvénic turbulence happily coexists with firehoses; critical balance maintained adaptively!

Conclusions

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1. Weakly collisional and collisionless magnetised $\beta \gtrsim 1$ plasmas often become susceptible to the firehose instability in the course of their macroscopic evolution, altering their basic physical properties.

2. Using analytic theory and hybrid-PIC simulations, we have established that, depending on the relative magnitude of the plasma β, the characteristic timescale τ of macroscopic evolution, and the ion Larmor frequency Ω_i, the saturation of the firehose instability in high-β plasma can result in two states that have qualitatively distinct thermodynamics and microphysics.

3. By contrast with the previously identified 'Alfvén-inhibiting' state, the newly identified `Alfvén-enabling' state, which is realised when $\tau > 30\Omega_i \beta^{3/2}$, can support Alfvén waves and Alfvénic turbulence because the magnetic tension associated with the plasma's macroscopic magnetic field is never completely negated by anisotropic pressure forces: $(\Delta_i)_{\rm sat} \approx -1.4\text{-}1.6/\beta > -2/\beta \implies v_{\rm A,eff} \approx 0.4 v_{\rm A}$.

4. The box-averaged collision frequency is $\nu_{\rm eff} \sim \beta_i/\tau$, in agreement with previous results, but certain sub-populations of particles experience collisions at a much greater (or smaller) rate depending on their velocity in the direction parallel to the magnetic field *(NB. Additional slides on effective firehose collision operator available on request!)*

5. The Alfvén-enabling state is the astrophysically relevant one!