Electromagnetic instabilities and plasma turbulence driven by electron-temperature gradient

T. G. Adkins[1], A. A. Schekochihin[1], P. G. Ivanov[1], and C. M. Roach[2]

[1] Rudolf Peierls Centre For Theoretical Physics,
University of Oxford, OXford, OX1 3PU, UK

[2] Culham Centre for Fusion Energy,
United Kingdom Atomic Energy Authority,
Abingdon, OX14 3EB, UK

13th Plasma Kinetics Working Meeting, Monday 01/08/2022

EUROfusion

This work has been carried out within the framework of the EUROfusion Consortium and has received funding from the Euratom research and training programme 2014-2018 and 2019-2020 under grant agreement No 633053. The views and opinions expressed herein do not necessarily reflect those of the European Commission.

Table of Contents

1. Motivation, orderings and equations
2. Linear instabilities: ETG and TAI
3. Turbulence and transport
4. Numerical results
5. Summary and future work

Table of Contents

1. Motivation, orderings and equations

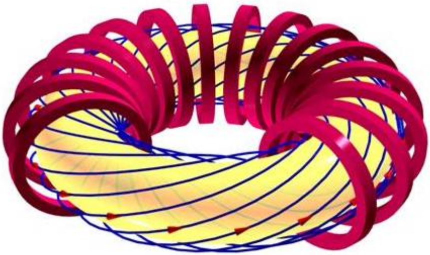
2. Linear instabilities: ETG and TAI

3. Turbulence and transport

4. Numerical results

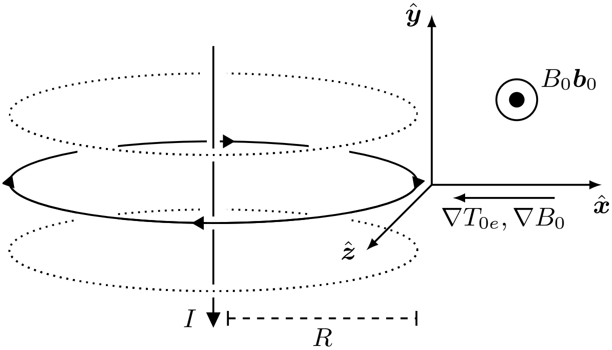
5. Summary and future work

Motivation



- Understanding (turbulent) heat transport in magnetically confined plasmas is crucial to the design of successful tokamak experiments
- Therefore, we need to determine the turbulent state of the plasma at saturation
- We focus on turbulence driven by the electron-temperature gradient (ETG), allowing for finite magnetic-field fluctuations within the context of a local slab model of a tokamak-like plasma
- We will discover a new instability in the electromagnetic regime, the so-called *thermo-Alfvénic instability* (TAI), whose physical mechanism hinges on the competition between ETG drive and rapid parallel streaming along perturbed magnetic-field lines

Local slab approximation



- Equilibrium gradients (constant throughout domain):

$$L_B^{-1} = -\frac{1}{B_0}\frac{\mathrm{d}B_0}{\mathrm{d}x}, \quad R^{-1} = -\left|\boldsymbol{b}_0 \cdot \nabla \boldsymbol{b}_0\right|, \quad L_T^{-1} = -\frac{1}{T_{0e}}\frac{\mathrm{d}T_{0e}}{\mathrm{d}x}.$$

Low-beta ordering

$$\omega \sim \underbrace{k_\parallel v_{\rm the}}_{\text{parallel streaming}} \sim \overbrace{\omega_{\rm KAW}}^{\text{kinetic Aflvén waves}} \sim \underbrace{\omega_{*e}}_{\rm ETG} \sim \overbrace{\omega_{de}}^{\text{Mag. drifts}} \sim \underbrace{k_\perp v_E}_{\boldsymbol{E}\times\boldsymbol{B}\text{ drifts}} \sim \overbrace{\nu_{ee} \sim \nu_{ei}}^{\text{collisions}},$$

- Lengthscales: flux-freezing scale is key

$$k_\perp d_e \sim 1, \quad k_\perp \rho_e \sim \sqrt{\beta_e}, \quad k_\parallel L \sim \sqrt{\beta_e}, \quad k_\parallel \lambda_{ei} \sim 1, \quad \frac{k_\parallel}{k_\perp} \sim \epsilon\sqrt{\beta_e},$$

- Amplitudes: effects of magnetic compression are negligible

$$\frac{e\phi}{T_{0e}} \sim \frac{\delta n_e}{n_{0e}} \sim \frac{\delta n_i}{n_{0i}} \sim \frac{\delta T_e}{T_{0e}} \sim \frac{\delta T_i}{T_{0i}} \sim \epsilon, \quad \frac{\delta \boldsymbol{B}_\perp}{B_0} \sim \epsilon\sqrt{\beta_e}, \quad \frac{\delta B_\parallel}{B_0} \sim \epsilon\beta_e.$$

- What follows is all derived in an asymptotic limit of gyrokinetics, for $\epsilon \ll \sqrt{\beta_e} \ll 1$.

Low-beta ordering

$$\omega \sim \underbrace{\kappa k_\parallel^2}_{\text{thermal conduction}} \sim \overbrace{\omega_{\text{KAW}}}^{\text{kinetic Aflvén waves}} \sim \underbrace{\omega_{*e}}_{\text{ETG}} \sim \overbrace{\omega_{de}}^{\text{Mag. drifts}} \sim \underbrace{k_\perp v_E}_{\boldsymbol{E}\times\boldsymbol{B}\text{ drifts}} \ll \overbrace{\nu_{ee} \sim \nu_{ei}}^{\text{collisions}},$$

- Lengthscales ($\chi^{-1} = \sqrt{\beta_e}\lambda_{ei}/L$):

$$k_\perp d_e \sim \chi^{-1}, \quad k_\perp \rho_e \sim \chi^{-1}\sqrt{\beta_e}, \quad k_\parallel L \sim \sqrt{\beta_e}, \quad k_\parallel \lambda_{ei} \sim \chi^{-1}, \quad \frac{k_\parallel}{k_\perp} \sim \chi\epsilon\sqrt{\beta_e},$$

- Amplitudes: effects of magnetic compression are negligible

$$\frac{e\phi}{T_{0e}} \sim \frac{\delta n_e}{n_{0e}} \sim \frac{\delta n_i}{n_{0i}} \sim \frac{\delta T_e}{T_{0e}} \sim \frac{\delta T_i}{T_{0i}} \sim \chi\epsilon, \quad \frac{\delta \boldsymbol{B}_\perp}{B_0} \sim \chi\epsilon\sqrt{\beta_e}, \quad \frac{\delta B_\parallel}{B_0} \sim \chi\epsilon\beta_e.$$

- Shall focus on the collisional limit, as the physics is more transparent. Important change is the replacement of parallel streaming with thermal conduction: $\kappa \sim v_{\text{the}}^2/\nu_{ei}$.

Collisional equations

- Implementing this ordering and taking the moments of the resultant hierarchy, we obtain the following fluid system:

$$
\begin{aligned}
&\frac{\mathrm{d}}{\mathrm{d}t}\frac{\delta n_e}{n_{0e}} + \nabla_\| u_{\|e} + \frac{\rho_e v_{\mathrm{the}}}{L_B}\frac{\partial}{\partial y}\left(\frac{\delta n_e}{n_{0e}} - \varphi + \frac{\delta T_e}{T_{0e}}\right) = 0,\\
&\frac{\mathrm{d}\mathcal{A}}{\mathrm{d}t} + \frac{v_{\mathrm{the}}}{2}\frac{\partial \varphi}{\partial z} = \frac{v_{\mathrm{the}}}{2}\nabla_\| \frac{\delta n_e}{n_{0e}} + \left(1+\frac{c_2}{c_1}\right)\frac{v_{\mathrm{the}}}{2}\nabla_\| \log T_e + \frac{\nu_{ei}}{c_1}\frac{u_{\|e}}{v_{\mathrm{the}}},\\
&\frac{\mathrm{d}}{\mathrm{d}t}\frac{\delta T_e}{T_{0e}} - \kappa \nabla_\|^2 \log T_e + \frac{2}{3}\left(1+\frac{c_2}{c_1}\right)\nabla_\| u_{\|e}\\
&\qquad + \frac{2}{3}\frac{\rho_e v_{\mathrm{the}}}{L_B}\frac{\partial}{\partial y}\left(\frac{\delta n_e}{n_{0e}} - \varphi + \frac{7}{2}\frac{\delta T_e}{T_{0e}}\right) + \frac{\rho_e v_{\mathrm{the}}}{2L_T}\frac{\partial \varphi}{\partial y} = 0,
\end{aligned}
$$

- Closed by quasineutrality and parallel Ampère's law:

$$
\frac{\delta n_e}{n_{0e}} = -\bar{\tau}^{-1}\varphi, \quad \varphi = \frac{e\phi}{T_{0e}}, \quad \bar{\tau} = \frac{\tau}{Z}, \quad \frac{u_{\|e}}{v_{\mathrm{the}}} = d_e^2 \nabla_\perp^2 \mathcal{A}, \quad \mathcal{A} = \frac{A_\|}{\rho_e B_0}.
$$

- The ion-charge-dependent constants c_1, c_2 and c_3 arise from the inversion of the Landau collision operator. For $Z = 1$, $c_1 \approx 1.95$, $c_2 \approx 1.39$, $c_3 \approx 3.16$.

Collisional equations

- Implementing this ordering and taking the moments of the resultant hierarchy, we obtain the following fluid system:

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\delta n_e}{n_{0e}} + \underbrace{\nabla_\parallel u_{\parallel e}}_{\text{parallel compressions}} + \underbrace{\frac{\rho_e v_{\mathrm{the}}}{L_B}\frac{\partial}{\partial y}\left(\frac{\delta n_e}{n_{0e}} - \varphi + \frac{\delta T_e}{T_{0e}}\right)}_{\text{magnetic drifts}} = 0,$$

$$\frac{\mathrm{d}\mathcal{A}}{\mathrm{d}t} + \frac{v_{\mathrm{the}}}{2}\frac{\partial \varphi}{\partial z} = \frac{v_{\mathrm{the}}}{2}\nabla_\parallel \frac{\delta n_e}{n_{0e}} + \left(1 + \frac{c_2}{c_1}\right)\frac{v_{\mathrm{the}}}{2}\nabla_\parallel \log T_e + \frac{\nu_{ei}}{c_1}\frac{u_{\parallel e}}{v_{\mathrm{the}}},$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\delta T_e}{T_{0e}} - \kappa \nabla_\parallel^2 \log T_e + \frac{2}{3}\left(1 + \frac{c_2}{c_1}\right)\nabla_\parallel u_{\parallel e}$$
$$+ \frac{2}{3}\frac{\rho_e v_{\mathrm{the}}}{L_B}\frac{\partial}{\partial y}\left(\frac{\delta n_e}{n_{0e}} - \varphi + \frac{7}{2}\frac{\delta T_e}{T_{0e}}\right) + \frac{\rho_e v_{\mathrm{the}}}{2 L_T}\frac{\partial \varphi}{\partial y} = 0,$$

- Closed by quasineutrality and parallel Ampère's law:

$$\frac{\delta n_e}{n_{0e}} = -\bar{\tau}^{-1}\varphi, \quad \varphi = \frac{e\phi}{T_{0e}}, \quad \bar{\tau} = \frac{\tau}{Z}, \quad \frac{u_{\parallel e}}{v_{\mathrm{the}}} = d_e^2 \nabla_\perp^2 \mathcal{A}, \quad \mathcal{A} = \frac{A_\parallel}{\rho_e B_0}.$$

- The ion-charge-dependent constants c_1, c_2 and c_3 arise from the inversion of the Landau collision operator. For $Z = 1$, $c_1 \approx 1.95$, $c_2 \approx 1.39$, $c_3 \approx 3.16$.

Collisional equations

- Implementing this ordering and taking the moments of the resultant hierarchy, we obtain the following fluid system:

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\delta n_e}{n_{0e}} + \nabla_\parallel u_{\parallel e} + \frac{\rho_e v_{\mathrm{th}e}}{L_B}\frac{\partial}{\partial y}\left(\frac{\delta n_e}{n_{0e}} - \varphi + \frac{\delta T_e}{T_{0e}}\right) = 0,$$

$$\underbrace{\frac{\mathrm{d}\mathcal{A}}{\mathrm{d}t} + \frac{v_{\mathrm{th}e}}{2}\frac{\partial \varphi}{\partial z}}_{\text{parallel electric field}} = \underbrace{\frac{v_{\mathrm{th}e}}{2}\nabla_\parallel \frac{\delta n_e}{n_{0e}} + \left(1 + \frac{c_2}{c_1}\right)\frac{v_{\mathrm{th}e}}{2}\nabla_\parallel \log T_e}_{\text{parallel pressure gradient}} + \underbrace{\frac{\nu_{ei}}{c_1}\frac{u_{\parallel e}}{v_{\mathrm{th}e}}}_{e\text{-}i \text{ friction}},$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\delta T_e}{T_{0e}} - \kappa \nabla_\parallel^2 \log T_e + \frac{2}{3}\left(1 + \frac{c_2}{c_1}\right)\nabla_\parallel u_{\parallel e}$$
$$+ \frac{2}{3}\frac{\rho_e v_{\mathrm{th}e}}{L_B}\frac{\partial}{\partial y}\left(\frac{\delta n_e}{n_{0e}} - \varphi + \frac{7}{2}\frac{\delta T_e}{T_{0e}}\right) + \frac{\rho_e v_{\mathrm{th}e}}{2L_T}\frac{\partial \varphi}{\partial y} = 0,$$

- Parallel gradient of the total temperature $T_e = T_{0e} + \delta T_e$:

$$\nabla_\parallel \log T_e = \nabla_\parallel \frac{\delta T_e}{T_{0e}} + \frac{\delta B_x}{B_0}\frac{1}{T_{0e}}\frac{\mathrm{d}T_{0e}}{\mathrm{d}x} = \nabla_\parallel \frac{\delta T_e}{T_{0e}} - \frac{\rho_e}{L_T}\frac{\partial \mathcal{A}}{\partial y}.$$

Collisional equations

- Implementing this ordering and taking the moments of the resultant hierarchy, we obtain the following fluid system:

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\delta n_e}{n_{0e}} + \nabla_\parallel u_{\parallel e} + \frac{\rho_e v_{\mathrm{th}e}}{L_B}\frac{\partial}{\partial y}\left(\frac{\delta n_e}{n_{0e}} - \varphi + \frac{\delta T_e}{T_{0e}}\right) = 0,$$

$$\frac{\mathrm{d}\mathcal{A}}{\mathrm{d}t} + \frac{v_{\mathrm{th}e}}{2}\frac{\partial \varphi}{\partial z} = \frac{v_{\mathrm{th}e}}{2}\nabla_\parallel \frac{\delta n_e}{n_{0e}} + \left(1 + \frac{c_2}{c_1}\right)\frac{v_{\mathrm{th}e}}{2}\nabla_\parallel \log T_e + \frac{\nu_{ei}}{c_1}\frac{u_{\parallel e}}{v_{\mathrm{th}e}},$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\delta T_e}{T_{0e}} - \underbrace{\kappa \nabla_\parallel^2 \log T_e}_{\text{thermal conduction}} + \underbrace{\frac{2}{3}\left(1 + \frac{c_2}{c_1}\right)\nabla_\parallel u_{\parallel e}}_{\text{compressional heating}}$$
$$+ \underbrace{\frac{2}{3}\frac{\rho_e v_{\mathrm{th}e}}{L_B}\frac{\partial}{\partial y}\left(\frac{\delta n_e}{n_{0e}} - \varphi + \frac{7}{2}\frac{\delta T_e}{T_{0e}}\right)}_{\text{magnetic drifts}} + \underbrace{\frac{\rho_e v_{\mathrm{th}e}}{2L_T}\frac{\partial \varphi}{\partial y}}_{\boldsymbol{E}\times\boldsymbol{B}\ \text{injection}} = 0, \quad \kappa = \frac{c_3 v_{\mathrm{th}e}^2}{3\nu_{ei}}$$

- Parallel gradient of the total temperature $T_e = T_{0e} + \delta T_e$:

$$\nabla_\parallel \log T_e = \nabla_\parallel \frac{\delta T_e}{T_{0e}} + \frac{\delta B_x}{B_0}\frac{1}{T_{0e}}\frac{\mathrm{d}T_{0e}}{\mathrm{d}x} = \nabla_\parallel \frac{\delta T_e}{T_{0e}} - \frac{\rho_e}{L_T}\frac{\partial \mathcal{A}}{\partial y}.$$

Collisional equations

- Implementing this ordering and taking the moments of the resultant hierarchy, we obtain the following fluid system:

$$
\begin{aligned}
&\frac{\mathrm{d}}{\mathrm{d}t}\frac{\delta n_e}{n_{0e}} + \nabla_\| u_{\|e} + \frac{\rho_e v_{\mathrm{the}}}{L_B}\frac{\partial}{\partial y}\left(\frac{\delta n_e}{n_{0e}} - \varphi + \frac{\delta T_e}{T_{0e}}\right) = 0, \\
&\frac{\mathrm{d}\mathcal{A}}{\mathrm{d}t} + \frac{v_{\mathrm{the}}}{2}\frac{\partial \varphi}{\partial z} = \frac{v_{\mathrm{the}}}{2}\nabla_\| \frac{\delta n_e}{n_{0e}} + \left(1 + \frac{c_2}{c_1}\right)\frac{v_{\mathrm{the}}}{2}\nabla_\| \log T_e + \frac{\nu_{ei}}{c_1}\frac{u_{\|e}}{v_{\mathrm{the}}}, \\
&\frac{\mathrm{d}}{\mathrm{d}t}\frac{\delta T_e}{T_{0e}} - \kappa \nabla_\|^2 \log T_e + \frac{2}{3}\left(1 + \frac{c_2}{c_1}\right)\nabla_\| u_{\|e} \\
&\qquad + \frac{2}{3}\frac{\rho_e v_{\mathrm{the}}}{L_B}\frac{\partial}{\partial y}\left(\frac{\delta n_e}{n_{0e}} - \varphi + \frac{7}{2}\frac{\delta T_e}{T_{0e}}\right) + \frac{\rho_e v_{\mathrm{the}}}{2L_T}\frac{\partial \varphi}{\partial y} = 0,
\end{aligned}
$$

- These form a closed, three-field system describing the dynamics of electrons in the presence of a temperature gradient and magnetic drifts.
- Support a range of temperature-gradient-driven instabilities, both electrostatic and electromagnetic, distinguished by whether the magnetic field lines are frozen into the electron flow.
- In what follows, we shall neglect to write the magnetic drifts in the temperature equation as they will never turn out to be physically important.

Table of Contents

1. Motivation, orderings and equations

2. Linear instabilities: ETG and TAI

3. Turbulence and transport

4. Numerical results

5. Summary and future work

Electrostatic regime

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\delta n_e}{n_{0e}} + \nabla_\parallel u_{\parallel e} + \frac{\rho_e v_{\mathrm{the}}}{L_B}\frac{\partial}{\partial y}\left(\frac{\delta n_e}{n_{0e}} - \varphi + \frac{\delta T_e}{T_{0e}}\right) = 0,$$

$$\frac{\mathrm{d}\mathcal{A}}{\mathrm{d}t} + \frac{v_{\mathrm{the}}}{2}\frac{\partial \varphi}{\partial z} = \frac{v_{\mathrm{the}}}{2}\nabla_\parallel \frac{\delta n_e}{n_{0e}} + \left(1 + \frac{c_2}{c_1}\right)\frac{v_{\mathrm{the}}}{2}\nabla_\parallel \log T_e + \frac{\nu_{ei}}{c_1}\frac{u_{\parallel e}}{v_{\mathrm{the}}},$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\delta T_e}{T_{0e}} - \kappa \nabla_\parallel^2 \log T_e + \frac{2}{3}\left(1 + \frac{c_2}{c_1}\right)\nabla_\parallel u_{\parallel e} + \frac{\rho_e v_{\mathrm{the}}}{2 L_T}\frac{\partial \varphi}{\partial y} = 0,$$

- Below the resistive (flux-freezing) scale ($\chi^{-1} = \sqrt{\beta_e}\lambda_{ei}/L$),

$$k_\perp d_e \chi \gg 1,$$

  $\Rightarrow$ magnetic field lines are no longer frozen into the electron velocity.
- Ignore inductive part of parallel electric field.
- Parallel gradient of total temperature becomes:

$$\nabla_\parallel \log T_e = \nabla_\parallel \frac{\delta T_e}{T_{0e}}.$$

  $\Rightarrow$ no contributions due to finite magnetic field perturbations

Electrostatic regime

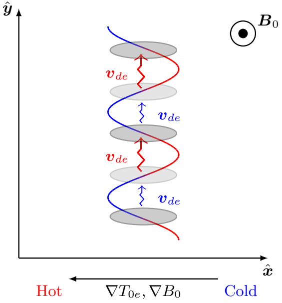
$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\delta n_e}{n_{0e}}+\nabla_\parallel u_{\parallel e}+\frac{\rho_e v_{\mathrm{th}e}}{L_B}\frac{\partial}{\partial y}\left(\frac{\delta n_e}{n_{0e}}-\varphi+\frac{\delta T_e}{T_{0e}}\right)=0,$$

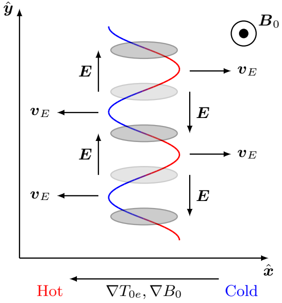
$$\frac{\mathrm{d}\mathcal{A}}{\mathrm{d}t}+\frac{v_{\mathrm{th}e}}{2}\frac{\partial\varphi}{\partial z}=\frac{v_{\mathrm{th}e}}{2}\nabla_\parallel\frac{\delta n_e}{n_{0e}}+\left(1+\frac{c_2}{c_1}\right)\frac{v_{\mathrm{th}e}}{2}\nabla_\parallel\log T_e+\frac{\nu_{ei}}{c_1}\frac{u_{\parallel e}}{v_{\mathrm{th}e}},$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\delta T_e}{T_{0e}}-\kappa\nabla_\parallel^2\log T_e+\frac{2}{3}\left(1+\frac{c_2}{c_1}\right)\nabla_\parallel u_{\parallel e}+\frac{\rho_e v_{\mathrm{th}e}}{2L_T}\frac{\partial\varphi}{\partial y}=0,$$

- Curvature-mediated ETG (2D interchange mode)

$$\kappa k_\parallel^2 \ll \omega_{de} \ll \omega \ll \omega_{*e}, \quad \Rightarrow \quad \omega=\pm i\left(2\omega_{de}\omega_{*e}\bar{\tau}\right)^{1/2}.$$





Electrostatic regime

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\delta n_e}{n_{0e}} + \nabla_\parallel u_{\parallel e} + \frac{\rho_e v_{\mathrm{the}}}{L_B}\frac{\partial}{\partial y}\left(\frac{\delta n_e}{n_{0e}} - \varphi + \frac{\delta T_e}{T_{0e}}\right) = 0,$$

$$\frac{\mathrm{d}\mathcal{A}}{\mathrm{d}t} + \frac{v_{\mathrm{the}}}{2}\frac{\partial\varphi}{\partial z} = \frac{v_{\mathrm{the}}}{2}\nabla_\parallel\frac{\delta n_e}{n_{0e}} + \left(1+\frac{c_2}{c_1}\right)\frac{v_{\mathrm{the}}}{2}\nabla_\parallel \log T_e + \frac{\nu_{ei}}{c_1}\frac{u_{\parallel e}}{v_{\mathrm{the}}} = 0,$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\delta T_e}{T_{0e}} - \kappa\nabla_\parallel^2 \log T_e + \frac{2}{3}\left(1+\frac{c_2}{c_1}\right)\nabla_\parallel u_{\parallel e} + \frac{\rho_e v_{\mathrm{the}}}{2L_T}\frac{\partial\varphi}{\partial y} = 0,$$

- Collisional slab ETG:

$$\omega_{de} \ll \kappa k_\parallel^2 \ll \omega \ll \omega_{*e}, \quad \Rightarrow \quad \omega = \pm\frac{1 - i\,\mathrm{sgn}(k_y)}{\sqrt{2}}\left(\frac{k_\parallel^2 v_{\mathrm{the}}^2|\omega_{*e}|\bar{\tau}}{2(c_1+c_2)\nu_{ei}}\right)^{1/2}.$$

- Equivalent of the usual (collisionless) slab ETG:

$$\omega_{de} \ll k_\parallel v_{\mathrm{the}} \ll \omega \ll \omega_{*e}, \quad \Rightarrow \quad \omega = \mathrm{sgn}(k_y)\left(-1, \frac{1}{2} \pm i\frac{\sqrt{3}}{2}\right)\left(\frac{k_\parallel^2 v_{\mathrm{the}}^2|\omega_{*e}|\bar{\tau}}{2}\right)^{1/3}$$

- Similar feedback mechanism (see Cowley 1991), though we shall not focus on that here.

Electromagnetic regime

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\delta n_e}{n_{0e}} + \nabla_\parallel u_{\parallel e} + \frac{\rho_e v_{\mathrm{th}e}}{L_B}\frac{\partial}{\partial y}\left(\frac{\delta n_e}{n_{0e}} - \varphi + \frac{\delta T_e}{T_{0e}}\right) = 0,$$

$$\frac{\mathrm{d}\mathcal{A}}{\mathrm{d}t} + \frac{v_{\mathrm{th}e}}{2}\frac{\partial \varphi}{\partial z} = \frac{v_{\mathrm{th}e}}{2}\nabla_\parallel \frac{\delta n_e}{n_{0e}} + \left(1 + \frac{c_2}{c_1}\right)\frac{v_{\mathrm{th}e}}{2}\nabla_\parallel \log T_e + \frac{\nu_{ei}}{c_1}\frac{u_{\parallel e}}{v_{\mathrm{th}e}},$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\delta T_e}{T_{0e}} - \kappa \nabla_\parallel^2 \log T_e + \frac{2}{3}\left(1 + \frac{c_2}{c_1}\right)\nabla_\parallel u_{\parallel e} + \frac{\rho_e v_{\mathrm{th}e}}{2L_T}\frac{\partial \varphi}{\partial y} = 0,$$

- Above the resistive (flux-freezing) scale ($\chi^{-1} = \sqrt{\beta_e}\lambda_{ei}/L$):

$$k_\perp d_e \chi \ll 1,$$

  $\Rightarrow \delta \boldsymbol{B}_\perp$ is created as electrons move across field lines and drag them along.
- Restore the inductive term and ignore resistive term.
- Magnetic field contributions to parallel gradients:

$$\nabla_\parallel \log T_e = \nabla_\parallel \frac{\delta T_e}{T_{0e}} + \frac{\delta B_x}{B_0}\frac{1}{T_{0e}}\frac{\mathrm{d}T_{0e}}{\mathrm{d}x} = \nabla_\parallel \frac{\delta T_e}{T_{0e}} - \frac{\rho_e}{L_T}\frac{\partial \mathcal{A}}{\partial y}.$$

  $\Rightarrow$ introduces another mechanism by which the perturbations can go unstable due to L_T.

Electromagnetic regime

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\delta n_e}{n_{0e}} + \nabla_\parallel u_{\parallel e} + \frac{\rho_e v_{\mathrm{the}}}{L_B}\frac{\partial}{\partial y}\left(\frac{\delta n_e}{n_{0e}} - \varphi + \frac{\delta T_e}{T_{0e}}\right) = 0,$$

$$\frac{\mathrm{d}\mathcal{A}}{\mathrm{d}t} + \frac{v_{\mathrm{the}}}{2}\frac{\partial \varphi}{\partial z} = \frac{v_{\mathrm{the}}}{2}\nabla_\parallel \frac{\delta n_e}{n_{0e}} + \left(1 + \frac{c_2}{c_1}\right)\frac{v_{\mathrm{the}}}{2}\nabla_\parallel \log T_e + \frac{\nu_{ei}}{c_1}\frac{u_{\parallel e}}{v_{\mathrm{the}}},$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\delta T_e}{T_{0e}} - \kappa \nabla_\parallel^2 \log T_e + \frac{2}{3}\left(1 + \frac{c_2}{c_1}\right)\nabla_\parallel u_{\parallel e} + \frac{\rho_e v_{\mathrm{the}}}{2L_T}\frac{\partial \varphi}{\partial y} = 0,$$

- Dominant thermal conduction = isothermal along perturbed field line:

$$\nabla_\parallel \log T_e = \nabla_\parallel \frac{\delta T_e}{T_{0e}} - \frac{\rho_e}{L_T}\frac{\partial \mathcal{A}}{\partial y} = 0,$$

- Curvature-mediated *thermo-Alfvénic instability* (cTAI):

$$\omega = \pm i \left[2\omega_{de}\omega_{*e}(1 + \bar{\tau})\right]^{1/2}.$$

  $\Rightarrow$ physically different, new instability.

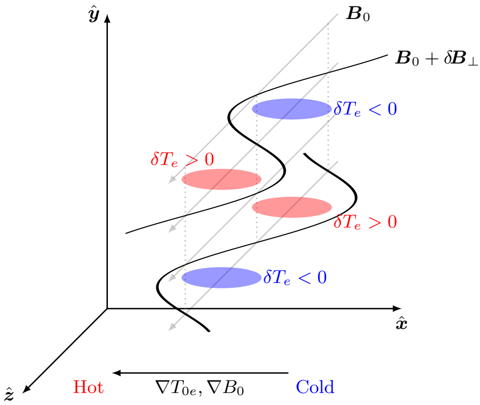
- Two key differences to cETG: (i) it relies on $k_\parallel \neq 0$, and (ii) it does not require the $\boldsymbol{E} \times \boldsymbol{B}$ feedback mechanism to be unstable.

Electromagnetic regime

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\delta n_e}{n_{0e}} + \nabla_\parallel u_{\parallel e} + \frac{\rho_e v_{\mathrm{th}e}}{L_B}\frac{\partial}{\partial y}\left(\frac{\delta n_e}{n_{0e}} - \varphi + \frac{\delta T_e}{T_{0e}}\right) = 0,$$

$$\frac{\mathrm{d}\mathcal{A}}{\mathrm{d}t} + \frac{v_{\mathrm{th}e}}{2}\frac{\partial \varphi}{\partial z} = \frac{v_{\mathrm{th}e}}{2}\nabla_\parallel \frac{\delta n_e}{n_{0e}} + \left(1 + \frac{c_2}{c_1}\right)\frac{v_{\mathrm{th}e}}{2}\nabla_\parallel \log T_e + \frac{\nu_{ei}}{c_1}\frac{u_{\parallel e}}{v_{\mathrm{th}e}},$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\delta T_e}{T_{0e}} - \kappa \nabla_\parallel^2 \log T_e + \frac{2}{3}\left(1 + \frac{c_2}{c_1}\right)\nabla_\parallel u_{\parallel e} + \frac{\rho_e v_{\mathrm{th}e}}{2L_T}\frac{\partial \varphi}{\partial y} = 0,$$



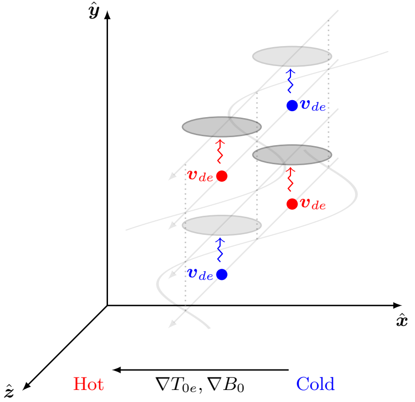
- A perturbation $\delta B_x = B_0 \rho_e \partial_y \mathcal{A}$ sets up a variation of total temp. along the perturbed field line as it makes excursions into hot and cold regions.
- Rapid thermal conduction along field lines creates a temperature perturbation that compensates for this.

Electromagnetic regime

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\delta n_e}{n_{0e}} + \nabla_\parallel u_{\parallel e} + \frac{\rho_e v_{\mathrm{the}}}{L_B}\frac{\partial}{\partial y}\left(\frac{\delta n_e}{n_{0e}} - \varphi + \frac{\delta T_e}{T_{0e}}\right) = 0,$$

$$\frac{\mathrm{d}\mathcal{A}}{\mathrm{d}t} + \frac{v_{\mathrm{the}}}{2}\frac{\partial \varphi}{\partial z} = \frac{v_{\mathrm{the}}}{2}\nabla_\parallel \frac{\delta n_e}{n_{0e}} + \left(1 + \frac{c_2}{c_1}\right)\frac{v_{\mathrm{the}}}{2}\nabla_\parallel \log T_e + \frac{\nu_{ei}}{c_1}\frac{u_{\parallel e}}{v_{\mathrm{the}}},$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\delta T_e}{T_{0e}} - \kappa \nabla_\parallel^2 \log T_e + \frac{2}{3}\left(1 + \frac{c_2}{c_1}\right)\nabla_\parallel u_{\parallel e} + \frac{\rho_e v_{\mathrm{the}}}{2L_T}\frac{\partial \varphi}{\partial y} = 0,$$



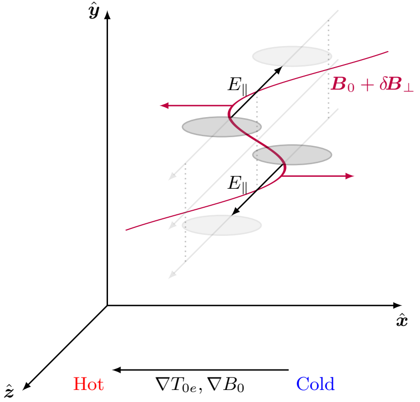
- Velocity dependence of magnetic drifts $\boldsymbol{v}_{de}$ creates an electron density perturbation (hot particles drift faster than cold ones).
- This electron density perturbation has both $k_y \neq 0$ and $k_\parallel \neq 0$.

Electromagnetic regime

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\delta n_e}{n_{0e}} + \nabla_\parallel u_{\parallel e} + \frac{\rho_e v_{\mathrm{th}e}}{L_B}\frac{\partial}{\partial y}\left(\frac{\delta n_e}{n_{0e}} - \varphi + \frac{\delta T_e}{T_{0e}}\right) = 0,$$

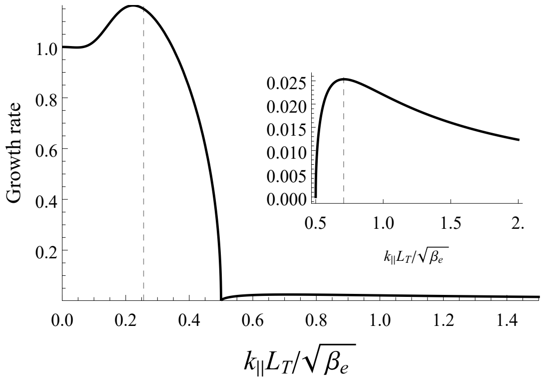
$$\frac{\mathrm{d}\mathcal{A}}{\mathrm{d}t} + \frac{v_{\mathrm{th}e}}{2}\frac{\partial \varphi}{\partial z} = \frac{v_{\mathrm{th}e}}{2}\nabla_\parallel \frac{\delta n_e}{n_{0e}} + \left(1 + \frac{c_2}{c_1}\right)\frac{v_{\mathrm{th}e}}{2}\nabla_\parallel \log T_e + \frac{\nu_{ei}}{c_1}\frac{u_{\parallel e}}{v_{\mathrm{th}e}},$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\delta T_e}{T_{0e}} - \kappa \nabla_\parallel^2 \log T_e + \frac{2}{3}\left(1 + \frac{c_2}{c_1}\right)\nabla_\parallel u_{\parallel e} + \frac{\rho_e v_{\mathrm{th}e}}{2L_T}\frac{\partial \varphi}{\partial y} = 0,$$



- The parallel density gradient must be balanced by the parallel electric field.
- Inductive part leads to an increase in δB_x, deforming the field line further into the hot and cold regions $\Rightarrow$ feedback.
- NB: δT_e and δB_x are frozen into different flows in this limit.

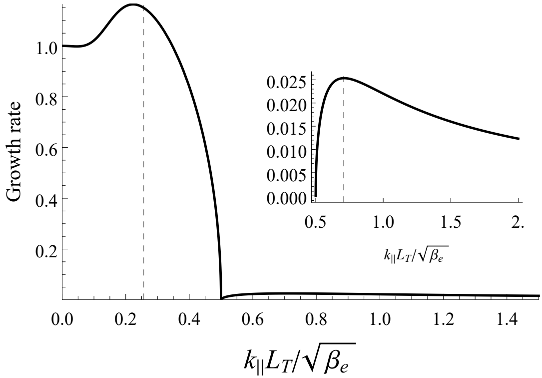
Electromagnetic regime



- Clear enhancement of cETG at finite $k_{\|}$:

$$\frac{k_{\|\max}L_T}{\sqrt{\beta_e}} = \left[\frac{81}{50}\frac{3+4\bar{\tau}}{(1+\bar{\tau})^2}\frac{L_T}{L_B}\right]^{1/6}\left(\frac{k_y^2 d_e}{k_\perp}\chi\right)^{1/3}.$$

Electromagnetic regime

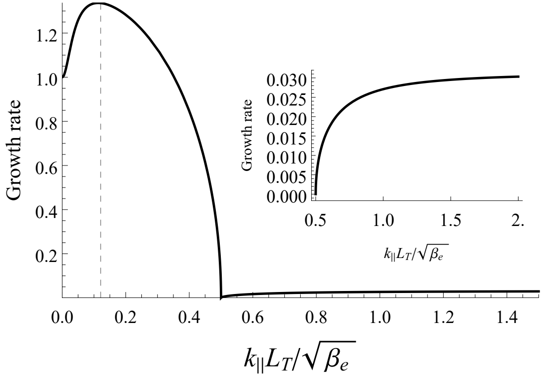


- For $\omega_{*e} \sim \kappa k_{\|}^2$, general TAI dispersion relation:

$$\omega^2 = -\left(2\omega_{de}\omega_{*e} - \omega_{\mathrm{KAW}}^2\right)\left(\bar{\tau} + \frac{1}{1+i\xi_*}\right), \quad \xi_* = \frac{\omega_{*e}}{\kappa k_{\|}^2}.$$

- Unstable even in the absence of any magnetic field gradient $\Rightarrow$ instability of KAWs, wherein the perturbed temperature gradient disrupts the KAW restoring force.

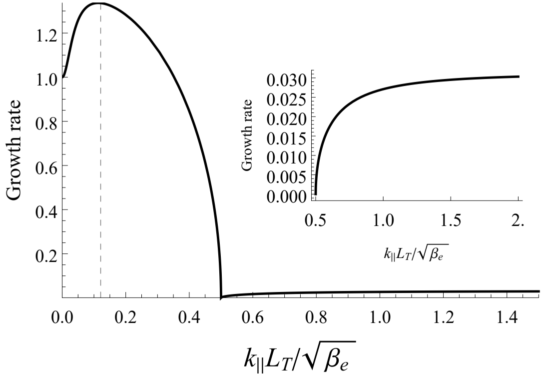
Electromagnetic regime



- For $\omega_{*e} \sim k_{\|} v_{\mathrm{the}}$, general TAI dispersion relation:

$$\omega^2 = -\left(2\omega_{de}\omega_{*e} - \omega_{\mathrm{KAW}}^2\right)\left(\bar{\tau} + \frac{1}{1+i\xi_*}\right), \quad \xi_* = \frac{\sqrt{\pi}}{2}\frac{\omega_{*e}}{k_{\|} v_{\mathrm{the}}}.$$

Electromagnetic regime



- Clear enhancement of cETG at finite $k_{\|}$:

$$\frac{k_{\|\max} L_T}{\sqrt{\beta_e}} = \left[\frac{\pi}{64}\frac{3+4\bar{\tau}}{(1+\bar{\tau})^2}\frac{L_T}{L_B}\right]^{1/4}\left(\frac{k_y^2 d_e}{k_\perp}\right)^{1/2}.$$

Electromagnetic regime

- The growth rate $\gamma = \mathrm{Im}(\omega)$ and the (real) frequency $\omega_r = \mathrm{Re}(\omega)$ of the growing TAI modes can be written as

$$\gamma^2 = \left|2\omega_{de}\omega_{*e} - \omega_{\mathrm{KAW}}^2\right| \bar{\tau}\, f_+(\xi_*), \quad \omega_r^2 = \left|2\omega_{de}\omega_{*e} - \omega_{\mathrm{KAW}}^2\right| \bar{\tau}\, f_-(\xi_*),$$

where

$$f_\pm(\xi_*) = \frac{1}{2\bar{\tau}} \left[\sqrt{\left(\bar{\tau} + \frac{1}{1+\xi_*^2}\right)^2 + \frac{\xi_*^2}{(1+\xi_*^2)^2}} \pm \mathrm{sgn}\left(2\omega_{de}\omega_{*e} - \omega_{\mathrm{KAW}}^2\right)\left(\bar{\tau} + \frac{1}{1+\xi_*^2}\right)\right],$$

and

$$\xi_* = \frac{\sqrt{\pi}}{2}\frac{\omega_{*e}}{k_\parallel v_{\mathrm{the}}}, \quad \xi_* = \frac{\omega_{*e}}{\kappa k_\parallel^2}.$$

- Both the growth rate and frequency vanish at a particular value of $k_\parallel$, given by

$$\frac{k_\parallel L_T}{\sqrt{\beta_e}} = \left(\frac{L_T}{L_B}\right)^{1/2} \frac{k_y}{k_\perp}.$$

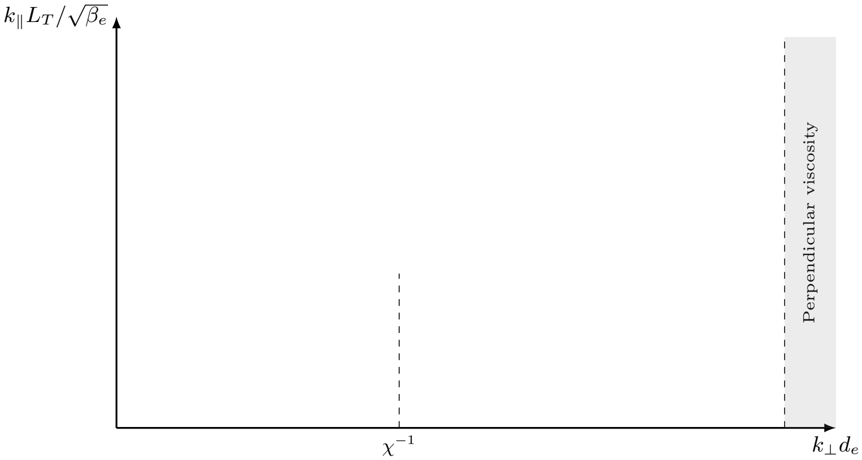
Wavenumber space portrait

- We want to map out the location of our instabilities and waves within our $\{k_\parallel, k_\perp\}$ wavenumber space.

Wavenumber space portrait

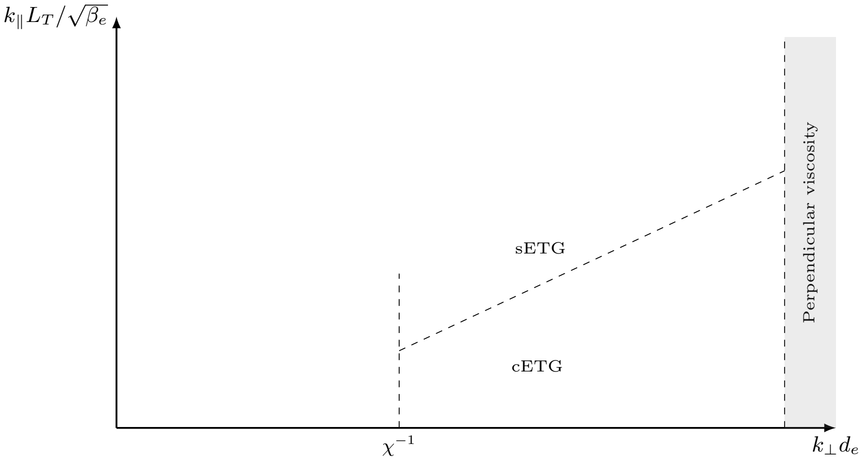
- Our instabilities and waves will be damped at ultraviolet wavenumbers by perpendicular viscosity.

Wavenumber space portrait



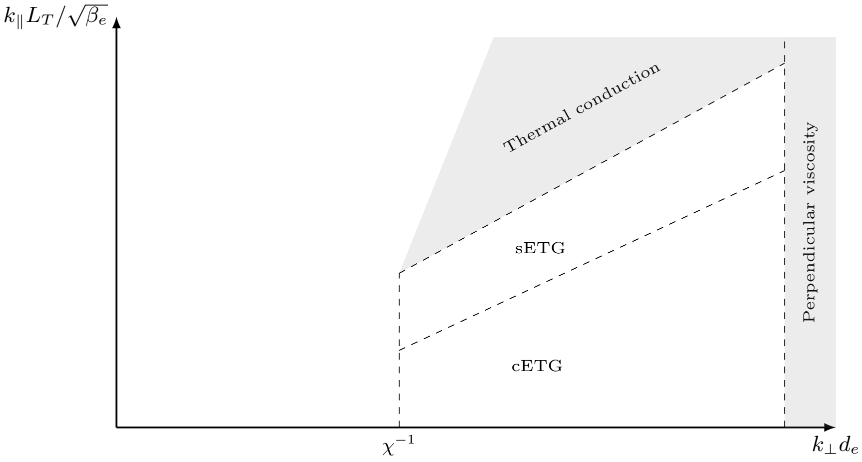
- The flux-freezing scale $k_\perp d_e \sim \chi^{-1}$ demarcates the transition between the electrostatic and electromagnetic regimes.

Wavenumber space portrait



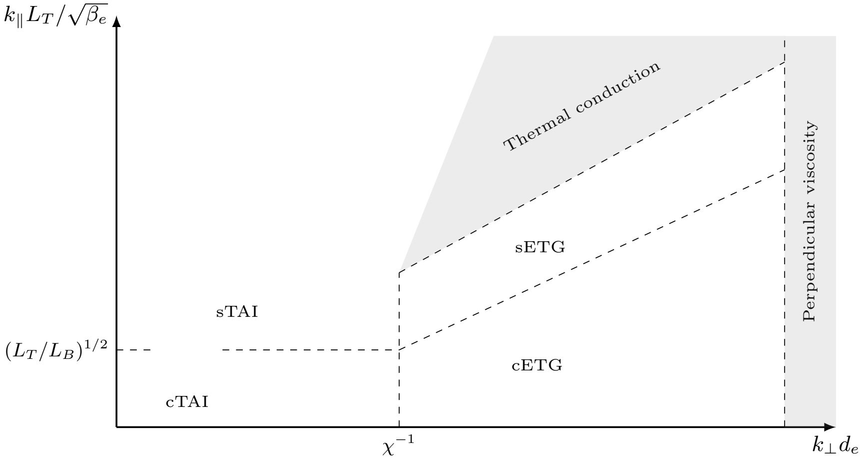
- On electrostatic scales, we have the cETG at lower parallel wavenumbers, which gradually transitions into the sETG at higher ones.

Wavenumber space portrait



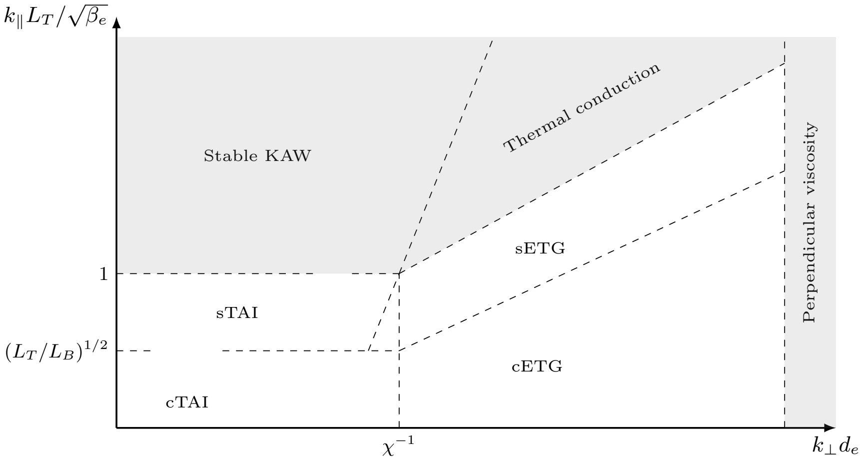
- The sETG is cut off at higher parallel wavenumbers by the effects of rapid thermal conduction that damps the temperature perturbation.

Wavenumber space portrait



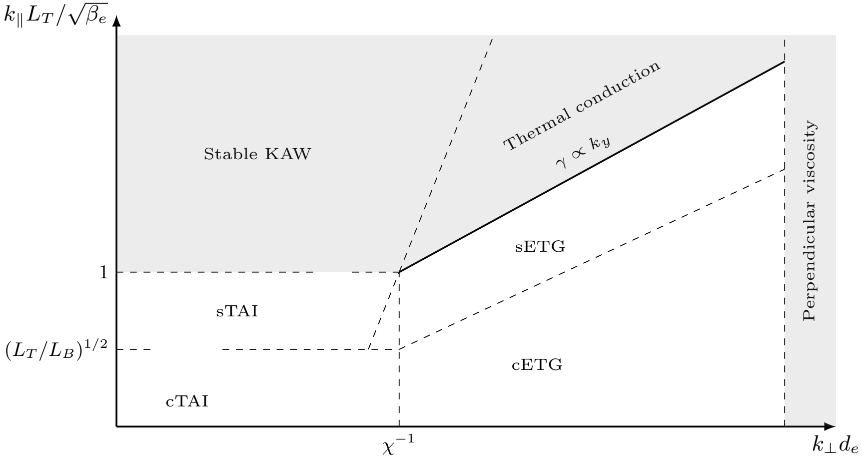
- On electromagnetic scales, we have the cTAI and sTAI, separated by the line $k_\parallel = k_{\parallel c}$.

Wavenumber space portrait



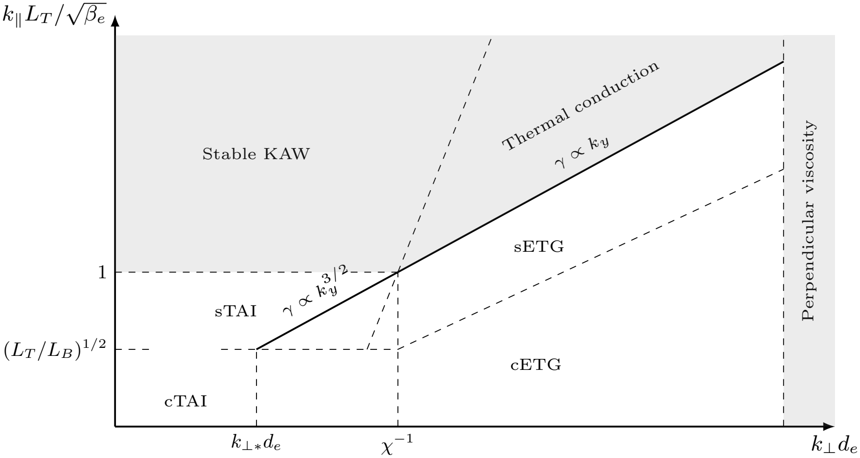
- The sTAI is cut off at higher parallel wavenumbers where the effects of parallel compression start to compete with the drive.

Wavenumber space portrait



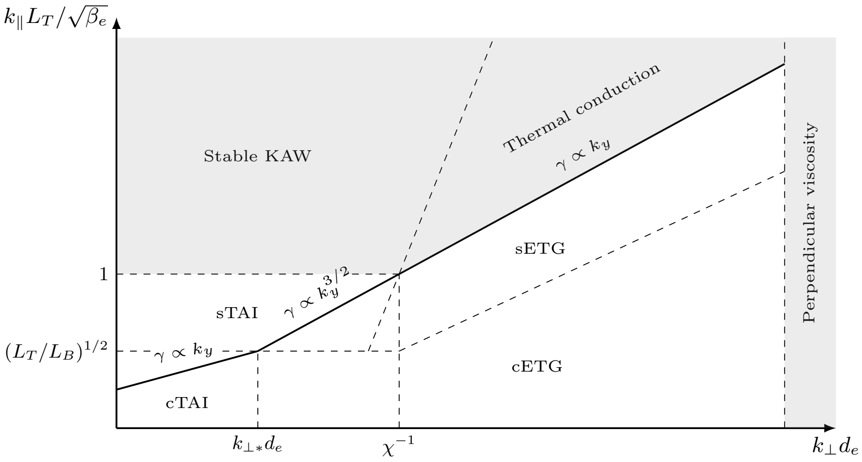
- The solid black line indicates the location of the maximum growth rate at each fixed $k_\perp$.

Wavenumber space portrait



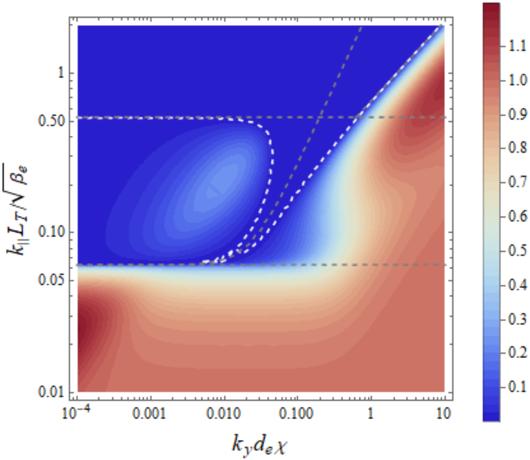
- The solid black line indicates the location of the maximum growth rate at each fixed $k_\perp$.

Wavenumber space portrait



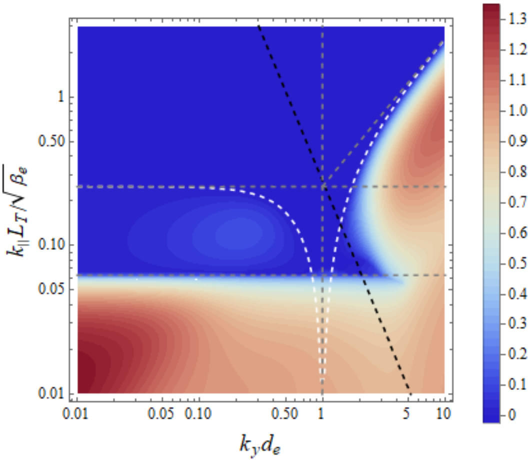
- The solid black line indicates the location of the maximum growth rate at each fixed $k_\perp$.

Wavenumber space portrait



Collisional, $L_B/L_T = 250$, $k_x d_e = 0$

Wavenumber space portrait



Collisionless, $L_B/L_T = 250$, $k_x d_e = 0$

Table of Contents

1. Motivation, orderings and equations

2. Linear instabilities: ETG and TAI

3. Turbulence and transport

4. Numerical results

5. Summary and future work

Free energy

- The system nonlinearly conserves the free energy

$$\frac{W}{n_{0e}T_{0e}} = \int \frac{\mathrm{d}^3\boldsymbol{r}}{V} \left[\frac{\varphi\bar{\tau}^{-1}\varphi}{2} + |d_e\nabla_\perp\mathcal{A}|^2 + \frac{1}{2}\frac{\delta n_e^2}{n_e^2} + \frac{3}{4}\frac{\delta T_e^2}{T_e^2}\right],$$

which is injected through equilibrium gradients and dissipated by collisions, viz.

$$\frac{1}{n_{0e}T_{0e}}\frac{\mathrm{d}W}{\mathrm{d}t} = \varepsilon - D, \quad \varepsilon = \frac{1}{L_T}\int\frac{\mathrm{d}^3\boldsymbol{r}}{V}\left(\frac{3}{2}\frac{\delta T_e}{T_{0e}}v_{Ex} + \frac{\delta q_e}{n_{0e}T_{0e}}\frac{\delta B_x}{B_0}\right),$$

where

$$v_{Ex} = -\frac{\rho_e v_{\mathrm{the}}}{2}\frac{\partial\varphi}{\partial y}, \quad \frac{\delta B_x}{B_0} = \rho_e\frac{\partial\mathcal{A}}{\partial y}, \quad \frac{\delta q_e}{n_{0e}T_{0e}} = -\frac{3}{2}\kappa\nabla_\parallel \log T_e.$$

- In the usual way, we can use this to calculate the constant flux of energy in the putative inertial range, and thus the turbulent heat-flux resulting from the cascade.
- We shall consider the example of electrostatic, slab ETG driven turbulence.

Electrostatic sETG turbulence

- We assume that there is no anistropy in the perpendicular plane, viz., $k_x \sim k_y \sim k_\perp$.
- We take the nonlinear turnover time to be given by the $\boldsymbol{E} \times \boldsymbol{B}$ advection rate:

$$t_{\mathrm{nl}}^{-1} \sim k_\perp v_E \sim \rho_e v_{\mathrm{the}} k_\perp^2 \varphi \sim \Omega_e (k_\perp \rho_e)^2 \varphi.$$

- It shall be useful to define the energy spectrum:

$$E^\varphi(k_z, k_x, k_y) = \frac{1}{2\bar{\tau}} \left(1 + \frac{1}{\bar{\tau}}\right) \left\langle |\varphi_{\boldsymbol{k}}|^2 \right\rangle_t .$$

- A Kolmogorov-style constant-flux argument leads to the scaling of the amplitudes in the inertial range:

$$\frac{\bar{\tau}^{-1}\varphi^2}{t_{\mathrm{nl}}} \sim \varepsilon = \mathrm{const} \quad \Rightarrow \quad \varphi \sim \left(\frac{\varepsilon}{\Omega_e}\right)^{1/3} (k_\perp \rho_e)^{-2/3} .$$

This scaling translates into the familiar perpendicular spectrum:

$$\boxed{\langle E^\varphi(k_z, k_\perp, \theta)\rangle_{k_z,\theta} \sim \frac{\varphi^2}{k_\perp} \propto k_\perp^{-7/3}}.$$

Electrostatic sETG turbulence

- We conjecture that in the inertial range, the following critical balance holds:

$$t_{\text{nl}}^{-1} \sim \omega \sim \frac{(k_\parallel v_{\text{the}})^2}{\nu_{ei}} \quad \Rightarrow \quad \boxed{k_\parallel \propto k_\perp^{2/3}}.$$

- At the outer scale, we have the balance between injection and nonlinearity

$$\Omega_e (k_\perp^o \rho_e)^2 \varphi^o \sim \frac{(k_\parallel^o v_{\text{the}})^2}{\nu_{ei}} \sim \omega_{*e}^o \quad \Rightarrow \quad \varphi^o \sim (k_\perp^o L_T)^{-1}, \quad \frac{k_\parallel^o L_T}{\sqrt{\beta_e}} \sim \left(k_y^o d_e \chi\right)^{1/2},$$

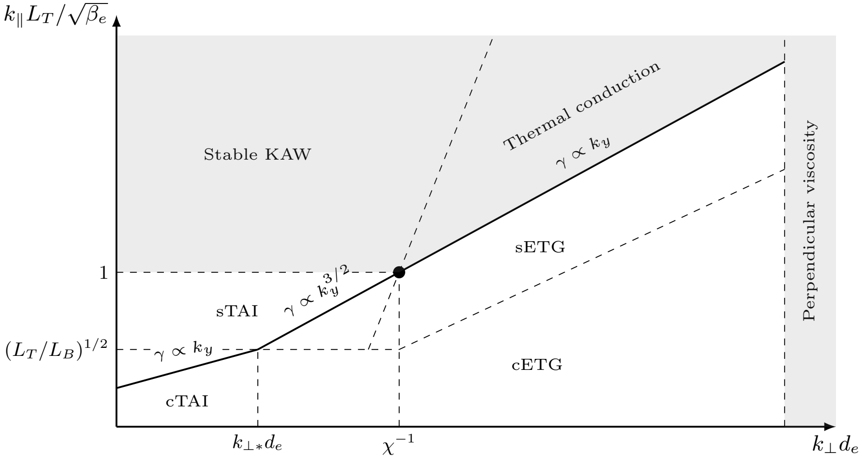
  and the energy injection is given by

$$\varepsilon \sim \omega_{*e}^o \varphi^o \frac{\delta T_e^o}{T_{0e}} \sim \frac{v_{\text{the}} \rho_e^2}{L_T^3 \sqrt{\beta_e}} (k_\perp^o d_e)^{-1}.$$

- We fix the relationship between the parallel and perpendicular outer-scale by conjecturing that $k_\parallel^o \sim L_z^{-1}$.
- Putting these results together, we obtain the following scaling of the heat-flux, for $Q_{\text{gB}} = n_{0e} T_{0e} v_{\text{the}} \left(\rho_e / L_B\right)^2$,

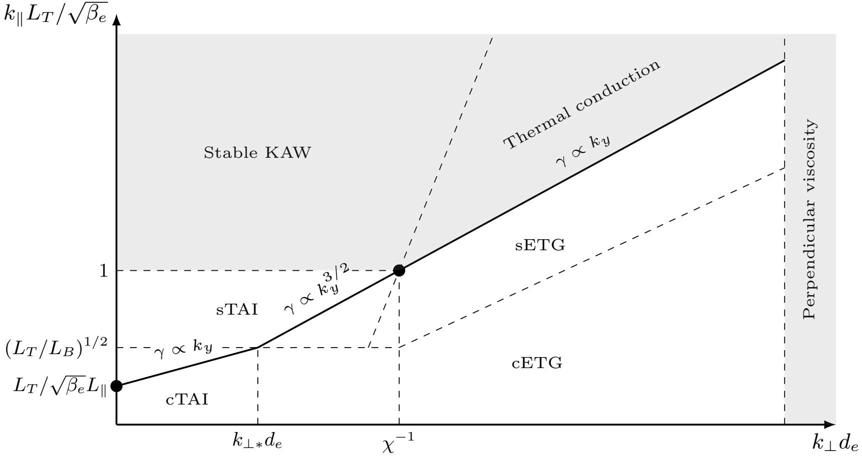
$$\boxed{Q_\nu^{\text{sETG}} \sim n_{0e} T_{0e} \varepsilon L_T \sim Q_{\text{gB}} \left(\frac{L_\parallel}{L_B}\right)^2 \left(\frac{L_B}{\lambda_{ei}}\right) \left(\frac{L_B}{L_T}\right)^3}.$$

heat-flux scalings in other regimes



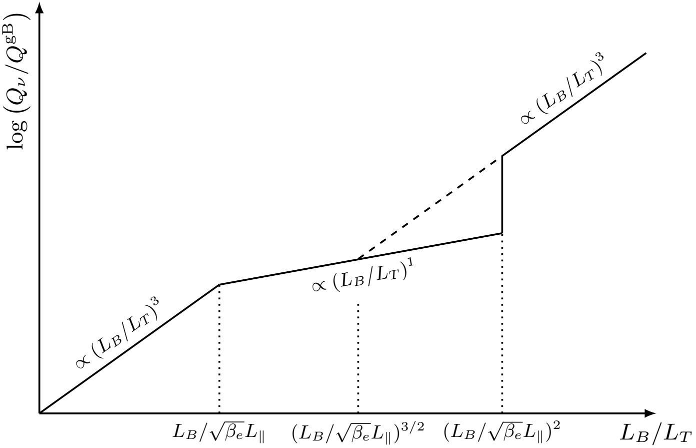
- The assumption that $k_\parallel^o \sim L_z^{-1}$ will no longer be satisfied when the outer scale reaches the flux-freezing scale, as it is here that the sETG is stabilised ⇒ change of regime

heat-flux scalings in other regimes



- Further scaling arguments show that the sTAI does not actually contribute to the cascade, meaning that the outer scale moves to even smaller parallel scales, eventually encountering the cTAI (which peaks at a specific parallel wavenumber).

heat-flux scalings in other regimes



- This implies that the effect of increasing β_e (or increasing $L_\parallel/L_B \sim \pi q$, as in the tokamak edge) is first to make the electron heat transport less stiff, as flux freezing pins down the ETG injection scale, and then to stiffen in back again, as the cTAI takes over.

Table of Contents

1. Motivation, orderings and equations

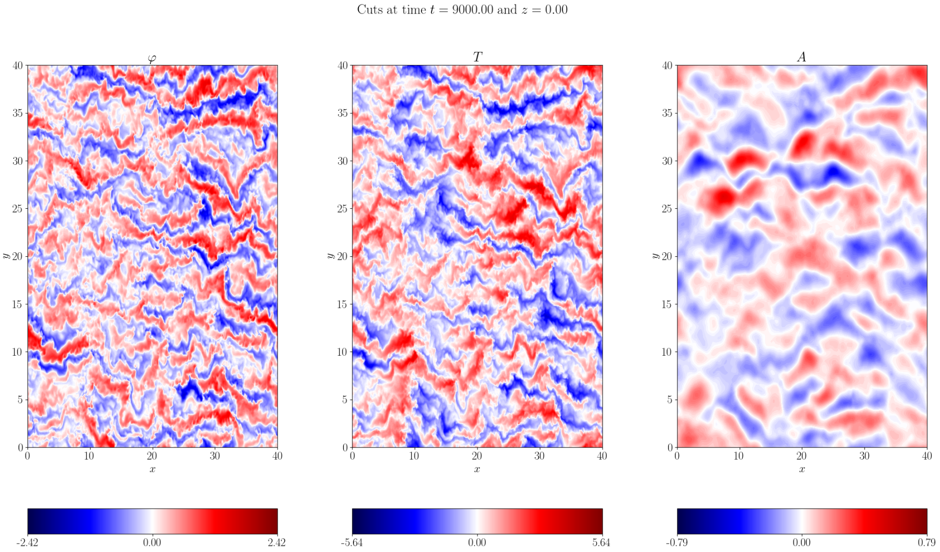
2. Linear instabilities: ETG and TAI

3. Turbulence and transport

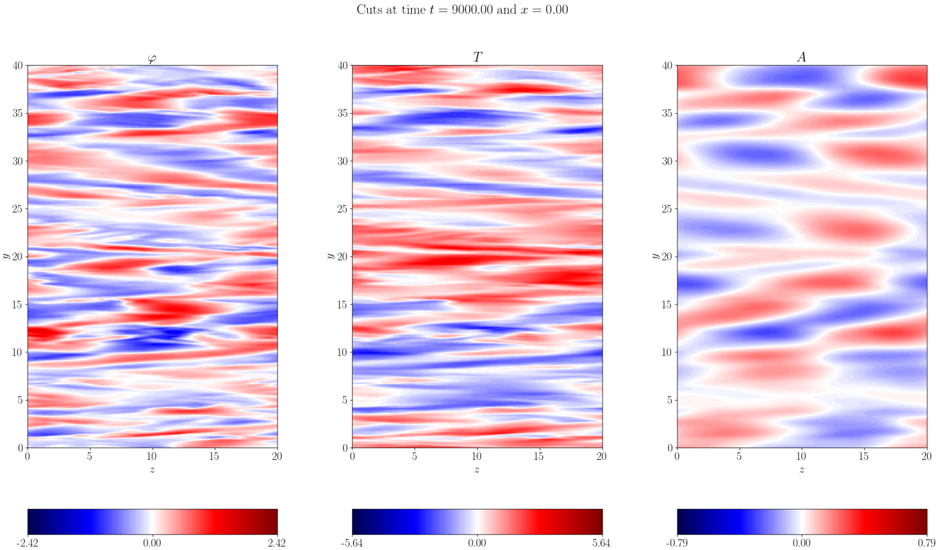
4. Numerical results

5. Summary and future work

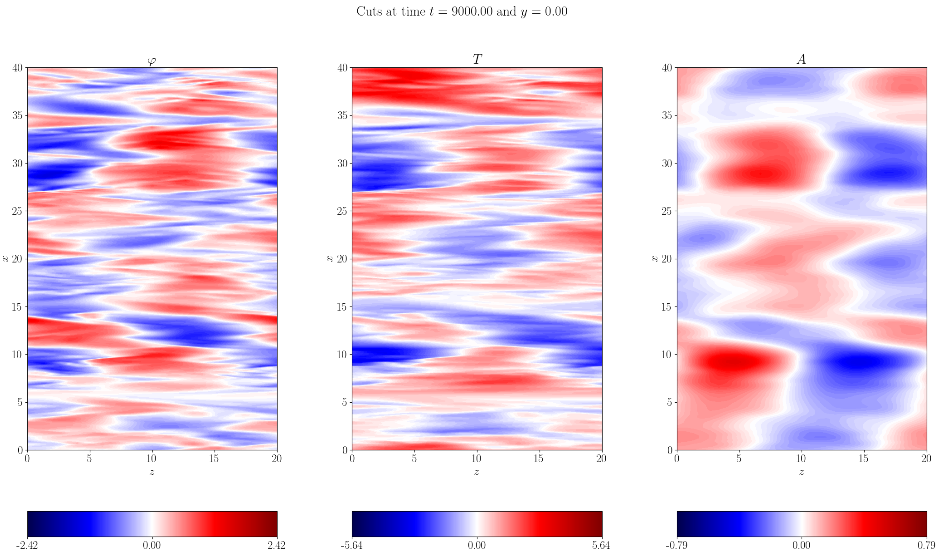
Real-space data



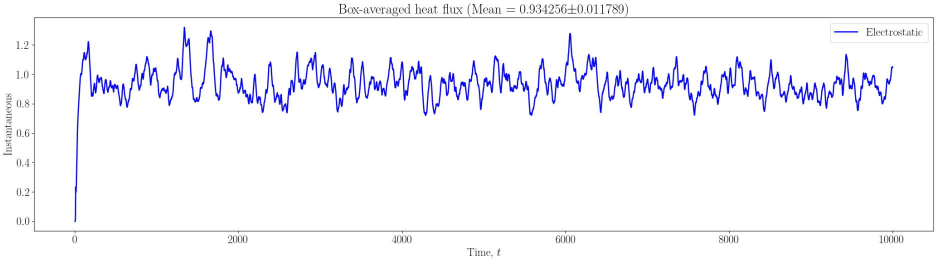
Real-space data



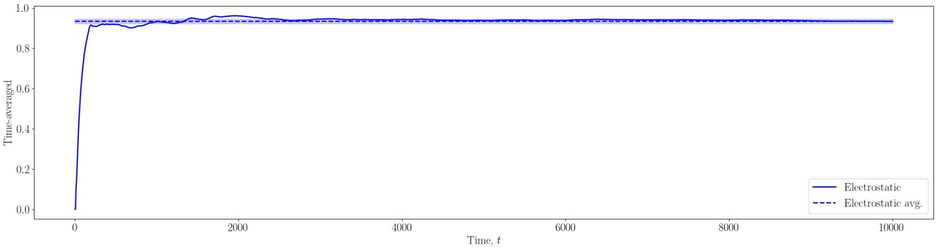
Real-space data



heat-flux

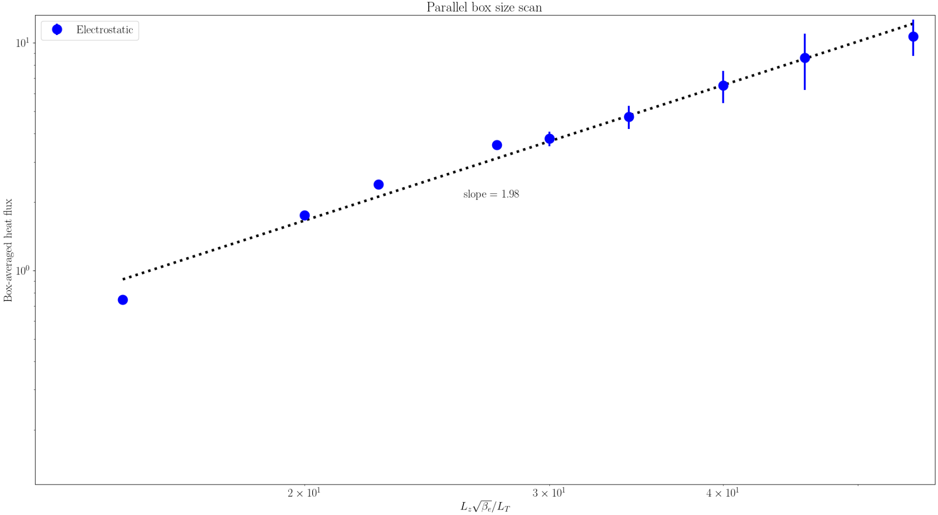


Box-averaged heat flux (Mean = 0.934256±0.011789)
Electrostatic
Instantaneous
1.2
1.0
0.8
0.6
0.4
0.2
0.0
0
2000
4000
6000
8000
10000
Time, t

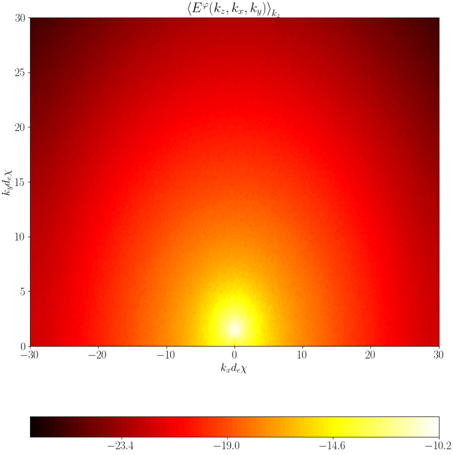


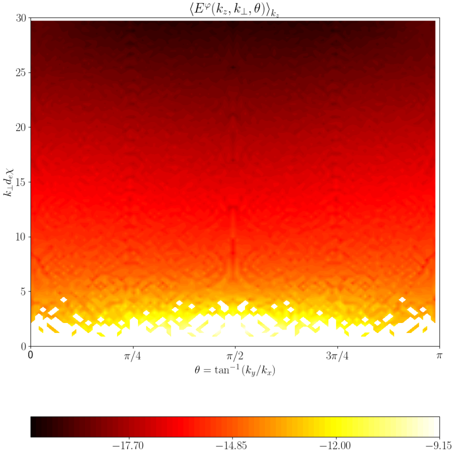
1.0
0.8
0.6
0.4
0.2
0.0
Time-averaged
Electrostatic
Electrostatic avg.
0
2000
4000
6000
8000
10000
Time, t

Scaling of time-averaged heat-flux with L_T

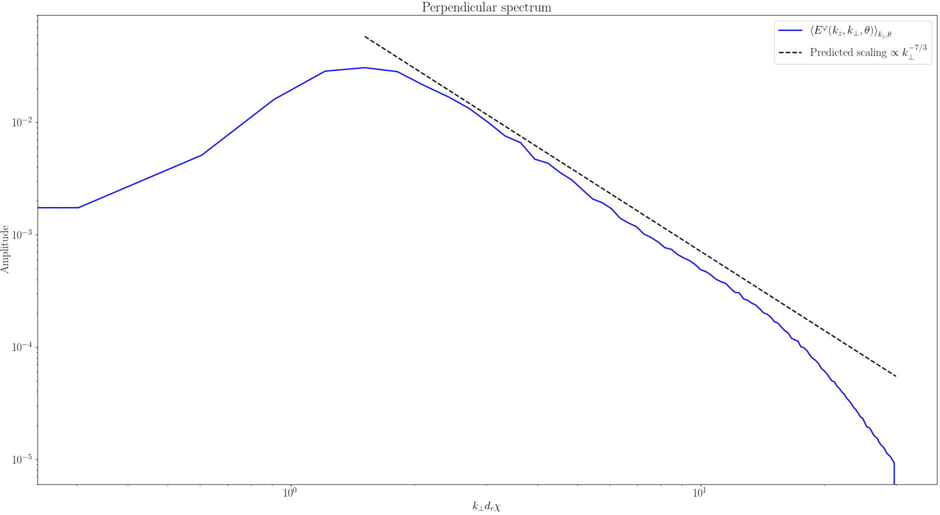


Perpendicular isotropy





Perpendicular spectrum



Critical balance

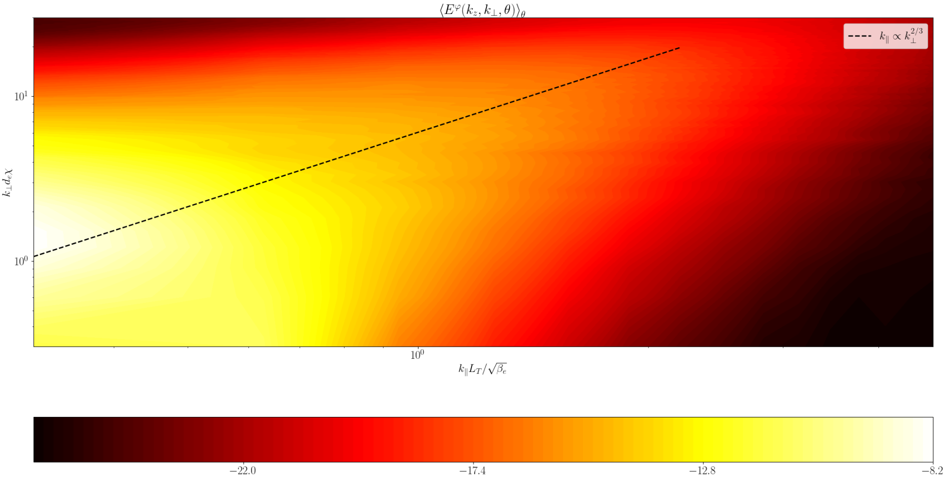


Table of Contents

1. Motivation, orderings and equations

2. Linear instabilities: ETG and TAI

3. Turbulence and transport

4. Numerical results

5. Summary and future work

Summary and future work

- We considered the turbulent state of a low-beta, magnetised plasma allowing electromagnetic perturbations and a variation of the equilibrium magnetic field
- In the electromagnetic regime, it was shown that the system supports the *thermo-Alfvénic instability (TAI)* that extracts free energy from the equilibrium temperature gradient through finite perturbations to the magnetic field direction.
- Scalings for the turbulent heat-flux driven by the electrons were derived, appealing to constant-flux arguments and critical balance.
- Numerical results confirm the predicted heat-flux scaling for the (electrostatic) slab ETG, $Q \sim L_z^2$, while work on the electromagnetic regime is ongoing.
- All of this is subject to generalisations to include the effects of kinetic ions, magnetic shear, toroidal geometry, elongation, triangularity, Shafranov shift, plasma shaping, trapped particles, impurities, fast particles, runaway electrons, etc.
- Paper available: https://arxiv.org/abs/2201.05670

Acknowledgements

We are indebted to G. Acton, S. Cowley, W. Dorland, M. Hardman, D. Hosking, L. Milanese, J. Parisi, and F. Parra for helpful discussions and suggestions at various states of this project. This work has been carried out within the framework of the EUROfusion Consortium and has received funding from the Euratom research and training programme 2014–2018 and 2019–2020 under Grant Agreement No. 633053, and from the UKRI Energy Programme (EP/T012250/1). The views and opinions expressed herein do not necessarily reflect those of the European Commission. TA was supported by a UK EPSRC studentship. The work of AAS was supported in part by UK EPSRC (EP/R034737/1).

Backup slides

- Ordering of frequencies:

$$\frac{\omega}{\Omega_e} \sim \epsilon\beta_e, \quad \frac{\omega}{\Omega_i} \sim \epsilon.$$

- Ordering of lengthscales ($L \sim L_{n_s} \sim L_{T_s} \sim L_B \sim R$):

$$k_\perp \rho_i \sim k_\perp d_e \sim 1, \quad k_\perp \rho_e \sim \sqrt{\beta_e}, \quad k_\parallel L \sim \sqrt{\beta_e},$$
$$k_\parallel \lambda_{ei} \sim 1, \quad \frac{k_\parallel}{k_\perp} \sim \epsilon\sqrt{\beta_e}.$$

- Ordering of fluctuation amplitudes:

$$\frac{e\phi}{T_{0e}} \sim \frac{\delta n_e}{n_{0e}} \sim \frac{\delta n_i}{n_{0i}} \sim \frac{\delta T_e}{T_{0e}} \sim \frac{\delta T_i}{T_{0i}} \sim \epsilon, \quad \frac{\delta \boldsymbol{B}_\perp}{B_0} \sim \epsilon\sqrt{\beta_e}, \quad \frac{\delta B_\parallel}{B_0} \sim \epsilon\beta_e.$$

- Non-adiabatic response of the ions:

$$\left(\frac{\mathrm{d}}{\mathrm{d}t} + \boldsymbol{v}_{di} \cdot \nabla_\perp\right) g_i + \frac{c}{B_0}\left\{\langle\phi\rangle_{\boldsymbol{R}_i} - \phi, g_i\right\} + \langle \boldsymbol{v}_E \rangle_{\boldsymbol{R}_i} \cdot \nabla_\perp f_{0i}$$
$$= C\left[g_i + \frac{q_i \langle\phi\rangle_{R_i}}{T_i} f_{0i}\right],$$

where

$$g_i = h_i - \frac{q_i \langle\phi\rangle_{R_i}}{T_i} f_{0i}.$$

- Quasineutrality and parallel Ampere's law:

$$\frac{\delta n_e}{n_e} = -\bar{\tau}^{-1}\varphi + \frac{1}{n_i}\int \mathrm{d}^3\boldsymbol{v}\ \langle g_i\rangle_{\boldsymbol{r}}, \quad \frac{u_{\parallel e}}{v_{\mathrm{the}}} = d_e^2 \nabla_\perp^2 \mathcal{A},$$

where we have defined the normalised variables $\varphi = e\phi/T_e$ and $\mathcal{A} = A_\parallel/\rho_e B_0$.

- The electrons are drift kinetic, since $k_\perp \rho_e \sim \sqrt{\beta_e} \ll 1$, vis.:

$$\left(\frac{\mathrm{d}}{\mathrm{d}t} + v_\parallel \nabla_\parallel + \boldsymbol{v}_{de} \cdot \nabla_\perp\right) \delta f_e = -\boldsymbol{v}_\chi \cdot \nabla_\perp f_{0e} - \frac{v_\parallel e E_\parallel}{T_e} + C[\delta f_e].$$

- We choose to expand δf_e in Hermite-Laguerre moments $g_{\ell,m}$:

$$g_{\ell,m}(\boldsymbol{r},t) = \frac{1}{n_{0e}} \int \mathrm{d}^3\boldsymbol{v}\, (-1)^\ell \frac{H_m(v_\parallel/v_{\mathrm{the}}) L_\ell(v_\perp^2/v_{\mathrm{the}}^2)}{\sqrt{2^m m!}}\, \delta f_e,$$

$$\delta f_e(\boldsymbol{r}, v_\parallel, v_\perp^2, t) = \sum_{\ell=0}^{\infty} \sum_{m=0}^{\infty} (-1)^\ell \frac{H_m(v_\parallel/v_{\mathrm{the}}) L_\ell(v_\perp^2/v_{\mathrm{the}}^2) f_{0e}}{\sqrt{2^m m!}}\, g_{\ell,m},$$

where

$$H_m = \text{Hermite polynomials of order } m,$$
$$L_\ell = \text{Laguerre polynomials of order } \ell.$$

- The Hermite-Laguerre transform allows us to express the electron gyrokinetic equation in terms of a series of 'fluid moments':

$$\frac{\mathrm{d}g_{\ell,m}}{\mathrm{d}t} + \frac{v_{\mathrm{the}}}{\sqrt{2}}\nabla_{\parallel}\left(\sqrt{m+1}\, g_{\ell,m+1} + \sqrt{m}\, g_{\ell,m-1}\right) - C[g_{\ell,m}] + \omega_{de}[g_{\ell,m}] = I_{\ell,m}.$$

- We have defined:

$$\frac{\mathrm{d}}{\mathrm{d}t} = \frac{\partial}{\partial t} + \boldsymbol{v}_E \cdot \nabla_{\perp} = \frac{\partial}{\partial t} + \frac{\rho_e v_{\mathrm{the}}}{2}\{\varphi, ...\},$$

$$\nabla_{\parallel} = \boldsymbol{b}\cdot\nabla = \frac{\partial}{\partial z} + \frac{\delta \boldsymbol{B}_{\perp}}{B_0}\cdot\nabla_{\perp} = \frac{\partial}{\partial z} - \rho_e\{\mathcal{A}, ...\},$$

  which are the time derivative in the frame moving with the $\boldsymbol{E}\times\boldsymbol{B}$ flow, and the derivative along the exact, perturbed magnetic field line respectively.

- Electron-electron and electron-ion collisions ($\nu_{ei} = \nu_{ee} + \nu_{ei}$)

$$
\begin{aligned}
C[g_{\ell,m}] = & -\nu_{ei}(m+2\ell)g_{\ell,m} + \nu_{ee} g_{0,1}\delta_{0,1} \\
& + \frac{1}{3}\nu_{ei}\left(\sqrt{2}g_{0,2} + 2g_{1,0}\right)\left(\sqrt{2}\delta_{0,2} + 2\delta_{1,0}\right).
\end{aligned}
$$

- Magnetic drifts:

$$
\begin{aligned}
\omega_{de}[g_{\ell,m}] = \frac{\rho_e v_{\mathrm{the}}}{2L_B}\frac{\partial}{\partial y}\Big[& \sqrt{(m+1)(m+2)}g_{\ell,m+2} + 2(m+\ell+1)g_{\ell,m} \\
& + \sqrt{m(m-1)}g_{\ell,m-2} + (\ell+1)g_{\ell+1,m} + \ell g_{\ell-1,m}\Big].
\end{aligned}
$$

- Energy/momentum injection:

$$
\begin{aligned}
I_{\ell,m} = & -\frac{\rho_e v_{\mathrm{the}}}{2L_{n_e}}\frac{\partial \varphi}{\partial y}\left[\delta_{0,0} + \eta_e\left(\delta_{1,0} + \frac{1}{\sqrt{2}}\delta_{0,2}\right)\right] \\
& + \frac{\sqrt{2}\rho_e v_{\mathrm{the}}}{2L_{n_e}}\frac{\partial \mathcal{A}}{\partial y}\left[\delta_{0,1} + \eta_e\left(\delta_{0,1} + \delta_{1,1} + \sqrt{\frac{3}{2}}\delta_{0,3}\right)\right] \\
& + \frac{v_{\mathrm{the}}}{\sqrt{2}}\left(\frac{2}{v_{\mathrm{the}}}\frac{\mathrm{d}\mathcal{A}}{\mathrm{d}t} + \frac{\partial \varphi}{\partial z}\right)\delta_{0,1} + \frac{\rho_e v_{\mathrm{the}}}{2L_B}\frac{\partial \varphi}{\partial y}\left[\sqrt{2}\delta_{0,2} + \delta_{1,0} + 2\delta_{0,0}\right].
\end{aligned}
$$

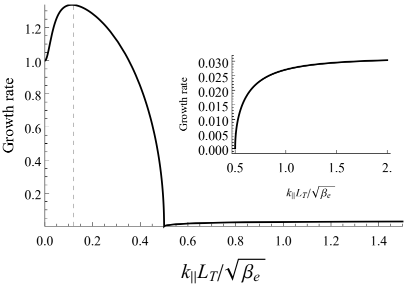
- Location of cTAI maximum:

$$\frac{k_{\parallel \max} L_T}{\sqrt{\beta_e}} = \begin{cases} \left[\frac{\pi}{64}\frac{3+4\bar{\tau}}{(1+\bar{\tau})^2}\frac{L_T}{L_B}\right]^{1/4}\left(\frac{k_y^2 d_e}{k_\perp}\right)^{1/2}, & \text{collisionless,} \\ \left[\frac{81}{50}\frac{3+4\bar{\tau}}{(1+\bar{\tau})^2}\frac{L_T}{L_B}\right]^{1/6}\left(\frac{k_y^2 d_e}{k_\perp}\chi\right)^{1/3}, & \text{collisional,} \end{cases}$$

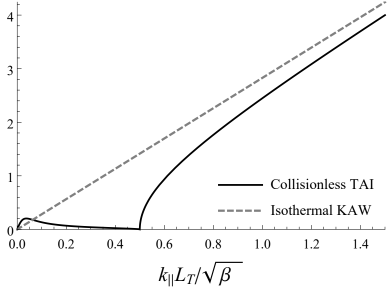
  in the collisionless and collisional limits, respectively.
- Transition wavenumber between isothermal and isobaric regimes:

$$k_{\perp *} d_e = \begin{cases} \frac{4}{\sqrt{\pi}}\left(\frac{L_T}{L_B}\right)^{1/2}, & \text{collisionless,} \\ \frac{5}{9}\frac{L_T}{L_B}\chi^{-1}, & \text{collisional,} \end{cases}$$

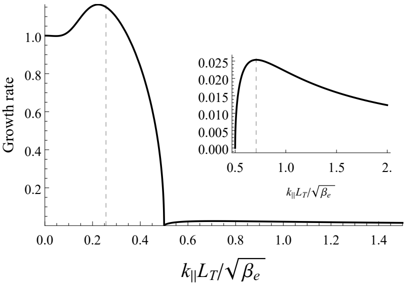
  with $\chi^{-1} = \sqrt{\beta_e}\lambda_{ei}/L_T$.



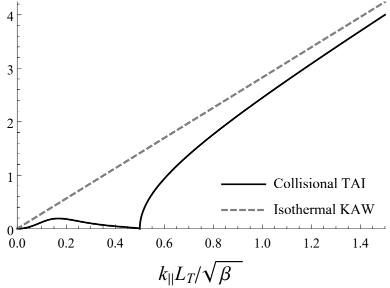
(a) $L_B/L_T = 4$, $k_y d_e = 0.1$



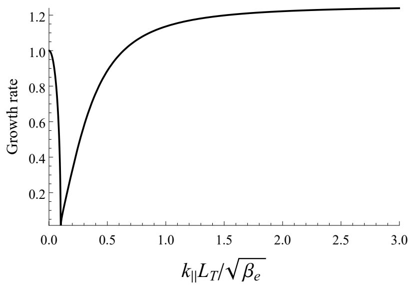
(b) $L_B/L_T = 4$, $k_y d_e = 0.1$



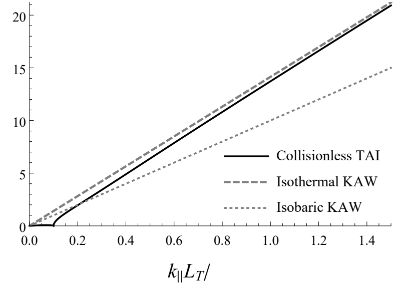
(c) $L_B/L_T = 4$, $k_y d_e \chi = 0.01$



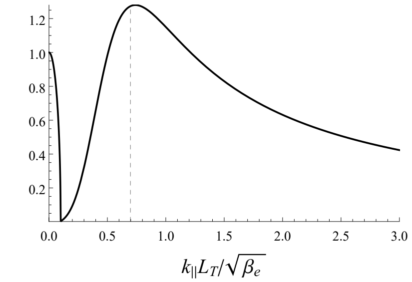
(d) $L_B/L_T = 4$, $k_y d_e \chi = 0.01$



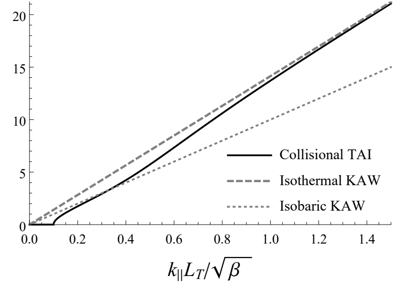
(a) $L_B/L_T = 100$, $k_y d_e = 0.8$



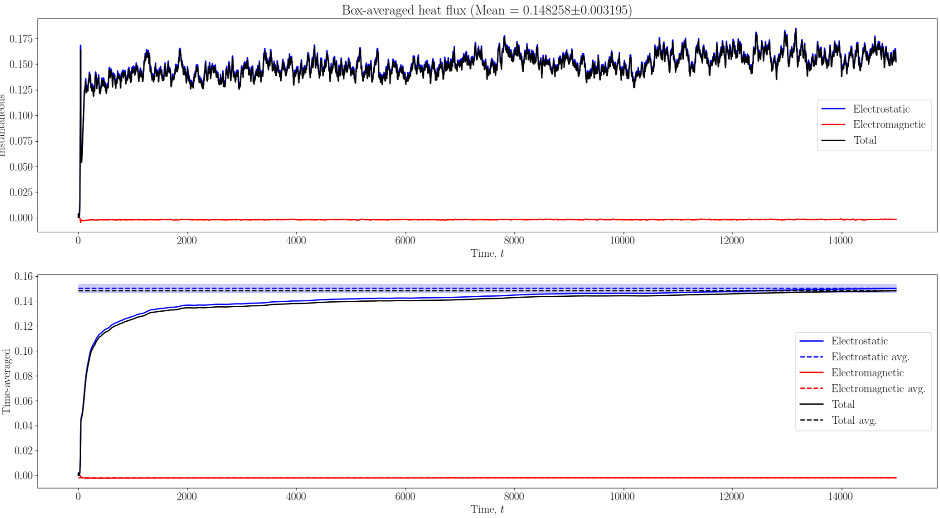
(b) $L_B/L_T = 100$, $k_y d_e = 0.8$



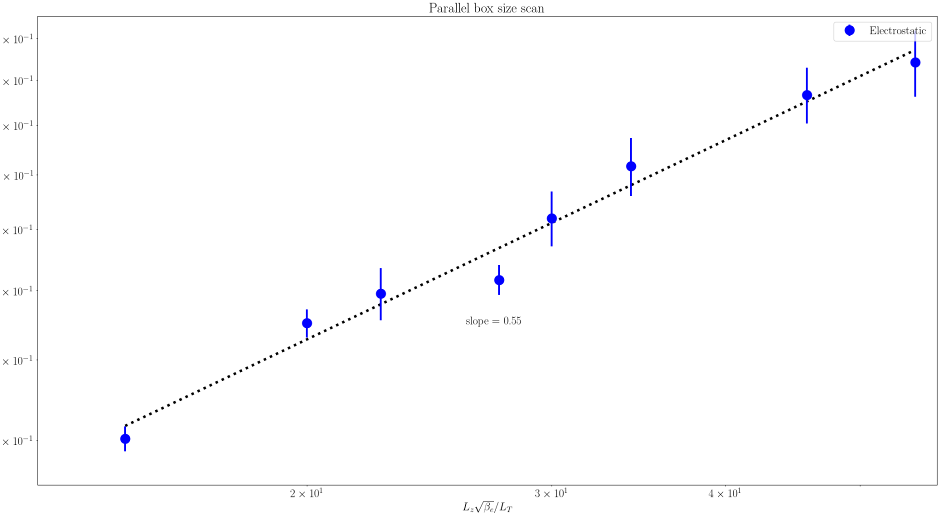
(c) $L_B/L_T = 100$, $k_y d_e \chi = 0.1$



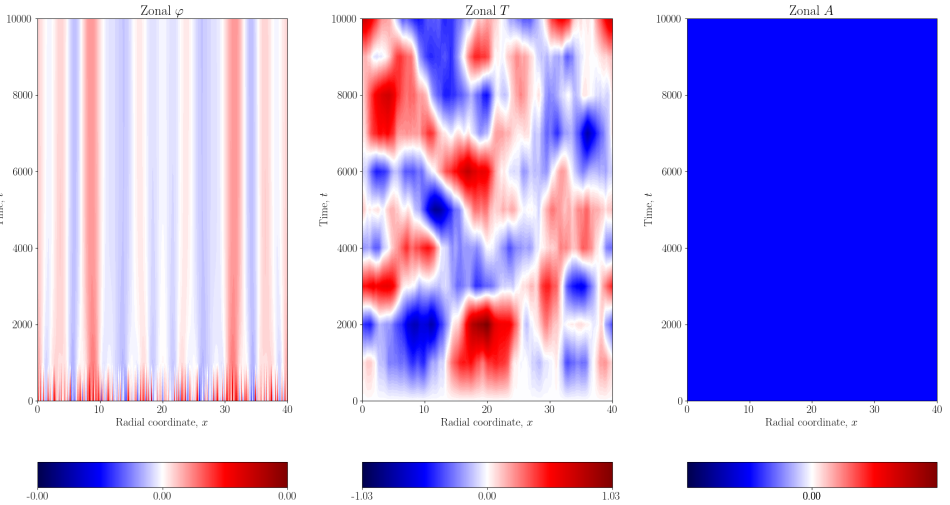
(d) $L_B/L_T = 100$, $k_y d_e \chi = 0.1$



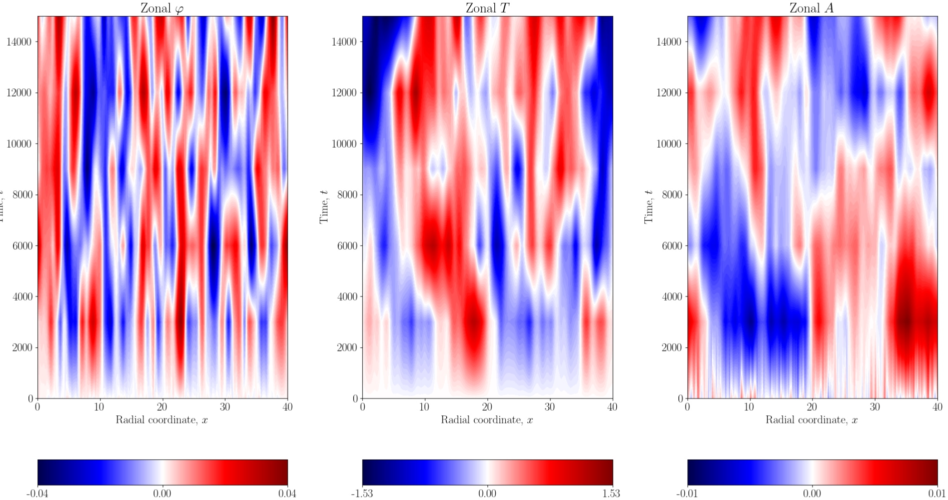
Box-averaged heat flux (Mean = 0.148258±0.003195)
Instantaneous
0.175
0.150
0.125
0.100
0.075
0.050
0.025
0.000
Electrostatic
Electromagnetic
Total
0
2000
4000
6000
8000
10000
12000
14000
Time, t
Time-averaged
0.16
0.14
0.12
0.10
0.08
0.06
0.04
0.02
0.00
Electrostatic
Electrostatic avg.
Electromagnetic
Electromagnetic avg.
Total
Total avg.
0
2000
4000
6000
8000
10000
12000
14000
Time, t



Parallel box size scan
Electrostatic
slope = 0.55
$\times 10^{-1}$
$\times 10^{-1}$
$\times 10^{-1}$
$\times 10^{-1}$
$\times 10^{-1}$
$\times 10^{-1}$
$\times 10^{-1}$
$\times 10^{-1}$
2×10^1
3×10^1
4×10^1
$L_z\sqrt{\beta_e}/L_T$



Zonal φ
Zonal T
Zonal A
Time, t
Radial coordinate, x
10000
8000
6000
4000
2000
0
0
10
20
30
40
-0.00
0.00
0.00
-1.03
0.00
1.03
0.00



Zonal φ
Zonal T
Zonal A
Time, t
Radial coordinate, x
0
2000
4000
6000
8000
10000
12000
14000
0
10
20
30
40
-0.04
0.00
0.04
-1.53
0.00
1.53
-0.01
0.00
0.01