Pressure Anisotropy in Collisionless Reconnection

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Outline

• Examples of Magnetic Reconnection
• Model for Electron Pressure Anisotropy
• Fixing the Fluid Equations (The Equations of State)
• Regimes of Reconnection
• Opportunities with TREX
• Conclusions
The Earth’s Magnetic Shield

- Coronal Mass Ejection (Billion Tons of Superhot Gas)
- Solar Wind
- Dayside Magnetopause
- Magnetotail
- Bow Shock
- Primary Reconnection Sites
Cluster observations on 2001-10-01.


Lobe: \( T_{e\infty} \approx 0.10 \text{ keV} \)

Inflow: \( T_{e\perp} \approx 0.10 \text{ keV}, \ T_{e\parallel} \approx 1 \text{ keV} \)

Exhaust: \( T_{e\perp} \approx T_{e\parallel} \approx 10 \text{ keV} = 100 \ T_{e\infty} \)
Another Cluster Event

Hwang et al., JGR 2013, Cluster, August 18, 2002

![Graphs and diagrams related to Cluster event]
Wind Spacecraft Observations in Distant Magnetotail, $60R_E$

• Measurements within the ion diffusion region reveal:
  Strong anisotropy in $f_e$

$P_{||} > P_{\perp}$

$B_\infty = 11.5\text{nT}, B_g = 6\text{nT}$

Wind trajectory

$E_{\text{y}}$
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Two-Fluid vs Kinetic Simulations

Isotropic pressure

Particle In Cell (PIC) simulation,

Kinetic Simulation

Out of plane current

$\text{Fluid: Isotropic Pressure}$

$\text{Fully Kinetic Simulation}$
Wind Spacecraft Observations in Distant Magnetotail, $60R_E$

- Measurements within the ion diffusion region reveal:
  Strong anisotropy in $f_e$

\[ p_\parallel > p_\perp \]
Electrons in an Expanding Flux Tube

Expanding Flux tube

-Trapped electron

Magnetic moment:

\[ \mu = \frac{mv_\perp^2}{2B} \]

⇒ mirror force:
Electrons in an Expanding Flux Tube

Expanding Flux tube

Trapped electron

Magnetic moment:

\[ \mu = \frac{mv_{\perp}^2}{2B} \]

\[ \Phi_\parallel(x) = \int_x^\infty \mathbf{E} \cdot d\mathbf{l} \]

\[ 0.5mv_{\parallel}^2 = e\Phi_\parallel \]
Electrons in an Expanding Flux Tube

Trapped:
\[ \mathcal{E}_\perp = \mu B = \mathcal{E}_\infty B / B_\infty \]
\[ \implies \mathcal{E}_\infty = \mu B_\infty \]

Passing:
\[ \mathcal{E} = \mathcal{E}_\infty + e\Phi_\parallel \]
\[ \implies \mathcal{E}_\infty = \mathcal{E} - e\Phi_\parallel \]

Vlasov:
\[ \frac{df}{dt} = 0 \]
\[ f(x, v) = f_\infty(\mathcal{E}_\infty) \]

\[ \Phi_\parallel(x) = \int_x^\infty E \cdot dl \]

\[ f(x, v) = \begin{cases} 
  f_\infty(\mathcal{E} - e\Phi_\parallel) , & \text{passing} \\
  f_\infty(\mu B_\infty) , & \text{trapped}
\end{cases} \]

J. Egedal et al., JGR (2009)
Formal Derivation using an “Ordering”

The drift kinetic equation:

\[
\frac{\partial f}{\partial t} + (v_\parallel + v_D) \cdot \nabla f + \left[ \mu \frac{\partial B}{\partial t} + e(v_\parallel + v_D) \cdot E \right] \frac{\partial f}{\partial \varepsilon} = 0
\]

Boundary conditions:

\[
B = B_\infty, \quad f = f_\infty(\varepsilon_\parallel, \varepsilon_\perp)
\]

Ordering:

\[
\nabla_\parallel \sim \frac{1}{L}, \quad \nabla_\perp \sim \frac{1}{d}, \quad \frac{\partial}{\partial t} \sim \frac{v_D}{d}
\]

\[
\frac{d}{L} \sim \delta, \quad \frac{v_D}{v_t} \sim \delta^2, \quad f = f_0 + \delta f_1 + \ldots
\]

\[
f_0(x, v) = \begin{cases} f_\infty(\varepsilon - e\Phi_\parallel), & \text{passing} \\ f_\infty(\mu B_\infty), & \text{trapped} \end{cases}
\]
Wind Spacecraft Observations in Distant Magnetotail, 60R\textsubscript{E}

\[ f(x, v) = \begin{cases} 
  f_\infty(\mathcal{E} - e\Phi_\parallel), & \text{passing} \\
  f_\infty(\mu B_\infty), & \text{trapped}
\end{cases} \]

\( \Phi_\parallel \sim 1 \text{ kV} \)
Field Structure at Full Mass Ratio (P. Pritchett)

Onset of magnetic reconnection in the presence of a normal magnetic field: Realistic ion to electron mass ratio

P. L. Pritchett


Figure 6. The electrostatic potential $e\Phi/T_e$ at time $\Omega_{ij} = 40$ for the simulation with $m_i/m_e = 1600$ in (a) the $x, z$ plane and (b) as a profile along $x$ at $z = 0$.

[19] Figure 8a shows the parallel potential $\Phi_\parallel$ computed from the field $E_\parallel$ by the definition $[Egedal et al., 2009]$

$$\Phi_\parallel(x) = \int_x^\infty E \cdot d\ell,$$

where the integration is carried out from the point $x$ along the magnetic field to the boundary of the simulation box. Note that $\Phi_\parallel$ contains contributions from both the electrostatic and inductive electric fields.

Figure 8. Structure at time $\Omega_{ij} = 40$ for the simulation with $m_i/m_e = 1600$ for (a) the parallel potential $e\Phi_\parallel/T_e$ and (b) the electron temperature anisotropy $T_{e\parallel}/T_{e\perp}$. 
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Upshot: New Fluid Closure (EoS)

\[ f(x, v) = \begin{cases} 
  f_\infty (E - e\Phi_\parallel), & \text{passing} \\
  f_\infty (\mu B_\infty), & \text{trapped}
\end{cases} \]

\[ \int \ldots d^3v \]

\[ n = n(B, \Phi_\parallel) \]
\[ p_\parallel = p_\parallel(B, \Phi_\parallel) \]
\[ p_\perp = p_\perp(B, \Phi_\parallel) \]

Eliminate \( \Phi_\parallel \) \( \Rightarrow \) \[ \begin{cases} 
  p_\parallel = p_\parallel(n, B) \\
  p_\perp = p_\perp(n, B)
\end{cases} \]

\[ p_\parallel \propto \frac{n^3}{B^2} \]
\[ p_\perp \propto nB \]

Smooth transition from Boltzmann to double adiabatic (Chew, Goldberger, FE… Low) scaling

A. Le et al., PRL (2009)
Confirmed in Kinetic Simulations

EoS previously confirmed in 2D simulations, now also in 3D simulations.
New *EoS* Implemented in Two-Fluid Code

Ohia et al., PRL, 2012

Out of plane current
Model for islands?

J. F. Drake¹, M. Swisdak¹, R. Ferro²

\[
\frac{\partial}{\partial t} f + \mathcal{O} \left( \frac{\partial}{\partial v_\parallel} v_\parallel - \frac{\partial}{\partial v_\perp} v_\perp^2 \right) f \\
= \nu \frac{\partial}{\partial \zeta} \left( 1 - \zeta^2 \right) \frac{\partial}{\partial \zeta} f - \frac{c_{\text{Au}}}{L} f + \dot{A}_T f_{\text{up}} \\
R = -\dot{B}/B
\]

Conserved quantities:
\[
\mu = m v_\perp^2 / B \\
\oint v_\parallel d\ell
\]

Area, flux, density:
Model for islands?

Conserved quantities:

\[ \mu = \frac{mv_{\perp}^2}{B} \]

\[ \oint v_\parallel d\ell \]

Area, flux, density:

Generalize:

B-constant, change n:

\[ \frac{\partial}{\partial t} f - \left( \frac{\partial \ln(B)}{\partial t} \right) \left( \frac{\partial}{\partial v_{\parallel}} v_{\parallel} - \frac{\partial}{\partial v_{\perp}^2} v_{\perp}^2 \right) f \]

\[ - \left( \frac{\partial \ln(n)}{\partial t} \right) \left( 1 - \frac{\partial}{\partial v_{\parallel}} v_{\parallel} \right) f = 0: \]

\[ = v \frac{\partial}{\partial \zeta} (1 - \zeta^2) \frac{\partial}{\partial \zeta} f \]

\[ + \frac{C_{A_{\parallel}p}}{L} f + \dot{A}_T f_{\perp} \]

In 2D = 0:

Integrates to CGL!

\[ p_\parallel \propto \frac{n^3}{B^2} \]

\[ p_\perp \propto nB \]
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EoS for Anti-Parallel Reconnection?

The electrons are magnetized in the inflow region:

Pitch angle diffusion is controlled by:

\[ \kappa = \sqrt{\frac{R_B}{\rho_e}} \]

Depends on \( B_g \) and \( m_i/m_e \).
Where does Guide-Field Reconnection Begin?

Kinetic simulation results at $m_i/m_e = 400$, [A Le et al., PRL 2013]
Where does Guide-Field Reconnection Begin?

Kinetic simulation results at $m_i / m_e = 1836$, [A Le et al., PRL 2013]
Regimes of the Electron Diffusion Region

$J_y$ in PIC simulations at $m_i/m_e = 1836$

Unexplored regime of reconnection, relevant to the MMS mission
Scaling Law for Electron Heating

Additional current term:
\[ J_{\perp \text{extra}} = \left[ (p_{\parallel} - p_{\perp})/B \right] \hat{b} \times \nabla \hat{b} \]

The magnetic tension is balanced by pressure anisotropy:
\[ p_{\parallel}(n, B) - p_{\perp}(n, B) = B^2/\mu_0 \]

Use EoS to get scaling laws:
\[ \beta_{e\infty} = \frac{\text{plasma pressure}}{\text{magnetic pressure}} \]

Magnetotail: \( \beta_{e\infty} = 0.003 \)
Simulation with $\beta_e \sim 0.003$
Spacecraft Distributions Reproduced

J. Egedal et al., Nature Physics (2012)
$\Phi_\parallel$ confines electrons, allowing sustained energization by $E_\perp$

Trapped region with pitch angle scattering

Heated by $v_d \cdot E_\perp$,

$$\frac{\partial v}{\partial t} \approx \frac{v_A}{\tau_b}$$
Generation of Super-Thermals

\[ \log_{10}(\text{# of electrons with energies above } 1.7mc^2) \]

Steady state:

\[ \log_{10}[f(E)] = \log_{10}[f(\gamma - 1)] \]

\[ \Gamma_l = dv f \frac{4\pi v^2}{\tau_b} R_{\text{loss}} \]

New electrons

Lost electrons

\[ f = \frac{A}{v^3} \exp \left( -\frac{v R_{\text{loss}}}{v_A} \right) \]
Generation of Super-Thermals

\[ \log_{10}(\text{# of electrons with energies above } 1.7mc^2) \]

Steady state:

\[ \Gamma_v = f \frac{4\pi v^2}{\tau_b} \frac{\partial v}{\partial t} \]
\[ \frac{\partial v}{\partial t} \approx \frac{v_{A}}{\tau_b} \]

New electrons

Lost electrons

\[ \Gamma_l = dv f \frac{4\pi v^2}{\tau_b} R_{\text{loss}} \]

\[ dv \frac{d\Gamma_v}{dv} = \Gamma_l \rightarrow f = \frac{A}{v^3} \exp \left( -\frac{v R_{\text{loss}}}{v_{A}} \right) \]
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Role of Collisions:

Full electron transit must be completed without collisions:

\[ \tau_e > \frac{d_i}{0.1v_A} \iff \nu_e/\omega_{ce} < 0.1 \frac{m_e}{m_i} \]

or

\[ S > 10^4 \frac{L}{d_i} \]
Reconnection setup in CRX

Insert holding field coils

MPDX (just delivered)
Key new hardware

- **Helmholtz Coils**
  - 2 × 80 turns, CW, 800A
  \[ B \approx 0.025 \text{T} \]

- **TF coil**
  - 96 turns,
  - CW, 800A
  \[ B = 0.015 \text{T} \at R=1\text{m} \]
  - Pulsed, 13kA
  \[ B = 0.25 \text{T} \at R=1\text{m} \]

- **Poloidal field coils**
  - 2 × 20 turns, pulsed, 5kA
  \[ B \approx 0.04 \text{T} \]

- **Reconnection drive coils**
  - 2 × 1 turns, 5kV, 10kA

**MPDX with TREX hardware**
Asymmetric reconnection in TREX

- Simple configuration using the HH-coils plus two internal coils
- This will be the first configuration to be implemented
Symmetric Inflow Configuration

TREX, poloidal magnetic fields

Collisional VPIC simulation
Role of Collisions, Moderate Guide-field Reconnection

- Continuous operation of magnetic coils
- Plasma pulsed at 1Hz
- $B_g = 20$ mT at 1m

<table>
<thead>
<tr>
<th></th>
<th>$n_e/10^{18}$ m$^{-3}$</th>
<th>$T_e$/eV</th>
<th>$B_r$/T</th>
<th>$B_g$/T</th>
<th>$L$/m</th>
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<tbody>
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<td>CRX</td>
<td>0.1-10</td>
<td>8-40</td>
<td>0.04</td>
<td>0-0.3</td>
<td>0.8-1.8</td>
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<td>MRX</td>
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<td>5-10</td>
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<td>8-30</td>
<td>0.01</td>
<td>0.1</td>
<td>0.3</td>
</tr>
</tbody>
</table>
Strong Guide-field Reconnection

- Pulsed operation of magnetic coils
- 3 min between pulses
- $B_g = 0.25\text{T at 1m}$

$\rho_s = \frac{(2m_iT_e)^{1/2}}{eB}$

= “ion sound Larmor Radius”
Conclusion

• The construction of the new Terrestrial Reconnnection EXperiment is well under way

• TREX is highly leveraged against earlier NSF investments through the use of the MPDX-vessel and plasma production

• TREX provides huge flexibility in available configurations, and the insert will allow for fast turn-around.

• Reconnection in an unprecedented range of plasma parameters to be explored

• Thanks to Cary and the MPDX team for a good start on TREX!