Poloidally varying non-fluctuating potentials and their effect on impurity peaking

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Outline

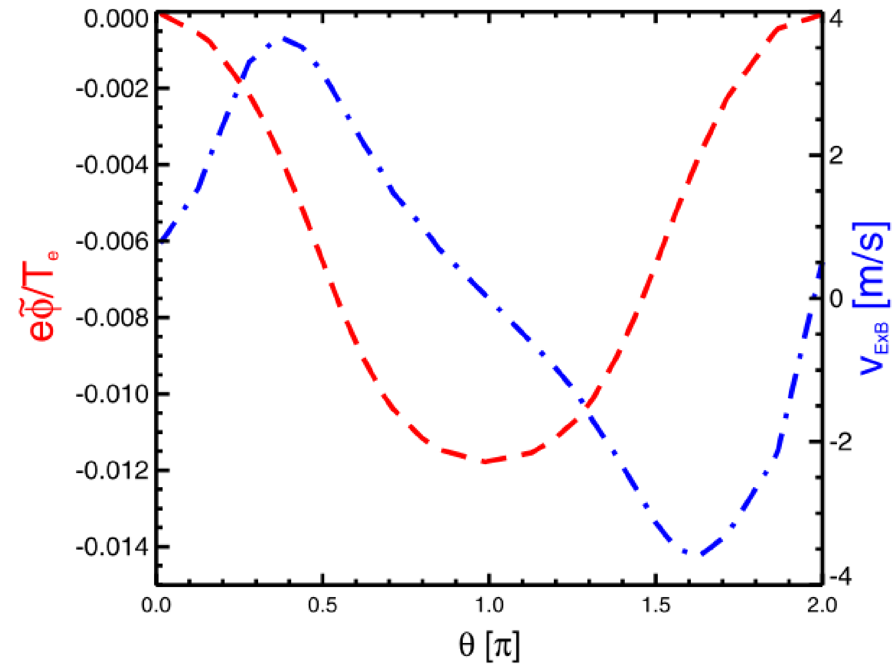
- Poloidal asymmetries in the core and edge
- Model for impurity peaking
- The effect of
  - $\mathbf{E}_\theta \times \mathbf{B}_\phi$ drifts
  - parallel compressibility
  - collisions

  on impurity peaking

Poloidal asymmetries in tokamaks

- Rotation, radio frequency heating, sources, neoclassical effects
- Poloidally varying electrostatic potential is too weak to modify main species dynamics
- Finite poloidal asymmetry in the impurity density
- $\mathbf{E}\times\mathbf{B}$ drift is independent of Z. Magnetic and diamagnetic drifts scale as $1/Z$
- Effects on turbulent impurity transport

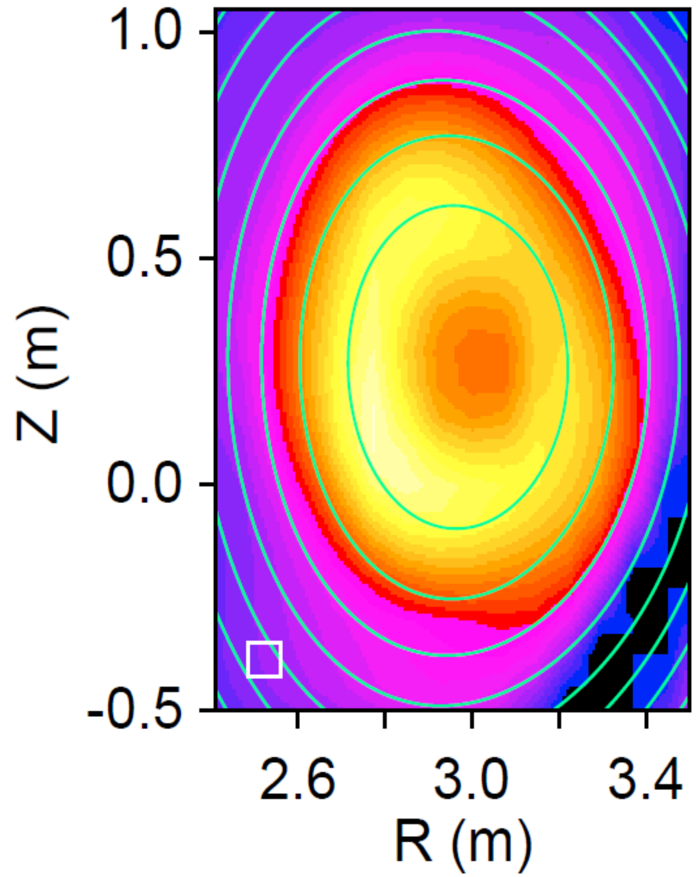
M.L. Reinke et al, PPCF **54** (2012) 045004



Poloidal variation of the non-fluctuating electrostatic potential $e[\phi(\theta) - \phi(0)]/T_{\rm e}$ and the resulting $\mathrm{E}\times\mathrm{B}$ flow velocity, under H minority heated ICRH. (Modeling)

RF induced asymmetries

- Heated species may be far from Maxwell-Boltzmann, and can build up a poloidally asymmetric distribution
- This might contribute to a reduced impurity peaking in RF-heated plasmas

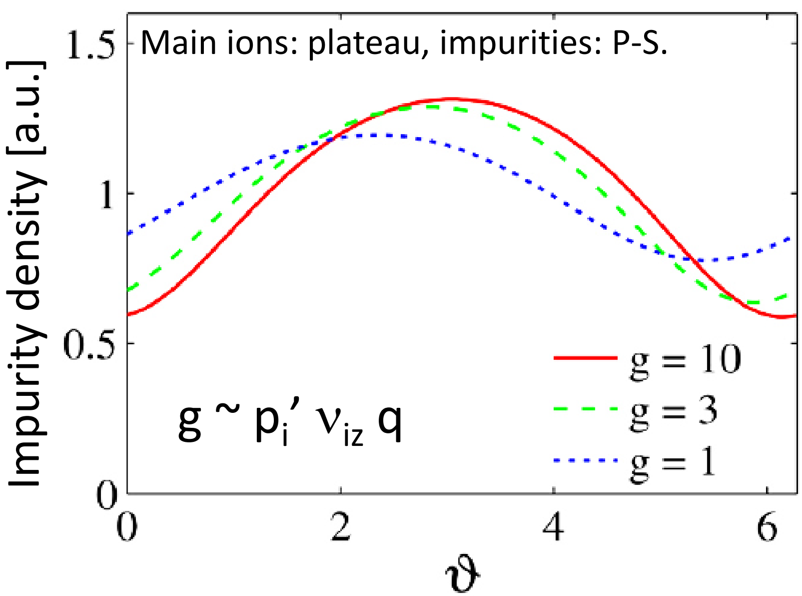


In-out asymmetry of Ni impurity in an ICRH heated JET discharge (SXR tomography). Ingesson et al, PPCF **42** (2000) 161

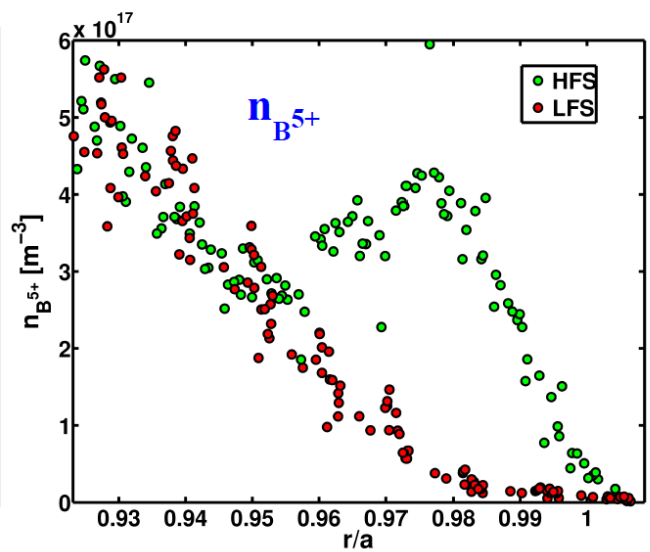
Neoclassical asymmetries in the edge

$$0 = -Zn_z e \nabla_{\|} \Phi - T_i \nabla_{\|} n_z + R_{zi\|}$$

- When $\rho_\theta \hat{\nu}_{ii} Z^2 / L_\perp \sim 1$, the impurity-ion friction can compete with the parallel pressure gradient, that leads to an accumulation of impurities at certain poloidal locations.



Landreman et al, PoP **18** (2011) 092507



R.M. Churchill, APS 2012

Strong in-out asymmetry of Boron in the edge of C-Mod. (Direct measurement, EDA H-mode; main ions in plateau)

Model of impurity peaking under poloidal asymmetries

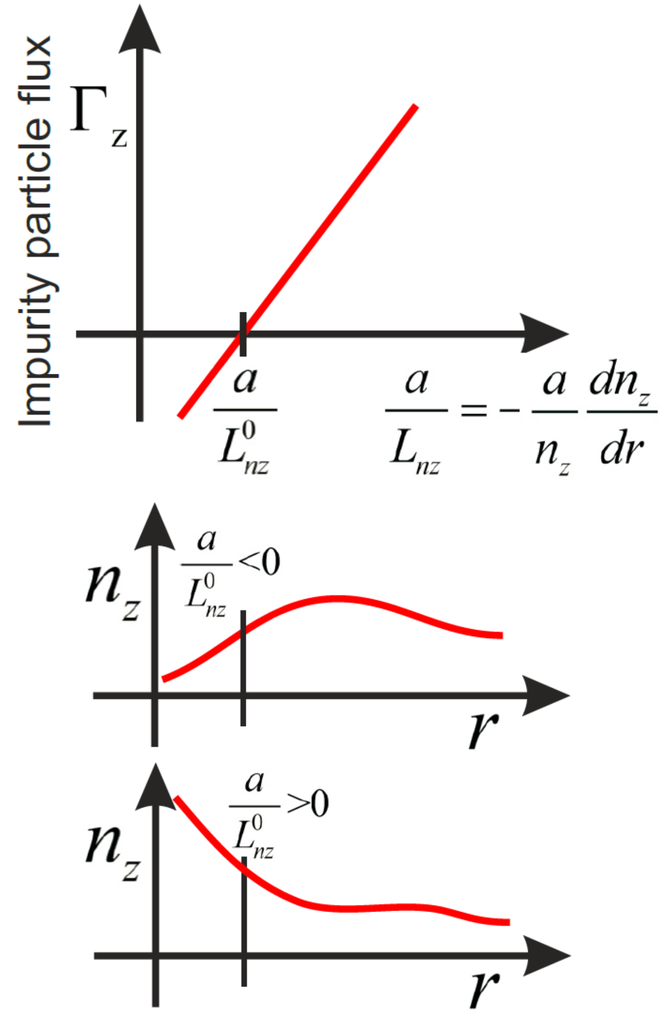
- Allow for $Ze\Delta\phi_E/T_z = \mathcal{O}(1)$
  i.e. the impurity density can be poloidally asymmetric
- In the impurity gyrokinetic equation include the $\mathbf{E}_\theta \times \mathbf{B}_\phi$ drift of impurities in the non-fluctuating electrostatic potential ϕ_E

$$\omega_E = -\frac{k_y}{B}\frac{s\vartheta}{r}\frac{\partial \phi_E}{\partial \vartheta}$$

- A „new" degree of freedom to respond to electrostatic perturbations; it can be comparable to magnetic drifts and parallel streaming for high enough Z

Model of impurity peaking under poloidal asymmetries

- Trace impurities. The main species are unaffected by poloidal asymmetries. Use GYRO simulations to get the linear mode characteristics $\omega, \phi(\vartheta)$
- No radial electric field effects and neoclassical transport
- Solving the linear gyrokinetic equation for impurities
- Looking for the steady state impurity density gradient (peaking factor)



Model of impurity peaking under poloidal asymmetries

- Parallel compressibility and finite Larmor radius (FLR) terms are retained (ion scale microinstabilities $k_y \rho_i \lesssim 1$)
- We assume that self-collisions dominate impurity collisions; $n_z Z^2 / n_e = \mathcal{O}(1)$
- The impurity collision frequency is allowed to be comparable to the mode frequency
- We use the properties of the full linearized impurity-impurity collision operator

Iterative solution of the impurity GK equation

- The linearized GK equation

$$\frac{v_\|}{qR}\frac{\partial g_z}{\partial \vartheta}\bigg|_{\mathcal{E},\mu} - i(\omega - \omega_{Dz} - \omega_E)g_z - C[g_z] = -i\frac{Zef_{z0}}{T_z}\left(\omega - \omega_{*z}^T\right)\phi J_0(z_z)$$

is solved perturbatively in $Z^{-1/2} \ll 1$

$$\omega_E/\omega,\ \omega_{Dz}/\omega,\ \omega_{*z}^T/\omega,\ \text{and}\ J_0(z_z) - 1 \approx -z_z^2/4$$

- are all treated as $\sim 1/Z$ small quantities

Iterative solution of the impurity GK equation

- To lowest order in the expansion

$$-i\omega g_0 - C_{zz}^{(l)}[g_0] = -i\omega Ze\phi f_{z0}/T_z$$

$$g_0 = Ze\phi f_{z0}/T_z \qquad \text{since } C_{zz}^{(l)}[g_0 \propto f_{z0}] \text{ vanishes}$$

- The lowest order result cancels with the adiabatic response
- To order $Z^{-1/2}$

$$v_\| \partial_\vartheta(g_0)/(qR) - i\omega g_1 - C_{zz}^{(l)}[g_1] = 0$$

$$g_1 = -iZef_{z0}v_\| \partial_\vartheta(\phi)/(T_z\omega qR) \qquad C_{zz}^{(l)}[g_1 \propto v_\| f_{z0}] = 0$$

- The above parts of g will not affect quasineutrality.

Iterative solution of the impurity GK equation

- The order Z^{-1} equation is of the form

$$g_2 - \frac{i}{\omega} C_{zz}^{(l)}[g_2] - X f_{z0} = 0$$

- We need only the density moment of g_2

$$\int d^3v \left[g_2 - X f_{z0} \right] = 0$$

- Thus we may use $X f_{z0}$ to evaluate the impurity particle flux. The zero flux condition yields

$$\frac{a}{L_{nz}^0} \langle \omega_a |\phi|^2 \mathcal{N} \rangle \Im \left[\frac{1}{\omega} \right] = \langle (2\hat{\omega}_D + \omega_E) |\phi|^2 \mathcal{N} \rangle \Im \left[\frac{1}{\omega} \right] + \frac{v_z^2}{2q^2 R_0^2} \left\langle \mathcal{N} \left| \frac{\partial \phi}{\partial \vartheta} \right|^2 \right\rangle \Im \left[\frac{1}{\omega^2} \right]$$

$$\hat{\omega}_D = -k_y T_z \mathcal{D}(\vartheta)/(ZeBR) \qquad \omega_a = -k_y T_z/(ZeBa)$$

$$\mathcal{N} = \exp(-Ze\phi_E/T_z)$$

Approximate expression for the peaking factor

- Order r/R corrections neglected
- Collisions and finite Larmor radius corrections do not affect the impurity particle flux to order $1/Z$
- Sinusoidal model for the asymmetries

$$\frac{Ze\phi_E}{T_z} = -K\cos(\theta - \delta)$$

Asymmetry strength (K)

Poloidal position of impurity density maximum (δ)

Approximate expression for the peaking factor

$$\frac{a}{L_{nz}^0} = 2\frac{a}{R_0}\langle \mathcal{D} \rangle_\phi + \frac{a}{r} sK \langle \theta \sin(\theta - \delta) \rangle_\phi$$
$$- \frac{2av_i}{(qR_0)^2 k_y \rho_i} \frac{Zm_i}{m_z} \frac{\omega_r}{\omega_r^2 + \gamma^2} \left\langle \left| \frac{\partial \phi}{\partial \theta} \right|^2 / |\phi|^2 \right\rangle_\phi$$

Magnetic drifts

$\mathbf{E}_\theta \times \mathbf{B}_\phi$ drift

Parallel dynamics

where

$$\langle \ldots \rangle_\phi = \langle \ldots \mathcal{N} |\phi|^2 \rangle / \langle \mathcal{N} |\phi|^2 \rangle \, ,$$

$$\mathcal{N} \equiv \exp[-Ze\phi_E/T_z] = \exp[K \cos(\theta - \delta)] \text{, and}$$

$$\mathcal{D}(\vartheta) = \cos\vartheta + s\vartheta \sin\vartheta$$

Approximate expression for the peaking factor

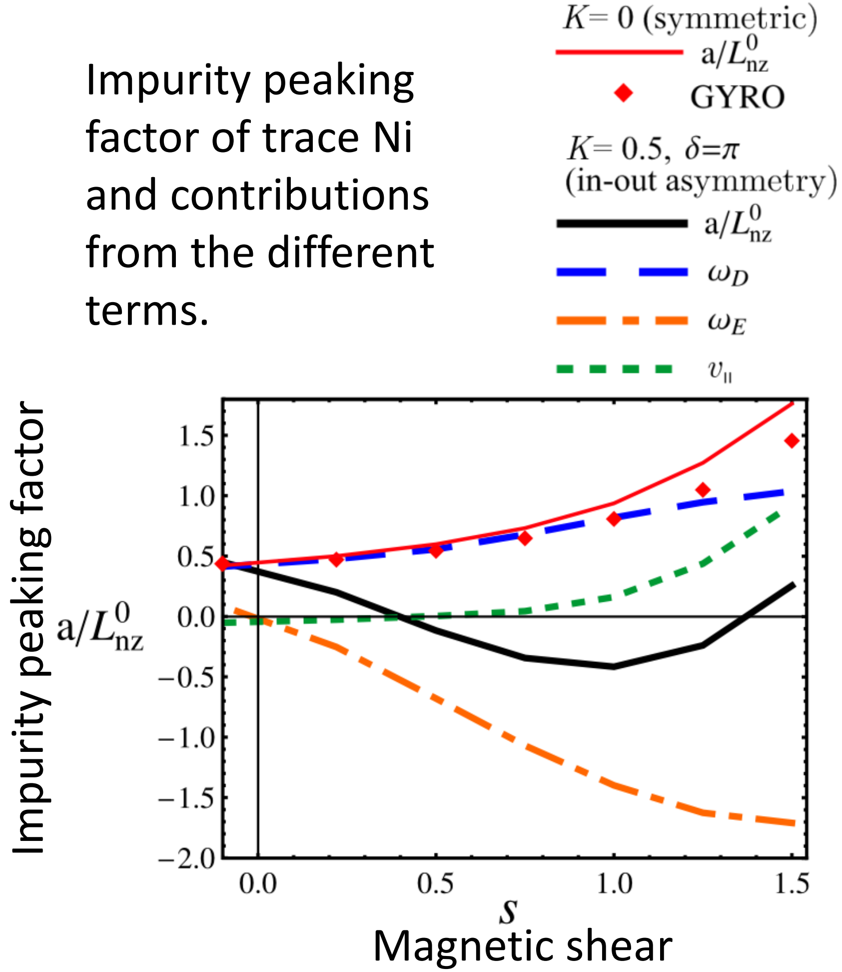
- The shear dependent part of the magnetic drift term and the $\mathbf{E}_\theta \times \mathbf{B}_\phi$ drift term can be combined as

$$(as/R_0)\langle\theta \sin\theta\rangle_\phi (2 \pm K/\epsilon)$$

- Asymmetry strength due to ICRH minority heating

$$K/\epsilon \approx \alpha_T Z X_m/(1+Z_{\rm eff})$$

Impurity peaking factor of trace Ni and contributions from the different terms.



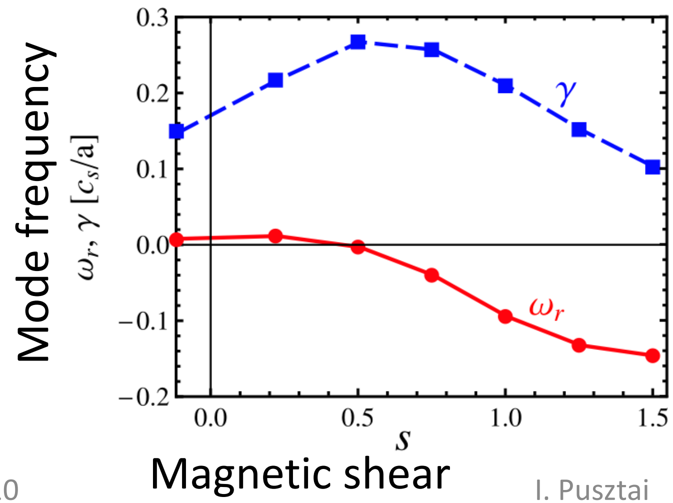
$$r/a = 0.3,\ R_0/a = 3,\ k_\theta\rho_s = 0.3,\ q = 1.7,\ a/L_{ne} = 1.5,\ T_i/T_e = 0.85,\ a/L_{Te} = 2,\ a/L_{Ti} = 2.5$$

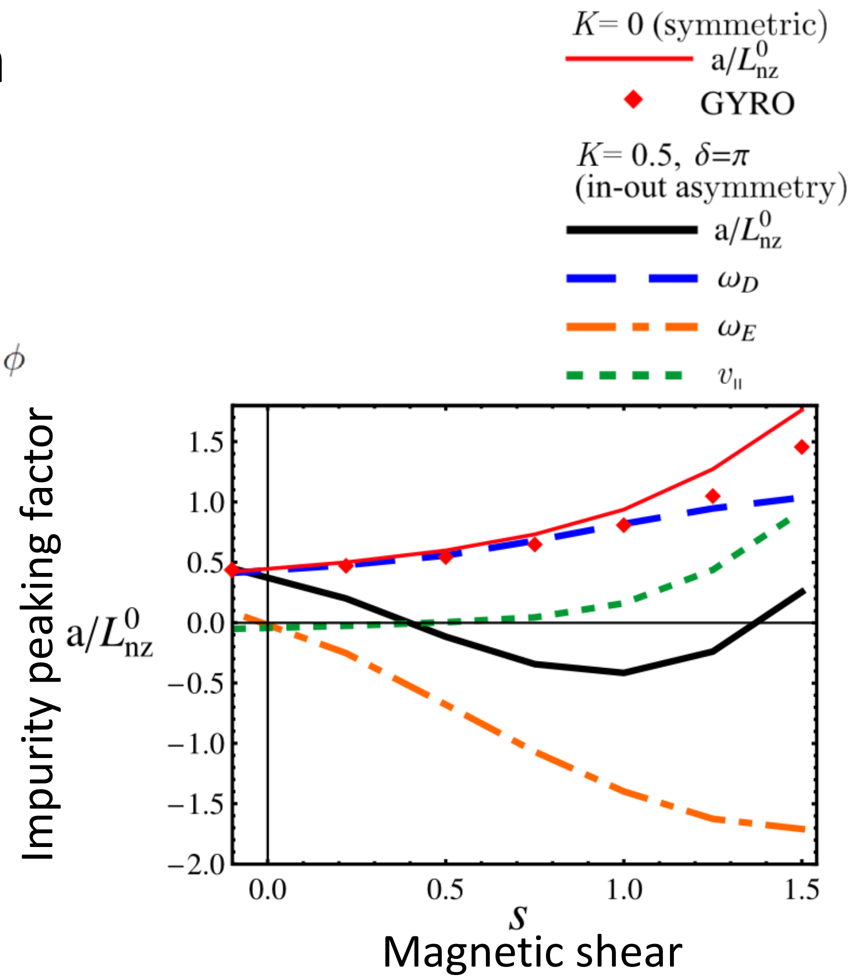
Approximate expression for the peaking factor

- Frequency dependence through parallel compressibility

$$-\frac{2av_i}{(qR_0)^2k_y\rho_i}\frac{Zm_i}{m_z}\frac{\omega_r}{\omega_r^2+\gamma^2}\left\langle\left|\frac{\partial\phi}{\partial\theta}\right|^2/|\phi|^2\right\rangle_\phi$$

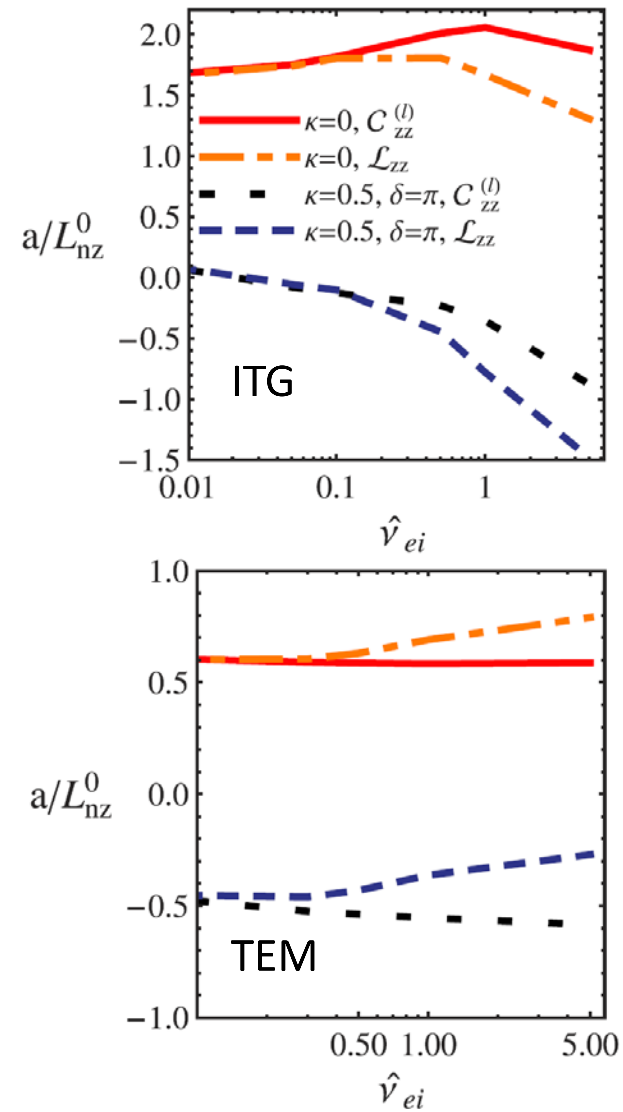
- Positive contribution if $\omega_r < 0$





When does momentum conservation of impurity self-collisions matter?

- Due to $C[g_1 \propto v_{||} f_{z0}] = 0$ collisional effects on the impurity particle flux do not appear to order $1/Z$
- When a Lorentz operator is used a factor $1/\left(1 + i\nu_D\left(x\right)/\omega\right)$ appears in the parallel compressibility contribution
- Momentum conservation of impurity self-collisions can be important in the edge



Conclusions

- A poloidally varying non-fluctuating electrostatic potential can significantly modify the peaking factor of high-Z impurities.
- The effect becomes important when the impurity charge is high enough that the magnetic drifts are reduced to the level of the $\mathbf{E}_\theta \times \mathbf{B}_\phi$ drift.
- A momentum conserving model for impurity self-collisions is important for an accurate evaluation of impurity transport when the collision frequency is comparable to the mode frequency or larger.

A few things to be addressed when applying the model to the edge

- The impurity transport is assumed to be purely turbulent.
- ϕ_E is externally given, and the poloidal variation of the impurity density is determined only by this parameter.
- Sources are neglected
- Finite Mach number effects are neglected.
- Include finite orbit width effects, especially in the steepest region of the pedestal.
- Evaluate the relevance of non-locality due to reduced separation between parallel and perpendicular transport times close to the separatrix.