Stellarator in a Box:

Understanding ITG turbulence in stellarator geometries

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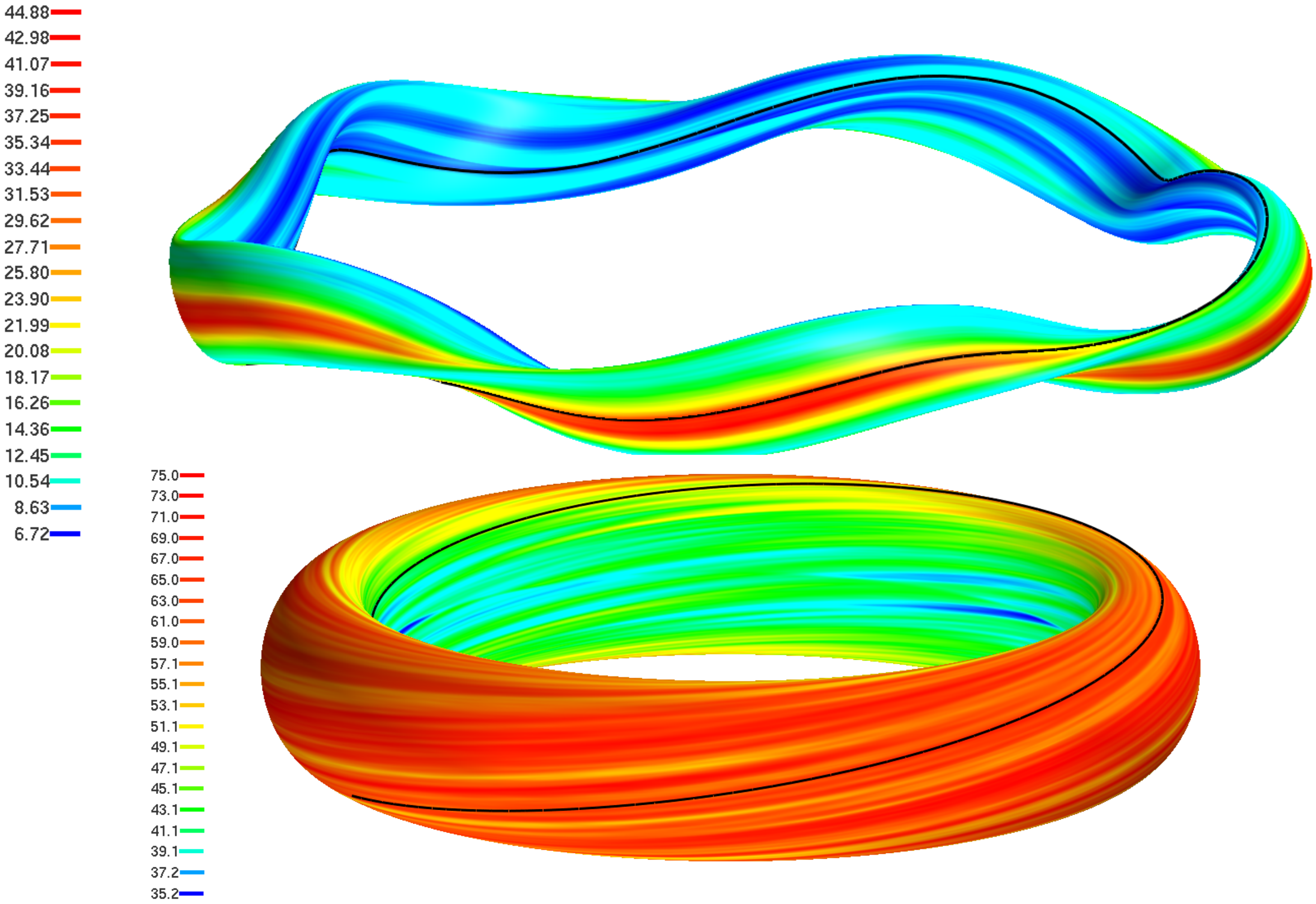
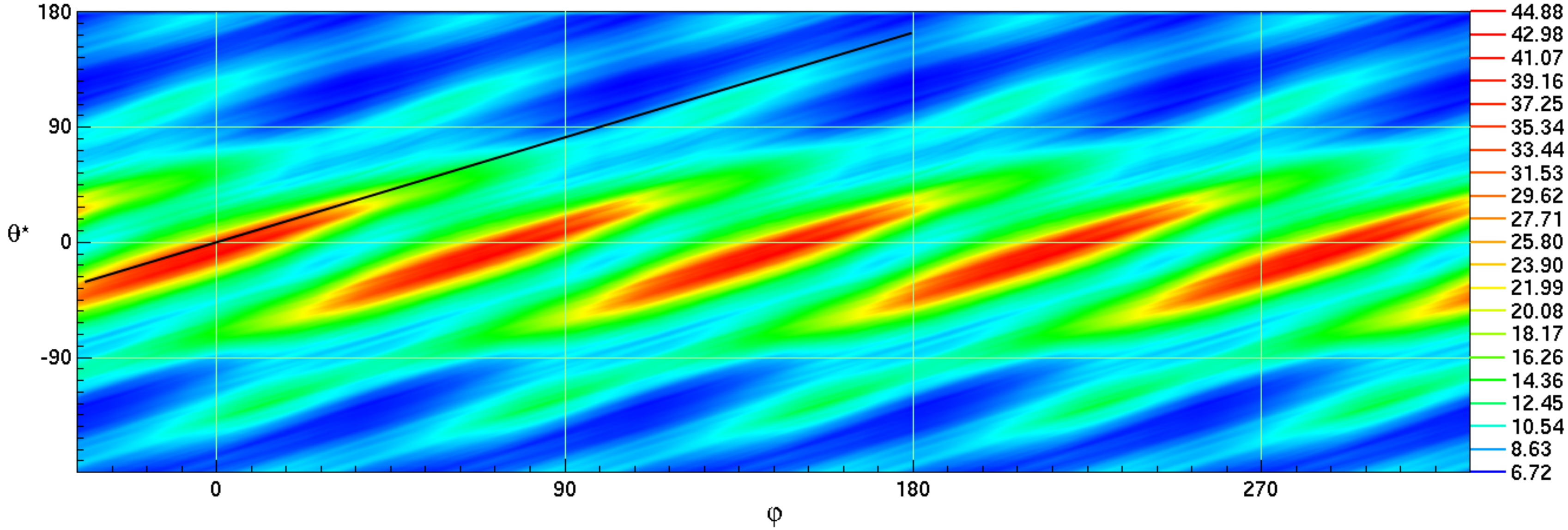
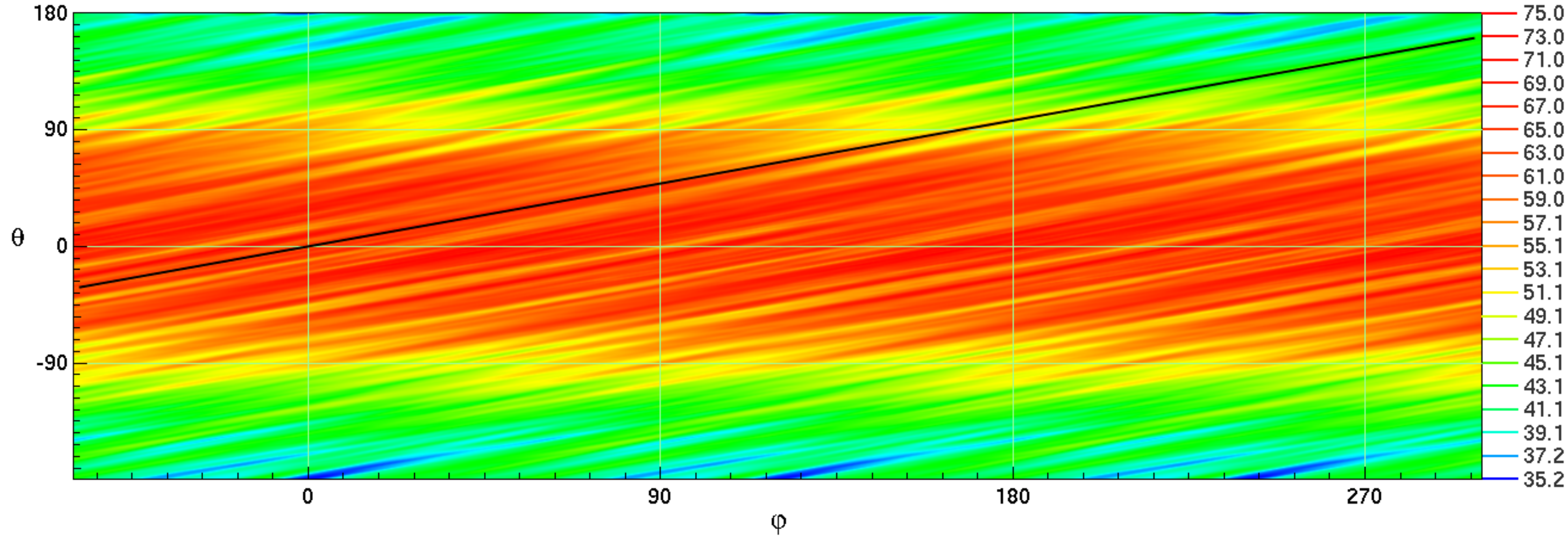
W7X vs. a Tokamak

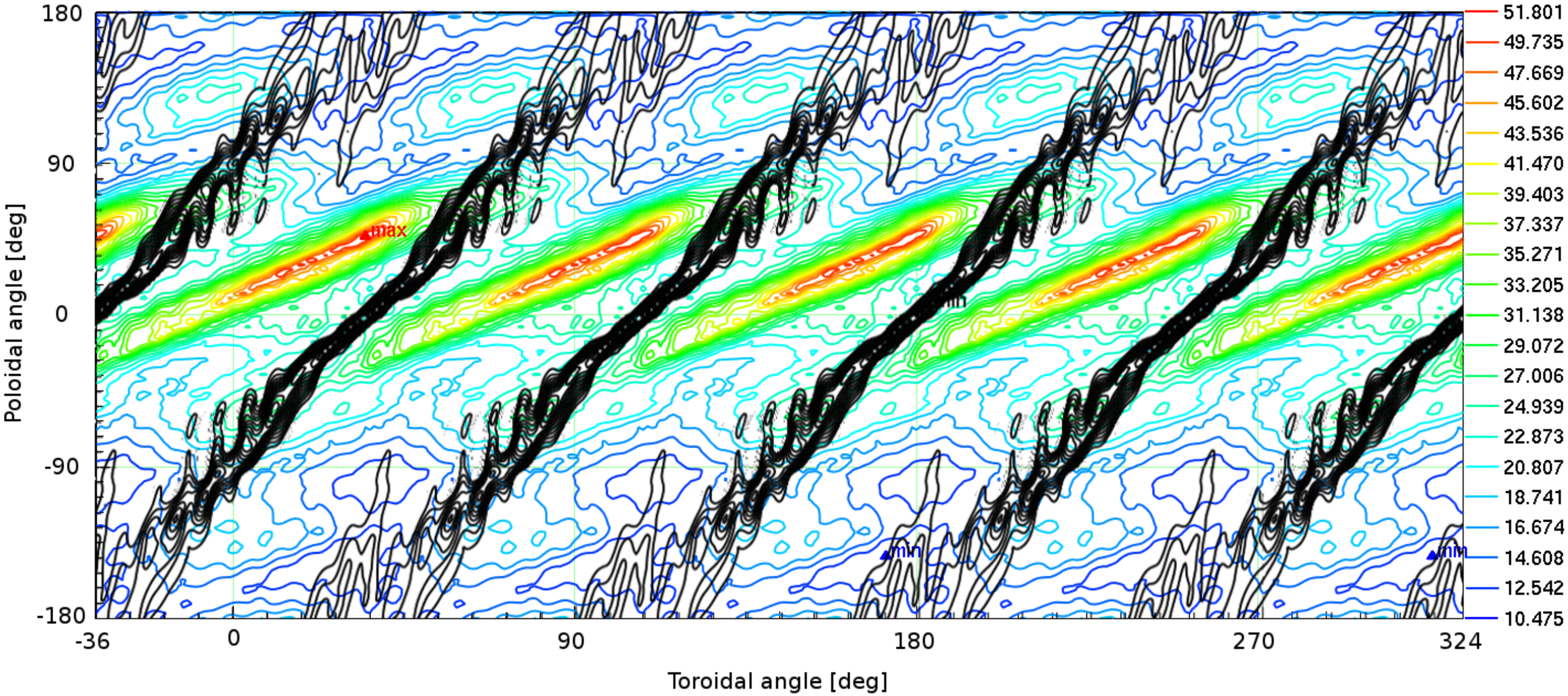


W7X vs. a Tokamak (r.m.s. ϕ)





Turbulence in stellarators is localized along the magnetic field (W7X)



Black contours show local magnetic shear

Credit: P. Xanthopoulos; see [P Helander et al 2012 Plasma Phys. Control. Fusion 54 124009]

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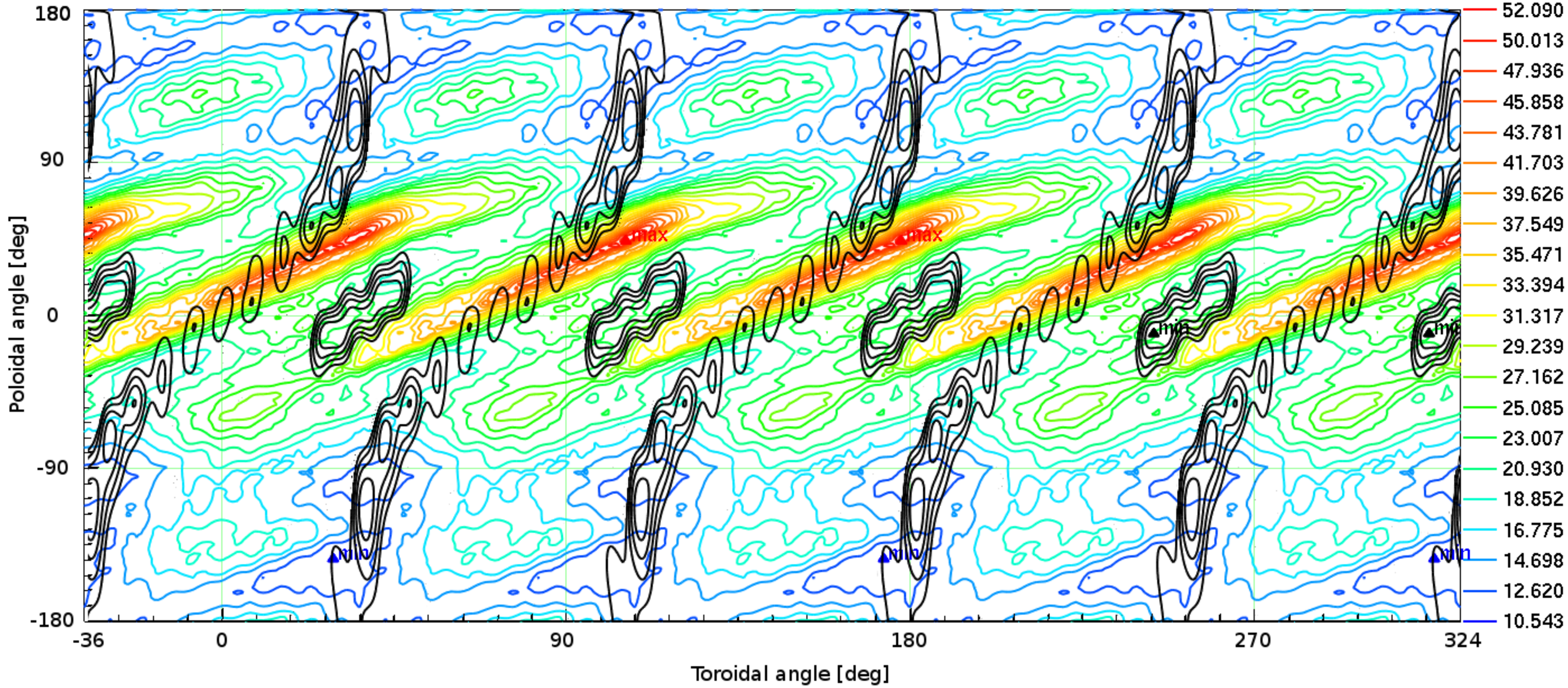


Black contours show normal curvature

Credit: P. Xanthopoulos; see [P Helander et al 2012 Plasma Phys. Control. Fusion 54 124009]

Overview

- Localization of Turbulence
- Part I: Local Magnetic Shear*
  - Model integral equation
  - Numerical experiments with Gene
- Part II: “Bad curvature” wells
  - “Boxing” or “Ballooning?”
  - SA-W7X: looks like a tokamak, feels like W7X
- Part III: Nonlinear theory (TBC)

*[R. E. Waltz and A. H. Boozer, Phys. Fluids B 5, 2201 (1993)]

Overture: The “Local” ITG Mode

Wave solution $\sim \exp(i(k_{\|} z + \mathbf{k}_{\perp} \cdot \mathbf{R}_{\perp}) - i\omega t)$
with dispersion relation:

$$D(\omega, \mathbf{k}, \omega_*, \omega_d, \eta) = 0$$

“**Toroidal branch**” ordering

- $\omega \sim \omega_* \sim \omega_d > k_{\|} v_{\mathrm{T}}$
- fininte parallel wavenumber is generally stabilizing

“**Slab branch**” ordering

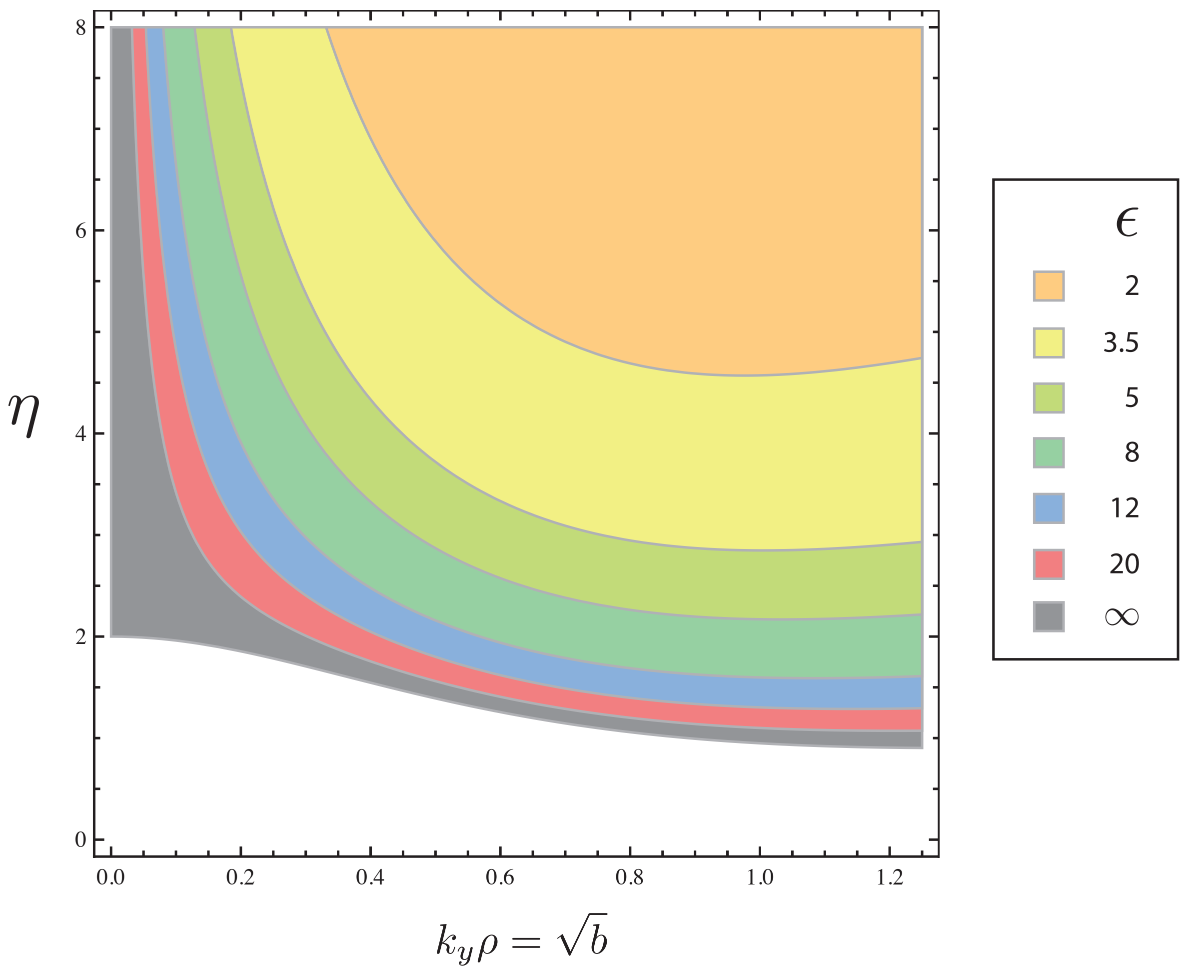
- $\omega \sim \omega_* \sim k_{\|} v_{\mathrm{T}} > \omega_d$
- most unstable mode is at finite parallel wavenumber

$$0 = \tau + 1 + \zeta Z(\zeta) + \Omega_* [(\eta/2 - 1) Z(\zeta) - \eta\zeta(\zeta Z(\zeta) + 1)]$$

$\xi = \omega/(k_{\|} v_{\mathrm{T}})$ *[Kadomtsev & Pogutse, Rev. Plasma Phys., Vol. 5 (1970)]*

Stabilizing Effect of Parallel "Localization" on Slab Mode

Conventional Slab ITG Stability Boundaries



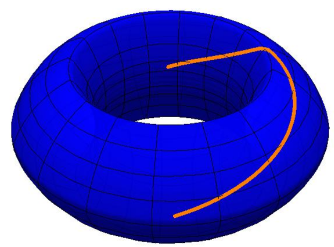
$$\omega_d/\omega_* = 0$$

$$\eta = \frac{L_n}{L_T}$$

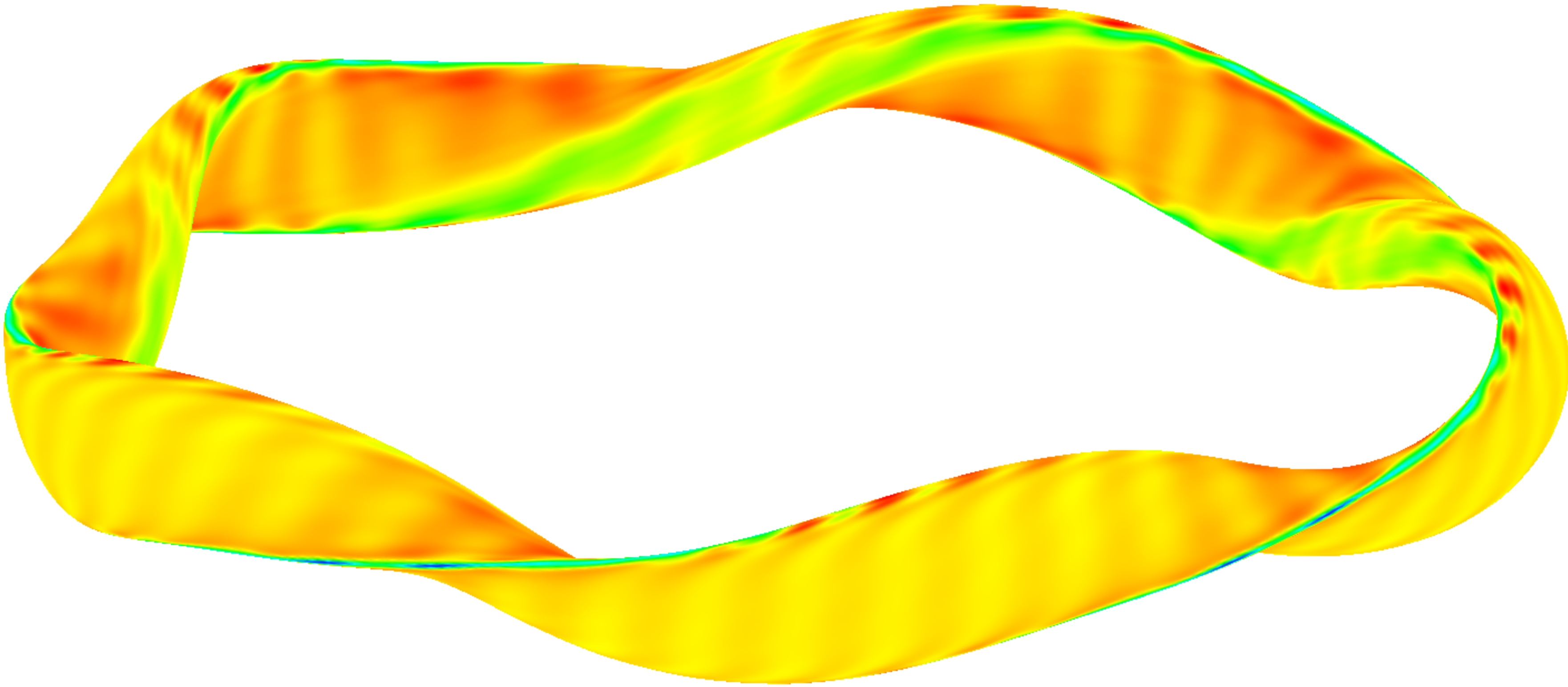
$$\epsilon = \frac{L_\parallel}{L_n} = \frac{\pi}{k_\parallel L_n}$$

Tokamak estimate:

$$k_\parallel \sim 1/(qR)$$



Part I: Local Magnetic Shear.



W7X

Theoretical Model*

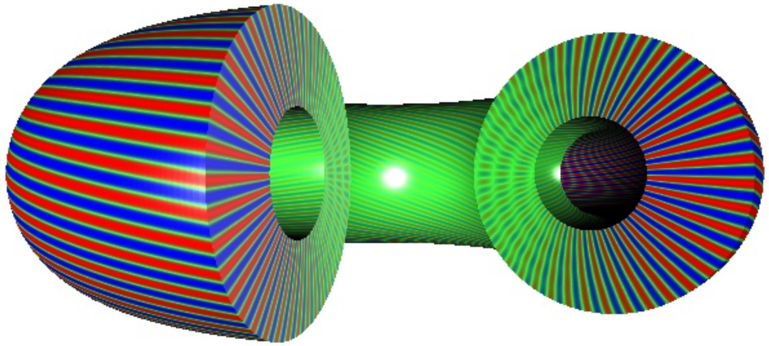
** [J W Connor et al, 1980 Plasma Phys. 22 757]*

Eikonal representation for non-adiabatic part of δf1:

$$g = \hat{g}(\ell)\exp(iS - i\omega t)$$

$$\boldsymbol{\nabla} S \equiv \mathbf{k}_{\perp} = k_{\alpha}\boldsymbol{\nabla}\alpha + k_{\psi}\boldsymbol{\nabla}\psi$$

"Ballooning"



[Source: http:GYRO]

Linear GK Equation:

$$v_{\|}\frac{\partial g}{\partial \ell} - i(\omega - \omega_d)g = -i\varphi J_0(\omega - \omega_*^{\mathrm{T}})f_0$$

Solution:

$$g(\ell) = g_0\exp(-i\sigma M(\ell_0,\ell)) - i\sigma(\omega - \omega_*^{\mathrm{T}})f_0\int_{\ell_0}^{\ell}\frac{J_0'}{|v_{\|}'|}\varphi(\ell')\exp(i\sigma M(\ell',\ell))d\ell'$$

$$\sigma = \mathrm{sign}(v_{\|}) \qquad M(\ell_0,\ell) = \int_{\ell_0}^{\ell}\frac{\omega - \omega_d(\ell')}{|v_{\|}(\ell')|}d\ell'$$

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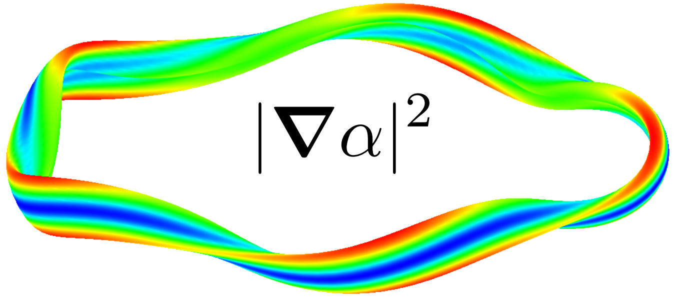

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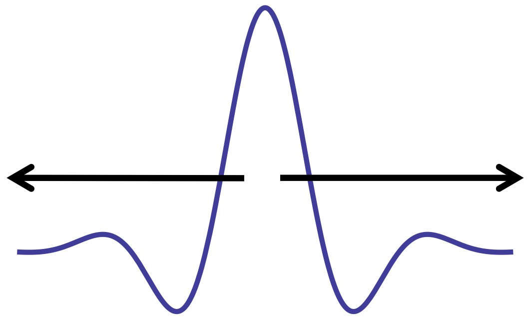
Theory…

Outgoing boundary conditions

$$g(v_{||} > 0, \ell = -\infty) = 0$$

$$g(v_{||} < 0, \ell = \infty) = 0$$



Solution becomes:

$$g(\ell) = -i\sigma(\omega - \omega_*^{\mathrm{T}}) f_0 \int_{-\sigma\infty}^{\ell} \frac{J_0'}{|v_{||}'|} \varphi(\ell') \exp(i\sigma M(\ell', \ell)) d\ell'$$

Approximation:

$$k_\perp(\ell) = \begin{cases} k_{\perp 0} & \text{if } |\ell| < L, \\ \infty & \text{if } |\ell| > L. \end{cases}$$

$$J_0' = 0 \text{ for } |\ell'| > L$$

Substitute solution into quasineutrality equation

$$\int d^3\mathbf{v} J_0 g = n(1 + \tau)\varphi$$

Integral Equation

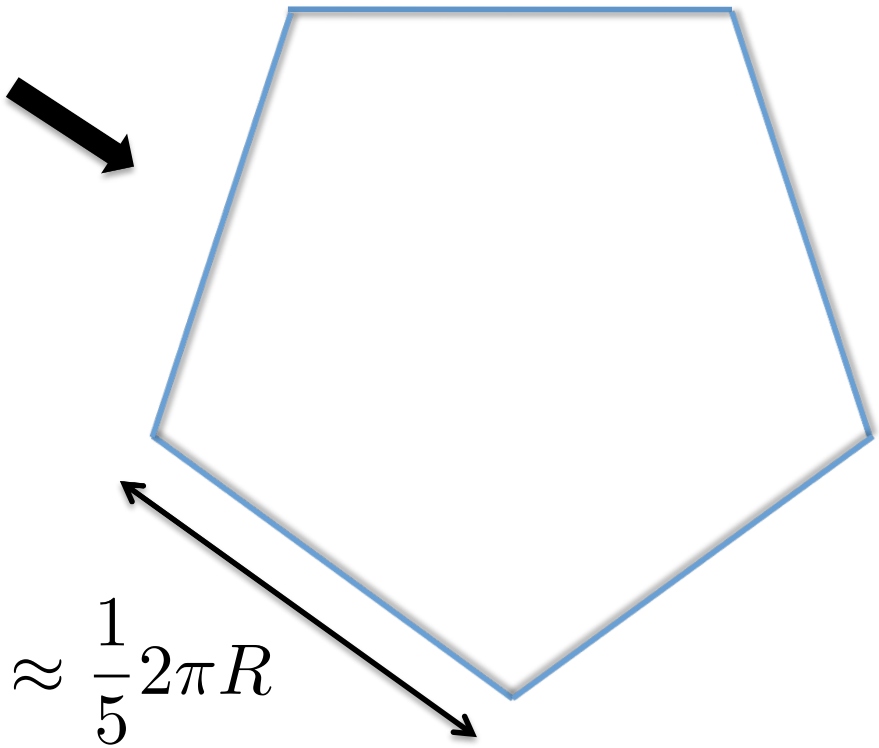
$$\boxed{\varphi(\ell) = \int_{-1}^{1} K(\ell', \ell, \omega)\varphi(\ell')d\ell'}$$

$$K(\ell', \ell, \omega) = \frac{i/\sqrt{\pi}}{1+\tau}\left[\eta\Omega_*\Gamma_0 F_1(\Omega|\ell - \ell'|) - ((\Omega - \Omega_*(1 - \eta b - \eta/2))\Gamma_0 - \eta\Omega_* b\Gamma_1)F_0(\Omega|\ell - \ell'|)\right]$$

$$F_n(y) = \int_0^\infty \exp(-x^2 + iy/x)\, x^{2n-1} dx \qquad \Gamma_n(b) = \exp(-b) I_n(b)$$

$\Omega = \omega L/v_{\mathrm{T}}$	$\Omega_* = \omega_* L/v_{\mathrm{T}}$	$\Omega_d = \omega_d L/v_{\mathrm{T}}$
$x_\perp = v_\perp/v_{\mathrm{T}}$	$x_\parallel = \lvert v_\parallel\rvert/v_{\mathrm{T}}$	$x^2 = x_\perp^2 + x_\parallel^2$

W7-X Parameters



Aspect Ratio $\frac{R}{a} \approx 11$

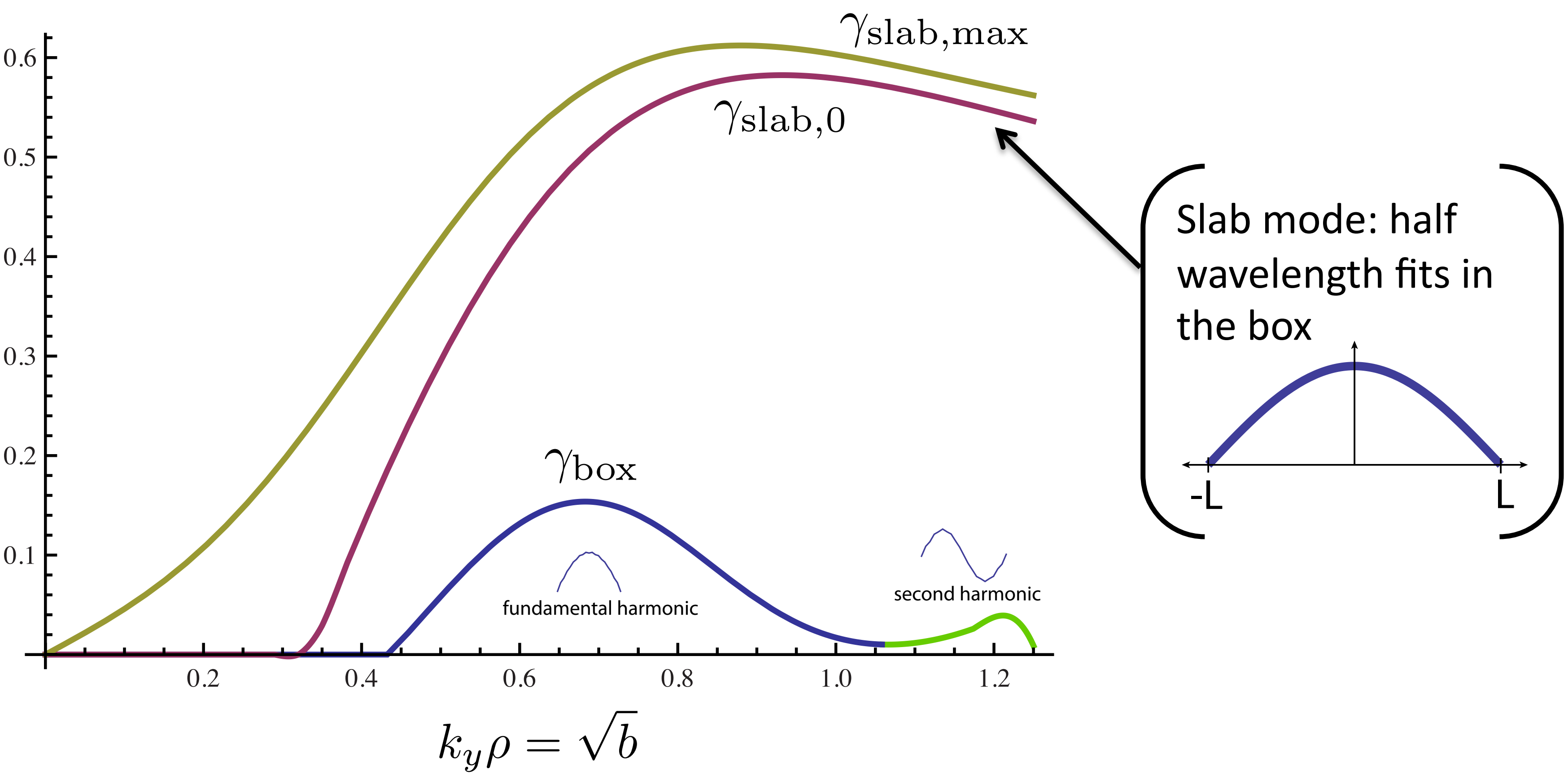
Also let $\frac{a}{L_n} \in [1, 3]$

So $\frac{L}{L_n} \in [7, 20]$ $\qquad 2L \approx \frac{1}{5} 2\pi R$

Growth rate curves (pure slab)

$$L/L_n = 7.0$$

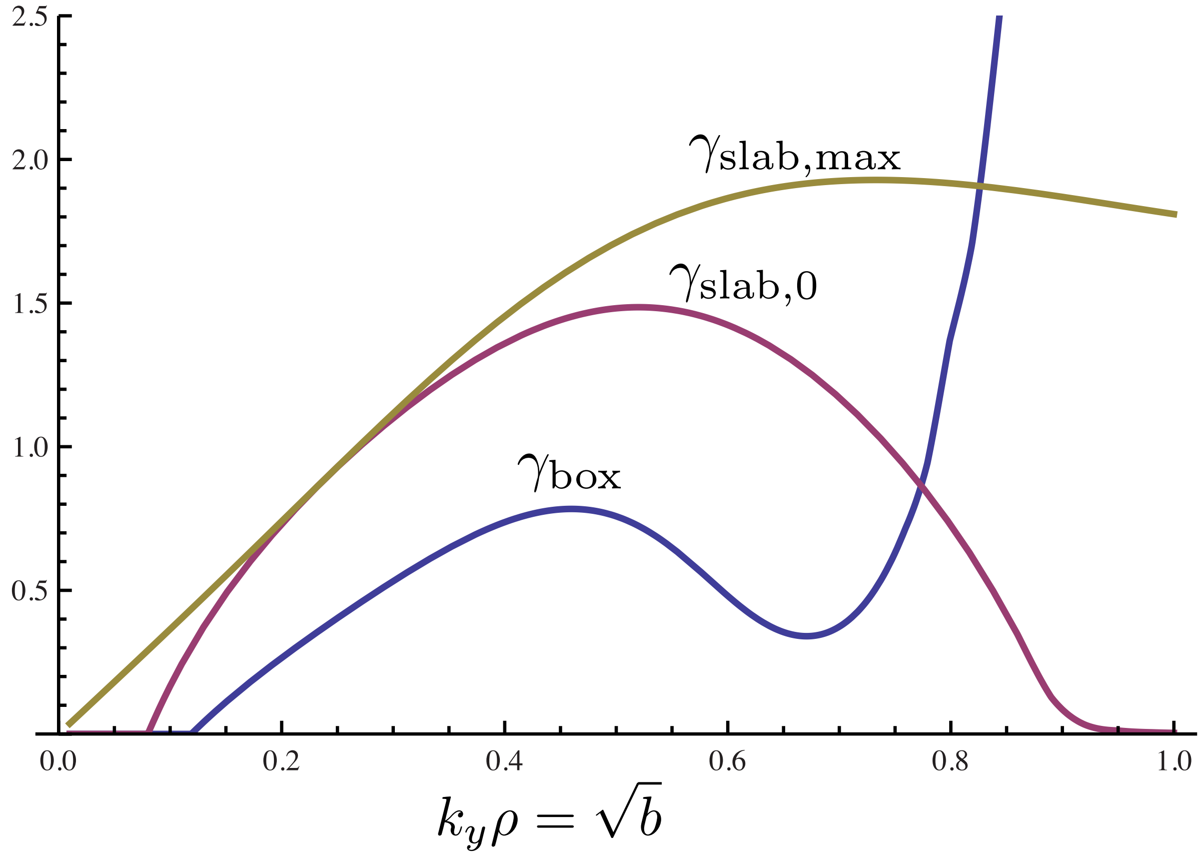
$$\eta = 3.0 \;\; (a/L_n = 1.0, a/L_T = 3.0)$$



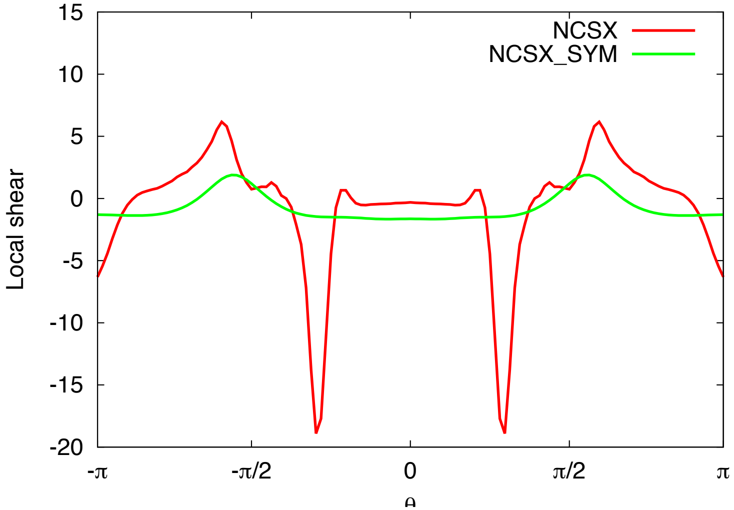
Growth rate curves (pure slab)

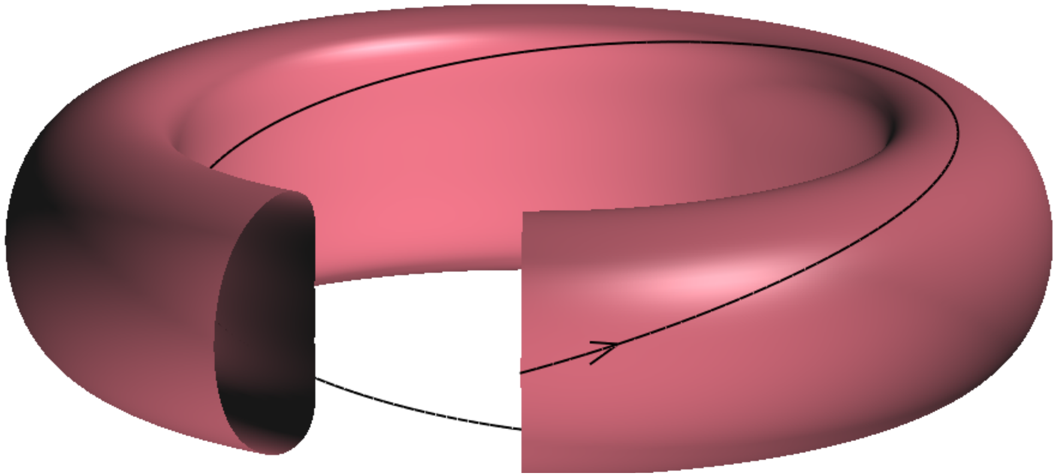
$$L/L_n = 7.0$$

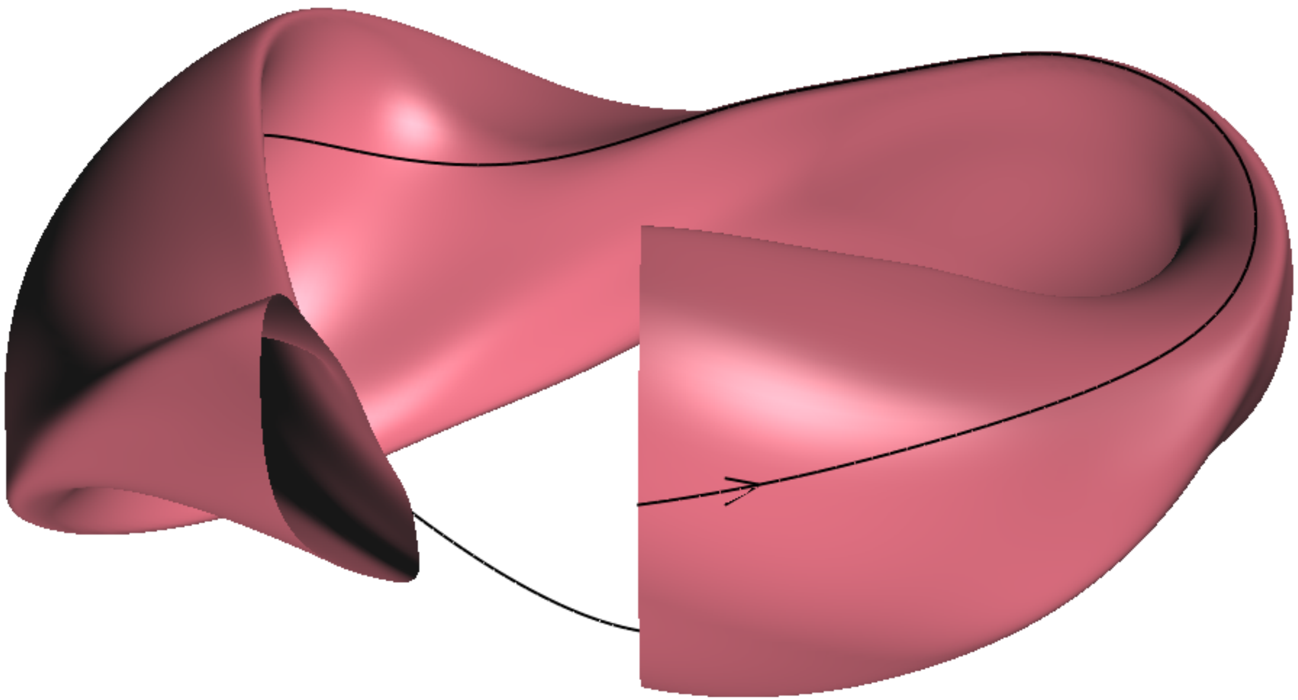
$$\eta = 9.0 \ \ (a/L_n = 1.0, a/L_T = 9.0)$$



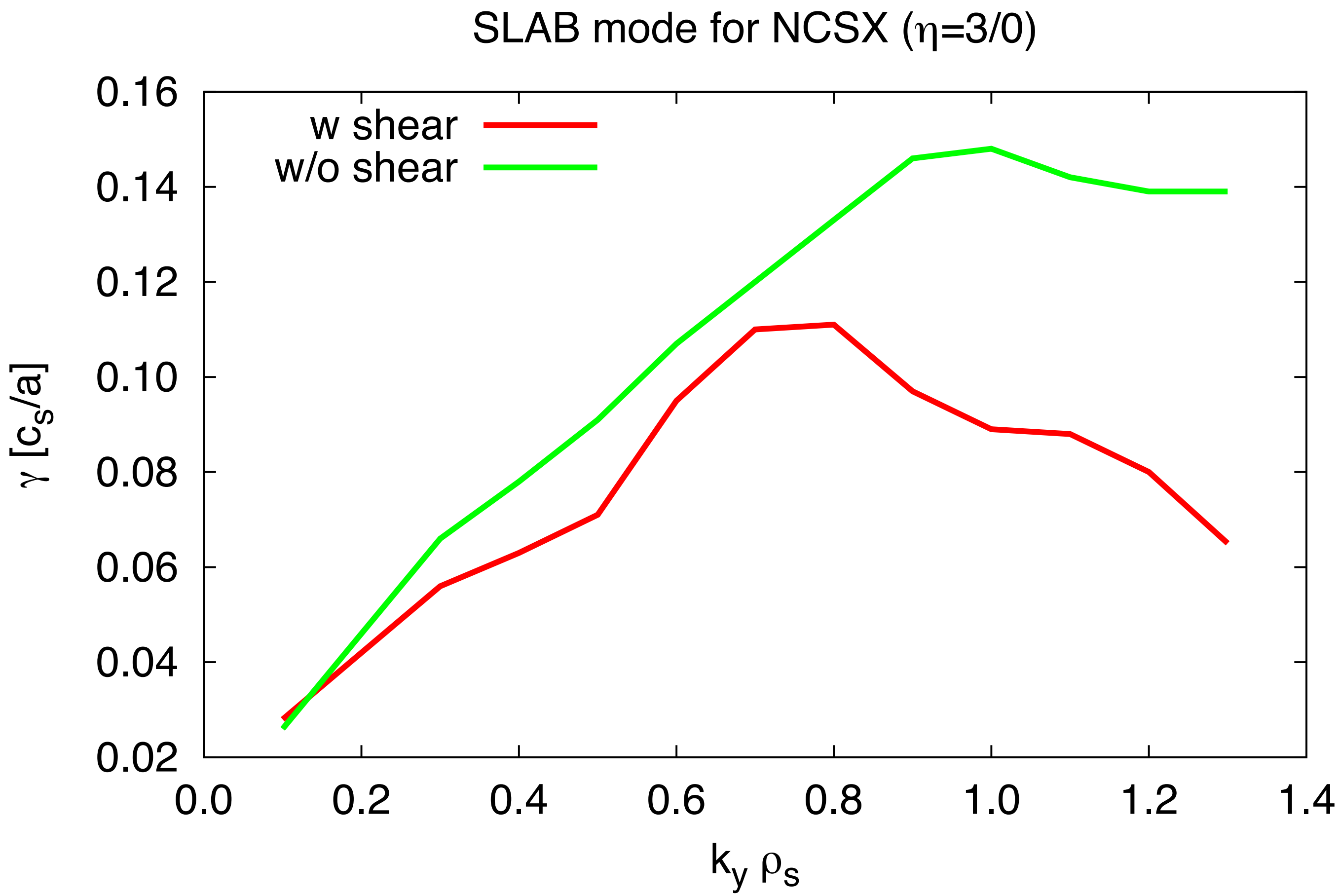
NCSX Linear Simulations



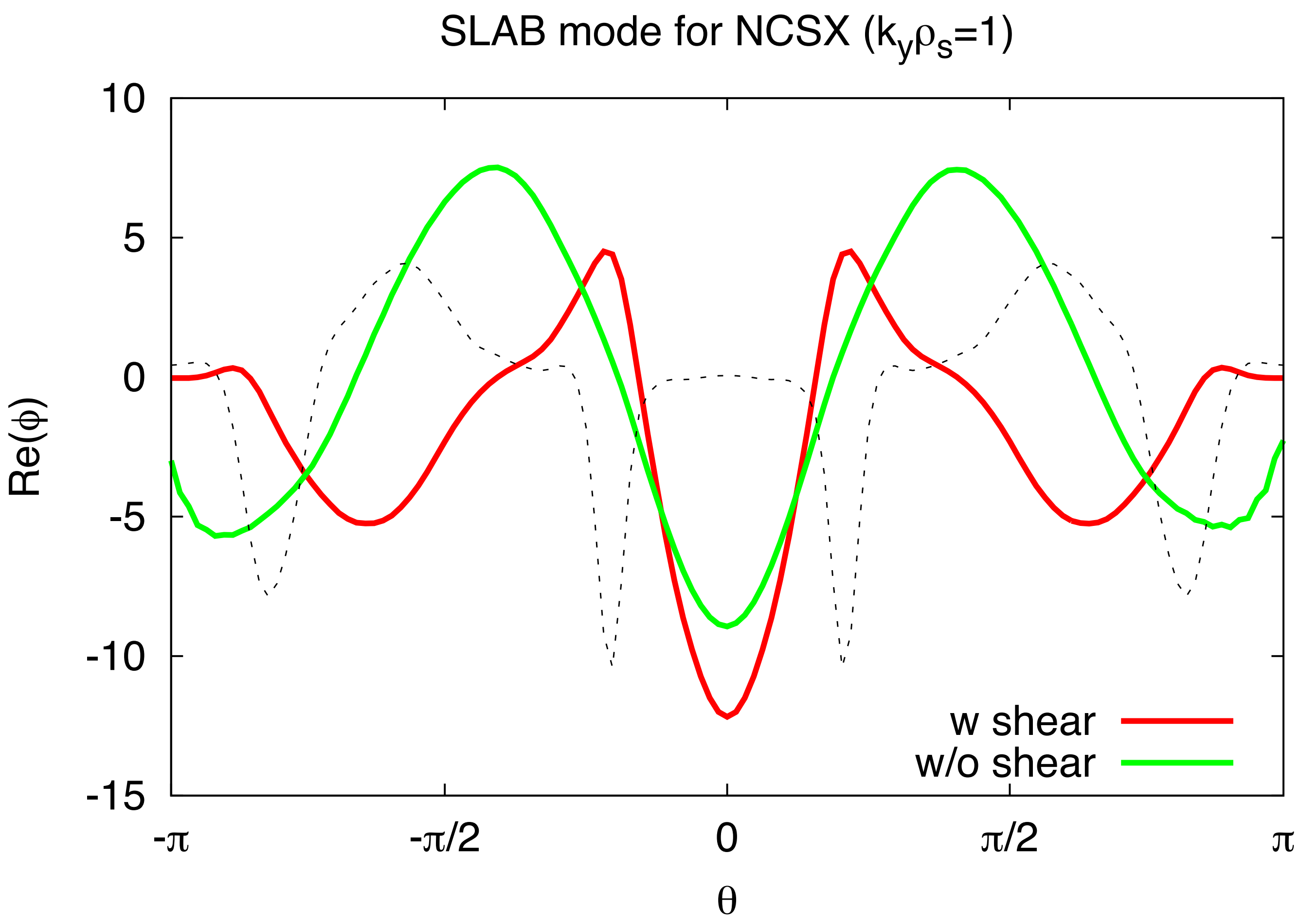




NCSX minus curvature



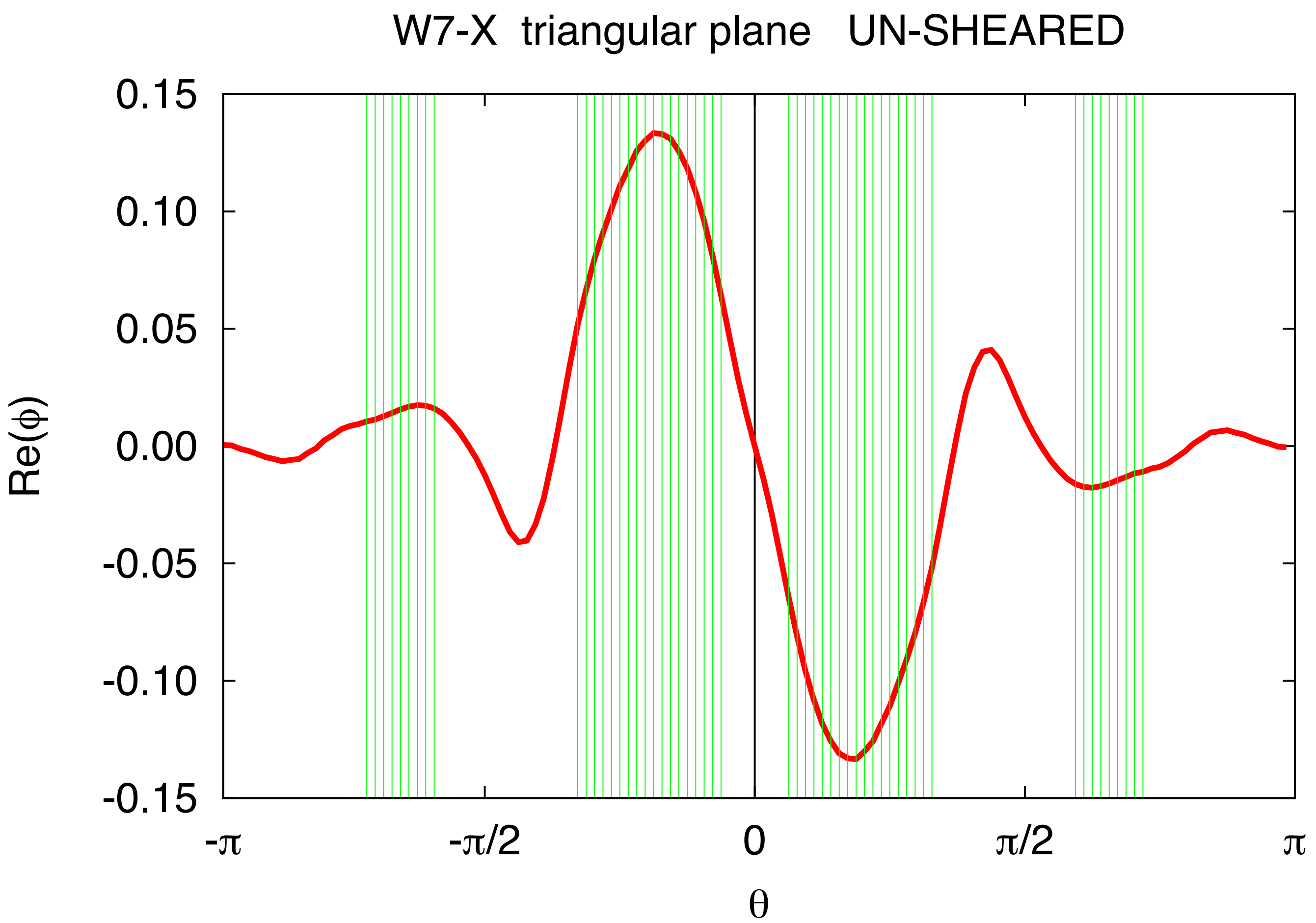
Eigenmode Structure for kρ ~ 1



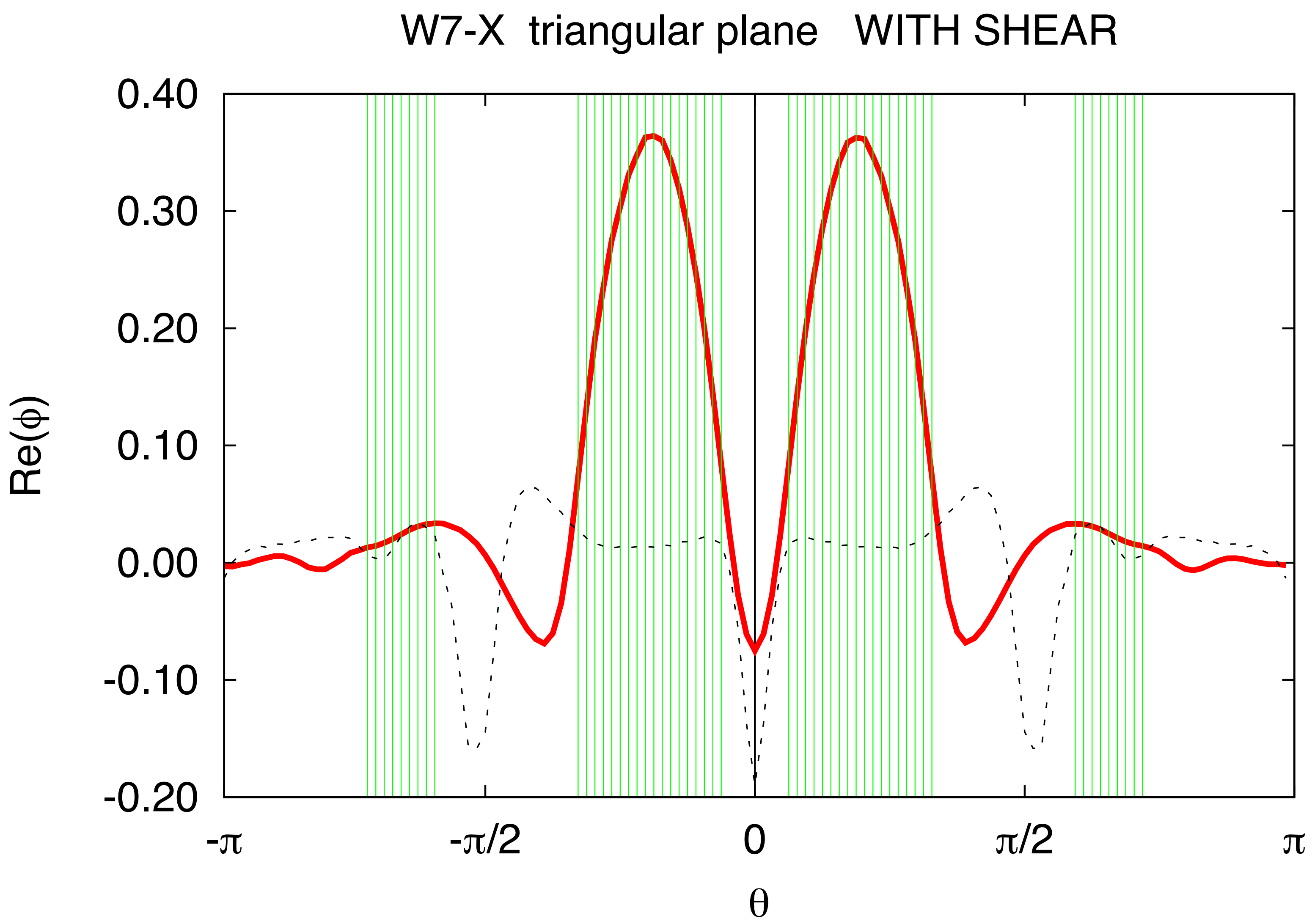
Linear Simulations using GENE W7-X (including curvature)

- center of flux tube: on the helical ridge, at the "triangular plane"

Eigenmode Structure for kρ ~ 1
W7-X No Shear

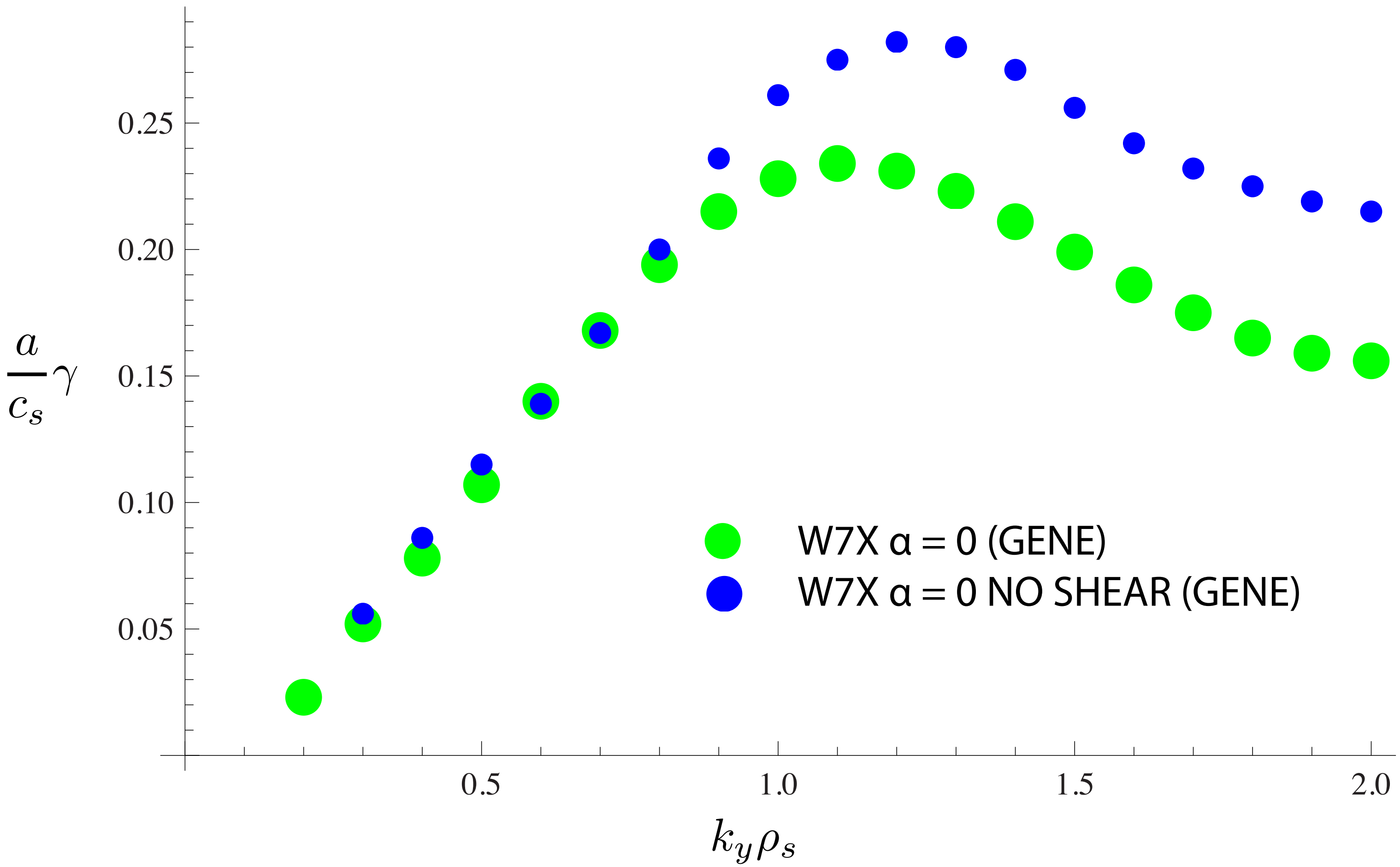


Eigenmode Structure for kρ ~ 1 W7-X with Shear



How much does shear "boxing" actually matter?

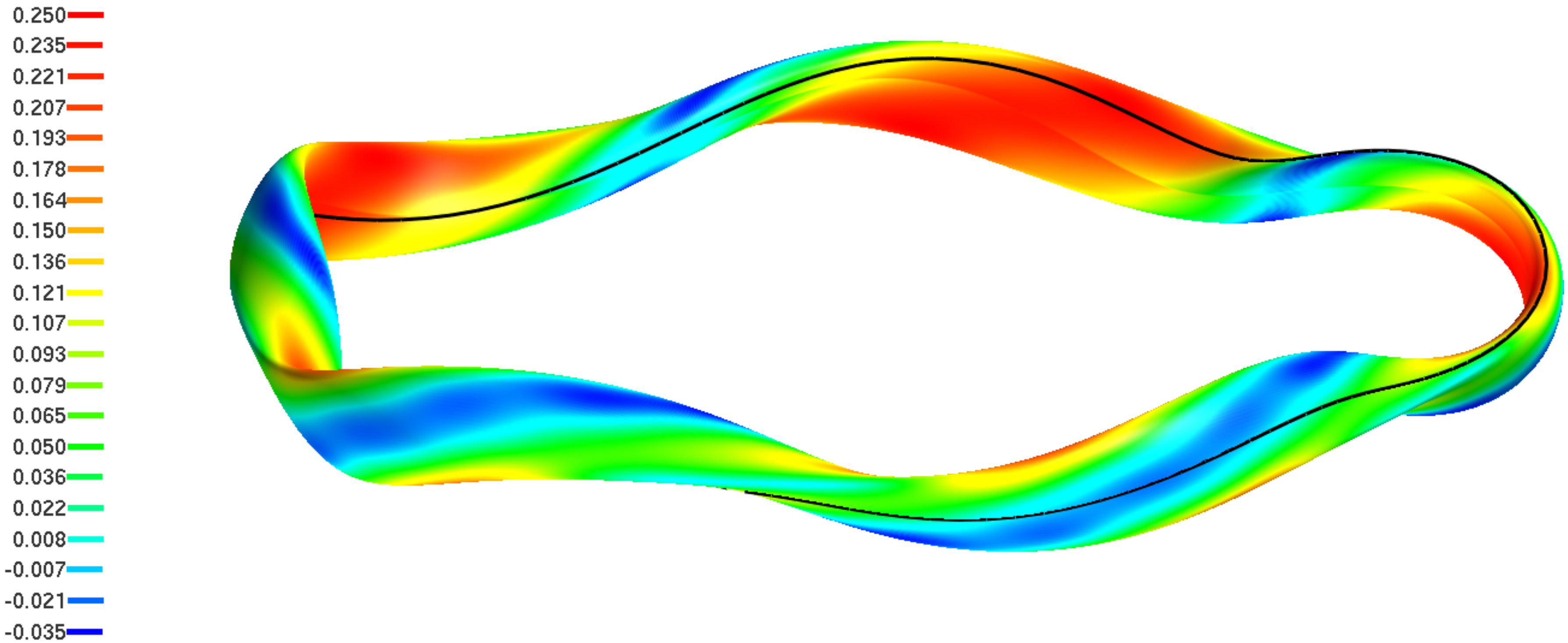
(a/L_n = 1.0, a/L_T = 3.0)



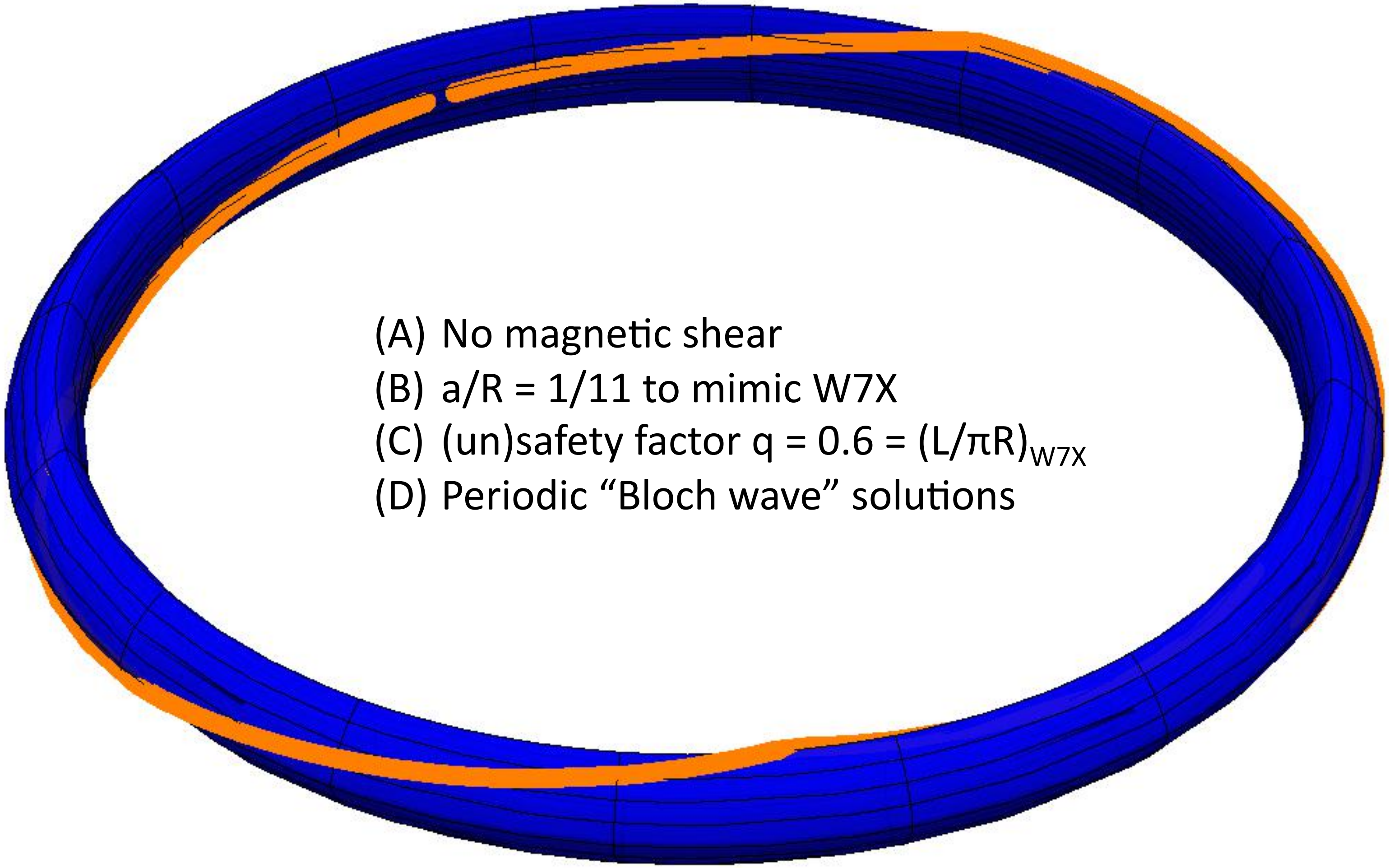
Part II: "Bad curvature"

- "Ballooning" = mode localization via global magnetic shear
- "Boxing" = mode localization via sudden shear spike
- **Option 3**: Curvature wells set the parallel extent (*i.e.* the usual rule-of-thumb $k_{||} = 1/qR$)

Curvature Landscape of W7X



“S-Alpha W7X”



(A) No magnetic shear
(B) $a/R = 1/11$ to mimic W7X
(C) (un)safety factor $q = 0.6 = (L/\pi R)_{W7X}$
(D) Periodic “Bloch wave” solutions

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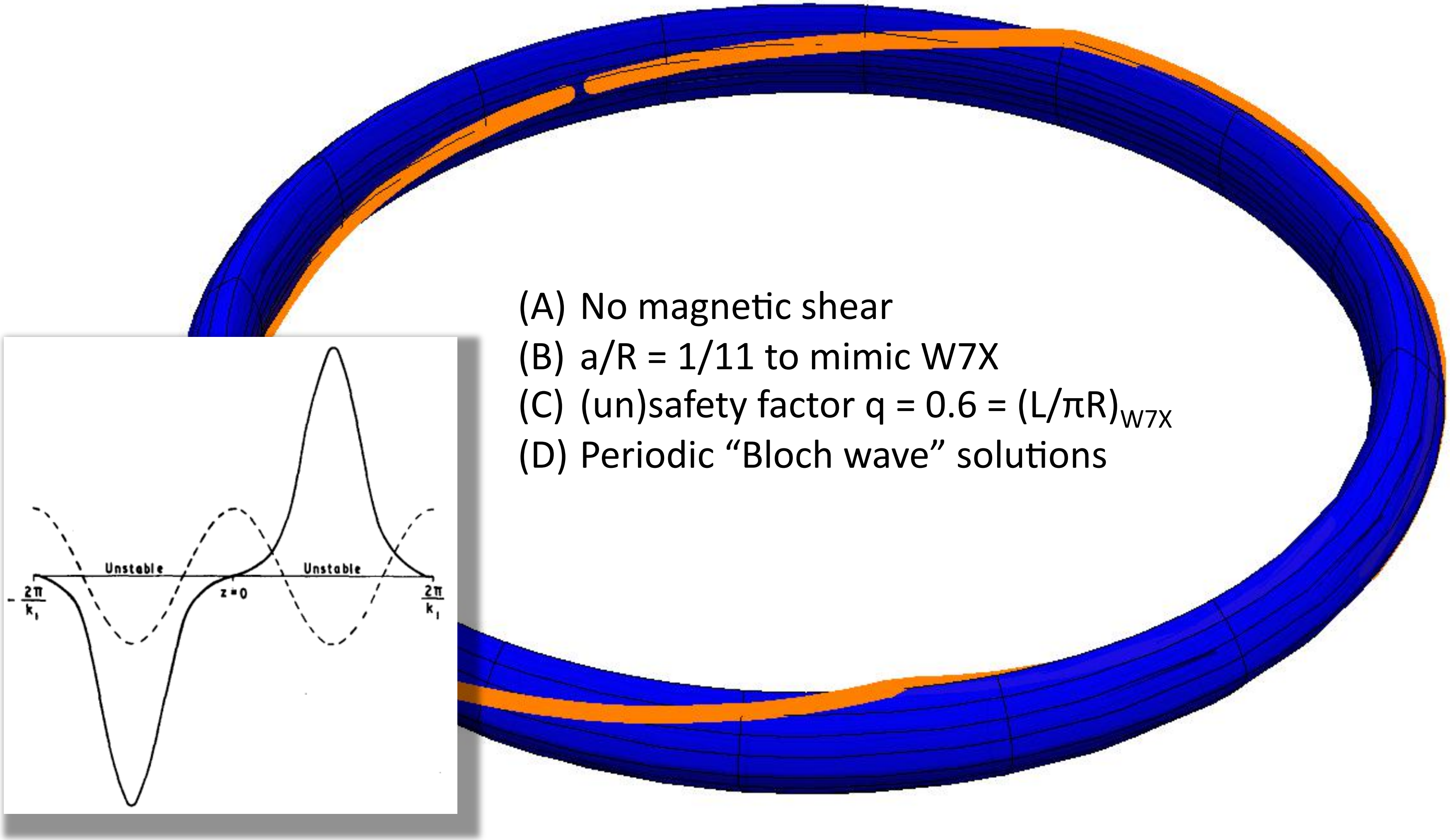

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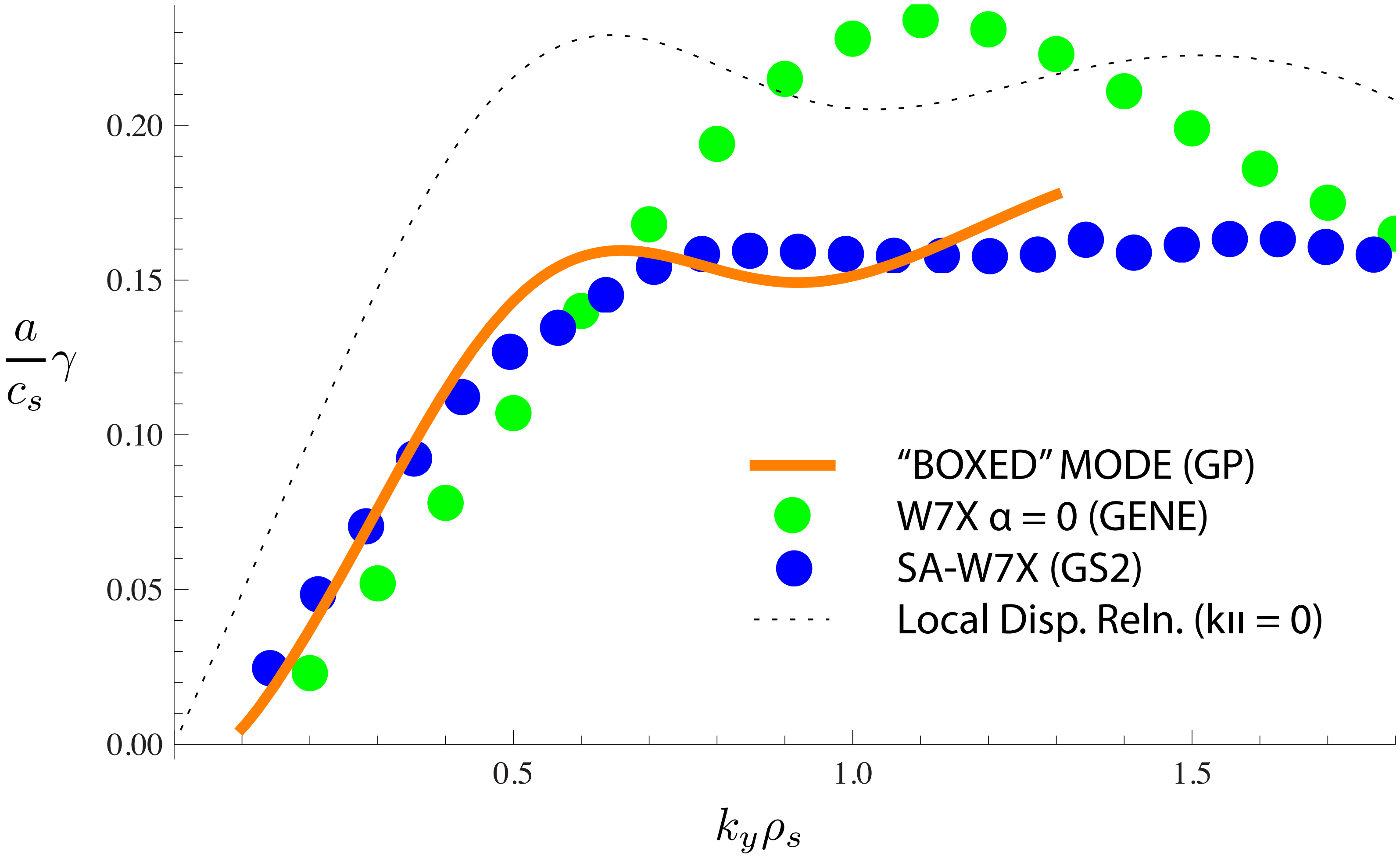
Jukes, Rohlena, Phys. Fluids 11, 891 (1968)

Growth Rate Comparison

(a/Ln = 1.0, a/LT = 3.0)



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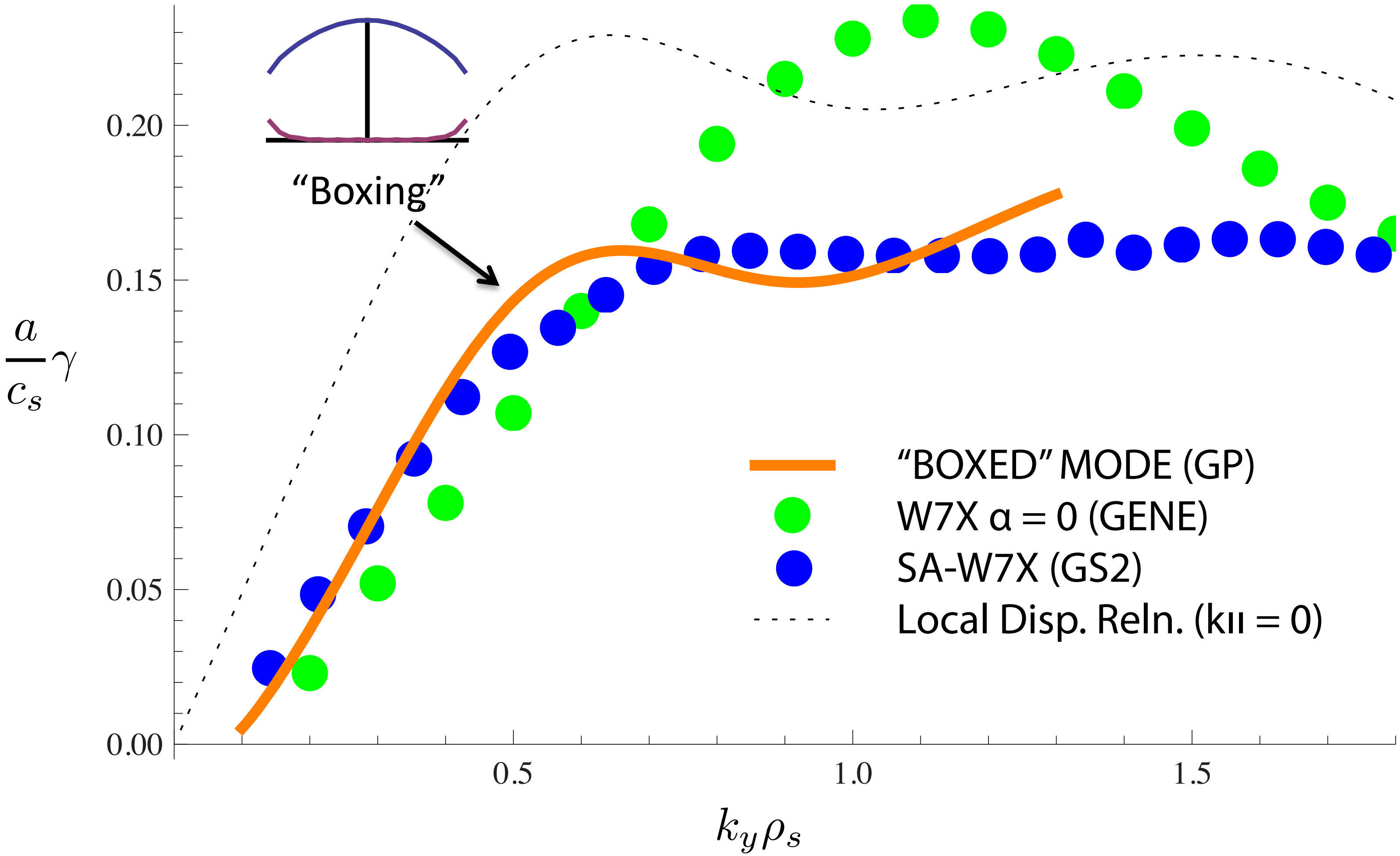

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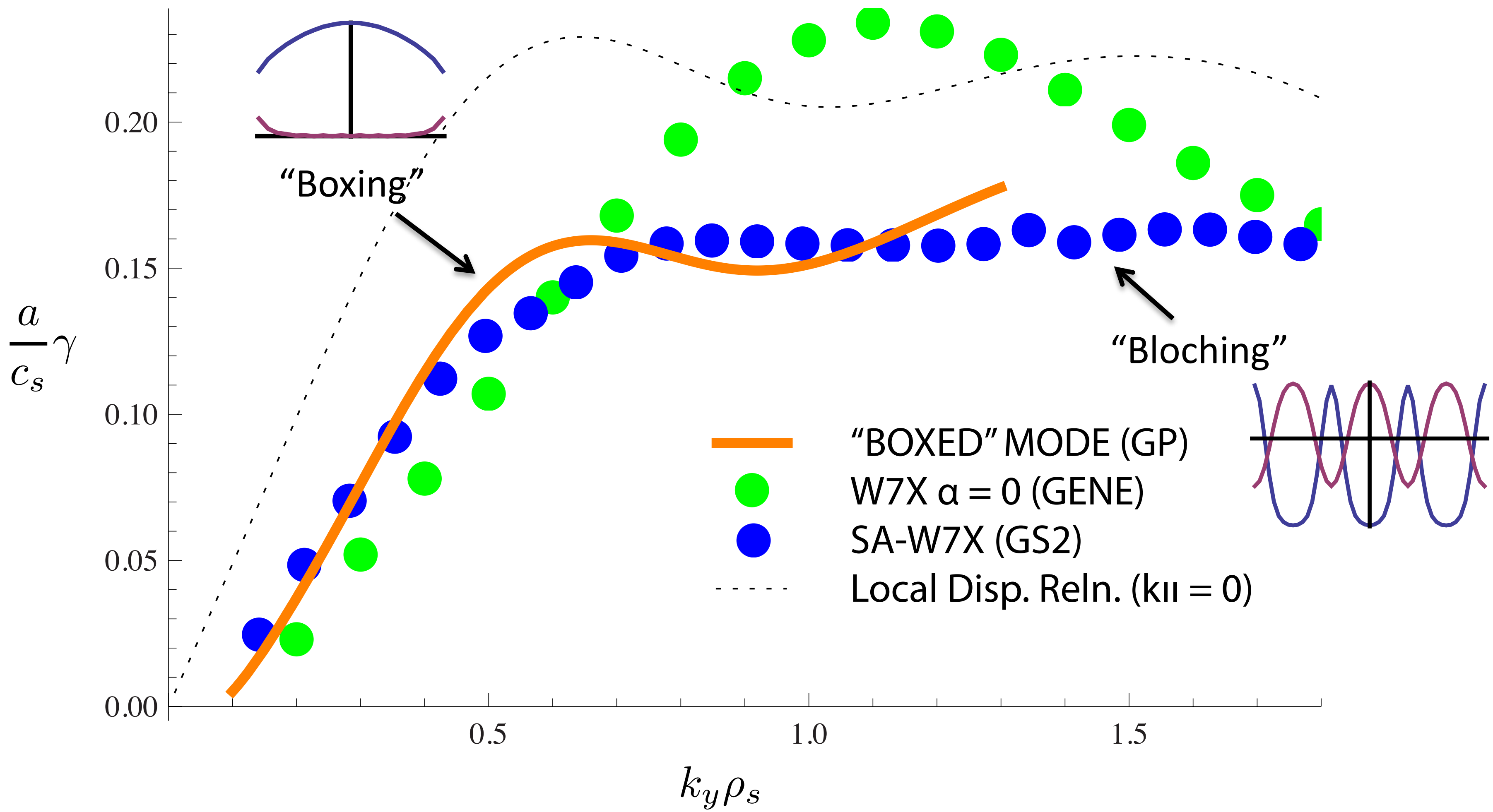

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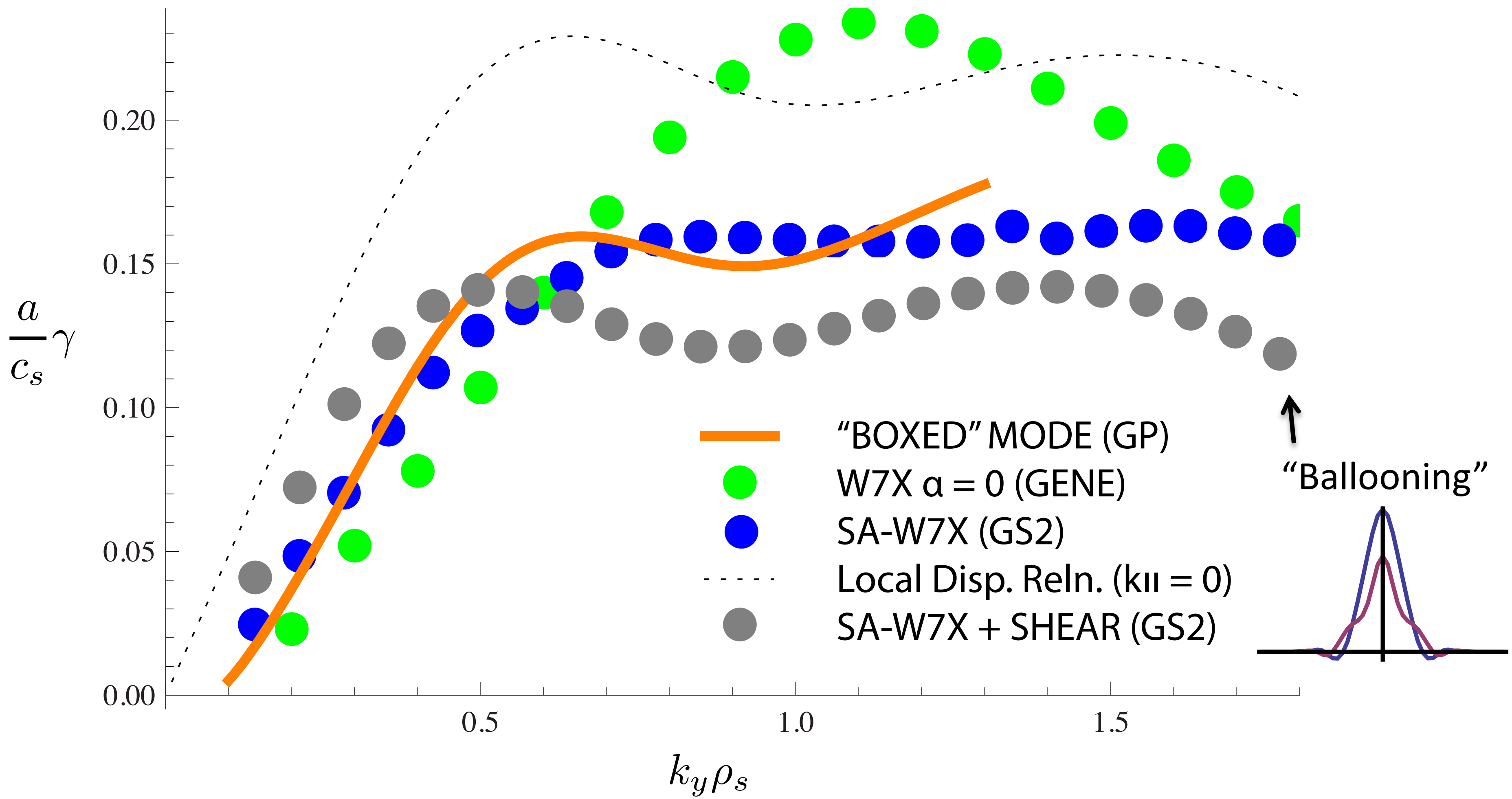
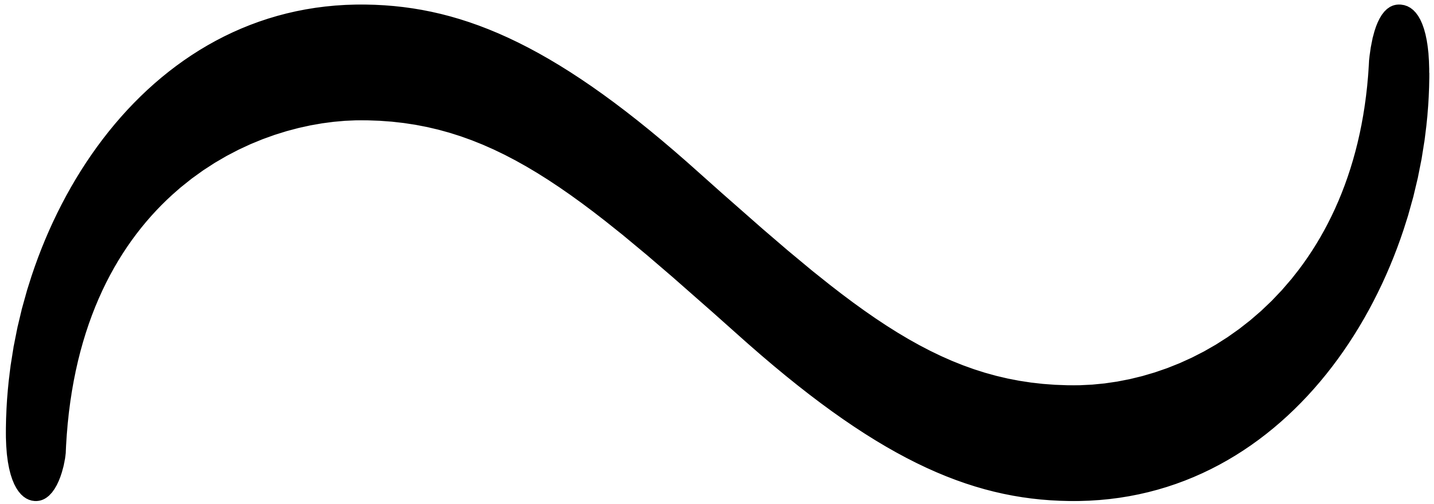


Part III: Nonlinear Theory

(a preview)



Conclusion

- Generally, magnetic geometry determines parallel variation of ITG turbulence in two ways – curvature and (local or global) magnetic shear
  - Theoretical model of shear "boxed" ITG mode demonstrates the potential for strong stabilization.
  - At large k, numerical simulations confirm significant stabilizing influence of local magnetic shear.
- W7X ITG mode behaves as conventional curvature ("toroidal branch") mode at low k, matching closely to a fictitious tokamak (s-alpha-W7X) ITG mode at the same parameter point.
- **Summary**: Simple "boxed" models capture the ITG mode in a flux tube. The modes match expectations from Tokamak calculation at a similar parameter point.
  - Exotic stellarator geometry effects do not seem to strongly effect the ITG mode.
  - We can understand the turbulence with existing theoretical machinery (+ epsilon)