Electric Field Effects in a Tokamak Pedestal

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Motivation: pedestal = transport barrier

- Higher energy content
- Larger energy confinement time

Existence of the pedestal associated with decreased transport and turbulence

Density pedestal results in strong radial electric field and electrostatically confined banana regime ions

Pedestal width \( w \sim \rho_{\text{pol}} \)
Particle orbits in pedestal

Strong radial electric field: \[(v_{||} \bar{n} + \bar{v}_E) \cdot \nabla \theta \approx (v_{||} + cI\Phi' / B) \bar{n} \cdot \nabla \theta\]

ExB drift \(\sim v_i \rho / w \sim v_i \rho / \rho_{pol} < v_{||}\), but geometry makes it comparable to poloidal projection of \(v_{||}\)
Overview of Pedestal Topics

- Version of gyrokinetics useful in pedestal: convenient for pedestal widths $w \sim$ ion poloidal gyroradius $\rho_{pol}$

- Neoclassical ion heat flux and ion flow in pedestal retaining finite radial electric field effects: must treat finite drift orbit effects
Gyrokinetic variables

\[ \mathbf{B} = I(\psi) \nabla \zeta + \nabla \zeta \times \nabla \psi = B \mathbf{n} \]

Canonical angular momentum

\[ \psi_\ast = \psi - (M_c / Z_e) R^2 \mathbf{n} \cdot \mathbf{v} \]

\[ = \psi + \Omega^{-1} \mathbf{n} \times \mathbf{v} \cdot \nabla \psi - I v_\parallel / \Omega \]

gyration drift

\[ (\rho / a) \psi \quad (\rho_{pol} / a) \psi \]

Toroidal angle \( \zeta_\ast \)

Poloidal angle \( \theta_\ast \)

Total energy \( E \)

Magnetic moment \( \mu \)

Gyrophase \( \varphi \)
**Axisymmetric gyrokinetic equation**

Axisymmetric \((\partial/\partial \zeta = 0)\) gyrokinetic equation

\[
\frac{\partial \langle f \rangle}{\partial t} + \langle \frac{d \theta}{dt} \rangle \frac{\partial \langle f \rangle}{\partial \theta_*} = \langle C\{f\} \rangle - \frac{Ze}{M} \frac{\partial \langle \Phi \rangle}{\partial t} \frac{\partial \langle f \rangle}{\partial E}
\]

Steady state \((\partial/\partial t = 0)\) to leading order in \(\rho_{\text{pol}}\): transit averaging in banana regime

\[\langle C\{f_*\} \rangle = 0\]

where \(\bar{Q} = \int \! d\tau Q / \int \! d\tau\) with \(d\tau = d\theta_* / \langle \dot{\theta}_* \rangle\)

**Are there non-Maxwellian solutions in pedestal?**

**Entropy production analysis: no!**

Pedestal ion temperature variation

In the banana regime $\partial f_*/\partial \theta_* = 0$ so $f_*(\psi_*, E, \mu)$

The only Maxwellian possible is

$$f_* = \eta \left( \frac{M}{2\pi T} \right)^{3/2} \exp \left( - \frac{Ze\Phi}{T} + \frac{M\omega^2 R^2}{2T} - \frac{Ze\omega\psi}{cT} \right) \exp \left[ - \frac{M(\bar{\nu} - \omega R\bar{\zeta})^2}{2T} \right]$$

where $\eta$, $\omega$, and $T$ are constants & $n$ is Maxwell-Boltzmann with

$$\omega = -c[\partial \Phi/\partial \psi + (T/Zen)\partial n/\partial \psi]$$

Non-isothermal modifications can only enter to next order in the $B_\rho/B$ expansion

$T, \eta, \omega$ must vary slowly compared to $\rho_{pol}$
Physical interpretation

In the core gradients are so weak ion departures from a flux surface are unimportant - can consider any given flux surface a closed system.

In the pedestal, gradients are as large as $1/\rho_{\text{pol}}$ so drift departures affect the equilibration of neighboring flux surfaces - the entire pedestal region is a closed system (rather than individual flux surfaces).
The $T_i$ gradient for the bulk ion in a DIII-D ECH H-mode pedestal is small relative to gradients in electron temperature and density.

DIII-D: Edge $T_i$ for bulk ion He$^{++}$

diiid 120239.01800 ne(bk) 4ti(r) 4te(bli) vs R midplane

- $B_{pol} = 0.44$ T
- $n_e(10^{19})$
- $4^{*}T_i$(keV)
- $4^{*}T_e$
- $\bar{n}_e = 4.4e19$
- $B_T = 1.77$ T
- $I_p = 1.66$ MA
- $q_{95} = 2.7$
- $\beta_N = 0.53$
- $\kappa = 1.8$
- LSN

The last closed flux surface

- The thermal ion full banana width is computed to be $2\rho_0 = 10$ mm for He$^{++}$ at the top of the density pedestal.
- The smooth spline fits to the data (solid lines) end at the LCFS as computed by EFIT. Note the clear break in slope for $T_i$ beyond the LCFS.
- In a nominally identical companion discharge we measured $T_i$ for the minor C$^6+$ impurity constituent. The $T_i$ profile for C$^6+$ has a very similar slope to that for He$^{++}$, but is $\sim 150$ eV greater in this region, probably because this discharge had an increase in $\beta_N$ of $\sim 10\%$ compared with the one shown here.

Pedestal pressure balance

Radial ion pressure balance using $\vec{V}_i = \omega_i R \vec{\zeta} + u_i \vec{B}$ gives

$$\omega_i \approx -c\left[ \frac{\partial \Phi}{\partial \psi} + \left( \frac{T_i}{Zen} \right) \frac{\partial n}{\partial \psi} \right]$$

subsonic pedestal ($w \sim \rho_{pol}$)

$$\frac{\omega_i}{\left[ \left( \frac{T_i}{en} \right) \frac{\partial n}{\partial \psi} \right]} \sim \frac{\omega_i R}{v_i} \ll 1 \quad \Rightarrow \quad \frac{\partial \Phi}{\partial \psi} = - \frac{T_i}{Zen} \frac{\partial n}{\partial \psi} > 0$$

pedestal electric field inward for subsonic ion flow

Radial electron pressure balance: $\vec{V}_e = \omega_e R \vec{\zeta} + u_e \vec{B}$

$$\omega_e = -c \left[ \frac{\partial \Phi}{\partial \psi} - (en)^{-1} \frac{\partial p_e}{\partial \psi} \right]$$

Electron pressure gradient adds to radial electric field making $\omega_e R \sim v_i$ so that $J_{ped} \sim env_i$ & co-current

Thus, the electric field balancing the $1/\rho_{pol}$ density gradient requires a stationary ion Maxwellian & large electron flow
Pedestal orderings & ExB drift effects

Drift departure $\rho_{\text{pol}}$ is of order pedestal width $w$

Finite drift orbits effects enter in leading order

Estimating $Ze\nabla\Phi \sim T/\rho_{\text{pol}}$ we note

$$\vec{v}_E \cdot \nabla \theta \sim v_{||} \vec{n} \cdot \nabla \theta$$

where $\vec{v}_E$ is the ExB drift velocity

Poloidal streaming $\sim$ ExB

Orbit localization from $\varepsilon << 1$

Decouple neoclassical & classical by assuming $\rho_{\text{pol}} >> \rho$
Ion motion for $\varepsilon = a/R << 1$

Assume a quadratic potential well and expand about $\psi_\ast - lu/\Omega$

$$\Phi = \Phi_\ast + \frac{Iv_\parallel}{\Omega} \Phi' + \frac{I^2 v_\parallel^2}{2\Omega^2} \Phi''$$

using $E$, $\mu$ and $\psi_\ast$ invariance while keeping $\Phi'$ find

$$\frac{1}{2} S(v_\parallel + u_\ast)^2 + \mu B - \frac{1}{2} S u_\ast^2 \approx \text{constant}$$

orbit squeezing

S = 1 + $cI^2\Phi''/B\Omega$

magnetic dipole energy

ExB energy

$u_\ast = cI\Phi_\ast'/SB$

S>0 (S<0) trapped particles outboard (inboard)

Denote equatorial plane crossing by “0” then

$$v_\parallel + u_\ast \approx (v_{\parallel 0} + u_0) \sqrt{1 - \kappa^2 \sin^2(\theta/2)}$$

$$\kappa^2 \approx \frac{4\varepsilon (\mu B_0 + u_{\ast 0}^2)}{S(v_{\parallel 0} + u_{\ast 0})^2} \approx 4\varepsilon S \frac{(\mu B_0 + u_0^2)}{(v_{\parallel 0} + u_0)^2}$$

with $\kappa^2 = 1$ the trapped-passing boundary & $u = cI\Phi'/B$
Trapped particle fraction

ExB drift:

i) Increases effective potential well depth: $\mu = 0$ trapped by $\Phi$

ii) Shifts the axis of symmetry of the trapped particle region - fewer trapped!

Trapped fraction decays exponentially if $u = c|\Phi'|/B > v_i$

Neoclassical and polarization phenomena strongly modified

Recall $u \approx (\rho_{pol}/\rho)v_E >> v_E$ so particle dynamics qualitatively changed by a subsonic ExB drift
Neoclassical ion heat flux & parallel flow

Gradient T drive only

\[ C_1 \{ g + f_M \frac{I_{v\|}Mv^2}{2\Omega T^2} \frac{\partial T}{\partial \psi} \} = 0 \]

Need a model for the collision operator - **must**
keep energy scatter as well as pitch angle scatter

\[ \Downarrow \]

Solve for \( g \)

\[ \Downarrow \]

Calculate quantities of interest by taking moments
of the distribution function
Collisions in the pedestal

Pitch-angle scattering is not sufficient to retain transitions across the trapped-passing boundary!

\[ \lambda = \frac{\mu B_0 + u_0^2}{W} = \frac{\kappa^2}{\kappa^2 + 2\epsilon} \]

\[ W(1 - \lambda B / B_0) = \frac{1}{2} S(v_{\|} + u)^2 \]

Convenient variables are \( \lambda \) and \( W \):

Pitch-angle scattering is not sufficient to retain transitions across the trapped-passing boundary!

*Kagan & Catto, to appear in PPCF*
Neoclassical parallel ion flow

Localized portion g - h higher order in $\varepsilon$

$$V_{\parallel i} = -\frac{cI}{B} \left( \frac{\partial \Phi}{\partial \psi} + \frac{1}{\text{Zen}} \frac{\partial p_i}{\partial \psi} \right) - \frac{7cI}{6ZeB_0} \frac{\partial T_i}{\partial \psi} J(u/v_i)$$

No orbit squeezing effect

J changes to Pfirsch-Schluter sign at $u/v_i \sim 0.6$

May help explain C-Mod flow measurements in pedestal

More pedestal bootstrap current
Pedestal impurity flow

Change in poloidal ion flow alters impurity flow

For Pfirsch-Schluter impurities & banana ions:

\[ V_{Z}^{\text{pol}} = V_{i}^{\text{pol}} - \frac{c I B_{\text{pol}}}{e B^{2}} \left( \frac{1}{n_{i}} \frac{\partial p_{p}}{\partial \psi} - \frac{1}{Z n_{Z}} \frac{\partial p_{z}}{\partial \psi} \right) \]

\[ V_{i}^{\text{pol}} \approx -\frac{7 c I B_{\text{pol}}}{6 e B_{0}^{2}} \frac{\partial T_{i}}{\partial \psi} J \left( \frac{u}{v_{i}} \right) \]

C-Mod pedestal flow:

Pfirsch-Schluter: ~ agree
Banana: problem - need \( E_{r} \)

Marr et al to appear PPCF
Pedestal ion heat flux

Modified ion flow:
\[
\langle \bar{q} \cdot \nabla \psi \rangle = -\frac{M \eta T}{Ze} \int d^3v \left( \frac{M v^2}{2T} - \frac{5}{2} \right) \frac{v_{||}}{B} C_1 \{g - h\}
\]

Evaluating:
\[
\langle \bar{q} \cdot \nabla \psi \rangle = -1.35 \sqrt{\epsilon_n} v_i \frac{I^2 T_i}{M \Omega^2} \frac{\partial T_i}{\partial \psi} \frac{G(u)}{\sqrt{S}}
\]

Radial ion heat flux and trapped population become exponentially small for \( u/v_i > 1 \)

Ion heat flux more sensitive to \( \Phi' \) than \( \Phi'' \)

conventional result
Summary

- Pedestal ions nearly isothermal ($\rho_{\text{pol}} \nabla T_i \ll 1$): subsonic ions electrostatically confined + magnetically confined electrons

- Banana regime ion heat flux reduced & poloidal ion flow can change sign in the pedestal due to $\Phi'$ as in C-Mo