Inertial Effects in Fast Nonlinear Magnetic Reconnection

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WPI, Vienna
17/2/2009

Outline

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   - Two-fluid effects
2. Motivation
3. Fast dispersive waves
   - Basic fluid equations for a pair plasma
   - Fixed Points
   - Viscosity-dominated regime ($\mu \neq 0, \eta = 0$)
   - Resistive regime
   - Numerical Validation
   - Pair Plasmas Results
4. Inertial regimes in EMHD
   - Hyper-resistive regime ($\eta = 0, d_e > 0$)

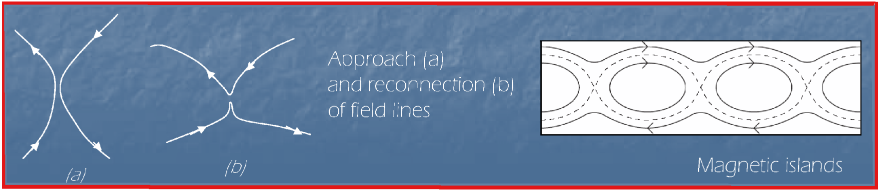
Abstract

Fast magnetic reconnection [1,2] is a fundamental process in astrophysical and laboratory plasmas whereby vast amounts of magnetic energy are released as thermal and kinetic energy in a relatively short period of time. Despite years of study, however, to-date there is no accepted theory able to satisfactorily explain all relevant aspects of this phenomenon. In this talk, after introducing basic concepts in magnetic reconnection, we will present the problematics related to two-fluid effects in collisional, semi collisional and collisionless regimes. Then we introduce a novel approach to attack the full nonlinear 2D problem, which reduces the fluid equations to a low-dimensional dynamical system [4]. In this way, we can analyze all reconnection regimes of interest [3,5,6], as well as predict and numerically validate their transitions. In particular, the achievement of a fast collisionless regime, followed by current collapses, will be shown.

1. J. Birn, et al., *Geophys. Res.* **106** (A3) (2001)
2. M. Ottaviani and F. Porcelli., *Phys. Plasmas.* **2** , 11 (1995)
3. L. Chacón, A. N. Simakov, V. S. Lukin, and A. Zocco *Phys. Rev. Lett.* **101** 025003 (2008)
4. L. Chacón, A. N. Simakov, and A. Zocco *Phys. Rev. Lett.* **99** 235001 (2007)
5. A. Zocco, L. Chacón, A. N. Simakov *Theory of Fusion plasmas.* Proceeding of the Joint Varenna-Lausanne International Workshop (2008)
6. A. Zocco, L. Chacón, A. N. Simakov, Current bifurcation and collapse in electron megnetohydrodynamics
7. A. N. Simakov and L. Chacón *Phys. Rev. Lett.* **101** 105003 (2008)

Introduction

- The term magnetic field like reconnection arose in physics of high conductivity plasmas
- Magnetic flux is “frozen” into the plasma for “small” resistivities
- In “real” plasmas magnetic field lines can split and reconnect across a current sheet
- Experiments and observations shows that reconnection looks like a relaxation process
- Sing: violent energy release
- Magnetic energy in converted suddenly into kinetic and thermal plasma energy

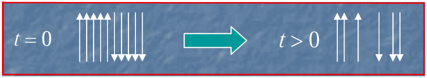


Introduction

- First major motivation ⇒ Physics of Solar Flares, magnetotail (Dungey, Giovanelli, Parker, Coppi, Laval, Pellat)

First Idea: Reconnection as a global diffusion

Slow reconnection rates



- Since: Magnetic Field is stirred into motion by fluid and Steep gradients are present

Sheet-like structures ⇒ Shorter Diffusive Times

Early '60s: Resistive Sweet-Parker mechanism

Does not work...anything!

$$\tau_{DIFF} \sim 10^{14} sec, \ \tau_{SP} \sim 10^{7} sec, \ \tau_{exp} \sim 10^{3} sec$$

Introduction

The problem of fast reconnection

Reconnection independent of the dissipative details

$$\gamma \sim \mathscr{D}^0$$

γ measure of growth rate of the reconnection instability
$\mathscr{D}$ diffusion coefficient

Attempts $\Rightarrow$magnetic turbulence, anomalous resistivity

One can

- Look at strictly collisionless limit of spontaneous instabilities (collisionless tearing modes)
- Look an ideal driver and study the nonlinear stages

Basic Idea

$\Downarrow$

Two-fluid nonideal corrections to Ohm's law can be effective impedance for electric fields

Frozen-in law

Induction equation in the ideal-MHD limit (plasma perfect conductor)

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B})$$

v fluid velocity

- For the magnetic flux Ψ s.t. $\mathbf{B} = \mathbf{z} \times \nabla\Psi$, $(\nabla \cdot \mathbf{B} = 0)$

$\eta \equiv 0$ ideal

$$\mathbf{E} + \frac{1}{c}\mathbf{v} \times \mathbf{B} = 0$$
$$\Downarrow$$
$$\frac{\partial \Psi}{\partial t} + \mathbf{v} \cdot \nabla\Psi = 0$$

Magnetic flux is frozen in the plasma

$\eta \neq 0$ resistive

$$\mathbf{E} + \frac{1}{c}\mathbf{v} \times \mathbf{B} = \eta \mathbf{J}$$
$$\Downarrow$$
$$\frac{\partial \Psi}{\partial t} + \mathbf{v} \cdot \nabla\Psi = \eta\Delta\Psi$$

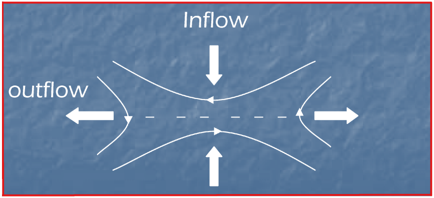
Magnetic flux can reconnect across a neutral line

Advection equation $\Rightarrow$ Advection-diffusion equation

and $E = \partial_t \Psi$ measures how the flux changes (eventually reconnects)

Phenomenology

2D Flow (Strong guide fields)

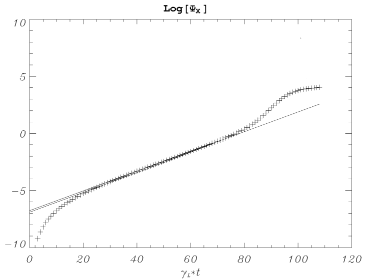


What we need

Neutral line $B(x = x_S) = 0$ (sheared magnetic field)

Resistive Ohm's law

Current sheet



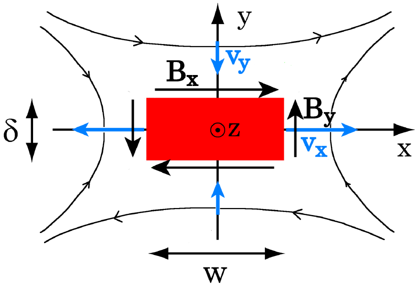
Nonlinear saturation of the reconnecting flux

Current layer separates two regions of opposite magnetic field

Magnetic Reconnection can take place

Driven Reconnection

$2D$ ***Steady Flow***



One fluid plasma, finite resistivity

Alfvén speed $V_A^2 = \frac{B_x^2}{4\pi n}$

Magnetic Reynolds $R_m = \frac{wV_A}{\mathscr{D}_m}$

Magnetic diffusivity $\mathscr{D}_m = \frac{c^2\eta}{4\pi}$

Continuity $\delta\, v_y = w\, v_x$

$$v_y B_x \sim \eta\, J_z \Rightarrow \frac{V_y}{v_x} \sim R_m^{-1/2} \sim \eta^{1/2}$$

Sweet-Parker reconnection rate: solar flares $\tau_{SP} \sim 10^7 sec$, $\tau_{esp} \sim 10^3 sec$.

There is one piece more of the puzzle!

Laboratory Plasmas

- Sawtooth oscillations: regular period reorganization of the core plasma surrounding the magnetic axis

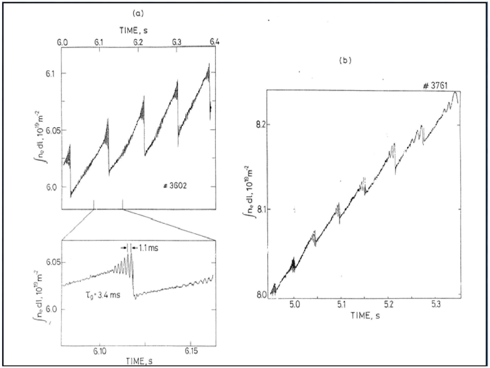
Three stages

- Ramp phase
- Precursor oscillation phase
- Collapse phase

JET high temperature discharges

- $n_e \sim 10^{19} m^{-3}$
- $T_e \sim 5\, keV$

Early discharges in JET



From Hastie (APSS, 1998)

Laboratory Plasmas

Theoretical Explanations

- 1973, long time evolution of the $m = 1$ MHD (internal kink) saturates at small amplitudes (Rosenbluth Dagazian Rutherford)
- 1976, with finite resistivity and geometrical properties of the field

$$\text{Reconnection} \Rightarrow \text{Temperature Collapse}$$

$$\tau_{Rec} = \tau_K \sim \eta^{1/2} \text{ Kadomtzev time scale}$$

- 1980-83: Kadomtzev model still survives until, q-profile flattening $q \simeq 1$ not observed!
- Furthermore: early 90's, high temperatures $\Rightarrow$ shorter time-scales then resistive (inertial effects, Wesson, 1990)

$$\gamma_0^{exp} \sim (40\,\mu s)^{-1} > \nu_{e,i} \sim (130\,\mu s)^{-1} \text{ fast sawteeth}$$

Two-fluid effects

- Generalized Ohm's law

$$\mathbf{E} + \mathbf{v_i} \times \mathbf{B} = \eta \mathbf{J} + \frac{d_e^2}{n} \frac{d\mathbf{J}}{dt}$$

η resistivity;

$d_e = c/\omega_{pl,e}$ electron skin depth $\Rightarrow$ finite electron inertia

Two-fluid effects

- Generalized Ohm's law

$$\mathbf{E} + \mathbf{v_i} \times \mathbf{B} = \eta \mathbf{J} + \frac{d_e^2}{n}\frac{d\mathbf{J}}{dt} + \eta_H \Delta \mathbf{J}$$

η resistivity;

$d_e = c/\omega_{pl,e}$ electron skin depth $\Rightarrow$ finite electron inertia

η_H perpendicular electron viscosity $\Rightarrow$ nongyrotropic effects

Two-fluid effects

- Generalized Ohm's law

$$\mathbf{E} + \mathbf{v_i} \times \mathbf{B} = \eta \mathbf{J} + \frac{d_e^2}{n}\frac{d\mathbf{J}}{dt} + \eta_H \Delta \mathbf{J} + \frac{d_i}{n}(\mathbf{J} \times \mathbf{B} - \nabla p)$$

η resistivity;

$d_e = c/\omega_{pl,e}$ electron skin depth $\Rightarrow$ finite electron inertia

η_H perpendicular electron viscosity $\Rightarrow$ nongyrotropic effects

$d_i = c/\omega_{pl,i}$ ionic skin depth $\Rightarrow$ ion inertia (kinetic waves)

Two-fluid effects

- Generalized Ohm's law

$$\mathbf{E} + \mathbf{v_i} \times \mathbf{B} = \eta \mathbf{J} + \frac{d_e^2}{n}\frac{d\mathbf{J}}{dt} + \eta_H \Delta \mathbf{J} + \frac{d_i}{n}\left(\mathbf{J} \times \mathbf{B} - \nabla p\right)$$

η resistivity; η_H perpendicular electron viscosity

$d_e = c/\omega_{pl,e}$ electron skin depth; $d_i = c/\omega_{pl,i}$ ionic skin depth

Different corrections $\Rightarrow$ Different regimes of reconnection

Much more complex dynamics

- electron inertia
- electron viscosity
- ion inertia
- fast dispersive waves

Motivation

- To develop a framework for the physics of the reconnection region ranging from resistive to two-fluid regimes
- To understand intrinsic limit of reconnection rates in all regimes of interest (2D, two-fluid)

Some basic issues

1. Role of fast dispersive waves
2. Fast reconnection and dissipation
3. Role of electron inertia

Approach

To write a non-linear reduced dynamical system for key quantities defining the reconnection region

From continuum equations (PDE)$\Rightarrow$ To ODE equations for discrete quantities

Pair plasma fluid continuum equations.

Equation of motion

$$mn(\partial_t \mathbf{v}_\pm + \mathbf{v}_\pm \cdot \nabla \mathbf{v}_\pm) = -\nabla p_\pm - \nabla \cdot \Pi_\pm \mp en\left(\mathbf{E} + \frac{1}{c}\mathbf{v}_\pm \times \mathbf{B}\right) \mp \Gamma$$

classical isothermal fluid

Fluid equations

$$\partial_t \mathbf{v} + \mathbf{v} \cdot \nabla \mathbf{v} + \frac{d_e^2}{4} \mathbf{j} \cdot \nabla \mathbf{j} - \mu \Delta \mathbf{v} = \frac{1}{2n}(\mathbf{j} \times \mathbf{B}) - \frac{1}{2n} \nabla p$$

$$\frac{d_e^2}{2}\left[\partial_t \mathbf{j} + \nabla \cdot (\mathbf{v}\,\mathbf{j} + \mathbf{j}\,\mathbf{v}) - \mu \Delta \mathbf{j}\right] = \mathbf{E} + \mathbf{v} \times \mathbf{B} - \eta \mathbf{j}$$

$\Gamma = -ne\eta(\mathbf{v}_+ - \mathbf{v}_-)$ friction
$\mathbf{v} = \frac{1}{2}(\mathbf{v}_+ + \mathbf{v}_-)$ c.m. velocity
$\mathbf{j} = ne(\mathbf{v}_+ - \mathbf{v}_-) = \frac{c}{4\pi} \nabla \times \mathbf{B}$
$p_+ = p_- = p/2,\ \mu_+ = \mu_- = \mu$
$\mathbf{B}_p = (B_x, B_y)$

$\Omega = \hat{\mathbf{z}} \cdot \nabla \times \mathbf{v}$ vorticity
$\mathbf{v} = \hat{\mathbf{z}} \times \nabla \Phi$ fluid velocity
$\nabla \cdot \mathbf{v} = 0$
$\mathscr{D} = \eta - \frac{\mu d_e^2}{2} \Delta$ Diffusion operator
$\mathbf{B}_p^* = \mathbf{B}_p + d_e^2 \nabla \times \nabla \times \mathbf{B}_p$

$\Rightarrow$

2D incompressible equations

$$\partial_t \mathbf{B}_p^* - \nabla \times \left(\mathbf{v} \times \mathbf{B}_p^*\right) = -\nabla \times \nabla(\mathscr{D}\mathbf{B}_p)$$

$$\partial_t \Omega + \mathbf{v} \cdot \nabla \Omega - \mu \Delta \Omega = \frac{1}{2} \mathbf{B}_p \cdot \nabla j_z$$

$$\Delta \varphi = \Omega$$

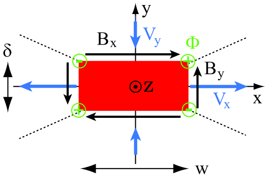
Linear dispersive waves $\omega = k_\parallel / \sqrt{2 + d_e^2 k^2} \Rightarrow$ NO FAST WAVES $\omega \sim k^2$

$d_e = c/(\omega_{pe} L)$ dimensionless electron skin depth, μ dimensionless viscosity

Zero-dimensional reconnection model

Symmetries

$B_x(-x,y,t) = B_x(x,y,t),\ B_x(x,-y,t) = -B_x(x,y,t)$
$B_y(x,-y,t) = B_y(x,y,t),\ B_y(-x,y,t) = -B_y(x,y,t)$
$\Phi(-x,y,t) = -\Phi(x,y,t),\ \Phi(x,-y,t) = -\Phi(x,y,t).$



At the symmetry axis

$$\partial_t B_x|_{x=0} + \partial_y(v_y B_x)|_{x=0} = -\partial_y(\mathscr{D} j_z)_{x=0} \quad (1)$$
$$\partial_t B_y|_{y=0} - \partial_x(v_x B_y)|_{y=0} = \partial_x(\mathscr{D} j_z)_{y=0} \quad (2)$$

Representation

- $B_x(x,y,t,\delta,w) = 2B_x(t)\frac{y}{\delta}\, g\left(\frac{x}{w}\right) e^{-2[(\frac{x}{w})^2+(\frac{y}{\delta})^2]}$
- $\Phi(x,y,t) = -\Phi(t)2\frac{x}{w}2\frac{y}{\delta} e^{-2[(\frac{x}{w})^2+(\frac{y}{\delta})^2]}$

internal profile, stagnation point, nonlinearities,
g even, and $B_y(x,y,t,\delta,w) = B_x(y,x,t,w,\delta)$

Procedure

- Integrate along the symmetry axis Eqs. (1)-(2)(Poloidal field)
- Integrate over the control volume the vorticity equation

Zero-dimensional reconnection model

Discrete Equations

$$\frac{dB_x^*}{dt} - B_x^*\frac{\dot\delta}{\delta} - \frac{\Phi B_x^*}{\delta w} = \mathscr{D}\left(\frac{B_y}{\delta w} - \frac{B_x}{\delta^2}\right),$$

$$\frac{dB_y^*}{dt} - B_y^*\frac{\dot w}{w} - \frac{\Phi B_y^*}{\delta w} = \mathscr{D}\left(\frac{B_x}{\delta w} - \frac{B_y}{w^2}\right),$$

$$\frac{d\Phi}{dt} - \Phi\left(\frac{\dot w}{w} + \frac{\dot\delta}{\delta}\right) + \frac{1}{2}\left(\frac{B_x}{w} + \frac{B_y}{\delta}\right)\left(\frac{B_y}{w} - \frac{B_x}{\delta}\right) =$$

$$\frac{\Phi^2}{\delta w}\left[\frac{1}{w^2} - \frac{1}{\delta^2}\right] - \mu\Phi\left(\frac{1}{\delta^2} + \frac{1}{w^2}\right)^2$$

$$B_x^* = B_x + \frac{d_e^2}{2}(B_x/\delta^2 - B_y/\delta w)$$

$$B_y^* = B_y + \frac{d_e^2}{2}(B_y/w^2 - B_x/\delta w)$$

$$\mathscr{D} = \eta + \frac{\mu d_e^2}{2}(\delta^{-2} + w^{-2})$$

$$\mathbf{v}\cdot\nabla\Omega \longrightarrow \frac{\Phi^2}{\delta w}\left[\frac{1}{\delta^2} - \frac{1}{w^2}\right]$$

$$\nabla\times\nabla\times\mathbf{B}_p \longrightarrow \left(\frac{B_y}{\delta w} - \frac{B_x}{\delta^2}\right)$$

$$\mathbf{B}\cdot\nabla j_z \longrightarrow \frac{1}{2}\left(\frac{B_x}{w} + \frac{B_y}{\delta}\right)\left(\frac{B_y}{w} - \frac{B_x}{\delta}\right)$$

Five Unknowns-Three equations (B_x and w chosen as parameters)

$$\Downarrow$$

Coupling to an external driver provides closure (through $\dot B_x$, $\dot B_y$)

Fixed Points ($\frac{d}{dt} \equiv 0$).

Master Equation for $\hat{d}_e = \frac{d_e}{\sqrt{2}\delta}$

$$1 + \hat{d}_e^{\,-2} = 2S^{-1}\left(\frac{w}{d_e}\right)^2\left[\frac{1}{1+\hat{d}_e^2} + \frac{S}{S_\mu}\right]^{1/2}, \ \xi^2 \ll 1, (d_e/w)^2 \ll 1$$

$S^{-1} = S_\eta^{-1} + S_\mu^{-1}\hat{d}_e^{\,2}(\xi^2+1) \approx S_\eta^{-1} + S_\mu^{-1}\hat{d}_e^{\,2}$
$\xi = \delta/w$
$S_\eta = B_x w/\eta$ Resistive Lundquist number
$S_H = B_x w/\mu$ Reynolds number

Associated reconnection rate

$$E_z = \mathscr{D} j_z = \mathscr{D}\left(\frac{B_x}{\delta} - \frac{B_y}{w}\right) \approx B_x^2 \frac{S^{-1}(\xi^{-1}-\xi)}{\left(1+2\hat{d}_e^2\xi^2\right)}$$

$\xi^2 \ll 1 \Rightarrow$ large E_z

Collisionless (viscous) steady state ($\mu \neq 0, \eta = 0$)

$$\xi^3 \approx \frac{S_\mu^{-1}}{\sqrt{2}} \frac{d_e}{w} \frac{1}{1+\hat{d}_e^2} \Rightarrow \frac{1}{\hat{d}_e^3} + \frac{1}{\hat{d}_e} \sim 2\frac{\mu w}{B_x d_e^2} \equiv 2\mu^*$$

magnetized regime $\delta/d_e \gtrsim 1$

$\delta/d_e \sim (\mu^*)^{1/3}$

inertial regime $\delta/d_e \lesssim 1$

$\delta/d_e \sim \mu^*$

$$j_z|_X \approx 2B_x/\delta \approx 2B_x^e/d_e\,, \text{ for } d_e > \delta \text{ where } B_x^e \equiv \hat{\mathbf{x}}\cdot\mathbf{B}(0, d_e/2)$$

magnetized regime

$\delta \sim (2\mu w\, d_e/B_x)^{1/3} > d_e$ ↘

inertial regime

$\delta \sim \sqrt{\mu w/(B_x^e)} < d_e$ ↙

$$E_z \approx \sqrt{2}\frac{B_{x,max}^2}{w} d_e$$

$B_{x,max} = max[B_x, B_x^e]$

Viscous sub-d_e layers can sustain dissipation-independent reconnection

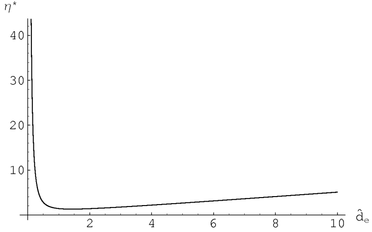
Resistive regime ($\eta \neq 0, \mu = 0$)

$$\frac{(1+\hat{d}_e^2)^{3/2}}{2\hat{d}_e^2} \approx \frac{\eta w}{B_x d_e^2} = \eta^*$$

Threshold

$\Downarrow$

No solutions for $\frac{\eta w}{B_x d_e^2} \lesssim 3\sqrt{3}/2$



magnetized regime $\delta/d_e \gtrsim 1$

$\delta \approx \delta_{SP} = \sqrt{\frac{\eta w}{B_x}}$; $E_z \approx \sqrt{\frac{\eta}{w}} B_x^{3/2}$

Sweet-Parker slow reconnection

inertial regime $\delta/d_e \lesssim 1$

$\left(\frac{\eta w}{\sqrt{2 B_x^e}}\right)^{1/2} \approx d_e, \ \forall \delta \lesssim d_e$

Resistivity cannot set any dissipative length scale below d_e

$\Downarrow$

The scale $\delta < d_e$ can be *arbitrarily small* and viscosity important!

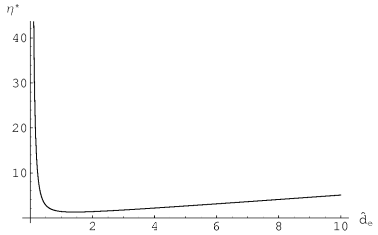
Resistive regime ($\eta \neq 0, \mu = 0$)

$$\frac{\left(1+\hat{d}_e^2\right)^{3/2}}{2\hat{d}_e^2} \approx \frac{\eta w}{B_x d_e^2} = \eta^*$$

Threshold

$\Downarrow$

No solutions for $\frac{\eta w}{B_x d_e^2} \lesssim 3\sqrt{3}/2$



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Sweet-Parker slow reconnection

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Resistivity cannot set any dissipative length scale below d_e

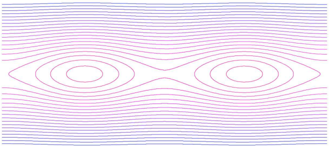
$\Downarrow$

The scale $\delta < d_e$ can be *arbitrarily small* and viscosity important!

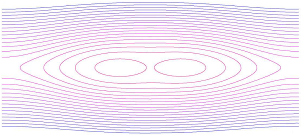
Numerical Validation

Test predictions with the island coalescence problem.

Equilibrium ideally unstable



Nonlinear stage: merged islands



magnetic equilibrium

$\Psi_0(x,y) = -\lambda \ln\left[\cosh\left(\frac{x}{\lambda}\right) + \varepsilon \cos\left(\frac{y}{\lambda}\right)\right]$

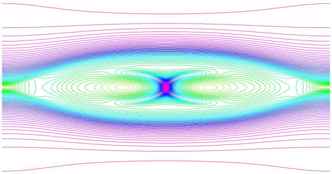
λ equilibrium length

ε island size Ideally Unstable

Diagnostics

- $\delta = 2\sqrt{2 \log 2}\, y_*$ FWHM of the current $j_z(0,y) = e^{-a(y/\delta)^2}$ with $\partial_y^2 j_z(0,y_*) = 0$
- w measured at the out-flow's maximum
- B_x measured up-stream $(0, \delta/2)$

Example of Induced Current Sheet (at maximum reconnection)



EMHD with $\eta \neq 0$, $d_e \neq 0$, $\eta_H \equiv 0$

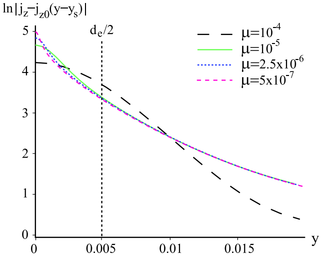
Ideal instability $\Rightarrow$ Driver of reconnection

Numerical Validation

Collisionless viscosity-dominated regime.

Viscous sub-layer

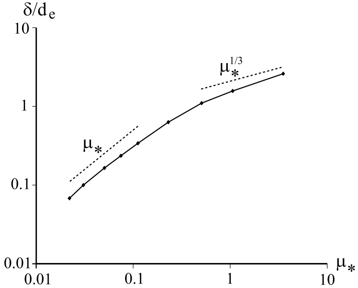
$\delta \sim \sqrt{\mu w/(B_x^e)} < d_e$



$y_s = y_O(t = t_{max}) - y_O(t = 0)$
$y_O(t)$ instantaneous island O-point position

Transition $\mu^* = 2\frac{\mu w}{B_x d_e^2} \sim 1,\ \delta \sim d_e$

from collisional $\delta/d_e > 1$ → to collisionless $\delta/d_e < 1$



δ/d_e Vs $\mu^* = \mu w/B_x d_e^2$ at time of maximum reconnection

Motivation

Some basic issues

1. Role of fast dispersive waves
2. Fast reconnection and dissipation
3. Role of electron inertia

Are not necessary to enable Fast (dissipation-independent) Reconnection

Motivation

Some basic issues

1. Role of fast dispersive waves
2. Fast reconnection and dissipation
3. Role of electron inertia

Generalized Ohm's law ions at rest, electron fluid ⇒ Electron Magnetohydrodynamics

$$\mathbf{E} + \mathbf{v_i} \times \mathbf{B} = \eta \mathbf{J} + \frac{d_e^2}{n}\frac{d\mathbf{J}}{dt} + \eta_H \Delta \mathbf{J}$$

EMHD continuum equations.

Two D ($\partial_z \equiv 0$) incompressible two fluid plasma with an ion neutralizing background ($\mathbf{v_i} \approx 0$)

$$\begin{aligned}
\frac{\partial B_x^*}{\partial t} - \nabla\cdot\left(\mathbf{j}_p B_x^* - \mathbf{B}_p^* j_x\right) &= -\eta\left(\frac{\partial^2 B_y}{\partial y \partial x} - \frac{\partial^2 B_x}{\partial y^2}\right) + \eta_H \Delta\left(\frac{\partial^2 B_y}{\partial y \partial x} - \frac{\partial^2 B_x}{\partial y^2}\right)\\
\frac{\partial B_y^*}{\partial t} - \nabla\cdot\left(\mathbf{j}_p B_y^* - \mathbf{B}_p^* j_y\right) &= -\eta\left(\frac{\partial^2 B_x}{\partial x \partial y} - \frac{\partial^2 B_y}{\partial x^2}\right) + \eta_H \Delta\left(\frac{\partial^2 B_x}{\partial x \partial y} - \frac{\partial^2 B_y}{\partial x^2}\right)\\
\frac{\partial B_z^*}{\partial t} + \mathbf{B}_p \cdot \nabla j_z &= -d_e^2 \left(\mathbf{j}_p \cdot \nabla\right) \Delta B_z + \eta \Delta B_z - \eta_H \Delta^2 B_z
\end{aligned}$$

$\mathbf{B}_p^* = (B_x^*, B_y^*), \mathbf{j}_p = (j_x, j_y) = -\hat{\mathbf{z}} \times \nabla B_z = -(v_x, v_y)$

$B_z^* = B_z - d_e^2 \Delta B_z$

η Resistivity, η_HElectron viscosity, d_eElectron skin depth

Properties

$$\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \mathscr{O} \underline{P}_z \begin{pmatrix} B_x eq. \\ B_y eq. \\ B_z eq. \end{pmatrix} = \begin{pmatrix} B_x eq. \\ B_y eq. \\ B_z eq. \end{pmatrix}$$

Operators

- $\underline{P}_z : B_z \mapsto -B_z$
- $\mathscr{O} : (\partial_x, \partial_y, B_x, B_y) \mapsto (\partial_y, \partial_x, B_y, B_x)$

EMHD discrete equations.

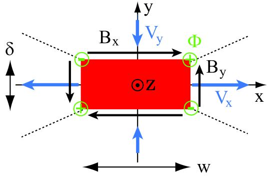
Discrete EMHD Equations

$$\frac{dB_x^*}{dt} - B_x^* \frac{\dot{\delta}}{\delta} - \frac{B_z B_x^*}{\delta w} = \mathscr{D}\left(\frac{B_y}{\delta w} - \frac{B_x}{\delta^2}\right)$$

$$\frac{dB_y^*}{dt} - B_y^* \frac{\dot{w}}{w} + \frac{B_z B_y^*}{\delta w} = \mathscr{D}\left(\frac{B_x}{\delta w} - \frac{B_y}{w^2}\right)$$

$$\frac{dB_z^*}{dt} - B_z^* \left(\frac{\dot{w}}{w} + \frac{\dot{\delta}}{\delta}\right) + \left(\frac{B_x}{w} + \frac{B_y}{\delta}\right)\left(\frac{B_y}{w} - \frac{B_x}{\delta}\right) =$$

$$-\mathscr{D}\left(\frac{1}{\delta^2} + \frac{1}{w^2}\right) B_z + d_e^2 \frac{B_z^2}{\delta w}\left(\frac{1}{w^2} - \frac{1}{\delta^2}\right)$$



$$B_x \equiv \hat{\mathbf{x}} \cdot \mathbf{B_p}(0, \delta/2)$$
$$B_y \equiv \hat{\mathbf{y}} \cdot \mathbf{B_p}(w/2, 0)$$
$$B_z \equiv -\hat{\mathbf{z}} \cdot \mathbf{B_z}(w/2, \delta/2)$$

$$B_x^* = B_x + d_e^2(B_x/\delta^2 - B_y/\delta w)$$
$$B_y^* = B_y + d_e^2(B_y/w^2 - B_x/\delta w)$$
$$B_z^* = B_z + d_e^2(\delta^{-2} + w^{-2})B_z$$
$$\mathscr{D} = \eta + \eta_H(\delta^{-2} + w^{-2})$$

Fixed Points ($\frac{d}{dt} \equiv 0$).

Current sheet aspect ratio ($\xi = \frac{\delta}{w}$) equation

$$\left\{ \frac{1+\hat{d}_e^2(1+\xi^2)}{1+2\hat{d}_e^2\xi^2} \right\}^2 = \frac{1}{S^2} \left\{ 1 + \frac{1}{\xi^2} + \frac{\hat{d}_e^2}{1+\hat{d}_e^2(1+\xi^2)} \left(\frac{\xi^2-1}{\xi} \right)^2 \right\}$$

$S^{-1} = S_\eta^{-1} + S_H^{-1}(\xi^{-2}+1)$, $\hat{d}_e = \frac{d_e}{\delta}$

$S_\eta = \sqrt{2}B_x/\eta$ Resistive Lundquist number

$S_H = \sqrt{2}B_x w^2/\eta_H$ Hyper-resistive Lundquist number

Centers in the parametric space $(\xi, \hat{d}_e)$

$$\delta_0 = \frac{d_e}{\hat{d}_e}, \;\; w_0 = \frac{d_e}{\hat{d}_e \xi}, \;\; \frac{B_z^0}{\sqrt{2}B_x} = S^{-1} \frac{\xi^{-1}-\xi}{1+\hat{d}_e^2(1+\xi^2)}$$

B_x,d_e define electron Alfvén speed $v_{A,e} = B_x/d_e$

Small aspect ratios ($\xi^2 \ll 1$)

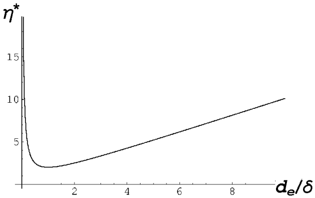
Reconnection Rate

$$E_z|_X = \mathscr{D} j_z|_X = \mathscr{D}\left(\frac{B_x}{\delta} - \frac{B_y}{w}\right) \Rightarrow E_z \approx \sqrt{2}\frac{B_x^2}{w}\{1+\hat{d}_e^2\}$$

$\xi \approx S^{-1}\frac{1}{1+\hat{d}_e^2}$, $\xi^2 \ll 1$

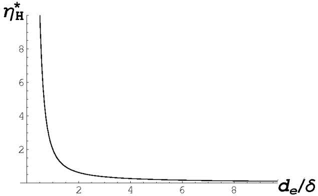
Inertial correction to the steady state current sheet equation

Resistivity-dominated regime



- $\hat{d}_e^{-1} + \hat{d}_e \approx \left(\frac{w}{d_e}\right)\frac{\eta}{\sqrt{2}B_x} = \eta^*$
- Threshold: $\frac{\eta w}{\sqrt{2}B_x} \gtrsim 2$

Hyper-resistivity dominated regime



- $\hat{d}_e^{-3} + \hat{d}_e^{-1} \approx \left(\frac{w}{d_e}\right)\frac{\eta_H}{\sqrt{2}B_x d_e^2} = \eta_H^*$

$\delta \lesssim d_e$ allowed

Outflows $v_x \approx B_z/\delta \sim B_x/\delta \leq B_{x,max}/d_e \equiv V_{A,e}$ bounded by the electron Alfvén speed

Hyper-resistive regime ($\eta = 0$, $d_e > 0$)

$$\xi \approx S^{-1} \frac{1}{1+\hat{d}_e^2} \Rightarrow \frac{1}{\hat{d}_e^3} + \frac{1}{\hat{d}_e} \sim \left(\frac{w}{d_e}\right) \frac{\eta_H}{\sqrt{2} B_x d_e^2} \equiv \eta_H^*$$

magnetized regime $\delta/d_e \gtrsim 1$

$\delta/d_e \sim (\eta_H^*)^{1/3}$

inertial regime $\delta/d_e \lesssim 1$

$\delta/d_e \sim \eta_H^*$

$$j_z|_X \approx 2B_x/\delta \approx 2B_x^e/d_e, \text{ for } d_e > \delta \text{ where } B_x^e \equiv \hat{\mathbf{x}} \cdot \mathbf{B}(0, d_e/2)$$

$B_{x,max} = max[B_x, B_x^e]$ magnetic field at the upstream boundary of induced current j_z

magnetized regime

$\delta \sim (\eta_H w/\sqrt{2} B_x)^{1/3} > d_e$

inertial regime

$\delta \sim \sqrt{\eta_H w/(B_x^e d_e)} < d_e$

$$E_z^H \approx \sqrt{2} \frac{B_{x,max}^2}{w}$$

Viscous sub-d_e layers can sustain dissipation-independent reconnection

Motivation

Some basic issues

1. Role of fast dispersive waves
2. Fast reconnection and dissipation
3. Role of electron inertia

We predict scaling laws for nonlinear current sheet solution *in all regimes of collisionality*

The maximum reconnection rate does not depend on electron inertia

Summary

- We developed a two-fluid theory for reconnection, in electron-positron plasmas, which features no fast dispersive waves, and includes resistivity and fluid viscosity
- These equations are related to the electron MHD model with electron inertia (which supports fast dispersive waves)
- A zero-dimensional model which describes key-quantities of the reconnection region is derived, solutions at time of maximum reconnection are found
- In resistivity-dominated regimes the current sheet layer can achieve arbitrarily small values $\delta < d_e$
- In viscosity-dominated regimes viscous layers $\delta < d_e$ develop and sustain dissipation-independent reconnection: $E \approx \frac{d_e}{w} B^2_{upstream}$ for pair plasmas, $E \approx \frac{1}{w} B^2_{upstream}$ for EMHD
- We gave a new framework of understanding for reconnection that apply in a wide range of physical regimes