Working Group Meeting Kinetic Instabilities, Plasma Turbulence and Magnetic Reconnection"
Wolfgang Pauli Institute, Vienna, 16-20 February 2009

ARE CASCADE AND INERTIAL RANGE IN TURBULENCE WELL POSED CONCEPTS?

from senescent idiot's fugitive essays on science (borrowed from Truesdell)

Many do not even want to listen to such or similar questions The subtitle is added to stir the audience even more as I intend to convince you that I am pretty serious.

Arkady Tsinober

Professor and Marie Curie Chair in Fundamental and Conceptual Aspects of Turbulent Flows
Institute for Mathematical Sciences and Department of Aeronautics, Imperial College London

Is "cascade" in genuine turbulence conceptually well defined notion or is it "mostly a pedagogical imagery"? Is there cascade in physical space? How meaningful is "cascade" of passive objects as described by linear equations? Is "cascade" Eulerian, Lagrangian, both or whatever? Do decompositions aid understanding or obscuring the physics of turbulence?

How inertial is inertial range? How clean is the decomposition on inertial and dissiaptive ranges? Is anomalous scaling an attribute of the ineratial range?

One gets an impression of little, randomly structured and distributed whirls in the fluid, with the cascade process consisting of the fission of the whirls into smaller ones, after the fashion of the Richardson poem. This picture seems to be drastically in conflict with what can be inferred about the qualitative structure of high-Reynolds-number turbulence from laboratory visualization techniques and from plausible application of Kelvin circulation theorem. KRAICHAN, 1974.

... the idea of conservative inertial cascade local in scale size is consistent *prima facie*, provided that the actual statistics do not differ strongly from Gaussian. It is another, and unsettled, matter to establish that K41 or a related theory actually describes what happens in NS flows. KRAICHNAN, 1991

A bit of histoiry and relaled

Phenomenology - The branch of a science that classifies and describes its phenomena without any attempt at explanation,

WEBSTER'S NEW WORLD DICTIONARY, 1962

WEATHER PREDICTION
BY
NUMERICAL PROCESS

BY

LEWIS F. RICHARDSON, B.A., F.R.MET.SOC., F:INST.P.
FORMERLY SUPERINTENDENT OF ESKDALEMUIR OBSERVATORY
LECTURER ON PHYSICS AT WESTMINSTER TRAINING COLLEGE

On the other hand we find that convectional motions are hindered by the formation of small eddies resembling those due to dynamical instability. Thus O. K. M. Douglas writing of observations from aeroplanes remarks : "The upward currents of large cumuli give rise to much turbulence within, below, and around the clouds, and the structure of the clouds is often very complex." One gets a similar impression when making a drawing of a rising cumulus from a fixed point; the details change before the sketch can be completed. We realize thus that: **big whirls have little whirls that feed on their velocity, and little whirls have lesser whirls and so on to viscosity— in the molecular sense...**

CAMBRIDGE
AT THE UNIVERSITY PRESS
1922

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Thus, because it is not possible to separate eddies into clearly defined classes according to the source of their energy; and as there is no object, for present purposes, in making a distinction based on size between cumulus eddies and eddies a few metres in diameter (since both are small compared with our coordinate chequer), therefore a single coefficient is used to represent the effect produced by eddies of all sizes and descriptions.

CAMBRIDGE
AT THE UNIVERSITY PRESS
1922 .

Do you see here a cascade?

…. even wrong theories may help in designing machines.

(FEYNMAN, 1996) Feynmann R., 1996 Lectures on Computation, Addison-Wesley.

An example how it was taught 60 years ago

Enrico Fermi, Notes on Thermodynamics and Statistics, The University of Chicago Press, Chicago and London Midway Reprint Edition, 1988; pp. 181-182. (1951 lectures)

CONVENTIONAL PHENOMENOLOGY AS IT APPEARS IN THE BOOK by ENRICO FERMI, Notes on Thermodynamics and Statistics, The University of Chicago Press, Chicago and London Midway Reprint Edition, 1988; pp. 181-182. (1951 lectures)

74 - Turbolence (Kolmogoroff Heisenberg, Onsager) 74 a

Gas kinetic viscosity (c = molecular velocity)

$$\eta_0 \approx \left(\frac{c}{\lambda}\right)(\rho\lambda)\left(\lambda\frac{dV}{dz}\right)\Big/\left(\frac{dV}{dz}\right) = \rho c \lambda$$

no of exchanges; mass exchanged; vel. difference of layers

For eddies of size ℓ & turbolent velocity v "apparent" viscosity

$$\eta_\ell = \rho v \ell$$

Power lost to viscosity per unit volume

$$\eta\left(\frac{\partial v}{\partial x}\right)^2$$

Power density lost by eddies ℓ to smaller eddies

$$\eta_\ell\left(\frac{v}{\ell}\right)^2 \approx \rho v \ell \frac{v^2}{\ell^2} = \rho\frac{v^3}{\ell}$$

Assuming constant flow of energy to smaller and smaller eddies

$$\rho\frac{v^3}{\ell} = \text{constant}$$

$$v \sim \ell^{1/3}$$

$$v \approx V\left(\frac{\ell}{L}\right)^{1/3}$$

V, L velocity and size of largest eddies

Smallest eddies, size s, are those for which true and turbolent viscosity become comparable

$$\rho c\lambda \approx \rho v_s s \approx \rho V\left(\frac{s}{L}\right)^{1/3} s$$

$$s \approx \left(\frac{c}{V}\right)^{3/4} L^{1/4} \lambda^{3/4}$$

Heisenbergs formulation

v is analyzed in Fourier components

Then: kin energy between k, $k+dk$ (wave numbers)

$$\sim k^{-5/3}\,dk$$

Proof

$$v = \sum a_k e^{ik\cdot r} \qquad a_k \sim k^{-\frac{11}{6}}$$

$$v_\ell^2 \sim \ell^{2/3} \sim \sum_{k>1/\ell} a_k^2 \approx \int_{1/\ell}^{\infty} a_k^2 k^2\,dk$$

$$k^{-2/3} \sim \int a_k^2 k^2\,dk \sim k^3 a_k^2$$

$$a_k^2 k^2 \sim k^{-5/3}$$

QED

An example how it is taught today

Courtesy Sasha Schekochihin

5 LECTURES ON TURBULENCE

A. Schekochihin
Cambridge 25 - 29 June 2007

So we now have the following picture

Scale

L — energy injected

↓

ℓ — energy transported

↓

ℓ_ν $(\sim L\,Re^{-3/4})$ — energy dissipated

Turbulence is multiscale disorder

$L \gg \ell \gg \ell_\nu$

inertial range

Our ability to make further progress hinges on the (philosophical) assumption that the physics of the inertial range are universal

Key insight: Richardson (1922) conjectured that the energy transfer is local in scale space, i.e. occurs via a cascade:

$$L \to \frac{L}{2} \to \frac{L}{4} \to \text{etc. to the visc. scale}$$

(you might think of motion at each scale going unstable and breaking up into smaller scales like the mean flow did at the outer scale).

First quantitative theory: Kolmogorov 1941

Some quotations

From the energetical point of view it is natural to imagine the process of turbulent mixing in the following way: the pulsations of the first order absorb the energy of the motion and pass it over successively to pulsations of higher orders. The energy of the finest pulsations is dispersed in the energy of heat due to viscosity. KOLMOGOROV 1941a

... it has not been pointed out that before that the subdivision of the energy must be a stepwise process, such that an n-fold increase of the wave-number is reached by a number of steps of the order $\log n$. For such a **cascade** mechanism that part of the energy density which is associated with large wave numbers should depend on the total volume rate of dissipation Q only. ONSAGER 1945

One gets an impression of little, randomly structured and distributed whirls in the fluid, with the cascade process consisting of the fission of the whirls into smaller ones, after the fashion of the Richardson poem. This picture seems to be drastically in conflict with what can be inferred about the qualitative structure of high- Reynolds-number turbulence from laboratory visualization techniques and from plausible application of Kelvin circulation theorem. KRAICHAN, 1974.

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The notion that turbulent flows are hierarchical and involve entities… of varying sizes is a common idea… This common notion underlies the concept of cascade, the third key element of turbulence theory. FRISCH AND ORSZAG, 1990.

All this cascade in Fourier space is a dream of linearized physicists. BETCHOV, 1993).

The tree examples [jet, boundary layer, and wake]… show that there is something wrong with this idea (the Richardson poem). In each case turbulence begins at small scales and grows larger: not the other way around. GIBSON, 1996

A warning from von Neumann 1949

Or is there a cascade in physical space?

The Richardson-Kolmogorov cascade picture was formulated in **physical** space and is used frequently without much distinction both in physical and Fourier space, as well as some others. However, it was VON NEUMANN (1949) (see also Onsager, 1949) who stressed that this process occurs not in physical space, but in Fourier space: ... *the system is "open" at both ends, energy is being supplied as well dissipated. The two "ends" do not, however, lie in ordinary space, but in its Fourier-transform . More specifically: The supply of energy occurs at the macroscopic end – it originates in the forced motions of macroscopic (bounding) bodies, or in the forced maintenance of (again macroscopic) pressure gradients. The dissipation, on the other hand, occurs mainly at the microscopic end, since it is ultimately due to molecular friction, and this is most effective in flow-patterns with high velocity gradients, that is, in small eddies...Thus the statistical aspect of turbulence is essentially that of transport phenomenon (of energy) – transport in the Fourier-transform space.*

That is, the nonlinear term in the Navier-Stokes equation redistributes energy among the Fourier modes not scales as is frequently claimed, unless the 'scale' is defined just as an inverse of the magnitude of the wave-number of a Fourier mode, which is not easy for everybody to swallow.

A natural question is then what does the nonlinear term in physical space do? Is energy transferred from large to small scales in physical space, i.e. is there a cascade in physical space? The answer to the last question depends on the definition of what is a 'scale' in physical space.

Recall that that there is no contribution from the nonlinear term in the total energy balance equation (and in a homogeneous/periodic flow it's contribution is null in both the total and the mean), since the nonlinear term (which includes or not the term with pressure) in the energy equation has the form of a spatial flux,

$$\partial\{u_j e\}/\partial x_j \; ; \qquad e = u^2/2 + p/\rho$$

In other words the nonlinear term redistributes the energy in physical space, but does it do more than that?

It is straightforward to see that in a statistically homogeneous turbulent flow the mean energy of volume of any scale is changing due to external forcing and dissipation only — there is no contribution in the mean of the nonlinear term, which includes the term with the pressure. That is, if one chooses to define a 'scale', l, in physical space as a fluid (or a fixed) volume, say, of order l^3, then in a statistically homogeneous flow there is no cascade in physical space in the sense that, in the mean, there is no energy exchange between different scales.

This happens because the nonlinear term in the energy equation has the form of a spatial flux, $\partial\{...\}/\partial x_j$, i.e. there is conservation of energy by non-linear terms. In other words, the nonlinear term redistributes the energy in physical space if the flow is statistically nonhomogeneous. So, generally, it is a ***misconception*** to interpret this or any other process involving spatial fluxes, $\partial\{...\}/\partial x_j$ (e.g. momentum flux), as a "cascade" in physical space.

Cascade versus decompositions/representations

It is a kind of trivial consequence
of nonlinearity and decompositions

Fourier transform ambiguity in turbulence... TENNEKES 1976
I think that the k *-space decomposition does actually obscure the physics.*
MOFFATT, 1990
See also LIEPMANN, 1962; LOHSE AND MÜLLER-GROELING, 1996

The reason for the above result on the absence of energy exchange between different scales in physical space is because no decomposition is involved in the above 'definition of scale'. Any decomposition (be it in physical space, Fourier or any other) brings the 'cascade' back to life since due to nonlinearity it results in interaction between its components ('cascade').

One of the problems with decompositions is that the nonlinear term redistributes the energy among the components of a particular decomposition in a different way for different decompositions, i.e. the energy exchange/transfer is decomposition dependent. This creates some discomfort as any physical process, should be invariant of particular decompositions of a turbulent field.

Coupled with a decomposition (of whatever form) - which is a good tool for linear problems – the nonlinearity results in interaction between its components ('cascade'). That is 'cascade' is not independent on the nature/form of the decomposition and , therefore, is not a good means for describing a physical process, since the latter cannot be decomposition (which is ours - not Natures') dependent .

Decompositions versus representations

‘Cascade' arising from a decomposition of the flow field viewed as a process of exchange of energy, momentum, etc. between the components of this decomposition is a **dynamical process**. This should be distinguished from “cascading processes” resulting from a **decomposition** of some quantity, e.g. dissipation, usually of its surrogate $(\partial u_1/\partial x_1)^2$, obtained from experimental signals. The former is **a dynamical process**, whereas the latter is a **representation** characterizing some aspects of the spatial and/or temporal structure of some flow characteristics.

In other words, 'structure' is not synonymous of 'process': it is the result of a process. Therefore, generally it is impossible to draw conclusions about the former from the information about the latter, though this is done quite frequently. For example, simple chaotic systems* with few degrees of freedom only produce also 'fine structure', possessing a continuous spectrum with a multitude of interacting modes. Of course, such a signal can be also cast in a multiplicative representation, but there is no 'cascade' whatsoever.

* E.g. three as in the Lorenz (1963) system or four in the forced spherical pendulum, Miles (1984), also Mullin(1993)

What are the ('small') scales in turbulent flows?

Avoiding decompositions

*Any decomposition results in a **bidirectional and direct (non locality)** interaction between 'small' and the 'large' scales (whatever this means) which is non-local (functional) both in space and time (i.e. history-dependent).*

There is an ambiguity in defining the meaning of the term‘small scales’ (or more generally ‘scales’ or ’eddies’ and consequently the meaning of the term ‘cascade’. The specific meaning of this term and associated interscale energy exchange/‘cascade’ (e.g. spectral energy transfer) is essentially decomposition/representation dependent. The only common thing in all decompositions/ representations (D/R) is that the small scales are always associated with the field of velocity derivatives (not necessarily of the first order). Therefore, it is naturally to look at this field as the one objectively (i.e. D/R independent) representing the small scales. Indeed, the dissipation is associated precisely with the symmetric part of the velocity derivative tensor $\partial u_i/\partial x_j$ — the rate of strain field s_{ij} both in Newtonian and non-Newtonian fluids, whereas vorticity $\omega_i = \varepsilon_{ijk}\partial u_j/\partial x_k$ is the anti-symmetric part.

The large scales are naturally characterized by the velocity field itself, **u**. This is justified also by the fact that sustaining turbulent flows requires energy input into flow, e.g. in case of a prescribed force, **F**, the power input is associated with this force, $\int \mathbf{F} \cdot \mathbf{u} dV$, i.e. with the velocity field, **u**. The advantage of the above 'definition' of small scales can be seen from the following.

While the mean contribution of the nonlinear term in the energy balance is vanishing, the nonlinearity definitely is producing vorticity and strain in physical space, since the mean enstrophy and strain production are strictly positive:

$$\langle \omega_i \omega_j s_{ij} \rangle > 0; \qquad \langle - s_{ij} s_{jk} s_{ki} \rangle > 0$$

SELF (!)-AMPLIFICATION OF VORTICITY AND STRAIN

$$(½)D\omega^2/Dt = \omega_i\omega_j S_{ij} + \nu\omega_i\Delta\omega_i + \varepsilon_{ijk}\omega_i\, \partial F_k/\partial x_j$$

$$(½)Ds^2/Dt = - s_{ij}s_{jk}s_{ki} - (1/4)\omega_i\omega_j s_{ij} - s_{ij}\partial^2 p/\partial x_i\partial x_j + \nu s_{ij}\Delta s_{ij} + s_{ij}F_{ij}$$

The property of self amplification of vorticity and strain is responsible for the fact the neither enstrophy ω^2 nor the total strain s^2 are inviscid invariants as is the kinetic energy u^2

That is 3-D turbulent flows have a natural tendency to create small scales. The velocity field (and its energy) arising in the process of (self) production of the field of velocity derivatives is the one which is associated with small scales. This process is what can be called as energy (and not only energy) transfer from large to small scales in physical space. The latter are not necessarily created via a stepwise turbulent 'cascade': it can be bypassed, and most probably is so in turbulent flows, for example via broad-band instabilities with highest growth rate at short wavelengths. More examples are given below

Small scales are not necessarily created from large scales via a stepwise turbulent 'cascade': it can be bypassed, and most probably is so in turbulent flows, for example via broad-band instabilities with highest growth rate at short wavelengths (Pierrehumbert and Widnall, 1982) or some other approximately single step process (Betchov 1976, Douady et al. 1991, Ott 1999; Shen and Warhaft 2000, Vincent and Meneguzzi 1994). The problem goes back to Townsend 1951: *...the postulated process differs from the ordinary type of turbulent energy transfer being fundamentally a single process*

An important example, is the complicated structure of vorticity (and passive objects) with power law spectrum, (multi)fractality and significant variations down to very small scale that can be produced by a single instability at much larger scale without any 'cascade' of successive instabilities arising in a simple fluid flow via a single instability only(!), Ott 1999.

Examples from transitional and partly turbulent flows

Abrupt transition

Pipe. Entrainment. Vortex breakdown. Turbulent spots.
Blow up instabilities. Bypass instabilities and transitions.
In all the above laminar fluid becomes turbulent in ‘no time’ without any cascade whatsoever.

Examples from transitional and partly turbulent flows

Transitional flows

ABRUPT TRANSITION

The transition between laminar and turbulent flows at the beginning and end of the turbulent region is abrupt relative to its duration.

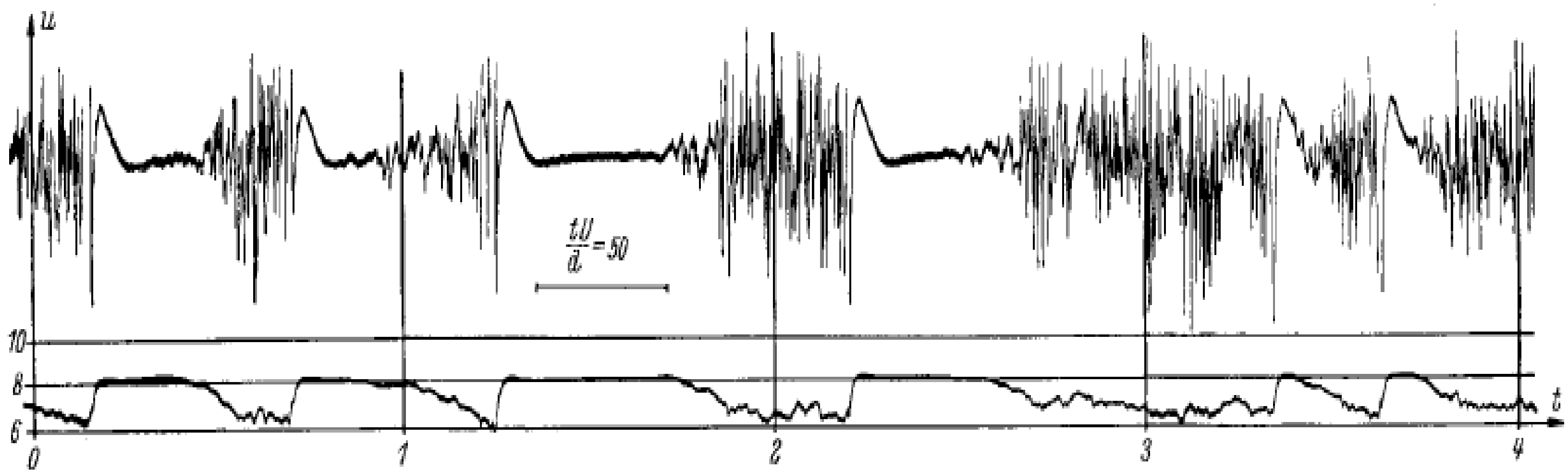


Abb. 6. Geschwindigkeitsschriebe in Rohrmitte bei $Re = 2550$. Rohreinlauf Nr. 1. Lauflänge $x/d = 322$. Obere Kurve: Aufzeichnung nur rascher Geschwindigkeitsschwankungen (Hitzdrahtstrom durch Wechselstromverstärker geleitet); untere Kurve: Gesamtverlauf der Geschwindigkeit (Hitzdrahtstrom durch Gleichspannungsverstärker geleitet). Geschwindigkeit u in m/sek.; Zeit t in sek.

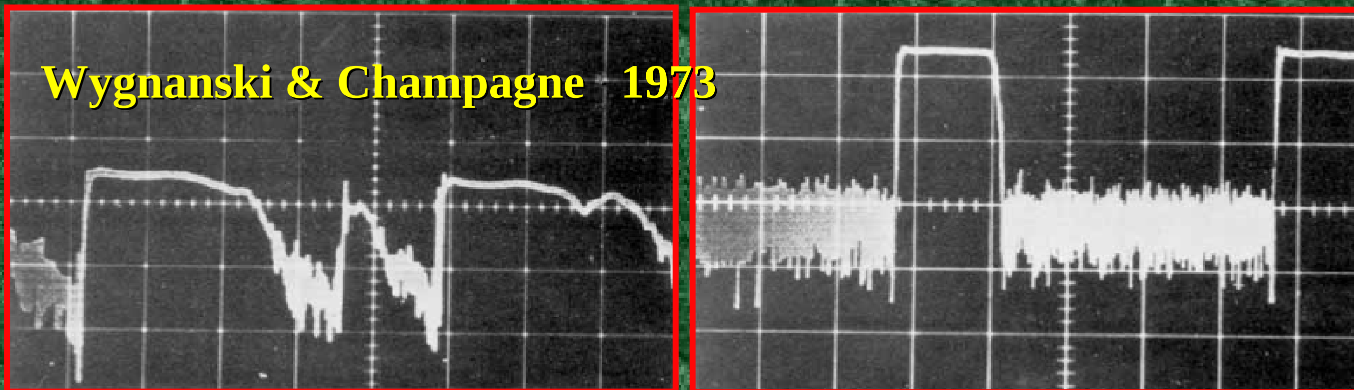
ROTTA, J. C.(1956) Experimenteller Beitrag zur Entstehung turbulenter Strömung im Rohr, *Ing. Arch.*, 24, No. 4, 258–281.

ABRUPT TRANSITION

The transition between laminar and turbulent flows at the beginning and end of the turbulent region is abrupt relative to its duration.

The transition is indeed frustratingly abrupt

s l u g s



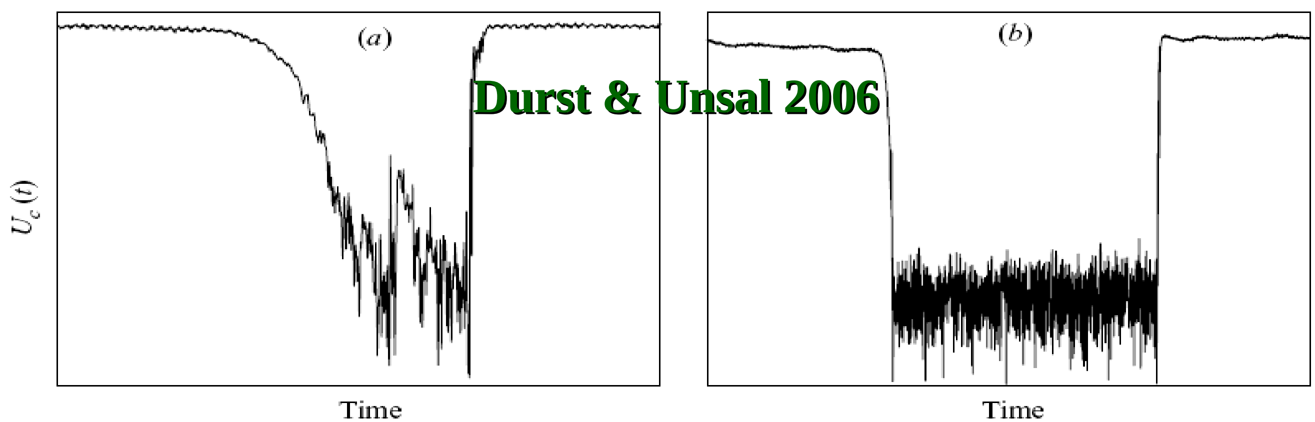
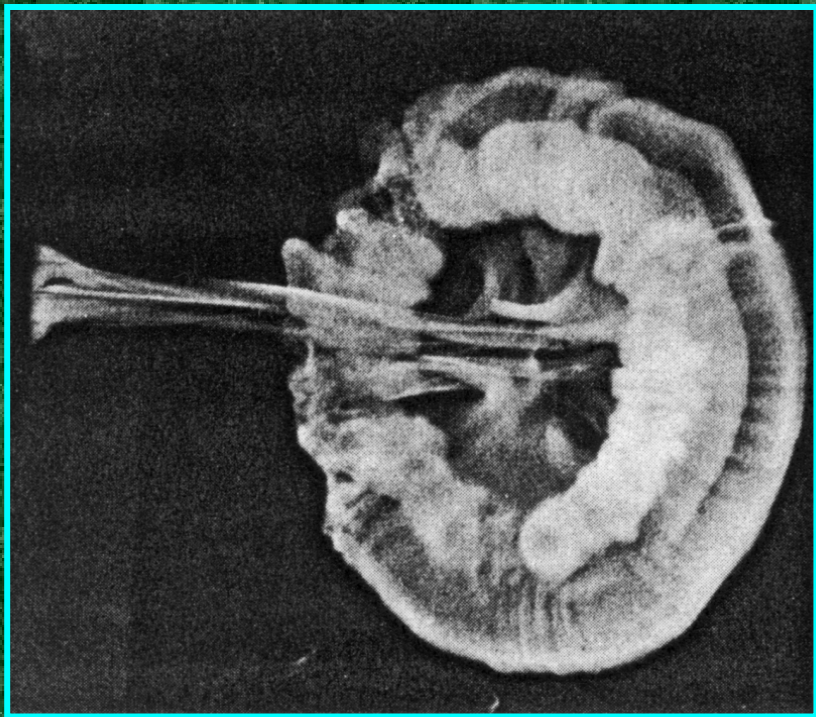


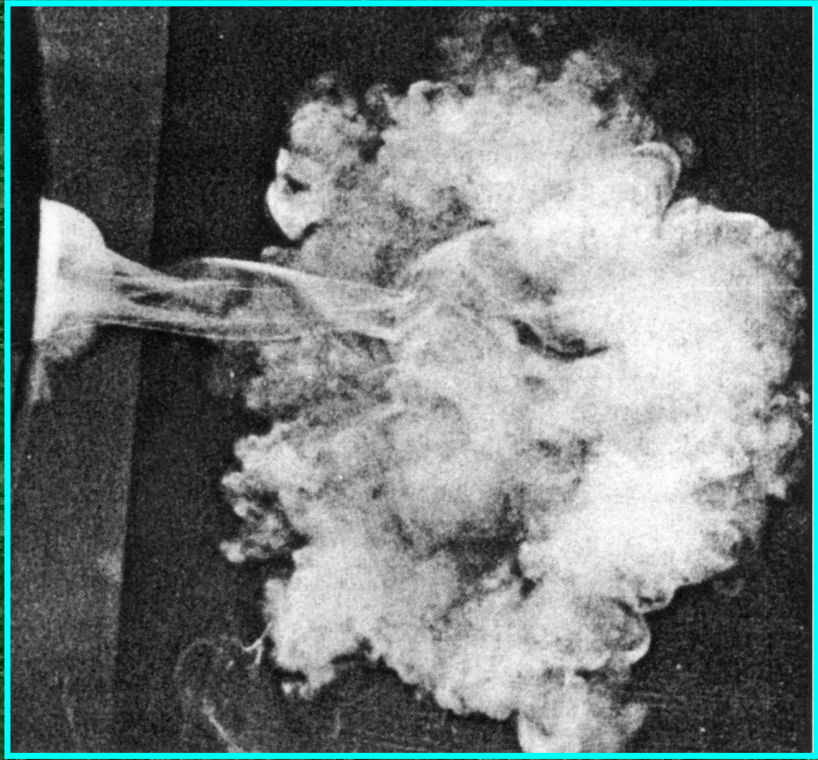
FIGURE 11. Centreline velocity time records as examples of (a) puff and (b) slug-type turbulent structures.

p u f f s

Abrupt Transition



A vortex ring impinging a wall becomes turbulent in no time as it approaches the wall



Examples from transitional and partly turbulent flows

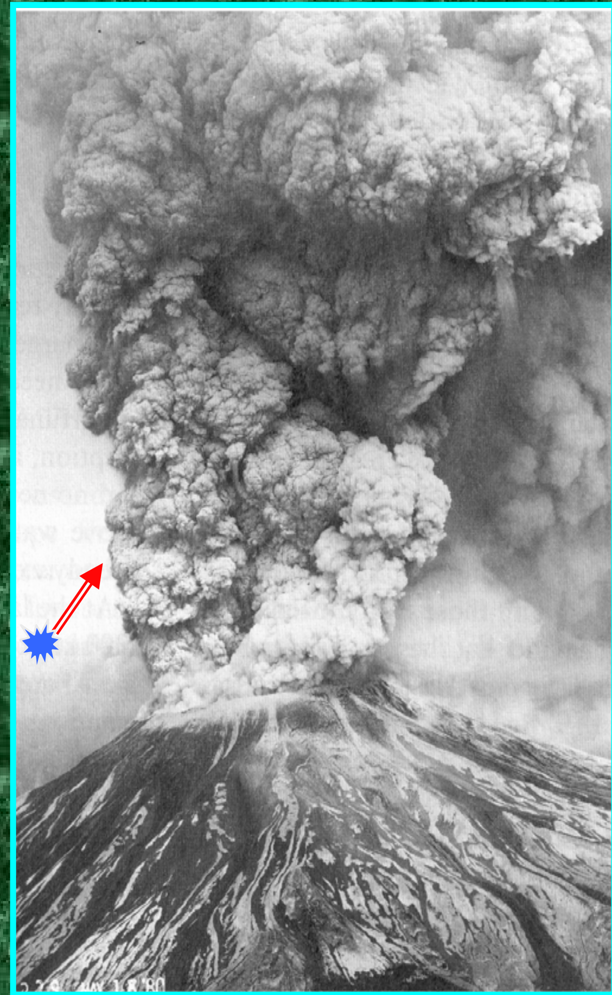
Partly turbulent flows

PTF - ENTRAINMENT

A turbulent jet from testing a Lockheed rocket engine in the Los Angeles hills

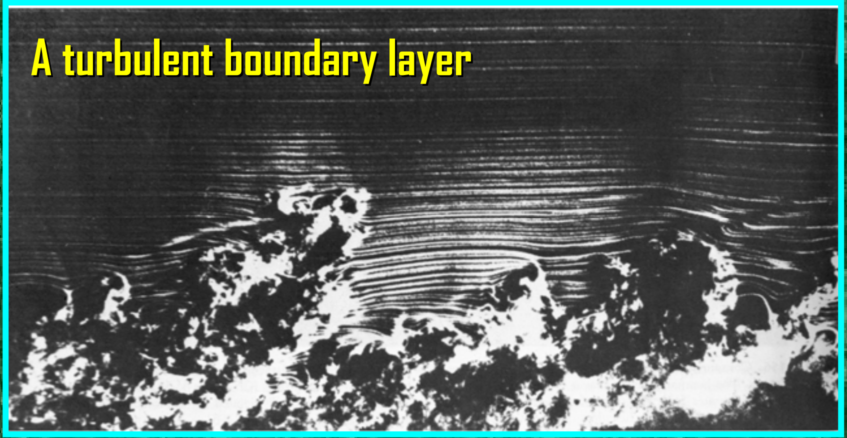
The laminar-turbulent "interface" is sharp so that fluid particles (note the Lagrangian aspect !) "are found" abruptly in a turbulent environment

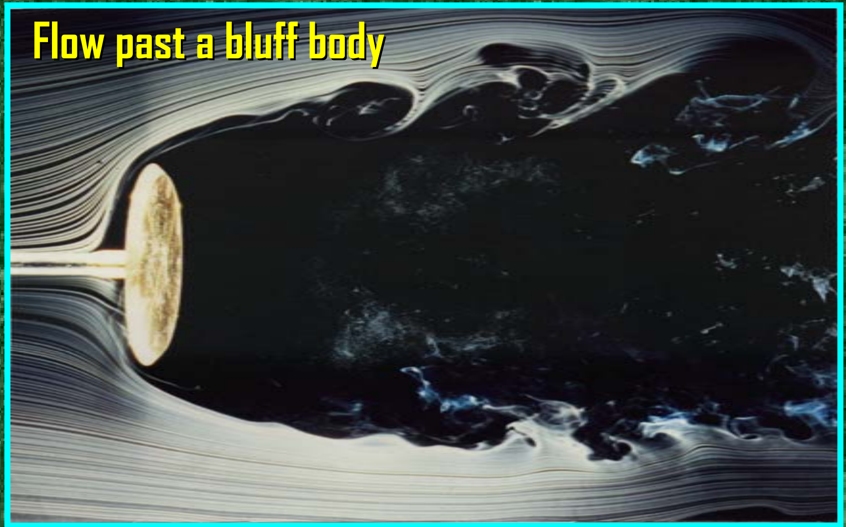
Mount St. Helen volcano on 18 May 1980

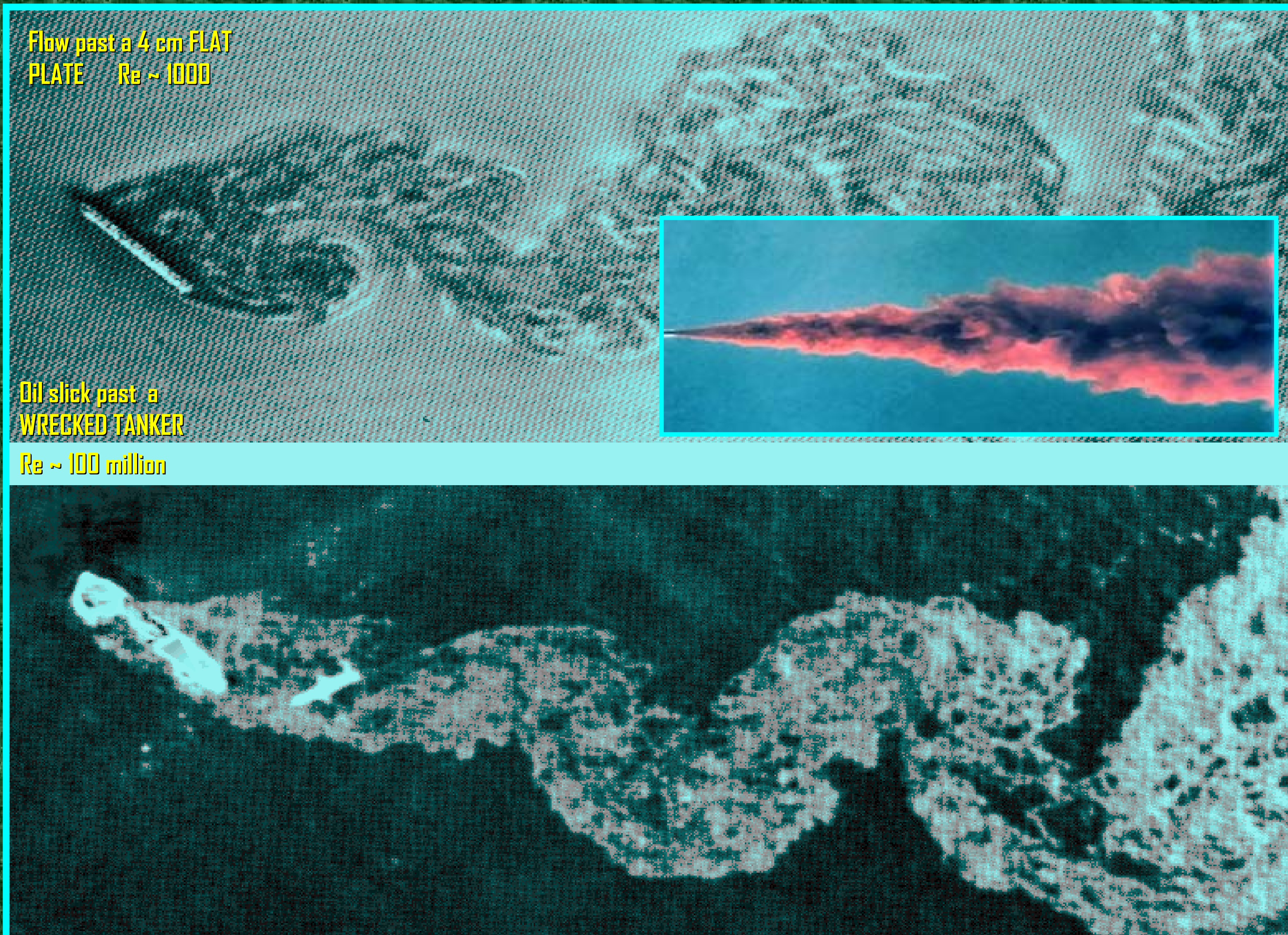


PARTLY TURBULENT FLOWS II

Coexistence of laminar and turbulent regions in the same flow

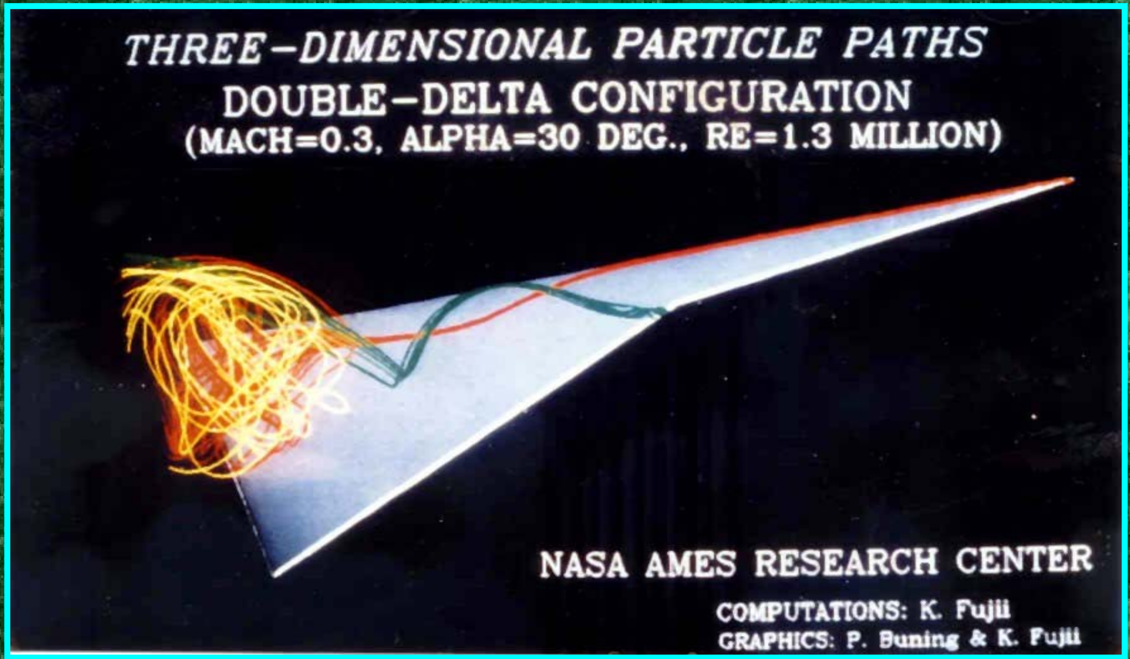


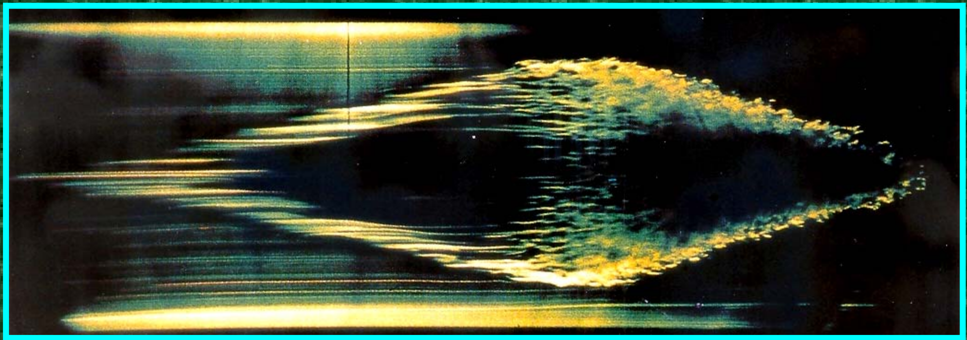


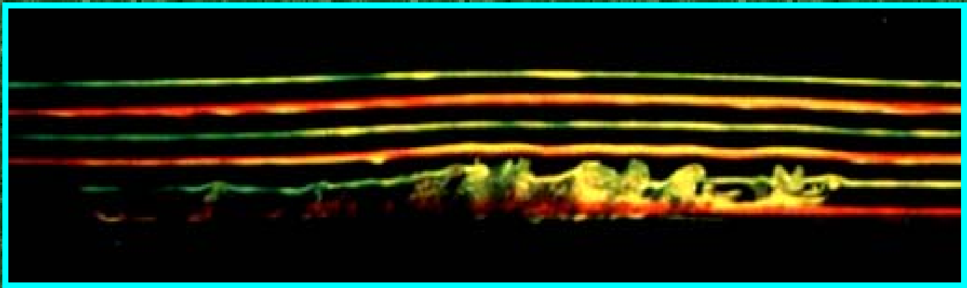

Flow past a 4 cm FLAT
PLATE Re ~ 1000
Oil slick past a
WRECKED TANKER
Re ~ 100 million

COEXISTENCE OF LAMINAR AND TURBULENT REGIONS IN THE SAME FLOW

Vortex breakdown







Coexistence of laminar and turbulent regions in the same flow

Turbulent spots

The front velocity is too small to explain the spot spreading.

Another example, is the complicated structure arising in a simple fluid flow via a single (!) instability with a power law spectrum, (multi)fractality and significant variations down to very small scales without any ‘cascade’ of successive instabilities. (REYL ET AL 1998, OTT, 1999).
This is true both the of the vorticity and passive scalar field resulting from a linear instability of such a flow.

The above ‘definition’ of small scales and self production of velocity derivatives) has a variety of consequences

1. Since the whole flow field (including velocity, which is mostly a large scale object) is determined entirely by the field of vorticity and or strain (namely, the velocity field is a functional of vorticity and/or strain $\mathbf{u} = \mathscr{F}\{\omega(\mathbf{x}, t)\}$, $\mathbf{u} = \mathscr{G}\{s_{ij}(\mathbf{x}, t)\}$), the production of vorticity and or strain 'reacts back' in creating the corresponding velocity field, i.e. the small scales are not just 'swept' by the large ones. Therefore, it is incorrect to treat the small scales as a kind of passive object (e.g. passive sink of energy) swept by the large scales or just 'slaved' to them.

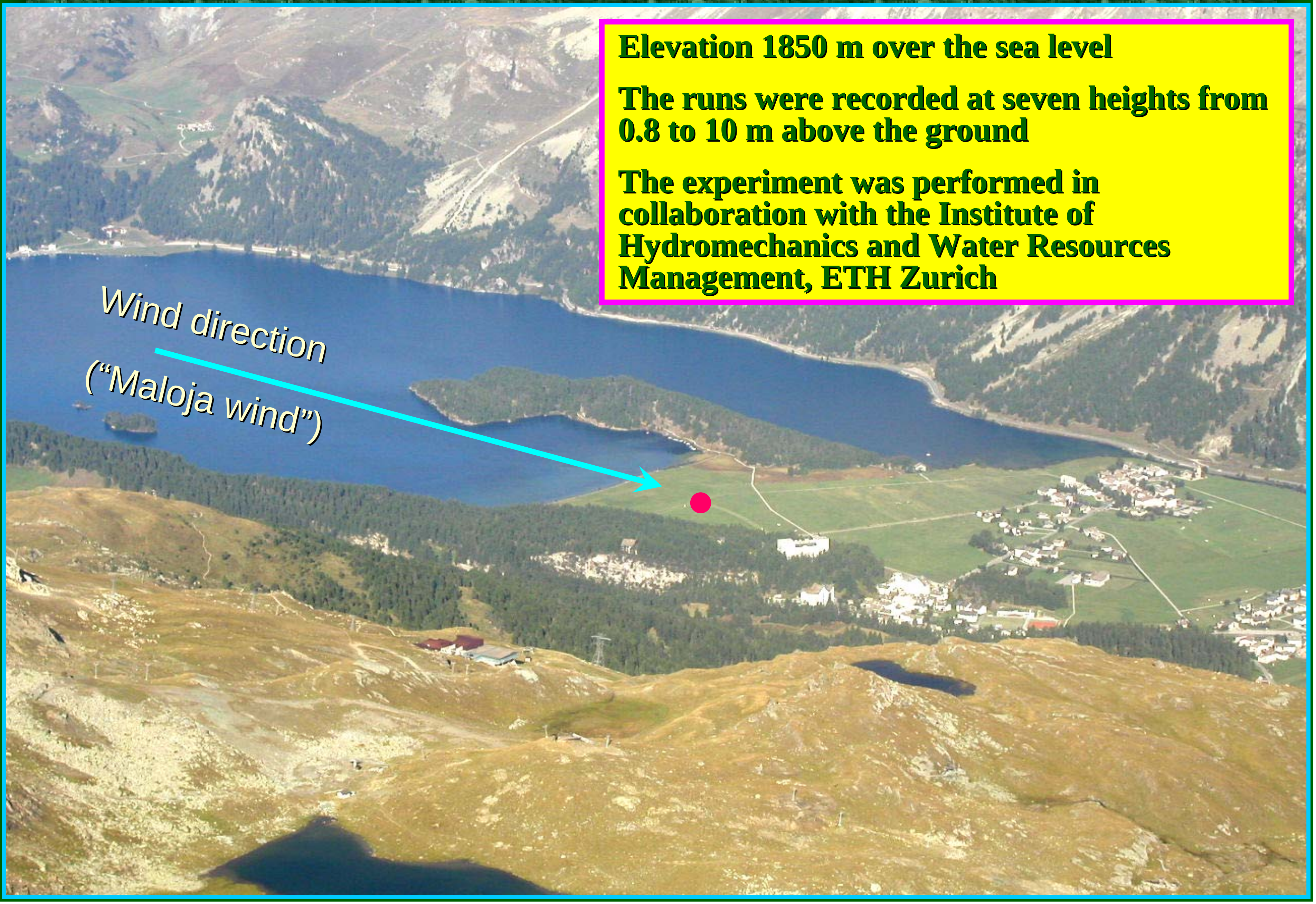
2. Due to nonlocality of the relations $\mathbf{u} = \mathscr{F}\{\omega(\mathbf{x}, t)\}$, $\mathbf{u} = \mathscr{G}\{s_{ij}(\mathbf{x}, t)\}$ mostly small scale vorticity and strain are, generally, creating also some large scale velocity. This and other aspects of nonlocality contradict the idea of cascade in physical space, which is local by definition (e.g. see Frisch, 1995, p.104). In particular, the frequently assumed statistical independence of large scales, such as structure functions $S_p(r) = \langle(\Delta u)^p\rangle$, $\Delta u \equiv [\mathbf{u}(\mathbf{x} + \mathbf{r}) - \mathbf{u}(\mathbf{x})]\cdot\mathbf{r}/r$, in the inertial range on the (nature) of dissipation, i.e. strain, stands in contradiction with the relation $\mathbf{u} = \mathscr{G}\{s_{ij}(\mathbf{x}, t)\}$ together with the process of self-production of strain in turbulent flows.

3. Inertial ($\omega_i \omega_j s_{ij}$, $- s_{ij} s_{jk} s_{ki}$) and viscous terms ($\nu \omega_i \nabla^2 \omega_i$, $\nu\, s_{ij} \nabla^2 s_{ij}$) do not act as if they were additive and independent - their interaction is crucial, e.g. the presence of viscosity changes qualitatively the nature of the enstrophy/strain production and the properties of the vorticity/strain field. This in turn means that it is important in the properties of the velocity (including structure functions) as the latter is fully determined by the field of vorticity/strain (kinematic nonlocality).

Is cascade local?
Nonlocality

Statistical dependence of small on large scales.

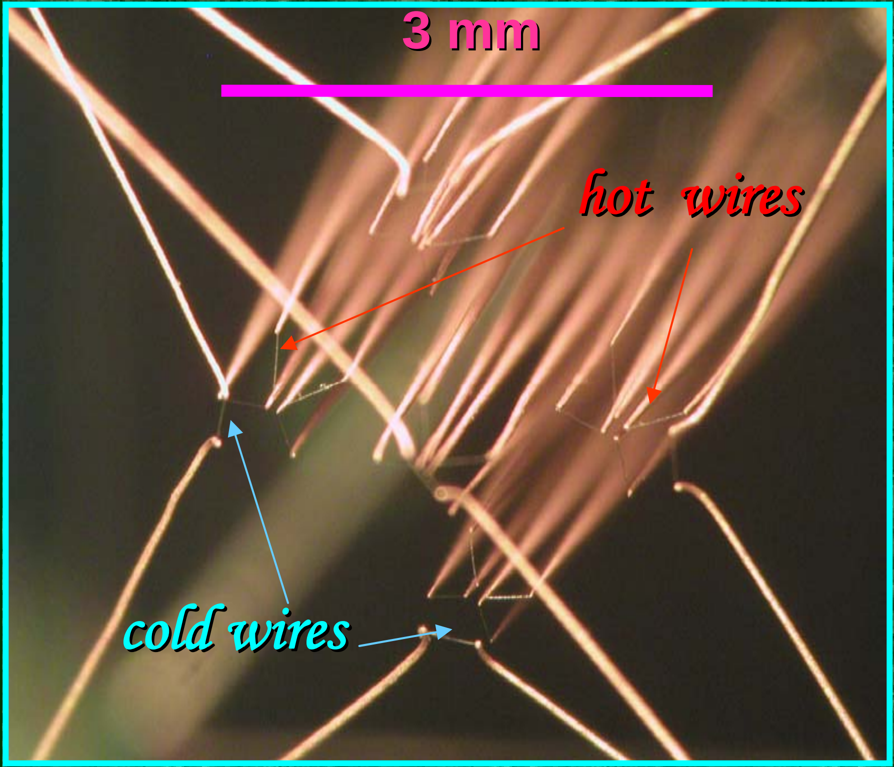
THE MARIA SILS SITE, SWITZERLAND



The Maria Sils Site, Switzerland

The calibration unit at 3m height

THE PROBE

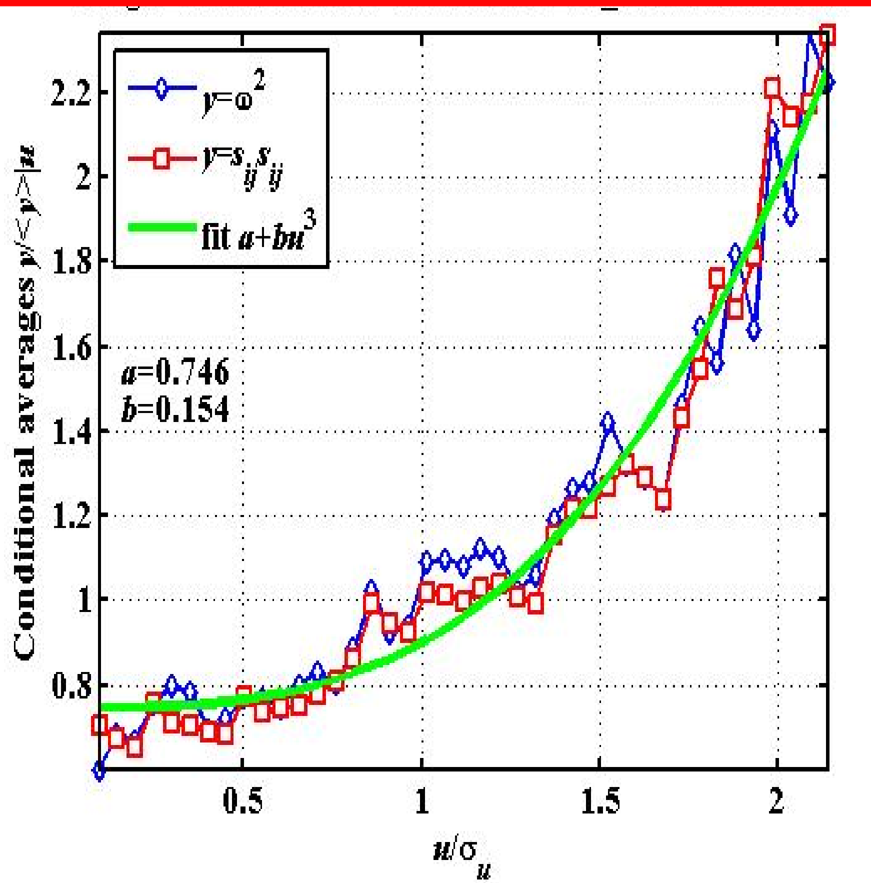


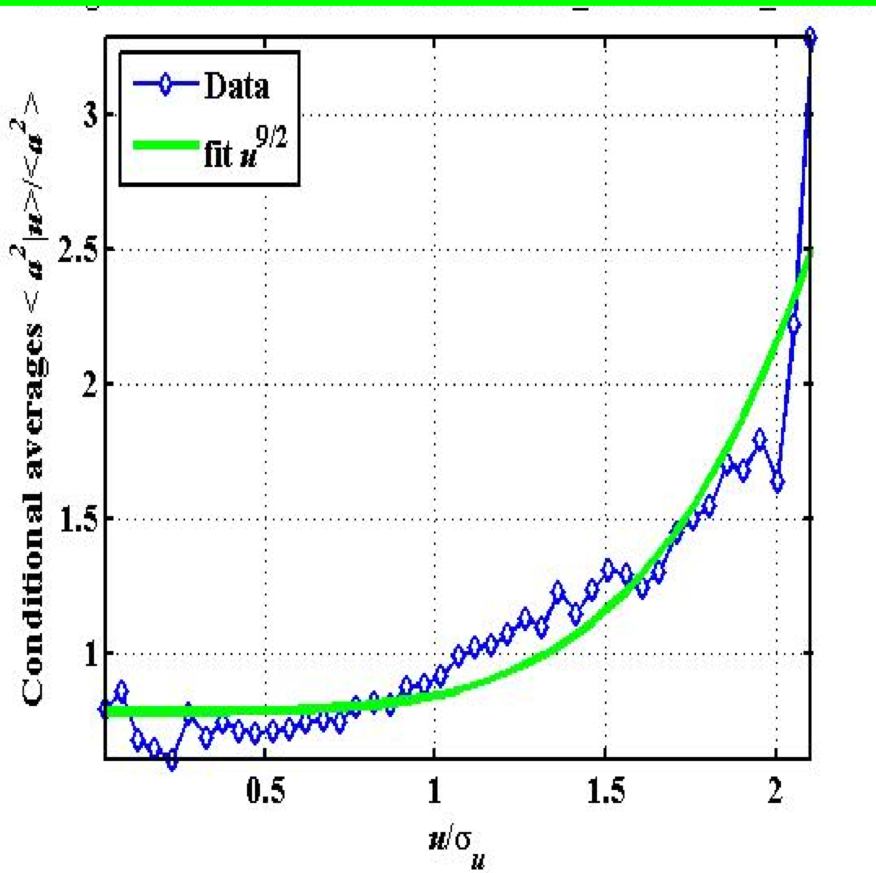
The tip of the probe

Manganin is used as a material for the sensor prongs instead of tungsten because the temperature coefficient of the electrical resistance of manganin is 400 times smaller than that of tungsten.

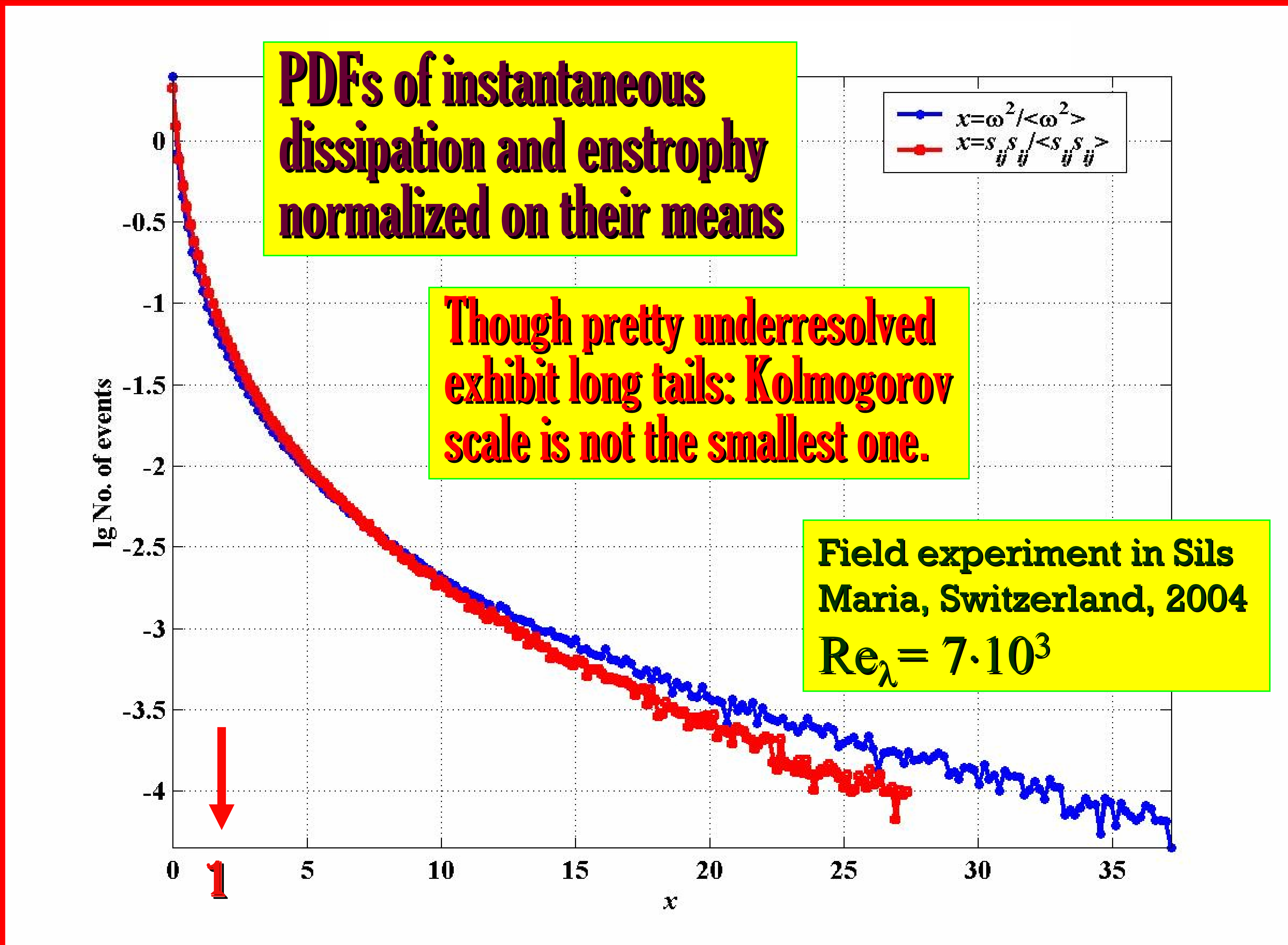
Statistical dependence of small on large scales.

Enstrophy ω^2, total strain s^2 and squared acceleration a^2 conditioned on magnitude of the velocity fluctuation vector, Field experiment, Sils-Maria, Switzerland, 2004, $Re_\lambda = 6800$ (Gulitskii et al. 2007, *J. Fluid Mech.*,, 589,)

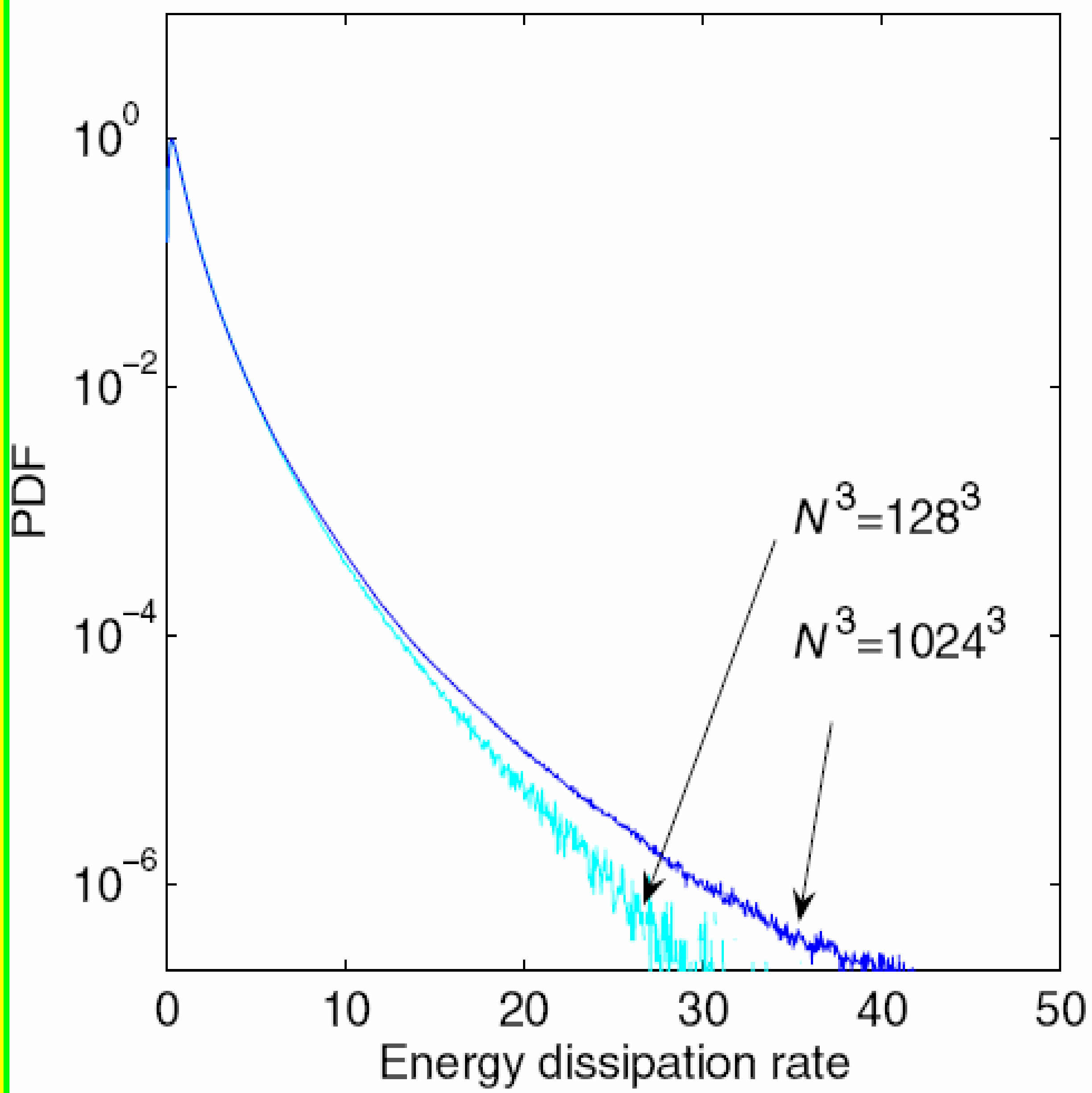




What about dependence of large on small scales?

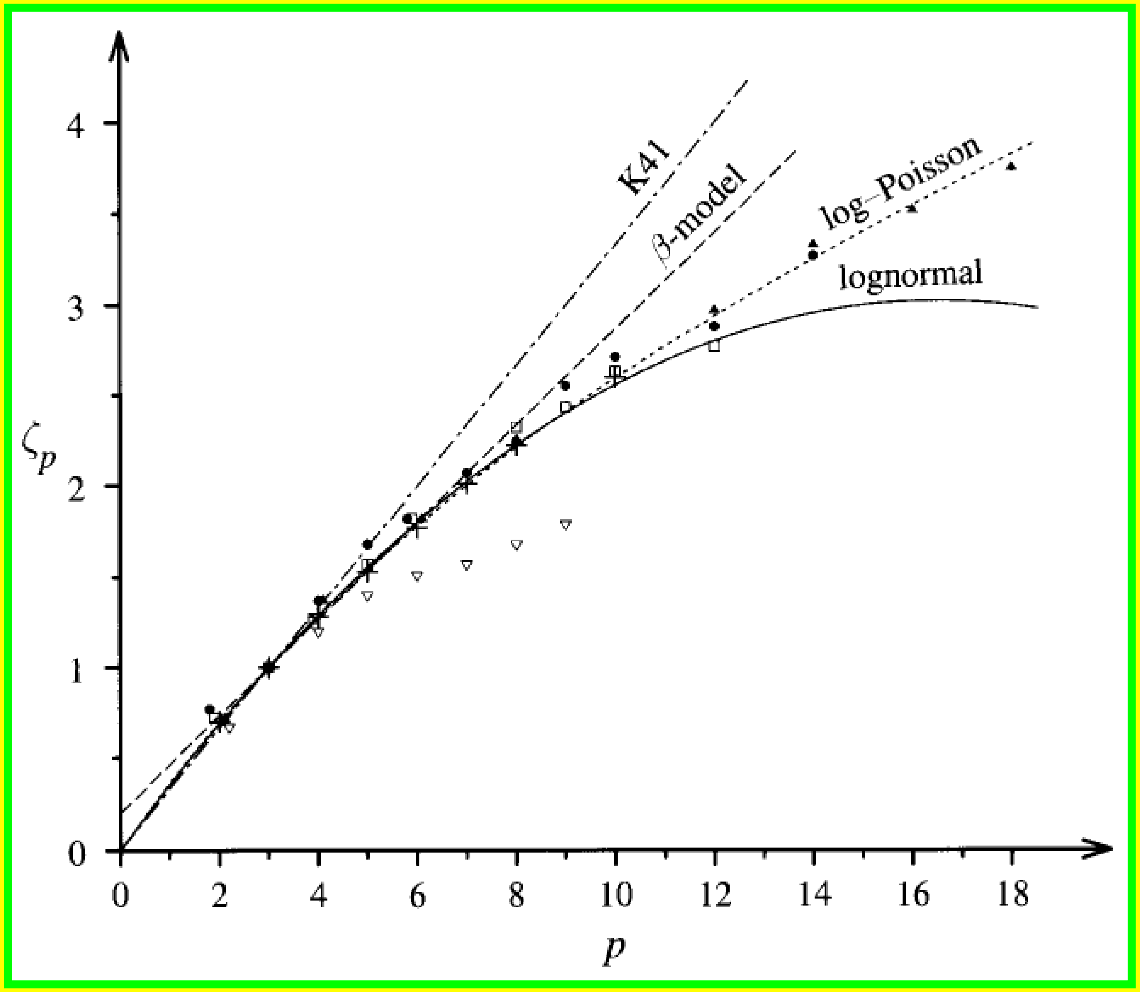


PDFs of instantaneous dissipation and enstrophy normalized on their means
Though pretty underresolved exhibit long tails: Kolmogorov scale is not the smallest one.
Field experiment in Sils Maria, Switzerland, 2004
$Re_\lambda = 7\cdot 10^3$
$x=\omega^2/\langle\omega^2\rangle$
$x=s_{ij}s_{ij}/\langle s_{ij}s_{ij}\rangle$
lg No. of events
x
0
-0.5
-1
-1.5
-2
-2.5
-3
-3.5
-4
0
1
5
10
15
20
25
30
35



The dissipation field is given in units of the mean energy dissipation rate EThe case with $k_{max}\eta_K$ = 1.2 (cyan curve), corresponding roughly to the standard resolution in a box of size N= 128, is compared with that of superfine resolution (blue curve, see also table 1). While the cores of both PDFs agree, deviations are manifest in the far tails. Both runs are for R_λ = 65.. Approximately 1.7 × 10^8 data points were processed for the analysis in the lowresolution run; the corresponding number for the high-resolution run was about 30 times larger.

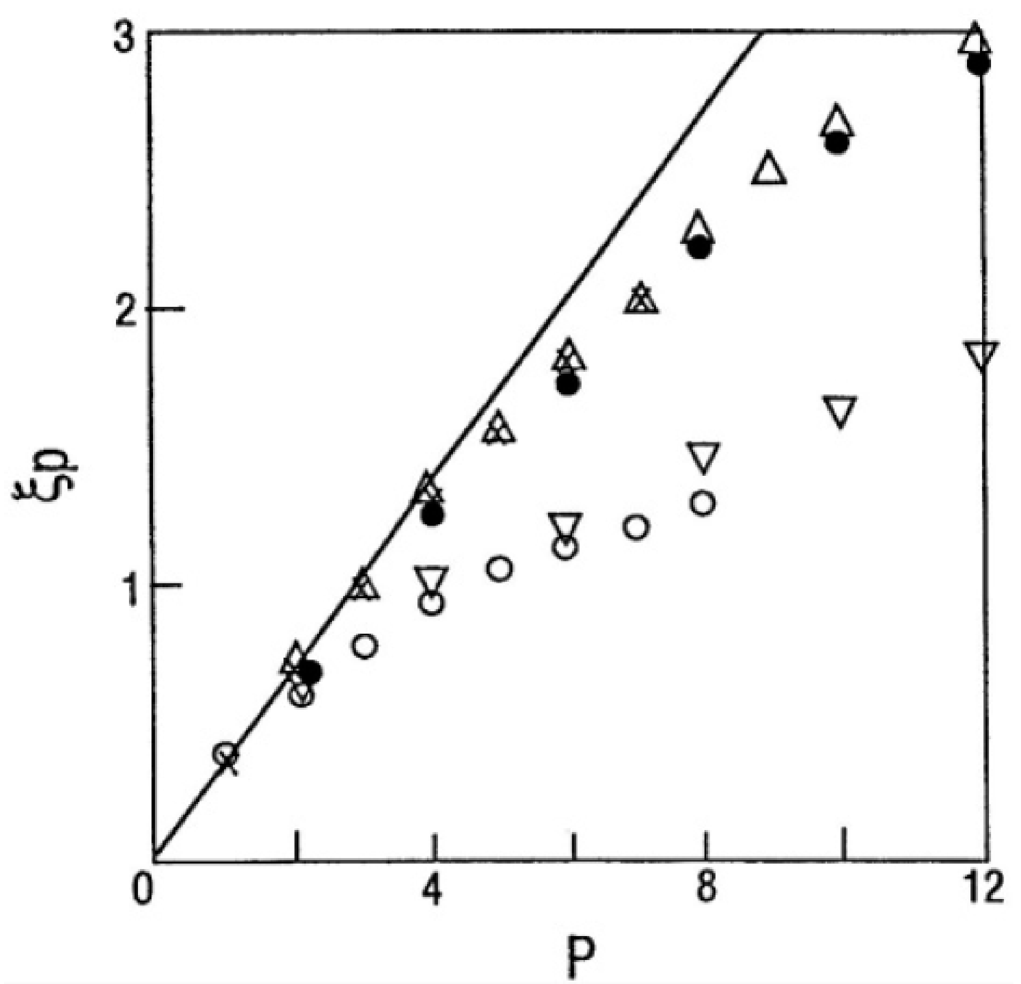
Schumacher et al., 2007



‘Anomalous’ scaling

from FRISCH 1995

Fig. 8.8. Exponent ζ_p of structure functions in the time domain of order p vs p. Inverted white triangles: data from Van Atta and Park (1972); black circles, white squares and black triangles: data from Anselmet, Gagne, Hopfinger and Antonia (1984) with $R_\lambda = 515, 536, 852$, respectively; + signs: S1 data processed by ‘ESS’ (see p. 131). Straight chain line: $\zeta_p = p/3$ (K41); dashed line: β-model (eq. (8.31)) with $D = 2.8$; solid line: lognormal model (eq. (8.122)) with $\mu = 0.2$; dotted line: log–Poisson model (eq. (8.141)).

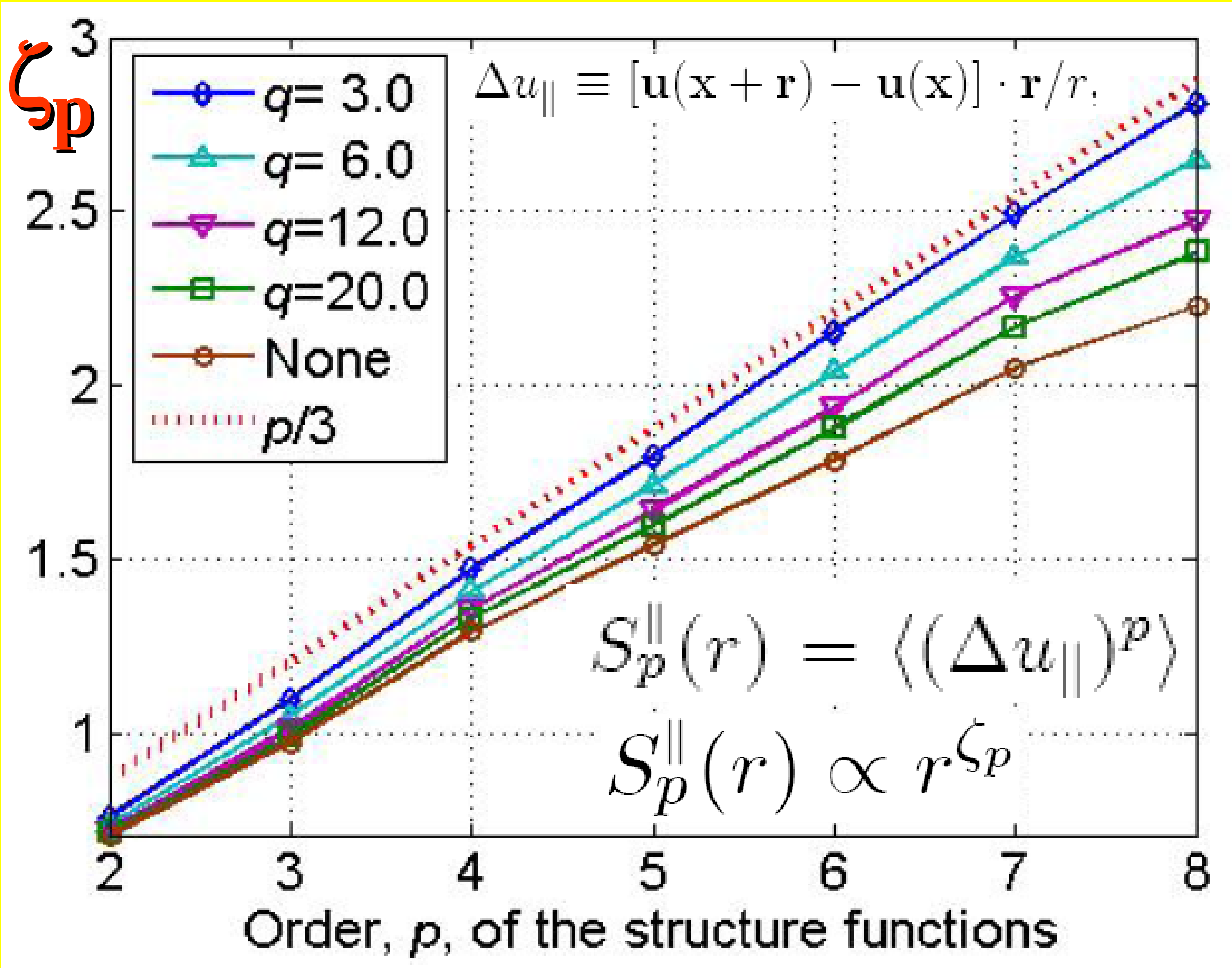


'Anomalous' scaling

from TSINOBER 2001

Figure 7.1. Exponents of structure functions for the longitudinal velocity component (△, •, ×) and temperature (▽, ∘); △ - Anselmet et al. (1984), ▽ – Antonia et al. (1984), ∘ – Ruiz-Chavaria et al. (1996), • – Vincent and Meneguzzi (1991); and exponents of structure functions for the transverse velocity component, × – Noullez et al. (1997). This figure is from Tsinober (1998b).

Scaling exponents, ζ_p, of structure functions for the longitudinal velocity component for the full data and the same data in which the strong dissipative events with different thresholds were removed.



An event is qualified as a strong dissipative if at least at one of its ends (x, x+r) the instantaneous dissipation

$\varepsilon > q \langle \varepsilon \rangle$

for $q > 1$.

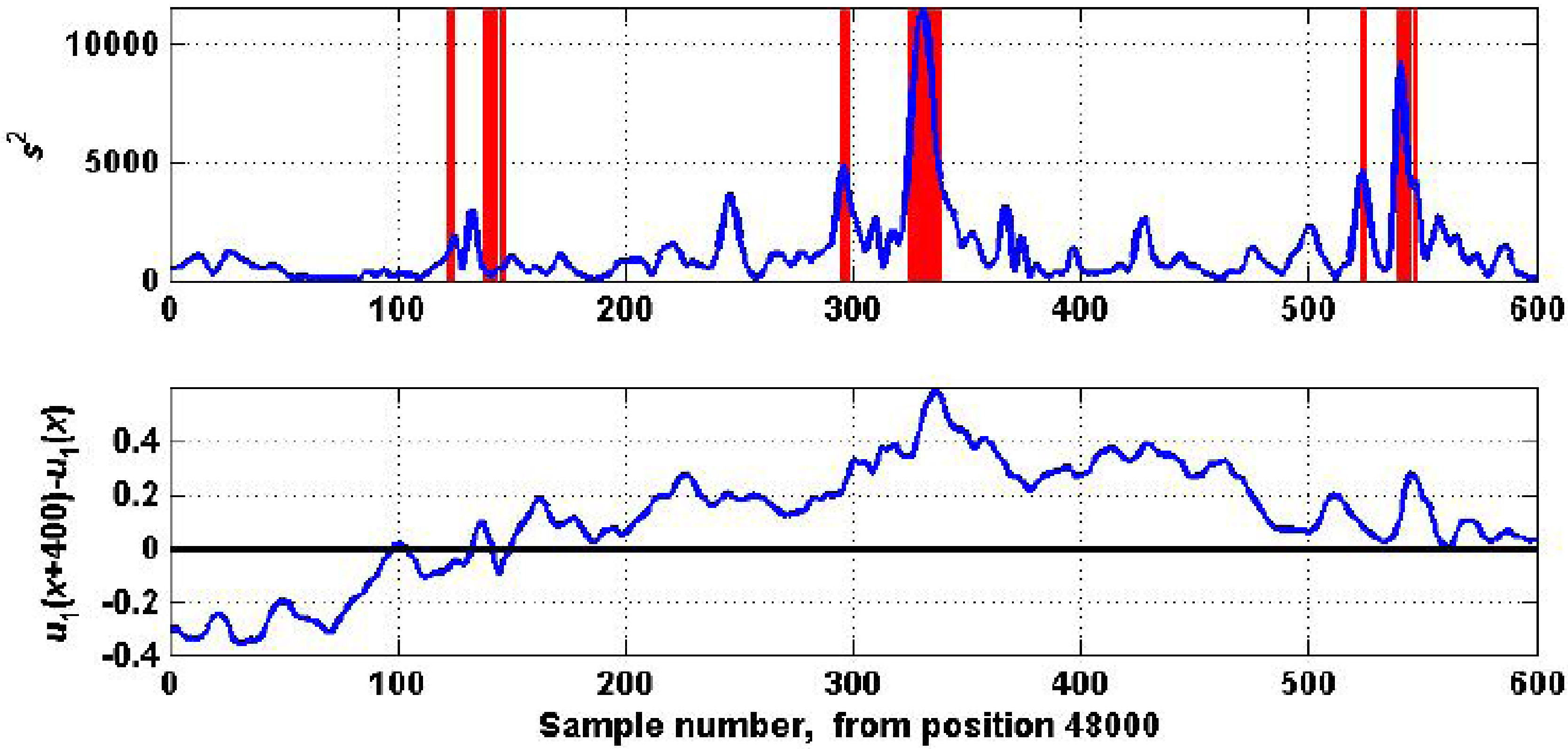


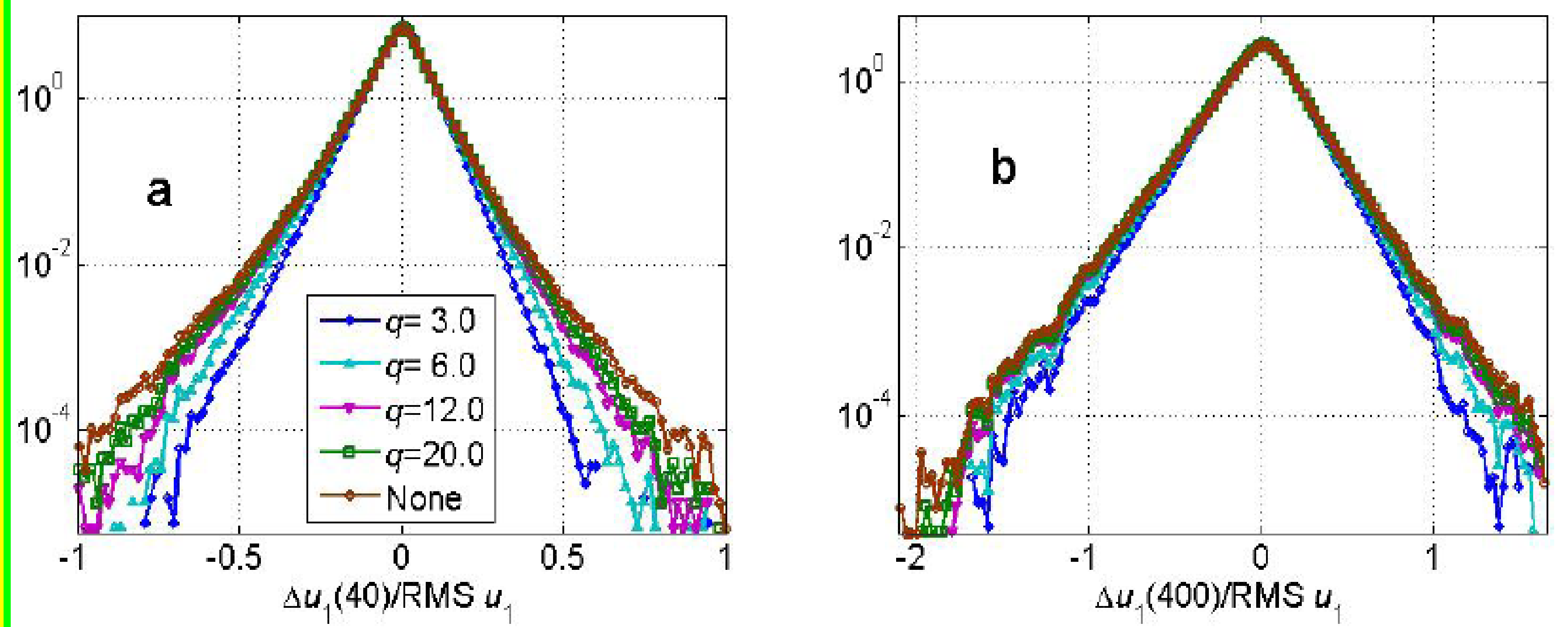
FIG. 2. Example of simultaneous time series of the squared magnitude of the rate of strain tensor, s^2, proportional to the dissipation ϵ (top) and the velocity increments, $\Delta u_1 \equiv u_1(x+r) - u_1(x)$ for $r = 400\eta$ (bottom). The marked segments correspond to the strong events, selected with the value of the threshold $q = 12$. This corresponds to the value of $s^2 \approx 4,000$.

The effect of the removal of the strong dissipative events is obviously much stronger for higher-order structure functions. For example, there are only 5% of dissipative events for $q = 6$ sitting mostly at tails of the PDF of $u_1(r)$ for r/ = 400 (i.e. deep in the 'inertial' range), which contribute about 36% to the total dissipation. These events contribute nearly 60% to the value of $S_8(r)$ at Re $\sim 10^4$.

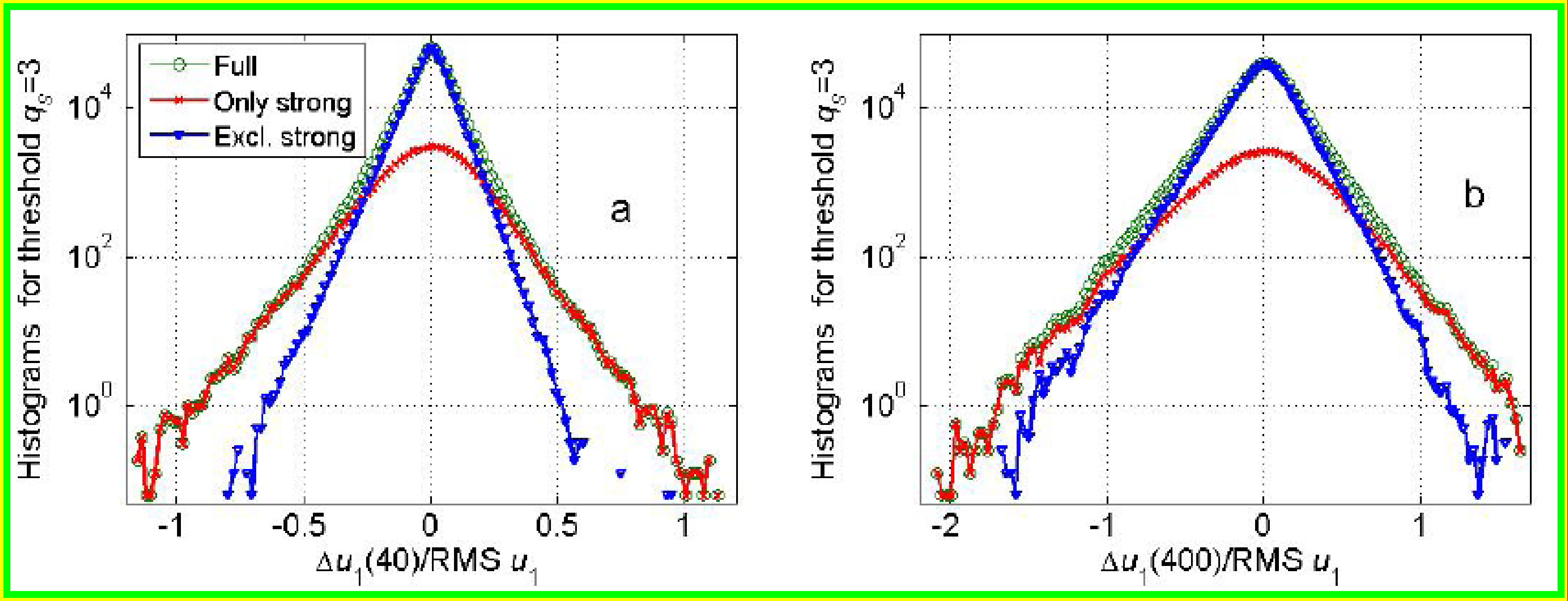
These same events change the S_2 (r) by about 11.5%, but contribute about 9% to S_3 (r)

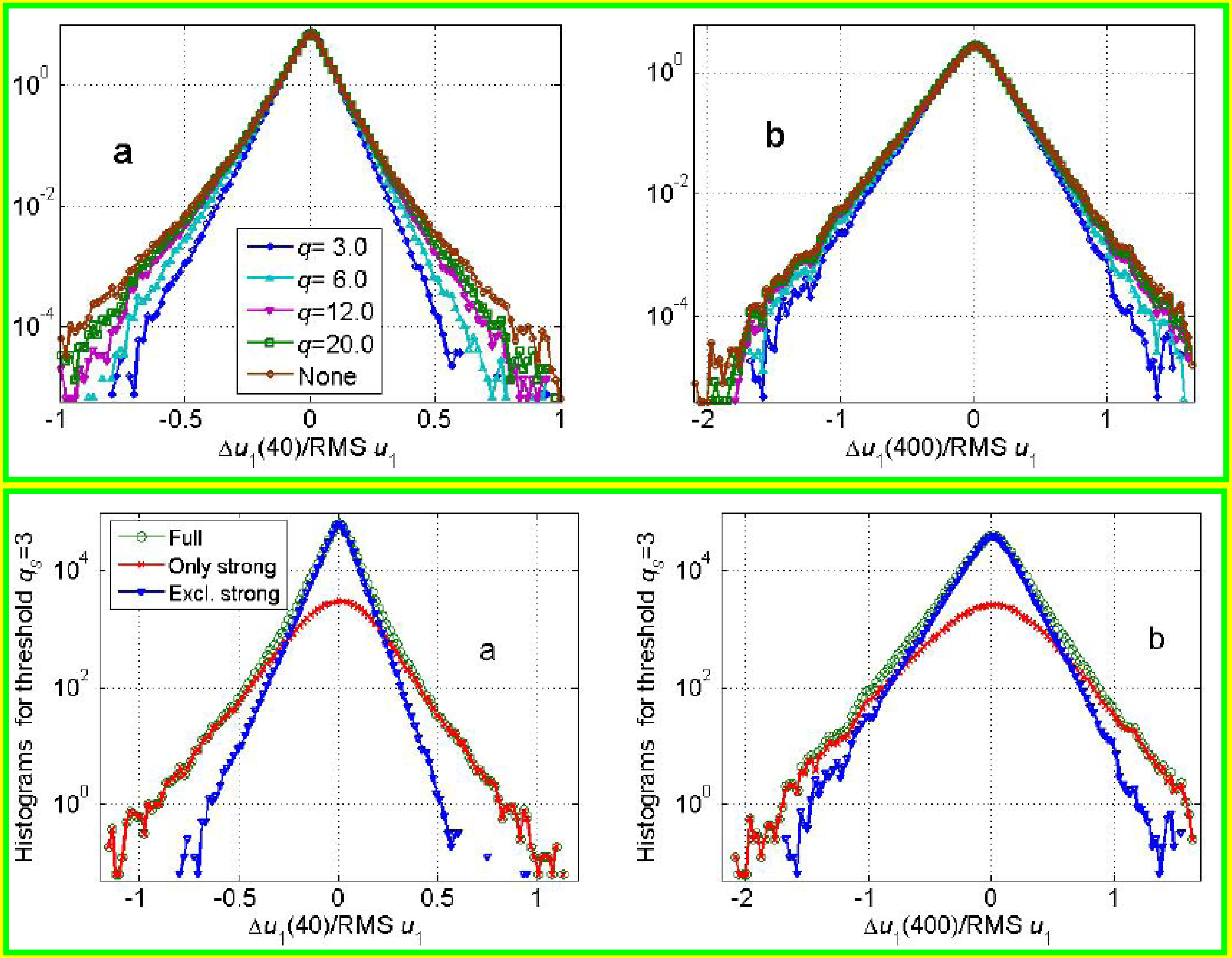
PDFs of the increments of the longitudinal velocity component for the full data and the same data in which the strong dissipative events with different thresholds were removed.. r/η = 40 corresponds to the lower edge of the inertial range. (a). r/η = 400 is deep in the inertial range (b)

An event is qualified as a strong dissipative if at least at one of its ends (x, x+r) the instantaneous dissipation $\varepsilon > q \langle \varepsilon \rangle$ for $q > 1$.

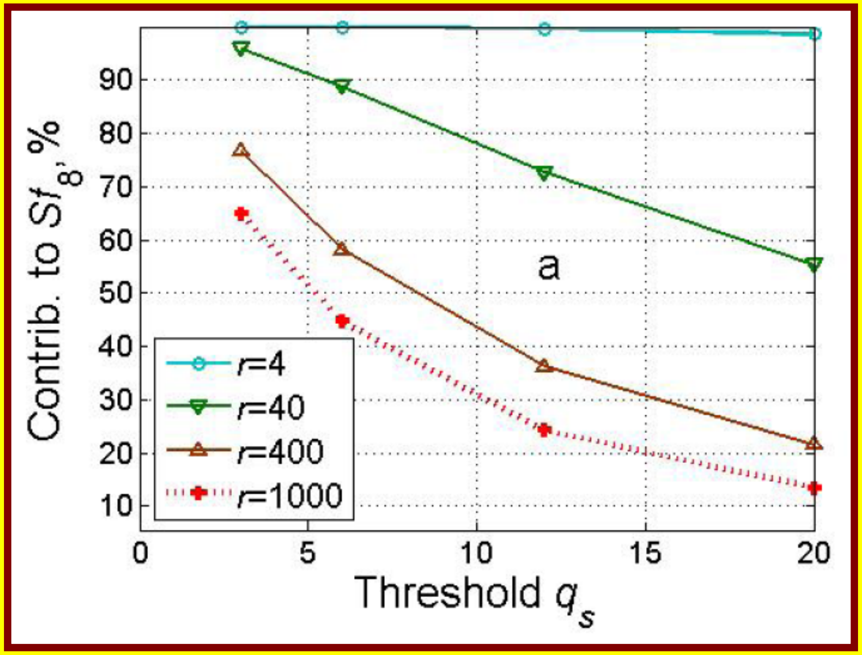


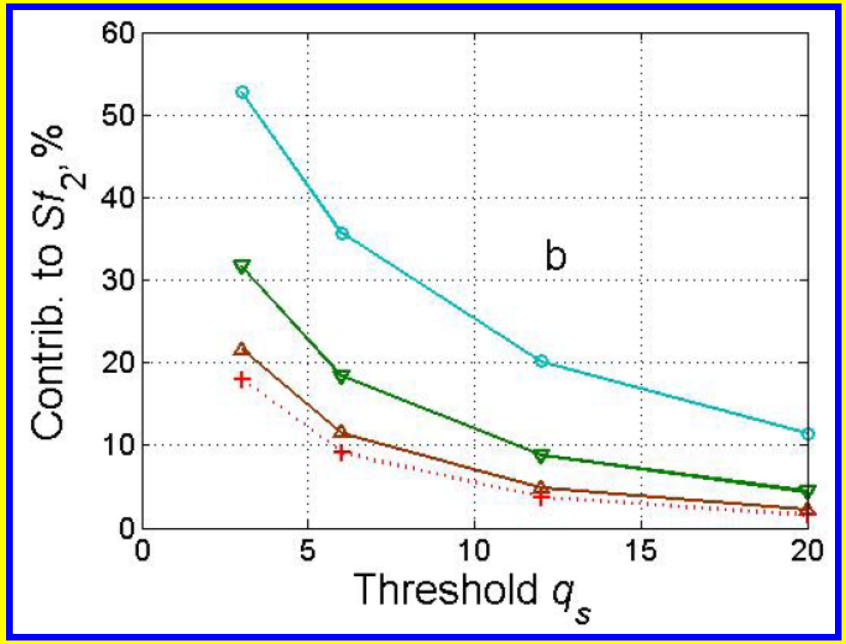
Histograms of the increments of the longitudinal velocity component for the full data and the same data in which the strong dissipative events with different thresholds were removed.. $r/\eta = 40$ corresponds to the lower edge of the inertial range. (a). $r/\eta = 400$ is deep in the inertial range (b)

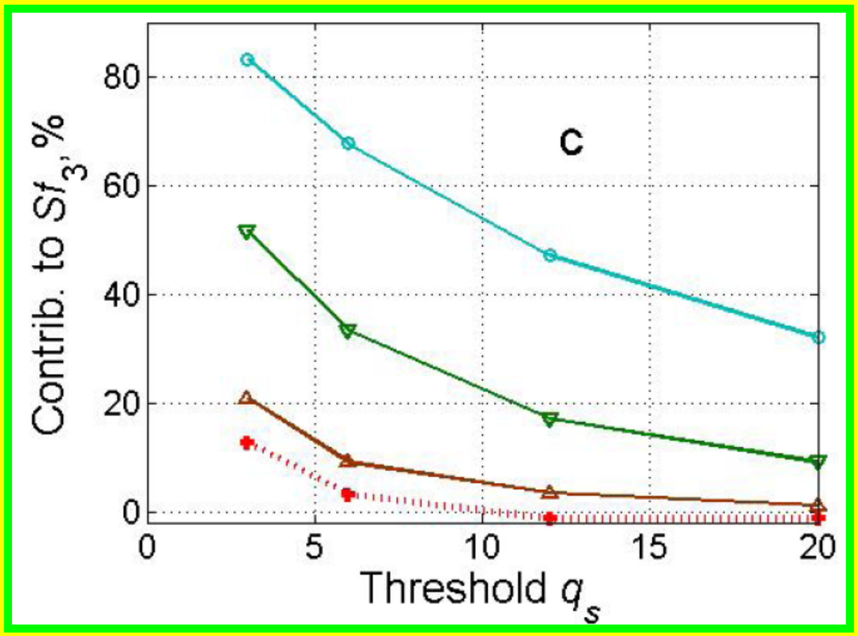




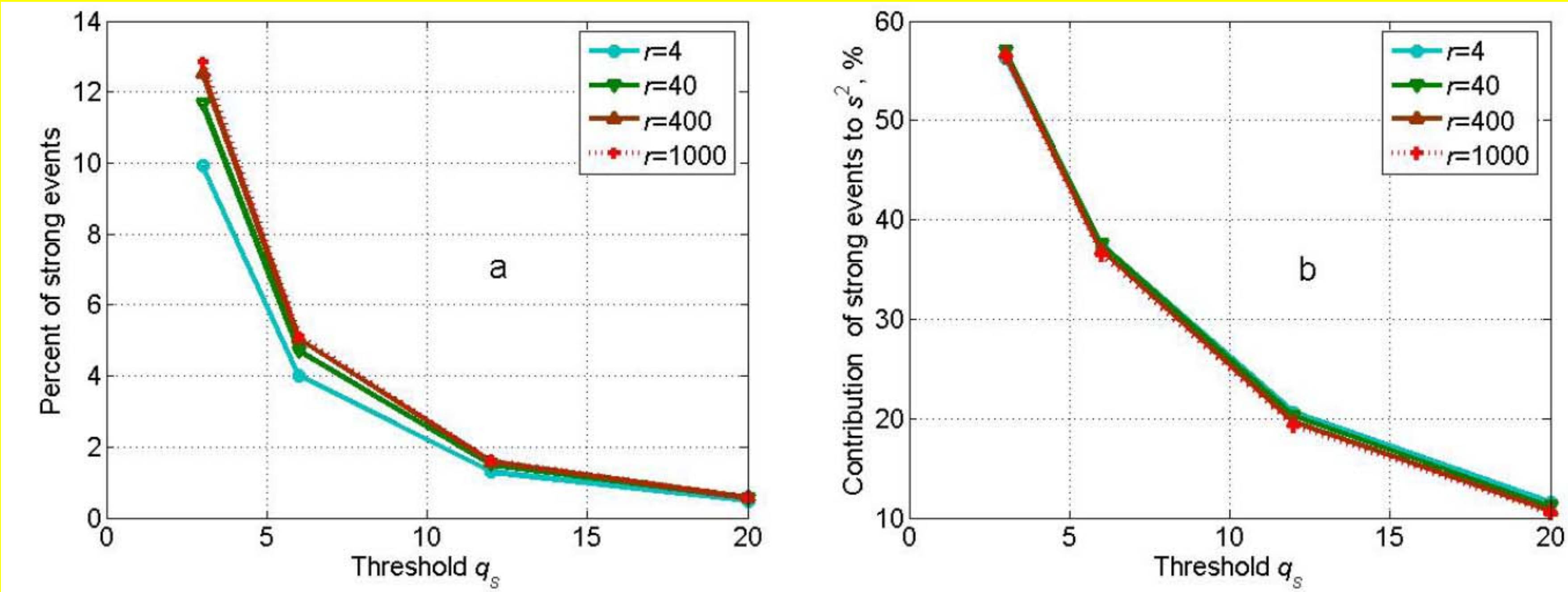
a
b
q= 3.0
q= 6.0
q=12.0
q=20.0
None
Δu1(40)/RMS u1
Δu1(400)/RMS u1
Full
Only strong
Excl. strong
Histograms for threshold qS=3







Contributions of the strong dissipative events, as defined above, to the eighth order structure function (a), the second-order structure function (b) and the third-order structure function (c) as a function of the threshold q for various separations r. Note that this contribution to the third order structure function is not negligible in the 'inertial range' !



Percent of the strong dissipative events as defined above (a) and their contribution to the total dissipation (b) as a function of the threshold q for various separations r.

Lagrangian setting and passive objects

Lagrangian setting

It seems natural to look at/for 'cascade' of passive objects in pure Lagrangian setting. However, in pure Lagrangian description the fluid particle acceleration is linear in the fuid particle displacement and the 'inertial' effects are manifested only by the term containing pressure - there are no terms like the advective terms $(\mathbf{u} \cdot \nabla)\mathbf{u}$ in pure Eulerian setting.

$$\begin{aligned}\partial^2 X_i/\partial^2 t \quad = \quad & [X_j, X_k, p] + \nu\{[X_2, X_3, [X_2, X_3, \partial X_i/\partial t] + \\ & [X_3, X_1, [X_3, X_1, \partial X_i/\partial t] + [X_1, X_2, [X_1, X_2, \partial X_i/\partial t]\end{aligned}$$

Here (i, j, k) means an even permutation of the indices $(1, 2, 3)$. The vector $\mathbf{X}(\mathbf{a}, t)$ is the particle position vector for a particle labeled by $\mathbf{a}$. Usually $\mathbf{a} \equiv \mathbf{X}(\mathbf{a}, \mathbf{t}_0)$, i.e. the initial positions of fluid particles are used as their labels. The expression $[A, B, C] \equiv \frac{\partial(A,B,C)}{\partial(a_1,a_2,a_3)}$ is an abbreviation for the Jacobian of the variables A, B, C in respect with variables a_1, a_2, a_3. We

The inertial interactions understood as $(\mathbf{u} \cdot \nabla)\mathbf{u}$ have a relative nature; they are eliminated in the transformation to the particle attached reference system

Therefore, the nonlinearity in the Lagrangian representation cannot be interpreted in terms of some cascade (as it cannot be maintained by pressure gradient alone) and it is far less clear (if at all) how one can employ decompositions even at the problematic level as done in pure Eulerian setting.

Similarly one can hardly speak about things like Reynolds decomposition and Reynolds stresses, turbuent kinetic energy production in shear flows in pure Lagrangian setting.

Also there is no sweeping of any kind at the outset as there are no terms like the advective terms $(\mathbf{u} \cdot \nabla)\{\ldots\}$ in pure Eulerian setting, so one has a problem speaking about the interaction between advective and diffusive processes in pure Largangian setting

Cascade of passive objects

Is there ‘enough’ analogy (*more on analogies in a separate lecture*) between genuine and ‘passive’ turbulence or the differences are essential? Nonlinear versus linear. Is extension of Kolmogorov phenomenology justified for systems governed by linear equations?

It is rather common, since OBUKHOV (1949) and CORRSIN (1951), to speak about cascade in case of a passive scalar and more recently passive . The main argument is from some analogy. Indeed, for instance in any random isotropic flow the rate of production of 'dissipation' (i.e. corresponding field of derivatives) of both passive scalars and passive vectors is essentially positive, which can be interpreted as a sort of `cascade'. However, the equations describing the behavour of passive objects are linear. Hence, there is no interaction between modes of whatever decomposition of the field of a passive object: the princilple of superposition is valid in case of passive objects*.

*Here by 'mode' is meant as a solution of the appropriate equation, e.g. of the advection-diffusion equation . Of course, there are many ways to use 'modes' that are not solutions of this equation, such as Fourier modes. In this case the Fourier modes do interact, since one of the coefficients of the advection-diffusion equation, the velocity field, is not constant. This interaction is interpreted frequently as a 'cascade' of passive objects. But, as mentioned, this interaction is decomposition dependent, and therefore is not appropriate for description of physical processes, which are invariant of our decompositions. There is a point concerning the behavior of an individual solution. Namely, the evolution of its energy spectrum is expected to exhibit positive energy transfer to higher wave numbers as a consequence of production of the field of derivatives of the passive field. Can one see this as a kind of 'cascade'? Even if the answer were affirmative it is a very different kind of cascade, if at all.

Therefore, it seems more appropriate to describe the process in terms of production of the field of derivatives of the passive object, which is performed by the velocity straining field, just like it is proposed above for the velocity field.

Hence the extension of Kolmogorov arguments and phenomenology to passive objects seems to be much less justified. No wonder that the phenomenological paradigms for the velocity field failed in most cases when applied to passive objects*. We are reminded that the 'analogy' between the passive objects and the active variables is, at best, very limited for several reasons, the main of which are the linear nature of 'passive' turbulence, Lagrangian chaos, the irreversible effect of the randomness of the velocity field on passive objects independently of the nature of this randomness, e.g. even a Gaussian one, and the one-way interaction between the velocity field and the field of a passive object .

*E.g. experiments by Villermaux et al. (2001) clearly show that this is the case. The behaviour of passive scalar in their experiments is distinctly nonlocal in the sense that the main mechanism responsible for mixiing involves direct interaction between large and small scales 'bypassing' the (nonexistent) cascade.

IS BELOW ANYTHING WRONG?

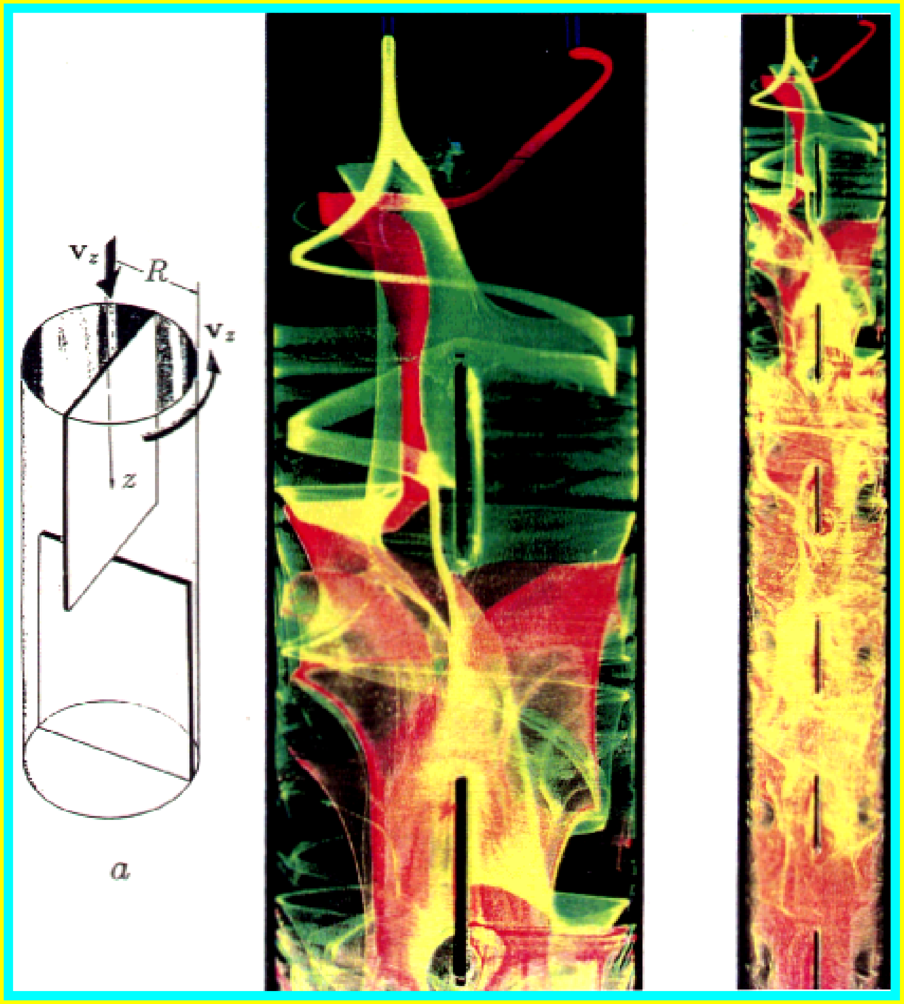
The mixed-derivative skewness $S_{u\theta}$

$$S_{u\theta} = \frac{\langle (\partial u_1/\partial x_1) \, (\partial \theta/\partial x_1)^2 \rangle}{\langle (\partial u_1/\partial x_1)^2 \rangle^{1/2} \langle (\partial \theta/\partial x_1)^2 \rangle}, \qquad (24)$$

is related to the nonlinear transfer of scalar variance to small scales and takes a value zero when there is no net cascade to higher wavenumbers.

A laminar Eulerian flow at Re~1 (E-Laminar) is Chaotic Lagrangian (L-turbulent)

Is there a cascade of passive objects?



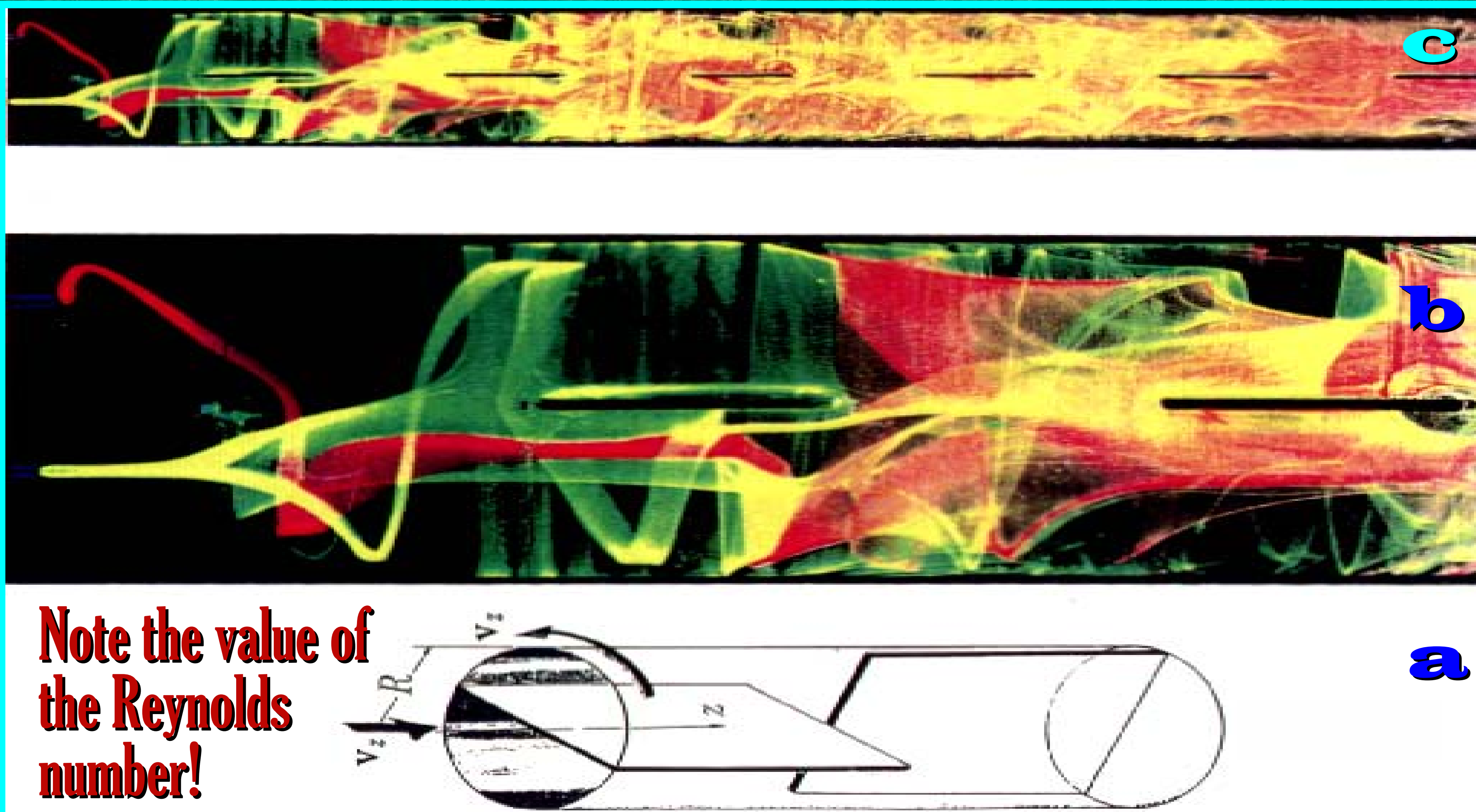
MIXING IN PMM, **Re ~ 1 (!)**

KUSH & OTTINO (1992)

RELEVANT TO MICROFLUIDICS with Re ~ 0 (!);

Linked twist maps (LTMs), Bernoulli mixing ...

The complexity and problematic aspects of the relation between the Lagrangian and Eulerian fields is seen in the example of Lagrangian (kinematic) chaos or Lagrangian turbulence (chaotic advection) with a priori prescribed and not random Eulerian velocity field (E-laminar). This is why Lagrangian description - being physically more transparent - is much more difficult than the Eulerian description. In such E-laminar but L-turbulent flows the Lagrangian statistics has no Eulerian counterpart, as in the flow shown at the left.



Note the value of the Reynolds number!

Mixing in PPM - partitioned-pipe mixer at very low Reynolds number. $\mathrm{Re}_{PPM:axial} = \langle v_z \rangle R/\nu = 0.3$ and $\mathrm{Re}_{PPM:cs} = v_R R/\nu = 1.8$; here $\langle v_z \rangle$ – average axial velocity and $v_R = \frac{1}{2}(|v_{1|\,\max} + |v_{1|\,\min})$ – characteristic cross-sectional velocity. $0 < \mathrm{Re}_{PPM:axial} < 0.8$ and $0 < \mathrm{Re}_{PPM:cs} < 0.8 < 8$. a) schematic of the PPM, b) is a close up of the upper part of c). From Kusch and Ottino (1992).

In lieu of conclusion

CHOOSE WHAT YOU LIKE MORE

Big whirls have little whirls,
Which feed on their velocity.
And little whirls have lesser whirls
And so on to viscosity –
In the molecular sense

RICHARDSON (1922)

Big whirls lack smaller whirls,
To feed on their velocity.
They crash and form the finest curls
Permitted by viscosity

BETCHOV (1976)

The notion that turbulent flows are hierarchical, which underlies the concept of cascade, though convenient, is more a reflection of the unavoidable (due to the nonlinear nature of the problem) hierarchical structure of models of turbulence and/or decompositions rather than reality. This is emphasized in the case of passive objects, whose evolution is governed by linear equations, with the velocity field entering multiplicatively in these equations, thus making them 'statistically nonlinear'.

I personally prefer the physical space rather than to live and die in some decomposition space.

The concept of inertial range is not well defined, e.g. in the context of 'anomalous' scaling behavior of higher order structure functions in the nominally defined inertial range. This is due to the contamination of the inertial range by strong dissipative events at whatever large Reynolds numbers. One of the consequences is that the 'decomposition' into inertial and dissipative ranges is not that nice, the anomalous scaling is not the attribute of the 'inertial range' in the conventional sense and corresponding consequences for what is called MF formalism. The 4/5 law is not a pure inertial law.

A similar problem seems to exist (because the data is still qualitative only) for passive scalars, so that the zero mode explanation may well be of not of the kind R^4 (the right result for the right reason)