Turbulence in dispersive MHD models

T. Passot, D. Laveder, L. Marradi, P.L. Sulem

Université de Nice Sophia Antipolis

CNRS

Observatoire de la Côte d'Azur, Nice, France

OUTLINE

- Introduction :

    - Evidence of **dispersive Alfvén waves** in the solar wind and the terrestrial magnetosheath and of the signature of **turbulence**
    - **Break of the spectrum at the ion gyroscale**: tentative explanations

- Forced **DNLS model**

- One-dimensional dynamics of the **forced Hall-MHD equation**

    - simulations in the parallel direction
    - simulations in oblique and quasi-transverse directions

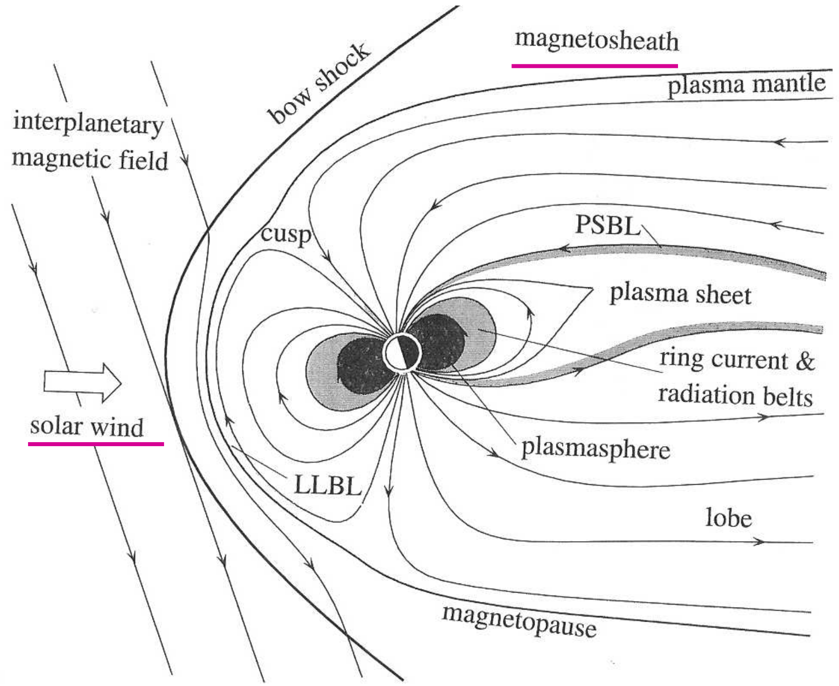
    ⇒ Global energy transfers, wavenumber spectra, mode indentification

- Preliminary 1D **Landau fluid simulations**

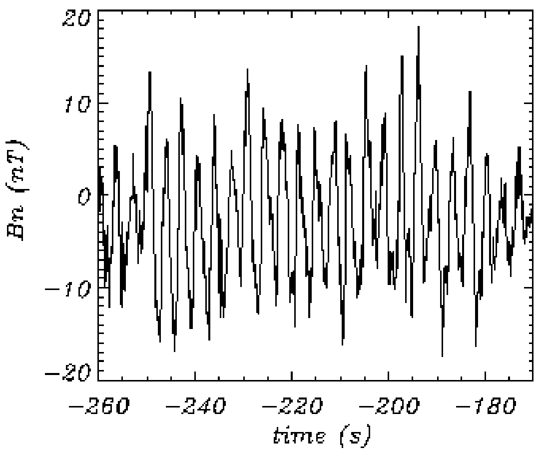
- Conclusion

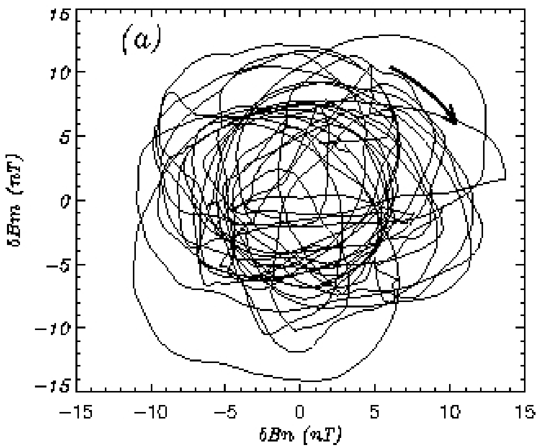
Evidence of DAWs

Quasi-monochromatic dispersive Alfvén waves are commonly observed in the solar wind and in the magnetosheath



Observation by CLUSTER satellites downstream the quasi-perpendicular bow shock
(Alexandrova et al., J. Geophys. Res., (2004, 2006))



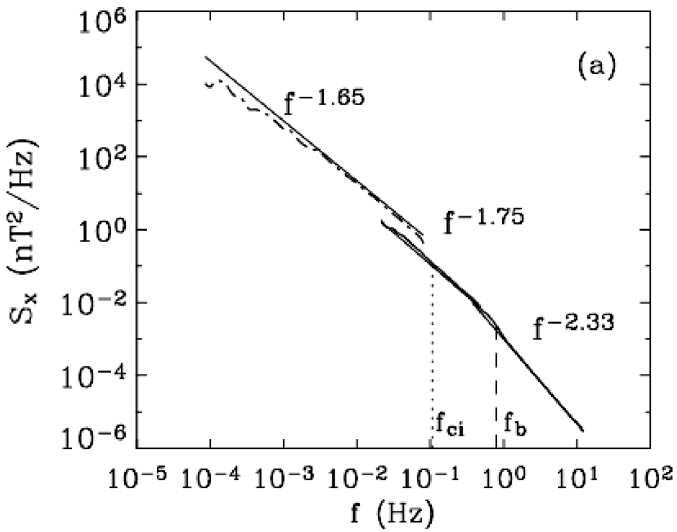


Presence of almost monochromatic left-hand circularly polarized Alfvén waves

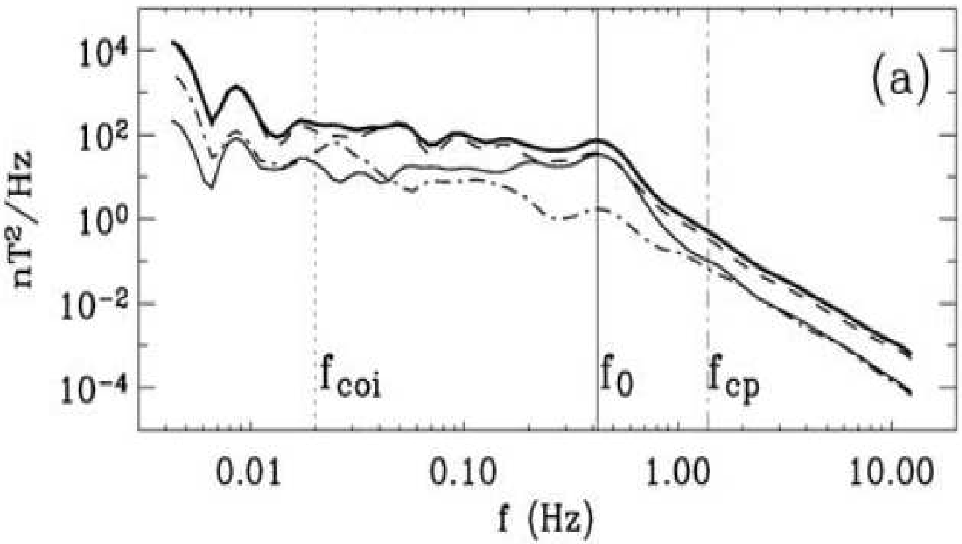
Evidence of turbulence

Space plasmas such as the solar wind or the magnetosheath are turbulent, magnetized plasmas with essentially no collisions. Observed cascade extends beyond the ion Larmor radius: kinetic effects play a significant role.

Range of observed frequency power law indices between -2 and -4.5 (**Leamon et al. 1998)**
RH polarized outward propagating waves (**Goldstein et al. JGR 94)**



Solar wind turbulent spectrum (Alexandrova et al., 2007)



Magnetic energy spectrum in the magnetosheath downstream of the bow shock (Alexandrova et al., JGR, 2006).

Presence of KAWs in the « dissipation range »?

The Alfvén wave cascade develops preferentially **perpendicularly to the ambient magnetic field.**

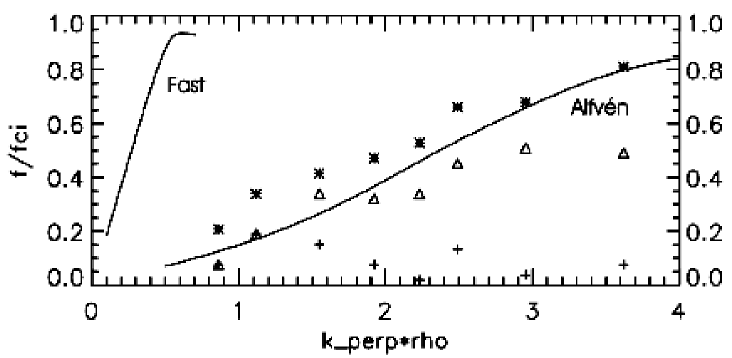
Assuming modes with frequencies larger than the ion gyrofrequency are decoupled, there should be a cascade of **Kinetic Alfvén waves** (slow modes are highly dissipative) at small scales, with a $k_\perp^{-7/3}$ spectrum.

The range of exponents for power laws could be attributed to:

a. collisionless damping, the true behavior being an exponential fall off, the observed power law being an artifact of instrumental sensitivity?

b. a competition with a dual cascade of entropy modes.

(Howes et al. JGR **113**, A05103 (2008), AIP, CP932 (2007), ApJ **651**, 590 (2006), Apj sup. submitted).



KAWs have been clearly identified using k-filtering technique in the cusp region (Sahraoui et al. AIP, 2007).

The nature of the fluctuations associated with the power spectrum at frequencies larger than the ion-gyrofrequency in the satellite frame is however not yet established in all situations.

Other tentative models for the « dissipation range »: I.

Types of waves

Based on linear kinetic theory, **whistler** wave cascade in the parallel direction or **magnetosonic** wave (and **KAW**) cascade in the quasi-transverse directions are possible as long as **β<2.5**.
Using a diffusion equation in wavenumber space with the linear time as the energy transfer time a k^{-3} spectrum is expected

$$\left(\frac{\partial E(k)}{\partial t}\right)_{\text{nonline}} = \frac{\partial}{\partial k}\left[\frac{\gamma k^4}{4\pi\tau_S(k)}\frac{\partial[k^{-2}E(k)]}{\partial k}\right]$$

(Leith (1967), Zhou & Matthaeus, JGR 95, 14881 (1990))
(Stawicki, Gary & Li, JGR **106**, 8273 (2001)).

2D PIC simulation of whistler turbulence shows preferential cascade towards perpendicular wavenumbers with steep power laws, <u>and no cascade in 1D</u> (Gary et al. GRL 35, L02104 (2008)).

Other tentative models for the « dissipation range »: II.

Dispersion leads to steepening:

Analytic insight:

Weak turbulence for **incompressible Hall MHD:**

For kd_i>>1 transfer essentially perpendicular to B_0 : $k_\perp^{-5/2}$

For kd_i<<1 transfer exclusively perpendicular to B_0: $k_\perp^{-2}$

(Galtier, JPP **72**, 721 (2006))

Numerical insight:

2D DNS of compressible HMHD : decaying turbulence shows steepening of the spectrum near the ion-cyclotron scale when the cross-helicity is high (Gosh et al. JGR **101**, 2493 (1996)).

With a shell model (without mean field):

The $k^{-5/3}$ AW cascade steepens to a $k^{-7/3}$ EMHD spectrum when magnetic energy dominates and to a $k^{-11/3}$ spectrum when kinetic energy dominates (Galtier & Buchlin, ApJ **656**, 560 (2007)).

Important role of nonlinearity in the Hall term.

Effect of waves versus dispersion: phenomenology

A **common formulation** to account for the inhibition of turbulent transfer due to the presence of waves, leading to shallower spectra (IK spectrum), and for the increase of the energy transfer rate due to dispersion, leading to steeper power laws.

$$\Pi = \frac{kE_k^B}{T_{\rm tr}}$$

$$T_{\rm tr} = \frac{T_{NL}^2}{T_W} = \frac{\omega(k)}{k^2 u_e^2}$$

← **Decay time of triple correlations proportional to the wave time. Nonlinear time governed by electron velocity.**

$$u_e = u_i - \frac{j}{en}$$

$$u_{i,k} = (kE_k^V)^{\frac{1}{2}} \text{ and } b_k = \frac{\omega}{k} u_{i,k}$$

Assumption of weak nonlinearity (or equipartition at large scales)

$$u_{e,k} \approx (kE_k^V)^{\frac{1}{2}} (1 + \frac{kd_i}{v_A}\frac{\omega}{k})$$

$$\frac{k^4 E_k^B E_k^V}{\omega}(1 + \frac{kd_i}{v_A}\frac{\omega}{k})^2 = \text{cte}$$

Constant flux

$$\text{at large scale } E_k^V = E_k^B \quad \text{and} \quad E_k^B \propto k^{-3/2} \text{ (IK spectrum)}$$

$$\text{at small scale } E_k^V = \frac{k^2}{\omega^2} E_k^B \quad \text{and} \quad E_k^B \propto k^{-2} \text{ (for whistlers) or } \quad E_k^B \propto k^{-3} \text{ (for ion} - \text{cyclotron waves)}$$

Further observations:

Recent analysis of the **spectrum break point** in the solar wind shows its location depends on a **combination of the fluctuation scale and its amplitude at that scale.**

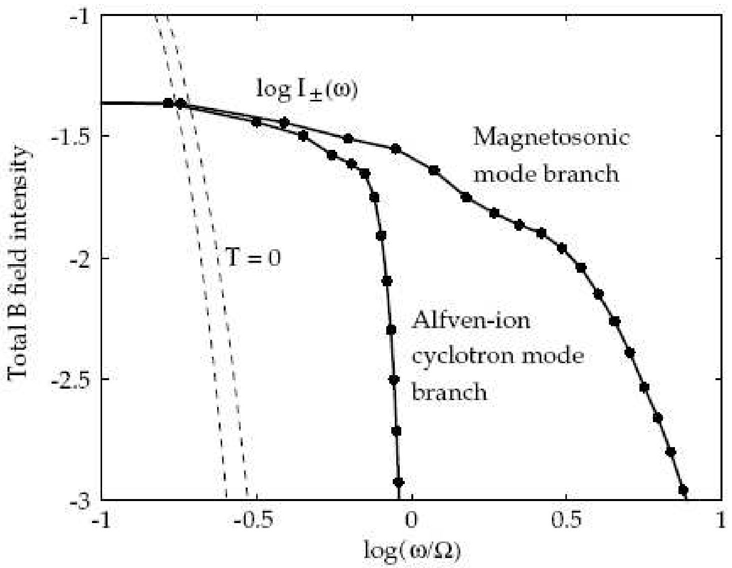
It is essentially a nonlinear process.

No theory seems to be able to explain all observations.
The correlation with ion inertial length is better than with the ion gyroscale.
(Markowskii et al. ApJ **675**, 1576 (2008)).

Alfvén wave cascade in 1D?

3D incompressible MHD leads to preferential transfer to small transverse scales.
Does this remain true in the presence of compressibility and wave dispersion?

It was shown theoretically that Vlasov equation supports a parallel AW weak turbulence cascade in 1D: 3-wave interactions mediated by ion-sound turbulence leads to transfer from large-scale AW to small-scale ion-cyclotron and magnetosonic whistler wave (Yoon, PPCF **50** 085007 (2008)).



What happens within the context of fluid models, easier to simulate numerically?
As a first step it is convenient to study this problem, with Hall-MHD and the Landau fluid model, i.e. a fluid model with linear Landau damping and FLR corrections.

Turbulence in DNLS

Parallel propagating Alfvén waves can develop solitonic structures, as seen in the context of DNLS, a large scale 1D reduction.

What happens in the presence of external forcing and dissipation?

$$\frac{\partial b}{\partial t} + \alpha \frac{\partial}{\partial x}\left(|b|^2 b\right) + i\delta \frac{\partial^2}{\partial x^2} b + \eta \frac{\partial^2}{\partial x^2} b = f$$

Similar results are obtained if the Newtonian viscous and/or magnetic diffusive term is replaced by Landau damping in the form:

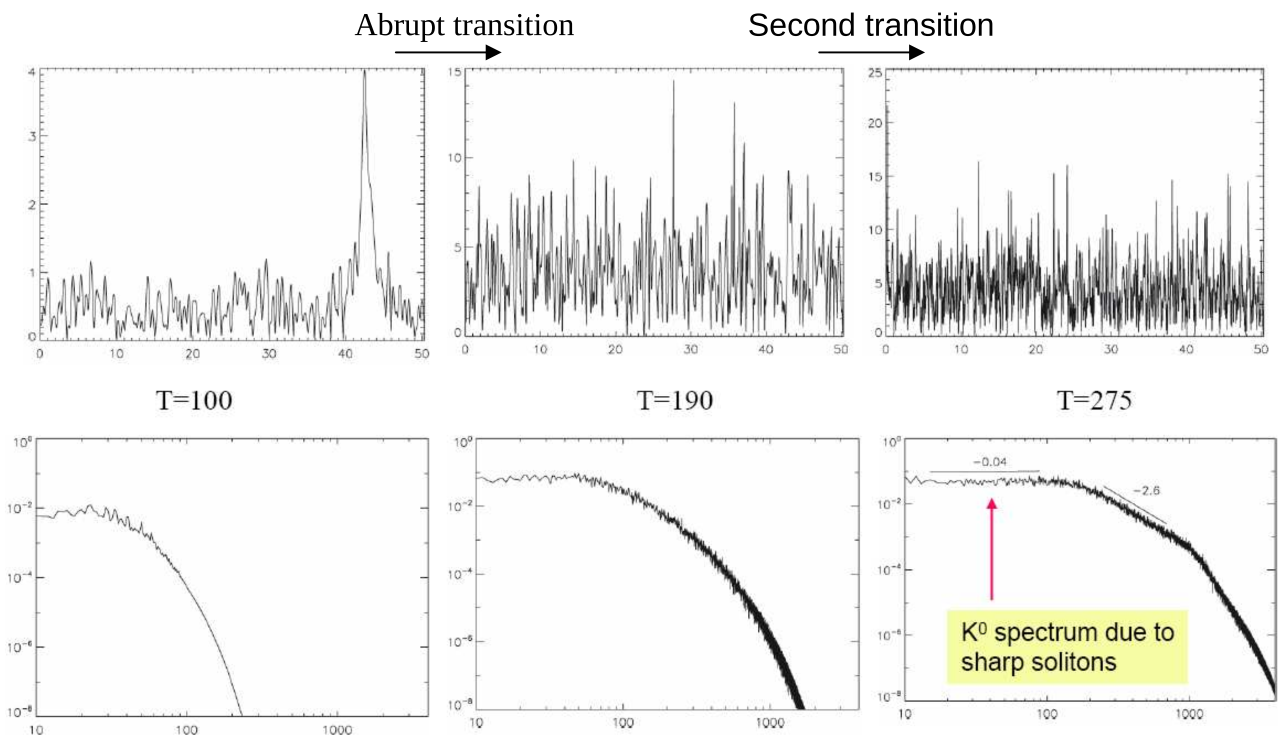
$$\frac{\partial b}{\partial t} + \frac{\partial}{\partial x}\left[|b|^2 b - \sigma b \mathcal{H}\left(|b|^2\right)\right] + i \frac{\partial^2 b}{\partial x^2} = 0 \quad \text{with} \quad \mathcal{H}\left\{V(x)\right\} = \frac{1}{\pi} P \int_{-\infty}^{+\infty} \frac{V(x')}{x' - x} dx'$$

Spatio-temporal chaos in DNLS

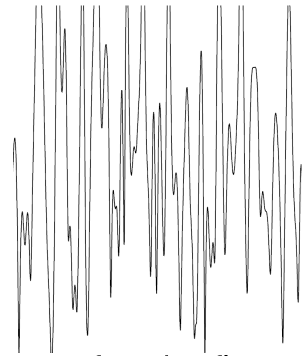
DNLS equation that is integrable cannot develop weak turbulence,
but can develop spatio-temporal chaos when subject to a (deterministic) driving and a very weak dissipation.

Initial condition : parallel soliton
Harmonic forcing at k=50
Very small dissipation
Energy increases and then saturates

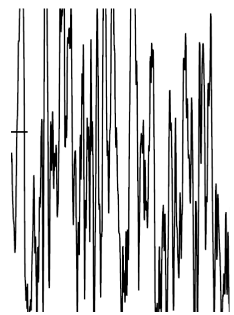
Problem first investigated by Buti and Nocera, Solar Wind 9, AIP (1999)



Enlargement and filtering



Signature of peaks: flat spectrum

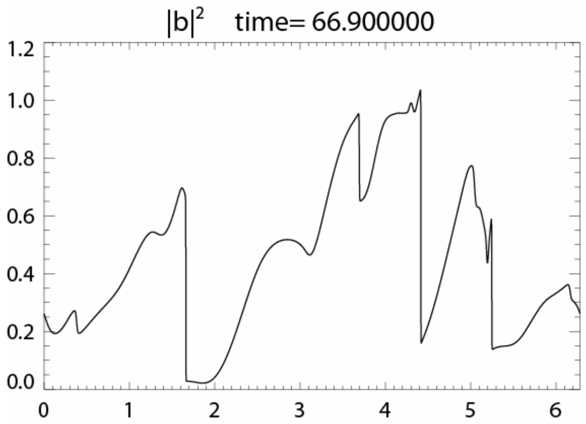


Signature of superimposed oscillations: steeper spectrum at smaller scales

In the presence of a strong enough random forcing (white noise at k=4), various kinds of turbulence can develop according to the ratio between dispersion and dissipation

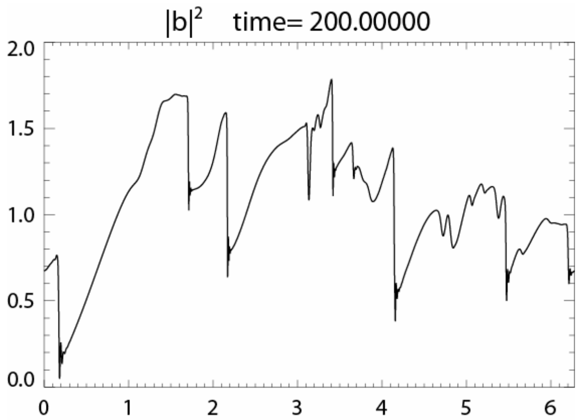
I. Small dispersion regime with significant dissipation

No dispersion



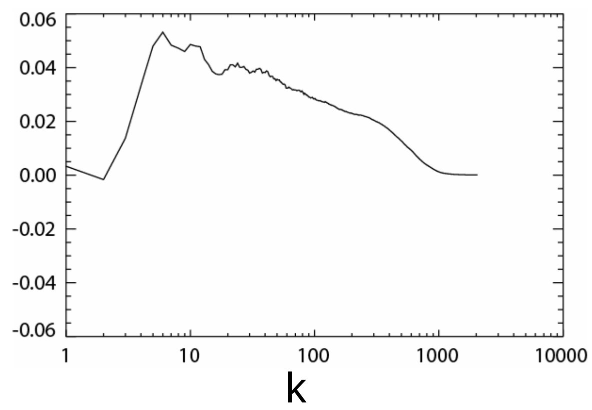
$\delta=0$, $\eta=2\times10^{-4}$

Small dispersion



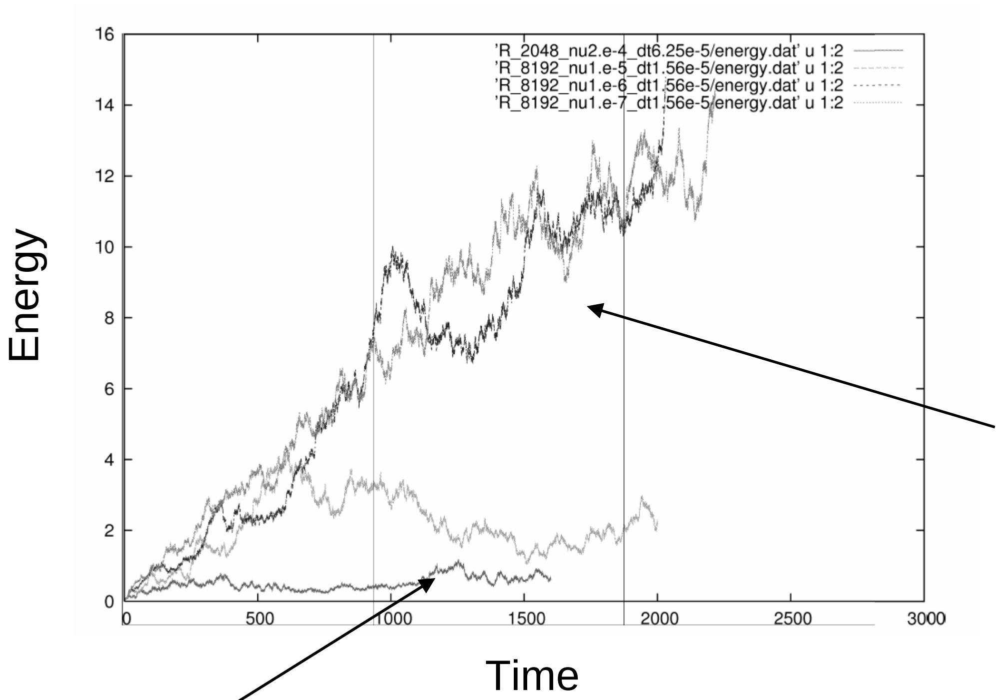
$\delta=1\times10^{-3}$, $\eta=2\times10^{-4}$

Direct energy transfer (as in HD turbulence)



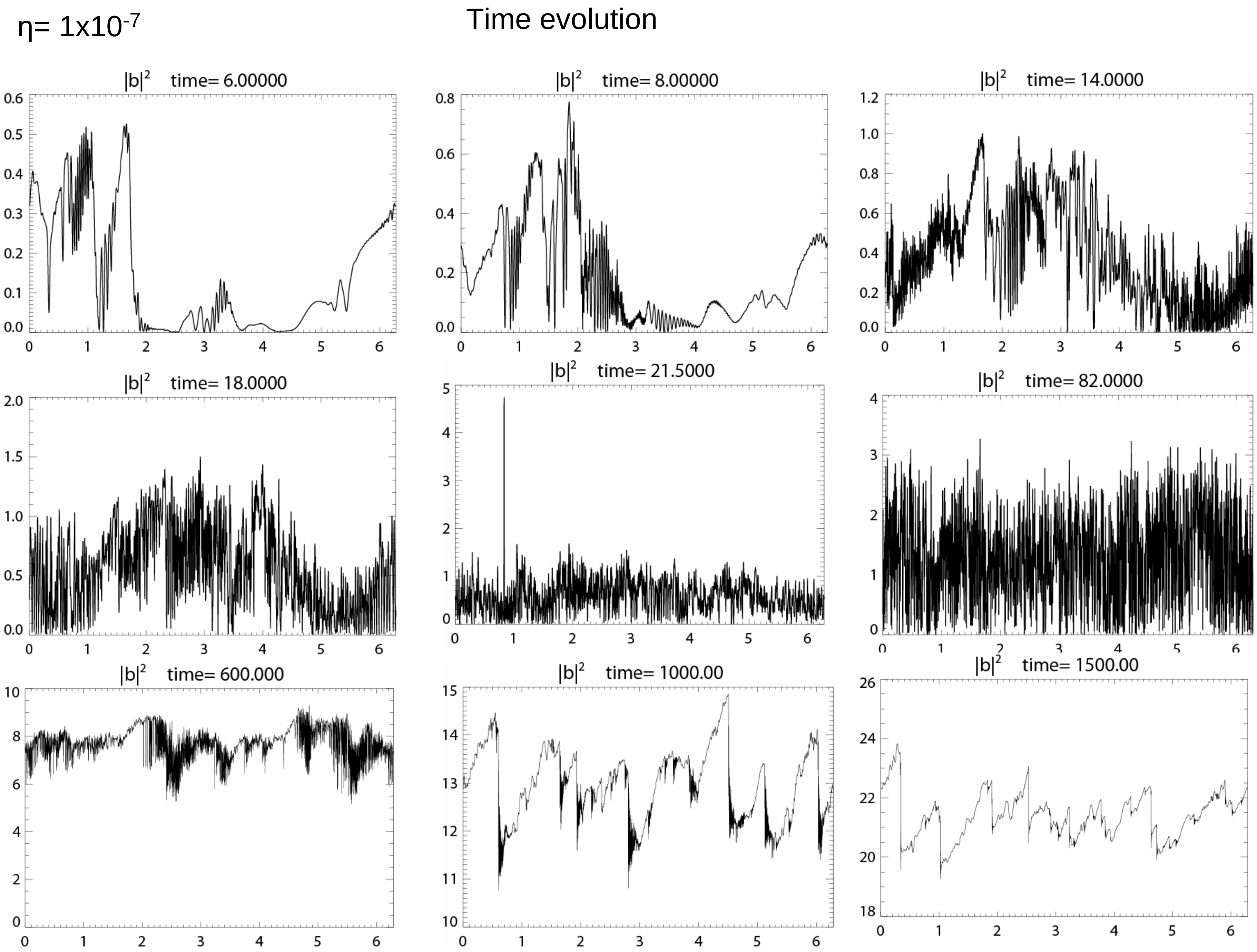
Total energy saturates in time

II. Small dispersion regime with very small dissipation

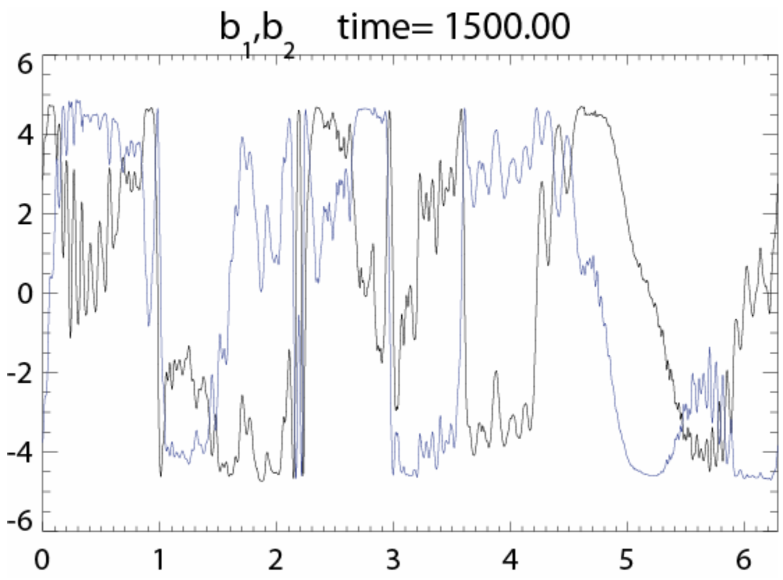


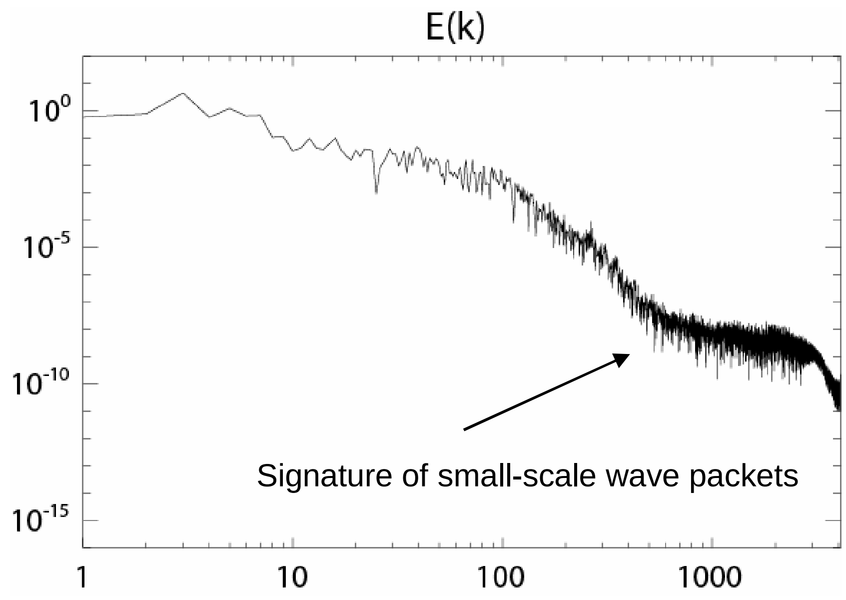
No saturation
of total energy

With larger viscosity, the energy saturates in time.



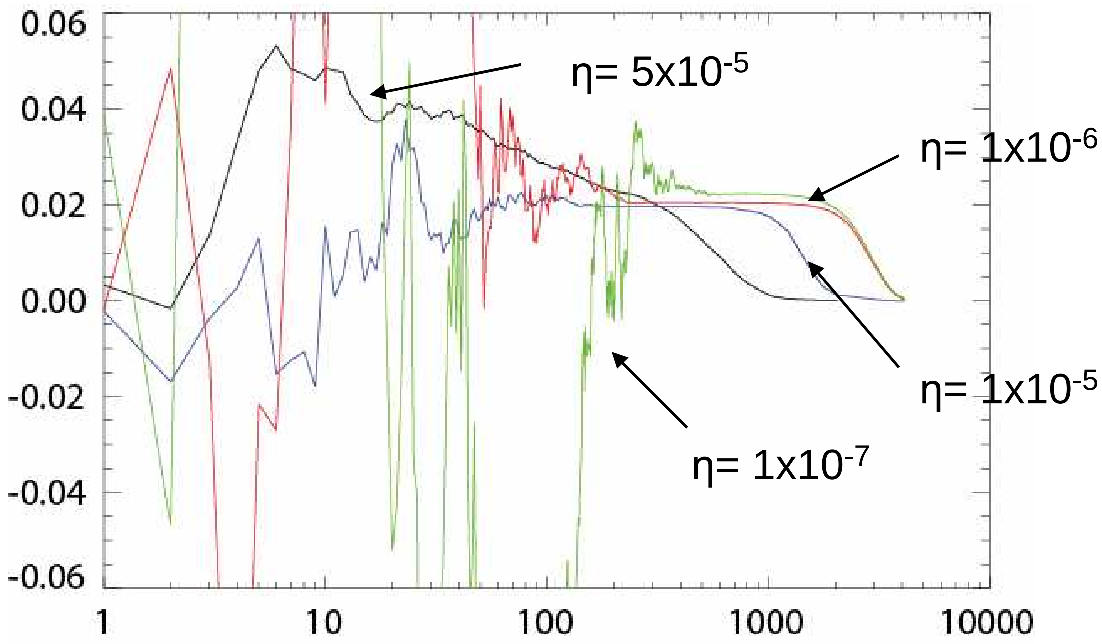
η= 1x10-7
Time evolution
|b|2 time= 6.00000
|b|2 time= 8.00000
|b|2 time= 14.0000
|b|2 time= 18.0000
|b|2 time= 21.5000
|b|2 time= 82.0000
|b|2 time= 600.000
|b|2 time= 1000.00
|b|2 time= 1500.00





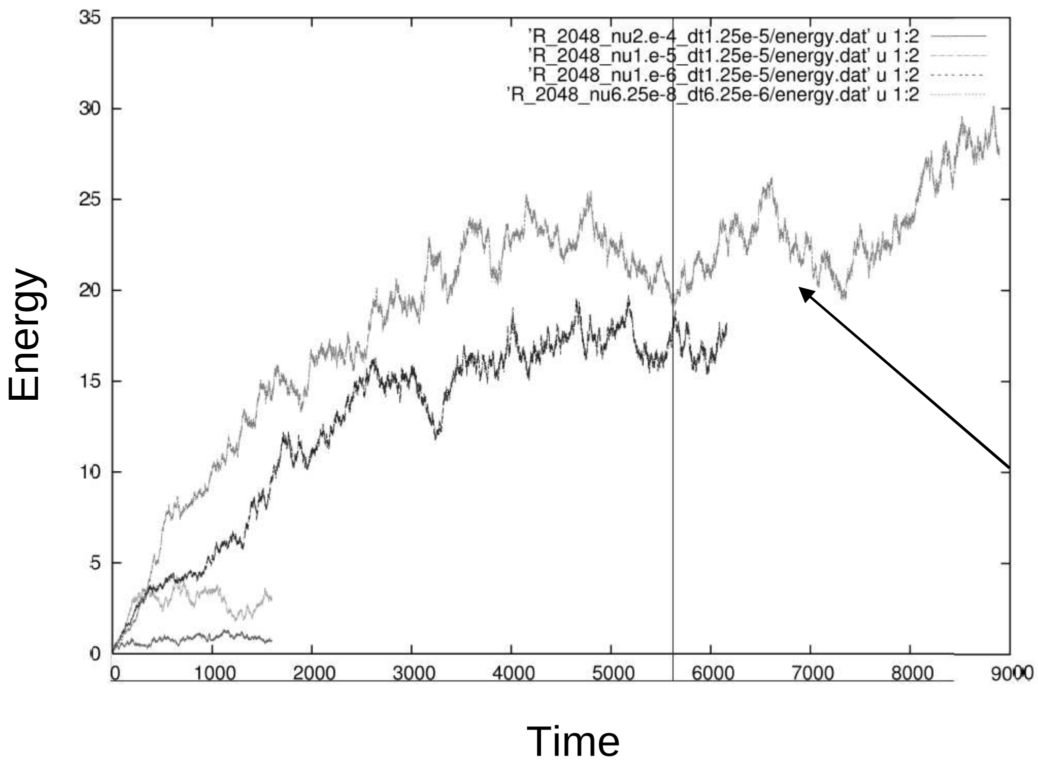
Typical energy transfers for decreasing viscosity:

The transfer is strongly intermittent at large scale.
The constant transfer range is associated with the small-scale wave packets.



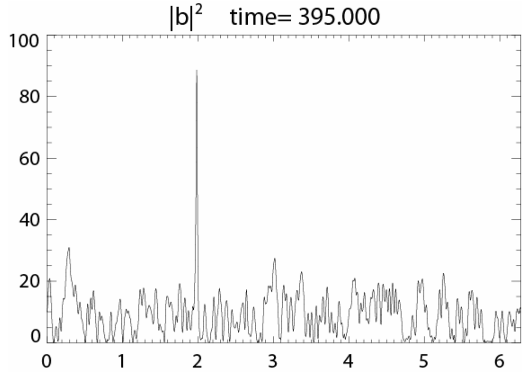
III. Large dispersion regime with very small dissipation

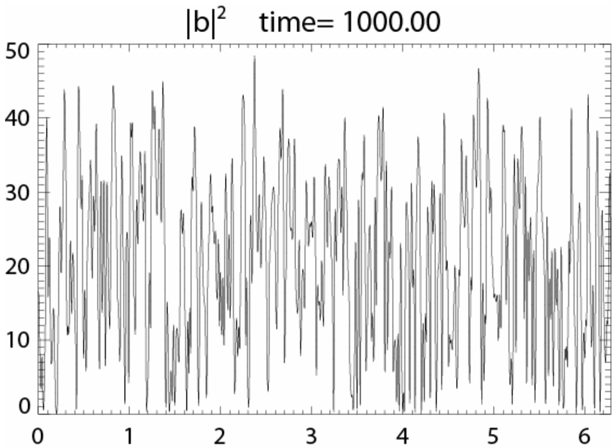
Energy versus time with $\delta=6.25\times10^{-2}$ and various viscosities

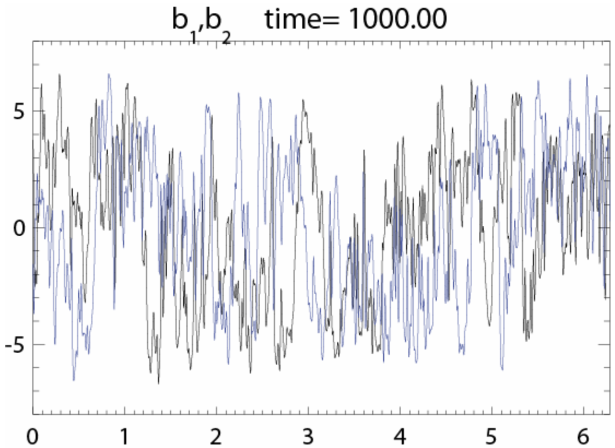


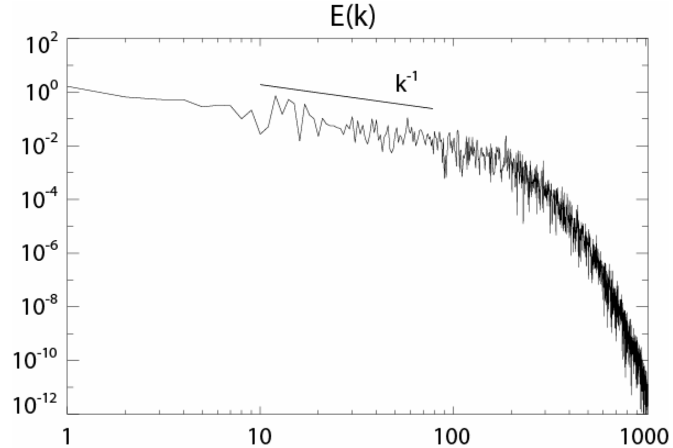
Energy saturates

$\delta=6.25\times10^{-2}$
$\eta=6.25\times10^{-8}$

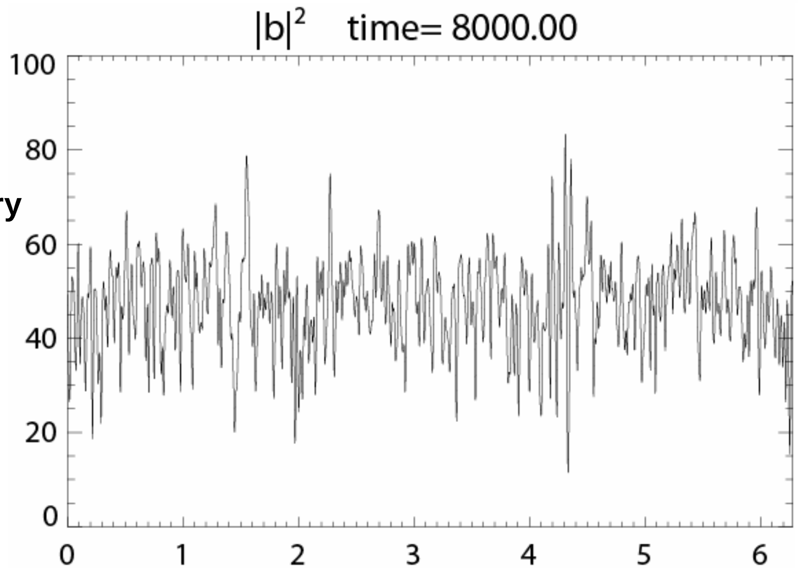


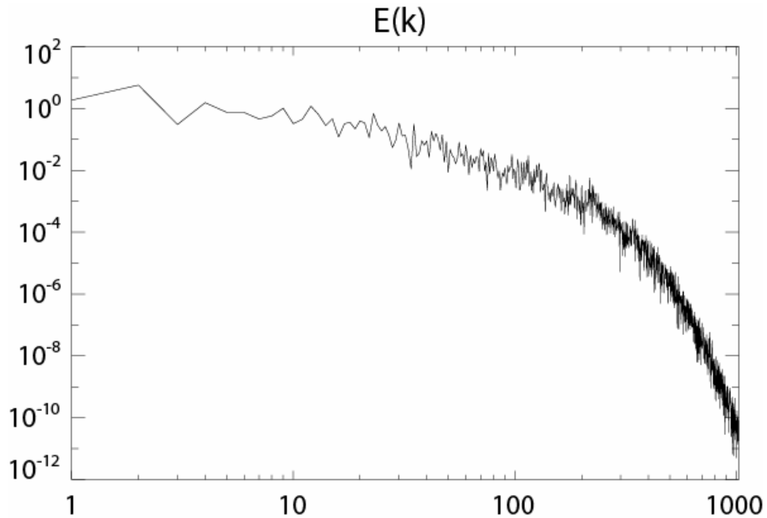




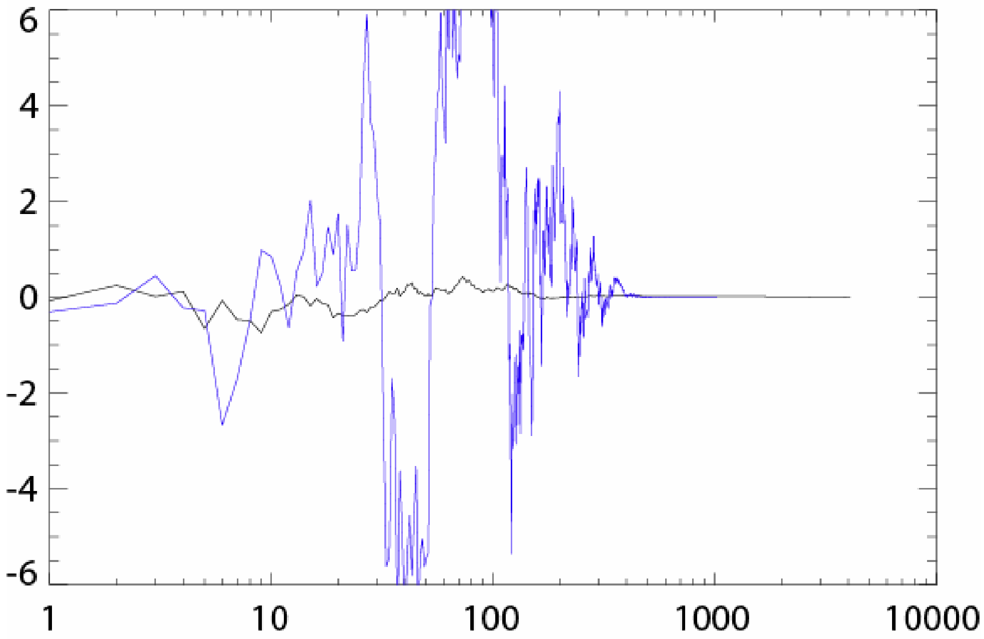


Almost stationary regime





In this regime, transfer is strongly intermittent at all scales and its fluctuations increase with dispersion



A detailed analysis of the statistic of transfer is necessary

Driven Hall-MHD system

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} = 0$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = -\frac{\beta}{\gamma\rho}\frac{\partial}{\partial x}\left(\rho^\gamma + \frac{|b|^2}{2}\right) + \frac{\mu_\parallel}{\rho}\frac{\partial^2(u)}{\partial x^2}$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} = \frac{b_x}{\rho}\frac{\partial b}{\partial x} + \frac{\mu_\perp}{\rho}\frac{\partial^2(v)}{\partial x^2} + f_v$$

$$\frac{\partial b}{\partial t} + \frac{\partial}{\partial x}(ub) = b_x\frac{\partial v}{\partial x} - ib_x\frac{\partial}{\partial x}(\frac{1}{\rho}\frac{\partial}{\partial x}b) + f_b + \kappa\frac{\partial^2(b)}{\partial x^2}$$

where $v = v_y + iv_z$, $b = b_y + ib_z$

Unit length = ion inertial length
x is the direction of propagation
$b_x = \cos(\theta)$ where θ is the angle (x, B_0)
f_v and f_b are white in time random noises centered on the mode k=4

We take β=2, γ=5/3 and

$$\kappa = \mu_\perp$$

$$\mu_\parallel \gg \mu_\perp$$

(to account for Landau damping of magnetosonic modes)

In oblique propagation only the viscosity in the z-direction is taken very small.
Desaliased spectral method with explicit temporal schemes (AB3).
Strong constraint on the time step ➔ very long simulations even in 1D.

Conditions of applicability of Hall-MHD:

It is a rigorous limit of collisionless kinetic theory for:

$T_i << T_e$
$\omega << \Omega_i$
$K_{||}\ v_{Thi} << \omega << k_{||}\ v_{The}$

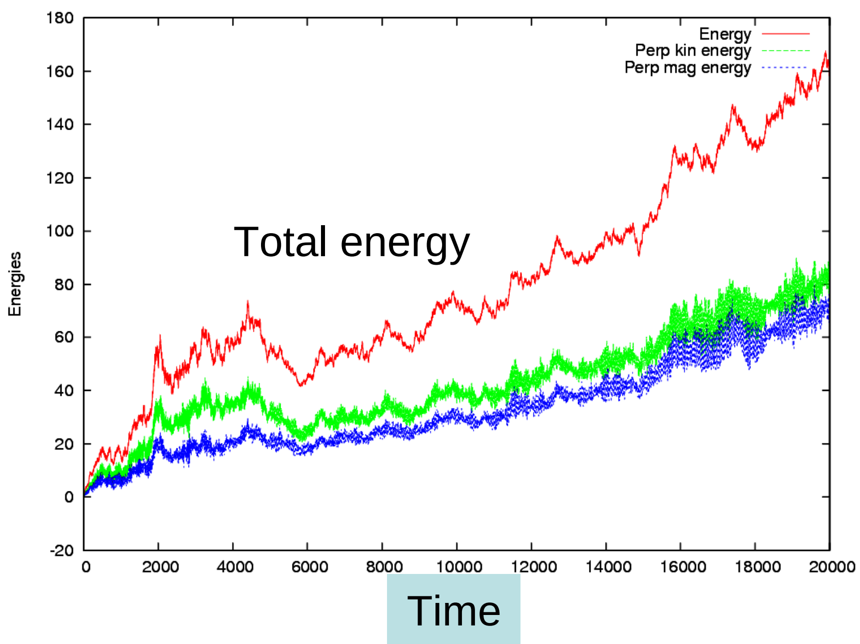
It correctly reproduces whistlers and KAWs for small to moderate β.

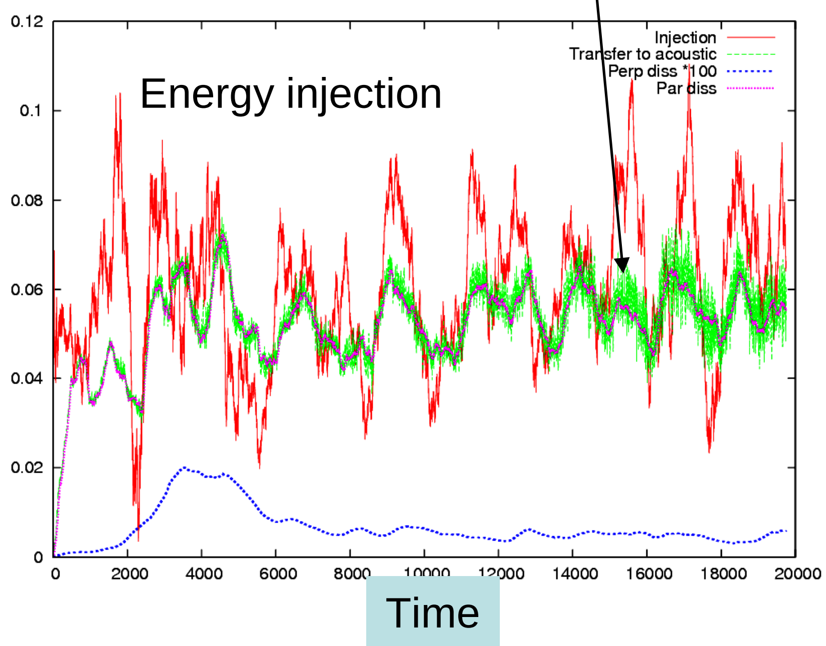
It contains waves that should be damped in a collisionless plasma and whose influence in the turbulent dynamics has to be evaluated

Parallel propagation in a large simulation domain: L=8*2π
Random forcing on the velocity field at k_f=0.5 k_{di}

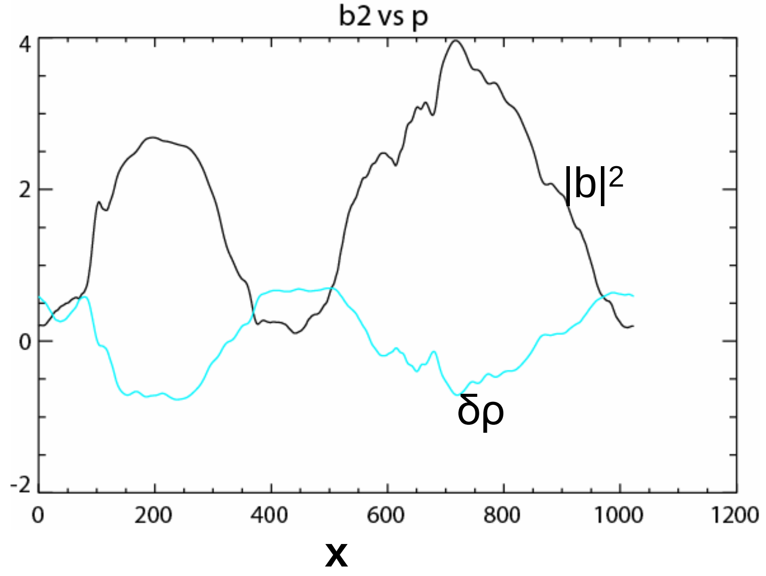
1. Total energy does not saturate in time due to the formation of « pressure balanced structures » at k=1.
2. At intermediate scales, the injected energy goes to the ion-acoustic mode and then dissipates.
3. Alfvenic dissipation is much smaller than magneto-acoustic dissipation.

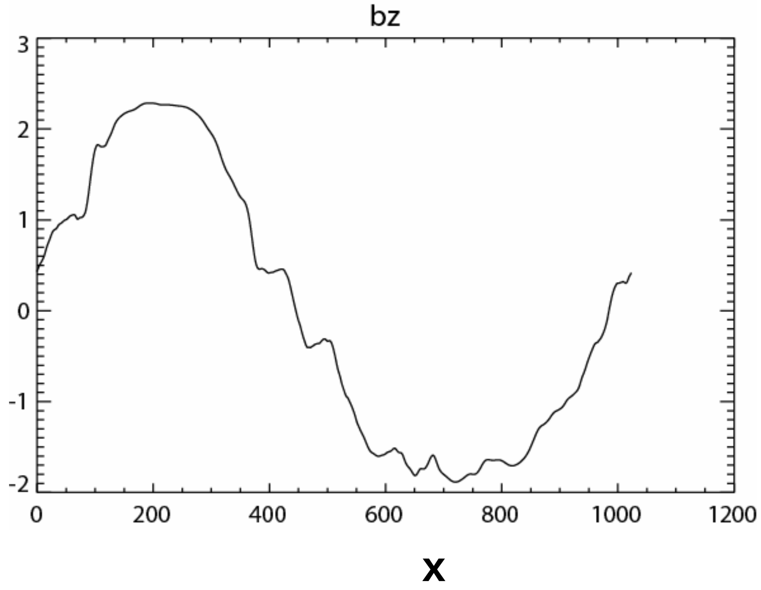
$$\frac{\partial E_\perp}{\partial t} = -\int \frac{|b|^2}{2} \frac{\partial u}{\partial x} dx$$



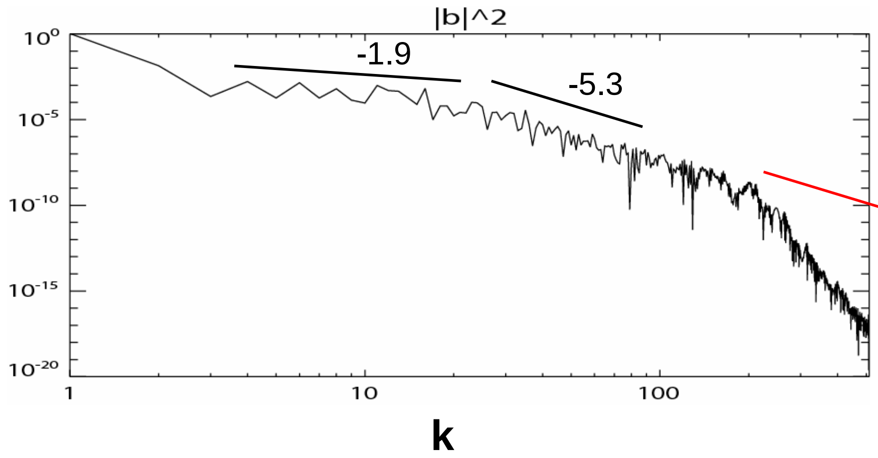


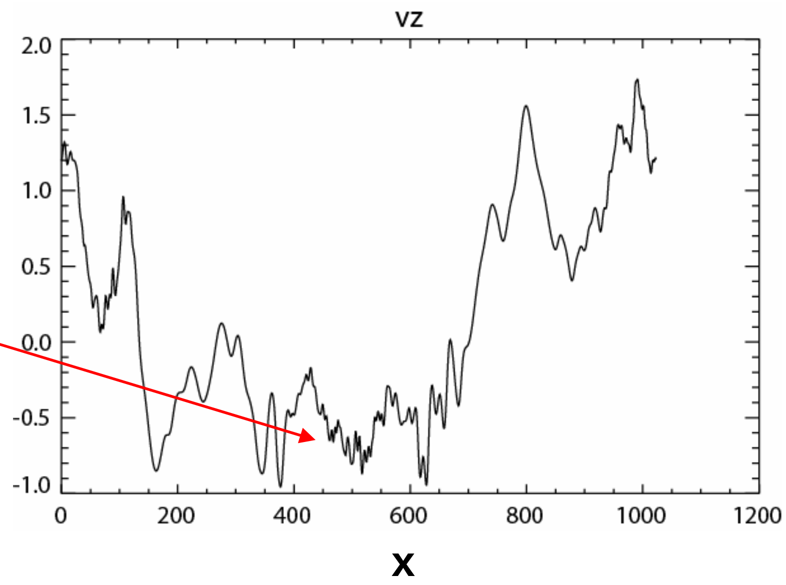
Pressure-balanced standing waves





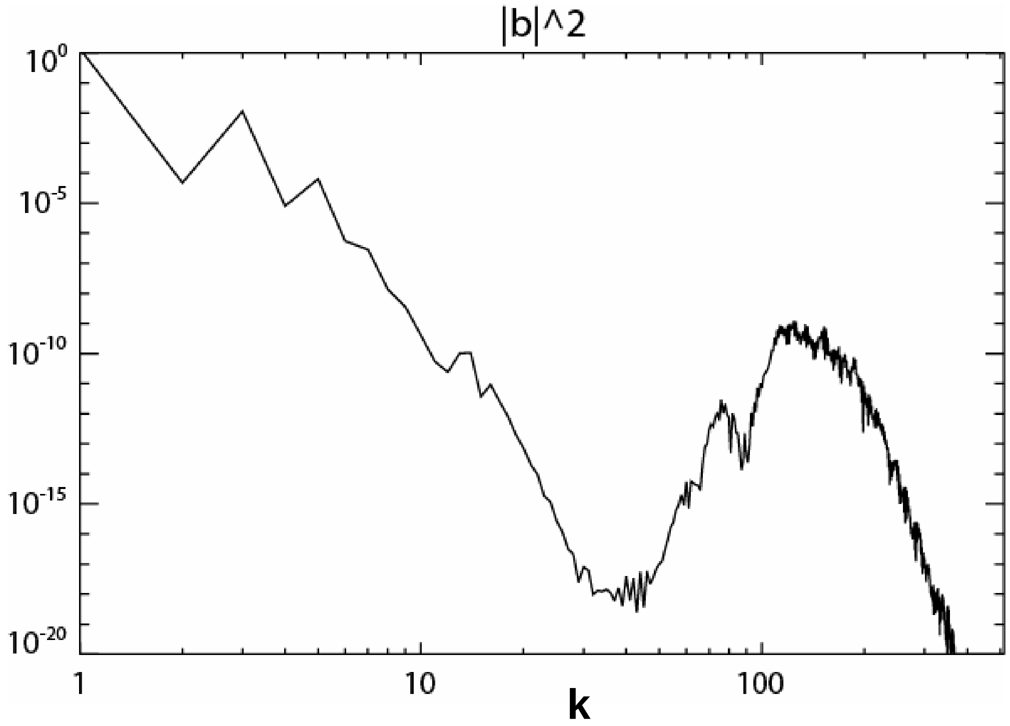
Second slope due to **cusp-like structures**
Small-scale oscillations due to **wave packets**





Forcing adjusted to keep a constant total energy:

In the case of large-scale parallel dynamics with velocity forcing, a spectral hole forms at intermediate scales, showing the **absence of Alfvén wave cascade.**



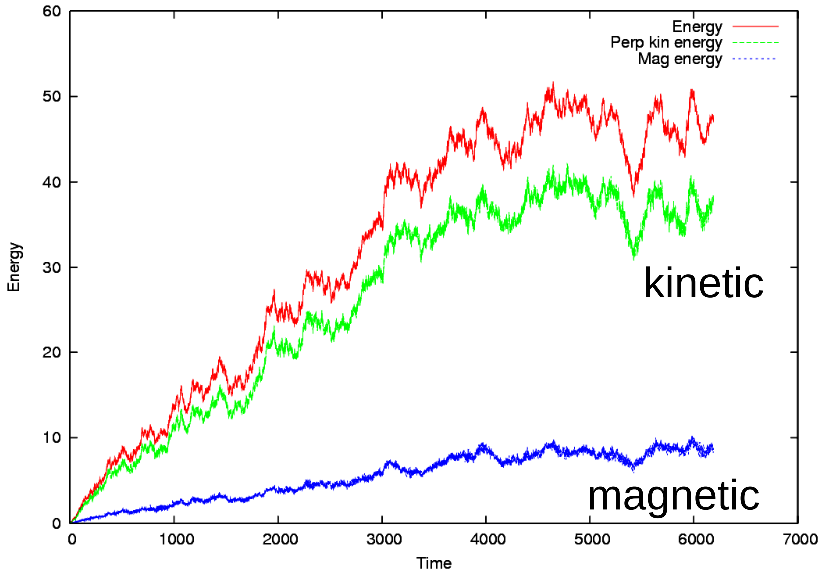
In all other cases, this forcing procedure does not change the dynamics.

Parallel propagation in a small simulation domain: L=2*2π Random forcing at $k_f=2k_{di}$: global energies

Two cases: forcing on the velocity or on the magnetic field.

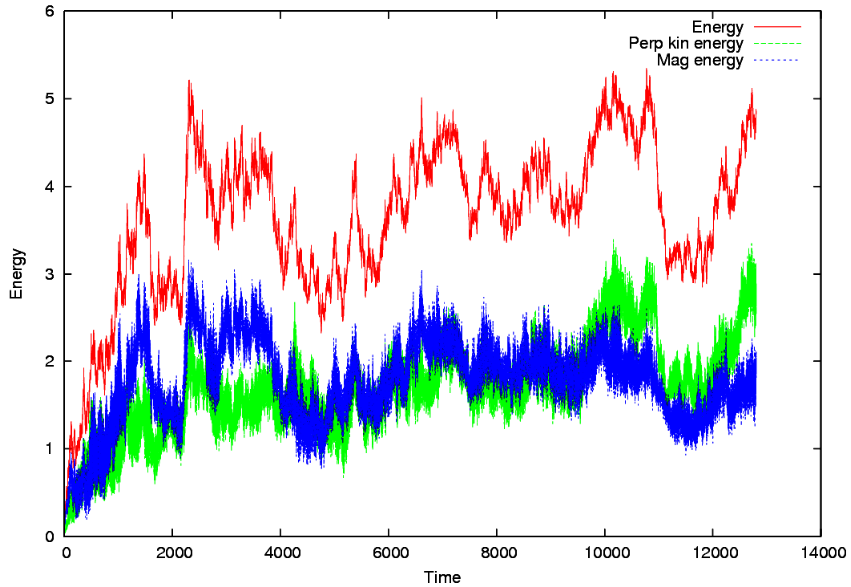
- Total energy saturates; no large-scale structures are formed (due to the strong dispersion together with large density fluctuations).
- Ion kinetic energy/ magnetic energy increases as dispersion increases (see also Gomez et al. PoP 2008).

Forcing on v



Domination of kinetic energy

Forcing on b

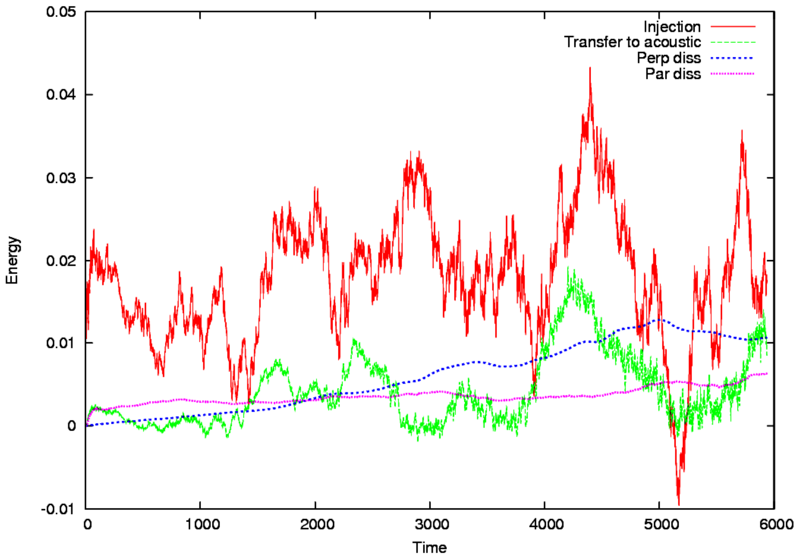


Approximate equipartition between kinetic and magnetic energies (at all scales)

Parallel propagation in a small simulation domain: L=2*2π
Random forcing at $k_f=2k_{di}$ (continued): transfer to MS modes

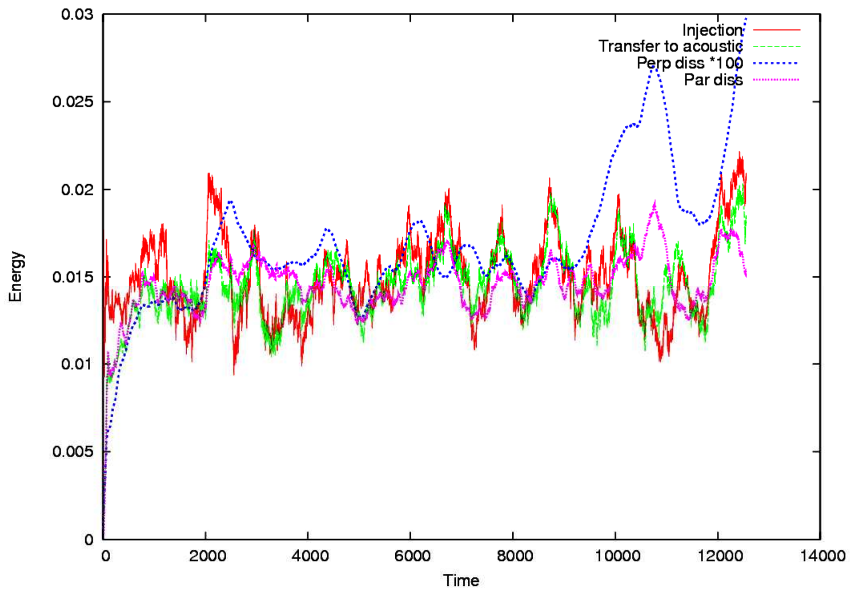
In this regime, magnetosonic modes do not develop shocks.

Forcing on v



Transfer to the acoustic modes << injected energy
Alfvenic dissipation is larger than magnetosonic dissipation by a factor 2.

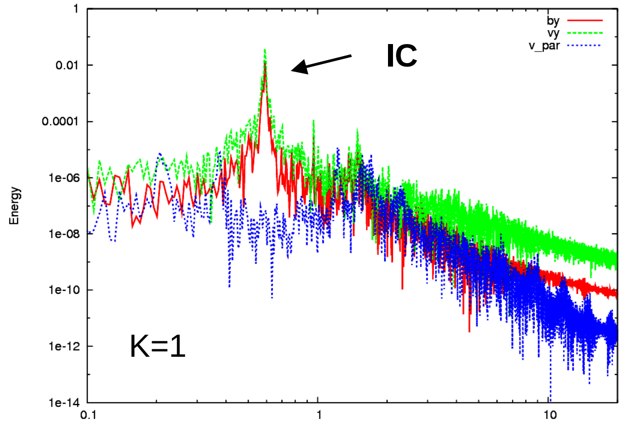
Forcing on b



Transfer to the acoustic modes=injected energy small
Magnetosonic dissipation dominates transverse dissipation by a factor 100.

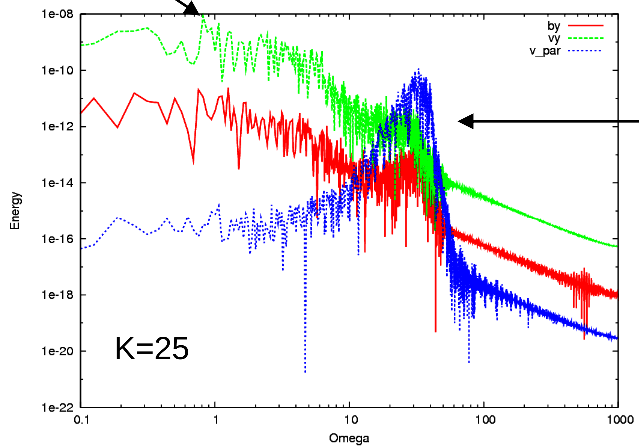
Parallel propagation in a small simulation domain: L=2*2π
Random forcing at $k_f=2k_{di}$ (continued) : Mode identification

Forcing on v



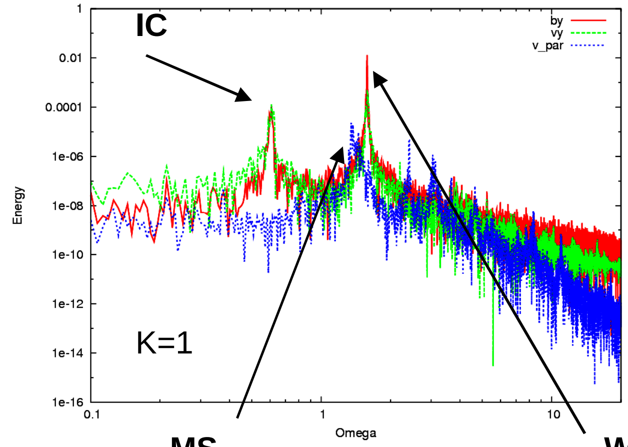
Omega

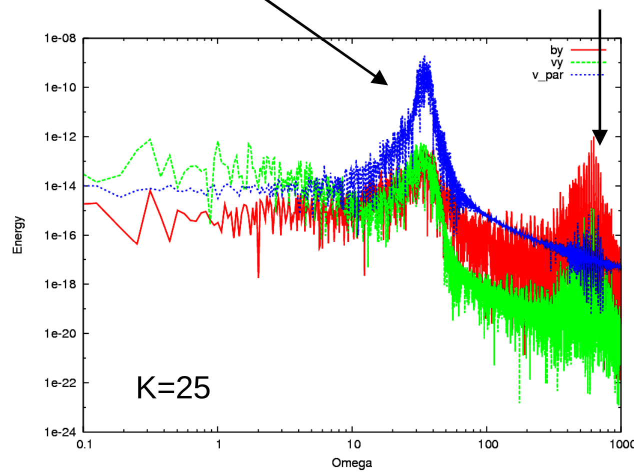
IC turbulence



Domination of ion-cyclotron waves.
Whistlers are subdominant.

Forcing on b

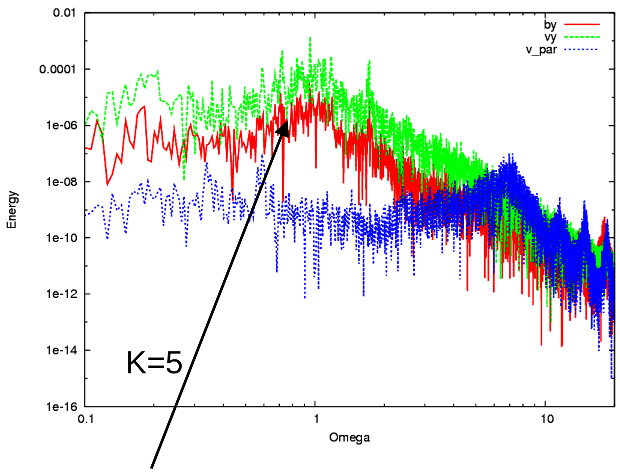




Signature of the presence of whistler waves in a weak turbulent regime

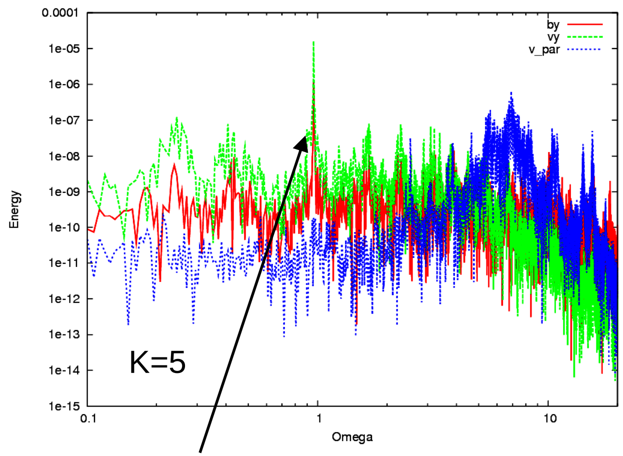
Turbulent broadening of the frequency peak

Forcing on v



IC turbulence

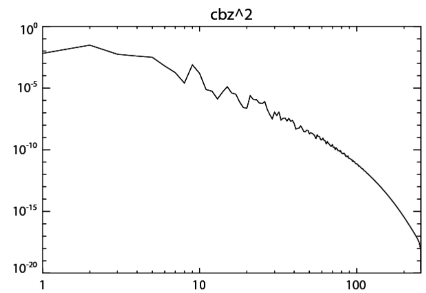
Forcing on b



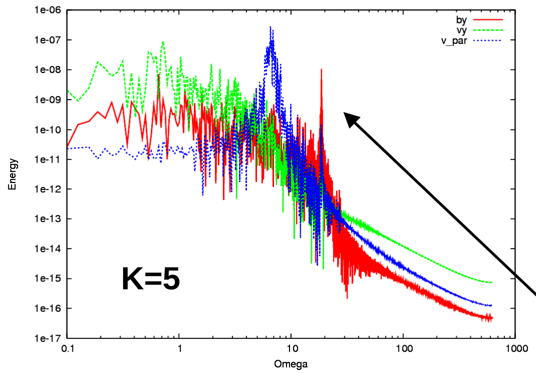
IC mode

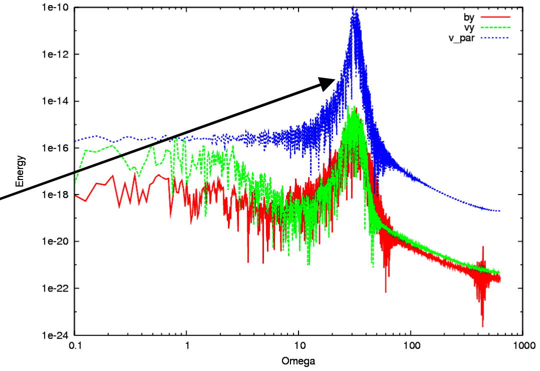
Oblique propagation (45^o)
Random forcing on the velocity field at k_f=2 k_{di}

1. Total energy does saturate; no large scale structures are formed.
2. All the injected energy goes to the parallel mode; small transverse dissipation



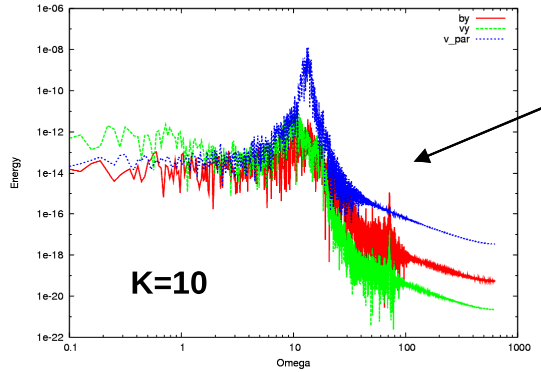
k^{-5} magnetic spectrum





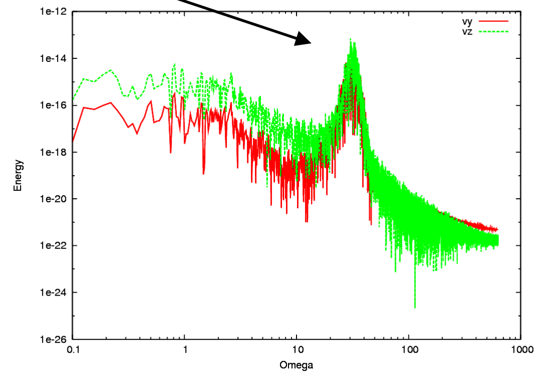
K=25

Domination of fluctuations associated with the intermediate mode (z-component)



Subdominant but visible whistler modes.

Whistlers are more intense when forcing on b field

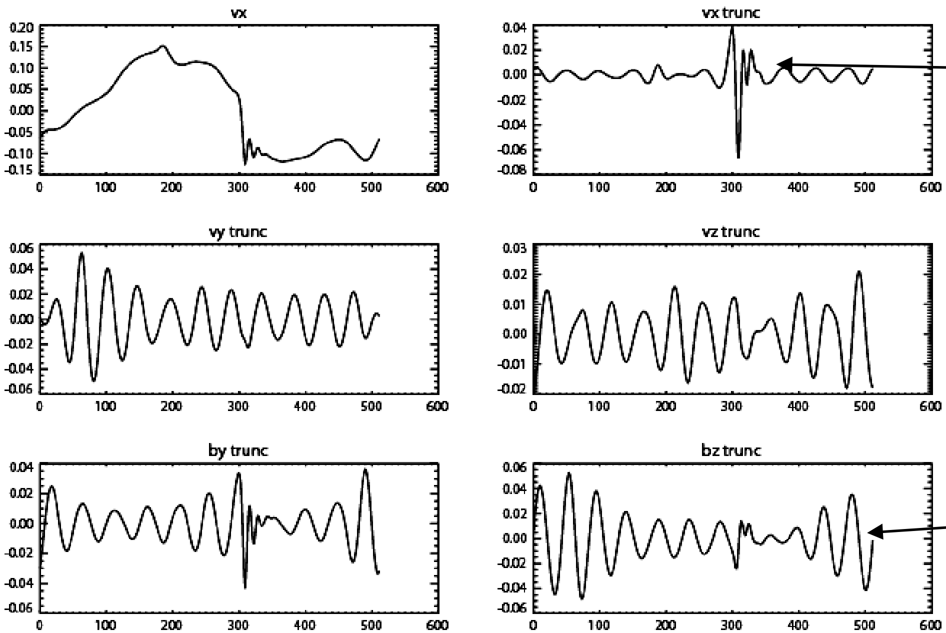


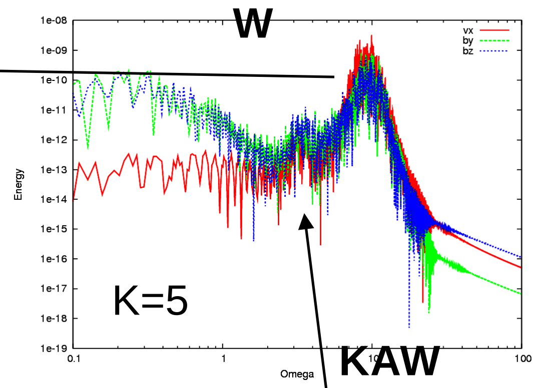
Oblique propagation (80º)
Random forcing on the velocity field at $k_f=2\ k_{di}$

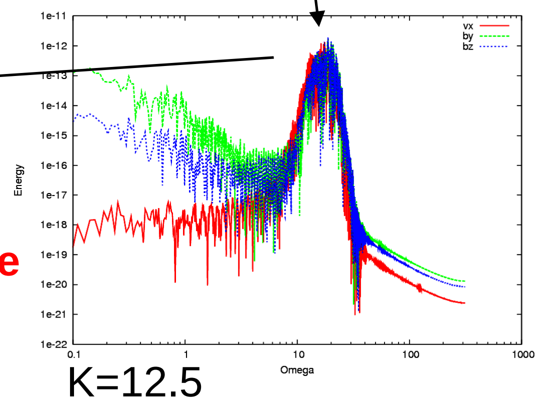
Presence of dispersive shocks at larger angles (and more intense compressibility at lower β).
Observations consistent with:

Spontaneous generation of small scales with anti-correlation density vs. magnetic intensity propagating perpendicular to B_0 especially at high β; breakdown of Alfvenicity (Servidio et al. PSS07)

Fields filtered at k>5

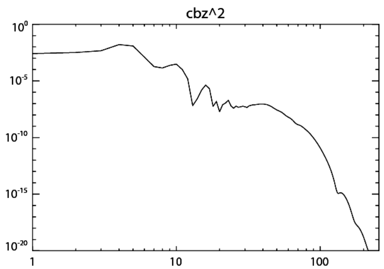






KAW turbulence

Spectrum dominated by structures



What happens with a more refined Landau fluid model including FLR dispersive effects and Landau damping?

Fluid hierarchy with 2 main ingredients:

(1) Closure relations at the level of the 4th order cumulants $\widetilde{r}_{\parallel\parallel}, \widetilde{r}_{\parallel\perp}, \widetilde{r}_{\perp\perp}$

Takes into account linear Landau damping

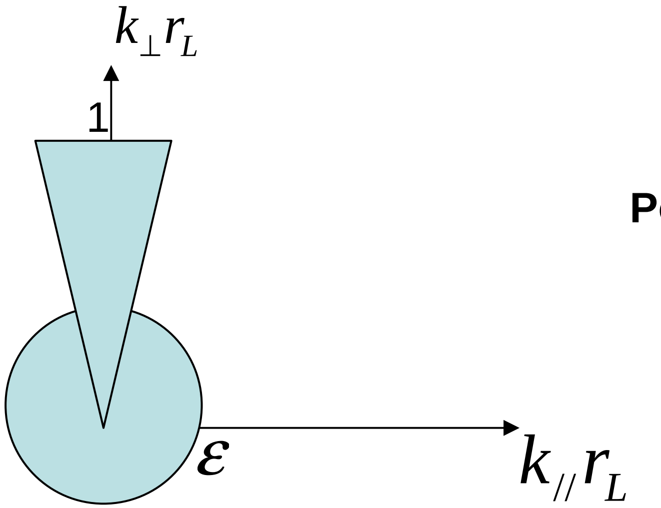
(2) FLR corrections (non-gyrotropic terms) to the various moments

This is obtained from the linear kinetic theory in the low-frequency limit.

$\omega/\Omega \sim \epsilon \ll 1$ gyrokinetic scaling:

- quasi-transverse fluctuations $(k_\parallel/k_\perp \sim \epsilon)$ with $k_\perp r_L \sim 1$
- hydrodynamic scales with $k_\parallel r_L \sim k_\perp r_L \sim \epsilon.$

r_L: ion Larmor radius



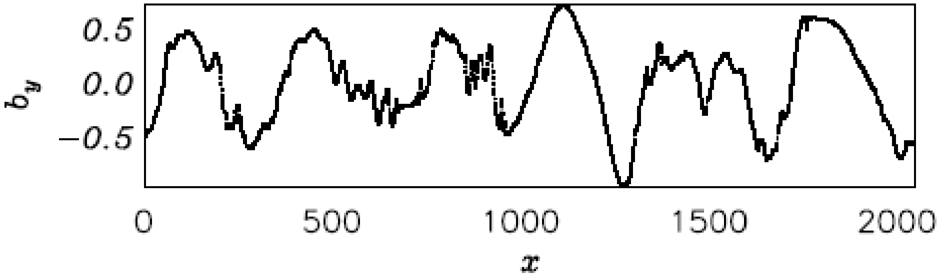
PoP 14, 082502 (2007)

A 1D SIMULATION

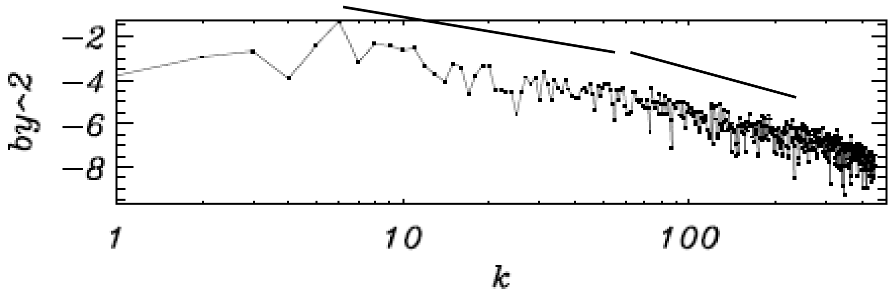
with

•Random forcing of the three velocity components between k=2 and 10, peaking at k=5, only on when the total energy falls below prescribed value.
•Angle of propagation: 84^o
•β=1, Te/Ti=5
•Size of the domain: 300* 2π
•**No extra dissipation**

Due to differential heating, a small amount of collisions is needed to let parallel and perpendicular pressures tend to the same mean values and thus avoid instabilities.



Turbulent cascade.
A break in the spectrum starts to develop at a **nonlinear** dispersive scale



Conclusions

- The forced DNLS equation exhibits three types of turbulence:
  - At small dispersion and large dissipation, a regime of dispersive shocks with a well-defined transfer of energy.
  - At small dispersion and small dissipation, a regime where the transfer is highly intermittent at large scale and constant as small scales (signature of wave packets) and where energy does not saturate in time.
  - At large dispersion and small dissipation, a quasi-stationary regime with highly intermittent transfer at all scales.

- In the Hall-MHD system:
  - In the parallel direction, there is absence of AW cascade at large scales, and evidence of ion-cyclotron turbulence and/or weak whistler wave turbulence at small scales.
  - In oblique directions, evidence of a turbulence of intermediate (AW) modes. Intensity of whistler modes increases with propagation angle.

Since IC waves are damped in a collisionless plasma, one expects turbulence to be dominated by **KAWs and Whistlers in oblique or quasi-perpendicular directions**, consistent with e.g. magnetosheath observation that 2D turbulence is preferred at small scale (Alexandrova et al. Ann. Geophys. 08).

Further questions to examine:

- It is important to extend this study in 3D and to take into account physical damping mechanisms.
- Is the important scale the ion gyroradius or the ion inertial length?
- Does the cascade proceed anisotropically all the way to the electron scale?
- Test the existence of weak turbulence for KAWs via three-wave decay: inverse cascade if $k_\perp \rho_i < 1$, forward cascade otherwise (with a steeper power law) (Voitenko, JPP **60**, 515 (1998)).