# *Collisionless and Collisional Tearing Mode in Gyrokinetics*

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**R. NUMATA**[†], T. Tatsuno, W. Dorland, B. N. Rogers[A], N. Loureiro[B]

† rnumata@umd.edu

Center for Multiscale Plasma Dynamics, The University of Maryland,

A) Dept. Physics, Dartmouth College,

B) UKAEA, Culham

# *Introduction*

- We study magnetic reconnection in the presence of a guide field using gyrokinetic code `AstroGK`.
- Guide field reconnections (or component reconnections) are often observed in astrophysical situations, not to mention in fusion experiments.
- Gyrokinetics includes various kinetic effects, such as FLR, electron inertia, tensorial pressures, important to understand collisionless magnetic reconnection.
- Gyrokinetics assumes strong guide field. Reconnection process may differ in gyrokinetics from anti-parallel reconnection (no guide field). Understanding of gyrokinetic reconnection complements that by weak guide field cases, and contributes to gain insights for how kinetic processes play roles in magnetic reconnection.
- Even though reconnection process occurs in collisionless situation, collisions are still important to smooth out velocity space structures.
- Relation between microscopic collisions and macroscopic resistivity is not trivial. We also intensively investigate the relation of them.

# AstroGK: *Basic equations*

The distribution function of particles is given by $f = \left(1 - \frac{q\phi}{T_0}\right) f_0 + h$, where $f_0 = n_0/(\sqrt{\pi} v_{\rm th})^3 \exp(-v^2/v_{\rm th}^2)$ is the Maxwellian, and the thermal velocity is given by $v_{\rm th} = \sqrt{2T_0/m}$. The equations to solve are the gyrokinetic equation for $h = h(\boldsymbol{R}, V_\perp, V_\parallel)$,

$$\frac{\partial h}{\partial t} + V_\parallel \frac{\partial h}{\partial Z} + \frac{1}{B_0} \{\langle \chi \rangle_{\boldsymbol{R}}, h\} - \langle C(h) \rangle_{\boldsymbol{R}} = q \frac{f_0}{T_0} \frac{\partial \langle \chi \rangle_{\boldsymbol{R}}}{\partial t}, \quad (1)$$

$\chi = \phi - \boldsymbol{v} \cdot \boldsymbol{A}$ and the field equations for $\phi(\boldsymbol{r})$, $A_\parallel(\boldsymbol{r})$, and $\delta B_\parallel(\boldsymbol{r})$,

$$\sum_s \left[ -\frac{q_s^2 n_{0s} \phi}{T_{0s}} + q_s \int \langle h_s \rangle_{\boldsymbol{r}} \mathrm{d}\boldsymbol{v} \right] = 0, \quad (2)$$

$$\nabla_\perp^2 A_\parallel = -\mu_0 \sum_s q_s \int \langle h_s \rangle_{\boldsymbol{r}} v_\parallel \mathrm{d}\boldsymbol{v} \quad (3)$$

$$B_0 \nabla_\perp \delta B_\parallel = -\mu_0 \nabla_\perp \cdot \sum_s \int \langle m \boldsymbol{v}_\perp \boldsymbol{v}_\perp h_s \rangle_{\boldsymbol{r}} \mathrm{d}\boldsymbol{v}. \quad (4)$$

# AstroGK: *Normalization*

**Time and Space**

$$t = \frac{a_0}{v_{\mathrm{th}0}} \hat{t} \qquad (v_{\mathrm{th}0} = \sqrt{2T_{00}/m_0}), \qquad z = a_0 \hat{z}, \qquad x = \rho_0 \hat{x}. \tag{5}$$

**Species temperature, mass, charge**

$$m_s = m_0 \hat{m}_s, \qquad T_{0s} = T_{00} \hat{T}_{0s}, \qquad q_s = q_0 \hat{q}_s. \tag{6}$$

**Fields**

$$\frac{a_0}{\rho_0} \frac{q_0 \phi}{T_{00}} = \hat{\phi}, \qquad \frac{a_0}{\rho_0} v_{\mathrm{th}0} \frac{q_0 A_\parallel}{T_{00}} = \hat{A}_\parallel, \qquad \frac{a_0}{\rho_0} \delta B_\parallel = B_0 \delta \hat{B}_\parallel. \tag{7}$$

**Distribution function**

$$h_s = \frac{\rho_0}{a_0} f_{0s} \hat{h}_s, \qquad \left(f_{0s} = \frac{1}{\pi^{3/2}} \frac{n_{0s}}{v_{\mathrm{th},s}^3} e^{-v^2/v_{\mathrm{th},s}^2}\right). \tag{8}$$

# Collision Operator

Recently, linearized collision operators for gyrokinetic simulations, which satisfies physical requirements are established and implemented in `AstroGK`. [Abel *et al*, Phys. Plasmas **15**, 122509 (2008), Barnes *et al*, submitted to Phys. Plasmas (2008).]

The operators are the pitch-angle scattering (Lorentz), the energy diffusion, and moments conserving corrections to those operators for like-particle collisions. Electron-ion collisions consists of pitch angle scattering by background ions and ion drag are also included.

We, here, mainly discuss the electron-ion collisions since it contributes to resistivity. The operator is given by (in Fourier space)

$$C_{\mathrm{ei}}(h_{\mathrm{e},\boldsymbol{k}}) = \nu_{\mathrm{ei}} \left(\frac{v_{\mathrm{th,e}}}{V}\right)^3 \left(\frac{1}{2}\frac{\partial}{\partial\xi}(1-\xi^2)\frac{\partial h_{\mathrm{e},\boldsymbol{k}}}{\partial\xi} - \frac{1}{4}(1+\xi^2)\frac{V^2}{v_{\mathrm{th,e}}^2}k_\perp^2\rho_{\mathrm{e}}^2 h_{\mathrm{e},\boldsymbol{k}} + \frac{2V_\parallel J_0(\alpha_{\mathrm{e}})u_{\parallel,\mathrm{i},\boldsymbol{k}}}{v_{\mathrm{th,e}}^2} f_{0\mathrm{e}}\right) \quad (9)$$

We examine how this collision operator relates with resistivity which decays the current.

# Spitzer Resistivity

From the fluid picture current decays due to collisional resistivity as

$$\frac{\partial J}{\partial t} = -\frac{\eta}{\mu_0} k^2 J, \tag{10}$$

and the decay rate is $\tau_{\mathrm{decay}}^{-1} = (\eta/\mu_0)k^2$. Using the Spitzer resistivity given by

$$\eta = \frac{m_{\mathrm{e}}}{1.98 \tau_{\mathrm{e}} n_{\mathrm{e}} e^2} \tag{11}$$

where $\tau_{\mathrm{e}} = 3\sqrt{\pi}/(4\nu_{\mathrm{ei}})$, the decay rate is casted into the following form,

$$\tau_{\mathrm{decay}}^{-1} = C \nu_{\mathrm{ei}} (d_{\mathrm{e}} k)^2 \tag{12}$$

where the constant $C = 4/(1.98 \times 3\sqrt{\pi}) \sim 0.380$. We will determine $C$ from numerical simulations.

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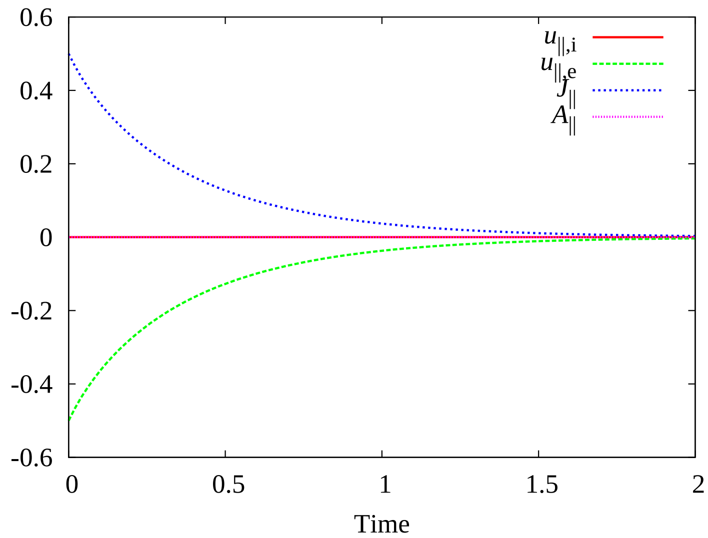
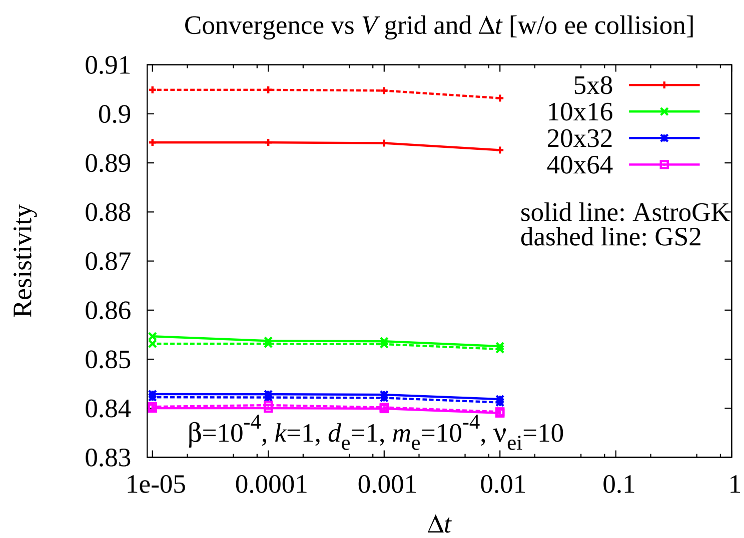
# Resistivity Estimate

We start the test with the following parameters $\nu_{\rm ei} = 10$, $\beta = 10^{-4}$, $k_\perp = 1$, $m_{\rm i} = 1$, $m_{\rm e} = 10^{-4}$, $n_{0\rm i} = 1$, $n_{0\rm e} = 1$, $q_{\rm i} = 1$, $q_{\rm e} = -1$ $T_{0\rm i} = 1$, $T_{0\rm e} = 1$, and $u_{\|,\rm e}(t=0) = -1$, $u_{\|,\rm i}(t=0) = 0$ (ion drag is off). For such a small $\beta$ value, the magnetic fluctuation and its temporal change is very small, and we may approximate

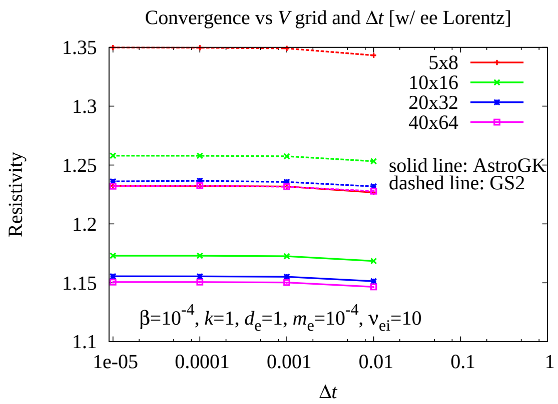
$$\frac{\partial h_{\rm e}}{\partial t} = C_{\rm ei}, \quad \frac{\partial h_{\rm i}}{\partial t} = 0. \tag{13}$$

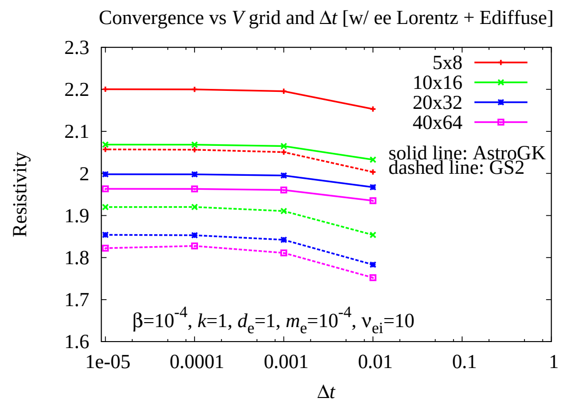


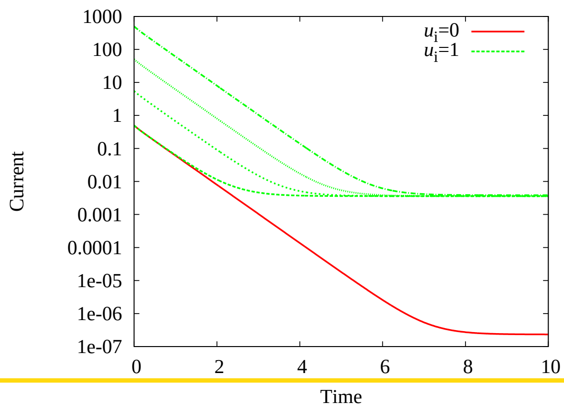
# Effects of e-e collisions and ion drag

We include ee collisions (Lorentz and energy diffusion) in addition to ei collision. Estimated $C$ is about $C \sim 1.15$ (w/ L), $C \sim 2$ (w/ L+E) ($C \sim 0.84$ w/o ee collisions)



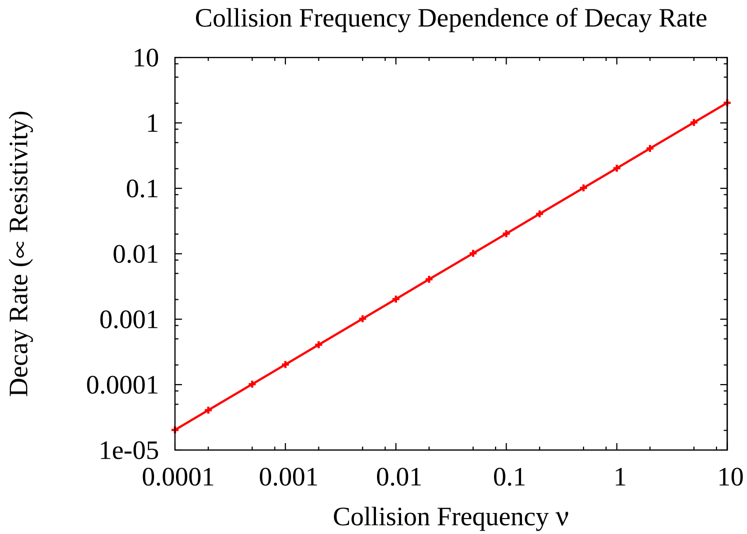



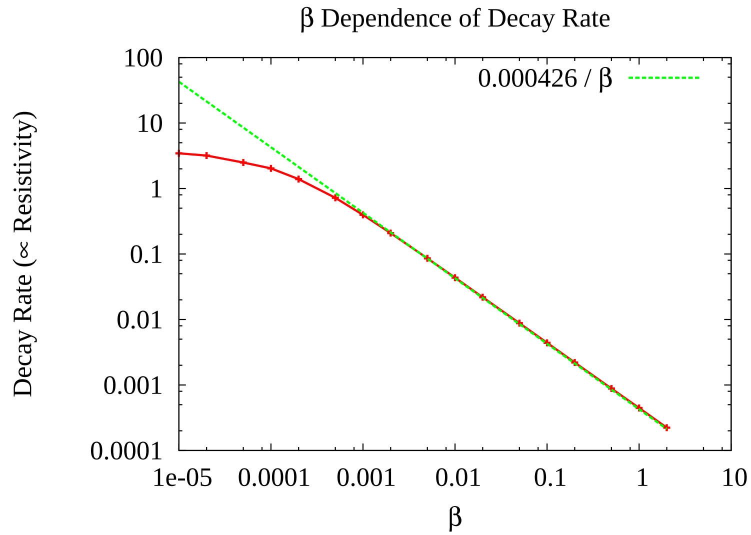

Even if we include the ion drag effect, current decay rate does not change as long as $J \gg J_\infty$.



# Parameter Dependence of Resistivity

Resistivity is proportional to $\nu_{\rm ei}$ and inversely proportional to $\beta$ as expected.




- Linearity to $\nu_{\rm ei}$ is exactly held
- By fitting $1/\beta$ function, we obtain $C \sim 0.426$, which is fairly close to Spitzer ($C \sim 0.380$).
- Conservation of momentum was relatively bad in `AstroGK`, which causes overestimate of resistivity. This is fixed very recently.
- $1/\beta$ does not fit for small $\beta \lesssim 0.001$.

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# *Collisional Tearing Mode Theory*

Time Scales

- Hydromagnetic time scale: $\tau_{\mathrm{H}} \equiv \tau_{\mathrm{A}}/(kLB_{0y}/B_0)$
- Resistive time scale: $\tau_{\mathrm{R}} \equiv \mu_0 L^2/\eta$

Assumptions

- Time scale separations

$$1/\tau_{\mathrm{R}} \ll \omega \ll 1/\tau_{\mathrm{H}} \qquad (14)$$

- Scale separations

$$\ell_\eta \ll a \qquad (15)$$

where $\ell_\eta$ is the resistive layer width

Dispersion relation

$$\Delta' a = -\frac{\pi}{8}\gamma^{5/4}\tau_{\mathrm{H}}^{1/2}\tau_{\mathrm{R}}^{3/4}\frac{\Gamma((\lambda^{3/2}-1)/4)}{\Gamma((\lambda^{3/2}+5)/4)} \qquad (16)$$

$\lambda = \gamma\tau_{\mathrm{H}}^{2/3}\tau_{\mathrm{R}}^{1/3}$

# Collisionless Tearing Mode Theory

- Mirnov *et al.*, Phys. Plasmas, **11**, 4468 (2004).

$$\frac{\Gamma^2 \rho_s}{G(\Gamma/\sqrt{\beta})} + \frac{2}{\Delta'} = \frac{2G(\Gamma/\sqrt{\beta})\delta}{\pi\Gamma} \tag{17}$$

  $\Gamma = \gamma\tau_A/(\rho_s k)$, $\delta^2 = d_e^2 + \eta/(\mu_0\gamma)$, $\beta = \mu_0(\gamma_e p_e^{(0)} + \gamma_i p_i^{(0)})/(B^{(0)})^2$, $G(x) = (\sqrt{x}/2)(\Gamma(1/4 + x/4)/\Gamma(3/4 + x/4))$.

- Fitzpatrick and Porcelli, Phys. Plasmas, **11**, 4713 (2004).

$$\frac{Q^2 d_\beta}{G(Q/c_\beta)} + \frac{2}{\Delta'} = \frac{2G(Q/c_\beta)d_e}{\pi Q} \tag{18}$$

  $Q = \gamma\tau_A/(d_\beta k)$, $d_\beta = c_\beta d_i$, $c_\beta = \sqrt{\beta/(1+\beta)}$, $\beta = \mu_0\gamma_e p_e^{(0)}/(B^{(0)})^2$. If $\beta << 1$, $d_\beta \to \rho_s$, and $Q \to \Gamma$. Thus, for cold ion and collisionless limit, the dispersion relation is same as Mirnov's.

Later Fitzpatrick and Porcelli removed $G$ in the RHS by taking into account gyroviscous cancellation. [PoP, **14**, 049902 (2007).]

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# Simulation Setting

Equilibrium profile

$$h_\mathrm{e} = h_{\mathrm{e},0} \cosh^{-2}\left(\frac{x}{a}\right) 2V_{||} \tag{19}$$

yields the fields $\phi = \delta B_{||} = 0$,

$$A_{||} = A_0 \cosh^{-2}\left(\frac{x}{a}\right) \tag{20}$$

$h_{\mathrm{e},0}$ (proportional to $A_0$) is determined such that it gives a desired $B_{0y}$.

Stability Index $\Delta'$

$$\Delta' a = 2\left(\frac{6\bar{k}^2 - 9}{\bar{k}(\bar{k}^2 - 4)} - \bar{k}\right) \tag{21}$$

$\bar{k}^2 = a^2 k^2 + 4$

# Simulation Setting

Parameters

$$n_{0\mathrm{e}} = n_{0\mathrm{i}} = 1, \qquad T_{0\mathrm{e}} = T_{0\mathrm{i}} = 1, \qquad -q_{\mathrm{e}} = q_{\mathrm{i}} = 1 \tag{22}$$

$$m_{\mathrm{e}} = 10^{-2}, \qquad m_{\mathrm{i}} = 1, \tag{23}$$

$$\beta_0 = 0.3 \tag{24}$$

which yield the following spatial scales

$$\rho_{\mathrm{i}} = 1 \qquad \rho_{\mathrm{e}} = 0.1 \tag{25}$$

$$d_{\mathrm{i}} = 1.8 \qquad d_{\mathrm{e}} = 0.18. \tag{26}$$

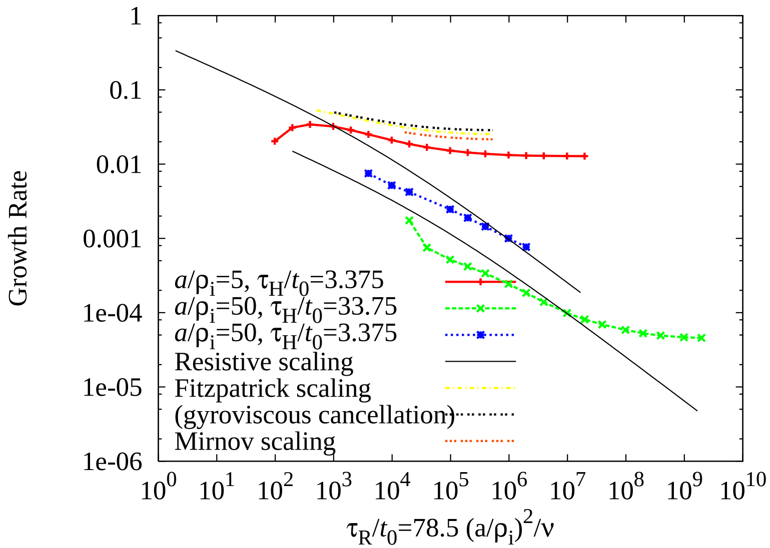
Interpretation of `AstroGK` time scales

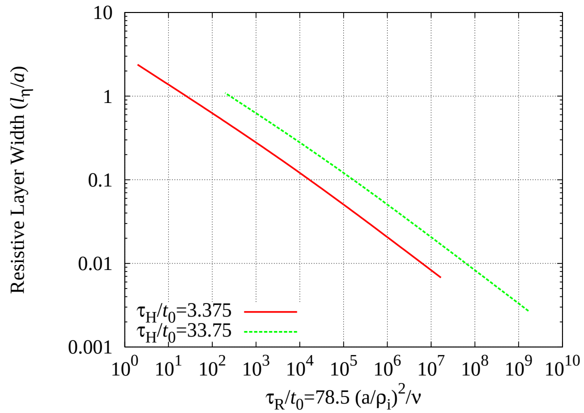
$$\tau_{\mathrm{H}}/t_0 = \frac{\sqrt{n_{0\mathrm{i}} m_{\mathrm{i}} \beta_0}}{k B_{0y}} \tag{27}$$

$$\tau_{\mathrm{R}}/t_0 = 2.63 \nu_{\mathrm{ei}}^{-1} a^2 \frac{n_{0\mathrm{e}} q_{\mathrm{e}}^2}{m_{\mathrm{e}}} \beta_0 \tag{28}$$

# Results

Lines are for different kinetic effects ($a/\rho_i = 5$ and $a/\rho_i = 50$)




- Growth rate is independent of collision frequency if it is small
- Growth rate does not scale like resistive scaling due to kinetic effect
- We must further reduce kinetic effect to observe resistive scaling scaling
- Collisionless scaling qualitatively fit to numerical results
- Red line does not change by ion temperature (result not shown)

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# *Summary*

- We have confirmed that e-i collisions in addition to full e-e collisions yield expected macroscopic behavior. The resistivity is quantitatively same as Spitzer's value.
- We have performed collisionless and collisional tearing mode simulations, and have scanned for $\nu_{\mathrm{ei}}$.
- We have observed transition from collisional regime (collision dependent growth rate) to collisionless regime (collision independent growth rate).
- Due to kinetic effects, growth rate does not fit to resistive scaling even in the collisional regime. Further decrease of kinetic effect (larger $a/\rho_{\mathrm{i}}$) needed.
- We have also compared the results with collisionless scaling by Fitzpatrick's and Mirnov's. Numerical results agree with the theories qualitatively, but are different by a factor.