Oscillatory migrating magnetic fields in simulations of helical turbulence in spherical domains

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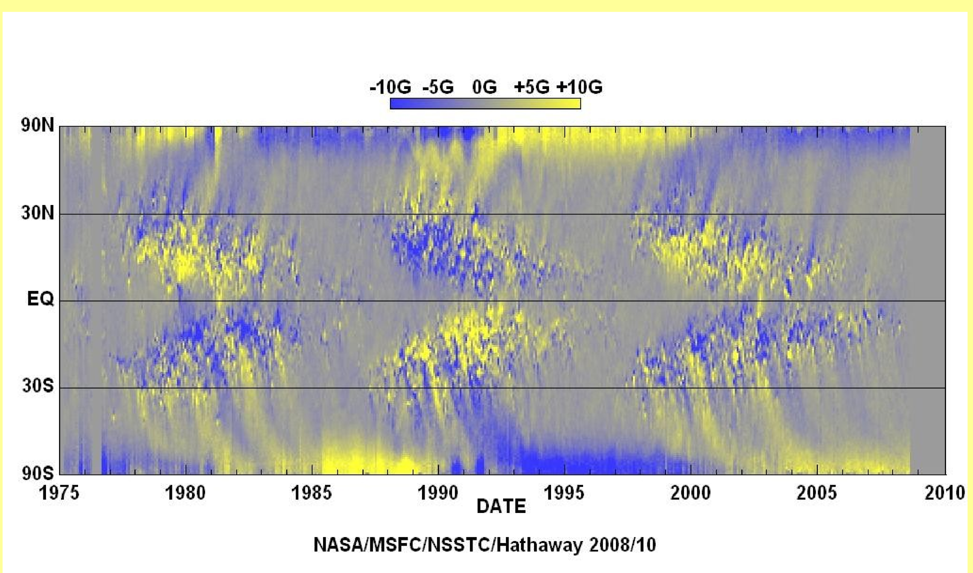
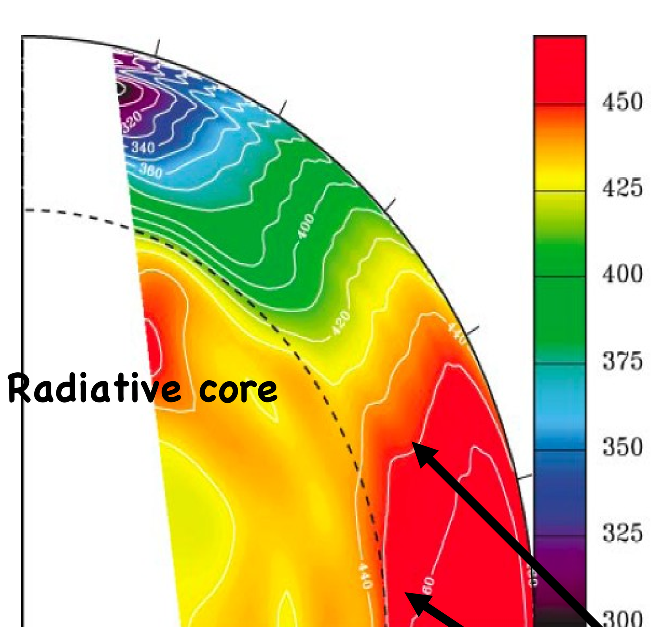
Solar Magnetic Field:



- *Oscillation* and polarity reversal, 22 year solar cycle
- Equator ward migration of sunspots.
- Poleward migration of diffusing field.
- Azimuthally averaged magnetic field.

Turbulence in the Sun:



Velocity Profile of the sun has been obtained by Helioseismology.

Important roles are played by rotation stratification and convection.

Radiative core

Convection zone

Tachocline

Observation from MDI (Schou et al 1998)

Estimates of Solar Magnetic Field:

- About 1G on surface, but about 2 kG at the sunspots.
- Equipartition field strength at the base of the convection zone is about 3 kG.
- Mean magnetic field (estimated from total magnetic flux that emerges from the surface during one cycle) is about 4 kG. (Galloway and Weiss, 1981)
- Peak magnetic field estimated from thin flux tube approximations is about 100kG (D'Silva and Chaudhuri, 1993)

Simulation:

Direct numerical simulation (DNS) of compressible MHD

$$\rho \frac{D\vec{U}}{Dt} = -\nabla p + \vec{J}\times\vec{B} + \vec{F}_{visc} + \vec{f}$$

$$\frac{\partial \rho}{\partial t} = -\nabla\cdot(\rho\vec{U})$$

$$\vec{J} = \nabla\times\vec{B}$$

$$\frac{\partial \vec{B}}{\partial t} = \nabla\times(\vec{U}\times\vec{B} - \eta\vec{J})$$

viscous force

Advective derivative

Magnetic diffusivity

Helical external force in spherical polar coordinates:

$$\nabla\times\vec{f} = \alpha\vec{f}, \quad \nabla\cdot\vec{f} = 0$$

$$\nabla^2\vec{f} = -\alpha^2\vec{f}$$

$$\nabla^2\psi + \alpha^2\psi = 0$$

$$\psi(r,\theta,\phi) = \left[a_l j_l(\alpha r) + b_l n_l(\alpha r)\right] Y_m^l(\theta,\phi)\, e^{im\phi}$$

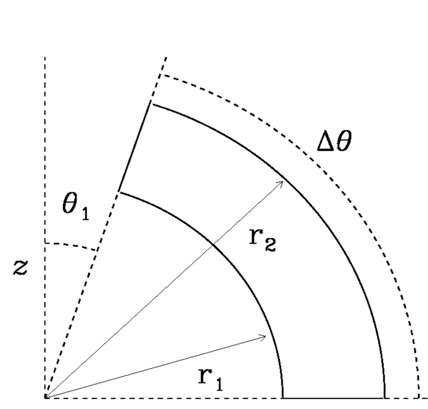
$$\vec{T} = \nabla\times(\hat{e}\psi), \quad \vec{S} = \frac{1}{\alpha}\nabla\times\vec{T}$$

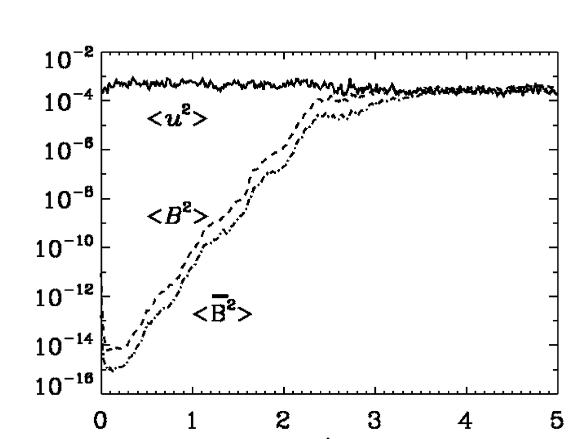
$$\vec{H} = \vec{T} + \vec{S}$$

- Chandrasekhar-Kendall functions.
- The coefficients are chosen to satisfy the right boundary conditions.

The coefficients and the unit vector is randomised to generate random helical forcing corresponding to a range of wave number.

Spherical wedge shaped domains:

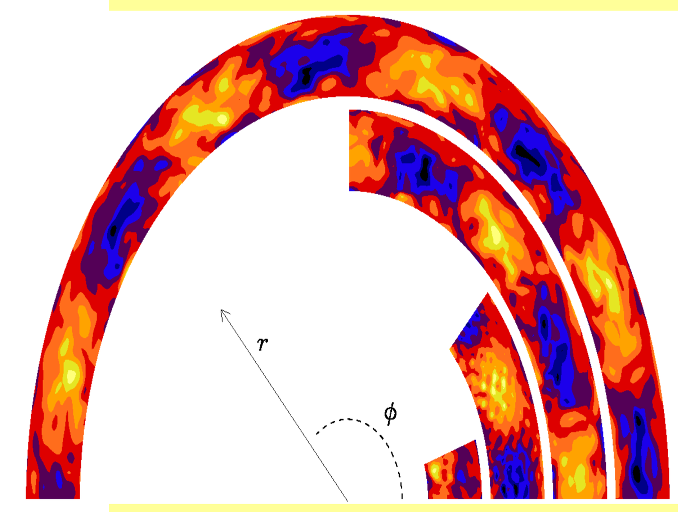


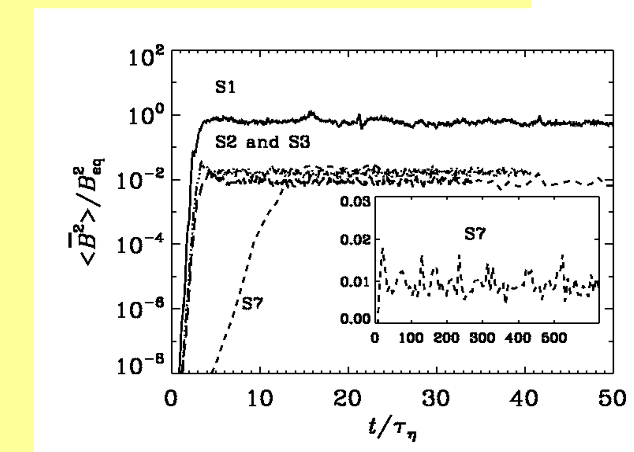


$$\frac{\overline{B}^2}{B_{eq}^2} = \frac{k_f^2}{k_1^2}\left[1 - e^{-2k_1^2\eta(t - t_{sat})}\right]$$

- External force injects positive kinetic helicity.
- Magnetic energy grows and reaches equipartition on slow dissipative scales.
- Limited by decay of small scale magnetic helicity due to magnetic diffusivity.

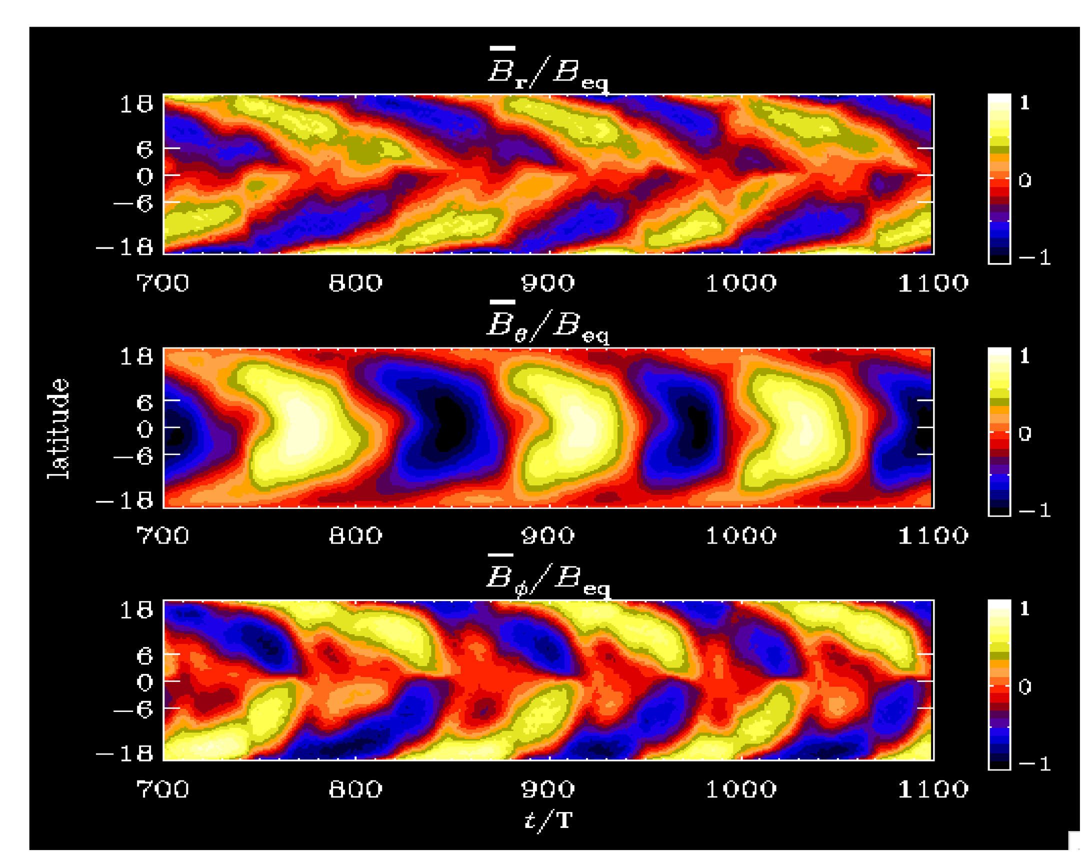
Different domain extents:



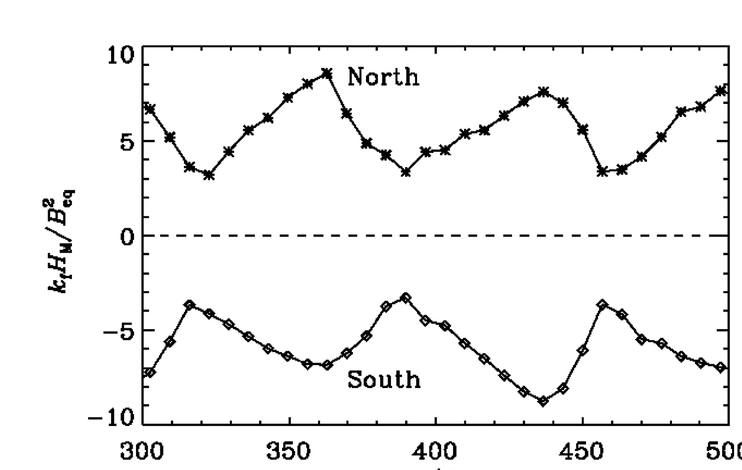


- The large scale magnetic field forms itself in cells along the azimuthal direction. Each cell has about unit aspect ratio.
- Cartesian simulations with similar aspect ratio shows similar behaviour.
- Extending the domain in azimuthal direction gives rise to the clusters repeating themselves.
- Extending the domain in the meridional direction gives no significant change.

Two hemispheres, two signs of kinetic helicity:



Gauge- independent magnetic helicity:



- We have no convection, rotation, and differential rotation, and quite small Reynolds number.
- How does the frequency of oscillations change with magnetic Reynolds number ? (This question is best answered in mean field simulations)
- To generate similar kinetic helicity from convective simulations and rotation will require rapid rotation.
- This is a model with minimum number of added ingredients which shows interesting dynamical behaviour of large scale magnetic field, e.g., equatorward migration, oscillations and polarity reversal.

Reference:

[1] Turbulent dynamos in spherical shell segments of varying geometrical extents. , D. Mitra, R. Tavakol, A. Brandenburg and D. Moss. (ArXiv:0812.3106)

[2] Oscillatory migrating magnetic fields in helical turbulence in spherical domains. D. Mitra, R. Tavakol, P. Kapyla and A. Brandenburg. (ArXiv:0901.2364)