Entropy cascade in gyrokinetic phase space

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outline

1. background
2. theoretical argument
3. simulation
4. summary

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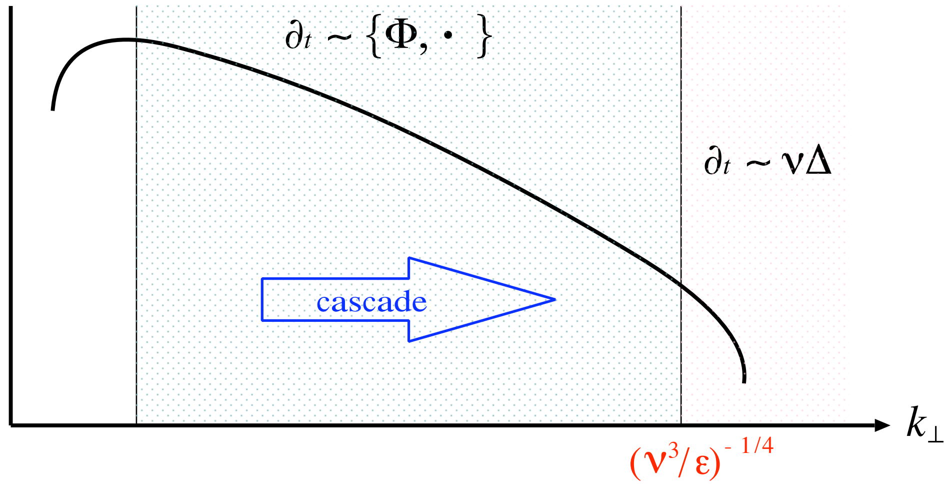
kinetic turbulence

plasma turbulence is mostly weakly-collisional

	fluid	kinetic
config. space	2D	4D
gov. eqn.	Navier-Stokes	GK
dissipation	parametric	collisional
fw cascad. quant.	enstrophy	entropy



fluid turbulence

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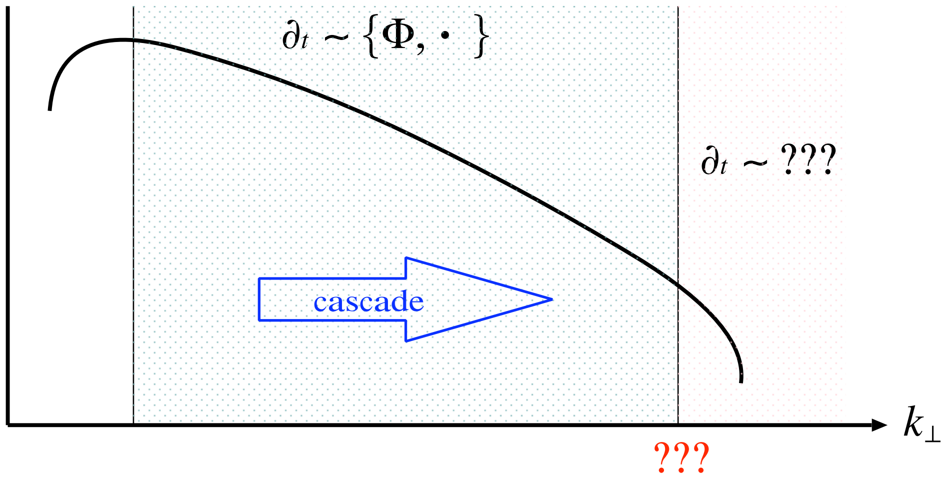


kinetic turbulence

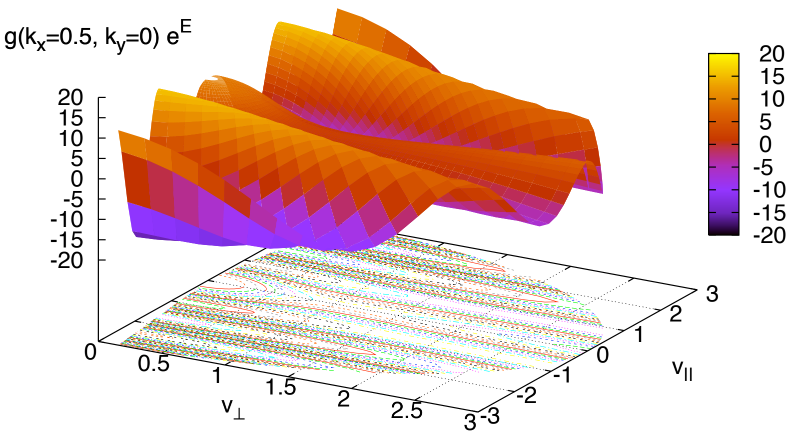
phase mixing

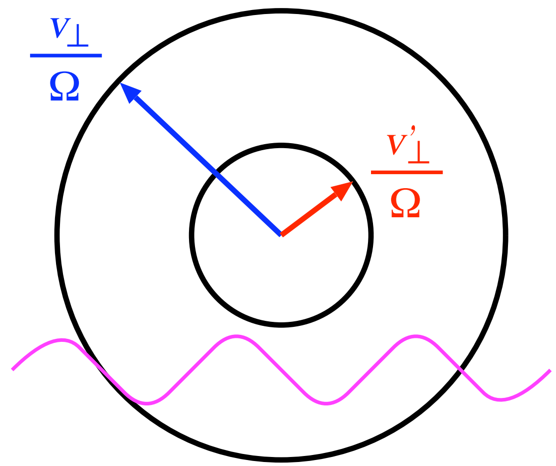
- linear
  (Landau damping)

$$\partial_t h_k \sim \mathrm{i}k_\parallel v_\parallel h_k$$
$$\Downarrow$$
$$h_k \sim \exp[\mathrm{i}k_\parallel v_\parallel t]$$

- nonlinear ($\delta v_\perp/\Omega \sim 1/k_\perp$)

$$\partial_t h_k \sim \sum_{k'} k'(k-k')J_0(k'v_\perp)\phi_{k'}h_{k-k'}$$
$$\Downarrow$$
$$h_k \sim f(k_\perp, v_\perp, t)$$





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ES gyrokinetic equations

Electrostatic dynamics of kinetic plasmas homogeneous along the field.

- GK ions ($\partial_z = 0$)

$$\frac{\partial h}{\partial t} - \nabla\langle\phi\rangle_{\boldsymbol{R}} \times \frac{\hat{z}}{B_0} \cdot \nabla h - \langle \mathcal{C}(h)\rangle_{\boldsymbol{R}} = \frac{qF_0}{T}\frac{\partial\langle\phi\rangle_{\boldsymbol{R}}}{\partial t}.$$

- Quasi-neutrality with no-response electrons ($Q_\phi = q^2 n_0/T_0$)

$$Q_\phi \phi = q \int \langle h\rangle_{\boldsymbol{r}} \, \mathrm{d}\boldsymbol{v}.$$

- Conserved quantity (w/o collisions)

$$W_{\mathrm{ES}} = \underbrace{\iint \frac{T_0\langle h^2\rangle_{\boldsymbol{r}}}{2F_0}\,\mathrm{d}\boldsymbol{v}\,\mathrm{d}\boldsymbol{r}}_{W_h} - \underbrace{\frac{q^2 n_0}{2T_0}\int \phi^2\,\mathrm{d}\boldsymbol{r}}_{W_\phi},$$

$$W_{\mathrm{2D}} = \sum_{\boldsymbol{k}} (1-\Gamma_0)|\phi_{\boldsymbol{k}}|^2.$$

no response?

electron GK eqn ($k_\perp \rho_e \ll 1$)

$$\frac{\partial h_e}{\partial t} + \frac{1}{B_0}\{\phi, h_e\} = \frac{q_e F_{0e}}{T_{0e}}\frac{\partial \phi}{\partial t} + \left(\frac{\partial h_e}{\partial t}\right)_c .$$

Assume

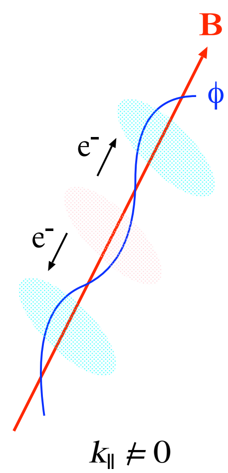
$$\omega \sim \frac{k_\perp^2 \phi}{B_0}, \quad k_\perp \lambda_{\mathrm{mfp}i} \sim \frac{1}{\epsilon}.$$

$$\frac{\nu_{ei}}{\omega} \sim \frac{B_0}{k_\perp^2 \phi}\frac{\nu_{ei}}{\nu_{ii}}\frac{v_{\mathrm{th}i}}{\lambda_{\mathrm{mfp}i}} \sim \sqrt{\frac{m_i}{m_e}}.$$

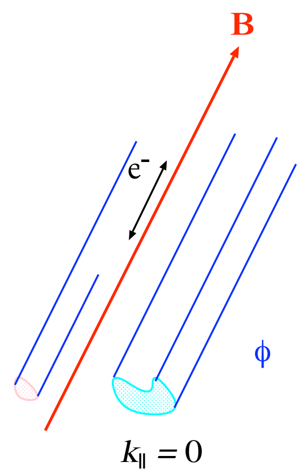
h_e is Maxwellian and the first order equation yields

$$\frac{d}{dt}\left(\frac{\delta n_e}{n_{0e}}\right) = 0.$$

Boltzmann resp.



no response



dimensional analysis

Entropy flux

$$w_h \sim \frac{v_{\rm th}^2}{\tau_\ell}\left(\frac{h v_{\rm th}^3}{n_0}\right)^2 = \text{const.}$$

Quasi-neutrality ($\Phi = \phi/B_0$, ℓ: perp. scale length $\ll \rho$)

$$\Phi \sim \rho v_{\rm th}\left(\frac{\ell}{\rho}\right)^{1/2}\frac{h v_{\rm th}^3}{n_0}\left(\frac{\delta v_\perp}{v_{\rm th}}\right)^{1/2} \sim \frac{v_{\rm th}^4}{n_0}h\ell.$$

Nonlinear decorrelation time

$$\partial_t \sim \{\langle\Phi\rangle_{\boldsymbol{R}},\cdot\} \quad \Leftrightarrow \quad \tau_\ell \sim \left(\frac{\rho}{\ell}\right)^{1/2}\frac{\ell^2}{\Phi}.$$

$$\Downarrow$$

Turbulent spectra

$$h \sim \ell^{1/6},\ \Phi \sim \ell^{7/6} \quad \Leftrightarrow \quad W_h \sim k_\perp^{-4/3}, W_\phi \sim k_\perp^{-10/3}$$

A. A. Schekochihin *et al.*, `arXiv: 0704.0044`

collisional cutoff

collisional dissipation

$$\nu \frac{v_{\mathrm{th}}^2}{\delta v^2} \sim \frac{1}{\tau_\ell}$$

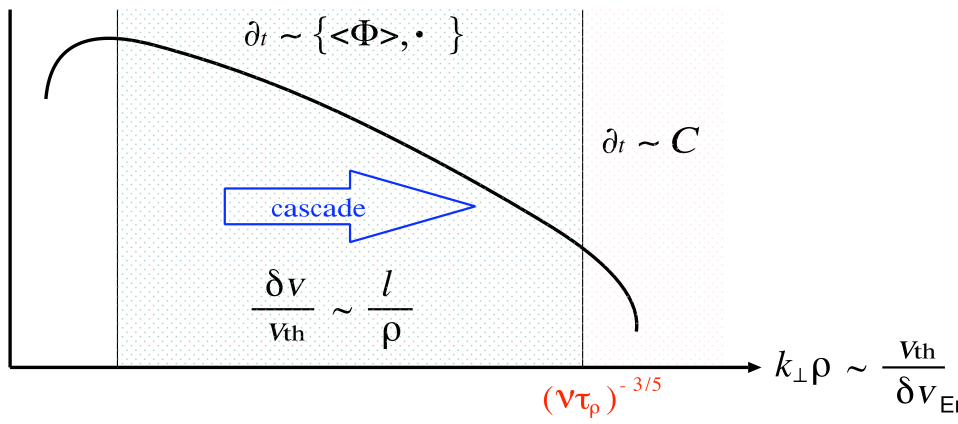
real and velocity space

$$\frac{\delta v}{v_{\mathrm{th}}} \sim \frac{\ell}{\rho}$$

$$\Downarrow$$

collisional cutoff (Dorland number)

$$Do^{-3/5} := \frac{\delta v}{v_{\mathrm{th}}} \sim \frac{\ell}{\rho} \sim (\nu \tau_\rho)^{3/5}$$



outline

AstroGK

developed and maintained by G. Howes, M. Barnes, R. Numata, W. Dorland and T. Tatsuno based on GS2.

- Fourier spectral in x-y (perp to field line)
- 2nd order centered FD in z (along field line)
- Legendre spectral integral in velocity space
- 2nd order implicit trapezoidal scheme (linear convection)
- 3rd order Adams-Bashforth scheme (nonlinear term)
- implicit Euler scheme
  (p-a scatt. $+$ energy diff. w/ mom. cons. collision)

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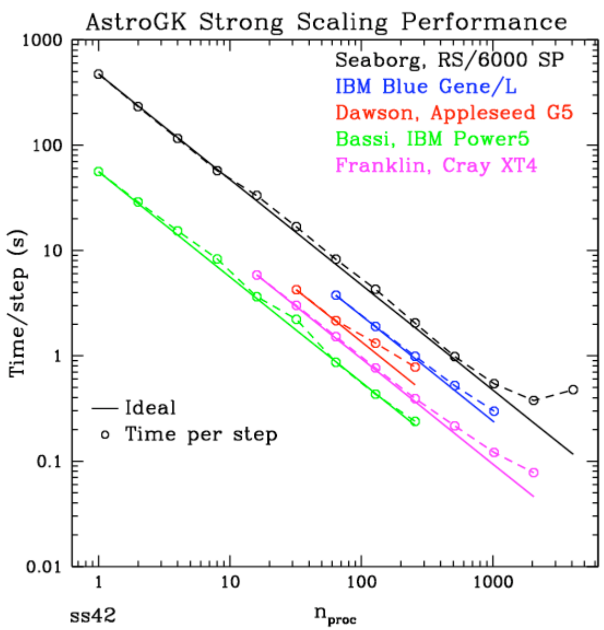
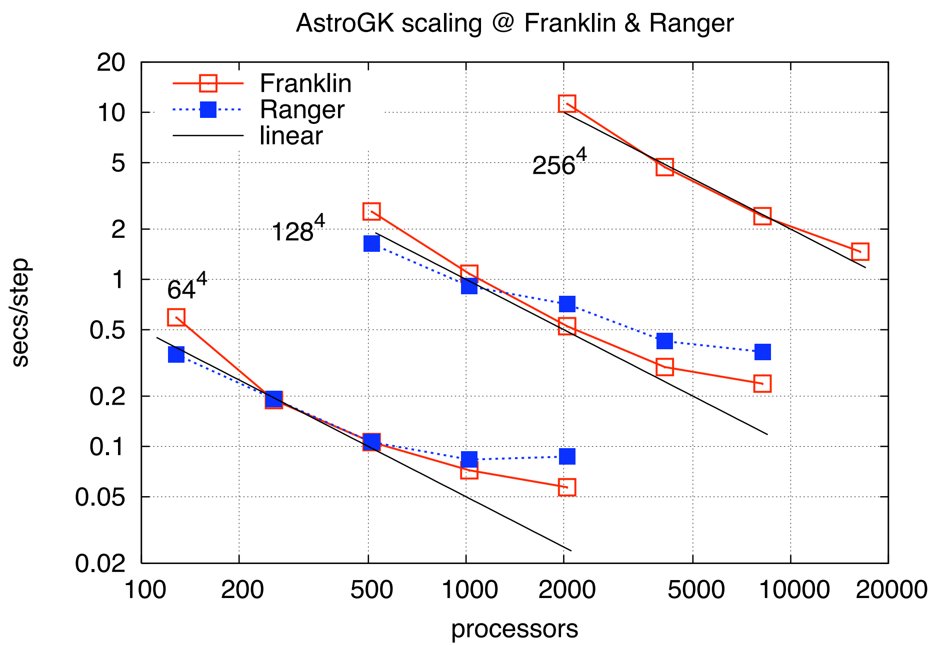
open source code: http://www.physics.uiowa.edu/˜ghowes/astrogk/

AstroGK strong scaling

- Recent upgrade improved parallel scaling
- Good scaling up to $16,384$ processors



old scaling



new scaling

collision operator

pitch-angle $+$ energy diffusion $+$ moments conserve

$$C(h_{\boldsymbol{k}}) = L(h_{\boldsymbol{k}}) + D(h_{\boldsymbol{k}}) + U_L(h_{\boldsymbol{k}}) + U_D(h_{\boldsymbol{k}}) + E(h_{\boldsymbol{k}})$$

where

$$L(h_{\boldsymbol{k}}) = \frac{\nu_D}{2}\frac{\partial}{\partial\xi}\left[(1-\xi^2)\frac{\partial h_{\boldsymbol{k}}}{\partial\xi}\right] - \frac{k^2v^2}{4\Omega_0^2}\nu_D(1+\xi^2)h_{\boldsymbol{k}}$$

$$D(h_{\boldsymbol{k}}) = \frac{1}{2v^2}\frac{\partial}{\partial v}\left(\nu_\parallel v^4 F_0 \frac{\partial}{\partial v}\frac{h_{\boldsymbol{k}}}{F_0}\right) - \frac{k^2v^2}{4\Omega_0^2}\nu_\parallel(1-\xi^2)h_{\boldsymbol{k}}$$

See Michael Barnes' poster.

I. Abel *et al*.: theory, submitted.

M. A. Barnes *et al*.: numerical implementation, submitted.

geometry & initial condition

- straight homogeneous slab: $L_x = L_y = 2\pi$.
- initial condition (decaying turbulence)

$$g = C(\cos x + \cos y)\, e^{-E},$$

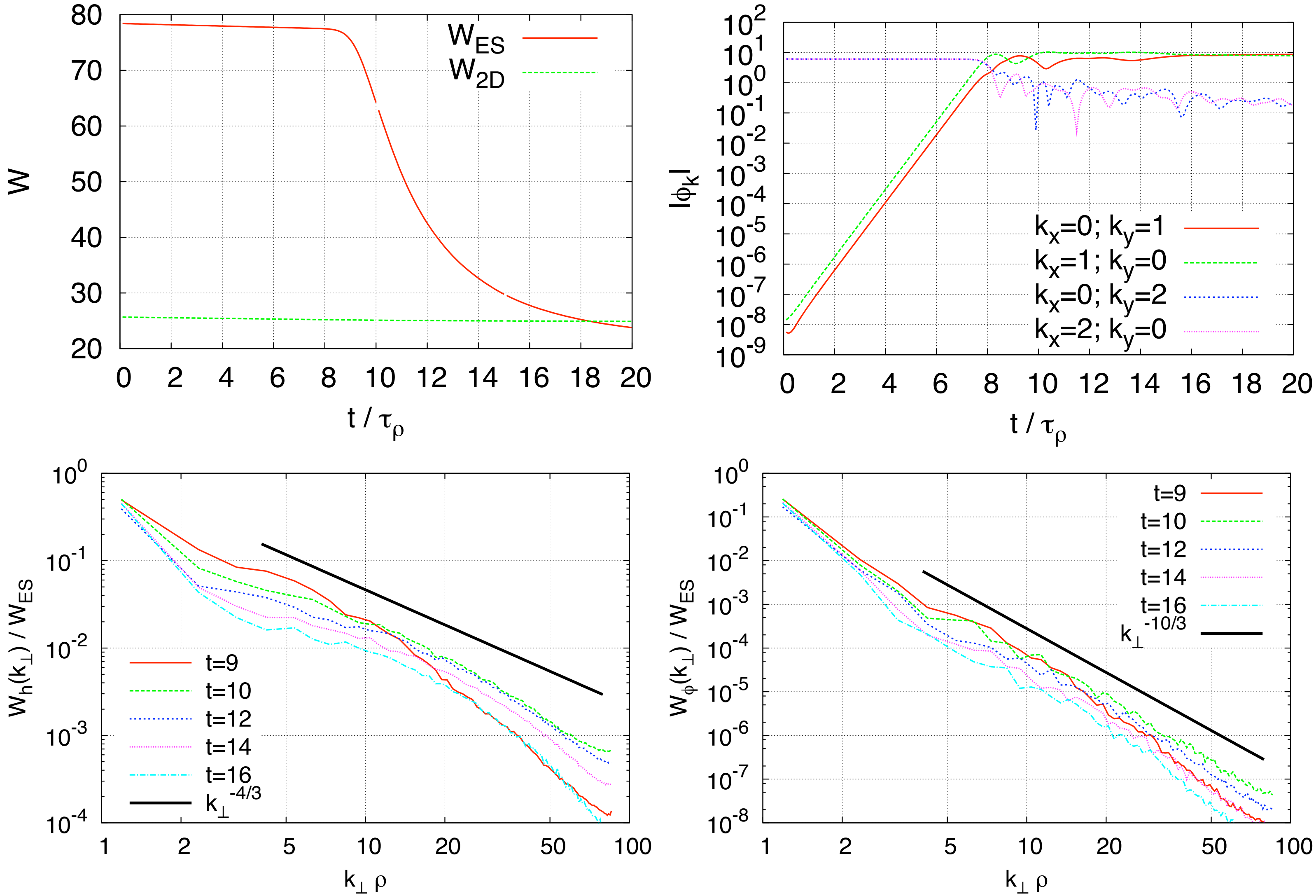
where C is an amplitude corresponding to $\tau_\rho \simeq 1$ and $g := h - qF_0\langle\phi\rangle_{\boldsymbol{R}}/T$.

- run table

case	ν_{ii}	$Do^{-3/5}$	resolution
(a)	0.01	15.85	$64^2 \times 32^2$
(b)	2×10^{-3}	41.63	$128^2 \times 48^2$
(c)	8×10^{-4}	72.13	$256^2 \times 72^2$

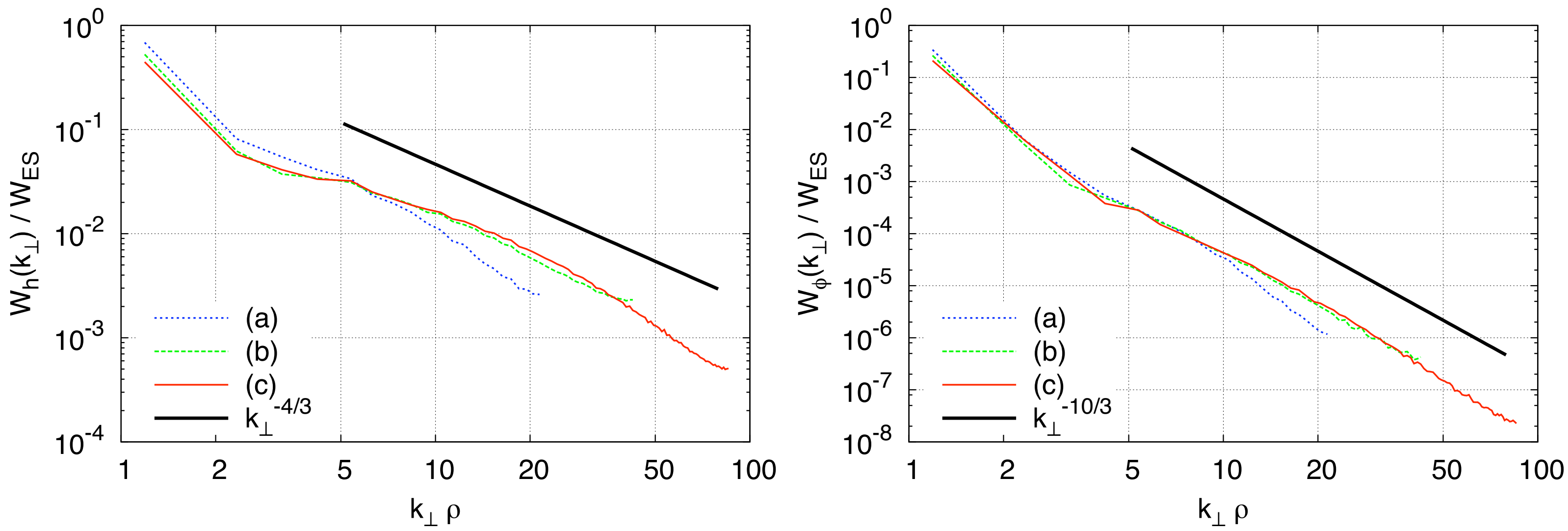
time evolution

Results from case (c): $256^2 \times 72^2$



averaged spectra

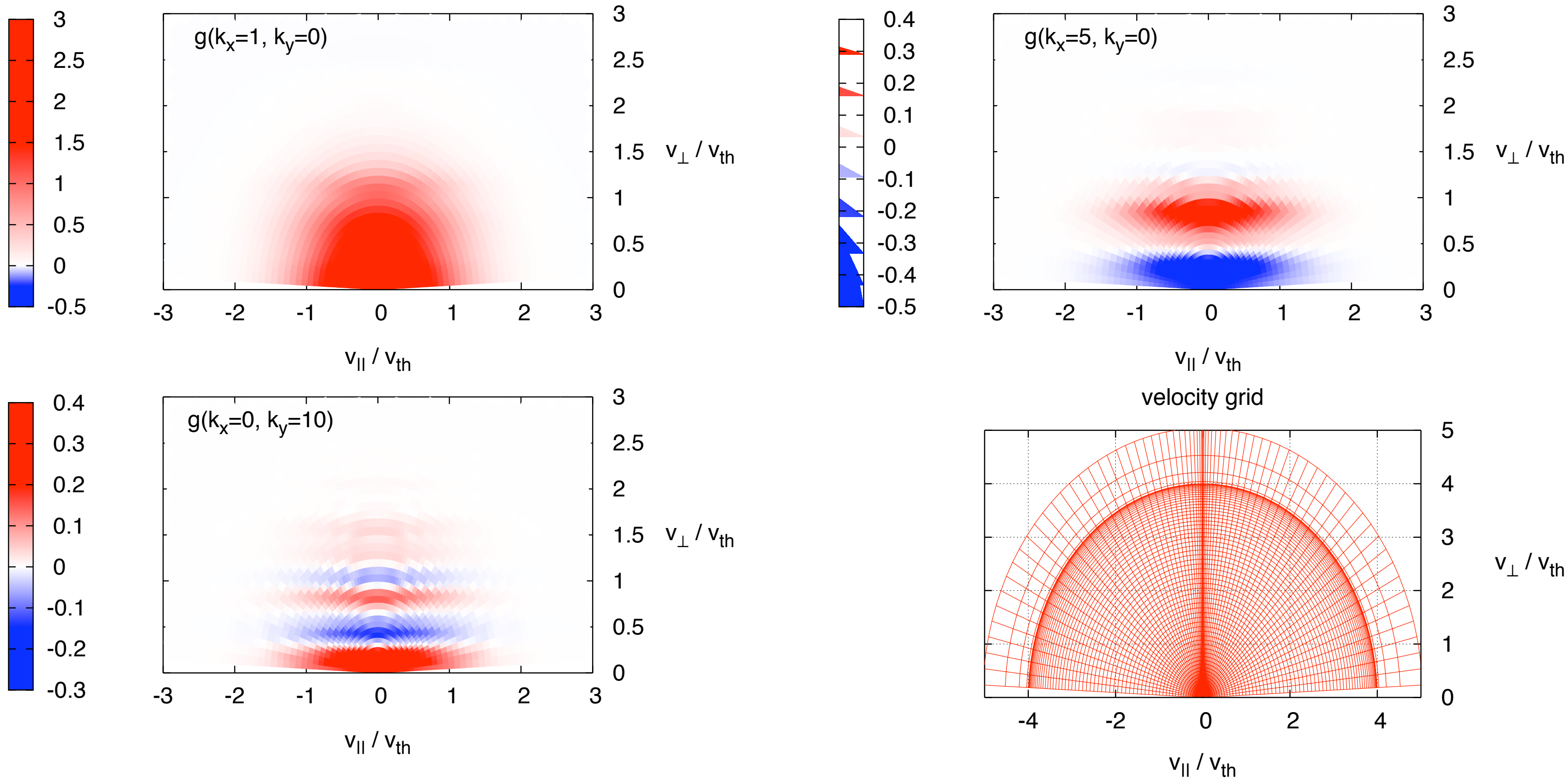
Wave number spectra averaged over $10 \leq t \leq 14$.



- potential spectra agrees perfect with theory
- dist func has steepening in high $k_\perp$ regime
  $\rightarrow$ probably from dissipation through velocity space?

velocity space structure

Snap shots @ $t = 10$



- Smaller structure for larger $k_\perp$

velocity space spectra — preliminary —

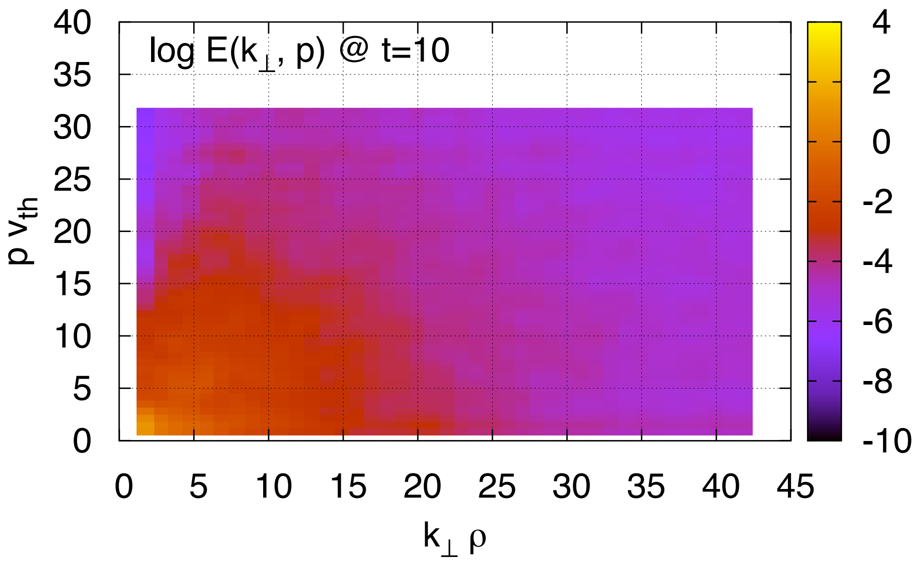
Hankel transform

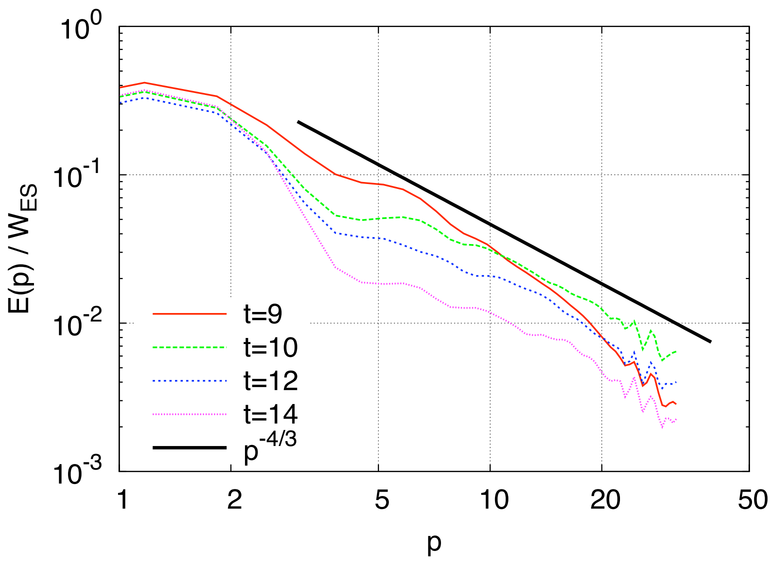
$$g_{\boldsymbol{k}}(p) = \int J_0(pv_\perp) g_{\boldsymbol{k}}(\boldsymbol{v})\, \mathrm{d}\boldsymbol{v}$$

Energy spectra

$$E_{\boldsymbol{k}}(p) = p\overline{|g_{\boldsymbol{k}}(p)|^2}$$

Data taken from case (b):





See Gabe Plunk's presentation.

summary

We have made 4D simulations of decaying entropy cascade

- first step to the understanding of turbulent dissipation
- energy spectra agree with theoretical prediction

$$W_h \sim k_\perp^{-4/3}, \qquad W_\phi \sim k_\perp^{-10/3}.$$

- observed nonlinear phase mixing
  - perpendicular phase mixing

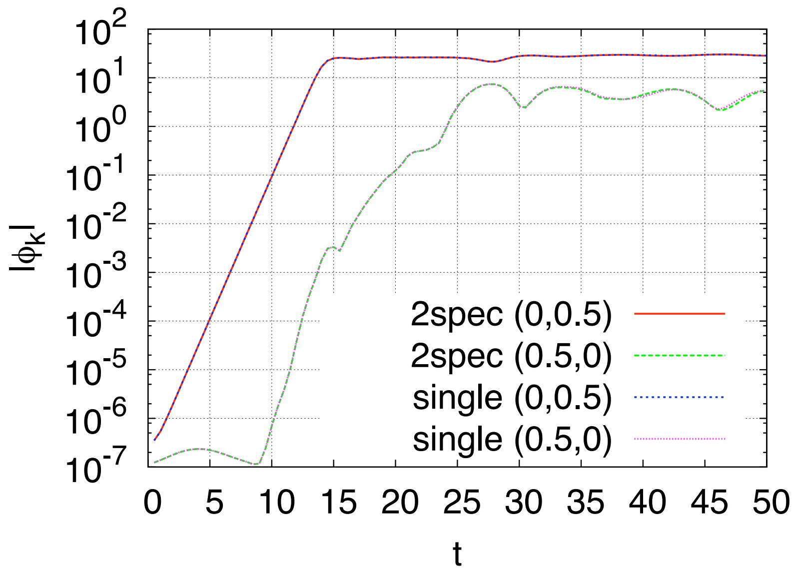
Future plan

- velocity space spectra
- effect of ITG/ETG — inverse cascade
- driven & 5D simulations

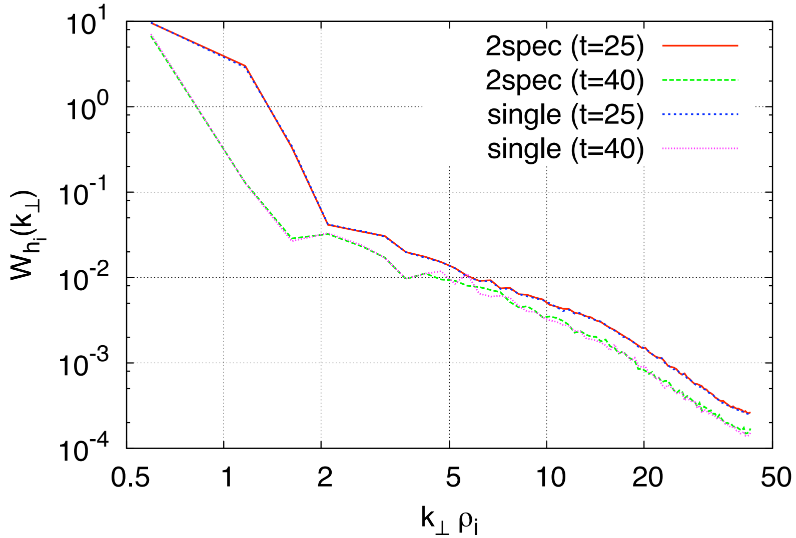
two-species runs

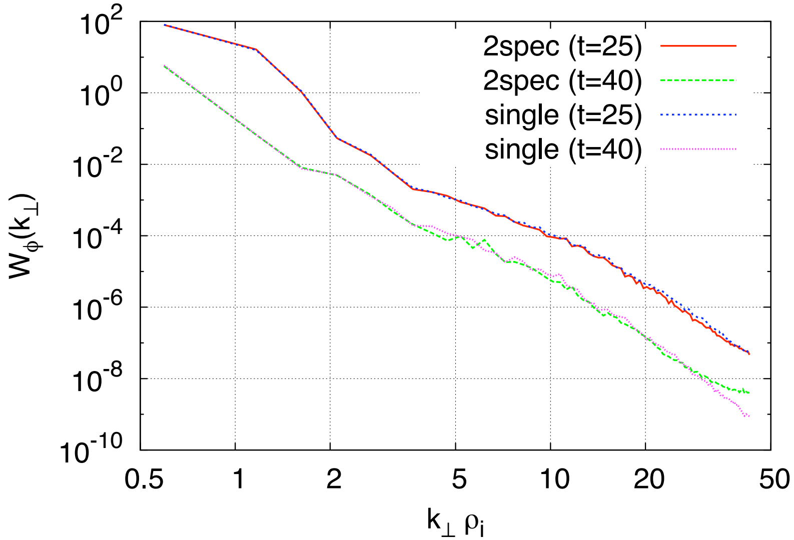
Parameters & time evolution

$$\frac{m_i}{m_e} \simeq 1836,$$
$$\delta n_{e,\mathrm{init}} = 0.$$



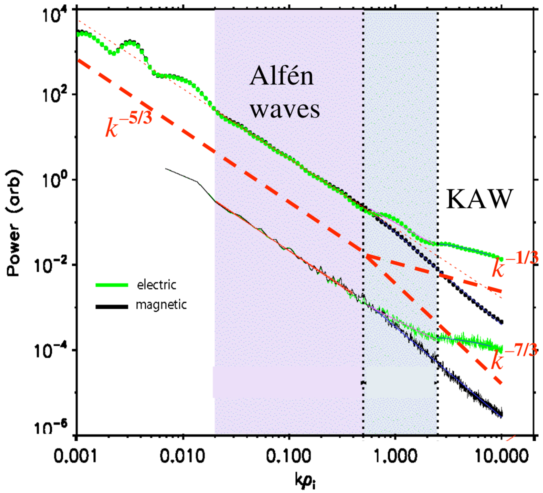
Snapshots of spectra



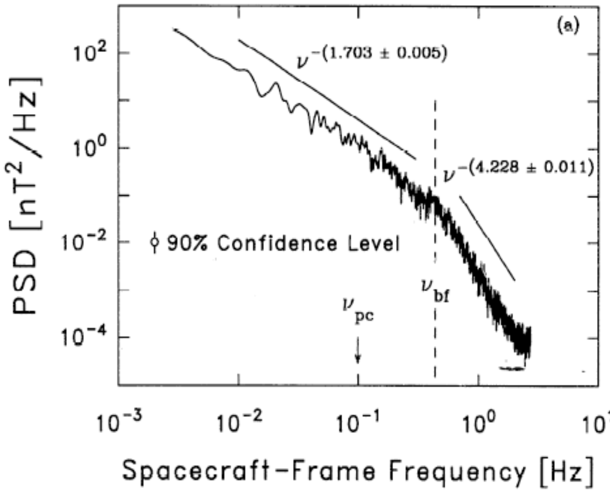


solar wind observations

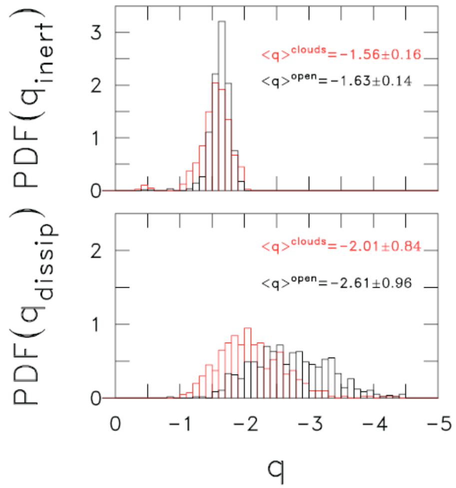
Dissipation range spectra varies on observations.



Bale *et al.*[a]



Leamon *et al.*[b]



Smith *et al.*[c]

- with or without KAW cascade?

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[a] Bale *et al.*, Phys. Rev. Lett. **94**, 215002 (2005).

[b] Leamon *et al.*, J. Geophys. Res. **103**, 4775 (1998).

[c] Smith *et al.*, Astrophys. J. **645**, L85 (2006).