Collisional Effects on Zonal Flows and Turbulent Transport

Gyrokinetic simulation of ITG turbulence

[Lin, Hahm, *et al.,* Phys.Plasmas (2000)]

Collisional damping of zonal flows

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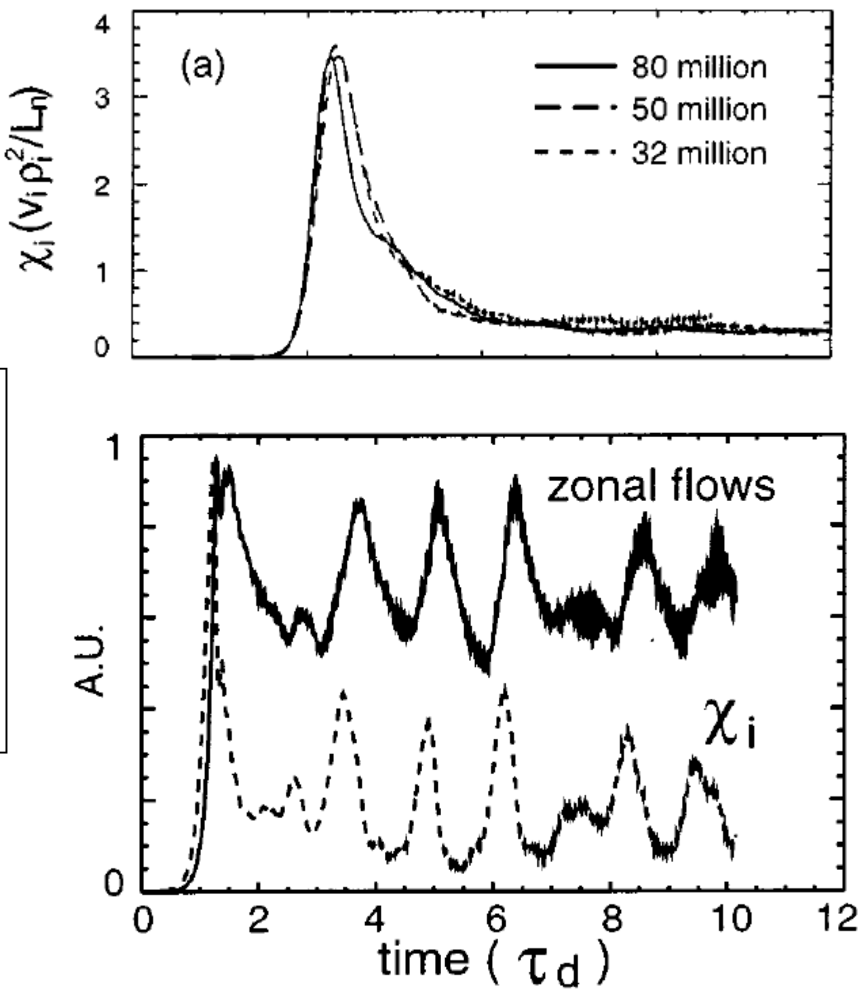
Enhancement of turbulent transport

without collisions



with collisions

Collision Operators (Linearized)

Ion collision $C_i = C_{ii}(\delta f_i) + C_{ie}(\delta f_i, \delta f_e)$

Electron collision $C_e = C_{ee}(\delta f_e) + C_{ei}(\delta f_e, \delta f_i)$

Conservation of particle numbers, momentum, and kinetic energy

$$\int d^3v\, C_{ab} = 0$$

$$\int d^3v\, m_a \mathbf{v} C_{ab} + \int d^3v\, m_b \mathbf{v} C_{ba} = 0$$

$$\int d^3v\, \frac{1}{2} m_a v^2 C_{ab} + \int d^3v\, \frac{1}{2} m_b v^2 C_{ba} = 0$$

Self-adjointness

$$\int d^3v \frac{\delta g_a}{F_{aM}} C_{aa}(\delta f_a) = \int d^3v \frac{\delta f_a}{F_{aM}} C_{aa}(\delta g_a) \quad (a = i, e)$$

$$T_i \int d^3v \frac{\delta g_i}{F_{iM}} C_{ie}(\delta f_i, \delta f_e) + T_e \int d^3v \frac{\delta g_e}{F_{eM}} C_{ei}(\delta f_e, \delta f_i)$$
$$= T_i \int d^3v \frac{\delta f_i}{F_{iM}} C_{ie}(\delta g_i, \delta g_e) + T_e \int d^3v \frac{\delta f_e}{F_{eM}} C_{ei}(\delta g_e, \delta g_i)$$

Collision Model

Electron-ion collision

$$C_{ei}(\delta f_e, \delta f_i) = \nu_D^e(v)\left(\frac{\partial}{\partial \mathbf{v}} \cdot \left[\left(v^2\mathbf{I} - \mathbf{v}\mathbf{v}\right) \cdot \frac{\partial \delta f_e}{\partial \mathbf{v}}\right] + \frac{m_e}{T_e}\mathbf{u}_i[\delta f_i] \cdot \mathbf{v}\, F_{eM}\right)$$

Ion flow velocity $\mathbf{u}_i[\delta f_i] = \int d^3v\, \delta f_i\, \mathbf{v}$

Ion-electron collision $C_{ie} \sim (m_e / m_i)^{1/2} C_{ii} << C_{ii}$

$$C_{ie}(\delta f_i, \delta f_e) = -\frac{\mathbf{v} \cdot \mathbf{F}_{ei}}{n_i T_i} F_{iM}$$

momentum-transfer part retained

Friction force $\mathbf{F}_{ei} = \int d^3v\, m_e \mathbf{v}\, C_{ei}(\delta f_e, \delta f_i) = -\int d^3v\, \delta f_e \nu_D^{ei}(v) m_e \mathbf{v} + \frac{m_e n_e}{\tau_{ei}} \mathbf{u}_i[\delta f_i]$

Like-particle collision $C_{aa}(\delta f_a) = C_{aa}^T(\delta f_a) + C_{aa}^F(\delta f_a) \qquad (a = e, i)$

Test-particle part **Field-particle part**

Like-particle collision $$C_{aa}(\delta f_a) = C_{aa}^T(\delta f_a) + C_{aa}^F(\delta f_a) \qquad (a = e, i)$$

Test-particle part **Field-particle part**

Test-particle part

$$C_{aa}^T(\delta f_a) = \frac{\nu_D^{aa}(v)}{2} \frac{\partial}{\partial \mathbf{v}} \cdot \left[\left(v^2 \mathbf{I} - \mathbf{v}\mathbf{v} \right) \cdot \frac{\partial \delta f_a}{\partial \mathbf{v}} \right] + \frac{1}{2} \frac{\partial}{\partial \mathbf{v}} \cdot \left(\nu_{\parallel}^{aa}(v) \mathbf{v}\mathbf{v} \cdot \frac{\partial \delta f_a}{\partial \mathbf{v}} \right)$$

Field-particle part

$$C_{aa}^F(\delta f_a) = F_{aM} \left[W_1(v)\, \mathbf{v} \cdot \frac{\int d^3v\, \mathbf{v} W_1(v) \delta f_a}{\int d^3v\, (v^2/3) W_1(v) F_{aM}} + W_2(v) v^2 \frac{\int d^3v\, v^2 W_2(v) \delta f_a}{\int d^3v\, v^4 W_2(v) F_{aM}} \right]$$

where

$$W_1(v) = \frac{m v^2}{T} \nu_{\parallel}(v) \qquad W_1(v) = 2\nu_D(v) - \left(2 m v^2 / T - 1 \right) \nu_{\parallel}(v)$$

$$\nu_D(v) = \frac{3\sqrt{\pi}}{4} \tau_{aa}^{-1} x^{-3} \left(\frac{(2x^2 - 1)\Phi(x) + x\Phi'(x)}{2x^2} \right)$$

$$\nu_{\parallel}(v) = \frac{3\sqrt{\pi}}{4} \tau_{aa}^{-1} x^{-3} \left(\frac{\Phi(x) - x\Phi'(x)}{x^2} \right) \qquad x^2 \equiv \frac{m v^2}{2T}$$

$$\Phi(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt$$ **Error function**

Collision Term in Gyrokinetic Equation

WKB (or Eikonal) representation $\phi(\mathbf{x}) = \sum \phi(\mathbf{k}_\perp)\exp[iS(\mathbf{x})]$ $\quad \delta f_a(\mathbf{x}) = \sum \delta f_a(\mathbf{k}_\perp)\exp[iS(\mathbf{x})]$

Perpendicular wave number vector $\mathbf{k}_\perp = \nabla S$

Perturbed particle distribution function

$$\delta f_a(\mathbf{k}_\perp) = -\frac{e_a}{T_a}\phi(\mathbf{k}_\perp)F_{aM} + h_a(\mathbf{k}_\perp)\exp(-i\mathbf{k}_\perp\cdot\rho_a)$$

Adiabatic part **Nonadiabatic part**

Gyrokinetic equation

$$\left(\frac{\partial}{\partial t} + i\mathbf{k}_\perp\cdot\mathbf{v}_D + v_\parallel \mathbf{b}\cdot\nabla_\parallel\right)h(\mathbf{k}_\perp) - \left\langle e^{i\mathbf{k}_\perp\cdot\rho}C\left[h(\mathbf{k}_\perp)e^{-i\mathbf{k}_\perp\cdot\rho}\right]\right\rangle_\varphi$$

$$= \frac{e}{T}F_M\left(\frac{\partial}{\partial t} + i\omega_*^T + v_\parallel \mathbf{b}\cdot\nabla_\parallel\right)\psi(\mathbf{k}_\perp) + \frac{c}{B}\sum_{k'+k''=k}\left[\mathbf{b}\cdot\left(\mathbf{k}'_\perp\times\mathbf{k}''_\perp\right)\right]\psi(\mathbf{k}'_\perp)h(\mathbf{k}''_\perp)$$

Gyrophase-averaged collision term $\left\langle e^{i\mathbf{k}_\perp\cdot\rho}C\left[h(\mathbf{k}_\perp)e^{-i\mathbf{k}_\perp\cdot\rho}\right]\right\rangle_\varphi$

$$\left\langle e^{i\mathbf{k}_\perp\cdot\rho}C_{aa}\left[h(\mathbf{k}_\perp)e^{-i\mathbf{k}_\perp\cdot\rho}\right]\right\rangle_\varphi$$

$$= \frac{\nu_D^{aa}(v)}{2}\frac{\partial}{\partial\mathbf{v}}\cdot\left[\left(v^2\mathbf{I}-\mathbf{vv}\right)\cdot\frac{\partial h(\mathbf{k}_\perp)}{\partial\mathbf{v}}\right] + \frac{1}{2}\frac{\partial}{\partial\mathbf{v}}\cdot\left(\nu_\parallel^{aa}(v)\mathbf{vv}\cdot\frac{\partial h(\mathbf{k}_\perp)}{\partial\mathbf{v}}\right) - \frac{\nu_D^{aa}(v)}{2}\frac{k_\perp^2}{\Omega^2}\left(v_\parallel^2+\frac{v_\perp^2}{2}\right)h(\mathbf{k}_\perp) - \frac{\nu_\parallel^{aa}(v)}{4}\frac{k_\perp^2 v_\perp^2}{\Omega^2}h(\mathbf{k}_\perp)$$

$$+ F_{aM}\left[J_0 v_\parallel W_1(v)\frac{\int d^3v\, J_0 v_\parallel W_1(v)h(\mathbf{k}_\perp)}{\int d^3v\,(v^2/3)W_1(v)F_{aM}} + J_1 v_\perp W_1(v)\frac{\int d^3v\, J_0 v_\perp W_1(v)h(\mathbf{k}_\perp)}{\int d^3v\,(v^2/3)W_1(v)F_{aM}} + J_0 v^2 W_2(v)\frac{\int d^3v\, J_0 v^2 W_2(v)h(\mathbf{k}_\perp)}{\int d^3v\, v^4 W_2(v)F_{aM}}\right]$$

$J_n = J_n(k_\perp\rho)$ $\quad (n = 0,1)$ **Bessel functions**

Collisional gyrokinetic equation for zonal-flow components

Gyrokinetic equation for zonal-flow components with wave number vector $\mathbf{k}_\perp = k_r \nabla r$

$$\frac{\partial \eta}{\partial t} - \overline{e^{iQ}\left\langle e^{i\mathbf{k}_\perp\cdot\boldsymbol{\rho}} C\left(\eta e^{-i\mathbf{k}_\perp\cdot\boldsymbol{\rho}} e^{-iQ}\right)\right\rangle_\varphi} = F_M \overline{\left[e^{iQ}\left(J_0 \frac{e}{T}\frac{\partial \phi}{\partial t} + S\right)\right]}$$

where $h(\mathbf{k}_\perp) = \eta e^{-iQ}, \quad Q = K v_\parallel / B, \quad K = k_\perp B_t / \Omega_p$ **S : nonlinear source term**

Laplace transform in *t* $\phi(p) = \int_0^\infty dt\, e^{-pt}\, \phi(t), \quad \eta(p) = \int_0^\infty dt\, e^{-pt}\, \eta(t)$

$$p\eta(p) - \overline{e^{iQ}\left\langle e^{i\mathbf{k}_\perp\cdot\boldsymbol{\rho}} C\left(\eta(p) e^{-i\mathbf{k}_\perp\cdot\boldsymbol{\rho}} e^{-iQ}\right)\right\rangle_\varphi} = p F_M \frac{e}{T}\overline{\left(e^{iQ} J_0\, \phi(p)\right)} + \overline{\left[e^{iQ}\left\{f^{(g)}(t=0) + F_M\, S(p)\right\}\right]}$$

Solution $\eta(p) = \dfrac{e\phi(p)}{T} G(p) + \dfrac{H(p)}{p}$

where *G*(*p*) and *H*(*p*) are defined by

$$G(p) - \frac{1}{p}\overline{e^{iQ}\left\langle e^{i\mathbf{k}_\perp\cdot\boldsymbol{\rho}} C\left(G(p) e^{-i\mathbf{k}_\perp\cdot\boldsymbol{\rho}} e^{-iQ}\right)\right\rangle_\varphi} = F_M \frac{e}{T}\overline{\left(e^{iQ} J_0\right)}$$

$$H(p) - \frac{1}{p}\overline{e^{iQ}\left\langle e^{i\mathbf{k}_\perp\cdot\boldsymbol{\rho}} C\left(H(p) e^{-i\mathbf{k}_\perp\cdot\boldsymbol{\rho}} e^{-iQ}\right)\right\rangle_\varphi} = \overline{\left[e^{iQ}\left\{f^{(g)}(t=0) + F_M\, S(p)\right\}\right]}$$

Zonal-Flow Potential

Poisson's equation

$$k_\perp^2 \phi = 4\pi e(\delta n_i - \delta n_e)$$

Density perturbation

$$\delta n = \int d^3v \left(\frac{e}{T} \phi F_M + h e^{-i\mathbf{k}_\perp \cdot \rho} \right)$$

Zonal-flow potential

$$\frac{e}{T_e} \phi_{\mathbf{k}_\perp}(p) = \frac{1}{p} \frac{\left[\sigma_{i\mathbf{k}_\perp}(p) - \sigma_{e\mathbf{k}_\perp}(p) \right]}{\left[(T_e/T_i) \chi_{i\mathbf{k}_\perp}(p) + \chi_{e\mathbf{k}_\perp}(p) + \left\langle k_\perp^2 \lambda_{De}^2 \right\rangle \right]}$$

where

$$\chi_{\mathbf{k}_\perp}(p) = 1 - \frac{1}{n_0} \left\langle \int d^3v \, J_0 \, e^{-iQ} G(p) \right\rangle$$

Shielding due to classical and neoclassical polarization

$$\sigma_{\mathbf{k}_\perp}(p) = \frac{1}{n_0} \left\langle \int d^3v \frac{1}{F_M} \left[G(p^*) \right]^* e^{iQ} \left\{ f_{\mathbf{k}_\perp}^{(g)}(t=0) + F_M \, S_{\mathbf{k}_\perp}(p) \right\} \right\rangle$$

Initial condition **Nonlinear source**

Inverse Laplace transform

$$\phi_{\mathbf{k}_\perp}(t) = \frac{1}{2\pi i} \int_\Gamma dp \, e^{pt} \phi_{\mathbf{k}_\perp}(p)$$

ITG-Mode Driven Zonal Flows

Wave number region $k_\perp a_i < 1$ $\qquad a_i \equiv (T_i / m_i)^{1/2} / \Omega_i$

Zonal-flow potential (GAM not included)

$$\frac{e}{T_i}\phi_{\mathbf{k}_\perp}(p) = \frac{\sigma_{i\mathbf{k}_\perp}(p)}{p\chi_{i\mathbf{k}_\perp}(p)}$$

Zonal-flow potential (part of contributions from initial conditions)

$$\frac{e\phi_{\mathbf{k}_\perp}(t)}{T_i} = \frac{\mathrm{K}(t)}{\langle k_\perp^2 a_i^2 \rangle}\left[\langle k_\perp^2 a_i^2 \rangle \frac{e\phi_{\mathbf{k}_\perp}(0)}{T_i} + iK_i \left\langle \frac{u_{i\|\mathbf{k}_\perp}(0)}{B} \right\rangle\right] + \frac{\mathrm{K}_0(t)}{\langle k_\perp^2 a_i^2 \rangle}\left[-iK_i \langle B^{-2} \rangle \langle u_{i\|\mathbf{k}_\perp}(0) B \rangle\right]$$

Ion poloidal zonal flow

$$\frac{u_{p\mathbf{k}}(t)}{B_p} = \mathrm{K}_0(t)\left[\left(1 + \frac{q^2}{\varepsilon^2}\right)\frac{\langle B^2 u_{p\mathbf{k}}(0)/B_p \rangle}{B_0^2} - \frac{q^2}{\varepsilon^2}\left\langle \frac{u_{p\mathbf{k}}(0)}{B_p} \right\rangle\right]$$

$$\mathrm{K}_0(t) = \frac{F(t)}{1 + 1.6 q^2/\varepsilon^{1/2}} \qquad \mathrm{K}(t) = \frac{\varepsilon^2}{q^2} + \mathrm{K}_0(t)$$

$$F(t) = \frac{2}{\sqrt{\pi}}\exp(t/\tau)\,\mathrm{Erfc}\left((t/\tau)^{1/2}\right) = 1 - \frac{2}{\sqrt{\pi}}(t/\tau)^{1/2} + (t/\tau) - \cdots$$

$$\tau = \left(1.6 + \frac{\varepsilon^{1/2}}{q^2}\right)^2 \frac{\left(\overline{\Lambda}\right)^3}{36\pi^2}\frac{\varepsilon}{\overline{\nu}_{Di}} \sim \varepsilon\tau_{ii}$$

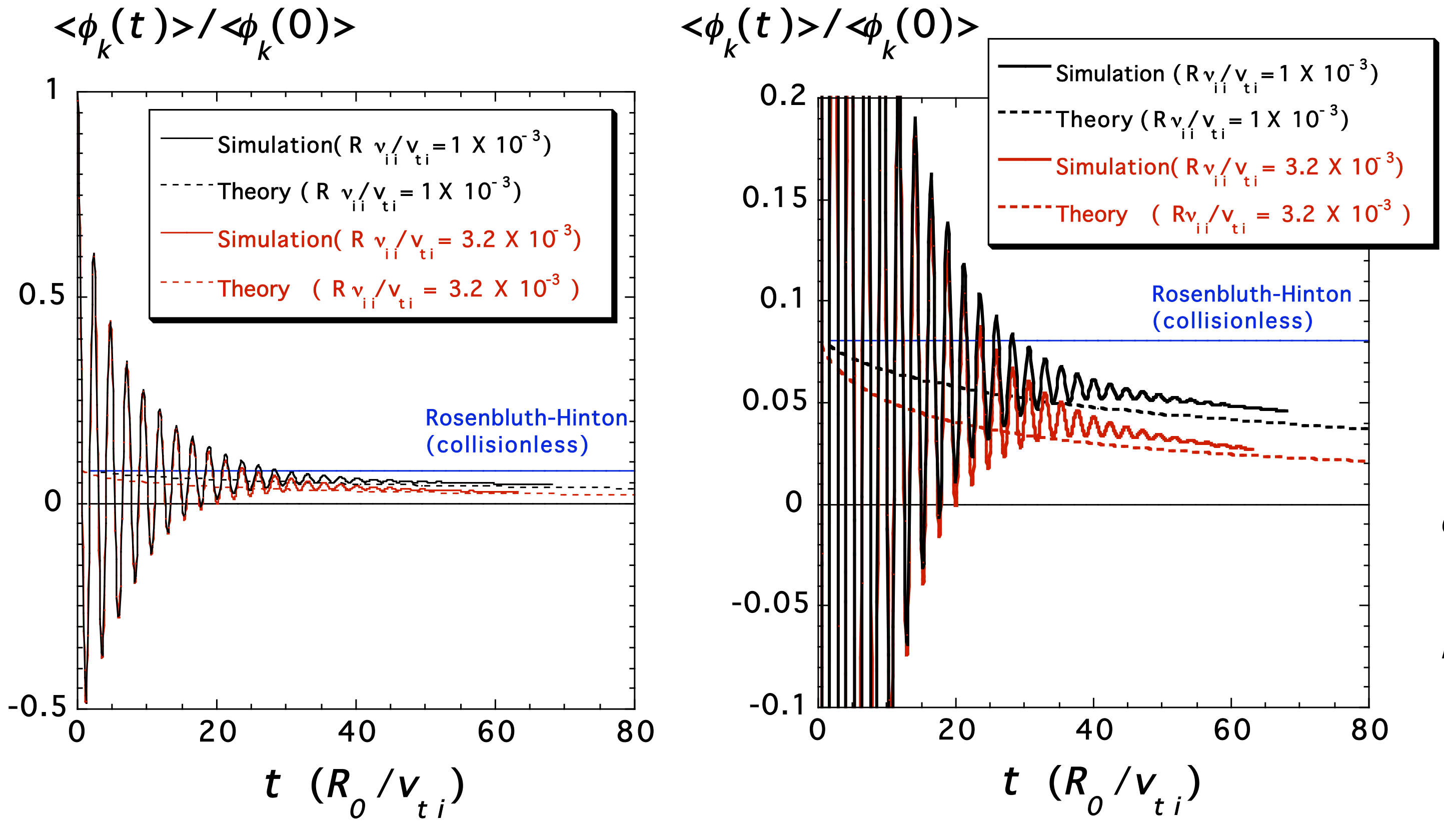
Collisional poloidal-flow decay time

Results from Gyrokinetic Vlasov Simulation

Gyrokinetic Vlasov Simulation using the test-particle collision model

$$C_{ii}^{T}(\delta f) = \frac{1}{2}\frac{\partial}{\partial \mathbf{v}} \cdot \left[\nu_D(v) v^2 F_M \frac{\partial}{\partial \mathbf{v}} \left(\frac{\delta f}{F_M} \right) \right]$$

Time evolution of ITG-mode-driven zonal flows in a tokamak



$q = 1.5$

$\varepsilon = 0.1$

$k_r \rho_{ti} = 0.13$