# Electron inertia effects in 2D driven reconnection in electron MHD

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Kinetic Equations, Numerical Approaches
and Fluid Models for Plasma Turbulence

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# Outline

1. Description of the reconnection region
   - Basic equations in EMHD
2. Steady state properties and linear stability
   - Fixed Points
   - Electron mass-less limit
   - Linear Stability
   - Finite Electron Inertia
3. Reconnection rates ($d_e > 0$)
   - Hyper-resistive regime ($\eta = 0$, $d_e > 0$)
   - Resistive regime ($\eta_H = 0$, $d_e > 0$)

# Abstract

We propose a reduced zero-dimensional dynamical model which builds upon Refs. [1,2], and describes the effects of electron inertia in electron MHD (EMHD) reconnection with resistive and viscous dissipation. Steady-state properties of the reconnection region are examined. In the resistivity-dominated regime, we find that the current sheet thickness is limited to the inertial length scale and no thinner structures can develop. Reconnection is slow and outflows are Alfvénic. A linear stability analysis suggests that elongated diffusion regions are preferred. On the other hand, in the viscosity-dominated regime, we find that linearly stable viscous layers, thinner than the electron inertial scale length, can develop to sustain reconnection. In this regime, the maximum reconnection rate is formally independent of dissipation and therefore potentially fast.

1. L. Chacón, A. N. Simakov, and A. Zocco *Phys. Rev. Lett.* **99** 235001 (20087)

2. L. Chacón, A. N. Simakov, V. S. Lukin, and A. Zocco *Phys. Rev. Lett.* **101** 025003 (2008)

# Motivations

- To develop a unified analytical model for the physics of the reconnection region ranging from resistive MHD to Hall regimes
- To understand intrinsic limit of reconnection rates in all regimes of interest (2D, two fluids)

Some basic questions

- Fast reconnection $\Rightarrow$ Dispersive waves?
- Fast reconection $\Rightarrow$ Dissipation independent?

**Approach**

To write a non linear dynamical system for key quantities defining the reconnection region

From continuum equations (PDE)$\Rightarrow$ To ODE equations for discretized quantities

# EMHD continuum equations.

Two D ($\partial_z \equiv 0$) incompressible two fluid plasma with an ion neutralizing background ($\mathbf{v_i} \approx 0$)

Component-form

$$\begin{aligned}
\frac{\partial B_x^*}{\partial t} - \nabla \cdot \left(\mathbf{j}_p B_x^* - \mathbf{B}_p^* j_x\right) &= -\eta \left(\frac{\partial^2 B_y}{\partial y \partial x} - \frac{\partial^2 B_x}{\partial y^2}\right) + \eta_H \Delta \left(\frac{\partial^2 B_y}{\partial y \partial x} - \frac{\partial^2 B_x}{\partial y^2}\right) \\
\frac{\partial B_y^*}{\partial t} - \nabla \cdot \left(\mathbf{j}_p B_y^* - \mathbf{B}_p^* j_y\right) &= -\eta \left(\frac{\partial^2 B_x}{\partial x \partial y} - \frac{\partial^2 B_y}{\partial x^2}\right) + \eta_H \Delta \left(\frac{\partial^2 B_x}{\partial x \partial y} - \frac{\partial^2 B_y}{\partial x^2}\right) \\
\frac{\partial B_z^*}{\partial t} + \mathbf{B}_p \cdot \nabla j_z &= -d_e^2 \left(\mathbf{j}_p \cdot \nabla\right) \Delta B_z + \eta \Delta B_z - \eta_H \Delta^2 B_z
\end{aligned}$$

$\partial_z \equiv 0$, $\mathbf{B}_p^* = (B_x^*, B_y^*)$, $\mathbf{j}_p = (j_x, j_y) = -\hat{\mathbf{z}} \times \nabla B_z = -(v_x, v_y)$

$B_x^* = B_x + d_e^2 \left(\frac{\partial^2 B_y}{\partial y \partial x} - \frac{\partial^2 B_x}{\partial y^2}\right)$, $B_y^* = B_y + d_e^2 \left(\frac{\partial^2 B_x}{\partial x \partial y} - \frac{\partial^2 B_y}{\partial x^2}\right)$, $B_z^* = B_z - d_e^2 \Delta B_z$

$\eta$ Resistivity, $\eta_H$ Electron viscosity, $d_e$ Electron skin depth

## Properties

$$\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \mathscr{O} \underline{P}_z \begin{pmatrix} B_x\, eq. \\ B_y\, eq. \\ B_z\, eq. \end{pmatrix} = \begin{pmatrix} B_x\, eq. \\ B_y\, eq. \\ B_z\, eq. \end{pmatrix}$$

## Operators

- $\underline{P}_z : B_z \mapsto -B_z$,
- $\mathscr{O} : (\partial_x, \partial_y, B_x, B_y) \mapsto (\partial_y, \partial_x, B_y, B_x)$

# EMHD discrete equations.

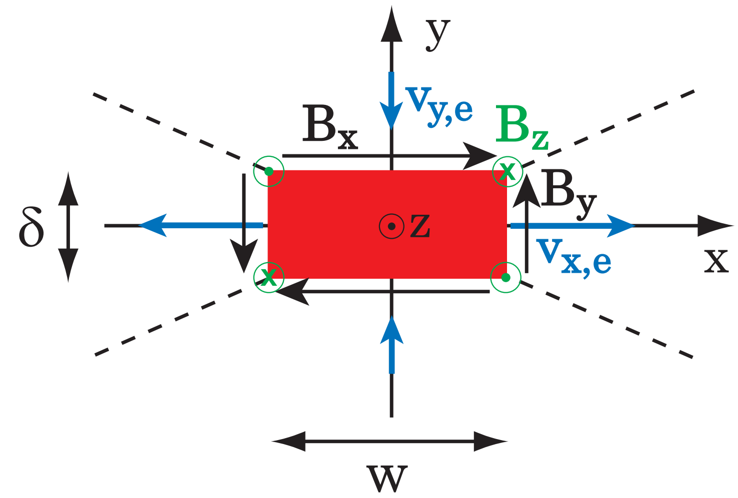
## Discrete EMHD Equations

$$\frac{dB_x^*}{dt} - B_x^* \frac{\dot\delta}{\delta} - \frac{B_z B_x^*}{\delta w} = \mathscr{D}\left(\frac{B_y}{\delta w} - \frac{B_x}{\delta^2}\right)$$

$$\frac{dB_y^*}{dt} - B_y^* \frac{\dot w}{w} + \frac{B_z B_y^*}{\delta w} = \mathscr{D}\left(\frac{B_x}{\delta w} - \frac{B_y}{w^2}\right)$$

$$\frac{dB_z^*}{dt} - B_z^* \left(\frac{\dot w}{w} + \frac{\dot\delta}{\delta}\right) + \left(\frac{B_x}{w} + \frac{B_y}{\delta}\right)\left(\frac{B_y}{w} - \frac{B_x}{\delta}\right) = -\mathscr{D}\left(\frac{1}{\delta^2} + \frac{1}{w^2}\right) B_z + d_e^2 \frac{B_z^2}{\delta w}\left(\frac{1}{w^2} - \frac{1}{\delta^2}\right)$$

$$B_x^* = B_x + d_e^2 (B_x/\delta^2 - B_y/\delta w)$$
$$B_y^* = B_y + d_e^2 (B_y/w^2 - B_x/\delta w)$$
$$B_z^* = B_z + d_e^2 (\delta^{-2} + w^{-2}) B_z$$
$$\mathscr{D} = \eta + \eta_H (\delta^{-2} + w^{-2})$$




$$B_x \equiv \hat{\mathbf{x}} \cdot \mathbf{B_p}(0, \delta/2)$$
$$B_y \equiv \hat{\mathbf{y}} \cdot \mathbf{B_p}(w/2, 0)$$
$$B_z \equiv -\hat{\mathbf{z}} \cdot \mathbf{B_z}(w/2, \delta/2)$$

Five Unknown-Three equations

$\Downarrow$

Coupling to the driver provides closure ( through $\dot B_x, \dot B_y$)

# Fixed Points ($\frac{d}{dt} \equiv 0$).

Equation for the current sheet aspect ratio $\xi = \frac{\delta}{w}$

$$\left\{ \frac{1+\hat{d}_e^2(1+\xi^2)}{1+2\hat{d}_e^2\xi^2} \right\}^2 = \frac{1}{S^2} \left\{ 1 + \frac{1}{\xi^2} + \frac{\hat{d}_e^2}{1+\hat{d}_e^2(1+\xi^2)} \left( \frac{\xi^2-1}{\xi} \right)^2 \right\}$$

$S^{-1} = S_\eta^{-1} + S_H^{-1}(\xi^{-2}+1)$, $\hat{d}_e = \frac{d_e}{\delta}$

$S_\eta = \sqrt{2}B_x/\eta$ Resistive Lundquist number

$S_H = \sqrt{2}B_x w^2/\eta_H$ Hyper-resistive Lundquist number

Centers in the parametric space $(\xi, \hat{d}_e)$

$$\delta_0 = \frac{d_e}{\hat{d}_e}, \; w_0 = \frac{d_e}{\hat{d}_e \xi}, \; \frac{B_z^0}{\sqrt{2}B_x} = S^{-1} \frac{\xi^{-1}-\xi}{1+\hat{d}_e^2(1+\xi^2)}$$

$B_x$,$d_e$define electron Alfvén speed $v_{A,e} = B_x/d_e$

# Fixed Points ($d_e \equiv 0$).

Equation for the current sheet aspect ratio $\xi = \frac{\delta}{w}$ [Chacón, Simakov, Zocco, PRL, 2008]

$$1 = \frac{1}{S^2}\left\{1 + \frac{1}{\xi^2}\right\}$$

$S^{-1} = S_\eta^{-1} + S_H^{-1}(\xi^{-2} + 1)$

$S_\eta = \sqrt{2}B_x/\eta \gg 1$ Resistive Lundquist number

$S_H = \sqrt{2}B_x w^2/\eta_H \gg 1$ Hyper-resistive Lundquist number

## $\eta \ll 1, \eta_H \equiv 0$

$\frac{B_y}{B_x} = \xi = (S_\eta^2 - 1)^{-1/2} \approx S_\eta^{-1} \ll 1$

$B_z \approx \sqrt{2}B_x$

$E_z \sim \frac{\eta}{\delta} B_x \Rightarrow E_z \sim \frac{B_x^2}{w} \sim \eta^0$

- Current sheets extremely elongated
- Unbounded electron outflows $v_{out} \sim B_z/\delta \sim \eta^{-1}$
- Consistent with $\omega_{whistler} = v_A d_i k k_\parallel \sim \gamma_\eta = \eta\, k^2$

## $\eta_H \ll 1, \eta \equiv 0$

$\frac{B_y}{B_x} = \xi = (S_H^{2/3} - 1)^{-1/2} \approx S_\eta^{-1/3}$

$B_z \approx \sqrt{2}B_x$

$E_z \sim \frac{\eta_H}{\delta^3} B_x \Rightarrow E_z \sim \frac{B_x^2}{w} \sim \eta_H^0$

- Both elongated and open X-point solutions are supported.
- Unbounded electron outflows $v_{out} \sim B_z/\delta \sim \eta_H^{-1/3}$
- Consistent with $\omega_{whistler} = v_A d_i k k_\parallel \sim \gamma_\eta = \eta_H k^4$

# Linear Stability

$\dot{B}_x, \dot{B}_y$ set the driver coupling $\Rightarrow \dot{B}_x = \dot{B}_y = 0$ "weak" driver (open boundary system)

$$\frac{d\hat{\chi}^i}{dt} = \mathscr{A}^i{}_j|_{(\delta_0, w_0, B_z^0)}\,\hat{\chi}^j + \mathscr{O}(\chi^2),\ \chi^2 \ll 1$$
$$\hat{\chi}^i = (\hat{\delta},\, \hat{w},\, \hat{B}_z)^T$$
$$\delta = \delta_0 + \hat{\delta} e^{\gamma t},\ w = w_0 + \hat{w} e^{\gamma t}$$
$$B_z = B_z^0 + \hat{B}_z e^{\gamma t}$$

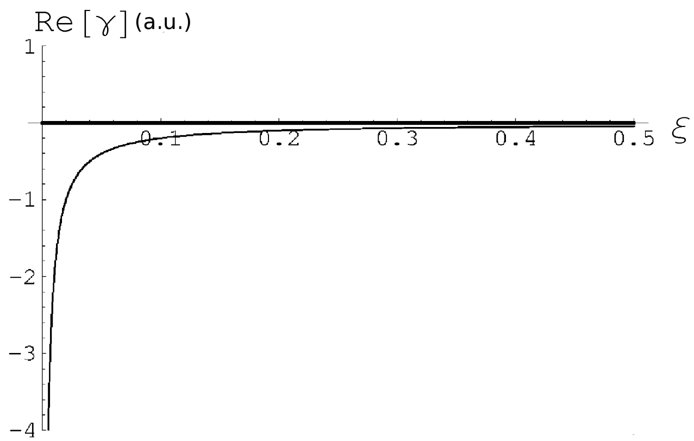
**Solve for**

$$det\, \mathscr{A}_{i,j}(\gamma, \xi, \hat{d}_e, B_x, d_e) = 0$$

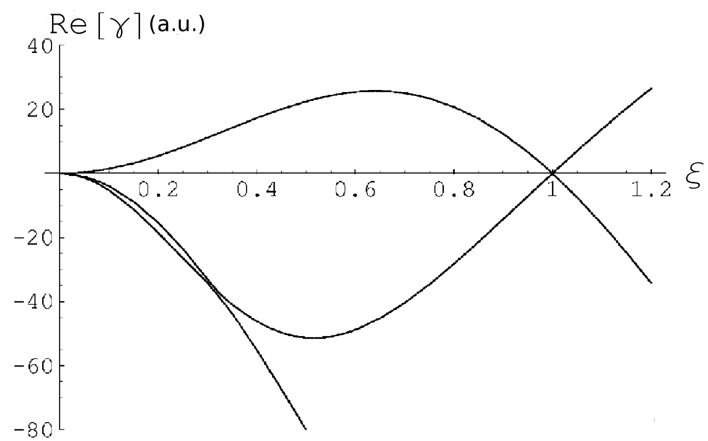
***Electron massless limit***

***Hyper-resistive***: elongated current sheets are preferred $\gamma \rightarrow 0 \Rightarrow \xi \rightarrow 0$

***Resistivie:*** $\xi < 1 \Rightarrow \gamma < 0$ marginal stability




Linear growth rate $(\eta \ll 1, \eta_H = 0)$

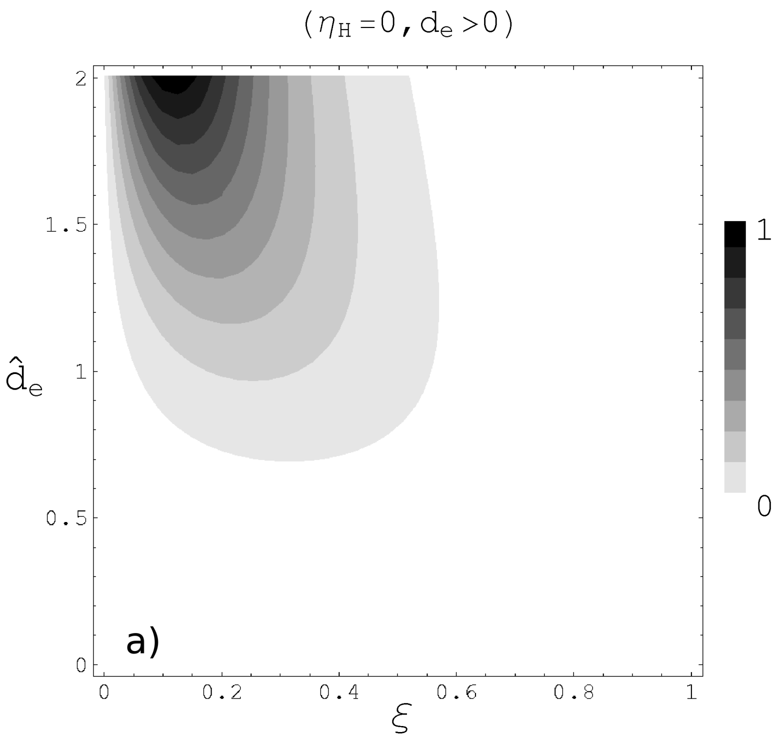



Linear growth rate $(\eta_H \ll 1, \eta = 0)$

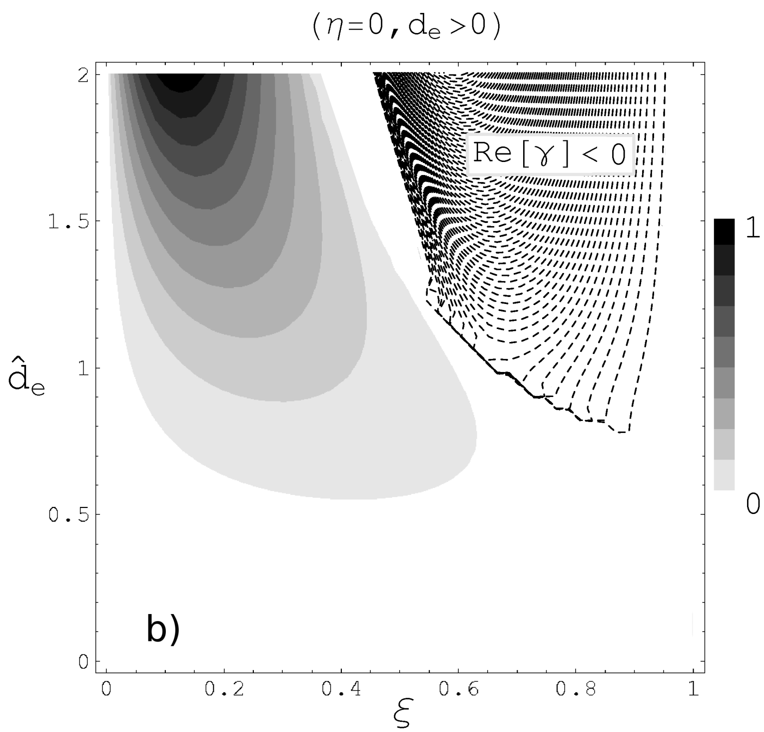
# Linear Stability ($d_e \neq 0$)

A new boundary of stability for $\hat{d}_e \gtrsim 1/\xi$ and $\xi < 1$

Elongated current sheets are still marginally stable $\gamma \to 0 \Rightarrow \xi \to 0$




Linear growth rate $(\xi, \hat{d}_e)$ space




Linear growth rate$(\xi, \hat{d}_e)$ space

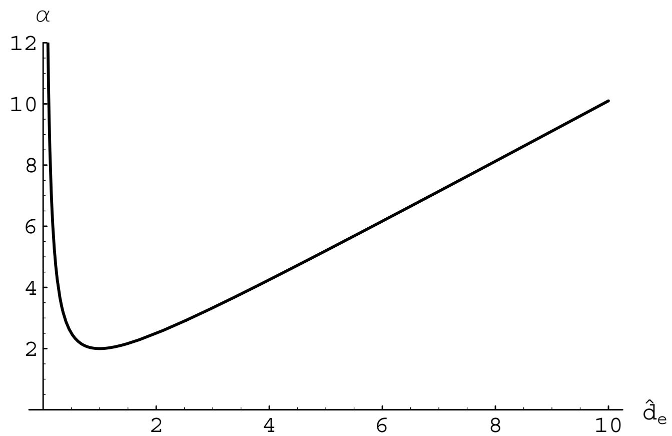
# Reconnection Rate ($d_e \neq 0$)

Electric field in the ignorable direction at the "$X$-point"

$$E_z|_X = \mathscr{D} j_z|_X \Rightarrow E_z \approx \sqrt{2}\frac{B_x^2}{w}\{1+\hat{d}_e^2\}$$

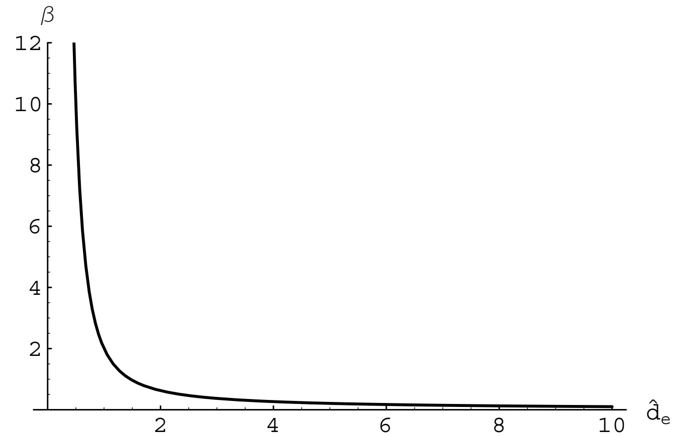
$j_z = B_x/\delta - B_y/w$ current density

$$\xi \approx S^{-1}\frac{1}{1+\hat{d}_e^2}$$ with inertial corrections (and $\xi^2 \ll 1$ )




- $\xi^2 \ll 1$ gives $\hat{d}_e^{-1} + \hat{d}_e = \left(\frac{w}{d_e}\right)\frac{\eta}{\sqrt{2}B_x} = \alpha$

threshold: $\alpha > 2$




- $\xi^2 \ll 1$ gives $\hat{d}_e^{-3} + \hat{d}_e^{-1} = \left(\frac{w}{d_e}\right)\frac{\eta_H}{\sqrt{2}B_x d_e^2} = \beta$

$\delta \lesssim d_e$ allowed

# Hyper-resistive regime ($\eta = 0$, $d_e > 0$)

$$\xi \approx S^{-1} \frac{1}{1+\hat{d}_e^2} \Rightarrow \frac{1}{\hat{d}_e^3} + \frac{1}{\hat{d}_e} \sim \left(\frac{w}{d_e}\right) \frac{\eta_H}{\sqrt{2} B_x d_e^2} \equiv \eta_H^*$$

**magnetized regime $\delta/d_e \gtrsim 1$**

$\delta/d_e \sim (\eta_H^*)^{1/3}$

**inertial regime $\delta/d_e \lesssim 1$**

$\delta/d_e \sim \eta_H^*$

$$j_z|_X \approx 2B_x/\delta \approx 2B_x^e/d_e \text{ ,where } B_x^e \equiv \hat{\mathbf{x}} \cdot \mathbf{B}(0, d_e/2)$$

Defining $B_{x,max} = max[B_x, B_x^e]$ the magnetic field at the upstream boundary of induced current $j_z$

***magnetized regime***

$$\delta \sim (\eta_H w/\sqrt{2} B_x)^{1/3}$$

***inertial regime***

$$\delta \sim \sqrt{\eta_H w/(B_x^e d_e)} < d_e$$

$$E_z^H \approx \sqrt{2} \frac{B_{x,max}^2}{w}$$

Viscous sub-$d_e$ layers can sustain dissipation-independent reconnection

# Resistive regime ($\eta_H = 0$, $d_e > 0$)

$$\xi \approx S^{-1}\frac{1}{1+\hat{d}_e^2} \Rightarrow \frac{1}{\hat{d}_e} + \hat{d}_e \sim \left(\frac{w}{d_e}\right)\frac{\eta}{\sqrt{2}B_x}$$

Threshold $\Rightarrow$No solutions for $\left(\frac{w}{d_e}\right)\frac{\eta}{\sqrt{2}B_x} \lesssim 2$

***magnetized regime*** $(\delta/d_e \gtrsim 1)$

$$\delta = \eta w/(\sqrt{2}B_x)$$
$$\Downarrow$$
$$\sqrt{2}B_x^2/w = \eta B_x/\delta \lesssim \eta B_x/d_e$$

***inertial regime*** $(\delta/d_e \lesssim 1)$

$$d_e = \eta w/(\sqrt{2}B_x^e)$$
$$\Downarrow$$
$$\sqrt{2}(B_x^e)^2/w = \eta B_x^e/d_e$$

$$E_z^\eta \lesssim \eta \frac{B_{x,max}}{d_e}$$

Resistive reconnection rate slower than Sweet-Parker

***Outflows***

Both regimes:$v_x \approx B_z/\delta \sim B_x/\delta \leq B_{x,max}/d_e \equiv V_{A,e}$

Bounded by the electron Alfvén speed

# Summary

- We included electron inertia effects in previous works on driven reconnection in EMHD.
- A zero dimensional reduced model which describes key quantities of the reconnection region is used
- In resistivity-dominated regimes reconnection rate drops down due to finite electron mass ($E_z \approx \frac{\eta}{d_e} B_x^{upstream}$).
- In viscosity-dominated regimes, viscous sub-layers develop and sustain fast reconnection ($E_z \approx B_x^{upstream}$)