Extension of gyrokinetics to transport time scales [in tokamaks]

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Introduction

- δf gyrokinetics has proven useful for calculating turbulence at $k_\perp \rho \sim 1$ and on saturation time scales

- Challenge: extending turbulence calculations to transport time scales (huge time scale separation)

- Special focus on axisymmetric E_r field
  - Important: $E \times B$ shear $\leftrightarrow$ turbulence decorrelation
  - Hard to compute due to axisymmetry

  $\Rightarrow$ undetermined toroidal rotation $\leftrightarrow$ undetermined E_r
  - Need toroidal angular momentum transport

Gyrokinetics

- Keep $k_\perp \rho \sim 1$
- Variables $\mathbf{R}$, E, μ and φ defined with $d\mathbf{R}/dt$, dE/dt, $d\mu/dt$ and $d\varphi/dt$ independent of φ

  $\Rightarrow$ fast gyromotion absorbed in GK variables
  - Here $d/dt \equiv$ Vlasov operator
- Need to find $f(\mathbf{r}, \mathbf{v}, t)$ from $f(\mathbf{R}, E, \mu, t)$
- Gyrokinetic variables are not constants of the motion! Gyrokinetics is an asymptotic expansion!
- Simplification: electrostatic gyrokinetics

  B slowly varying and time independent

Orderings

- Small parameter $\delta = \frac{\rho}{L} \sim \frac{\omega}{\Omega} \sim \frac{\nu}{\Omega} << 1$
- f and ϕ have $k_\perp \rho \sim 1$ but $k_{||} L \sim 1$
- For $k_\perp L \sim 1$, $e\phi/T \sim 1$ and $f \approx f_M \equiv$ Maxwellian
- For $k_\perp \rho \sim 1$, $e\phi_k/T \sim f_k / f_M \sim \delta$
- For general $k_\perp$, $\frac{e\phi_k}{T} \sim \frac{f_k}{f_M} \sim \frac{1}{k_\perp L}$
  - $\nabla \phi_k \sim T/eL$ and $\nabla f_k \sim \nabla f_M$
  - Drift ordering $v_{ExB} \sim \delta v_i << v_i$

Gyrokinetic variable $\mathbf{R}$

- Define $\mathbf{R}$ such that $d\mathbf{R}/dt = \langle d\mathbf{R}/dt \rangle$ + negligible where $d/dt \equiv \partial/\partial t + \mathbf{v}\cdot\nabla - Ze/M\,\nabla\phi\cdot\nabla_v - \Omega\partial/\partial\varphi_0$
  - Here $\langle \ldots \rangle \equiv$ gyroaverage holding $\mathbf{R}$, E, μ, t fixed
- $\mathbf{R} = \mathbf{r} + \mathbf{R}_1 + \mathbf{R}_2$, $\mathbf{R}_1 = O(\delta L)$ and $\mathbf{R}_2 = O(\delta^2 L)$
  - To first order $\dot{\mathbf{R}} \cong \dot{\mathbf{r}} + \dot{\mathbf{R}}_1 \cong \mathbf{v} - \Omega\,\partial\mathbf{R}_1/\partial\varphi_0$
- Imposing $d\mathbf{R}/dt = \langle d\mathbf{R}/dt \rangle$ to first order,

$$\dot{\mathbf{R}} \cong \dot{\mathbf{r}} - \Omega\partial\mathbf{R}_1/\partial\varphi_0 = \left\langle \dot{\mathbf{R}} \right\rangle \cong \left\langle \dot{\mathbf{r}} \right\rangle$$

- Then, $\mathbf{R}_1 = \Omega^{-1}\int d\varphi_0\left(\dot{\mathbf{r}} - \left\langle \dot{\mathbf{r}} \right\rangle\right) = \Omega^{-1}\mathbf{v}\times\hat{\mathbf{b}}$
- Similarly, $\mathbf{R}_2 = \Omega^{-1}\int d\varphi_0\left(\dot{\mathbf{r}} + \dot{\mathbf{R}}_1 - \left\langle \dot{\mathbf{r}} + \dot{\mathbf{R}}_1 \right\rangle\right) = \ldots$

Gyrokinetic variable $\mathbf{R}$

- Gyrocenter position $\mathbf{R}$ (using $\mathbf{R}_1$ and $\mathbf{R}_2$)

$$\frac{d\mathbf{R}}{dt} \cong \left\langle \frac{d\mathbf{R}}{dt} \right\rangle = u\hat{\mathbf{b}}(\mathbf{R}) + \mathbf{v}_d \quad (= \text{gyrocenter motion})$$

  - Parallel velocity defined by $u^2/2 + \mu B(\mathbf{R}) = E$
  - Drift velocity $\mathbf{v}_d = \mathbf{v}_M - \frac{c}{B} \nabla_{\mathbf{R}} \langle \phi \rangle \times \hat{\mathbf{b}}$

$$\mathbf{v}_M = \frac{\mu}{\Omega(\mathbf{R})} \hat{\mathbf{b}}(\mathbf{R}) \times \nabla_{\mathbf{R}} B(\mathbf{R}) + \frac{u^2}{\Omega(\mathbf{R})} \hat{\mathbf{b}}(\mathbf{R}) \times \boldsymbol{\kappa}(\mathbf{R})$$

with $\langle \phi \rangle = \frac{1}{2\pi} \oint d\varphi \, \phi(\mathbf{R} - \mathbf{R}_1(\varphi) - \mathbf{R}_2(\varphi), t)$

Gyrokinetic variables E, μ *and* φ

- Kinetic energy $E = \upsilon^2/2 + E_1 + E_2$,

$$\frac{dE}{dt} \cong \left\langle \frac{dE}{dt} \right\rangle = -\frac{Ze}{M}\left[u\hat{\mathbf{b}}(\mathbf{R}) + \mathbf{v}_M\right] \cdot \nabla_{\mathbf{R}} \langle \phi \rangle$$

where $E_1 = \dfrac{Ze\tilde{\phi}}{M}$, with $\tilde{\phi} = \phi - \langle \phi \rangle$

$$E_2 = \frac{c}{B}\frac{\partial \tilde{\Phi}}{\partial t}, \text{ with } \tilde{\Phi} = \int^{\varphi} d\varphi' \, \tilde{\phi}(\varphi')$$

- Magnetic moment $\mu = \upsilon_\perp^2/2B + \mu_1$ such that $\langle d\mu/dt \rangle = 0$ to requisite order
  - Only need μ_1, f is f_M to lowest order
- Gyrophase $\varphi = \varphi_0 + \varphi_1$, with $\mathbf{v}_\perp = \upsilon_\perp \left(\hat{\mathbf{e}}_1 \cos\varphi_0 + \hat{\mathbf{e}}_2 \sin\varphi_0 \right)$
  - Only need φ_1, gyrokinetics $\Rightarrow \varphi$ - dependence weak

Fokker-Planck equation

- Fokker-Planck equation

$$\frac{\partial f}{\partial t}+\dot{\mathbf{R}}\cdot\nabla_{\mathbf{R}}f+\dot{E}\frac{\partial f}{\partial E}+\dot{\mu}\frac{\partial f}{\partial \mu}+\dot{\varphi}\frac{\partial f}{\partial \varphi}=C\{f\}$$

- Gyroaveraging

$$\frac{\partial\langle f\rangle}{\partial t}+(u\hat{\mathbf{b}}+\mathbf{v}_d)\cdot\left(\nabla_{\mathbf{R}}\langle f\rangle-\frac{Ze}{M}\nabla_{\mathbf{R}}\langle\phi\rangle\frac{\partial\langle f\rangle}{\partial E}\right)=\langle C\{f\}\rangle$$

- Also possible to use parallel velocity and to write in conservative form
- Gyrophase dependent piece

$$\tilde{f}=f-\langle f\rangle\cong-\Omega^{-1}\int^{\varphi}d\varphi'\left(C\{f\}-\langle C\{f\}\rangle\right)=O(\delta\nu/\Omega f_M)$$

Quasineutrality (QN): $Zn_i = n_e$

- Taylor expanding f_i

$$f_i(\mathbf{R}, E, \mu, t) \cong f_i(\mathbf{r} + \Omega^{-1}\mathbf{v}\times\hat{\mathbf{b}}, E_0, \mu_0, t) - \frac{Ze\tilde{\phi}}{T_i} f_M + \ldots$$

- For electrons (ITG ordering), $n_e = n_0 + \frac{en_0}{T_e}\left(\phi - \langle\phi\rangle_\theta\right)$
  - Here $\langle\ldots\rangle_\theta \equiv$ flux surface average
- For $k_\perp \rho \sim 1$ and to $O(\delta n)$,

$$\frac{Z^2 e}{T_i}\int d^3v\, \tilde{\phi} f_M + \frac{en_0}{T_e}\left(\phi - \langle\phi\rangle_\theta\right) = Z\hat{N}_i - n_0$$

  - Here $\hat{N}_i = \int d^3v\, f_i(\mathbf{r} + \Omega^{-1}\mathbf{v}\times\hat{\mathbf{b}}, E_0, \mu_0, t)$
- For $k_\perp L \sim 1$ and axisymmetry, QN independent of $\langle\phi\rangle_\theta$ to $O(\delta^2 n)$ due to INTRINSIC AMBIPOLARITY

Intrinsic ambipolarity

- In axisymmetric systems and for $k_\perp L \sim 1$, n and T evolution does not depend on or in any way determines $\langle \phi \rangle_\theta$ through $O(\delta^2)$
- Symmetry $\Rightarrow$ free toroidal rotation of flux surfaces $\Rightarrow$ rotation ~ $\partial\phi/\partial\psi \Rightarrow$ free $\langle \phi \rangle_\theta$ (without viscosity)
  - No poloidal symmetry $\Rightarrow$ relation between $V_{i\|}$ & $\partial\phi/\partial\psi$
- Viscosity ($\pi_{\psi\zeta}$) $\Rightarrow$ toroidal momentum transport $\Rightarrow$ constraint on rotation and $\phi(\psi)$
  - $\pi_{\psi\zeta} \sim$ transport coeff. $\times \nabla(nM\mathbf{V}) \sim (\delta\rho v_i)\times(nM\delta v_i/L) \sim \delta^3 p$

Intrinsic ambipolarity & viscosity

- Flux surface average of toroidal angular momentum conservation ($\partial/\partial t = 0$)

$$\frac{1}{V'}\frac{\partial}{\partial \psi}V'\left\langle cR^2\nabla\psi\cdot\overleftrightarrow{\boldsymbol{\pi}}_i\cdot\nabla\zeta\right\rangle_\theta = \left\langle R^2(\mathbf{J}\times\mathbf{B})\cdot\nabla\zeta\right\rangle_\theta = \left\langle \mathbf{J}\cdot\nabla\psi\right\rangle_\theta = 0$$

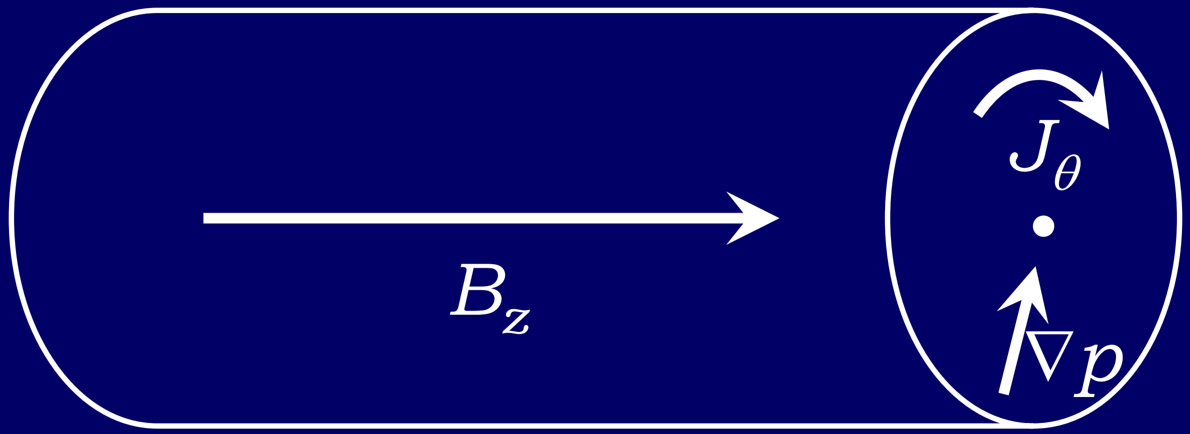
Here $V' \equiv$ volume of a flux surface

$$\overleftrightarrow{\boldsymbol{\pi}}_i = \overleftrightarrow{\mathbf{P}}_i - p_{i\perp}\overleftrightarrow{\mathbf{I}} - \left(p_{i||} - p_{i\perp}\right)\hat{\mathbf{b}}\hat{\mathbf{b}},\ \text{where}\ \overleftrightarrow{\mathbf{P}}_i = \int d^3v\, f_i M\mathbf{v}\mathbf{v}$$

- $\langle\phi\rangle_\theta$ solved from $\left\langle R^2\nabla\psi\cdot\overleftrightarrow{\boldsymbol{\pi}}_i\cdot\nabla\zeta\right\rangle_\theta = 0$

- In general stellarators, no direction similar to ζ
  $\Rightarrow$ it is enough to keep $(p_{||} - p_\perp)$

θ - pinch



- For $k_\perp L \sim 1$, steady-state θ - pinch, to $O(\delta^2)$

$$\langle \dot{\mathbf{R}} \rangle \cdot \left(\nabla_{\mathbf{R}} \langle f_i \rangle - \frac{Ze}{M} \nabla_{\mathbf{R}} \langle \phi \rangle \frac{\partial \langle f_i \rangle}{\partial E} \right) = \langle C\{f_i\} \rangle$$

- For axisymmetric solutions, $\langle C\{f_i\} \rangle = 0$

θ - pinch solution

- With Krook $C\{f\} = -\nu(f - f_M)$ and $\langle \ldots \rangle$ to $O(\delta^2 f_M)$

$$\langle f_i \rangle = \langle f_M \rangle = f_{M0}\left[1 - \frac{Mv_\perp^2}{2p_i}\nabla\cdot\left(\frac{cn_i}{B\Omega}\nabla_\perp\phi\right) + \left(2 - \frac{Mv_\perp^2}{2T_i}\right)\frac{Mc^2}{2T_i B^2}\left|\nabla_\perp\phi\right|^2 + \ldots\right]$$

with $f_M = n_i\left(\frac{M}{2\pi T_i}\right)^{3/2}\exp\left(-\frac{M(\mathbf{v}-\mathbf{V}_i)^2}{2T_i}\right)$, $f_{M0} = n_i\left(\frac{M}{2\pi T_i}\right)^{3/2}\exp\left(-\frac{ME}{T_i}\right)$

- To find $\langle\phi\rangle_\theta$, QN needed to $O(\delta^2 n)$ (valid for any n_e)

$$-\nabla\cdot\left(\frac{Zcn_i}{B\Omega}\nabla_\perp\phi\right) + \frac{Zn_i Mc^2}{2T_i B^2}\left|\nabla_\perp\phi\right|^2 = Z\hat{N}_i - n_e$$

with $\hat{N}_i = \int d^3v\,\bar{f}_i(\mathbf{r}, E_0, \mu_0, t)\left(1 + \frac{v_{||}}{\Omega}\hat{\mathbf{b}}\cdot\nabla\times\hat{\mathbf{b}}\right) + \left(\overleftrightarrow{\mathbf{I}} - \hat{\mathbf{b}}\hat{\mathbf{b}}\right) : \frac{\nabla\nabla p_i}{2M\Omega^2}$

θ - pinch and tokamak potential

- Substitute f_i into QN $\Rightarrow$ $\boxed{0 = 0}$ $\Rightarrow$ any $\langle\phi\rangle_\theta$ satisfies!
- Typically, $\delta^2 f_M$ terms MISSING $\Rightarrow$ non physical $\langle\phi\rangle_\theta$
  But even with $\delta^2 f_M$ terms,
  1. $f_i(t=0) = f_{M0}\left[1 - \frac{Mv_\perp^2}{2p_i}\nabla\cdot\left(\frac{cn_i}{B\Omega}\nabla_\perp G\right) + \ldots\right]$, G arbitrary
  2. Use QN to find : $\phi = G$
  3. f_i steady-state solution: $f_i = f_i(t=0)$ and $\phi = G$ at any t
- Only need f_i to $O(\delta^2 f_M)$ if using $\langle J_r\rangle_\theta = 0 \Rightarrow \pi_{r\theta} = 0$

$$Ze\frac{\partial\phi}{\partial r} + \frac{1}{n_i}\frac{\partial p_i}{\partial r} = rB\int dr\frac{3}{rB}\frac{\partial T_i}{\partial r}\left[\frac{5}{3}\frac{\partial}{\partial r}\ln B - \frac{\partial}{\partial r}\ln\left(\frac{p_i}{r}\frac{\partial T_i}{\partial r}\right)\right] \sim \frac{\partial T_i}{\partial r}$$

- Same in tokamaks. In both, $\pi_{\psi\zeta} = 0$ gives $\langle\phi\rangle_\theta$ at $\delta^3 p$

Vorticity for δf models

- For $k_\perp \rho \sim 1$, $\partial/\partial t$ of quasineutrality

$$\frac{\partial \rho_{cp}}{\partial t} - \nabla \cdot \left[\frac{c\rho_{cp}}{B} \nabla \phi \times \hat{\mathbf{b}} \right] = \nabla \cdot \left[J_{||}\hat{\mathbf{b}} + \mathbf{J}_{dg} + \tilde{\mathbf{J}}_i \right] + \text{coll.}$$

where ρ_{cp} is the polarization charge density

$$\rho_{cp} = -\int d^3v \, \frac{Z^2 e^2 \tilde{\phi}}{T_i} f_M \xrightarrow[k_\perp \rho \to 0]{} \nabla \cdot \left(\frac{Zecn_i}{B\Omega} \nabla_\perp \phi \right)$$

$$\mathbf{J}_{dg} \approx \frac{cp_\perp}{B} \hat{\mathbf{b}} \hat{\mathbf{b}} \cdot \nabla \times \hat{\mathbf{b}} + \frac{cp_\perp}{B^2} \hat{\mathbf{b}} \times \nabla B + \frac{cp_{||}}{B} \hat{\mathbf{b}} \times \boldsymbol{\kappa}$$

- Must construct algorithm to recover intrinsic ambipolarity $\Rightarrow \langle \phi \rangle_\theta$ constant in time

$$\frac{\partial \langle \rho_{cp} \rangle_\theta}{\partial t} = \langle \nabla \cdot (\ldots) \rangle_\theta = 0 \ \text{ at } \ k_\perp L \sim 1$$

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Moment approach

- Solve a gyrokinetic equation for $(f - f_M)$
- Find n_e, $\mathbf{V}_i$, $\mathbf{J}$, T_i, T_e and ϕ from moment equations

$$\frac{\partial n_e}{\partial t} + \nabla \cdot \left(n_e \mathbf{V}_e\right) = 0$$

$$\nabla \cdot \mathbf{J} = 0$$

$$\frac{\partial}{\partial t}\left(n_i M \mathbf{V}_i\right) + \nabla \cdot \left[p_\perp \overleftrightarrow{\mathbf{I}} + (p_{||} - p_\perp)\hat{\mathbf{b}}\hat{\mathbf{b}} + \overleftrightarrow{\boldsymbol{\pi}}_i\right] = \frac{1}{c}\mathbf{J} \times \mathbf{B}$$

$$\nabla \cdot \left[p_{e\perp} \overleftrightarrow{\mathbf{I}} + (p_{e||} - p_{e\perp})\hat{\mathbf{b}}\hat{\mathbf{b}}\right] = e n_e \nabla \phi - \frac{e n_e}{c}\mathbf{V}_e \times \mathbf{B} + \mathbf{F}_{ei}$$

$$\frac{\partial}{\partial t}\left(\frac{3}{2} n_i T_i\right) + \nabla \cdot \mathbf{Q}_i = -Z e n_i \mathbf{V}_i \cdot \nabla \phi - W_{ei}$$

$$\frac{\partial}{\partial t}\left(\frac{3}{2} n_e T_e\right) + \nabla \cdot \mathbf{Q}_e = e n_e \mathbf{V}_e \cdot \nabla \phi + W_{ei}$$

Energy and momentum fluxes

- Moments of the Fokker Planck equation $\Rightarrow$ higher order results than direct evaluation of $\mathbf{Q}_i$, $\mathbf{Q}_e$ and $\overleftrightarrow{\boldsymbol{\pi}}_i$
- Using the $M\upsilon^2\mathbf{v}/2$ moment

$$\mathbf{Q}_{i\perp} = \frac{1}{\Omega}\hat{\mathbf{b}}\times\nabla\cdot\left[\int d^3\upsilon\frac{M\upsilon^2}{2}\mathbf{vv}f_i\right] - \left(\frac{1}{2}p_{i||} + 2p_{i\perp}\right)\frac{c}{B}\nabla\phi\times\hat{\mathbf{b}}$$

$$+\frac{c}{B}\hat{\mathbf{b}}\times(\overleftrightarrow{\boldsymbol{\pi}}_i\cdot\nabla\phi) - \frac{1}{\Omega}\hat{\mathbf{b}}\times\int d^3\upsilon\frac{M\upsilon^2}{2}\mathbf{v}C\{f_i\} - \frac{\partial}{\partial t}\left[\frac{1}{\Omega}\mathbf{Q}_{i\perp}^{(0)}\times\hat{\mathbf{b}}\right]$$

- Using the $M\mathbf{vv}$ moment,

$$\overleftrightarrow{\boldsymbol{\pi}}_i = \frac{1}{4\Omega}\left[\hat{\mathbf{b}}\times\overleftrightarrow{\mathbf{K}}\cdot\left(\overleftrightarrow{\mathbf{I}} + 3\hat{\mathbf{b}}\hat{\mathbf{b}}\right) - \left(\overleftrightarrow{\mathbf{I}} + 3\hat{\mathbf{b}}\hat{\mathbf{b}}\right)\cdot\overleftrightarrow{\mathbf{K}}\times\hat{\mathbf{b}}\right]$$

$$\overleftrightarrow{\mathbf{K}} = \nabla\cdot\left[\int d^3\upsilon\, M\mathbf{vvv}f_i\right] + Zen_i(\nabla\phi\,\mathbf{V}_i + \mathbf{V}_i\nabla\phi) - \int d^3\upsilon\, M\mathbf{vv}\,C\{f_i\} + \frac{\partial\overleftrightarrow{\boldsymbol{\pi}}_i^{(0)}}{\partial t}$$

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Vorticity in general

- Plugging $\mathbf{J}_\perp$ from momentum into $\nabla\cdot\mathbf{J}=0$

$$\frac{\partial\varpi}{\partial t}=\nabla\cdot\left[J_{||}\hat{\mathbf{b}}+\mathbf{J}_d+\frac{c}{B}\hat{\mathbf{b}}\times(\nabla\cdot\overleftrightarrow{\boldsymbol{\pi}}_i)\right]$$

$$\text{Vorticity}\equiv\varpi=\nabla\cdot\left(\frac{Ze}{\Omega}n_i\mathbf{V}_i\times\hat{\mathbf{b}}\right)\cong\nabla\cdot\left[\frac{Zecn_i}{B\Omega}\nabla_\perp\phi+\frac{c}{B\Omega}(\nabla\cdot\overleftrightarrow{\mathbf{P}}_i)\Big|_\perp\right]$$

$$\mathbf{J}_d=\frac{cp_\perp}{B}\hat{\mathbf{b}}\hat{\mathbf{b}}\cdot\nabla\times\hat{\mathbf{b}}+\frac{cp_\perp}{B^2}\hat{\mathbf{b}}\times\nabla B+\frac{cp_{||}}{B}\hat{\mathbf{b}}\times\boldsymbol{\kappa}$$

- Electron dynamics along $\mathbf{B}$ determine $\hat{\mathbf{b}}\cdot\nabla\phi$
  - $J_{||}$ must adapt to provide the right value of $\hat{\mathbf{b}}\cdot\nabla\phi$
- $\overleftrightarrow{\boldsymbol{\pi}}_i$ matters for $k_\perp\rho\sim 1$ due to gradients $\sim\rho^{-1}$
- $\overleftrightarrow{\boldsymbol{\pi}}_i$ matters for $k_\perp L\sim 1$ determines rotation $\Rightarrow\langle\phi\rangle_\theta$

$\langle\text{Vorticity}\rangle_\theta$ gives conservation of toroidal momentum

Tokamak vs. Stellarator

- Vorticity

$$\frac{\partial \varpi}{\partial t} = \nabla \cdot \left[J_{||}\hat{\mathbf{b}} + \mathbf{J}_d + \frac{c}{B}\hat{\mathbf{b}} \times (\nabla \cdot \overleftrightarrow{\boldsymbol{\pi}}_i) \right]$$

$$\mathbf{J}_d = \frac{cp_\perp}{B}\hat{\mathbf{b}}\hat{\mathbf{b}} \cdot \nabla \times \hat{\mathbf{b}} + \frac{cp_\perp}{B^2}\hat{\mathbf{b}} \times \nabla B + \frac{cp_{||}}{B}\hat{\mathbf{b}} \times \boldsymbol{\kappa}$$

- For tokamaks, $\partial\varpi/\partial t = F(\pi_i) = O(\delta^4 e n_e v_i / L)$
  $\Rightarrow \Delta t \sim L^2/D_{gB} \sim L/\delta^2 v_i$

- For stellarators, $\partial\varpi/\partial t = F(J_d) = O(\delta^2 e n_e v_i / L)$
  $\Rightarrow \Delta t \sim L/v_i$

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Comparison with full f approach

- Full f solves for all time and length scales!!

  Moment approach can separate scales
- Current gyrokinetic equations only good to $O(\delta)$ but need $O(\delta^2)$ to see radial transport

  Do full f models retain the right radial transport?
- For $k_\perp L \sim 1$, take moment G of gyrokinetic equation

$$\frac{\partial}{\partial t}\int d^3v\, G f_i + \nabla\cdot\left\{\int d^3v\, G f_i\left[u\hat{\mathbf{b}}(\mathbf{R}) + \mathbf{v}_d\right]\cdot\nabla_{\mathbf{R}}\mathbf{r}\right\} = \int d^3v\, G\frac{Ze}{M}\frac{\partial\tilde{\phi}}{\partial t}\left(\frac{\partial f_i}{\partial E} + \frac{1}{B}\frac{\partial f_i}{\partial \mu}\right)$$

$$+\int d^3v\, f_i\left[\left.\frac{\partial G}{\partial t}\right|_{\mathbf{r},\mathbf{v}} + \dot{\mathbf{R}}\cdot\nabla_{\mathbf{R}}G + \dot{E}\frac{\partial G}{\partial E}\right] + \int d^3v\, G\langle C\{f_i\}\rangle$$

Particle and energy transport in full f

- Flux surface averaging moment $Mv^2/2 = ME_0$

$$\frac{\partial}{\partial t}\left\langle \frac{3}{2} n_i T_i \right\rangle_\theta + \frac{1}{V'}\frac{\partial}{\partial \psi} V' \left\langle \int d^3v \, \frac{Mv^2}{2} f_i \left(\mathbf{v}_{M0} - \frac{c}{B}\nabla\phi \times \hat{\mathbf{b}} \right) \cdot \nabla\psi \right\rangle_\theta =$$

$$- Ze \left\langle \nabla\phi \cdot \int d^3v \, f_i \left(v_{||}\hat{\mathbf{b}} + \mathbf{v}_{M0} \right) \right\rangle_\theta + \ldots$$

  Except for neoclassical pieces, it coincides with moment approach! Similar for particle transport
- Full f models retain the turbulent radial particle and energy transport at $k_\perp L \sim 1$
- BUT do not keep the particle radial transport to high enough order to compute $\langle \phi \rangle_\theta$

Conclusions

- Intrinsic ambipolarity means f_i to $O(\delta^2 f_M)$ doesn't determine $\langle\phi\rangle_\theta$ in axisymmetric, $k_\perp L \sim 1$ limit
  - Explicit demonstration given for θ - pinch
- Typically, gyrokinetic f only to $O(\delta) \Rightarrow$ faulty $\langle\phi\rangle_\theta$
- First solution: vorticity equation for δf codes that insures $\partial\rho_{cp}/\partial t = 0$ for axisymmetry and $k_\perp L \sim 1$
- Second solution: moment approach
  - Moment approach only requires f_i to $O(\delta^2 f_M)$
- Full f codes may recover the right particle and energy transport but not the right electric field

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