FINITE DRIFT ORBIT EFFECTS IN A TOKAMAK PEDESTAL

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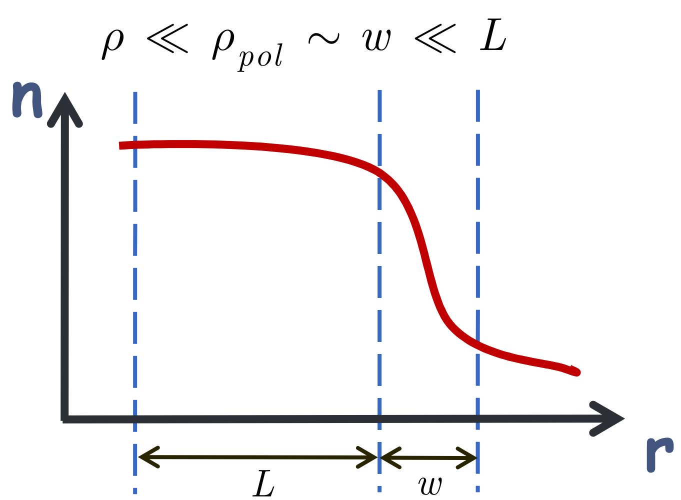
Agenda

- Gyrokinetic treatment of background scales of order poloidal ion gyroradius
- Ion temperature and pressure balance in the pedestal and internal transport barrier regions
- Zonal flow in pedestal

Gyrokinetic orderings in pedestal



Small parameter:

$$\rho / \rho_{pol} \ll 1$$

Charasteristic frequency is taken to be that of drift waves:

$$\omega \sim \omega_* \equiv k_\perp \rho \frac{v_{th}}{w}$$

Strong perpendicular gradients are allowed for small components of the distribution function and potential:

$$\frac{f_k}{f_0} \sim \frac{Ze\phi_k}{T} \sim \frac{1}{k_\perp w}$$

This ordering allows $k_\perp \rho \sim 1$

Axisymmetric $\vec{B}$ variables

Canonical angular momentum

$$\psi_* = \psi - \frac{Mc}{Ze} R\vec{v}\cdot\hat{\zeta} = \psi + \frac{\vec{v}\times\hat{n}}{\Omega}\cdot\nabla\psi - \frac{Iv_{||}}{\Omega} \quad \left(\vec{B}_{tok} = I\nabla\zeta + \nabla\zeta\times\nabla\psi = B\hat{n}\right)$$

$(\rho/L)\psi$ classical

$(\rho_{pol}/L)\psi$ neoclassical

Toroidal angle	ζ
Poloidal angle	θ
Total energy	E
Magnetic moment	μ
Gyrophase	φ

First order corrections

$$\left\langle \dot{\psi}_* \right\rangle_\varphi \approx -c\frac{\partial \bar{\phi}}{\partial \zeta_*} \sim \delta^2 \Omega \psi_*$$

— small enough so that no correction is needed

$$\theta_* = \theta + \frac{\vec{v}\times\hat{n}}{\Omega}\cdot\nabla\theta \qquad \left\langle \dot{\theta}_* \right\rangle_\varphi \approx \left(v_\parallel^* \hat{n}_* + \vec{v}_d\right)\cdot(\nabla\theta)_* - \frac{I v_\parallel}{\Omega}\frac{\partial\left(v_\parallel \hat{n}\cdot\nabla\theta\right)}{\partial\psi} \approx \left(v_\parallel^* \hat{n}_* + \vec{v}_E\right)\cdot(\nabla\theta)_*$$

$$\zeta_* = \zeta + \frac{\vec{v}\times\hat{n}}{\Omega}\cdot\nabla\zeta \qquad \left\langle \dot{\zeta}_* \right\rangle_\varphi \approx \left(\frac{I v_\parallel}{B R^2}\right)_* + \vec{v}_d\cdot\nabla\zeta - \frac{I v_\parallel}{\Omega}\frac{\partial}{\partial\psi}\left(\frac{I v_\parallel}{B R^2}\right)$$

pedestal ordering

$$\mu_* = \frac{v_\perp^2}{2B} - \frac{1}{B}\vec{v}_\perp\cdot\vec{v}_M - \frac{v_\parallel}{4B\Omega}\left[\vec{v}_\perp(\vec{v}\times\hat{n}) + (\vec{v}\times\hat{n})\vec{v}_\perp\right] : \nabla\hat{n} - \frac{v_\parallel v_\perp^2}{2B\Omega}\hat{n}\cdot\nabla\times\hat{n} + \frac{Ze}{MB}\tilde{\phi} \qquad \left\langle \dot{\mu}_* \right\rangle_\varphi \approx 0$$

Here we may take $\varphi_* = \varphi$ and $\dot{\varphi} = -\Omega$ to the requisite order

In the above formulas we defined $\vec{v}_d \equiv \frac{v_\parallel^2}{\Omega}\hat{n}\times(\hat{n}\cdot\nabla\hat{n}) + \frac{\mu}{\Omega}\hat{n}\times\nabla B - \frac{c}{B}\nabla\bar{\phi}\times\hat{n}$

Axisymmetric gyrokinetic equation and isothermal tokamak equilibrium

For $\partial/\partial\zeta = 0$ kinetic equation in the new variables is given by

$$\frac{\partial \bar{f}}{\partial t} + \langle \dot{\theta}_* \rangle \frac{\partial \bar{f}}{\partial \theta_*} = \langle C\{f\} \rangle - \frac{Ze}{M}\frac{\partial \bar{\phi}}{\partial t}\frac{\partial \bar{f}}{\partial E}$$

For $\partial/\partial t = 0$, $f = f(\psi_*, E)$ makes the left side exactly vanish

To make the collision operator vanish f has to be Maxwellian

Therefore, we find an exact solution to the above equation:

$$f_* = \eta\left(\frac{M}{2\pi T}\right)^{3/2} \exp\left(-\frac{Ze\phi}{T} + \frac{M\omega^2 R^2}{2T} - \frac{Ze}{cT}\omega\psi\right)\exp\left(-\frac{M\left(\vec{v} - \omega R\hat{\zeta}\right)^2}{2T}\right)$$

where T, ω and η are constants

toroidally rotating Maxwellian

In terms of the new variables:

$$f_* = \eta\left(\frac{M}{2\pi T}\right)^{3/2} e^{-\frac{ME}{T} - \frac{Ze}{cT}\omega\psi_*}$$

Solubility constraint for a non-isothermal tokamak

Let us now analyze the steady state still assuming $\dfrac{\partial}{\partial \zeta} = 0$

Setting $\dfrac{\partial}{\partial t} = 0$ and transit averaging:

$$\overline{\langle C\{f\}\rangle} = 0$$

full nonlinear equation

where $\bar{Q} \equiv \oint Q d\tau \,/\, \oint d\tau$ with $d\tau \equiv d\theta \,/\, \langle \dot{\theta}_* \rangle$

Are there non-Maxwellian solutions in pedestal?

Entropy production analysis: no!

ION TEMPERATURE VARIATION ACROSS THE PEDESTAL

$$\left\langle \dot{\theta}_* \right\rangle \frac{\partial f_0}{\partial \theta_*} = \left\langle C\{f_0\} \right\rangle$$

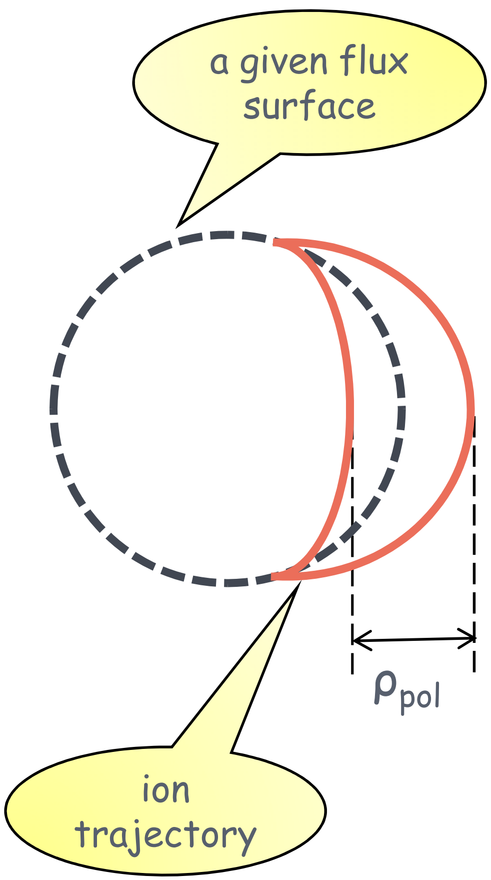
f_0 is Maxwellian

In the banana regime f_0 can not depend on θ_* but only on E, ψ_* and μ

Combining these two statements we conclude that pedestal plasma is essentially isothermal !!!

That is, T_i must vary slowly compared to ρ_{pol}

Physical interpretation



In the core plasma gradients are so weak that ions departures from a flux surface are not important and we can consider any given flux surface a closed system.

In the pedestal gradients are as large as $1/\rho_{pol}$ and therefore these departures affect the equilibrating of the neighboring flux surfaces. Thus, it is the entire pedestal region that is a closed system rather than its individual flux surfaces.

PRESSURE BALANCE IN PEDESTAL

radial ion pressure balance $\left(\vec{V}_i = \omega_i R^2 \nabla\zeta\right)$

$$\omega_i = -c\frac{d\phi}{d\psi} - \frac{cT_i}{en}\frac{dn}{d\psi}$$

in pedestal (w~ρ_{pol}) $\omega_i \Big/ \frac{cT_i}{en}\frac{dn}{d\psi} \sim \frac{\omega_i R}{v_{th}^{(i)}} \ll 1$ ⇨ $\frac{d\phi}{d\psi} > 0$ as $\frac{dn}{d\psi} < 0$

that is, pedestal electric field is inward for subsonic ion flow

radial electron pressure balance

$$\omega_e = -c\frac{d\phi}{d\psi} + \frac{c}{en}\frac{dp_e}{d\psi}$$

$\frac{dp_e}{d\psi} < 0$ hence it adds to $\frac{d\phi}{d\psi}$ to make $\omega_e \sim v_{th}^{(i)}/R$

$$J_{ped} \approx -en\omega_e R \sim env_{th}^{(i)}$$

Thus, electric potential that provides 1/ρ_{pol} density gradient can only be sustained by large *electron* flow

Corrections to the leading order distribution function

Let us assume $\overline{f} = f_*(\psi_*, E) + g(\psi_*, \theta_*, \mu_*, E, t)$ with $g << f_*$ and $\partial g / \partial \zeta = 0$

Let $\overline{\phi} = \phi_0 + \phi_1$, where ϕ_0 is the equilibrium potential and ϕ_1 stands for its zonal flow perturbation with $\partial \phi_1 / \partial t \gg \partial \phi_0 / \partial t \to 0$.

Assuming $\phi_1 = \hat{\phi} e^{iG(\psi)}$, defining $\phi_* \equiv \hat{\phi} e^{iG(\psi_*)}, \vec{k}_\perp \equiv \nabla G$, and $Q \equiv \frac{Iv_{||}}{\Omega} G'$

finite poloidal gyroradius effect

and transit averaging we obtain

$$\frac{\partial g}{\partial t} - \overline{\left\langle C_l \{ g + \frac{Iv_{||}}{\Omega} f_M \frac{Mv^2}{2T^2} \frac{\partial T}{\partial \psi} \} \right\rangle} = \frac{Ze}{T} \frac{\partial \phi_*}{\partial t} f_M \overline{J_0\left(\frac{k_\perp v_\perp}{\Omega}\right) e^{iQ}}$$

neoclassical drive

zonal flow drive

Neoclassical polarization

Density response to the perturbation of the potential:

$$n_1 = \frac{Ze}{T}\phi_1 \left\langle \int d^3v f_M \left(e^{-iQ}\overline{e^{iQ}} - 1 \right) \right\rangle_\theta$$

where collisions and FLR effects are neglected

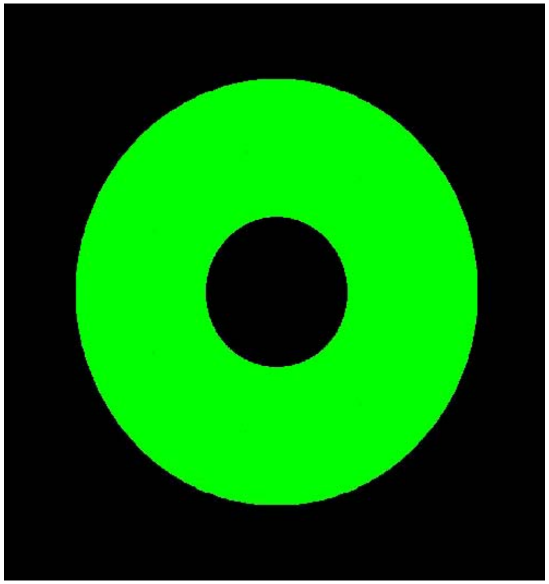
Rosenbluth-Hinton (zero electric field) limit

$$n_1 = \frac{Ze}{T}\phi_1 \left\langle \int d^3v f_M \left(i\overline{Q} - iQ - \frac{Q^2 - 2Q\overline{Q} + \overline{Q^2}}{2} \right) \right\rangle_\theta$$, where $Q \equiv \frac{Iv_{||}}{\Omega} G' \sim k_\perp \rho_{pol}$

In the absence of the electric field $\overline{v_{||}}$ is an odd function of $v_{||}$ so that the terms of the first order in Q vanish.

It is no longer the case in pedestal as there is a preferred direction of rotation in the poloidal plane due to ExB drift. Consequently, in our case, terms linear in Q contribute to the density response that makes neoclassical polarization complex. Thus, there is now a spatial phase shift between density and potential perturbations

Time evolution of zonal flow



J. Candy & R. Waltz

Free ion density accumulated by plasma turbulence drives zonal flow whose potential evolves so that

$$\frac{\phi_1(t\to\infty)}{\phi_1(t=0)} = \frac{\varepsilon_{k,cl}^{pol}}{\varepsilon_{k,cl}^{pol}+\varepsilon_{k,nl}^{pol}}$$

$$\varepsilon_{k,cl}^{pol}\Big|_{R\&H} \approx \frac{\omega_{pi}^2}{\omega_{ci}^2} \qquad \varepsilon_{k,nl}^{pol}\Big|_{R\&H} \approx 1.6\frac{\omega_{pi}^2}{\omega_{ci}^2}\frac{q^2}{\sqrt{\varepsilon}}$$

From the above

$$\frac{\phi_1(t\to\infty)}{\phi_1(t=0)} = \frac{k_\perp^2\rho_i^2}{k_\perp^2\rho_i^2+\frac{1}{n_0}\left\langle\int d^3v f_M\left(i\bar{Q}-iQ-\frac{Q^2-2Q\bar{Q}+\overline{Q^2}}{2}\right)\right\rangle_\theta}$$

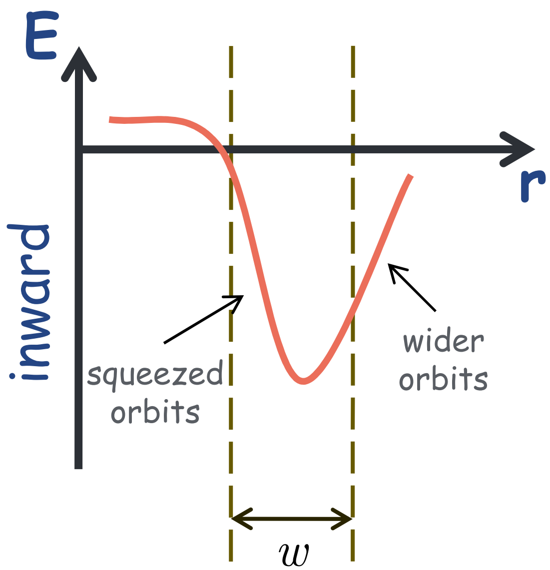
classical polarization

Particle orbits in pedestal

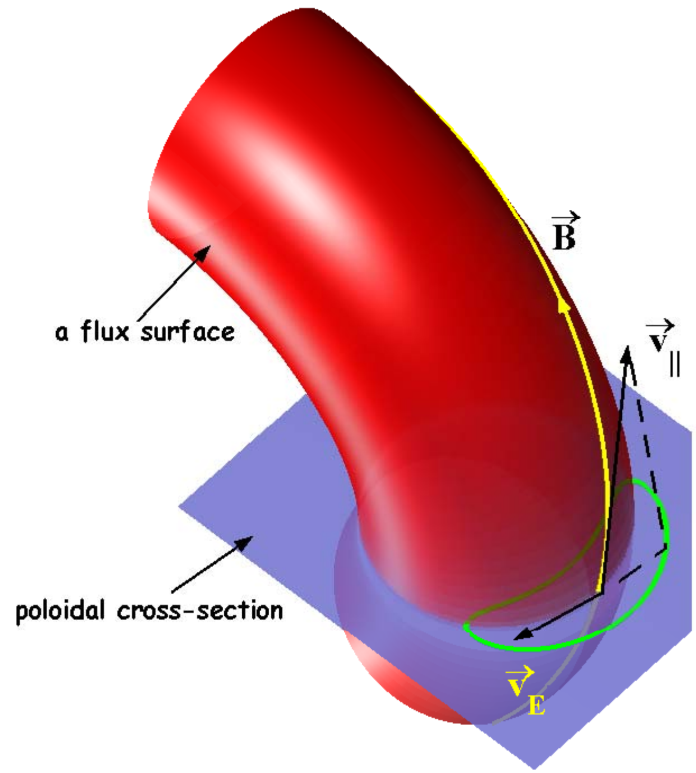
Boltzmann relation

Strong electric field

$$|Ze\nabla\phi/T| \sim 1/w \sim 1/\rho_{pol}$$



$$\langle\dot{\theta}_*\rangle \approx (v_{\parallel} + cI\phi'(\psi)/B)\hat{n}\cdot\nabla\theta \approx \dot{\theta}$$



ExB drift is of order $v_{th}(\rho/\rho_{pol}) \ll v_{||}$, but due to the geometrical factors its contribution to the poloidal velocity is comparable to that of $v_{||}$

Energy conservation

Assume a quadratic potential well

$$\phi = \alpha + \phi_*'(\psi - \psi_*) + \frac{\phi_*''}{2}(\psi - \psi_*)^2$$

then, using μ and ψ_* invariance we can write energy conservation as

$$\frac{\left[\dot{\theta}/(\hat{n}\cdot\nabla\theta)\right]^2}{2S} - \frac{\left(cI\phi_*'/B\right)^2}{2S} + \mu B = const.$$

$$S \equiv 1 + cI^2\phi''/B\Omega$$

orbit squeezing

ExB energy

magnetic dipole energy

If S<0 trapped particles reside on the inside of a tokamak. If S>0 – on the outside. For S>0, the maximum initial angular velocity at which particle can be trapped is given by

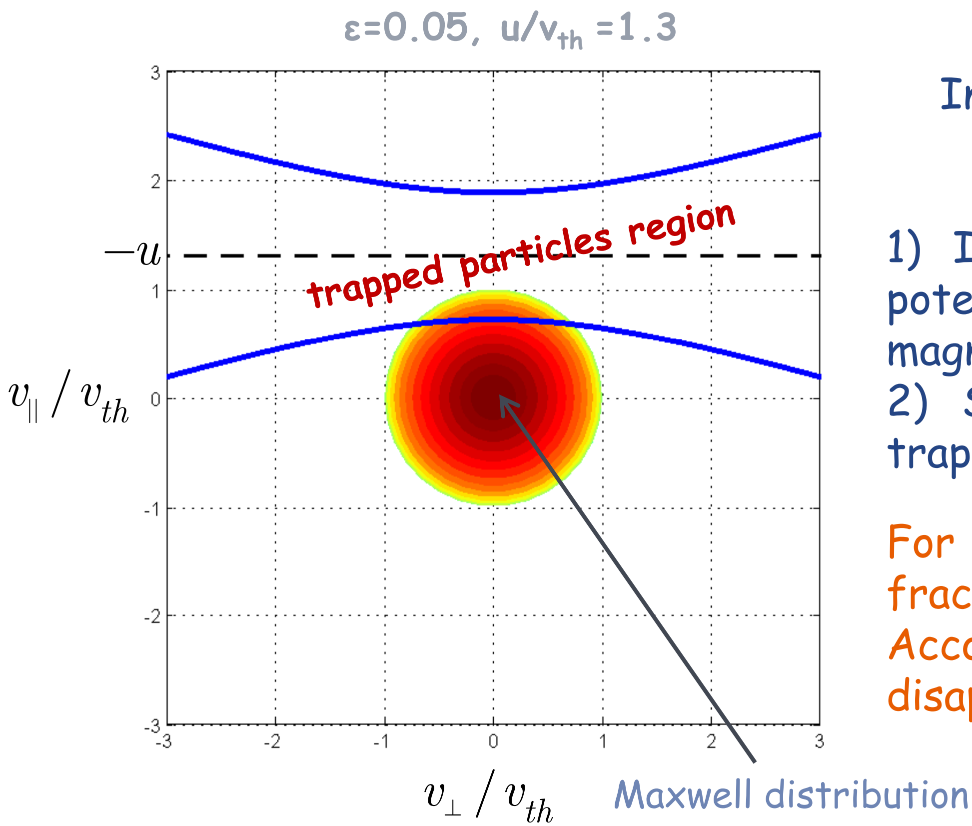
$$\left(\dot{\theta}_0 qR\right)^2_{\max} = 4\varepsilon S\left[\left(cI\phi_*'/SB\right)^2 + \mu B_0\right]$$

For S=1 (Φ''=0) this can be rewritten as

$$\left[\left(v_{\|0}\right)_{\max} + u\right]^2 \approx 4\varepsilon\left[\mu B_0 + u^2\right],$$ where $$u \equiv cI\phi_*'/B_0 \approx \left(\rho_{pol}/\rho\right)v_E$$

where the subscript "0" corresponds to the outboard equatorial plane (θ=0)

Trapped particles fraction



In the absence of orbit squeezing (S=1), ExB drift has the following effects:

1) Increases the depth of the effective potential well – now particles with no magnetic moment can be trapped.
2) Shifts the axis of symmetry of the trapped particles region.

For small enough ε the trapped particles fraction decays exponentially as |u| grows. Accordingly, neoclassical polarization should disappear in the large electric field limit

Notice, that $u \approx (\rho_{pol}/\rho) v_E \gg v_E$ and therefore particle dynamics can be significantly affected even by the ExB drift much less than v_{th}.

Evaluation of the zonal flow residual 1

Need to evaluate

$$Y \equiv \frac{1}{n_0}\left\langle \int_\psi d^3v f_M \left(i\bar{Q} - iQ - \frac{Q^2 - 2Q\bar{Q} + \overline{Q^2}}{2} \right) \right\rangle$$

Transit averages are to be performed holding ψ* fixed while the outer integral has to be calculated at a fixed ψ

Also, Q has to be redefined

$$Q \equiv \frac{Iv_{\|}}{\Omega} G' \rightarrow \frac{I\left(v_{\|} + u\right)}{\Omega} G'$$

as now particles of interest are <u>localized</u> around $v_{\|} + u = 0$

Using that $u + v_{\|} = \pm\left(u + v_{\|0}\right)\sqrt{1 - \kappa^2 \sin^2\left(\theta/2\right)}$, where $\kappa^2 = 4\varepsilon \frac{u^2 + \mu B_0}{\left(u + v_{\|0}\right)^2}$

we obtain

$$\bar{Q} = \left(G' I / \Omega\right) \frac{\pi\left(v_{\|0} + u\right)}{2K(\kappa)}$$

$$\overline{Q^2} = \left(G' I / \Omega\right)^2 \left(v_{\|0} + u\right)^2 \frac{E(\kappa)}{K(\kappa)}$$

EVALUATION OF THE ZONAL FLOW RESIDUAL 2

$$Y \equiv \frac{1}{n_0}\left\langle \int_\psi d^3v f_M \left(i\bar{Q} - iQ - \frac{Q^2 - 2Q\bar{Q} + \overline{Q^2}}{2} \right) \right\rangle$$

$\bar{Q}$ and $\overline{Q^2}$ are found in terms of $v_{||0}$ and κ while the outer integral is over d^3v

Therefore, it is convenient to switch to κ^2 and $v_{||0} + u$ variables

with Jacobean of the transformation given by
$$\frac{2\pi dv_\perp dv_{||}}{d\kappa^2 d\left(v_{||0} + u\right)} = \frac{\pi B\left(v_{||0} + u\right)^2}{2\varepsilon B_0\sqrt{1 - \kappa^2 \sin^2(\theta/2)}}$$

Then, after some algebraic manipulations we obtain

$$\frac{Y}{Y_{R\&H}} = \left(1 + \frac{i\varepsilon u\left(2M/T\right)^{1/2}}{qk_\perp\rho_i}\right) e^{-Mu^2/2T} \frac{4}{3\sqrt{\pi}} \int_0^\infty dy e^{-y}\left(y + \frac{iMu^2}{T}\right)^{3/2}$$

The zonal flow residual with the orbit squeezing effect retained

$$\frac{Y}{Y_{R\&H}} = \left(1 + \frac{i\varepsilon u_0 (2M/T)^{1/2}}{qk_\perp \rho_i}\right) \frac{e^{-Mu_0^2/2T}}{S^{3/2}} \frac{4}{3\sqrt{\pi}} \int_0^\infty dy e^{-y} \left(y + \frac{iMu_0^2}{T}\right)^{3/2}$$

where $u_0 \equiv -cI\phi'(\psi)/B_0$

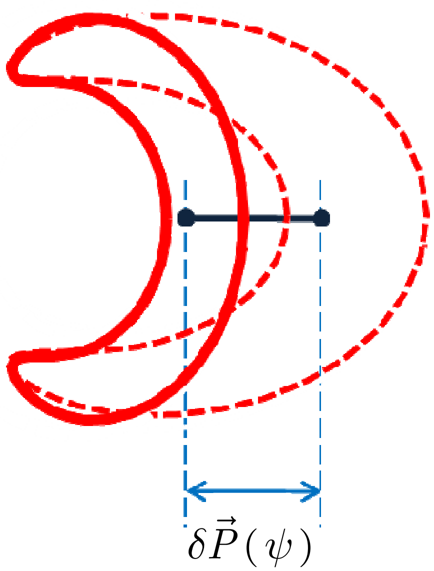
for $u_0 \to 0$
$$\frac{Y}{Y_{R\&H}} = \left(1 + \frac{i\varepsilon u_0 (2M/T)^{1/2}}{qk_\perp \rho_i}\right) \frac{1 + Mu_0^2/2T}{S^{3/2}}$$

for $u_0 \to \infty$
$$\frac{Y}{Y_{R\&H}} = \left(1 + \frac{i\varepsilon u_0 (2M/T)^{1/2}}{qk_\perp \rho_i}\right) \left(\frac{Mu_0^2}{T}\right)^{3/2} \frac{4}{3\sqrt{\pi}} \frac{e^{-Mu_0^2/2T}}{S^{3/2}}$$

exponential decay

In the strong electric field limit $\phi_1(t \to \infty) = \phi_1(t = 0)$

Neoclassical polarization in a single particle picture



A dipole moment gained by a particle on a given flux surface due to electric field perturbation

$$\delta\vec{P}(\psi) = \alpha\delta\vec{E}(\psi)$$

Density response of a flux surface

$$\delta\rho^{pol} = -\nabla\cdot\left(n_0\delta\vec{P}\right) \propto -\frac{d}{d\psi}\left[n_0\delta P(\psi)\right]$$

Assuming an eikonal form: $\delta\phi = \hat{\phi}e^{iG(\psi)}$, $\delta E = ik\hat{\phi}e^{iG(\psi)}$ so that

R&H piece

$$\delta\rho^{pol} = \underbrace{-ik\delta\phi\frac{d}{d\psi}(\alpha n_0)}_{Q} + \underbrace{k^2\delta\phi\alpha n_0}_{Q^2}$$

Summary

- Gyrokinetic formalism developed retains finite Larmor radius effects as well as finite poloidal gyroradius effects
- Pedestal plasma is nearly isothermal ($\rho_{pol}\nabla T_i \ll 1$) and sustains sharp density gradients due to electron dynamics
- The zonal flow residual is evaluated in pedestal
  - Spatial phase shift between initial and final zonal flow potentials is observed
  - The zonal flow residual is sensitive to electric field (u) and its shear (S)
  - Neoclassical shielding vanishes in strong electric field