Modulational Instability of Drift Waves and Generation of Zonal Jets

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WPI-Vienna, Sept 18 2008

DW-ZF paradigm (Balk, Nazarenko, Zakharov, 1990), Linear Modulational Instabilty (Gill, 1973), Nonlinear Modulational Instability (Manin, Nazarenko, 1994), Appl to LH transitions (Diamond et a 2000).

Outline

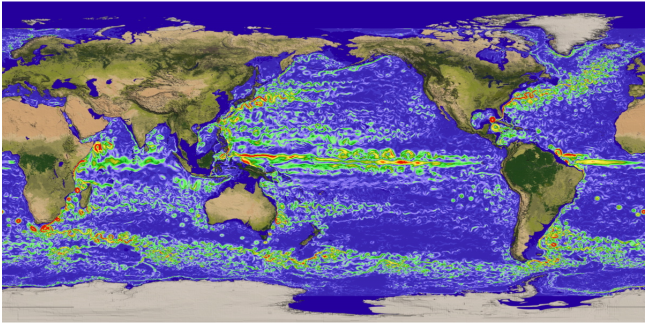
1. Introduction
    - Zonal Jets in Atmospheres, Oceans and Plasmas
    - Charney-Hasegawa-Mima Model
2. Zonalisation via Modulational Instability of Rossby/Drift Waves
    - Linear Stability Analysis of a Travelling Wave
    - Role of Nonlinearity
    - Weakly Nonlinear Regime
    - Instability of a strong wave and transition to turbulence
    - Instability of a weak drift wave.
3. Conclusions and Future Work

Zonal Jets on Gas Giants

Zonal turbulence on Jupiter (NASA)

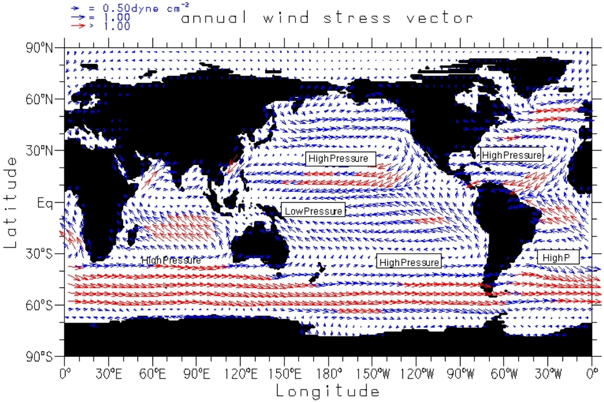
Rings and zonal bands of Saturn (NASA)

Zonal Jets in Earth's Atmosphere and Oceans



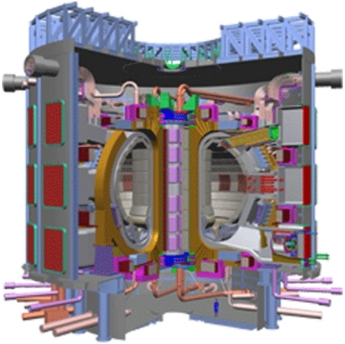
Eddy-resolving simulation of Earths oceans (Earth Simulator Center/JAMSTEC)

Zonal Jets in Earth's Atmosphere and Oceans



Average oceanic winds on Earth (QSCAT)

Zonal Flows in Tokamaks



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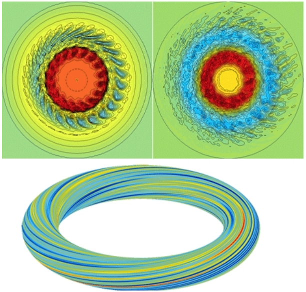


Plasma turbulence (L. Villard)

Charney-Hasegawa-Mima Model

Very minimal, but allowed to discover the mechanism for the LH transitions (generation of zonal jets which suppress drift turbulence) and similar transport blocking by jets in geophysics.

GFD: quasi-geostrophic equation (Charney 1949):

$$\partial_t (\Delta\psi - F\psi) + \beta\partial_x\psi - (\partial_y\psi)(\partial_x\Delta\psi) + (\partial_x\psi)(\partial_y\Delta\psi) = 0,$$

Plasmas: Hasegawa-Mima (1978) equation for electric potential, ϕ:

$$\partial_t \left(\rho_s^{-2}\phi - \Delta\phi\right) + v_d\partial_y\phi + (\partial_y\phi)(\partial_x\Delta\phi) - (\partial_x\phi)(\partial_y\Delta\phi) = 0,$$

Correspondence: $(x, y, \phi) \to (y, x, -\psi)$, $\rho_s^{-2} \to F$, $v_d \to -\beta$.

Rossby/Drift Wave Solutions of CHM Equation

CHM equation supports linear waves, known as Rossby/drift waves:

$$\psi_0(\mathbf{x}, t) = \Psi_0 e^{i(\mathbf{k}\cdot\mathbf{x}-\omega(\mathbf{k})t)} + \bar{\Psi}_0 e^{-i(\mathbf{k}\cdot\mathbf{x}-\omega(\mathbf{k})t)} \tag{1}$$

(Anisotropic) dispersion relation:

$$\omega(\mathbf{k}) = -\frac{\beta k_x}{k^2 + F}, \tag{2}$$

where $\mathbf{k} = (k_x, k_y)$ and $|\mathbf{k}|$. Such waves are exact solutions of the full nonlinear equation.
Are they stable? (Lorenz 1972, Gill 1973)

$$\psi(\mathbf{x}, 0) = \psi_0(\mathbf{x}) + \epsilon\, \psi_1(\mathbf{x}) \tag{3}$$

The perturbation consists of three components,

$$\psi_1(\mathbf{x}) = \psi_Z(\mathbf{x}) + \psi^+(\mathbf{x}) + \psi^-(\mathbf{x}), \tag{4}$$

a zonal mode, $\psi_Z(\mathbf{x})$, and two "sideband" modes, $\psi^+(\mathbf{x})$ and $\psi^-(\mathbf{x})$. These are defined as:

$$\begin{aligned} \psi_Z(\mathbf{x}) &= ae^{i\mathbf{q}\cdot\mathbf{x}} + \bar{a}e^{-i\mathbf{q}\cdot\mathbf{x}} & (5)\\ \psi^+(\mathbf{x}) &= b^+e^{i\mathbf{p}_+\cdot\mathbf{x}} + \bar{b}^+e^{-i\mathbf{p}_+\cdot\mathbf{x}} & (6)\\ \psi^-(\mathbf{x}) &= b^-e^{i\mathbf{p}_-\cdot\mathbf{x}} + \bar{b}^-e^{-i\mathbf{p}_-\cdot\mathbf{x}} & (7) \end{aligned}$$

where $\mathbf{q}$ is the zonal wave-vector, $p_\pm = \mathbf{k} \pm \mathbf{q}$ and a and $b^\pm$ are the amplitudes of the constituent modes of the perturbation.

Linear stability analysis

Perturbation propagates with a frequency Ω determined by:

$$(q^2 + F)\Omega + \beta q_x + |\Psi_0|^2 \, |\mathbf{k} \times \mathbf{q}|^2 \, (k^2 - q^2) \times$$
$$\left[\frac{p_+^2 - k^2}{(p_+^2 + F)(\Omega + \omega) + \beta p_{+x}} - \frac{p_-^2 - k^2}{(p_-^2 + F)(\Omega - \omega) + \beta p_{-x}} \right] = 0$$

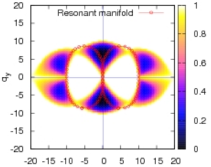
Dimensionless parameter,

$$M = \frac{\Psi_0 k^3}{\beta} \tag{8}$$

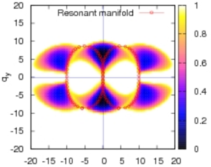
Ratio of nonlinear to linear terms at the carrier wave scale.

- $M \to \infty$: Euler limit (Rayleigh instability)
- $M \to 0$: wave turbulence limit (resonant wave interaction)

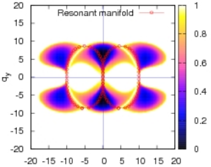
Structure of instability as a function of M (F=0)



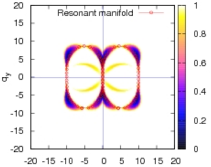
M=100



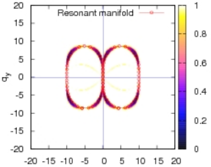
M=10



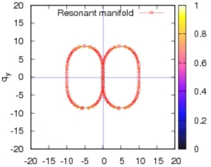
M=5



M=1



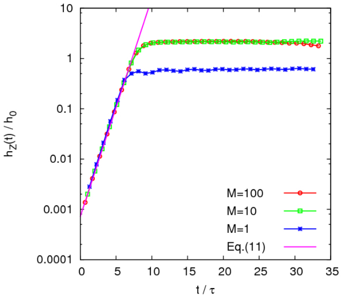
M=0.5



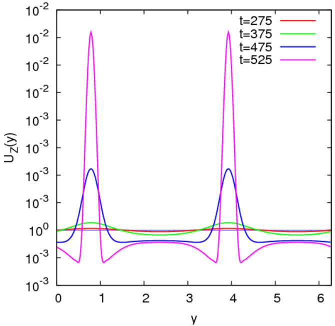
M=0.1

Unstable region collapses onto the resonant curve. For small *M* the most unstable disturbance is not zonal.

Nonlinear stage of modulational instability



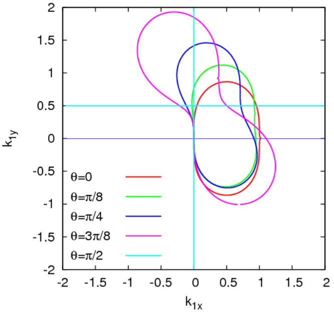
Growth of $|\psi_{\mathbf{q}}|^2$ compared to predictions of linear stability.



Zonal velocity profile (averaged over x).

Pinching of jets predicted by Manin & SN, 1994. Transport barriers.

Weakly Nonlinear regime: $M \to 0$

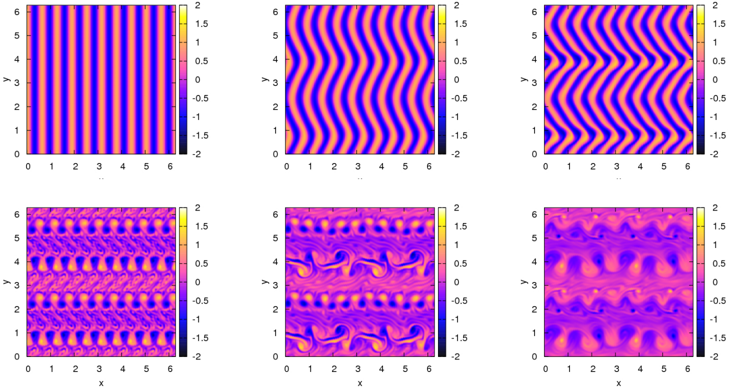


Resonant manifolds for various orientations of **k**.

- In limit $M \to 0$ instability concentrated on (anisotropic) resonant manifolds:

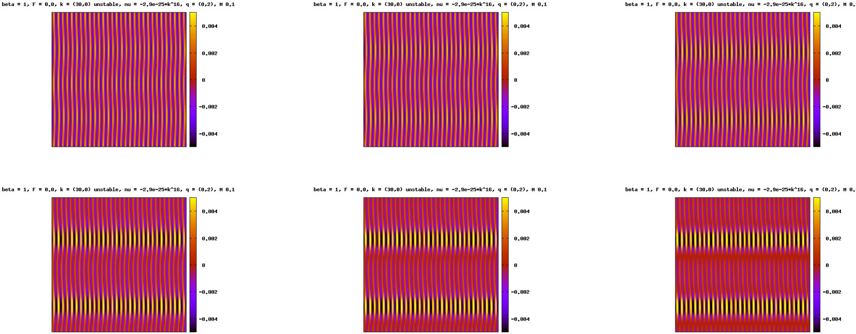
$$\begin{aligned} \mathbf{k} + \mathbf{k}_1 &= \mathbf{k}_2 \\ \omega(\mathbf{k}) + \omega(\mathbf{k}_1) &= \omega(\mathbf{k}_2) \end{aligned}$$

Strong wave case ($M = 10$): transition to turbulence



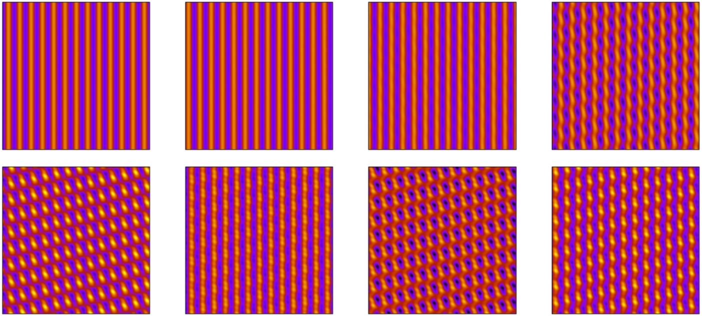
Jet consists of a vortex street. It breaks via a vortex pairing instability, *NOT* Kelvin-Helmholtz (as for x-independent jet).

Weak wave case ($M = 0.1$): zonal disturbance.



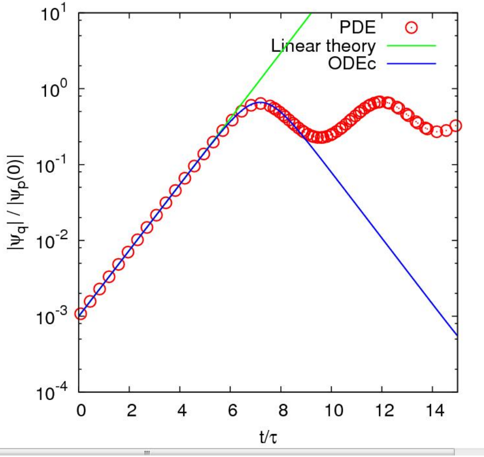
Original drift wave experiences self-focusing, but it preserves its wave identity. Zonal jets are also narrow and located in the high wave amplitude regions.

Weak wave ($M = 0.1$): off-axis disturbance.



Unstable but recursive (periodic).

Weak wave ($M = 0.1$): off-axis disturbance.



Conclusions

- Modulational instability of a travelling drift wave exists for any nonlinearity M.
- Most unstable disturbance is zonal for large M's and an inclined wave for small M.
- Two limits : Euler limit for $M \gg 1$ vs weak resonance interaction for $M \ll 1$.
- Zonal jets are mostly eastward due to the β-effect.
- Nonlinear pinching of jets (for any M). Simplest model for the transport barriers.
- Effect of the finite gyroradius?
- Role of the modulational instability in cases with a broad initial range of scales?