Introduction to Cosmology

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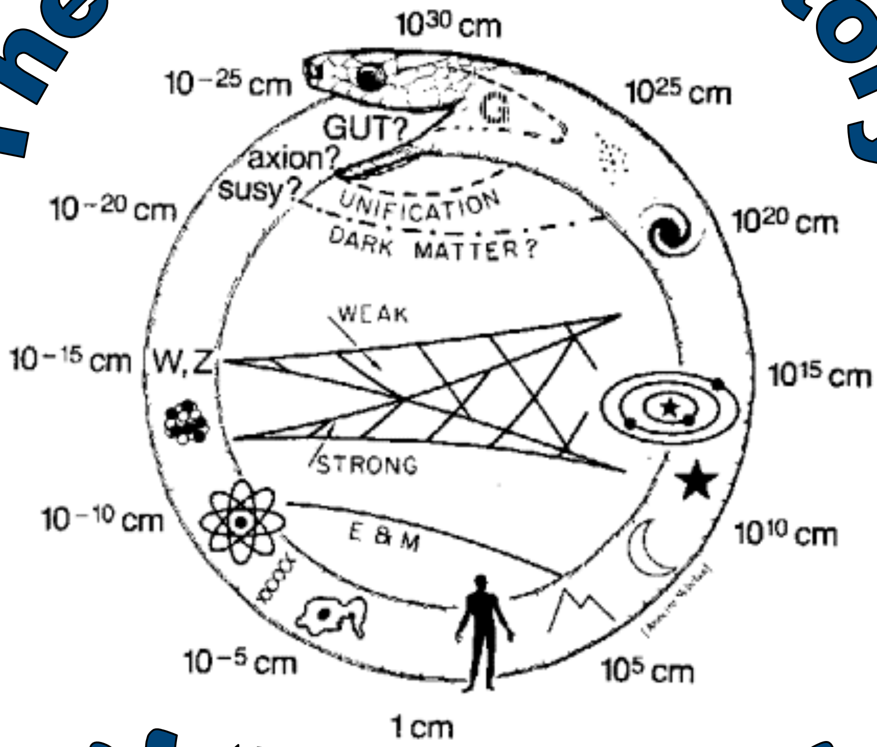
- **Seeing the edge of the Universe:** From speculation to science
- **Constructing the Universe:** Relativistic world models
- **The history of the Universe:** Decoupling of the relic radiation and nucleosynthesis of the light elements
- **The content of the Universe:** Dark matter & dark energy
- **Making sense of the Universe:** Fundamental physics & cosmology

http://www-thphys.physics.ox.ac.uk/user/SubirSarkar/cernlectures.html

Lecture 3

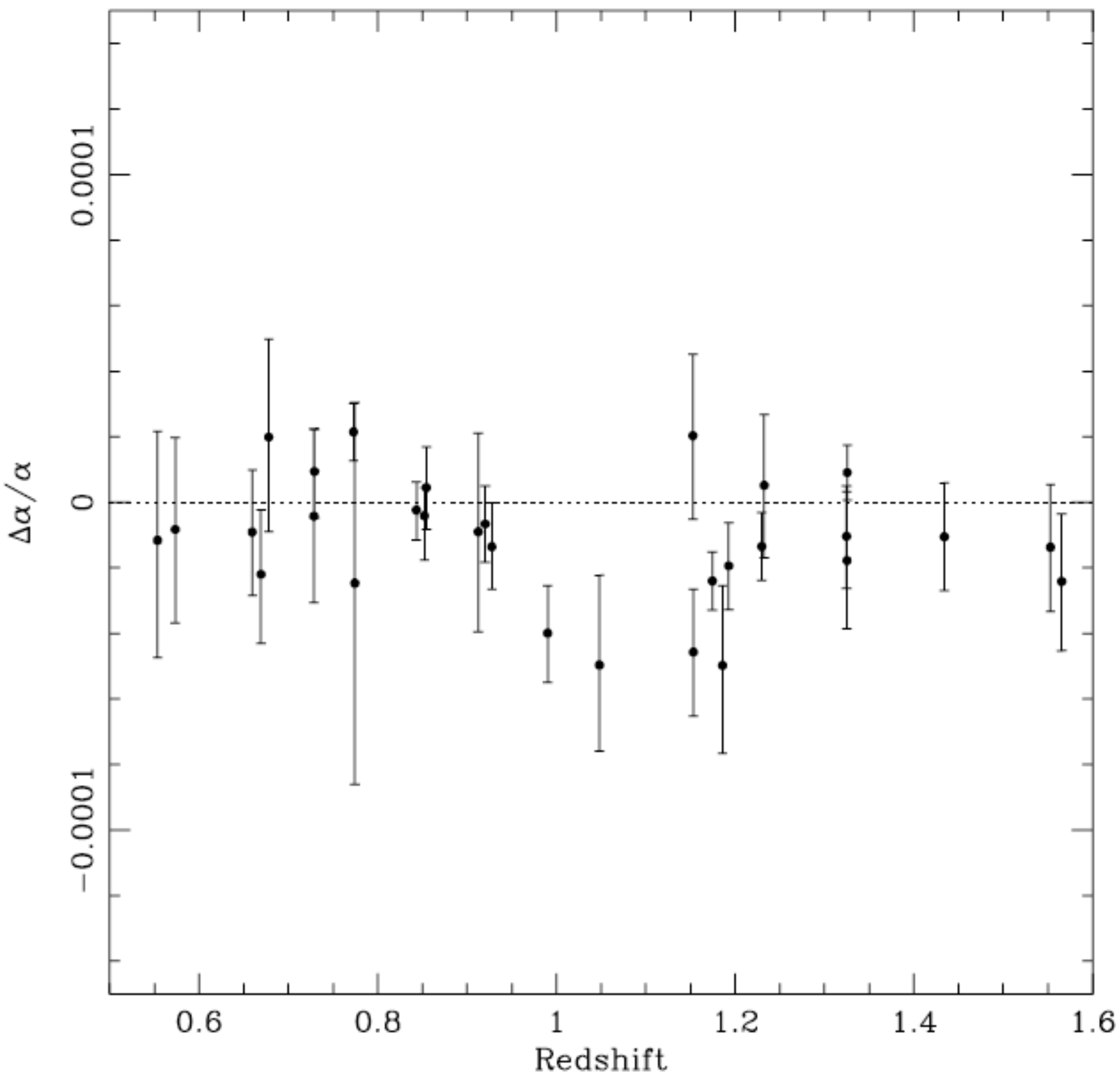
The thermal history of the universe



of the universe

Decoupling of the relic radiation and synthesis of the light elements

We can check *experimentally* that physical constants such as α have had ~the same values far back in our past as they do now



Webb *et al* (1999)

So we are entitled to extrapolate back with confidence …

Knowing the **equation of state**, we can solve the Friedman equation …

For matter: $\frac{d}{dt}(\rho a^3) = 0 \quad \Rightarrow \quad \rho = \rho_0 / a^3 = \rho_0 (1+z)^3$

so we get $\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G \rho_0}{3a^3}$ i.e. $a(t) = \left(\frac{t}{t_0}\right)^{2/3}$

For radiation: $\frac{d}{dt}(\rho a^4) = 0 \quad \Rightarrow \quad \rho = \rho_0 / a^4 = \rho_0 (1+z)^4$

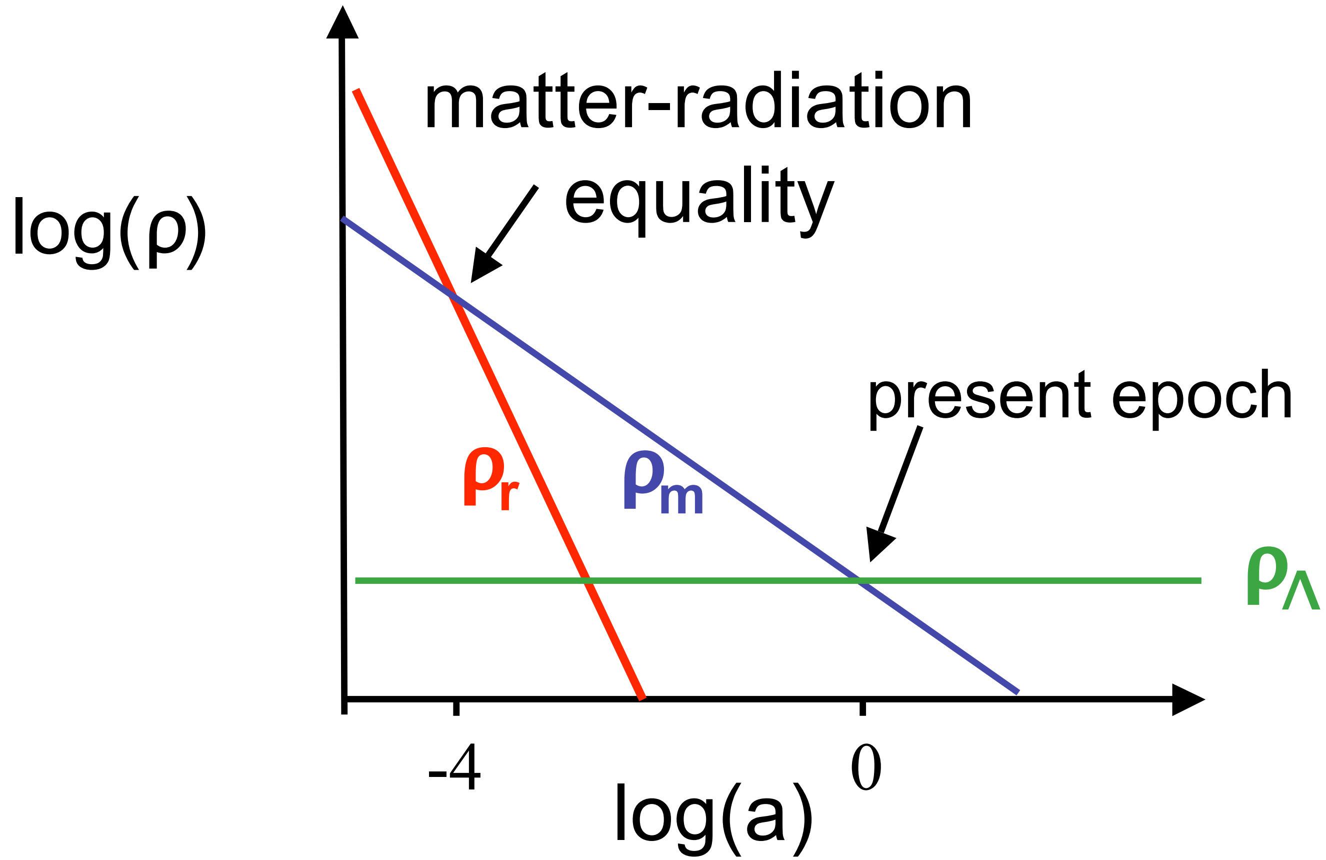
So radiation will dominate over other components as we go to early times

$a(t) = \left(\frac{t}{t_0}\right)^{1/2} \quad \Rightarrow \quad \rho_{\mathrm{r}} \propto t^{-2}$ **Radiation-dominated era**

But at $a_{\mathrm{eq}} = \rho_{\mathrm{r},0} / \rho_{\mathrm{m},0}$ the matter density will come to dominate …

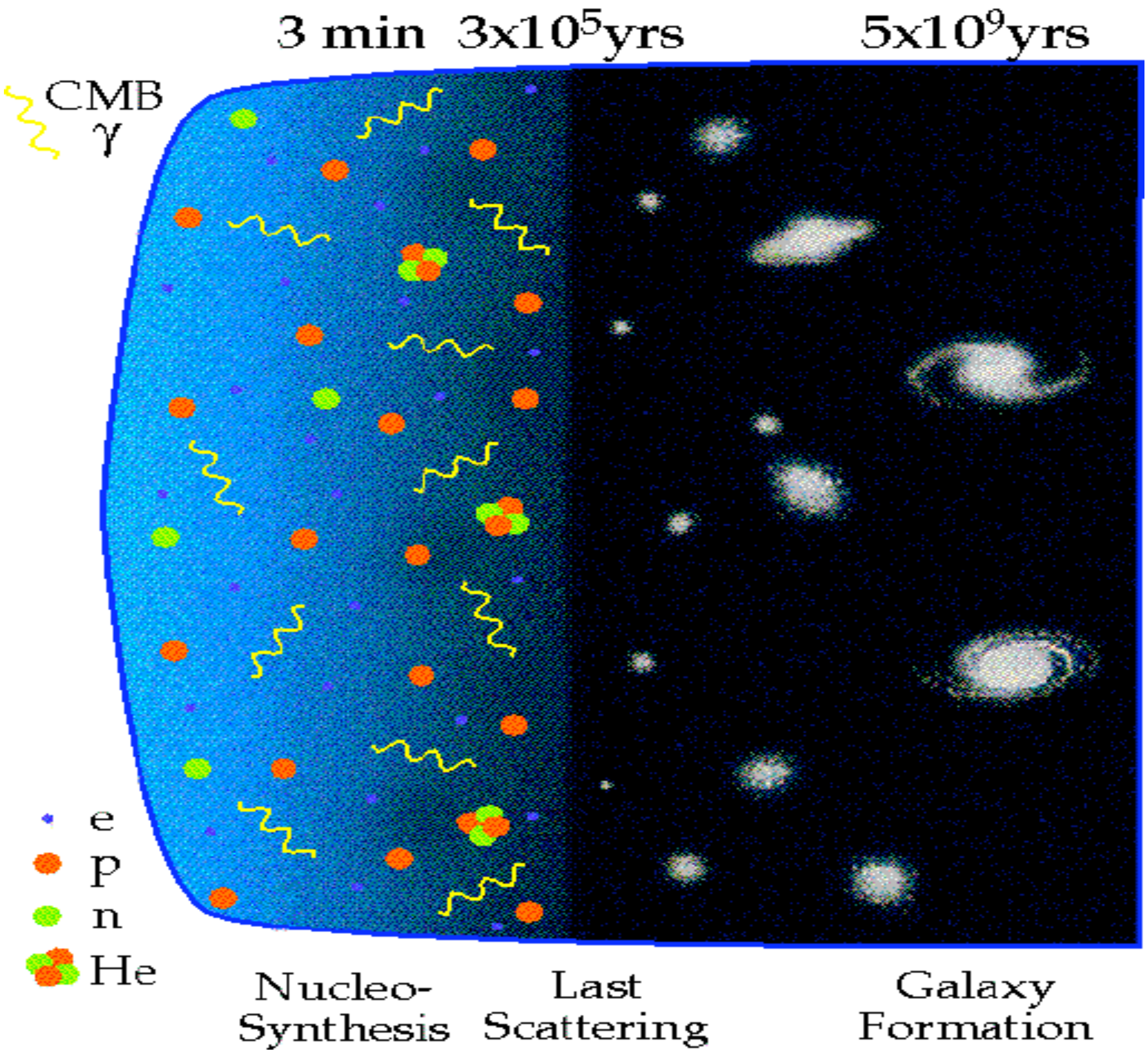
Note that $\rho_{\mathrm{m}} \propto t^{-2}$ as well during the **Matter-dominated era**

Evolution of different energy components during the evolution of the Universe



Very recently (at z ~ 1), the expansion seems to be accelerating under the influence of a cosmological constant $\Lambda \sim H^2$ of which more later …

(Very) Brief History



The Standard Model of the Early Universe

Thermodynamics of ultra-relativistic plasma:

Number density: $$n = \frac{\xi(3)}{\pi^2} g'(T) T^3$$

Energy density: $$\rho = 3p = \frac{\pi^2}{30} g(T) T^4$$

Entropy density: $$s \equiv \frac{p+\rho}{T} = \frac{2\pi^2}{45} g(T) T^3$$

Where, the number of relativistic degrees of freedom *sum* over all bosons and fermions with appropriate weight:

$$g'(T) = g_b(T) + \frac{3}{4} g_f(T)$$

$$g(T) = g_b(T) + \frac{7}{8} g_f(T)$$

In the absence of dissipative processes (e.g. phase transitions which generate entropy) the **comoving entropy** is *conserved*:

$$\frac{d}{dt}(sa^3) = 0 \quad \Rightarrow \quad s \propto 1/a^3 \qquad \text{i.e.} \quad T \propto 1/a$$

At early times the curvature term becomes negligible (compared to radiation) so the Friedmann equation simplifies to:

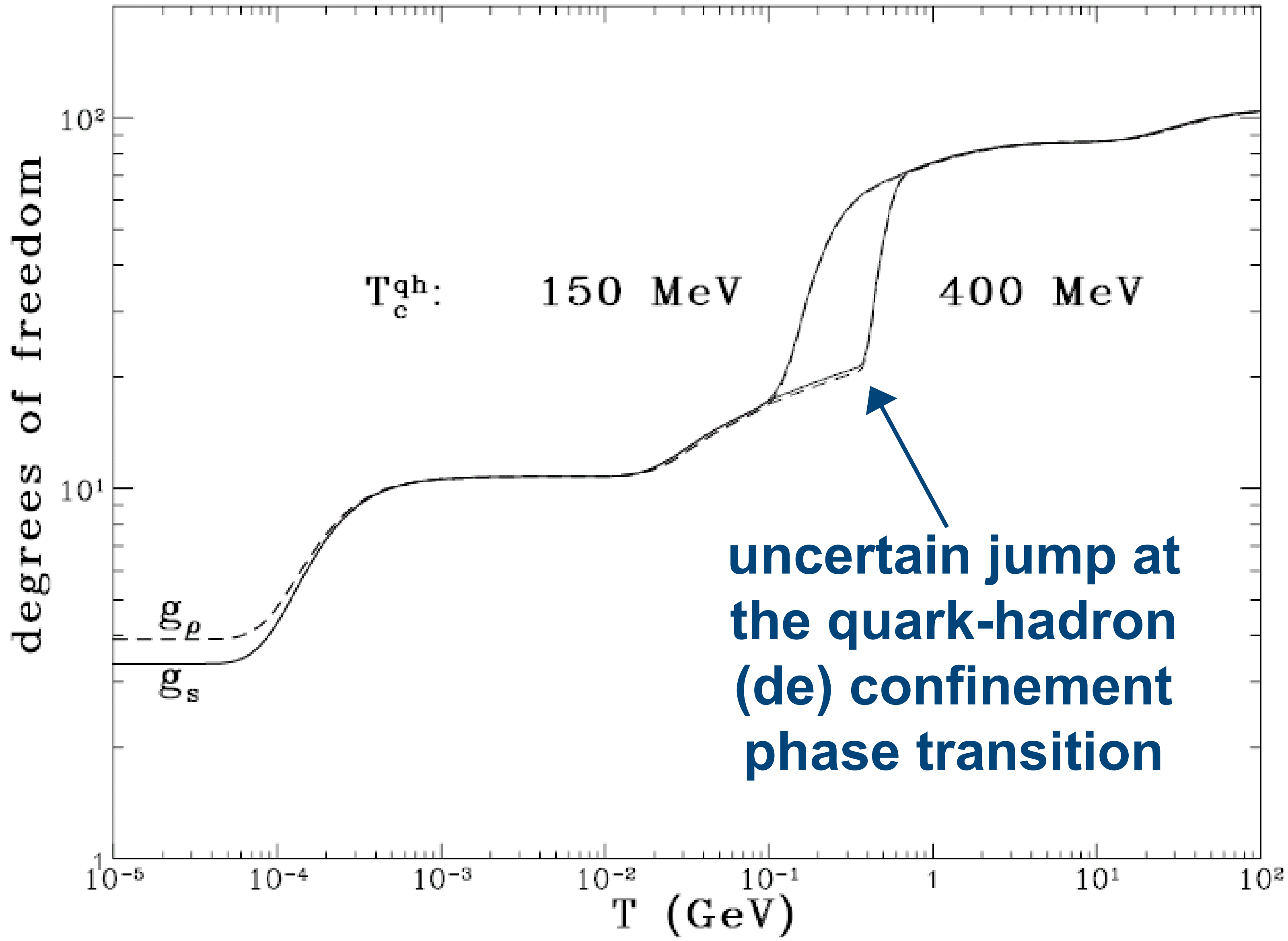
$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G\rho}{3}$$

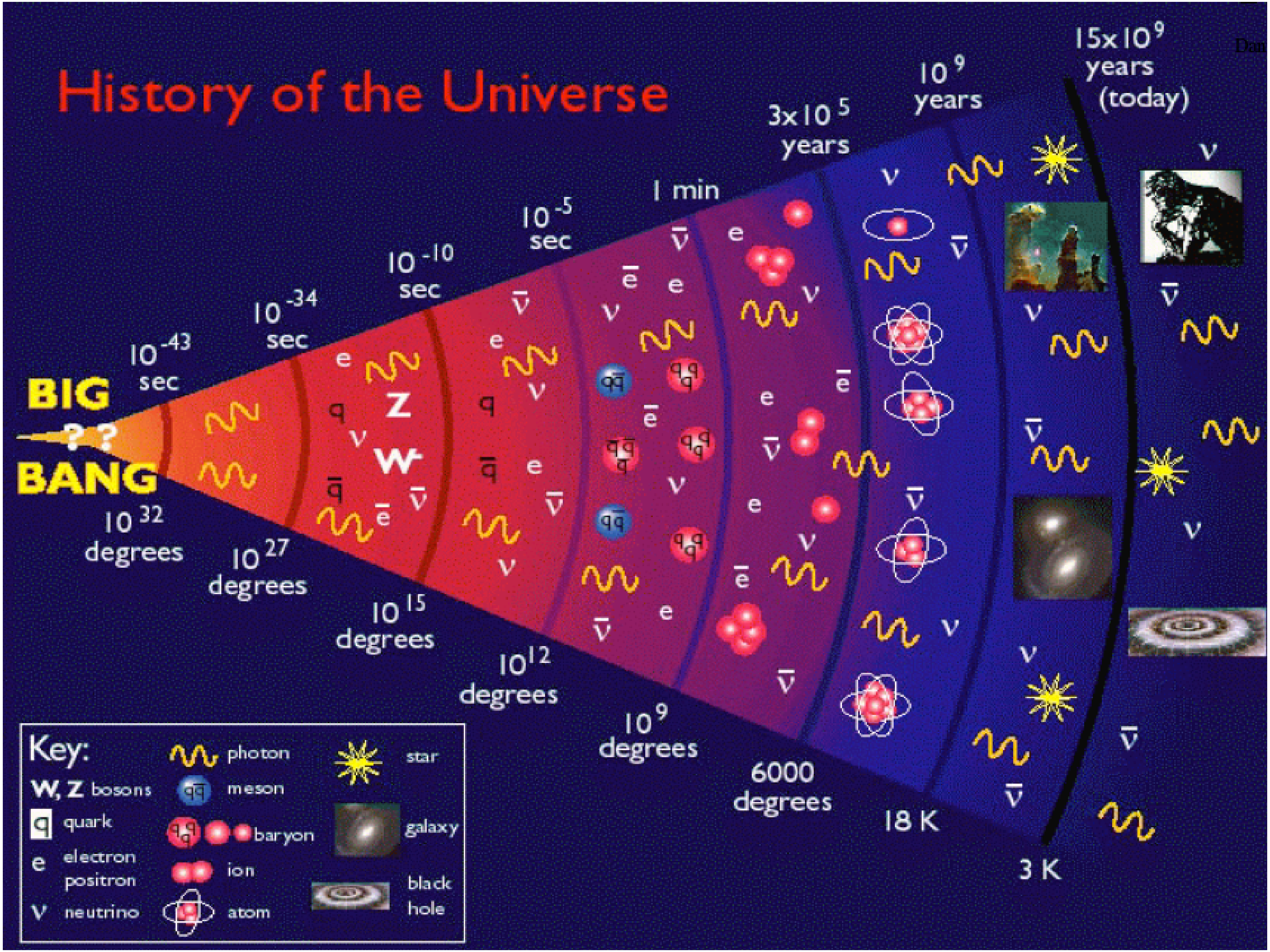
Integrating this yields the time-temperature relationship:

$$t\,(\mathrm{s}) = 2.42\, g^{-1/2}\, (T/\mathrm{MeV})^{-2}$$

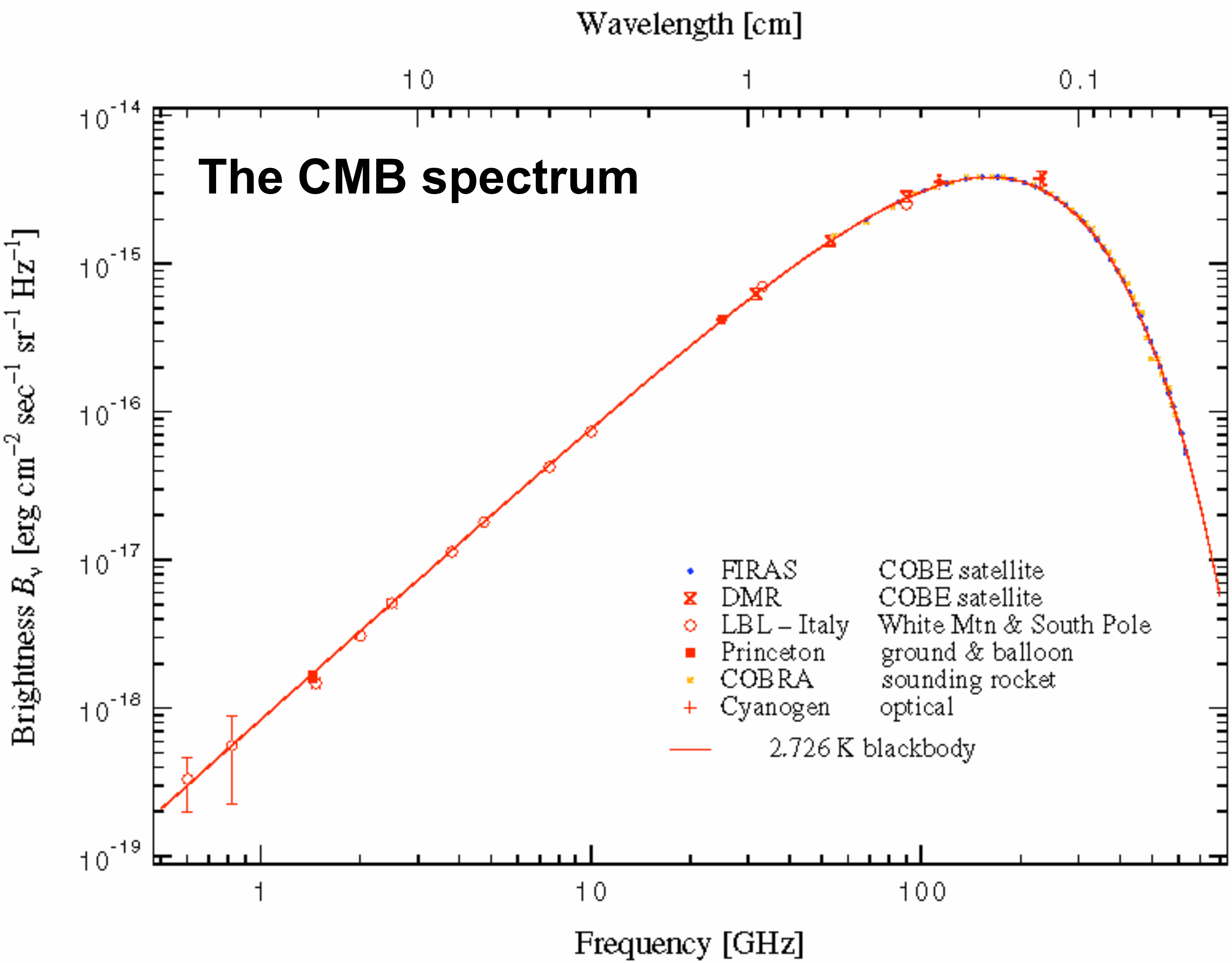
So we can work out when events of *physical significance* occurred in our past (according to the **Standard Model** of particle physics)

To get this right we need to count all the bosons and fermions contributing to the relativistic degrees of freedom … and take into account our uncertain knowledge of possible phase transitions





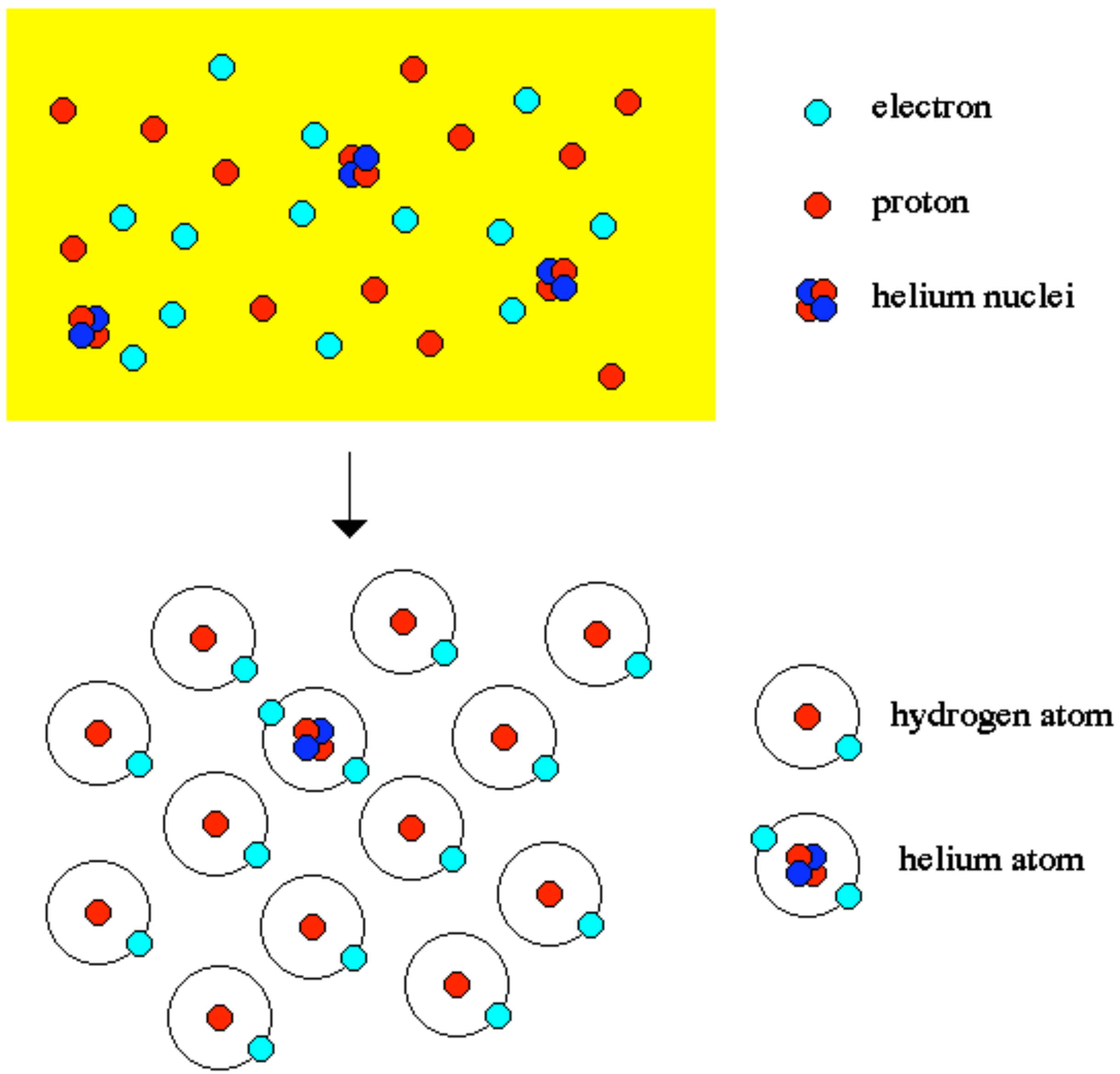
History of the Universe
BIG BANG
??
10^{-43} sec
10^{-34} sec
10^{-10} sec
10^{-5} sec
1 min
3×10^{5} years
10^{9} years
15×10^{9} years (today)
10^{32} degrees
10^{27} degrees
10^{15} degrees
10^{12} degrees
10^{9} degrees
6000 degrees
18 K
3 K
Key:
W, Z bosons
q quark
e electron positron
ν neutrino
photon
meson
baryon
ion
atom
star
galaxy
black hole

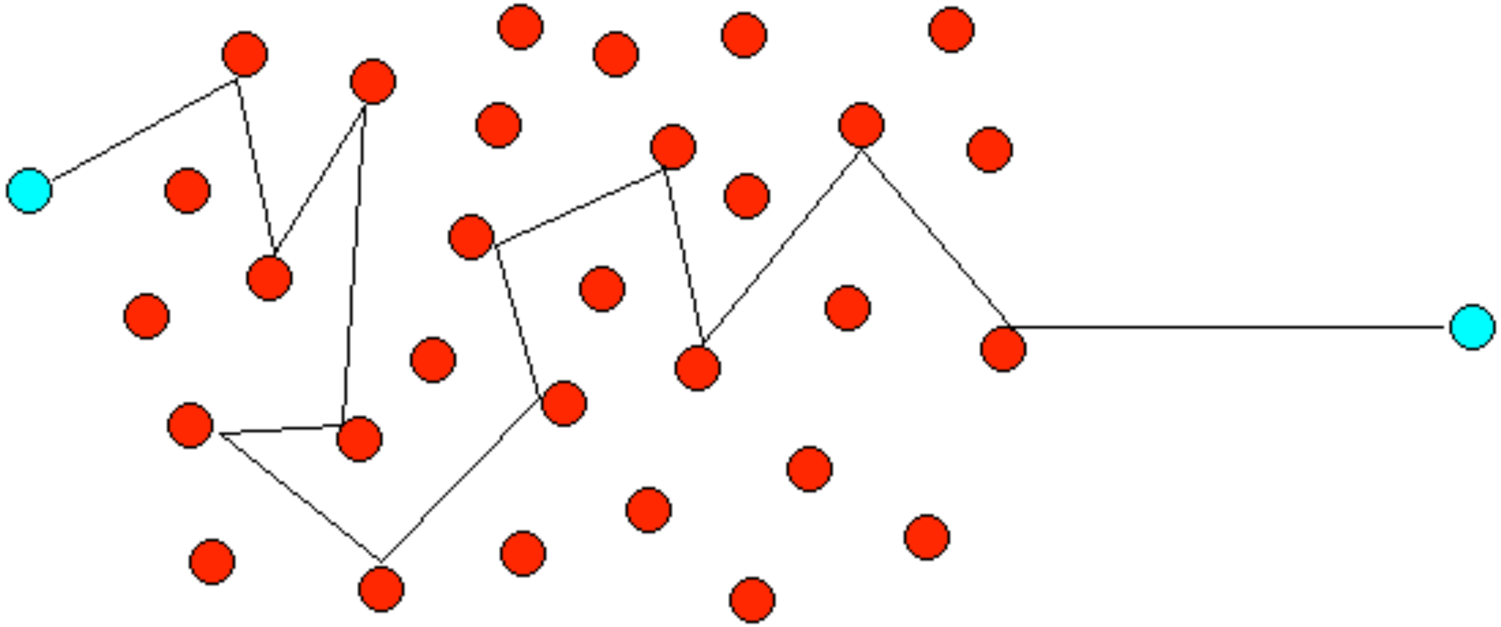


Such a perfect blackbody is testimony to our hot, dense past

Recombination

As the Universe expands and cools, protons and electrons combine to form hydrogen (the most abundant element). And helium nuclei combine with electrons to form helium atoms. This process is called recombination.





the Universe is opaque at high densities (the mean free path of a photon is very short), as the density drops with time, the Universe becomes transparent (the mean free path of a photon becomes very large).

The ionisation fraction x_e drops rapidly at (re)combination so the Thomson scattering rate also decreases sharply below the Hubble expansion rate … this defines a ***last scattering surface*** for the relic photons … which we see today as the cosmic microwave background

Interaction between photons and matter

Thomson scattering on electrons: $\gamma + e \rightarrow \gamma + e$

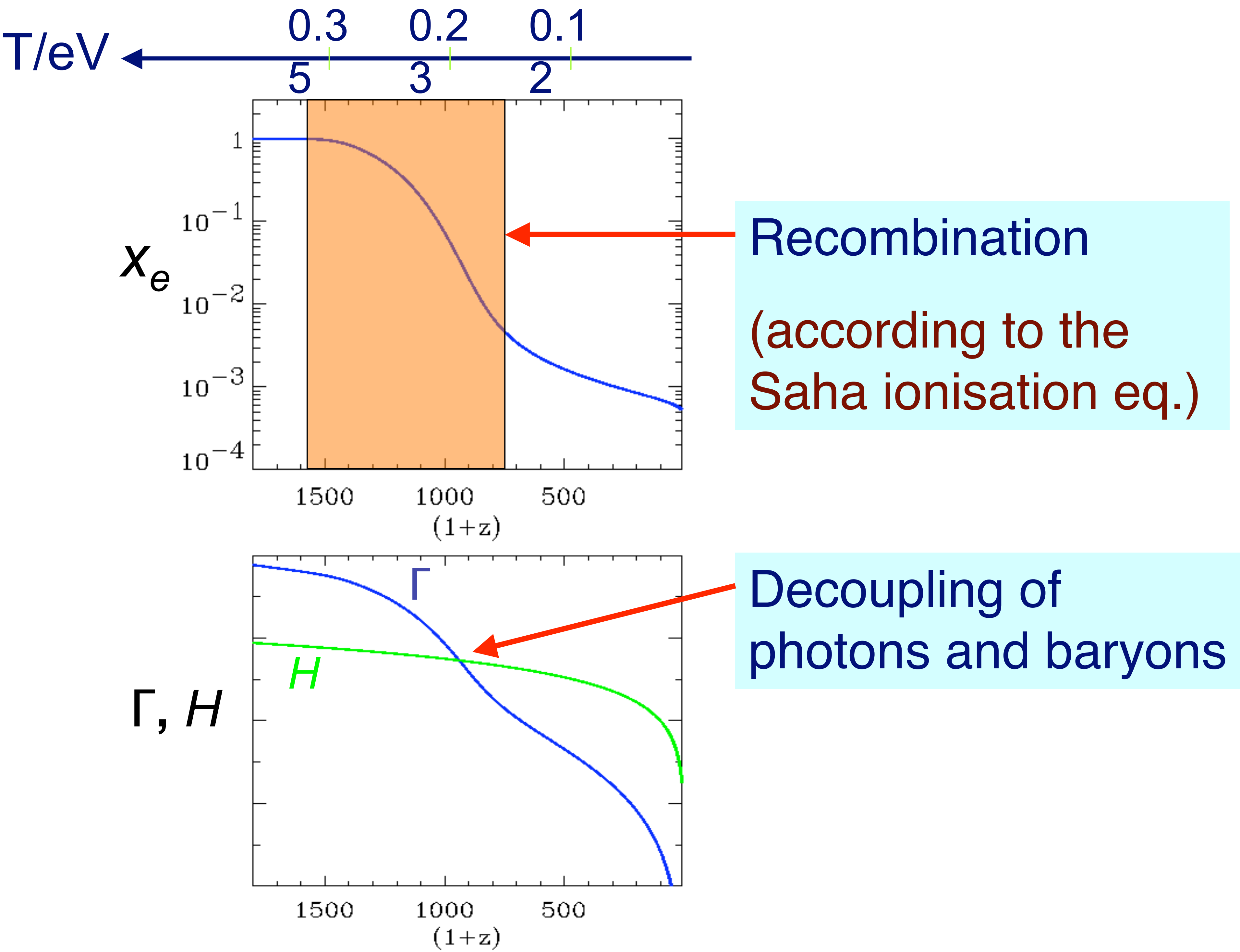
Interaction rate for photons:

$$\Gamma_{\text{Thomson}} = n_e \langle \sigma_T |v| \rangle \propto x_e T^3 \sigma_T$$

Expansion rate of the universe (MD era): $H \propto T^{3/2}$

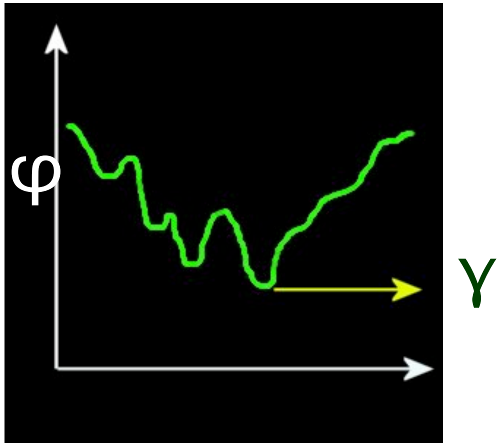
$\Gamma_{\text{Thomson}} > H$: Photons and matter *in equilibrium*

$\Gamma_{\text{Thomson}} < H$: Photons and matter stop interacting

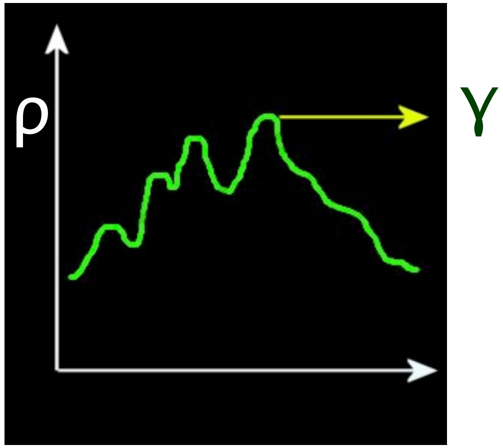


T/eV
0.3
0.2
0.1
5
3
2
x_e
1
10^{-1}
10^{-2}
10^{-3}
10^{-4}
1500
1000
500
(1+z)
Recombination
(according to the Saha ionisation eq.)
Γ
H
Γ, H
1500
1000
500
(1+z)
Decoupling of photons and baryons

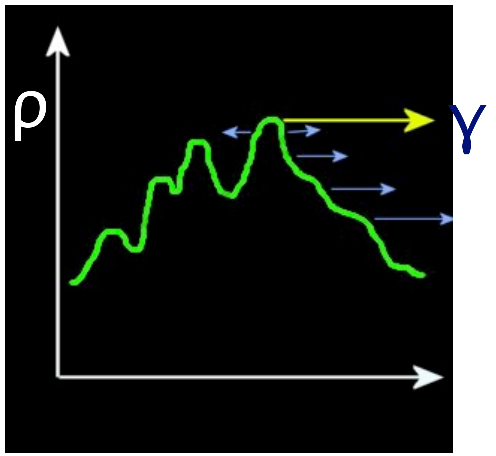
Fluctuations in the matter density → fluctuations in the CMB temperature



Photons are redshifted as they move out of gravitational potential wells

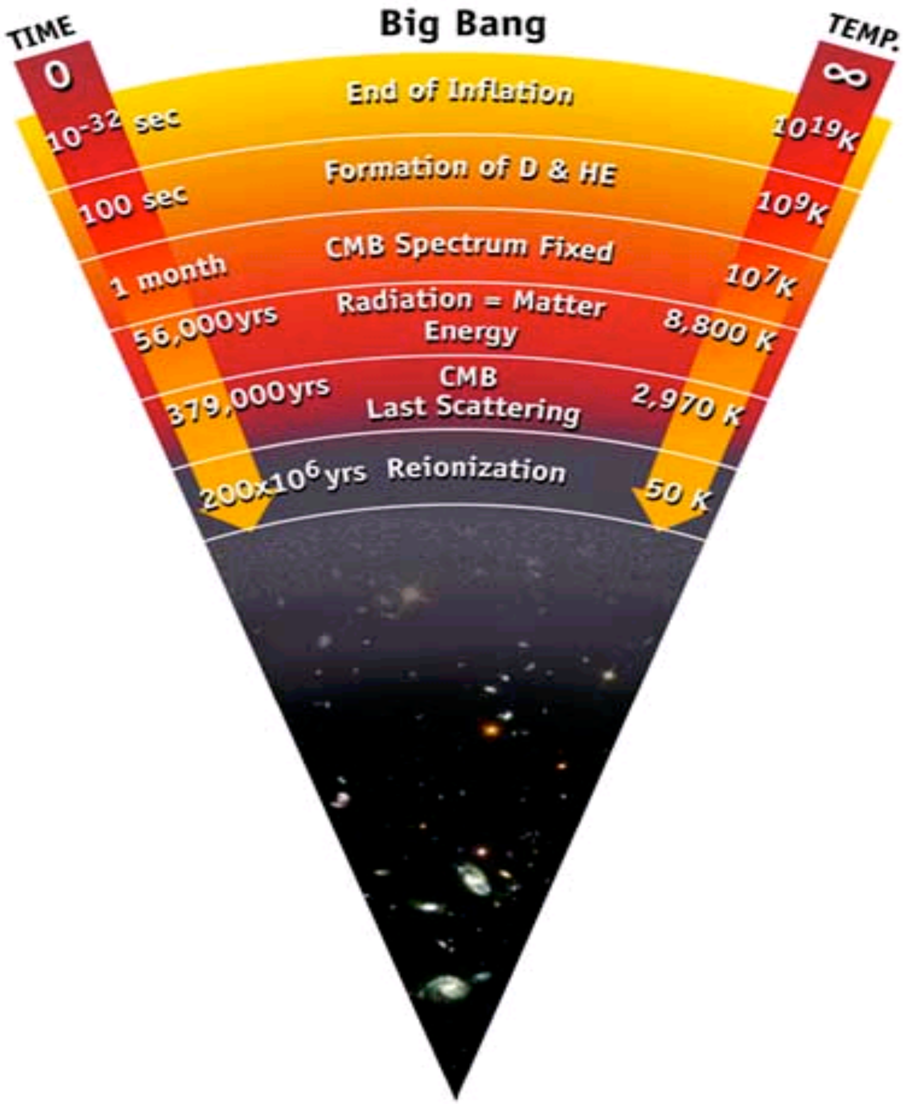


Dense regions have higher temperature
⇒ photons have higher energy



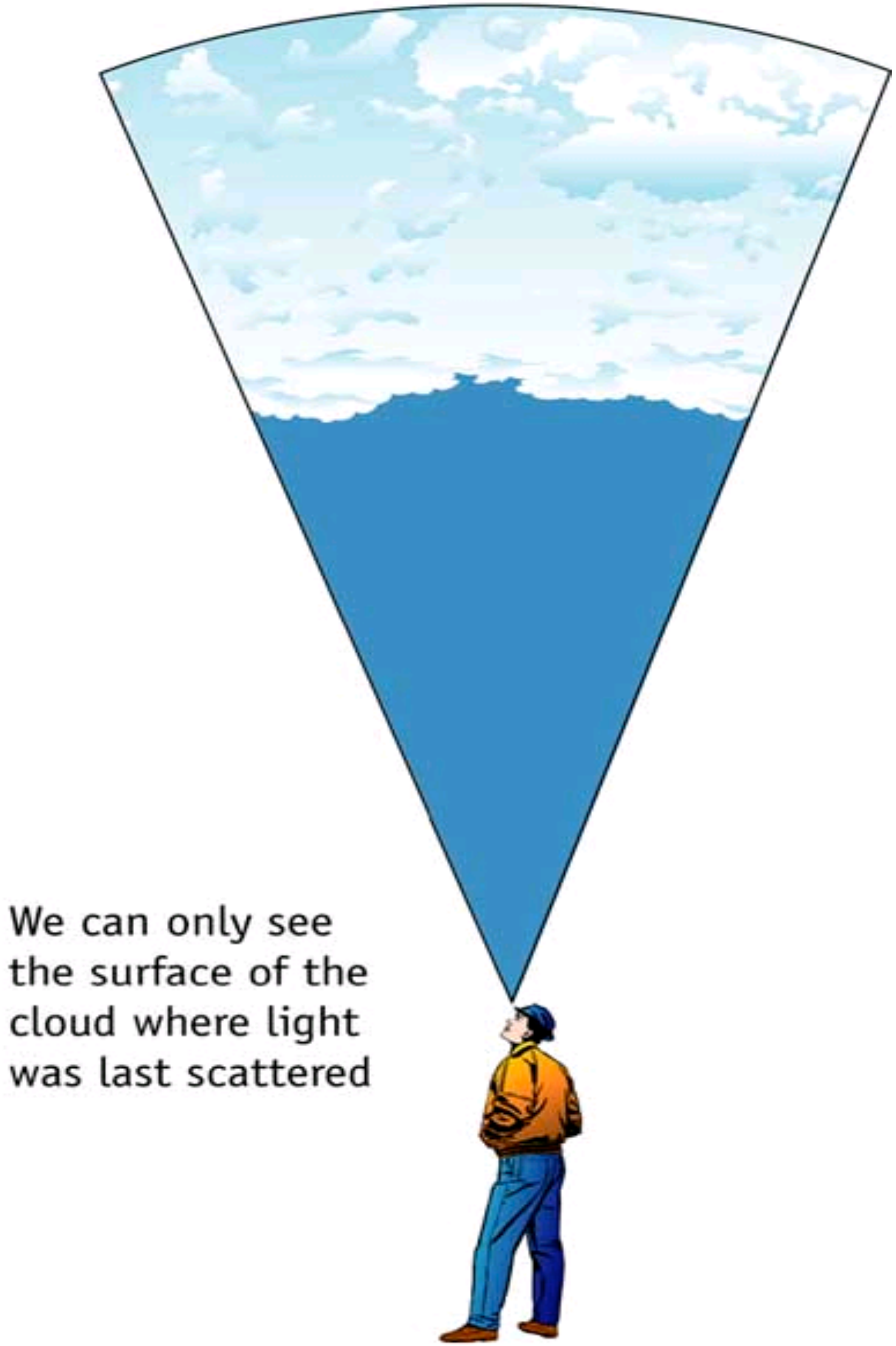
Photons emitted from a moving surface are red/blue-shifted

Fortunately the effects do not *quite* cancel so the CMB carries a memory of the past



PRESENT
13.7 Billion Years after the Big Bang

The cosmic microwave background Radiation's "surface of last scatter" is analogous to the light coming through the clouds to our eye on a cloudy day.

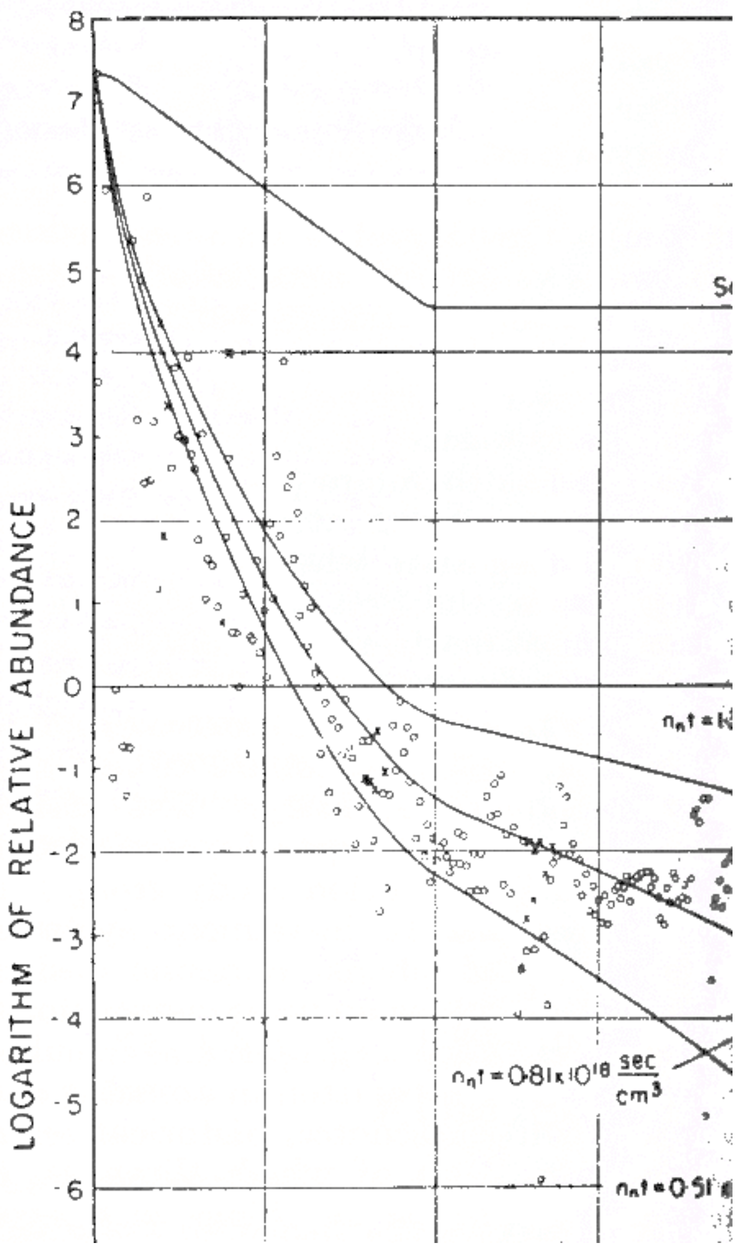


Synthesis of the light elements

THE EVOLUTION OF THE UNIVERSE

By Dr. G. GAMOW
George Washington University, Washington, D.C.

THE discovery of the red shift in the spectra of distant stellar galaxies revealed the important fact that our universe is in the state of uniform expansion, and raised an interesting question as to whether the present features of the universe could be understood as the result of its evolutionary development, which must have started a few thousand million years ago from a homogeneous state of extremely high density and temperature. We conclude first of all that the relative abundances of various atomic species (which were found to be essentially the same all over the observed region of the universe) must represent the most ancient archæological document pertaining to the history of the universe. These abundances must have been established during the earliest stages of expansion when the temperature of the primordial matter was still sufficiently high to permit nuclear transformations to run through the entire range of chemical elements. It is also interesting to notice that the observed relative amounts of natural radioactive elements suggest that their nuclei must have been formed (presumably along with all other stable nuclei) rather soon after the beginning of the universal



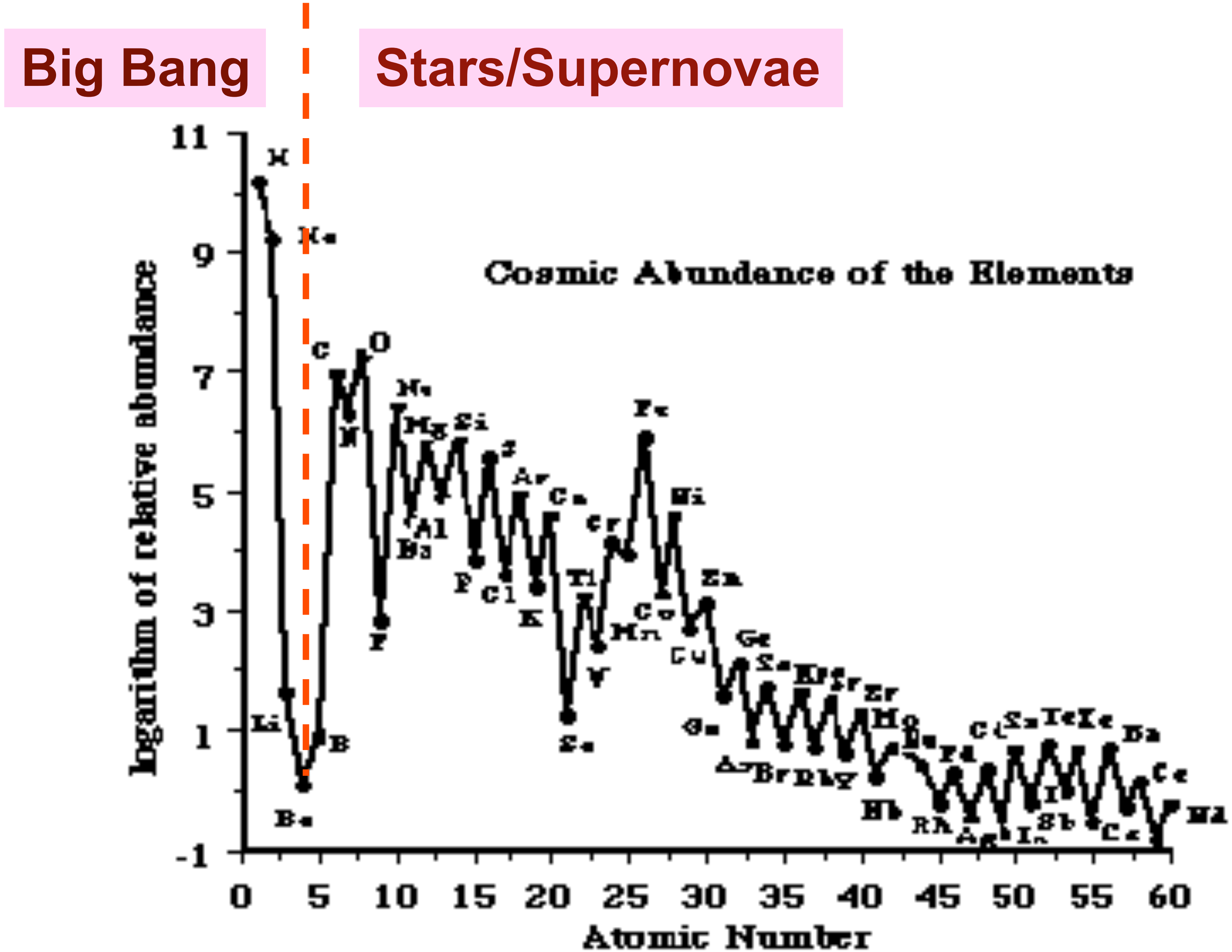
Periodic Table of the Elements

	IA	IIA	IIIB	IVB	VB	VIB	VIIB	VII	VII	VII	IB	IB	IIIA	IVA	VA	VIA	VIIA	0
	1 H																	2 He
2	3 Li	Be											5 B	6 C	7 N	8 O	9 F	10 Ne
3	11 Na	12 Mg											13 Al	14 Si	15 P	16 S	17 Cl	18 Ar
4	19 K	20 Ca	21 Sc	22 Ti	23 V	24 Cr	25 Mn	26 Fe	27 Co	28 Ni	29 Cu	30 Zn	31 Ga	32 Ge	33 As	34 Se	35 Br	36 Kr
5	37 Rb	38 Sr	39 Y	40 Zr	41 Nb	42 Mo	43 Tc	44 Ru	45 Rh	46 Pd	47 Ag	48 Cd	49 In	50 Sn	51 Sb	52 Te	53 I	54 Xe
6	55 Cs	56 Ba	57 *La	72 Hf	73 Ta	74 W	75 Re	76 Os	77 Ir	78 Pt	79 Au	80 Hg	81 Tl	82 Pb	83 Bi	84 Po	85 At	86 Rn
7	87 Fr	88 Ra	89 +Ac	104 Rf	105 Ha	106 106	107 107	108 108	109 109	110 110	111 111	112 112						

Naming conventions of new elements

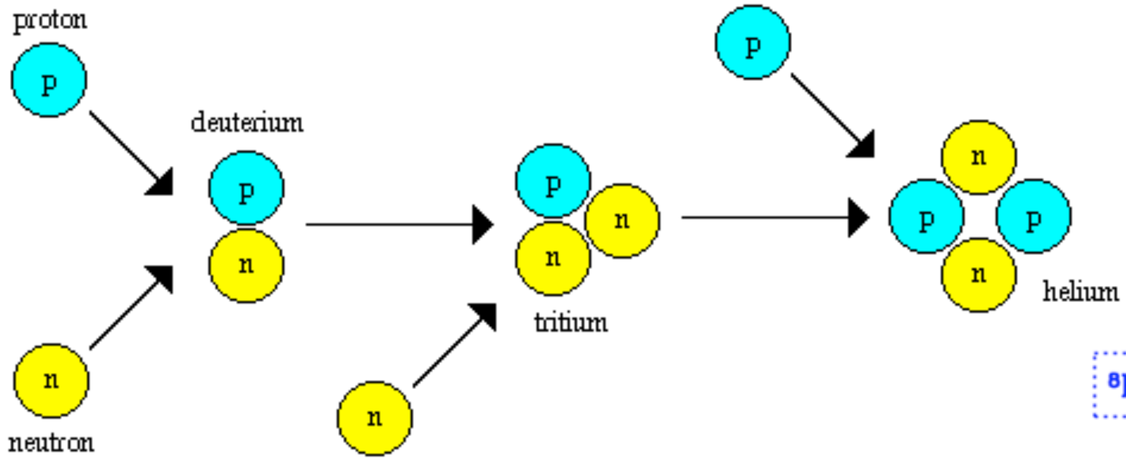
* Lanthanide Series	58 Ce	59 Pr	60 Nd	61 Pm	62 Sm	63 Eu	64 Gd	65 Tb	66 Dy	67 Ho	68 Er	69 Tm	70 Yb	71 Lu
+ Actinide Series	90 Th	91 Pa	92 U	93 Np	94 Pu	95 Am	96 Cm	97 Bk	98 Cf	99 Es	100 Fm	101 Md	102 No	103 Lr

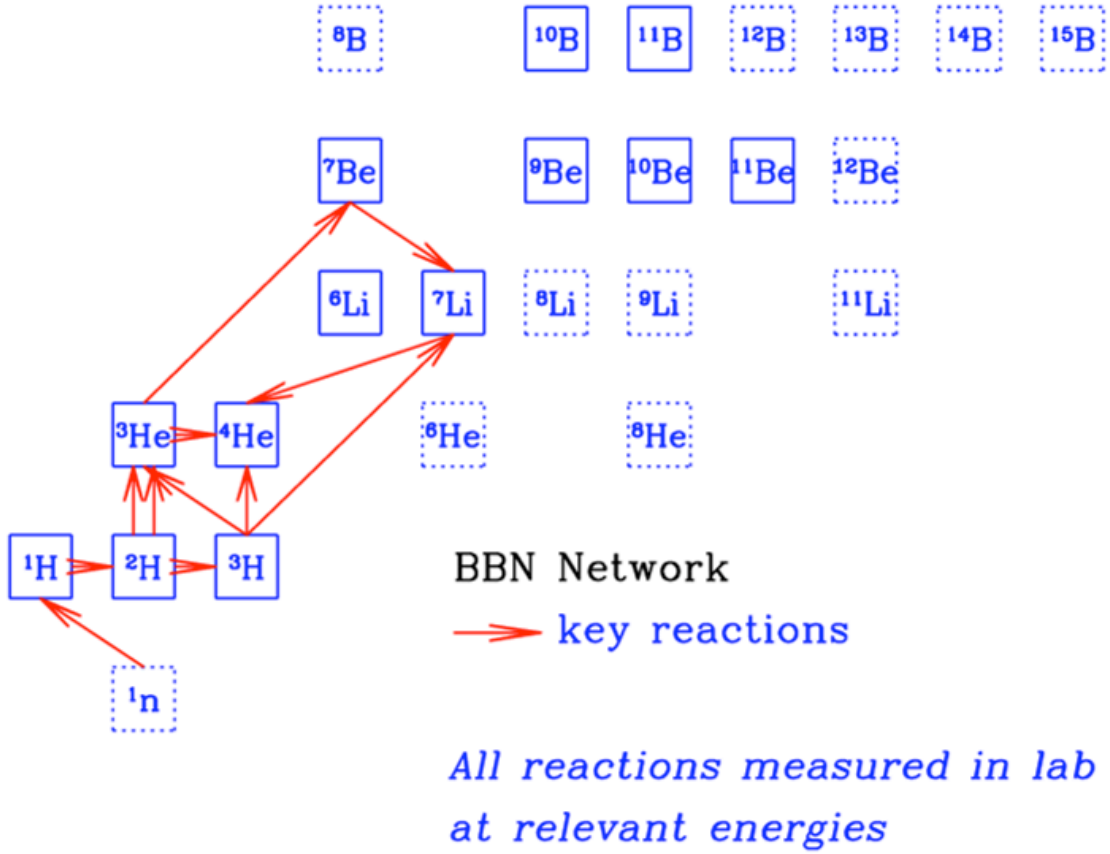
Where did all the elements come from?



Nucleosynthesis

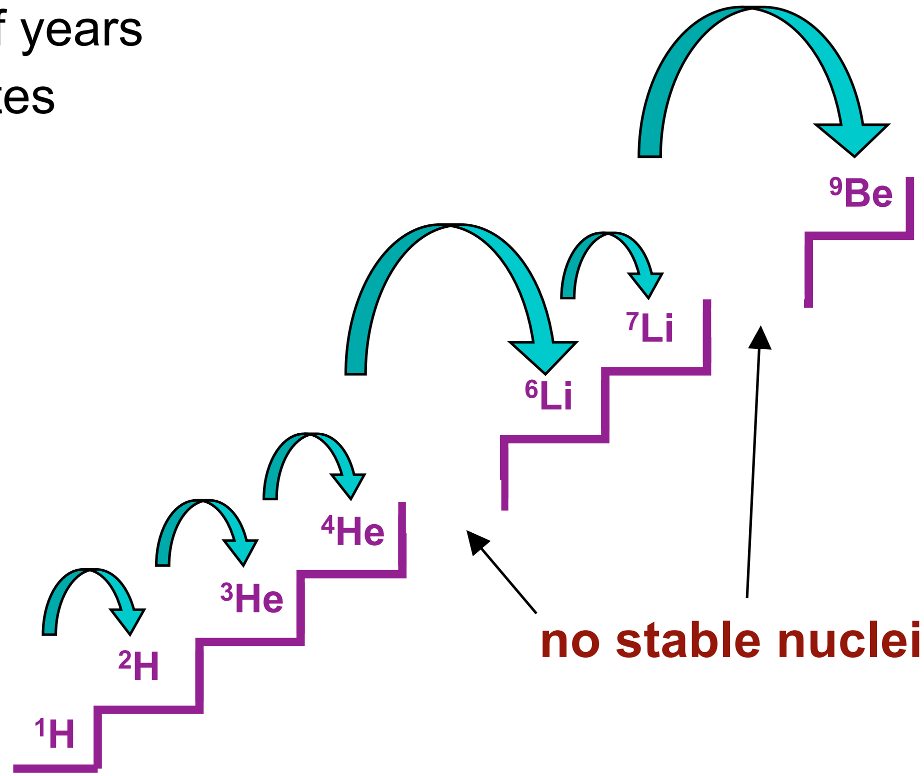
as the Universe cools, protons and neutrons can fuse to form heavier atomic nuclei





Primodial versus Stellar Nucleosynthesis

- **Timescale**
  - » Stellar Nucleosynthesis (SN): billions of years
  - » Primordial Nucleosynthesis (PN): minutes
- **Temperature evolution**
  - » SN: slow increase over time
  - » PN: rapid cooling
- **Density**
  - » SN: 100 g/cm^3
  - » PN: 10^{-5} g/cm^3 (like air!)
- **Photon to baryon ratio**
  - » SN: less than 1 photon per baryon
  - » PN: billions of photons per baryon



The lack of stable elements with masses 5 and 8 make it hard for primordial nucleosynthesis to synthesise elements beyond Helium

Conditions in the Early Universe:

$$T \gtrsim 1 \text{ MeV}$$

$$\rho = \frac{\pi^2}{30}(2 + \frac{7}{2} + \frac{7}{4}N_\nu)T^4$$

$$\eta = n_B/n_\gamma \sim 10^{-10}$$

β-Equilibrium maintained by weak interactions

Freeze-out at ~ 1 MeV determined by the competition of expansion rate $H \sim T^2/M_p$ and the weak interaction rate $\Gamma \sim G_F^2 T^5$

$$n + e^+ \leftrightarrow p + \bar{\nu}_e$$

$$n + \nu_e \leftrightarrow p + e^-$$

$$n \leftrightarrow p + e^- + \bar{\nu}_e$$

At freezeout n/p fixed modulo free neutron decay, $(n/p) \simeq 1/6 \rightarrow 1/7$

Nucleosynthesis Delayed (Deuterium Bottleneck)

$p + n \rightarrow \mathbf{D} + \gamma$ $\qquad \Gamma_p \sim n_B \sigma$

$p + n \leftarrow \mathbf{D} + \gamma$ $\qquad \Gamma_d \sim n_\gamma \sigma e^{-E_B/T}$

Nucleosynthesis begins when $\Gamma_p \sim \Gamma_d$

$\frac{n_\gamma}{n_B} e^{-E_B/T} \sim 1$ $\qquad$ **@** $T \sim 0.1$ **MeV**

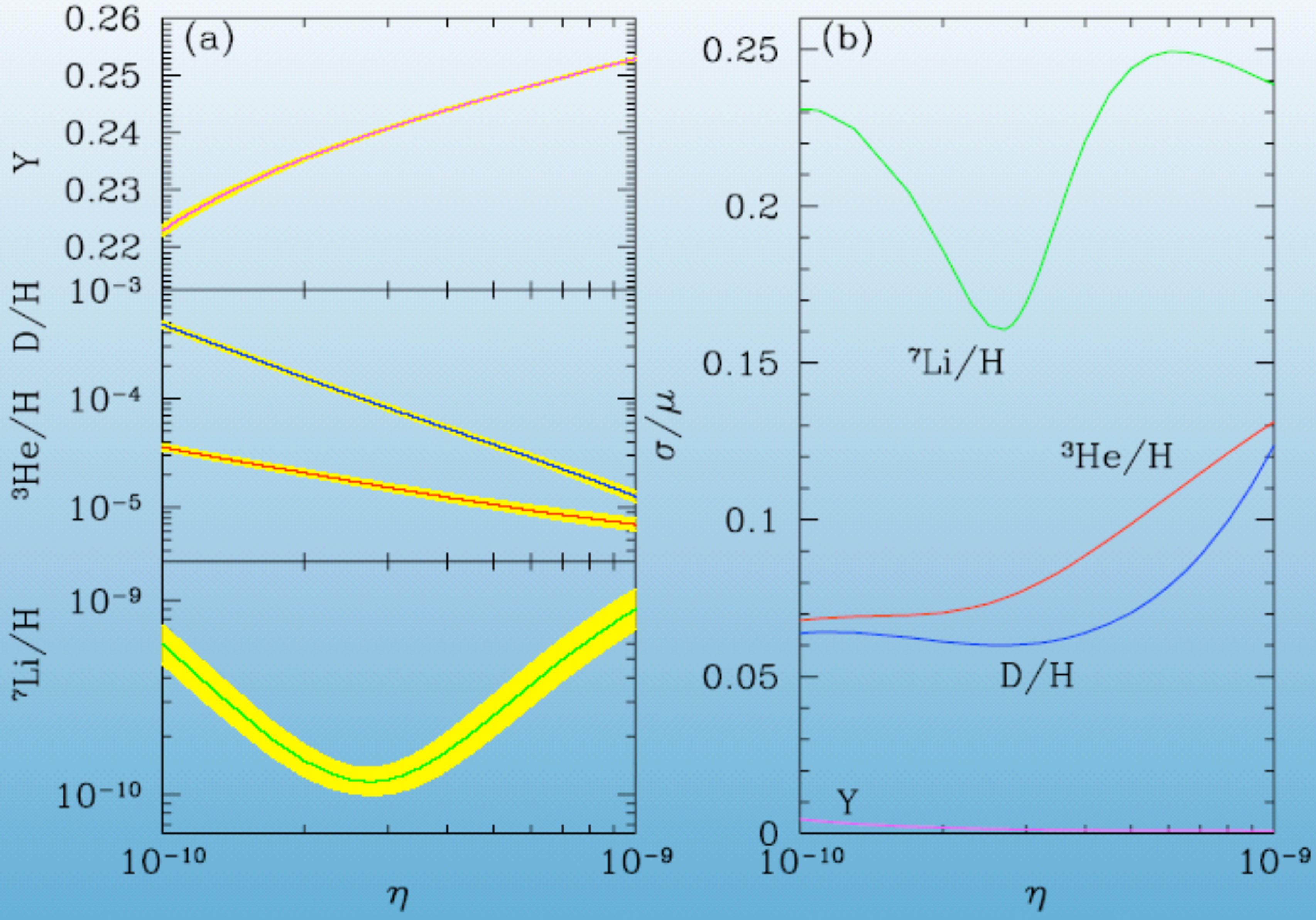
All neutrons $\rightarrow$ ^{4}He

$$Y_p = \frac{2(n/p)}{1 + (n/p)} \simeq 25\%$$

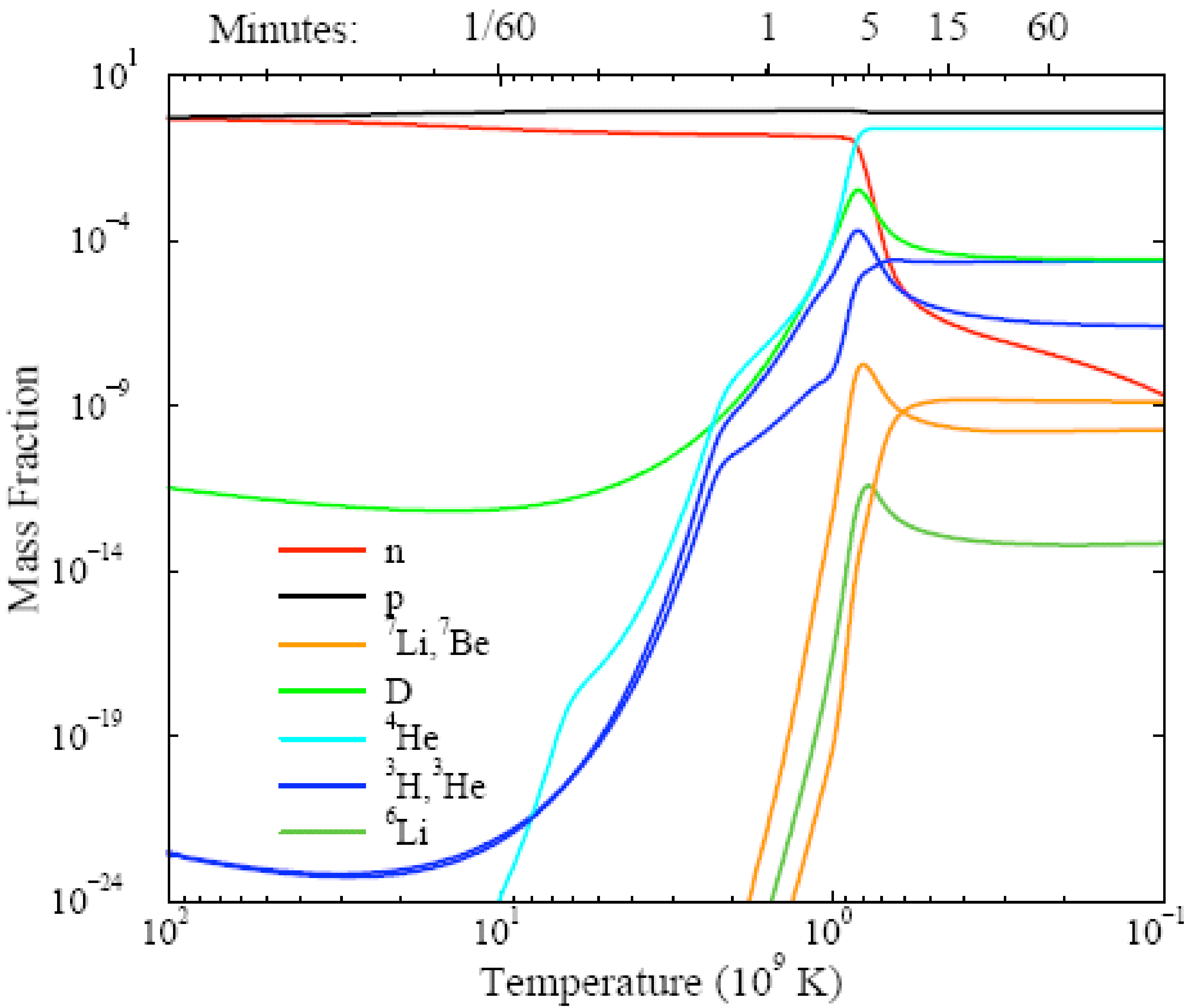
Remainder:

D, ^{3}He $\sim 10^{-5}$ and ^{7}Li $\sim 10^{-10}$ by number

BBN predictions



line widths $\Rightarrow$ theoretical uncertainties (neutron lifetime + nuclear cross sections)



Minutes: 1/60 1 5 15 60
Mass Fraction
10^{1}
10^{-4}
10^{-9}
10^{-14}
10^{-19}
10^{-24}
Temperature (10^9 K)
10^{2} 10^{1} 10^{0} 10^{-1}
n
p
^{7}Li, ^{7}Be
D
^{4}He
^{3}H, ^{3}He
^{6}Li

Primordial nucleosynthesis

- **Time < 15 s, Temperature > 3 x 10^9 K**
  - universe is soup of protons, electrons and other particles … so hot that nuclei are blasted apart by high energy photons as soon as they form

- **Time = 15s, Temperature = 3 x 10^9 K**
  - Still too hot for Deuterium to survive
  - Cool enough for Helium to survive, but too few building blocks

- **Time = 3 min, Temperature = 10^9 K**
  - Deuterium survives and is quickly fused into He
  - no stable nuclei with 5 or 8 nucleons, and this restricts formation of elements heavier than Helium
  - trace amounts of Lithium are formed

- **Time = 35 min, Temperature = 3 x 10^7 K**
  - nucleosynthesis essentially complete
  - Still hot enough to fuse Helium, but density too low for appreciable fusion

Model makes precise predictions about the relative abundances of the light elements ^2H, ^3He, ^4He and ^7Li, as a function of the nucleon density

Nucleosynthesis *without* a computer

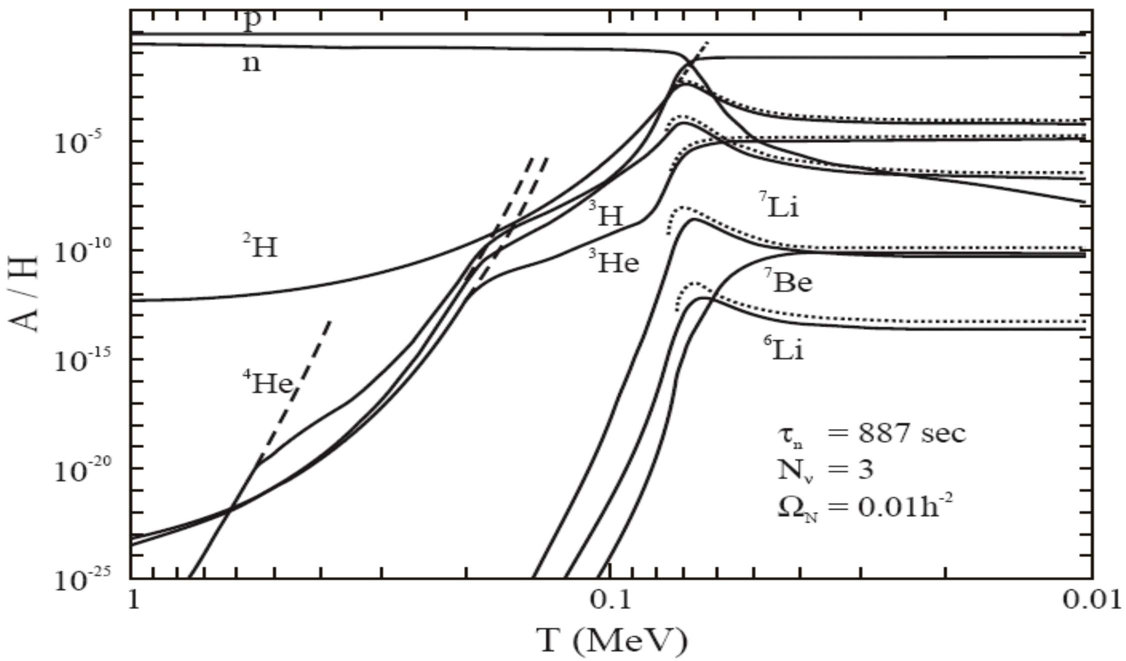
$$\frac{\mathrm{d}X}{\mathrm{d}t} = \underbrace{J(t)}_{\text{source}} - \underbrace{\Gamma(t)}_{\text{sink}}X \quad \Rightarrow \quad X^{\text{eq}} = \frac{J(t)}{\Gamma(t)}$$

… but general solution is:

$$X(t) = \exp\left(-\int_{t_\mathrm{i}}^{t} \mathrm{d}t'\, \Gamma(t')\right)\left[X(t_\mathrm{i}) + \int_{t_\mathrm{i}}^{t} \mathrm{d}t'\, J(t')\, \exp\left(-\int_{t_\mathrm{i}}^{t} \mathrm{d}t''\, \Gamma(t'')\right)\right]$$

If $\left|\frac{\dot{J}}{J} - \frac{\dot{\Gamma}}{\Gamma}\right| \ll \Gamma$ … then abundances approach equilibrium values

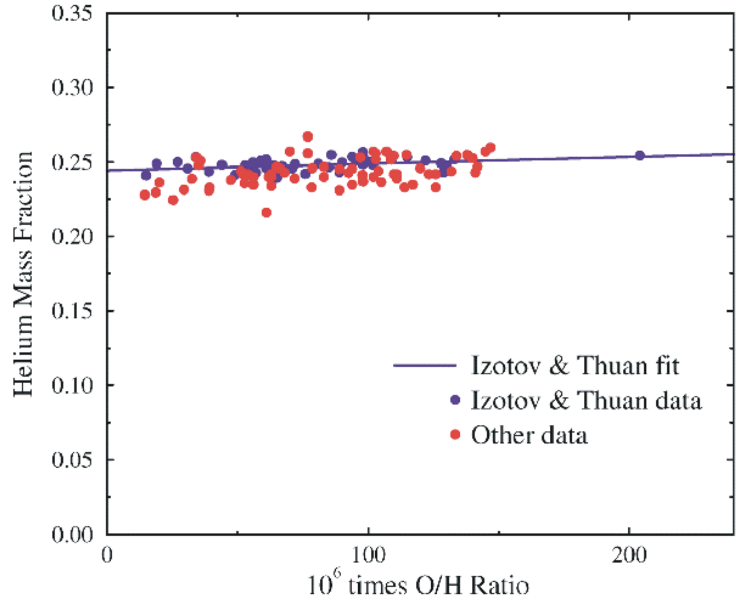
Freeze-out occurs when: $\Gamma \simeq H \quad \Rightarrow \quad X(t \to \infty) \simeq X^{\text{eq}}(t_{\text{fr}}) = \frac{J(t_{\text{fr}})}{\Gamma(t_{\text{fr}})}$



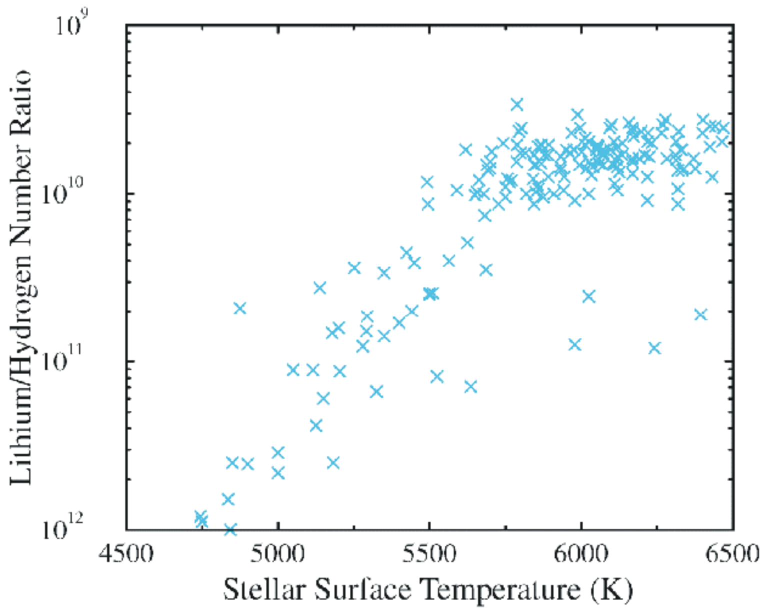
………….. analytic solution

Observations of the light elements He and Li

- **Helium Abundance**
  - » measured in extragalactic HII regions with lowest observed abundances of heavier elements like Oxygen and Nitrogen (i.e. smallest levels of contamination from stellar nucleosynthesis)

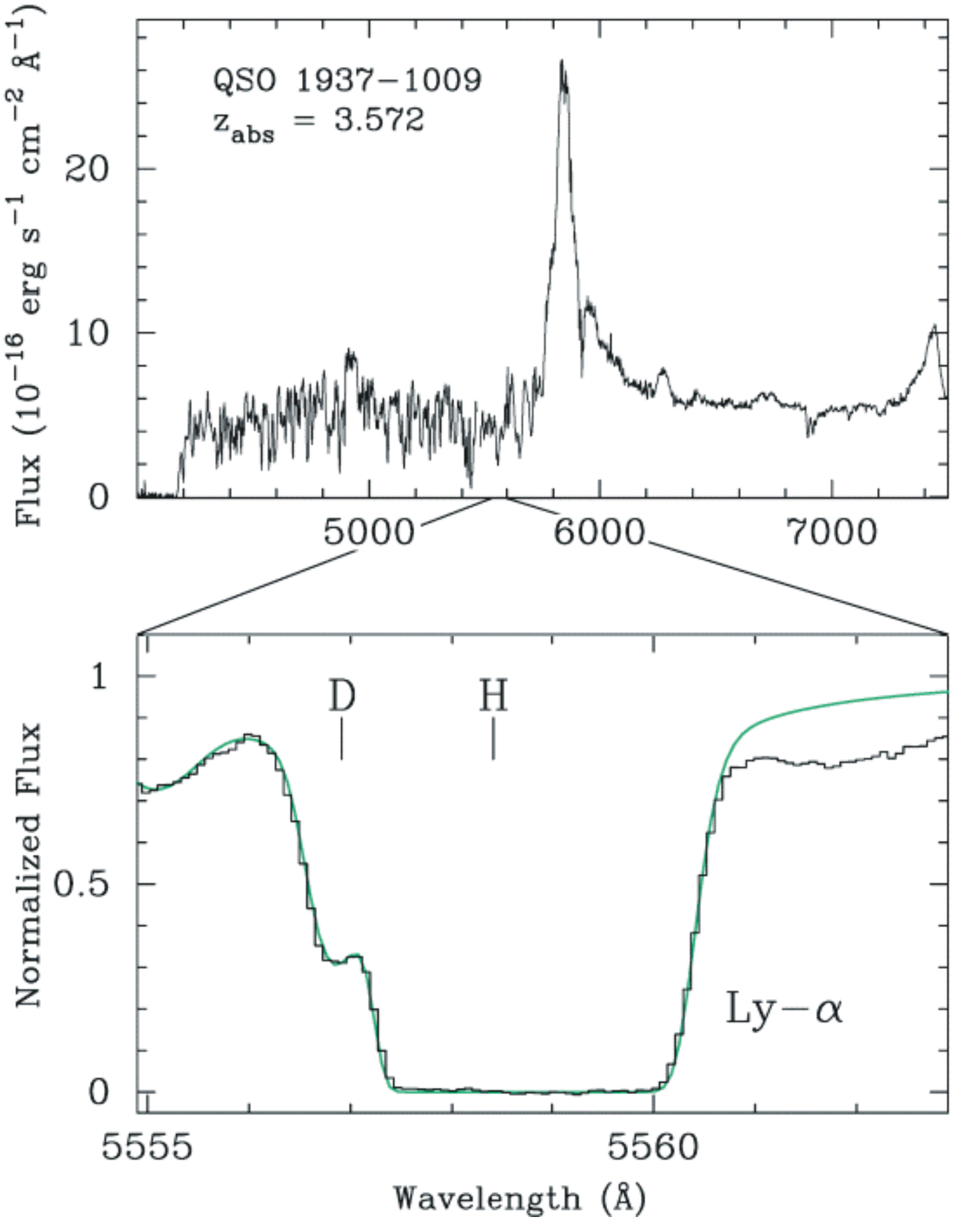


- **Lithium Abundance**
  - » measured in Pop II stars in the Halo
  - » Lithium is very easily destroyed hence observe the transition from low mass stars (lower surface temperature) whose core material is well mixed by convection to higher mass stars (higher surface temp) where mixing of core is not efficient



Observations of the light element Deuterium

- very easily destroyed in stars
- so look for deuterium in low density clouds of gas seen in absorption along the lines of sight to distant quasars
- differences between Hydrogen and Deuterium nucleus cause a small change in the energies of electron transitions, shifting their absorption lines apart
- this allows the Deuterium to Hydrogen ratio to be measured
- gas cloud absorbed the quasar light when the universe was only 10% its current age



Comparison with *inferred* primordial abundances

^{4}He observed in extragalctic HII regions:

abundance by mass = 25%

^{7}Li observed in the atmospheres of dwarf halo stars:

abundance by number = 10^{-10}

D observed in quasar absorption systems (and locally):

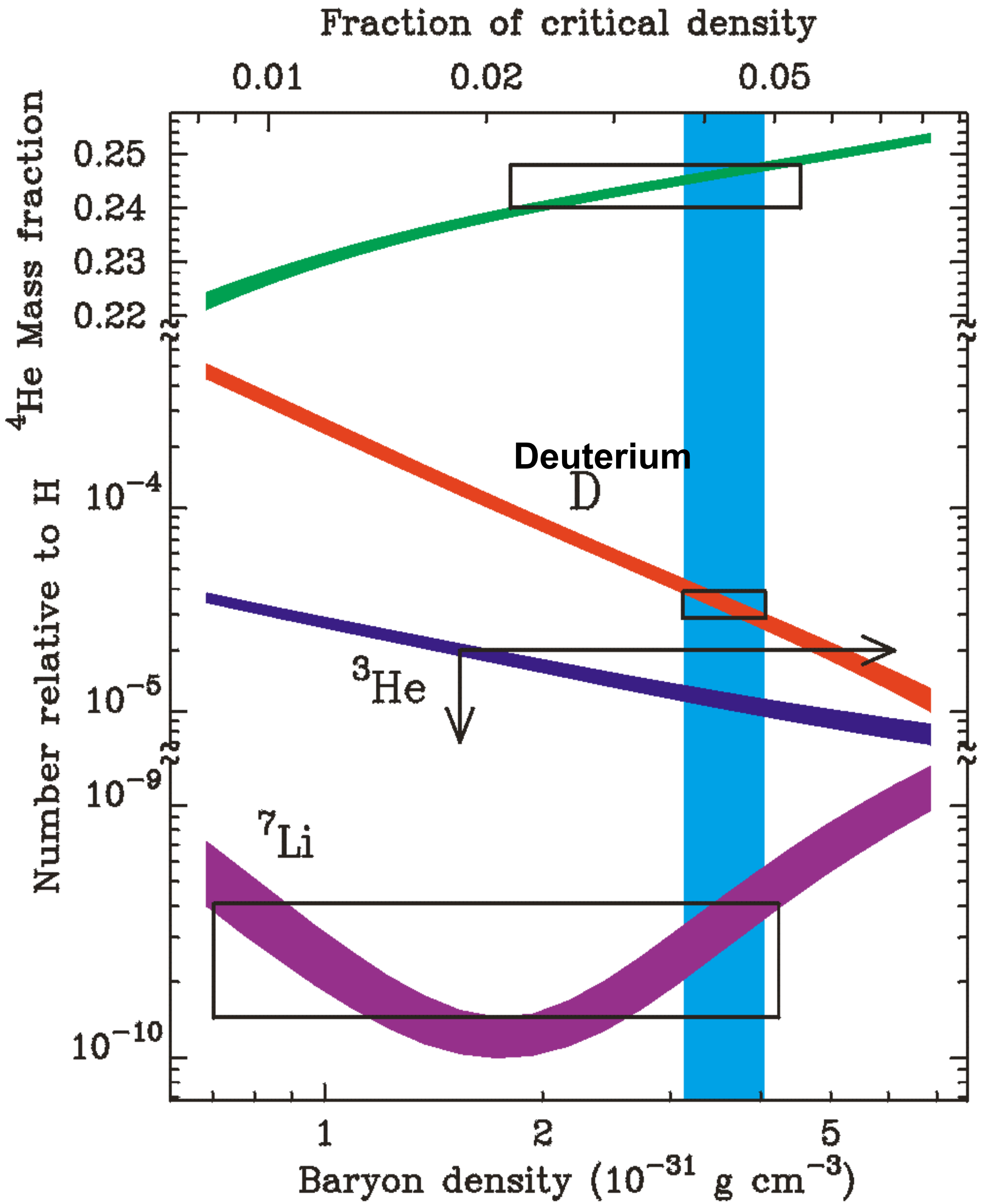
abundance by number = 3×10^{-5}

^{3}He in solar wind, in meteorites, and in the ISM:

abundance by number = 10^{-5}

Cosmic Concordance

- **Primordial nucleosynthesis**
  - » explains observed light element abundances if the density of normal matter (baryons) in the universe lies around 3.5×10^{-31} g/cm^3 or 0.21 hydrogen atoms per cubic meter
- **Precise observational test**
  - » independent measurements of abundances of four different light elements lead to consistent constraints on the density of normal matter
  - » provides confidence that Big Bang nucleosynthesis provides a correct explanation of the formation of the light elements.

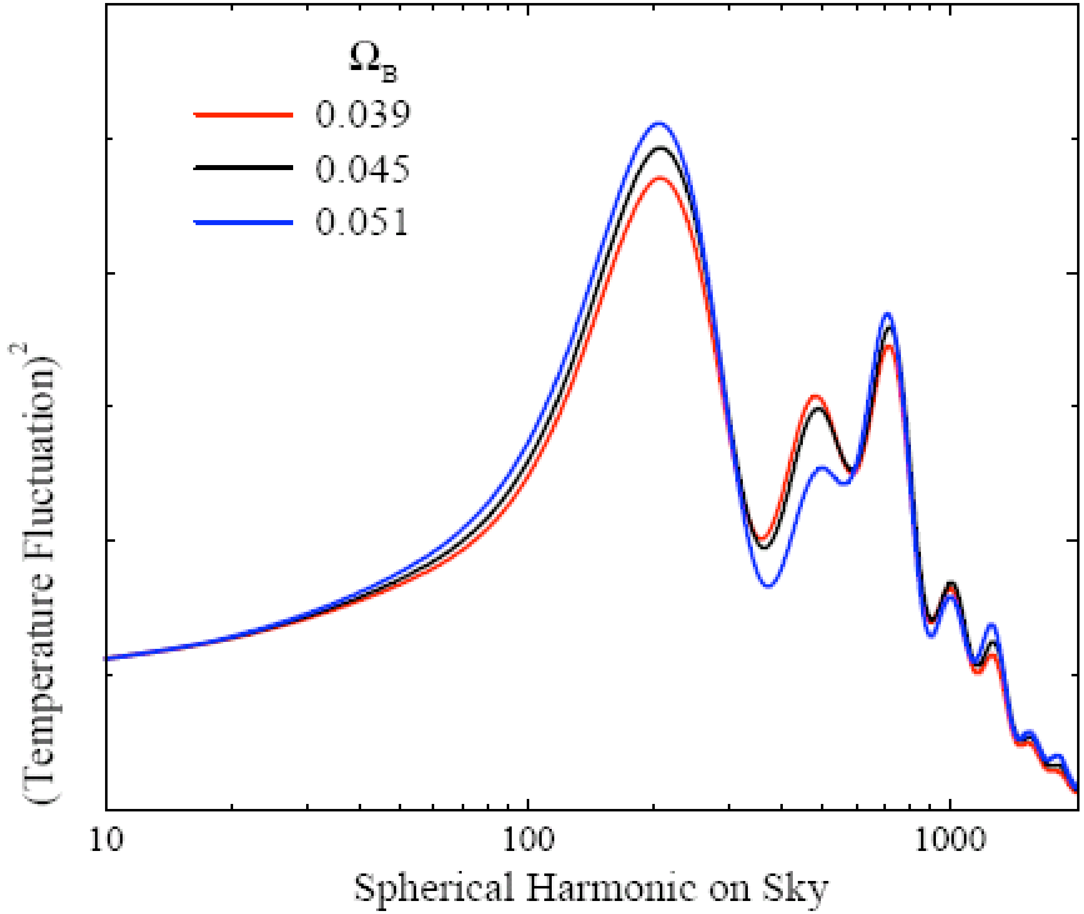


The Cosmic Microwave Background as a *baryometer*

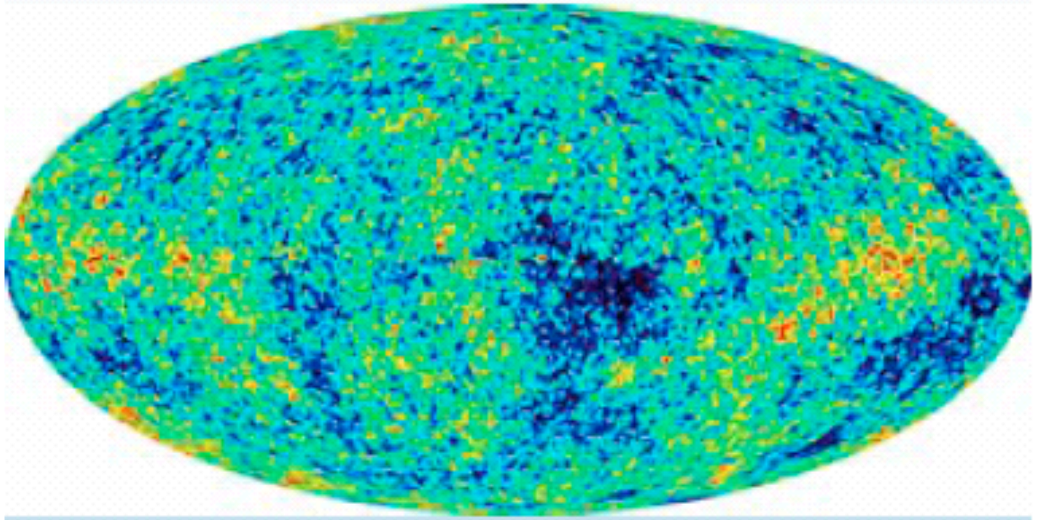
ΔT_ℓ provide *independent* measure of $\Omega_B h^2$

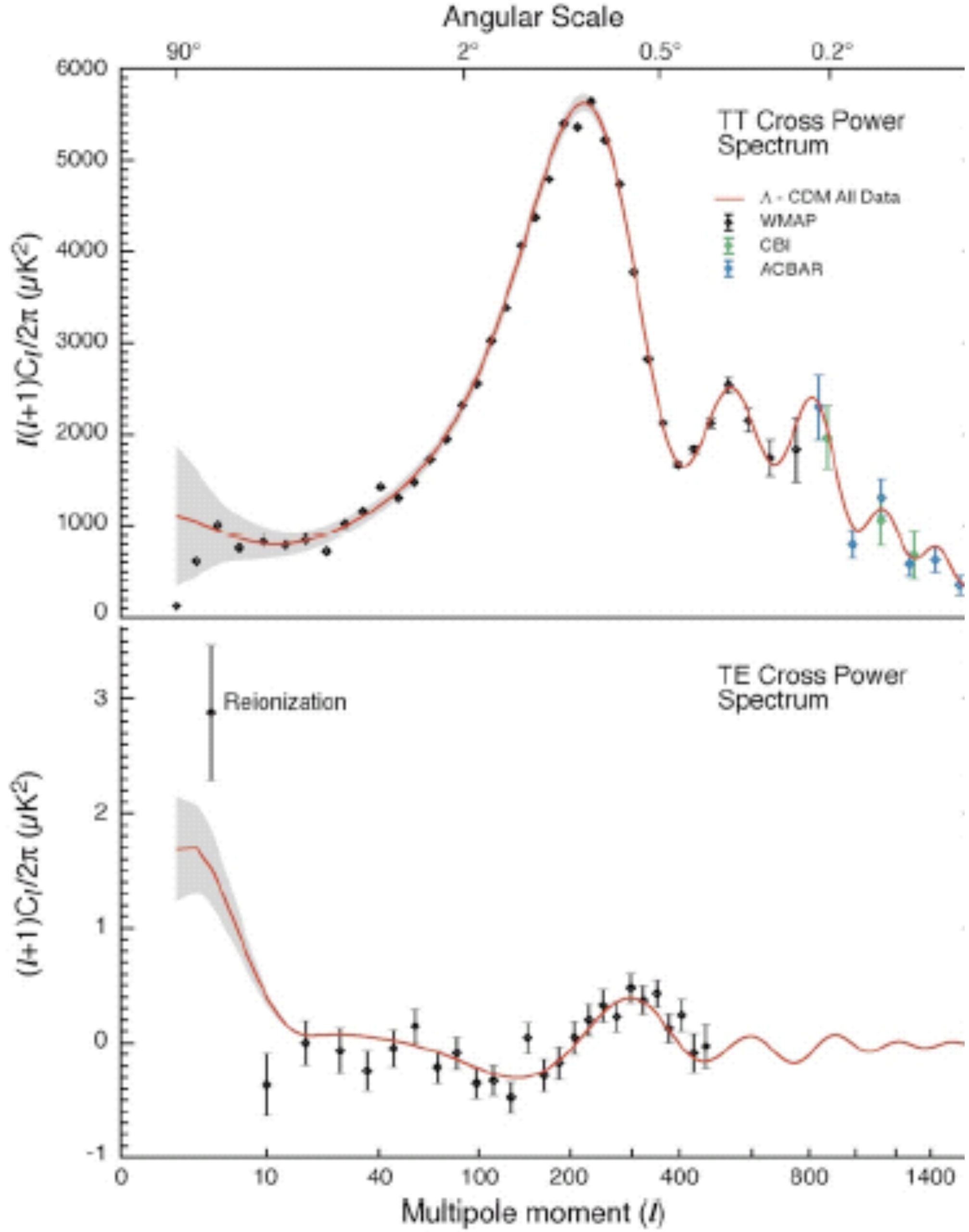
Acoustic oscillations in (coupled photon-baryon fluids Imprint features at < 1^o in angular power spectrum

Peak positions and heights sensitive to cosmological parameters e.g.
Ratio of 2nd peak/1st peak
⇒ baryon density



BBN vs CMB determinations of baryon density → fundamental test of cosmology and thermal history at z ~ 10^3 - 10^{10}





WMAP best fit (WMAPext + 2dFGRS + Lyman α +running sp. index)

$$\Omega_B h^2 = 0.0224 \pm 0.0009$$

$$\eta_{10} = 6.14 \pm 0.25$$

BBN versus CMB

η_{BBN} *agrees with* η_{CMB}

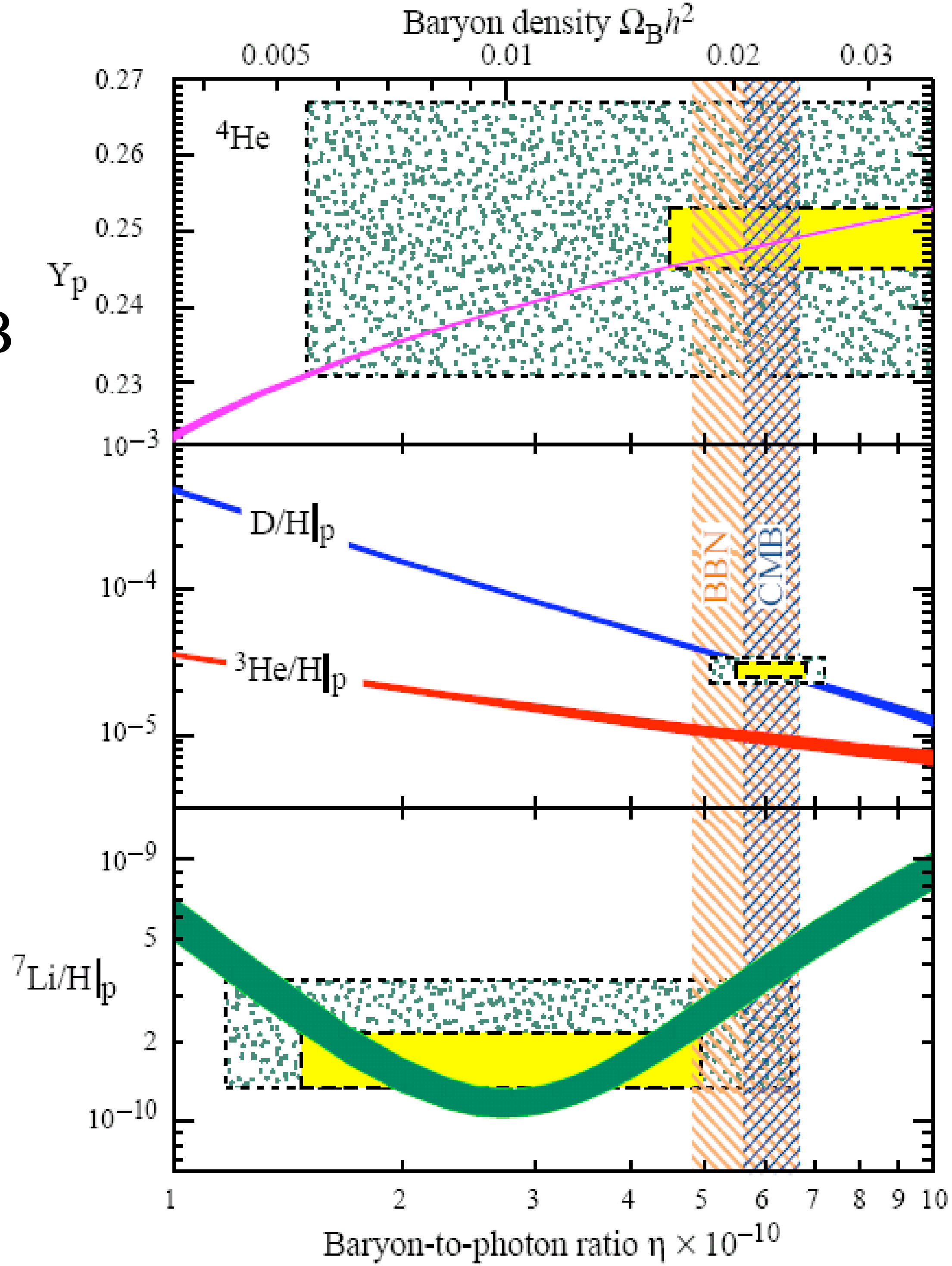
Confirms the case for (two kinds of) dark matter

Baryonic dark matter
(warm-hot intergalactic medium, Ly-α clouds…)

+

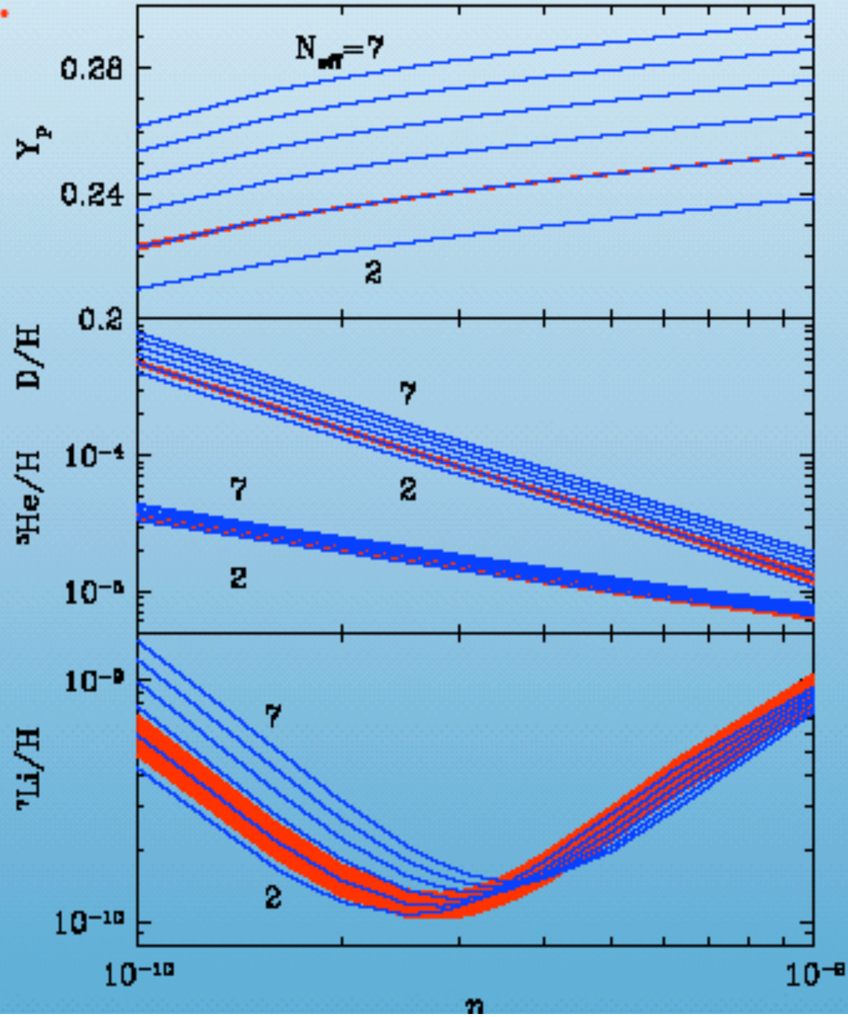
Non-baryonic dark matter
(neutralino? axion? …)

(Fields & Sarkar, PDG 2008)



Limits on Particle Properties

- BBN Concordance rests on balance between interaction rates and expansion rate.
- Allows one to set constraints on:
  - Particle Types
  - Particle Interactions
  - Particle Masses
  - Fundamental Parameters



Constraints from balance of weak rates vs Hubble rate

$$G_F^2 T^5 \sim \Gamma(T_f) \sim H(T_f) \sim \sqrt{G_N N} T_f^2$$

through He abunance

$$\frac{n}{p} \sim e^{-\Delta m/T} \qquad \text{fixed at freezeout} \qquad Y \sim \frac{2(n/p)}{1+(n/p)}$$

Sets constraints on G_F, G_N, N, etc.

Note *n-p* mass difference is sensitive to both em and strong interactions, hence ^{4}He abundance is *exponentially* sensitive to *all* coupling strengths

Conversely obtain bound of < few % on *any* additional contribution to energy density driving expansion (over the Standard Model value)

"Neutrino counting"

Light element abundances are sensitive to expansion history during BBN

$$H^2 \sim G\rho_{\mathrm{rel}}$$

$\Rightarrow$ observed values constrain relativistic energy density

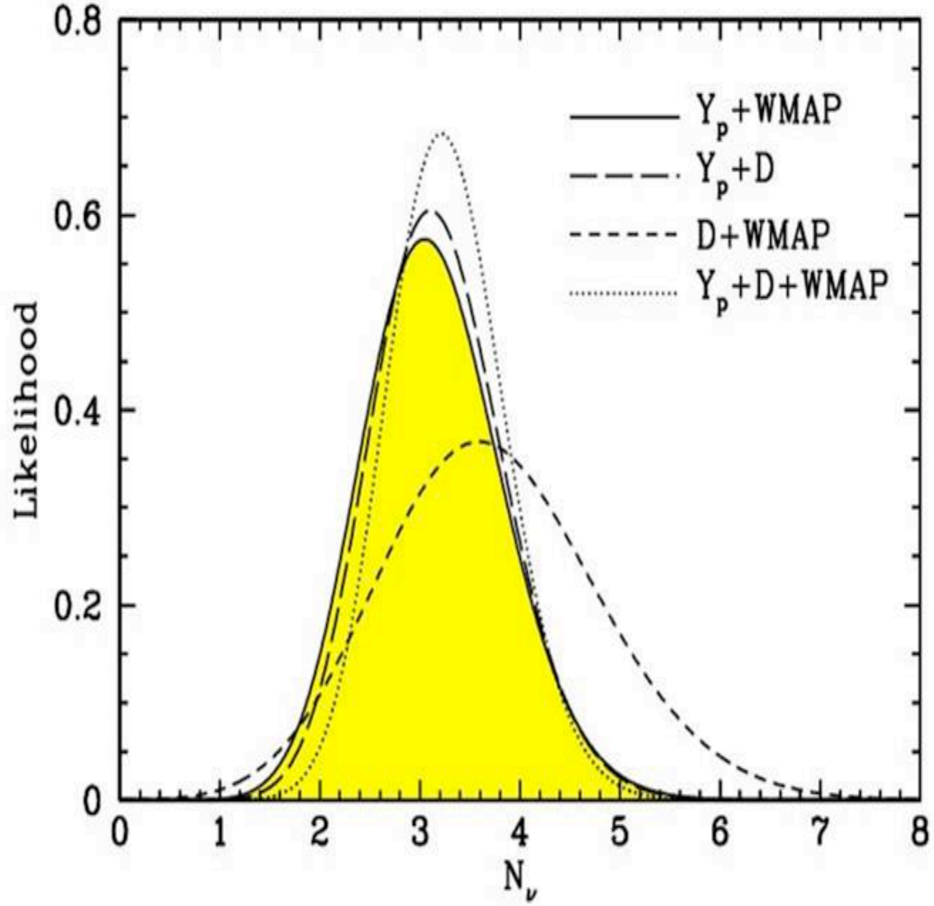
$$\rho_{\mathrm{rel}} \equiv \rho_{\mathrm{EM}} + N_{\nu,\mathrm{eff}}\, \rho_{\nu\bar{\nu}}$$

Pre-CMB:

^{4}He as probe, other elements give η

With η from CMB:

- All abundances can be used
- ^{4}He still sharpest probe
- D competitive if measured to 3%



This constrains sterile neutrinos (and other hypothetical particles) which do *not* couple to the Z^0 … *complementary* to laboratory bounds e.g from LEP