Slides
Condensed Matter Physics
Revision Lecture 2

2. The $\omega(k)$ dispersion relation for the vibrations of a one-dimensional linear diatomic lattice in which alternate atoms have masses of M_1 and M_2 ($M_1 > M_2$) is given by the expression

$$\omega^2(k) = C\left(\frac{1}{M_1} + \frac{1}{M_2}\right) \pm C\left[\left(\frac{1}{M_1} + \frac{1}{M_2}\right)^2 - \frac{4\sin^2(\frac{1}{2}kb)}{M_1 M_2}\right]^{1/2},$$

where C is the force constant and b is the lattice constant. Use this expression to derive the acoustic phonon sound velocity, the angular frequency of the acoustic and optical phonon branches at the Brillouin Zone boundary and the angular frequency of the optical phonon mode at $k = 0$. Give a simple explanation of the differences between the acoustic and optical modes of vibration, illustrating your answer with an appropriate diagram for modes close to $k = 0$. [9]

Draw a labelled diagram of $\omega(k)$, including in your figure the Brillouin zone boundaries. Show also the $\omega(k)$ relation for light on your diagram. Draw a sketch of the density of phonon states for this chain. [8]

Which sections of the $\omega(k)$ relation might it be possible to measure by studying the interactions of the crystal vibrations with light? [3]

3. The density of states per unit volume in the conduction band of a semiconductor is given by

$$D_{\mathrm{e}}(E) = \frac{1}{2\pi^2}\left(\frac{2m_{\mathrm{e}}^*}{\hbar^2}\right)^{3/2}(E - E_{\mathrm{g}})^{1/2},$$

where m_{e}^* is the electron effective mass, E is the electron energy and E_{g} is the energy at the bottom of the conduction band. Hence show that the number, n, of electrons in the conduction band of a semiconductor is given by

$$n = 2\left(\frac{m_{\mathrm{e}}^* k_B T}{2\pi\hbar^2}\right)^{3/2}\exp\left(\frac{\mu - E_{\mathrm{g}}}{k_B T}\right),$$

where μ is the chemical potential and T is the temperature. [5]

What is the corresponding expression $D_{\mathrm{h}}(E)$ for a valence band with a maximum at $E = 0$? Using $D_{\mathrm{h}}(E)$ derive an expression for the density of holes, p. Using your expressions for n and p derive an expression for the temperature dependence of the chemical potential of an intrinsic semiconductor in terms of m_{e}^*, m_{h}^* (the hole effective mass), and E_{g}. [8]

A semiconductor with a band gap of $1\,\mathrm{eV}$ and effective masses $m_{\mathrm{e}}^* = 0.1 m_e$ and $m_{\mathrm{h}}^* = 0.4 m_e$ is doped with a concentration of $1 \times 10^{22}\,\mathrm{m}^{-3}$ shallow donor impurities. How does this doping affect the chemical potential at room temperature? Draw a diagram to show the qualitative temperature dependence of the chemical potential for this material from $10\,\mathrm{K}$ to $900\,\mathrm{K}$. [7]

$$\left[\int_0^\infty x^{1/2} e^{-x}\, dx = \frac{\sqrt{\pi}}{2}\right]$$

2007 Exam:

4. Explain what is meant by an *exchange interaction*. What are the physical conditions under which this leads to the formation of a ferromagnet? [5]

Describe the use of the molecular field approximation to describe the average magnetisation properties of a ferromagnetic material with a nearest-neighbour exchange interaction of the form $J\mathbf{S}_i.\mathbf{S}_j$, where J is negative, with z nearest neighbours. Using the Curie's law approximation, show how this can be used to derive an expression for the magnetisation as a function of temperature that predicts a divergence in the susceptibility at a critical temperature T_c, and hence the formation of a ferromagnetic state. [10]

Explain why domains frequently form in ferromagnetic materials and relate this to the different hysteresis loops that are observed for different alloys of iron. [5]