This talk is based on work carried out from 2005 - present

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Talk Structure

1. Introduction and Motivation
2. Moduli Stabilisation and LARGE Volume
3. Stringy Model Building
4. Hyperweak Gauge Groups
5. Axions and the Strong CP Problem
Nature likes hierarchies:

- The Planck scale, $M_P = 2.4 \times 10^{18}\text{GeV}$.
- The GUT scale, $M_{GUT} \sim 3 \times 10^{16}\text{GeV}$.
- The inflationary scale, $M \sim 10^{13} \rightarrow 10^{16}\text{GeV}$.
- The axion scale, $10^9\text{GeV} \lesssim f_a \lesssim 10^{12}\text{GeV}$
- The weak scale: $M_W \sim 100\text{GeV}$
- The fermion masses, $m_e \sim 0.5\text{MeV} \rightarrow m_t \sim 170\text{GeV}$.
- The neutrino mass scale, $0.05\text{eV} \lesssim m_\nu \lesssim 0.3\text{eV}$.
- The cosmological constant, $\Lambda \sim (10^{-3}\text{eV})^4$

These demand an explanation!
Maladies of Particle Physics

We can also ask

1. Why $SU(3) \times SU(2) \times U(1)$?
2. Why three generations?
3. What sets $\alpha_{EW}$, $\alpha_{strong}$, $\alpha_Y$?
4. Where does the flavour structure come from?
5. Why four dimensions?

None of these questions can be answered within the Standard Model.

To address them, a more fundamental approach is needed.
String theory represents a candidate theory of quantum gravity.

It has given important insights into algebraic geometry, black holes and quantum field theory.

Can string theory do the same for particle physics?

If nature is stringy, string theory should give insights into all the fundamental problems mentioned previously.

String phenomenology aims to use string theory to address these fundamental problems of particle physics.
String Phenomenology

- String theory lives in ten dimensions.
- The world is four-dimensional, so ten-dimensional string theory needs to be compactified.
- To preserve $\mathcal{N} = 1$ supersymmetry, we compactify on a six-dimensional Calabi-Yau manifold - Ricci-flat and Kähler.
- All scales, matter, particle spectra and couplings come from the geometry of the extra dimensions.
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String Phenomenology
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There are two basic tasks of string phenomenology:

1. How to stabilise the geometry of the extra dimensions.
   \[\rightarrow\text{Moduli stabilisation}\]

2. How to obtain a Standard Model matter spectrum in four dimensions.
   \[\rightarrow\text{Model building}\]

Fully realistic models must accomplish both.
String Phenomenology

- String compactifications generically have moduli parametrising the geometry.
- Moduli are naively massless scalars with large classical vevs. The vevs determine the scales and couplings of the 4d field theory.
- Moduli are uncharged and interact gravitationally. Such massless scalars generate long-range, unphysical fifth forces.
- It is essential to generate potentials for moduli and stabilise them.
- The LARGE volume models are an appealing scenario of moduli stabilisation.
Moduli Stabilisation: Fluxes

In IIB string theory, the moduli definitions are

- **Kähler moduli:**
  \[ T_i = \tau_i + i c_i, \quad \text{where} \quad \tau_i = \int_{\Sigma_4} e^{-\phi} \sqrt{g} \quad \text{and} \quad c_i = \int_{\Sigma_4} C_4. \]
  \( T_i \) are complexified 4-cycle volumes.

- **Complex structure moduli:**
  \( U_i \) are the geometric complex structure moduli of the Calabi-Yau.

- **Dilaton modulus:**
  \[ S = \frac{1}{g_s} + i c_0 \] combines the string coupling and the RR 0-form.
Moduli Stabilisation: Fluxes

We work in (orientifolds of) type IIB string theory with D3 and D7 branes.

The IIB field content includes 3-form field strengths $F_3$ and $H_3$ from the RR and NS-NS sectors.

$$G_3 = F_3 + SH_3.$$ 

The extra dimensions contain

1. D3/D7 D-branes
2. O3/O7 orientifold planes
3. 3-form fluxes $G_3 = F_3 + SH_3$. 

Under these conditions it can be shown \cite{GKP2001} that the 10-D metric is

\[ ds_{10}^2 = e^{2A(y)} \eta_{\mu\nu} dx^\mu dx^\nu + e^{-2A(y)} \tilde{g}_{MN} dy^M dy^N \]

The metric is \textit{warped Calabi-Yau}.

The warp factor scales as

\[ e^{2A(y)} \sim 1 + \frac{1}{V^{2/3}} \]

and vanishes in the infinite volume limit.

The dilaton and complex structure moduli are fixed by the fluxes. The Kähler moduli are not fixed.
Moduli Stabilisation: Fluxes

- We want to work in 4-dimensional supergravity.
- The fluxes carry energy generating a potential for the moduli associated with these cycles.
- This energy is expressed through a superpotential

\[ W = \int (F_3 + iSH_3) \wedge \Omega \equiv \int G_3 \wedge \Omega. \]

- This generates a potential for the dilaton and complex structure moduli.
The effective supergravity theory is

\[
K = -2 \ln(V) - \ln \left( i \int \Omega \wedge \bar{\Omega} \right) - \ln(S + \bar{S})
\]

\[
W = \int (F_3 + iSH_3) \wedge \Omega \equiv \int G_3 \wedge \Omega.
\]

This stabilises the dilaton and complex structure moduli.

\[
D_S W = D_U W = 0.
\]

\[
W = \int G_3 \wedge \Omega = W_0.
\]
Moduli Stabilisation: Fluxes

The theory has an important no-scale property.

\[ \hat{K} = -2 \ln \left( \mathcal{V}(T + \bar{T}) \right) - \ln \left( i \int \Omega \wedge \bar{\Omega}(U) \right) - \ln (S + \bar{S}) , \]

\[ W = \int G_3 \wedge \Omega (S, U). \]

\[ V = e^{\hat{K}} \left( \sum_{U,S} \hat{K}^{\alpha \bar{\beta}} D_\alpha W D_{\bar{\beta}} \bar{W} + \sum_T \hat{K}^{i \bar{j}} D_i W D_{\bar{j}} \bar{W} - 3|W|^2 \right) \]

\[ = e^{\hat{K}} \left( \sum_{U,S} \hat{K}^{\alpha \bar{\beta}} D_\alpha W D_{\bar{\beta}} \bar{W} \right) = 0. \]
Moduli Stabilisation: Fluxes

\[ \hat{K} = -2 \ln \left( \mathcal{V}(T_i + \bar{T}_i) \right), \]
\[ W = W_0. \]
\[ V = e^{\hat{K}} \left( \sum_{T} \hat{K}_{\bar{i}j} D_i W D_{\bar{j}} \bar{W} - 3|W|^2 \right) \]
\[ = 0 \]

No-scale model:
- \( \) vanishing vacuum energy
- \( \) broken susy
- \( T \) unstabilised

No-scale is broken perturbatively and non-pertubatively.
Moduli Stabilisation: KKLT

\[ \hat{K} = -2 \ln (\mathcal{V}) - \ln \left( i \int \Omega \wedge \bar{\Omega} \right) - \ln (S + \bar{S}), \]

\[ W = \int G_3 \wedge \Omega + \sum_i A_i e^{-a_i T_i}. \]

Non-perturbative effects (D3-instantons / gaugino condensation) allow the \( T \)-moduli to be stabilised by solving \( D_T W = 0 \).

For consistency, this requires

\[ W_0 = \left\langle \int G_3 \wedge \Omega \right\rangle \ll 1. \]
\[ \hat{K} = -2 \ln (\mathcal{V}), \]
\[ W = W_0 + \sum_i A_i e^{-a_i T_i}. \]

Solving \( D_T W = \partial_T W + (\partial_T K)W = 0 \) gives

\[ \text{Re}(T) \sim \frac{1}{a} \ln(W_0) \]

For \( \text{Re}(T) \) to be large, \( W_0 \) must be enormously small.
KKLT stabilisation has three phenomenological problems:

1. No susy hierarchy: fluxes prefer $W_0 \sim 1$ and $m_{3/2} \gg 1\text{TeV}$.

2. Susy breaking not well controlled - depends entirely on uplifting.

3. $\alpha'$ expansion not well controlled - volume is small and there are large flux backreaction effects.
\[ \tilde{K} = -2 \ln \left( V + \frac{\xi}{2g_s^{3/2}} \right), \quad \left( \xi = \frac{\chi(M) \zeta(3)}{2(2\pi)^3} \right) \]

\[ W = W_0 + \sum_i A_i e^{-a_i T_i}. \]

- Include perturbative as well as non-perturbative corrections to the scalar potential.
- Add the leading $\alpha'$ corrections to the Kähler potential.
- This leads to dramatic changes in the large-volume vacuum structure.
Moduli Stabilisation: LARGE Volume

The simplest model $\mathbb{P}^4_{[1,1,1,6,9]}$ has two Kähler moduli.

$$V = \left( \frac{T_b + \bar{T}_b}{2} \right)^{3/2} - \left( \frac{T_s + \bar{T}_s}{2} \right)^{3/2} \equiv \left( \tau_b^{3/2} - \tau_s^{3/2} \right).$$

If we compute the scalar potential, we get for $V \gg 1$,

$$V = \frac{\sqrt{\tau_s a_s^2 | A_s |^2} e^{-2a_s \tau_s}}{V} - \frac{a_s | A_s W_0 | \tau_s e^{-a_s \tau_s}}{V^2} + \frac{\xi | W_0 |^2}{g_s^{3/2} V^3}.$$
\[ V = \frac{\sqrt{\tau_s a_s^2 A_s^2 e^{-2a_s \tau_s}}}{V} \Bigg( - \frac{a_s A_s W_0 \tau_s e^{-a_s \tau_s}}{V^2} \Bigg) + \frac{\xi |W_0|^2}{g_s^{3/2} V^3}. \]

Integrate out \( \tau_s \)

\[ V = \frac{|W_0|^2 (\ln V)^{3/2}}{V^3} + \frac{\xi |W_0|^2}{g_s^{3/2} V^3}. \]

A minimum exists at

\[ V \sim |W_0|e^{a_s \tau_s}, \quad \tau_s \sim \frac{\xi^{2/3}}{g_s}. \]

This minimum is non-supersymmetric AdS and at exponentially large volume.
The minimum of the potential is non-supersymmetric AdS and at exponentially large volume.

The large volume lowers the string scale and gravitino mass through

\[ m_s = \frac{M_P}{\sqrt{V}}, \quad m_{3/2} = \frac{M_P W_0}{V}. \]

To solve the gauge hierarchy problem, need \( V \sim 10^{15} \).

D7-branes wrapped on small cycle carry the Standard Model: need \( T_s \sim 20(2\pi \sqrt{\alpha'})^4 \).

The vacuum is pseudo no-scale and breaks susy.
Moduli Stabilisation: LARGE Volume

The mass scales present are:

- **Planck scale:** $M_P = 2.4 \times 10^{18}\text{GeV}$.
- **String scale:** $M_S \sim \frac{M_P}{\sqrt{V}} \sim 10^{11}\text{GeV}$.
- **KK scale:** $M_{KK} \sim \frac{M_P}{V^{2/3}} \sim 10^9\text{GeV}$.
- **Gravitino mass:** $m_{3/2} \sim \frac{M_P}{V} \sim 1\text{TeV}$.
- **Moduli:** $m_U, m_{TS}, m_S \sim m_{3/2} \sim 1\text{TeV}$.
- **Volume modulus:** $m_{\tau_b} \sim \frac{M_P}{V^{3/2}} \sim 1\text{MeV}$.
Model Building
Model Building

- Necessary to combine moduli stabilisation and semi-realistic chiral matter spectra.
- Standard Model must be local and live on the ‘hole in the cheese’
- Local geometry consists of magnetised D7 branes wrapped on a collapsing 4-cycle.
- Mathematically, a collapsing 4-cycle is a del Pezzo surface.
- Model-building must come from D-branes on a (resolved) del Pezzo singularity.
Model Building

There are two limits:

1. The geometric limit:
   The del Pezzo is blown up to finite size.
   The stability conditions are geometric \((J \wedge F = 0)\).

2. The singular limit:
   The del Pezzo is collapsed to give a del Pezzo singularity.
   Both ‘branes’ and ‘antibranes’ are supersymmetric.
Model Building

We work at the singular limit. Why?

- Moduli stabilisation of ‘Standard Model’ cycle requires D-terms.
- D-terms drive del Pezzo cycle to vanishing limit at edge of Kähler cone.
- Geometric branes cross lines of marginal stability and become unstable.
- We must therefore use the appropriate supersymmetric branes at singularities!
Model Building

We use the quivers for del Pezzo $n$ ($dP_n$) singularities. The $dP_0 \equiv \mathbb{C}^3 / \mathbb{Z}_3$ quiver is

This has an $SU(3)$ family symmetry for $33$ interactions.
The $dP_1 \equiv \mathbb{C}^3/\mathbb{Z}_3$ quiver is

This has an $SU(2) \times U(1)$ family symmetry for 33 interactions.
Model Building

A Standard-Model like spectrum for $dP_0$:
A Pati-Salam-Model like spectrum for $dP_0$: 

![Diagram](image_url)
Both these models have (33) Yukawa textures

\[ Y_{ijk} \sim \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & M \\ 0 & -M & 0 \end{pmatrix}. \]

The \((M, M, 0)\) Yukawa structure follows from the \(SU(3)\) family symmetry.

This can be improved by going to models based on \(dP_1\) singularities.

Here the family symmetry is reduced to \(SU(2) \times U(1)\).
Model Building

A Standard-Model like spectrum for $dP_1$:

\[Q, L, H, d, D, R, U, H_u, Z, E_R, Y_j, L, H_d, D_R, X_i\]

(0) (3) (6)

The Large Volume Scenario: Towards Realistic String Vacua – p. 37/56
For models based on $dP_1$, the (33) superpotential is

$$W = \epsilon_{ij} X_i Y_j Z_3 - \epsilon_{ij} X_i Y_3 Z_k + \frac{\Phi}{\Lambda} X_3 \epsilon_{ij} Y_i Z_j.$$

The $SU(3)$ family symmetry is broken down to $SU(2) \times U(1)$. The Yukawa mass spectrum is now $(M, m, 0)$, where $m \sim \langle \frac{\Phi}{\Lambda} \rangle$.

In contrast to $dP_0$ this now allows a heavy third generation.
None of the above models are perfect. However

- It is easy to get quasi-realistic spectra and couplings.
- Three generations of chiral SM matter arises naturally (cf heterotic string!).
- Flavour symmetries for Yukawas arise naturally.
- Relatively few exotics are present.

**Punchline**: local model-building gives attractive SM-like models with relative ease.
Model Building

$Z_3$ singularity

Orientifold action

$Z_3$ singularity

MSSM

The Large Volume Scenario: Towards Realistic String Vacua – p. 40/56
Hyper-Weak Gauge Groups

A Standard-Model like spectrum for $dP_1$:
Hyper-Weak Gauge Groups

- In LARGE volume models, the Standard Model is a local construction and branes only wrap small cycles.
- There are also bulk cycles associated to the overall volume. These have cycle size

\[ \tau_b \sim V^{2/3} \sim 10^{10}. \]

- There is no reason not to have D7 branes wrapping these cycles!
- The gauge coupling for such branes is

\[ \frac{g^2}{4\pi} = \frac{1}{\tau_b}. \]

with \( g \sim 10^{-4}. \)
Hyper-Weak Gauge Groups

\[ V = 10^{15} l^6 \]

BULK

BLOW–UP

\[ \begin{align*}
Q_L & \quad U(2) \\
Q_R & \quad U(3) \\
e_L & \quad U(1) \\
e_R & \quad U(1)
\end{align*} \]
Hyper-Weak Gauge Groups

In LARGE volume models it is a generic expectation that there will exist additional gauge groups with very weak coupling

$$\alpha^{-1} \sim 10^9.$$ 

Two phenomenological questions to ask:

1. How heavy is the hyper-weak $Z'$ gauge boson?
2. How does Standard Model matter couple to the hyper-weak force?
Hyper-Weak Gauge Groups

Hyper-weak $Z'$ may get a mass by a Higgs mechanism in either visible or hidden sectors.

- If hyperweak gauge group is broken by weak-scale vevs,

  $$M_{Z'} \sim gv \sim 10^{-4} \times 10 \rightarrow 100 \text{GeV} \sim 1 \rightarrow 10 \text{MeV}.$$  

- If hyperweak gauge group is broken by chiral condensate $\langle \bar{q}q \rangle \sim \Lambda_Q^3$,

  $$M_{Z'} \sim gv \sim 10^{-4} \times 100 \text{MeV} \sim 10 \text{keV}.$$  

- If hyperweak gauge group is broken by hidden sector physics $\langle \phi_{hid} \rangle \sim v$, $M_{Z'} \sim gv \sim 10^{-4} \times v$.  

The Large Volume Scenario : Towards Realistic String Vacua – p. 45/5
There may also be kinetic mixing between the new gauge boson and the photon (cf $Z/\gamma$ mixing).

If the new gauge boson is light, the mixing allows the new boson to couple to electromagnetic currents:

$$\mathcal{L}_{int} = \frac{(\bar{\psi} \gamma^\mu \psi) A_\mu}{\sqrt{1 - \lambda^2}} - \frac{(\bar{\psi} \gamma^\mu \psi) \lambda Z'_\mu}{\sqrt{1 - \lambda^2}}$$

where $\lambda$ is the mixing parameter.

This can give milli-charged fermions under the new gauge boson $Z'$. 

The Large Volume Scenario : Towards Realistic String Vacua – p. 46/56
Hyper-Weak Gauge Groups

Bounds on an MeV-scale gauge boson: axial and vector couplings $g_V(\bar{\psi} \gamma^\mu \psi)Z'_\mu$ or $g_A(\bar{\psi} \gamma^5 \gamma^\mu \psi)Z'_\mu$.

<table>
<thead>
<tr>
<th>Coupling</th>
<th>Bound</th>
<th>Experimental measurement</th>
</tr>
</thead>
<tbody>
<tr>
<td>$g^V_e$</td>
<td>$10^{-4} m_U$</td>
<td>$g_e - 2$</td>
</tr>
<tr>
<td>$g^A_e$</td>
<td>$5 \times 10^{-5} m_U$</td>
<td>$g_e - 2$</td>
</tr>
<tr>
<td>$g^V_\mu$</td>
<td>$10^{-3}$</td>
<td>$g_\mu - 2$</td>
</tr>
<tr>
<td>$g^A_\mu$</td>
<td>$5 \times 10^{-6} m_U$</td>
<td>$g_\mu - 2$</td>
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<td>$</td>
<td>g_e g_\nu</td>
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<tr>
<td>$g^A_{c(b)}$</td>
<td>$10^{-6} m_U$</td>
<td>$B(\psi(\Upsilon) \rightarrow \gamma +$ invisible$)$</td>
</tr>
<tr>
<td>$</td>
<td>g_e g_\gamma</td>
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</tbody>
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(Based on Fayet hep-ph/0702176)
Axions are a well-motivated solution to the strong CP problem of the Standard Model.

The Lagrangian for QCD contains a term

$$\mathcal{L}_{QCD} = \frac{1}{4\pi g^2} \int d^4 x F_{\mu\nu}^a F^{a,\mu\nu} + \frac{\theta}{8\pi^2} \int F^a \wedge F^a.$$ 

The $\theta$ term

$$\mathcal{L}_\theta = \frac{\theta}{8\pi^2} \int F^a \wedge F^a.$$ 

gives rise to the strong CP problem.

This term vanishes in perturbation theory and is only non-vanishing for non-perturbative instanton computations.
Axions

$\theta$ is an angle and in principle takes any value $\theta \in (-\pi, \pi)$.

It can be shown that non-vanishing $\theta$ generates an electric dipole moment for the neutron.

The measured absence of a neutron electric dipole moment implies that

$$|\theta_{QCD}| \lesssim 10^{-10}.$$ 

No symmetry of the Standard Model requires $|\theta|$ to be so small.

The problem of why $|\theta_{QCD}| \ll \pi$ is the strong CP problem of the Standard Model.
Axions

The best solution to the strong CP problem is due to Peccei and Quinn.

The angle $\theta$ is promoted to a dynamical field with Lagrangian

$$\mathcal{L} = \frac{1}{2} \partial_\mu \theta \partial^\mu \theta + \int \frac{\theta}{8\pi^2 f_a} F^a \wedge F^a.$$ 

$f_a$ is called the axion decay constant and measures the strength of the non-renormalisable axion-matter coupling.
Axions

Non-perturbative QCD effects generate a potential for $\theta$,

$$V_\theta \sim \Lambda_{QCD}^4 \left( 1 - \cos \left( \frac{\theta}{f_a} \right) \right) = \frac{\Lambda_{QCD}^4}{2 f_a^2} \left( \theta^2 + \ldots \right)$$

The axion field $\theta$ gets a mass

$$m_\theta = \frac{\Lambda_{QCD}^2}{f_a} \sim 0.01 \text{eV} \left( \frac{10^9 \text{GeV}}{f_a} \right)$$

Axion phenomenology depends entirely on the value of $f_a$. 
Constraints from supernova cooling and direct searches imply \( f_a \gtrsim 10^9 \text{GeV} \) (axions cannot couple too strongly to matter).

Avoiding the overproduction of axion dark matter during a hot big bang prefers \( f_a \lesssim 10^{12} \text{GeV} \) (axions cannot couple too weakly to matter).

There exists an axion ‘allowed window’,

\[
10^9 \text{GeV} \lesssim f_a \lesssim 10^{12} \text{GeV}.
\]
In string theory, the axionic coupling comes from the Chern-Simons interaction

\[ \int_{\mathcal{M}_4 \times \Sigma_4} F \wedge F \wedge C_4 \rightarrow \int_{\mathcal{M}_4} \frac{c}{f_a} F \wedge F. \]

\[ f_a \] measures the axion-matter coupling.
Axions

- The axion coupling is a local coupling and does not see the overall volume.
- The coupling is set by the string scale:

\[ f_a \sim m_s \sim \frac{M_P}{\sqrt{V}}. \]

- This generates the axion scale,

\[ f_a \sim \frac{M_P}{\sqrt{V}} \sim 10^{11} \text{GeV}. \]
**Axions**

TeV-scale supersymmetry required

\[ M_{\text{susy}} \sim m_{3/2} = \frac{M_P}{\mathcal{V}} = 1\text{TeV}. \]

This gives \( \mathcal{V} \sim 10^{15} l_s^6 \).

The axion scale is therefore

\[ f_a \sim \sqrt{M_{\text{susy}}M_P} \sim 10^{11}\text{GeV} \]

The existence of an axion in the allowed window correlates to the existence of supersymmetry at the TeV scale.
Conclusions

- String theory may be relevant for explaining the many hierarchical scales present in nature.
- LARGE volume models are an attractive method of generating hierarchies and allow for semi-realistic matter spectra.
- These models have interesting consequences for
  1. Low-energy supersymmetry
  2. Model-building using branes at singularities
  3. Possible new hyper-weak gauge groups (e.g. gauged \(B - L\), \(\alpha_{B-L} \lesssim 10^{-10}\)).
  4. QCD axions in the allowed window