Hierarchy Problems in String Theory: An Overview of the LARGE Volume Scenario

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Amsterdam, June 2008

Papers

Moduli Stabilisation: hep-th/0502058 (Balasubramanian, Berglund, JC, Quevedo),
hep-th/0505076 (JC, Quevedo, Suruliz),
arXiv:0704.0737 (Berg, Haack, Pajer),
arXiv:0708.1873, 0805.1029 (Cicoli, JC, Quevedo),
arXiv:0711.3389 (Blumenhagen, Moster, Plauschinn)

Soft terms: hep-th/0505076 (JC. Quevedo, Suruliz),
hep-th/0605141 (JC, Quevedo),
hep-th/0609180 (JC, Cremades, Quevedo),
hep-th/0610129 (JC, Abdussalam, Quevedo, Suruliz),
arXiv:0704.0737 (Berg, Haack, Pajer),
arXiv:0704.3403 (JC, Kom, Suruliz, Allanach, Quevedo).

Cosmology: hep-th/0509012, arXiv:0705.3460 (JC, Quevedo),
astro-ph/0605371, arXiv:0712.1875 (Simon *et al*), hep-th/0612197 (Bond *et al*),
arXiv:0712.1260 (Misra, Shukla)

Axions and Neutrino Masses: hep-th/0602233 (JC), hep-ph/0611144 (JC, Cremades)

Thanks to my collaborators:

Shehu Abdussalam, Ben Allanach, Vijay Balasubramanian, Per Berglund, Michele Cicoli, Daniel Cremades, Chun-Hay (Steve) Kom, Fernando Quevedo, Kerim Suruliz

Talk Structure

- Hierarchies in Nature
- String Phenomenology and LARGE Volume Models
- Supersymmetry Breaking
- Axions
- Neutrino Masses
- A New Scale
- Conclusions

Hierarchies in Nature

Nature likes hierarchies:

- The Planck scale, $M_P = 2.4 \times 10^{18}$GeV.
- The GUT/inflation scale, $M \sim 10^{16}$GeV.
- The axion scale, 10^9GeV $\lesssim f_a \lesssim 10^{12}$GeV
- The weak scale : $M_W \sim 100$GeV
- The QCD scale $\Lambda_{QCD} \sim 200$MeV
- The neutrino mass scale, 0.05eV $\lesssim m_\nu \lesssim 0.3$eV.
- The cosmological constant, $\Lambda \sim (10^{-3}\text{eV})^4$

These demand an explanation!

Hierarchies in Nature

This talk will argue that

- an intermediate string scale $m_s \sim 10^{11}$GeV
- stabilised exponentially large extra dimensions ($\mathcal{V} \sim 10^{15} l_s^6$).

explains the axionic, weak and neutrino hierarchies.

Different hierarchies will come as different powers of the (large) volume.

Moduli Stabilisation

- String theory lives in ten dimensions.
- Compactify on a Calabi-Yau manifold to give a four-dimensional theory.
- The geometry determines the four-dimensional particle spectrum.
- The spectrum always includes uncharged scalar particles - moduli - describing the size and shape of the extra dimensions.

Moduli Stabilisation

- Moduli are naively massless scalars which couple gravitationally.
- These generate fifth forces and so must be given masses.
- Generating potentials for moduli is the field of moduli stabilisation.
- This talk is on the large-volume models which represent a particular moduli stabilisation scenario.

Moduli Stabilisation: Fluxes

- Fluxes carry an energy density which generates a potential for the cycle moduli.
- In IIB compactifications, 3-form fluxes generate a superpotential

$$
\begin{aligned}
K &= -2\ln(\mathcal{V}) - \ln\left(i\int \Omega \wedge \bar{\Omega}\right) - \ln(S+\bar{S}) \\
W &= \int (F_3 + iSH_3)\wedge\Omega \equiv \int G_3 \wedge \Omega.
\end{aligned}
$$

- This stabilises the dilaton and complex structure moduli.

$$D_S W = D_U W = 0.$$

$$W = \int G_3 \wedge \Omega = W_0.$$

Moduli Stabilisation: Fluxes

$$\begin{aligned}
\hat{K} &= -2\ln\left(\mathcal{V}(T_i + \bar{T}_i)\right), \\
W &= W_0 . \\
V &= e^{\hat{K}} \left(\sum_T \hat{K}^{i\bar{j}} D_i W D_{\bar{j}} \bar{W} - 3|W|^2 \right) \\
&= 0
\end{aligned}$$

No-scale model :

- vanishing vacuum energy
- broken susy
- T unstabilised

No-scale is broken perturbatively and non-pertubatively.

Moduli Stabilisation: KKLT

$$\hat{K} = -2\ln(\mathcal{V}),$$
$$W = W_0 + \sum_i A_i e^{-a_i T_i}.$$

- Non-perturbative ADS superpotential.
- The T-moduli are stabilised by solving $D_T W = 0$.
- This gives a susy AdS vacuum which is uplifted by anti-branes/magnetic fluxes/IASD 3-form fluxes/F-terms/something else.
- Susy breaking is sourced by the uplift.

Moduli Stabilisation: KKLT

KKLT stabilisation has three phenomenological problems:

1. No susy hierarchy: fluxes prefer $W_0 \sim 1$ and $m_{3/2} \gg 1$TeV.
2. Susy breaking not well controlled - depends entirely on uplifting.
3. α' expansion not well controlled - volume is small and there are large flux backreaction effects.

Moduli Stabilisation: Large-Volume

$$\hat{K} = -2\ln\left(\mathcal{V} + \frac{\xi}{2g_s^{3/2}}\right), \qquad \left(\xi = \frac{\chi(\mathcal{M})\zeta(3)}{2(2\pi)^3 g_s^{3/2}}\right)$$

$$W = W_0 + \sum_i A_i e^{-a_i T_i}.$$

- Include perturbative as well as non-perturbative corrections to the scalar potential.
- Add the leading α' corrections to the Kähler potential (Becker-Becker-Haack-Louis).
- These descend from the $\mathcal{R}^4$ term in 10 dimensions.
- This leads to dramatic changes in the large-volume vacuum structure.

Moduli Stabilisation: Large-Volume

The simplest model $\mathbb{P}^4_{[1,1,1,6,9]}$ has two Kähler moduli.

$$\mathcal{V} = \left(\frac{T_b + \bar{T}_b}{2}\right)^{3/2} - \left(\frac{T_s + \bar{T}_s}{2}\right)^{3/2} \equiv \left(\tau_b^{3/2} - \tau_s^{3/2}\right).$$

If we compute the scalar potential, we get for $\mathcal{V} \gg 1$,

$$V = \frac{\sqrt{\tau_s} a_s^2 |A_s|^2 e^{-2a_s\tau_s}}{\mathcal{V}} - \frac{a_s |A_s W_0| \tau_s e^{-a_s\tau_s}}{\mathcal{V}^2} + \frac{\xi |W_0|^2}{g_s^{3/2}\mathcal{V}^3}.$$

Moduli Stabilisation: Large-Volume

$$V = \underbrace{\frac{\sqrt{\tau_s} a_s^2 |A_s|^2 e^{-2a_s\tau_s}}{\mathcal{V}} - \frac{a_s |A_s W_0| \tau_s e^{-a_s\tau_s}}{\mathcal{V}^2}}_{\text{Integrate out } \tau_s} + \frac{\xi |W_0|^2}{g_s^{3/2}\mathcal{V}^3}.$$

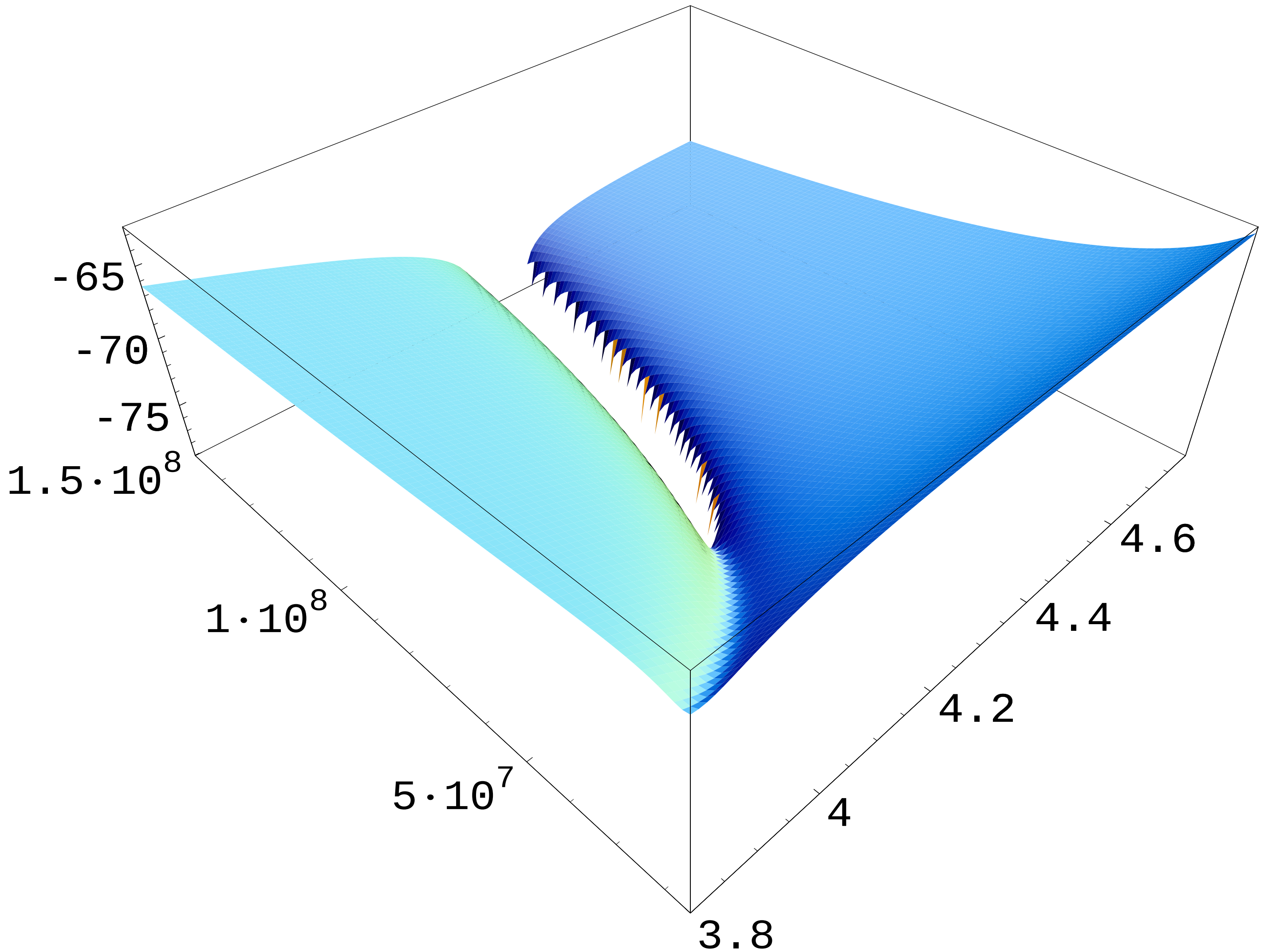
$$V = -\frac{|W_0|^2 \left(\ln \mathcal{V}\right)^{3/2}}{\mathcal{V}^3} + \frac{\xi |W_0|^2}{g_s^{3/2}\mathcal{V}^3}.$$

A minimum exists at

$$\mathcal{V} \sim |W_0| e^{a_s \tau_s}, \qquad \tau_s \sim \frac{\xi^{2/3}}{g_s}.$$

This minimum is non-supersymmetric AdS and at exponentially large volume.

Moduli Stabilisation: Large-Volume



Moduli Stabilisation: Large-Volume

Higher α' corrections are suppressed by more powers of volume.

Example:

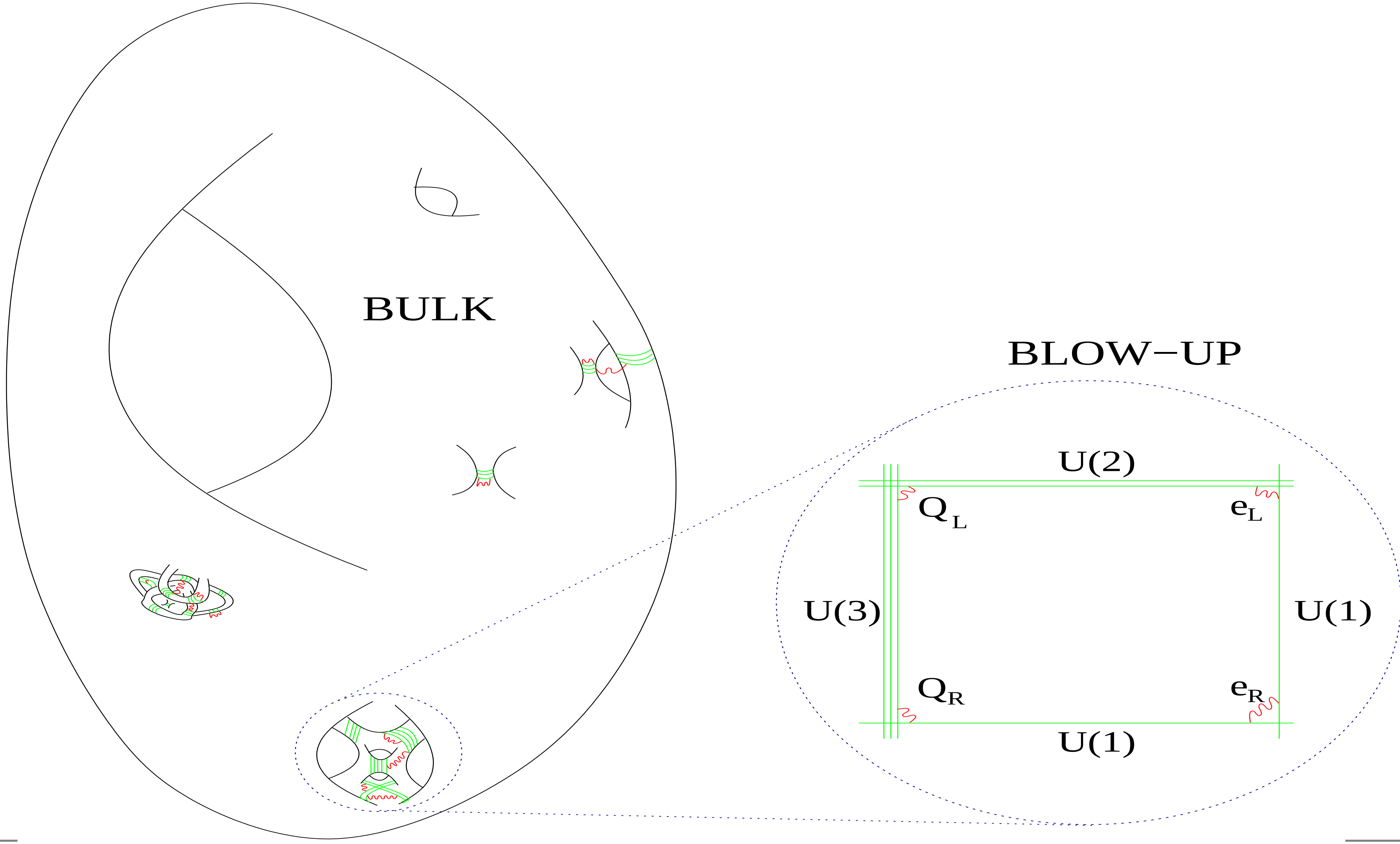
$$\begin{aligned}
\int d^{10}x\sqrt{g}\mathcal{G}_3^2\mathcal{R}^3 \quad &: \quad \int d^{10}x\sqrt{g}\mathcal{R}^4 \\
\int d^4x\sqrt{g_4}\left(\int d^6x\sqrt{g_6}\mathcal{G}_3^2\mathcal{R}^3\right) \quad &: \quad \int d^4x\sqrt{g_4}\left(\int d^6x\sqrt{g_6}\mathcal{R}^4\right) \\
\int d^4x\sqrt{g_4}\left(\mathcal{V}\times\mathcal{V}^{-1}\times\mathcal{V}^{-1}\right) \quad &: \quad \int d^4x\sqrt{g_4}\left(\mathcal{V}\times\mathcal{V}^{-4/3}\right) \\
\int d^4x\sqrt{g_4}\left(\mathcal{V}^{-1}\right) \quad &: \quad \int d^4x\sqrt{g_4}\left(\mathcal{V}^{-1/3}\right)
\end{aligned}$$

Moduli Stabilisation: Large-Volume

Loop corrections are suppressed by more powers of volume: there exists an 'extended no scale structure'

$$
\begin{aligned}
W &= W_0, \\
K_{full} &= K_{tree} + K_{loop} + K_{\alpha'} \\
&= -3\ln(T+\bar{T}) + \underbrace{\frac{c_1}{(T+\bar{T})(S+\bar{S})}}_{loop} + \underbrace{\frac{c_2(S+\bar{S})^{3/2}}{(T+\bar{T})^{3/2}}}_{\alpha'}. \\
V_{full} &= V_{tree} + V_{loop} + V_{\alpha'} \\
&= \underbrace{0}_{tree} + \underbrace{\frac{c_2(S+\bar{S})^{3/2}}{(T+\bar{T})^{3/2}}}_{\alpha'} + \underbrace{\frac{c_1}{(S+\bar{S})(T+\bar{T})^2}}_{loop}
\end{aligned}
$$

Moduli Stabilisation: Large-Volume



Moduli Stabilisation: Large-Volume

- The stabilised volume is exponentially large.
- The large volume lowers the string scale and gravitino mass through

$$m_s = \frac{M_P}{\sqrt{\mathcal{V}}}, \qquad m_{3/2} = \frac{M_P}{\mathcal{V}}.$$

- To solve the gauge hierarchy problem, need $\mathcal{V} \sim 10^{15}$.
- D7-branes wrapped on small cycle carry the Standard Model: need $T_s \sim 20(2\pi\sqrt{\alpha'})^4$.
- The vacuum is pseudo no-scale and breaks susy...

Moduli Stabilisation: Large-Volume

The mass scales present are:

Planck scale:	$M_P = 2.4 \times 10^{18}$GeV.
String scale:	$M_S \sim \frac{M_P}{\sqrt{\mathcal{V}}} \sim 10^{11}$GeV.
KK scale	$M_{KK} \sim \frac{M_P}{\mathcal{V}^{2/3}} \sim 10^9$GeV.
Gravitino mass	$m_{3/2} \sim \frac{M_P}{\mathcal{V}} \sim 30$TeV.
Small modulus	$m_{\tau_s} \sim m_{3/2} \ln\left(\frac{M_P}{m_{3/2}}\right) \sim 1000$TeV.
Complex structure moduli	$m_U \sim m_{3/2} \sim 30$TeV.
Soft terms	$m_{susy} \sim \frac{m_{3/2}}{\ln(M_P/m_{3/2})} \sim 1$TeV.
Volume modulus	$m_{\tau_b} \sim \frac{M_P}{\mathcal{V}^{3/2}} \sim 1$MeV.

SUSY Breaking and Soft Terms

Supersymmetry will (hopefully) be discovered at the LHC.

It is parametrised by

- Soft scalar masses, $m_i^2 \phi_i^2$
- Gaugino masses, $M_a \lambda^a \lambda^a$,
- Trilinear scalar A-terms, $A_{\alpha\beta\gamma}\phi^\alpha\phi^\beta\phi^\gamma$
- B-terms, BH_1H_2.

SUSY Breaking and Soft Terms

The fact that the gravitino mass is $m_{3/2} \sim 1$TeV means supersymmetry is broken at a hierachically low scale.

- Soft breaking terms can be computed.
- These follow from the F-terms of the Kähler moduli and their coupling to matter.
- The soft terms are the input into phenomenological studies of low-scale supersymmetry.

Mirror Mediation

- These soft terms are flavour-universal.

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- An invalid argument:

  In gravity mediation flavour and susy breaking are both Planck-scale physics.

  Therefore susy breaking is sensitive to flavour

  Therefore squark masses are non-universal.

Mirror Mediation

- These soft terms are flavour-universal.
- An invalid argument:

  In gravity mediation flavour and susy breaking are both Planck-scale physics.

  Therefore susy breaking is sensitive to flavour

  Therefore squark masses are non-universal.
- In string theory, we have Kähler (T) and complex structure (U) moduli. These are decoupled at leading order.

$$\mathcal{K} = -2\ln\left(\mathcal{V}(T)\right) - \ln\left(i\int \Omega \wedge \bar{\Omega}(U)\right) - \ln(S+\bar{S}).$$

- The kinetic terms for T and U fields do not mix.

Mirror Mediation

- Due to the shift symmetry $T \rightarrow T + i\epsilon$, the T moduli make no perturbative appearance in the superpotential.
- It is the U moduli that source flavour...

$$W = \ldots + \frac{1}{6} Y_{\alpha\beta\gamma}(U) C^{\alpha} C^{\beta} C^{\gamma} + \ldots .$$

- ...and the T moduli that break supersymmetry,

$$D_T W \neq 0, F^T \neq 0, \qquad D_U W = 0, F^U = 0.$$

- At leading order, susy breaking (Kähler moduli) and flavour (complex structure moduli) decouple.

Axions

- Axions are a well-motivated solution to the strong CP problem.
- The QCD Lagrangian is

$$\mathcal{L}_{QCD} = \frac{1}{g^2} \int d^4x F^a_{\mu\nu} F^{a,\mu\nu} + \theta \int F^a \wedge F^a.$$

- The strong CP problem:

  Naively $\theta \in (-\pi, \pi)$ - experimentally $|\theta| \lesssim 10^{-10}$.

- The axionic (Peccei-Quinn) solution is to promote θ to a dynamical field, $\theta(x)$.

Axions

- The canonical Lagrangian for θ is

$$\mathcal{L} = \frac{1}{2}\partial_\mu \theta \partial^\mu \theta + \int \frac{\theta}{f_a} F^a \wedge F^a.$$

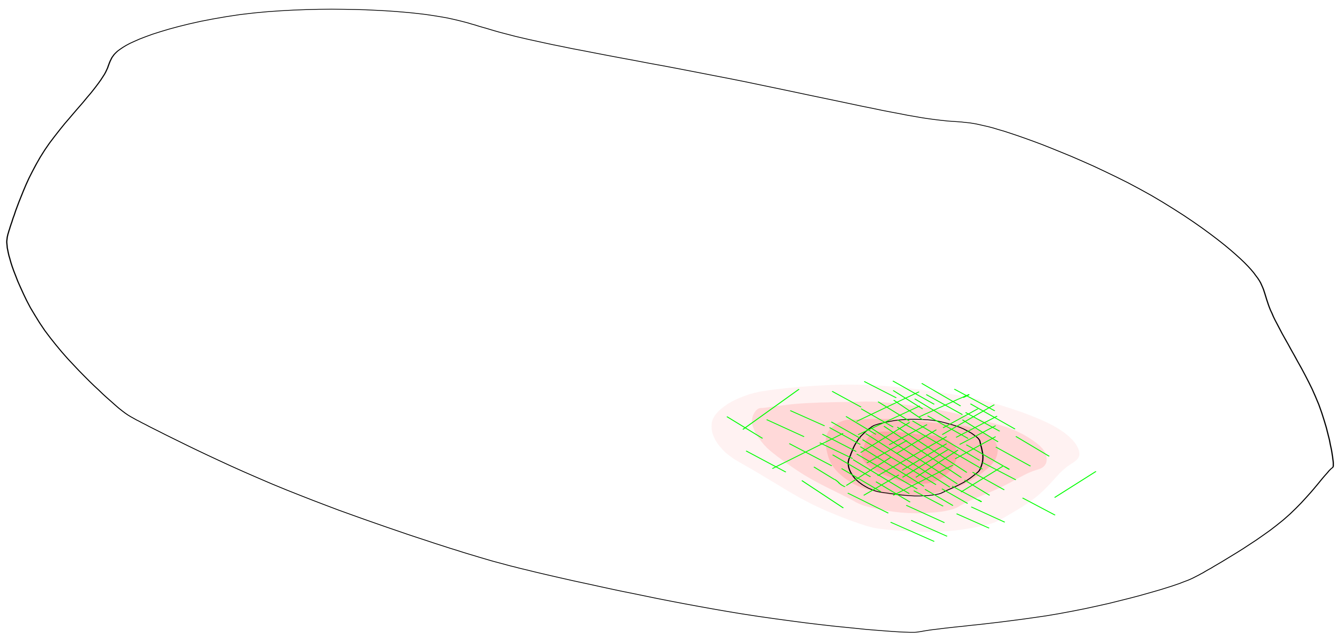
f_a is the axionic decay constant.

- Constraints on supernova cooling and direct searches imply $f_a \gtrsim 10^9$GeV.
- Avoiding the overproduction of axion dark matter prefers $f_a \lesssim 10^{12}$GeV.
- There exists an axion 'allowed window',

$$10^9\text{GeV} \lesssim f_a \lesssim 10^{12}\text{GeV}.$$

Axions

- For D7 branes, the axionic coupling comes from the RR form in the brane Chern-Simons action.
- The axion decay constant f_a measures the coupling of the axion to matter.



Axions

- The coupling of the axion to matter is a local coupling and does not see the overall volume.
- This coupling can only see the string scale:

$$f_a \sim m_s \sim \frac{M_P}{\sqrt{\mathcal{V}}}.$$

(This is confirmed by a full analysis)

- This generates the axion scale,

$$f_a \sim \frac{M_P}{\sqrt{\mathcal{V}}} \sim 10^{11}\text{GeV}.$$

Neutrino Masses

- Neutrino masses exist:

$$0.05\text{eV} \lesssim m_\nu^H \lesssim 0.3\text{eV}.$$

- In the seesaw mechanism, this corresponds to a Majorana mass scale for right-handed neutrinos

$$M_{\nu_R} \sim 3 \times 10^{14}\text{GeV}.$$

- Equivalently, this is the suppression scale Λ of the dimension five MSSM operator

$$\begin{aligned} \mathcal{O}_{m_\nu} &= \frac{1}{\Lambda} H_2 H_2 L L \\ \Rightarrow m_\nu &= 0.1\text{eV} \left(\sin^2\beta \times \frac{3 \times 10^{14}\text{GeV}}{\Lambda} \right). \end{aligned}$$

Neutrino Masses

Neutrino masses imply a scale $\Lambda \sim (\text{a few}) \times 10^{14}$GeV which is

- not the Planck scale 10^{18}GeV
- not the GUT scale 10^{16}GeV
- not the intermediate scale 10^{11}GeV
- not the TeV scale 10^{3}GeV

Can the intermediate-scale string give a quantitative understanding of this scale?

Neutrino Masses

The low-energy theory is

$$W = Y_{\alpha\beta\gamma} C^\alpha C^\beta C^\gamma + \frac{Z_{\alpha\beta\gamma\delta}}{M_P} C^\alpha C^\beta C^\gamma C^\delta + \frac{\lambda}{M_P} H_2 H_2 L L,$$

$$K = \hat{K}(\Phi, \bar{\Phi}) + \tilde{K}_{\alpha\bar{\beta}} \Phi^\alpha \bar{\Phi}^{\bar{\beta}} + \dots$$

We focus particularly on the term

$$\frac{\lambda}{M_P} H_2 H_2 L L \in W$$

This term generates neutrino masses when the Higgs gets a vev.

Neutrino Masses

The physical normalised operator is

$$e^{\hat{K}/2}\frac{\lambda}{M_P}\frac{H_2 H_2 L L}{\left(\tilde{K}_{H\bar{H}}\tilde{K}_{H\bar{H}}\tilde{K}_{L\bar{L}}\tilde{K}_{L\bar{L}}\right)^{\frac{1}{2}}}$$

We know

$$\hat{K} = -2\ln(\mathcal{V})$$

To compute the physical suppression scale, we need the volume scaling of $\tilde{K}_{H\bar{H}}$ and $\tilde{K}_{L\bar{L}}$.

Neutrino Masses

We can compute this using the Yukawa couplings.

The physical Yukawa couplings are

$$\hat{Y}_{\alpha\beta\gamma} = e^{\hat{K}/2} \frac{Y_{\alpha\beta\gamma}}{\sqrt{K_{\alpha\bar{\alpha}} K_{\beta\bar{\beta}} K_{\gamma\bar{\gamma}}}}$$

Geometry implies these couplings are local.

We know

- $Y_{\alpha\beta\gamma}$ does not depend on $\mathcal{V}$.
- The overall term $e^{\hat{K}/2} \sim \frac{1}{\mathcal{V}}$.

Neutrino Masses

The local interactions care only about the local geometry and decouple from the bulk volume.

Physical locality then implies the physical Yukawa couplings $\hat{Y}_{\alpha\beta\gamma}$ do not depend on the bulk volume.

For local fields C^{α} the relation

$$\hat{Y}_{\alpha\beta\gamma} = e^{\hat{K}/2} \frac{Y_{\alpha\beta\gamma}}{\sqrt{K_{\alpha\bar{\alpha}} K_{\beta\bar{\beta}} K_{\gamma\bar{\gamma}}}}$$

implies

$$\tilde{K}_{\alpha\bar{\alpha}} \sim \frac{1}{\mathcal{V}^{2/3}}.$$

Neutrino Masses

- Using the large-volume result $\tilde{K}_\alpha \sim \frac{\tau_s^{1/3}}{\mathcal{V}^{2/3}}$, the physical coupling becomes

$$\frac{\lambda \mathcal{V}^{1/3}}{\tau_s^{2/3} M_P} \langle H_2 H_2 \rangle LL.$$

- Use $\mathcal{V} \sim 10^{15}$ (to get $m_{3/2} \sim 1\text{TeV}$) and $\tau_s \sim 10$:

$$\frac{\lambda}{10^{14}\text{GeV}} \langle H_2 H_2 \rangle LL$$

- With $\langle H_2 \rangle = \frac{v}{\sqrt{2}} = 174\text{GeV}$, this gives

$$m_\nu = \lambda (0.3\,\text{eV}).$$

Neutrino Masses

Integrating out string / KK states generates a dimension-five operator suppressed by

$$\text{(string)} \qquad \mathcal{V}^{-1/12} \times \frac{\mathcal{V}^{\frac{1}{2}}}{M_P} \times \mathcal{V}^{-1/12} \sim \frac{\mathcal{V}^{1/3}}{M_P}$$

$$\text{(KK)} \qquad \mathcal{V}^{-1/6} \times \frac{\mathcal{V}^{2/3}}{M_P} \times \mathcal{V}^{-1/6} \sim \frac{\mathcal{V}^{1/3}}{M_P}$$

- Integrating out heavy states of mass M does not produce operators suppressed by M^{-1}.
- The dimension-five suppression scale is independent of the masses of the heavy states integrated out.

A new scale....

- The volume modulus χ always has a mass

$$m_\chi \sim \frac{m_{3/2}^{3/2}}{M_P^{\frac{1}{2}}} \sim 1\text{MeV}.$$

This is a *totally robust* prediction of these models.

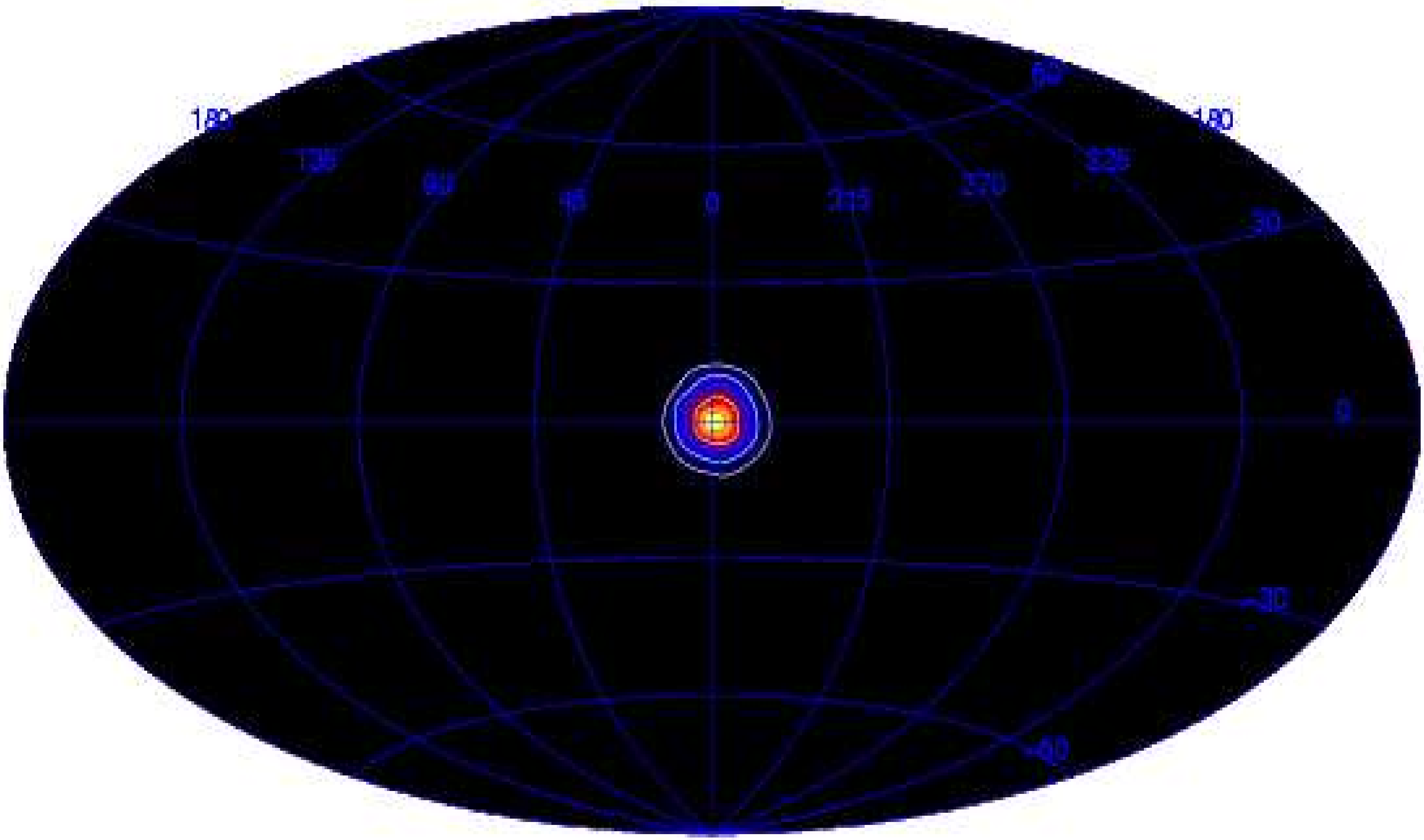
- This particle can decay via $\chi \to 2\gamma$ and $\chi \to e^+e^-$.

One can show

$$\begin{aligned} \tau_\chi &\sim 10^{27} s, \\ Br(\chi \to e^+e^-) &\sim \frac{\ln(M_P/m_{3/2})^2}{20} Br(\chi \to 2\gamma). \end{aligned}$$

A new scale...

The sky at 511keV (as was...)



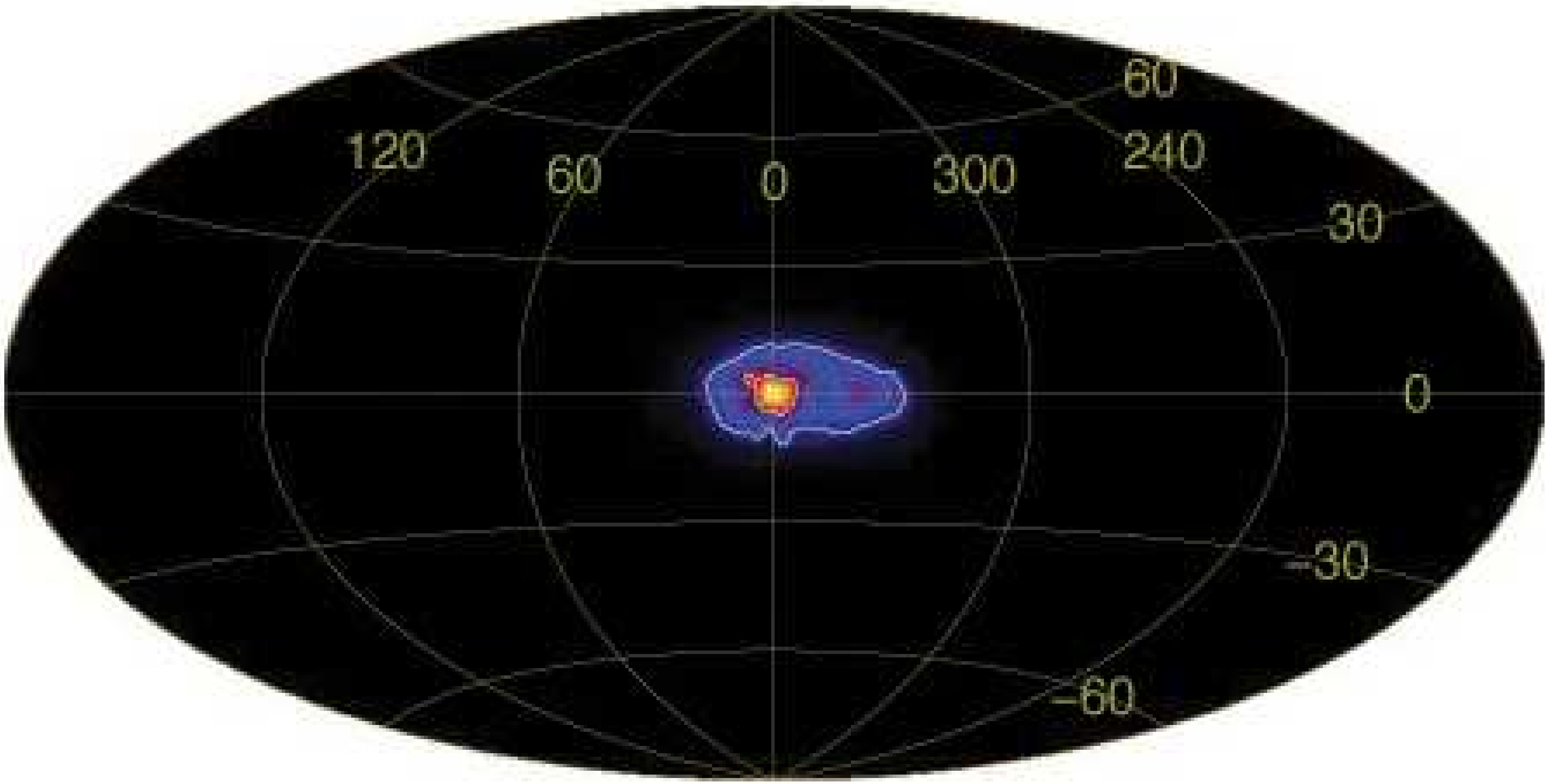
(INTEGRAL/SPI, ESA, 2005)

A new scale...

- There is a large flux of positrons from the galactic centre.
- The astrophysical origin of these positrons was not well know, hinting at new physics around 1 MeV.
- If present, this could arise from light dark matter annihilating or decaying in the galactic centre.
- However....

A new scale...

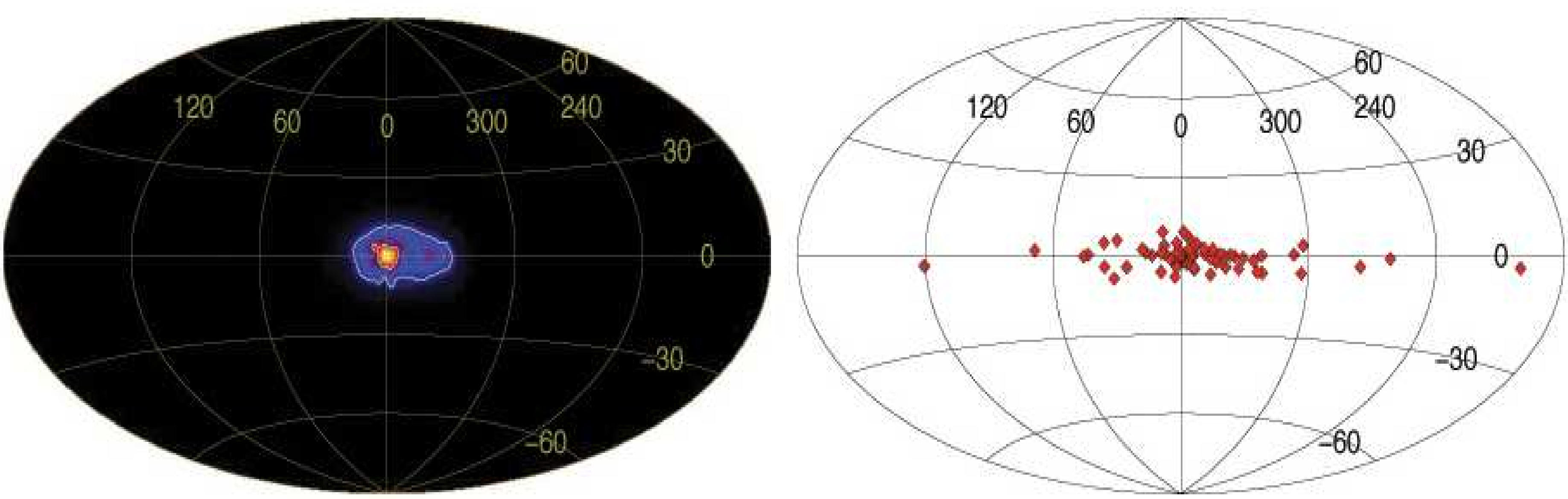
The sky at 511keV (now...)



(INTEGRAL/SPI, ESA (January 2008))

A new scale...

- The positron distribution is now asymmetric and does not look like a dark matter distribution.
- It also correlates with the distribution of low-mass hard X-ray binaries:



A new scale....

- The decays of the volume modulus would contribute both to the cosmic gamma-ray background and to the 511keV flux.
- Non-observation constrains the abundance of the volume modulus to

$$\Omega_\chi \lesssim 10^{-4}.$$

- At best, can contribute a small fraction of dark matter.

Large Volumes are Power-ful

In large-volume models, an exponentially large volume naturally appears ($\mathcal{V} \sim e^{\frac{c}{g_s}}$). This generates scales

- Susy-breaking: $m_{soft} \sim \frac{M_P}{\mathcal{V}} \sim 10^3$GeV
- Axions: $f_a \sim \frac{M_P}{\sqrt{\mathcal{V}}} \sim 10^{11}$GeV
- Neutrinos/dim-5 operators: $\Lambda \sim \frac{M_P}{\mathcal{V}^{1/3}} \sim 10^{14}$GeV
- A new scale at $m \sim \frac{M_P}{\mathcal{V}^{3/2}} \sim 1$MeV.
- All four scales (plus flavour universality) are yoked in an attractive fashion.
- The origin of all four scales is the exponentially large volume.