Astroparticle Physics: Problem Sheet 1

1. Working in natural units ($\hbar = c = 1$), so everything is expressed in appropriate powers (positive or negative) of GeV, evaluate the following.

- $8\pi G$, where G is Newton's constant
- The Fermi scale, G_F , describing the strength of weak interactions.
- The radius of the proton, 10^{-15} m.
- The Schwarzschild radius of a solar mass black hole $(M = 1M_{\odot})$
- The Schwarschild radius of a super-massive black hole $(M = 10^6 M_{\odot})$
- The radial size of both the Milky Way and also that of dwarf galaxies
- The Compton wavelength of an electron (m = 0.5MeV) and the Compton wavelength of a neutrino (take $M = 10^{-3}$ eV)
- The plasma mass of the photon in galactic space (you can take $n_e = 10^{-12} \text{cm}^{-3}$)
- The temperature of the cosmic microwave background (take T = 2.73K)

2. Estimate the rate of Higgs boson production in the upper atmosphere, caused by the interaction of high-energy protons from cosmic rays with stationary protons. Take initiative in looking up the magnitude of Higgs boson production cross-sections! First work out the arrival flux of cosmic rays protons with energies above the threshold for Higgs boson production.

3. Consider the example in the lectures of a scalar dark matter particle with Lagrangian

$$\mathcal{L} = \frac{1}{2} \partial_{\mu} \phi \partial^{\mu} \phi - \frac{1}{2} m^2 \phi^2 + \frac{\lambda \phi}{M_P} F_{\mu\nu} F^{\mu\nu}$$

Treating this as a decaying dark matter candidate, compute the flux of photons arriving on Earth from dark matter decay at a telescope with a field of view of 1 degree in diameter and an area of 1 square metre. (You should look up an estimate for the dark matter density but may assume that no photons are absorbed along the line of sight.)

4. For the case of question 3, work out the velocity dispersion of the arriving photon line and hence work out the induced broadening of the dark matter decay line. Consider these three possible places to search for dark matter decay: the center of the Milky Way, the Draco dwarf galaxy and the centre of the Perseus cluster of galaxies. Comment on the *advantages* and *disadvantgaes* of each as a location to look for decaying dark matter.

5. Work out the details of the argument sketched in the lectures that considerations of Fermi degeneracy and galactic confinement imply that the mass of fermionic dark matter has to satisfy $m \gtrsim 1 \text{eV}$.