Yang-Mills Matrix Integrals

The quantum mechanics obtained by dimensional reduction of Yang-Mills field theories to one dimension was first studied in the 1980s [1] when it was discovered that the supersymmetric theory (SSYM) in ten dimensions with gauge group $SU(N)$ could be regarded in the large N limit as a light-cone regularization of the super-membrane [2]. In the ensuing period many remarkable discoveries were made in string theory. All the known super-string theories have been found to be related to each other by various duality transformations and to 11-dimensional supergravity by compactification, and the role of D-branes as solitons in string theories opened another window on non-perturbative string physics. The relationships between these various string theories clearly implies that they are all different perturbative limits of an over-arching theory, christened M-theory. It was conjectured by BFSS [3] that the regularized supermembrane theory provided by the large N limit of SSYM quantum mechanics, now known as M(atrix) Theory, represents M-theory in the light-cone gauge. Simultaneously IKKT proposed that the dimensional reduction of $D = 10$ $SU(N)$ SSYM to zero dimensions (which is of course the reduction of the quantum mechanics by its remaining dimension) describes in the large N limit the type IIB superstring [4]. These developments have generated renewed interest in Yang-Mills quantum mechanics and the Yang-Mills matrix integrals which occur both when studying aspects of the quantum mechanics and in the IKKT model.

The Yang-Mills matrix integral partition function, which is obtained by dimensionally reducing the Euclidean SSYM action from D down to zero dimensions, is given by

$$\mathcal{Z}_{D,G} = \int \prod_{\mu=1}^{D} dX_\mu \prod_{\alpha=1}^{\mathcal{N}} d\psi_\alpha \exp\left(\sum_{\mu,\nu} \mathrm{Tr}\ [X_\mu, X_\nu]^2 + \mathrm{Tr}\, \psi_\alpha [\Gamma^\mu_{\alpha\beta} X_\mu, \psi_\beta] \right). \tag{1}$$

The traceless hermitian matrix fields X_μ and ψ_α (respectively bosonic and fermionic) are in the Lie algebra $\mathcal{G}$ of the gauge group G. In two recent papers we have developed methods which enabled us to give the first rigorous proofs of various properties of these models [5, 6]. These results are important because they underpin various other methods which have been employed in studying these models. We have established which of these matrix models exist in the sense of having finite partition functions and at least some sensible correlation functions. Our classification covers all variants of the model with and without supersymmetry and all compact gauge groups.

We now propose to focus more explicitly on the large N limit in the $SU(N)$ case which is the model of direct relevance to M-theory. These models are difficult to treat analytically for various reasons; for example there is no small parameter in the model and therefore no well-regulated perturbation expansion. The physics is inherently non-perturbative. Our interest is in establishing the behaviour of correlation functions in the large N limit by rigorous methods. This is an important topic because it underlies the whole question of whether the matrix theories really are a representation of M-theory, and therefore central to the subject, or just some backwater in mathematical physics.

References

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