Renormalisation Group & Critical Phenomena Hilary 2005 Prof. J. Cardy

Problem Set 1

The solutions to these problems should be handed in by **Friday, February 18** for marking. They should be sent to Dr. P. Calabrese in Theoretical Physics. They will be reviewed at a problems class at a time TBA, which will be conducted by Dr. Calabrese. These problems are all from the book and are numbered accordingly. Hints for doing them can be found on my web page, but try them without first. Any other questions before the due date may be addressed to me at j.cardy1@physics.ox.ac.uk.

2.2) Show that the mean field equations are exact in the thermodynamic limit $N \to \infty$ for the *infinite-range* Ising model, with hamiltonian $\mathcal{H} = -(J/N)\sum_{r,r'} s(r)s(r')$, where the sum is over *all* pairs of spins (r, r'). [Hint: write the partition function as a Gaussian integral, carry out the trace over the spins, and use the method of steepest descent.]

2.5) Work out the mean field theory for the spin-1 Ising model (the Blume–Capel model) given by the hamiltonian

$$\mathcal{H} = -\tfrac{1}{2}\sum_{r,r'} J(r-r')s(r)s(r') + \Delta\sum_{r} s(r)^2$$

where $s(r)$ takes the values 0 or ± 1. [Hint: do not try to approximate the $s(r)^2$ term.] Show that there is a point on the critical line where the coefficient of the M^4 term in the expansion of the mean field free energy vanishes. This corresponds to a tricritical point. What are the Landau values for the various standard critical exponents at this point?

2.7) Repeat the mean field calculation of the correlation function in the Ising model, for the case $T < T_c$, still in zero applied field. In the relation $\xi \sim \xi_0|t|^{-\nu}$, what is the ratio of the amplitudes ξ_0 above and below T_c, at least in mean field theory?

3.1) The one-dimensional Ising model in a magnetic field has the reduced hamiltonian $\mathcal{H} = -K\sum_j s_j s_{j+1} - h\sum_j s_j$. By summing over every other spin, show that you can define an RG transformation with $b = 2$. It is useful to express this in terms of the variables $x = e^{-2K}$ and $y = e^h$. Sketch the flows in the (x, y) plane and indicate the fixed points.

3.5) Suppose that the infinitesimal RG equations have the form $dg_i/d\ell = -\beta_i(\{g\})$ where the right hand sides have a perturbative expansion of the form $\beta_i = -y_i g_i + \sum_{jk} c_{ijk} g_j g_k +$

$\cdots$. Show that, in general, it is possible to define, order by order in the gs, *non-linear* scaling variables $g_i' \equiv g_i + \sum_{jk} d_{ijk} g_j g_k + \cdots$ so that the RG equations simplify to $dg_i'/d\ell = y_i g_i'$ exactly, with no higher order terms. Under what circumstances does such a transformation fail (to the order stated)? If there was a non-trivial fixed point at some finite value of the g_i in terms of the old variables, what has happened to it in terms of the non-linear scaling variables?