Green functions of $T^{\mu\nu}$ during weak coupling hydrodynamization

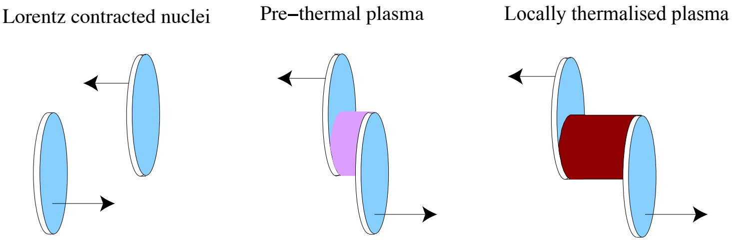
Aleksi Kurkela

AK, Mazeliauskas, Paquet, Schlichting, Teaney, in progress
Keegan, AK, Mazeliauskas, Teaney JHEP 1608 (2016) 171
AK, Zhu PRL 115 (2015) 18, 182301; AK, Lu PRL 113 (2014) 18, 182301
AK, Moore JHEP 1111 (2011) 120
AK, Moore JHEP 1112 (2011) 044

Universitetet i Stavanger

Oxford, March 2017

Motivation?



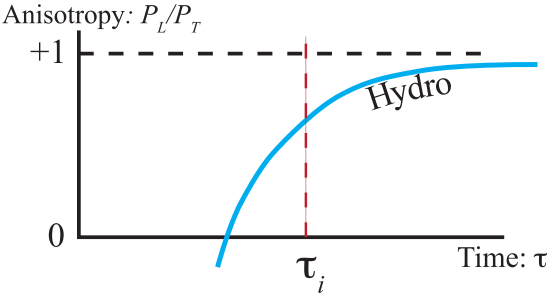
- Soft physics of HIC described by relativistic hydrodynamics

$$\partial_\mu T^{\mu\nu} = 0$$

- Gradient expansion around local thermal equilibrium

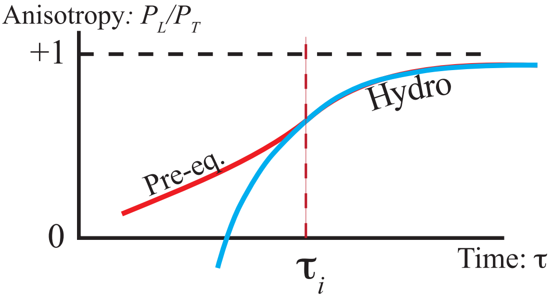
$$T^{\mu\nu} = T^{\mu\nu}_{\text{eq.}} - \eta 2\nabla^{<\mu} u^{\nu>} + \ldots$$

Motivation?



- At early times *pre-equilibrium* evolution
- Hydro simulations start at *intialization time* τ_i

Motivation:



- If prethermal evolution converges smoothly to hydro, independence of unphysical τ_i

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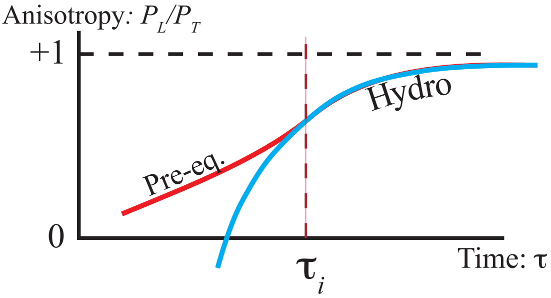

- If prethermal evolution converges smoothly to hydro, independence of unphysical τ_i
- In most current pheno: either free streaming, or nothing at all

Motivation:

- In AA collisions: pre-equilibrium evolution $\sim 10\%$ of the evolution
    - Pre-equilibrium evolution major uncertainty affects η/s, etc

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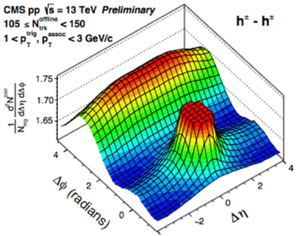

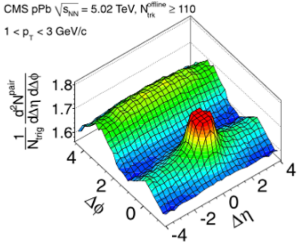

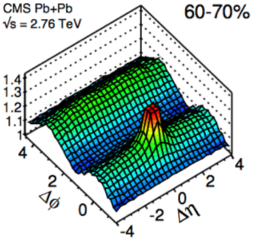

Motivation:

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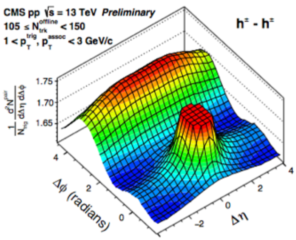
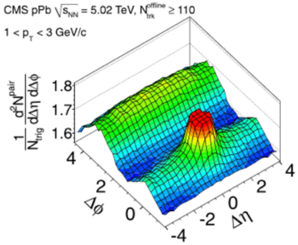
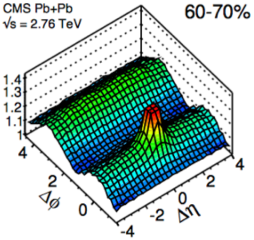
  - Pre-equilibrium evolution major uncertainty affects η/s, etc

- In pA collisions: currently no quantitative description
  - even if the system becomes hydrodynamical, "pre-equilibrium" evolution $\mathcal{O}(1)$ of the evolution







Motivation:

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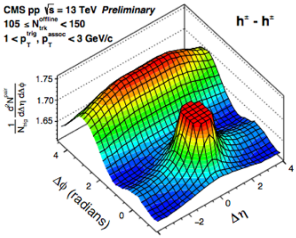
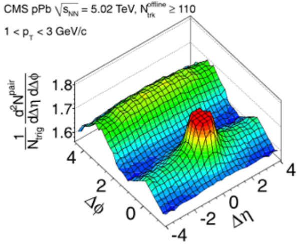
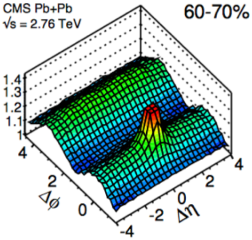
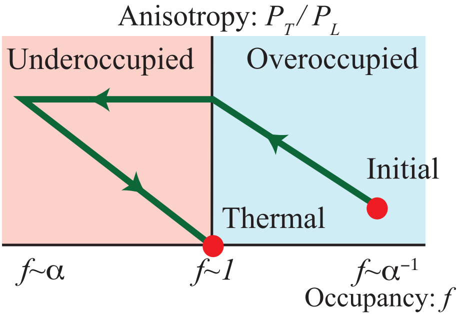
- pp collisions: ?????







Hydrodynamization in weak coupling



- Color Glass Condensate: Initial condition overoccupied

  McLerran, Venugopalan PRD49 (1994) , PRD49 (1994); Gelis et. al Int.J.Mod.Phys. E16 (2007), Ann.Rev.Nucl.Part.Sci. 60 (2010)

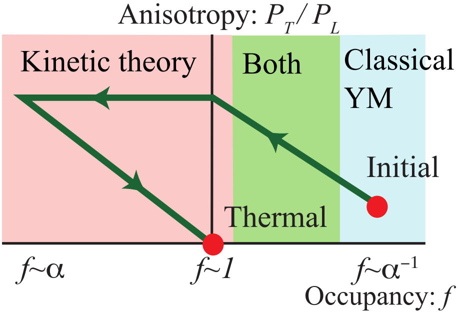
$$f(Q_s) \sim 1/\alpha_s, \qquad Q_s \sim 2\text{GeV}$$

- Expansion makes system underoccupied before thermalizing

  Baier et al PLB502 (2001)

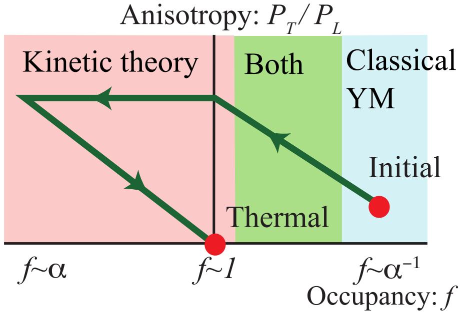
$$f(Q_s) \ll 1$$

Hydrodynamization in weak coupling



- Degrees of freedom:
  - $f \gg 1$: Classical Yang-Mills theory (CYM)
  - $f \ll 1/\alpha_s$: (Semi-)classical particles, Eff. Kinetic Theory (EKT)

Hydrodynamization in weak coupling

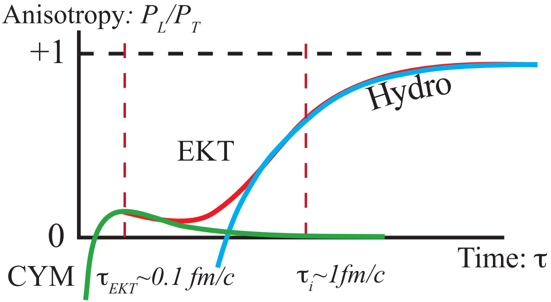


- Transmutation of fields to particles: Field-particle duality
  Son, Mueller PLB582 (2004) 279-287; Jeon PRC72 (2005) 014907; Mathieu et al EPJ. C74 (2014) 2873; AK et al PRD89 (2014) 7, 074036

$$1 \ll f \ll 1/\alpha_s$$

- "Bottom-up thermalization" of underoccupied system

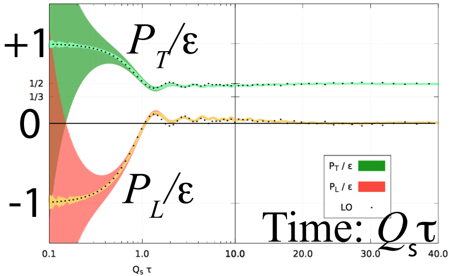
Strategy at weak coupling



Strategy: Switch from CYM to EKT at τ_{EKT}, $1 \ll f \ll 1/\alpha_s$

From EKT to hydro at τ_i, $P_L/P_T \sim 1$

Early times $0 < Q_s\tau \lesssim 1$: classical evolution



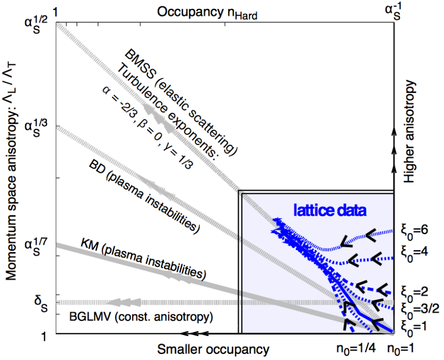
Epelbaum & Gelis, PRL. 111 (2013) 23230

- Melting of the coherent boost invariant CGC fields

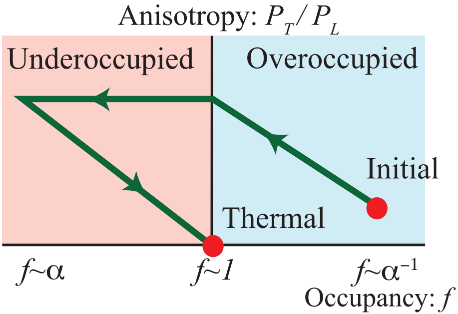
Initial condition from CGC: MV-model, JIMWLK

  - After $\tau \sim 1/Q_s$, fields decohere, $P_L > 0$

Later times $Q_s\tau > 1$: classical evolution



Berges et al. Phys.Rev. D89 (2014) 7, 074011



- Numerical demonstration of overoccupied part of the diagram
- Classical theory never thermalizes or isotropizes
- Before $f \sim 1$, must switch to kinetic theory

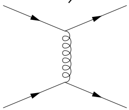
Outline

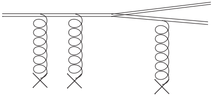
- Effective kinetic theory
- Hydrodynamization and thermalization at weak coupling in effective kinetic theory
- Apples to apples comparison of weak and strong coupling hydrodynamization
- Green functions of $T^{\mu\nu}$ in during hydrodynamization and phenomenological application to HIC

Effective kinetic theory of Arnold, Moore, Yaffe

JHEP 0301 (2003) 030

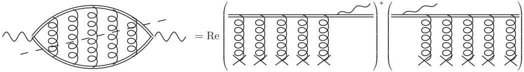
$$\frac{df}{dt} = -C_{2\leftrightarrow 2}[f] - C_{1\leftrightarrow 2}[f]$$





- Soft and collinear divergences lead to nontrivial matrix elements

soft: screening, Hard-loop; collinear: LPM, ladder resum



- No free parameters; LO accurate in the $\alpha_s \to 0$, $\alpha_s f \to 0$ limit, for

$$\Delta t \sim \omega^{-1} > \text{Typical scattering time } \sim 1/(\alpha^2 T)$$

,

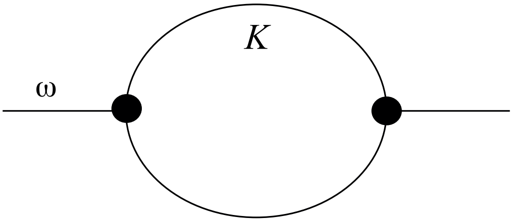
- Caveat: in anistropic systems screening complicated. Here with isotropic screening. Also no fermions here

plasma instabilities, ...

Why kinetic theory needed?

LO spectral function in unresummed pert-theory

$$\rho_{\phi^2\phi^2}(\omega,k) \sim \int \frac{d^4k}{(2\pi)^4}\left(1+n(-k^0+\omega)\right)(1+n(k^0))\rho(k,-k^0+\omega)\rho(k,k^0)$$



Jeon PRD47 (1993)

Why kinetic theory needed?

LO spectral function in unresummed pert-theory

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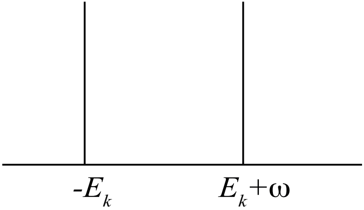
- Free spectral function

$$\rho_{\text{free}} = \text{sign}(k^0)2\pi\delta(-(k^0)^2+k^2+m^2)$$

- No overlap if $\omega < 2m$



$$k^0 = -\sqrt{k^2+m^2}, \qquad k^0 = \sqrt{k^2+m^2}+\omega$$

Jeon PRD47 (1993)

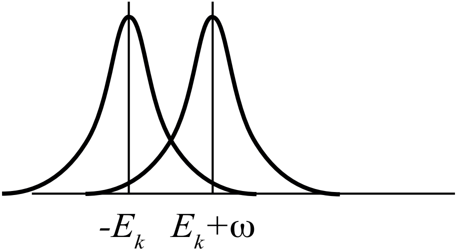
Why kinetic theory needed?

- In interactive theory

$$\rho(k^0,k) \approx \frac{4k^0\Gamma_k}{\left[(k^0)^2 - E_k^2\right]^2 + 4(k^0\Gamma_k)^2}$$

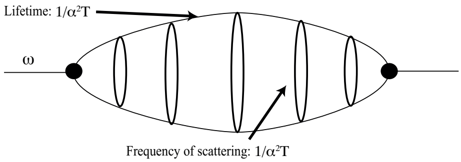
Smooth limit

$$\lim_{\omega\to 0}\frac{\rho(0,\omega)}{\omega} \sim \int \frac{d^4k}{(2\pi)^4} n(E_k)(1+n(E_k))\frac{1}{E_k^2\Gamma_k}$$



- In weak coupling $\Gamma_k \sim \alpha^2 T$
- coupling in the denominator $\to$ resummation needed

Why kinetic theory needed?



- Physical reason: Both lines long lived $(\alpha^2 T)^{-1}$, of the order or scattering time

- Diagrammatic resummation (in $\lambda\phi^4$)

  Jeon PRD52 (1995)

- Interpretation of the diagrammatic resummation in terms of effective kinetic theory

  Jeon, Yaffe PRD53 (1996)

- Generalization to gauge theories through power counting

  Arnold et al. JHEP 0301 (2003) 030

Outline

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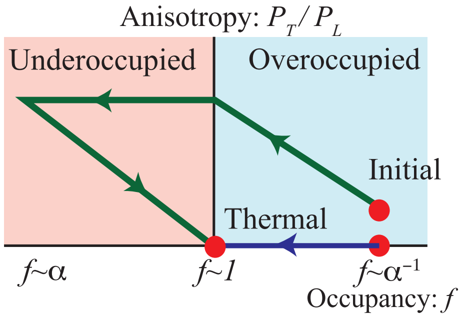
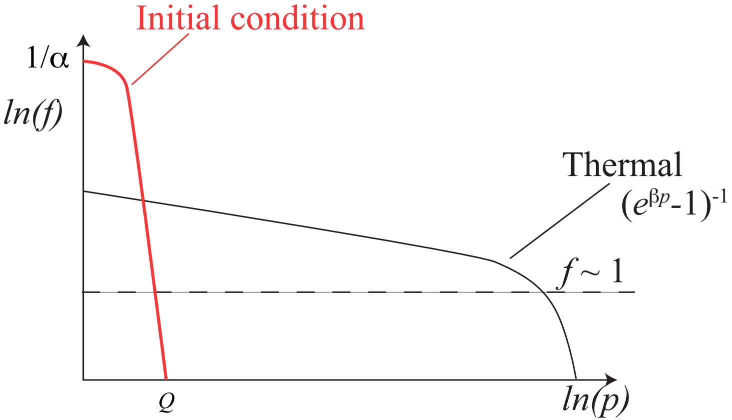


- Isotropic overoccupied: Transmutation of d.o.f's
- Isotropic underoccupied: Radiative break-up
- Effect of longitudinal expansion: Hydrodynamization

Overoccupied cascade

AK, Moore JHEP 1112 (2011) 044

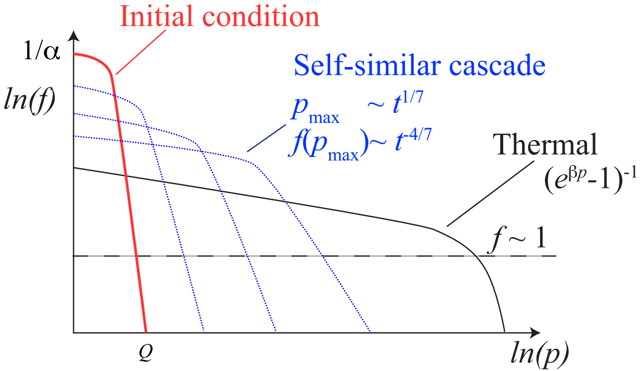
What happens if you have too many soft gluons, $f \sim 1/\alpha_s$.



Overoccupied cascade

AK, Moore JHEP 1112 (2011) 044

What happens if you have too many soft gluons, $f \sim 1/\alpha_s$.
No longitudinal expansion.

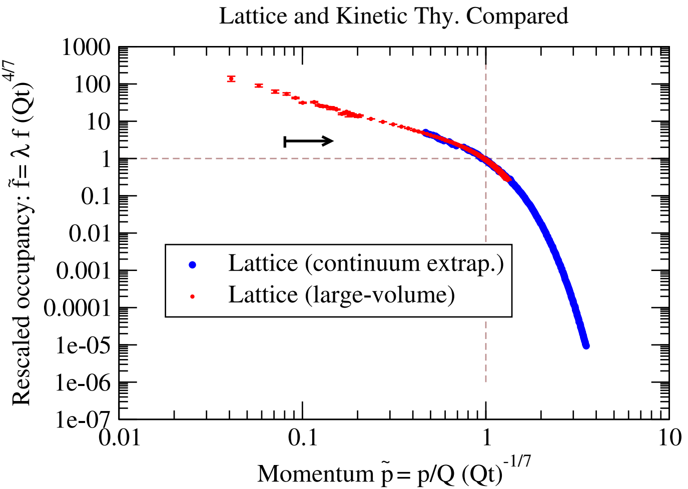


$$\tau_{\rm init} \sim [\sigma n(1+f)]^{-1} \sim \left(\frac{Q}{T}\right)^7 \frac{1}{\alpha_s^2 T} \ll \frac{1}{\alpha_s^2 T} \sim \tau_{\rm them}.$$

c.f. Bokuslawski's talk

Overoccupied cascade

AK, Lu, Moore, PRD89 (2014) 7, 074036



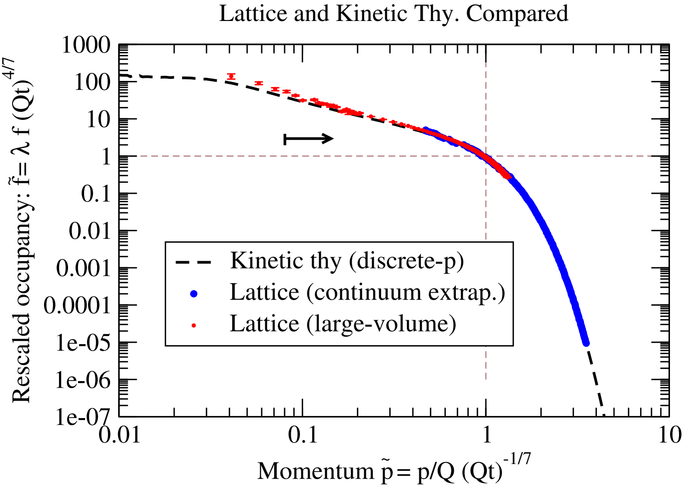
Form of cascade from classical lattice simulation,

$$1 \ll f \lesssim 1/\alpha_s$$

Large-volume: (Qa)=0.2, (QL)=51.2, Cont. extr.: down to (Qa)=0.1, (QL)=25.6, Qt=2000, $\tilde{m} = 0.08$

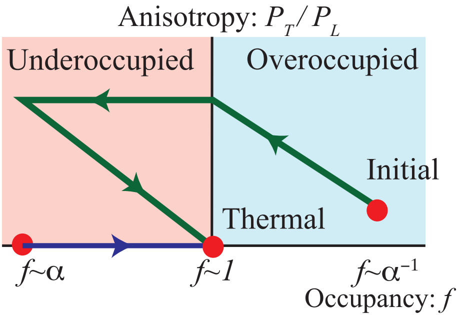
Overoccupied cascade

AK, Lu, Moore, PRD89 (2014) 7, 074036



Same system, very different degrees of freedom

$$1 \lesssim f \ll 1/\alpha_s$$



- Isotropic overoccupied: Transmutation of d.o.f's
- Isotropic underoccupied: Bottom-up thermalization
- Effect of longitudinal expansion: Hydrodynamization

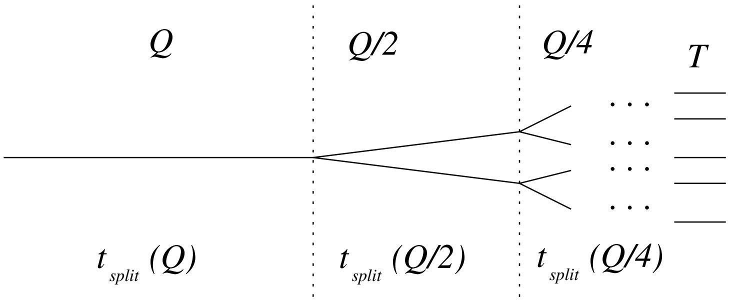
Bottom-up thermalization

- Hard particles emit soft radiation: creation of a soft thermal bath
  - Soft bath starts to dominate dynamics (screening, scattering, etc.)

- Hard particles undergo radiative break-up
  - System thermalizes in a time scale it takes to quench a jet of momentum Q

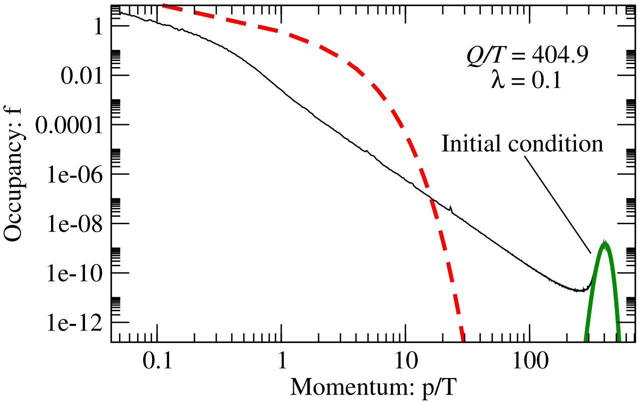
AK, Moore 1107.5050

$$t_{\mathrm{eq}} \sim \left(\frac{Q}{T}\right)^{1/2} \frac{1}{\lambda T^2}$$



Bottom-up thermalization

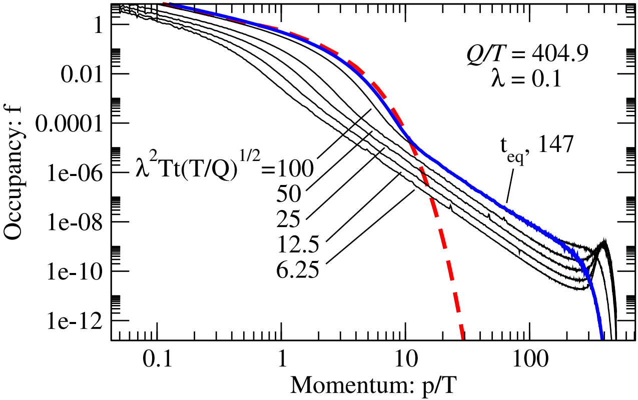
AK, Lu, PRL 113 (2014) 18, 182301



- Start with an underoccupied initial condition $p \sim Q$
- after a very short time, an IR bath is created (1$\leftrightarrow$ 2–processes)

Bottom-up thermalization

AK, Lu, PRL 113 (2014) 18, 182301



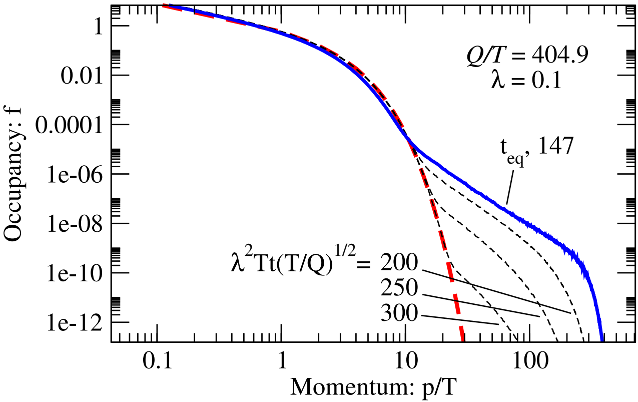
- More energy flows to the IR, temperature increases, "Bottom-up"
- When "bottom" reaches final T, "up" is quenched

AK, Moore JHEP 1112 (2011) 044

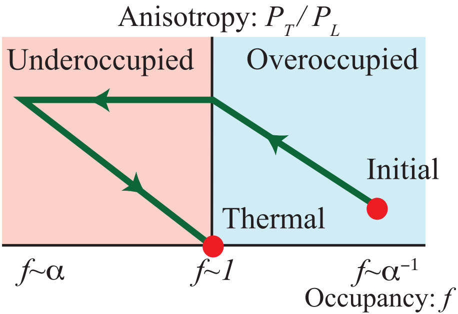
$$t_{\text{eq}} \sim (Q/T)^{1/2} \frac{1}{\alpha_s^2 T}$$

Bottom-up thermalization

AK, Lu, PRL 113 (2014) 18, 182301



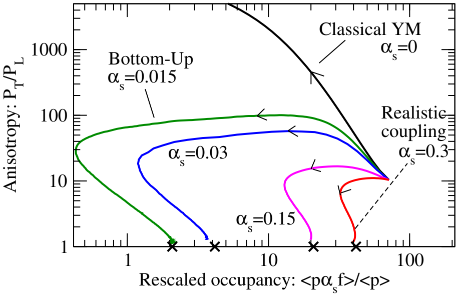
- Hardest scales reach equilibrium last.

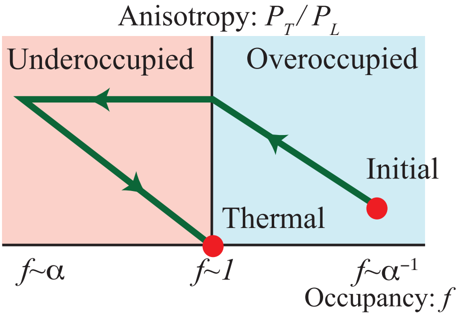


- Isotropic overoccupied: Transmutation of d.o.f's
- Isotropic underoccupied: Radiative break-up
- Application to HIC: effect of longitudinal expansion

Route to equilibrium in EKT

AK, Zhu, PRL 115 (2015) 18, 182301





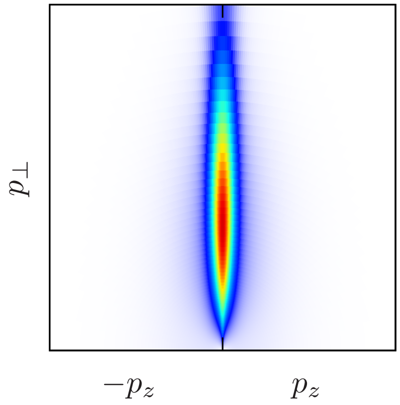
- Initial condition ($f \sim 1/\alpha_s$) from classical field thy calculation

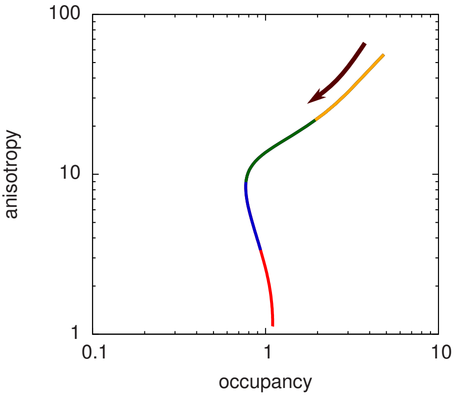
  Lappi PLB703 (2011) 325-330

- In the classical limit ($\alpha_s \to 0, \alpha_s f$ fixed), no thermalization
- At small values of couplings, clear Bottom-Up behaviour
- Features become less defined as α_s grows

Bottom-up thermalization

$p^2 f(p_\perp, p_z)$





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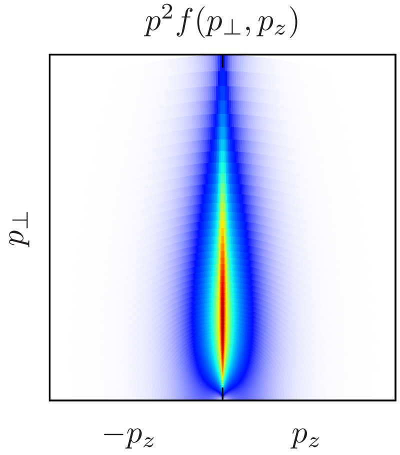

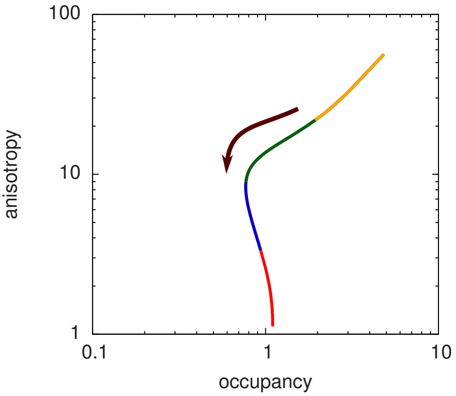

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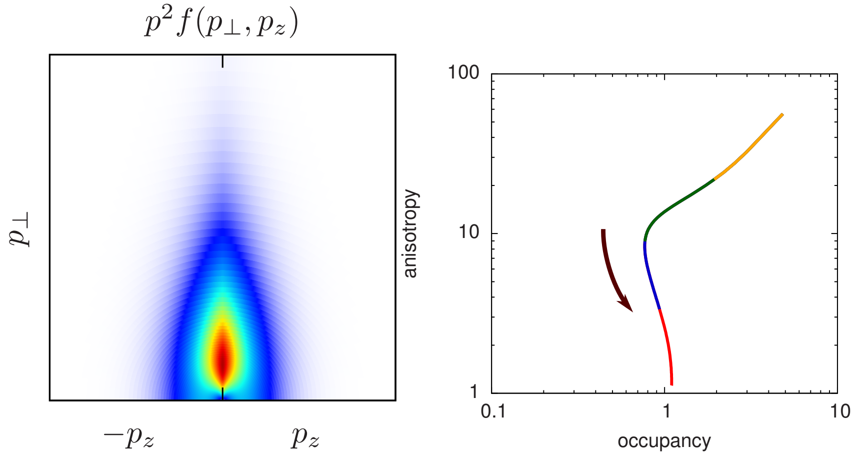

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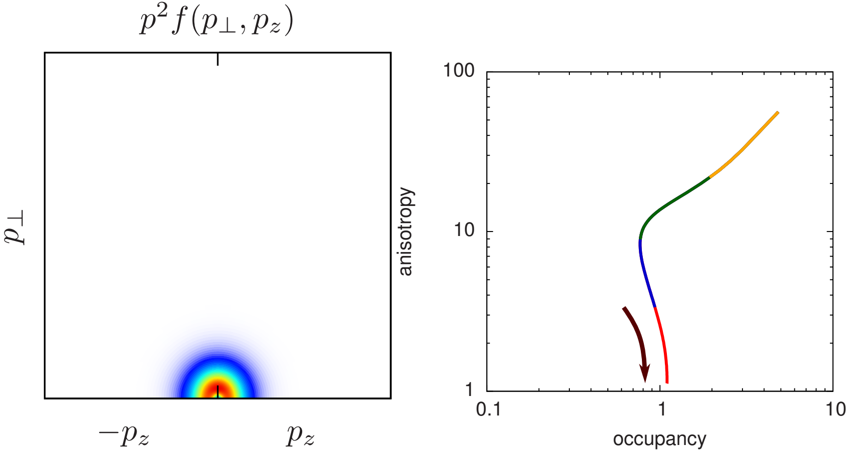
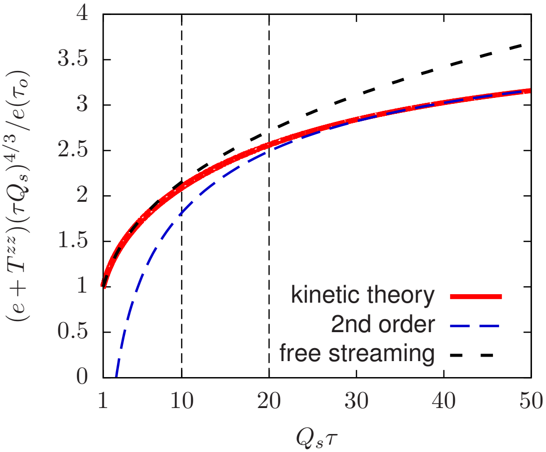


Smooth appreach to hydrodynamics



AK, Zhu, PRL 115 (2015) 18, 182301

- Kinetic theory smoothly and automatically goes to hydrodynamics

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- Green functions of $T^{\mu\nu}$ in during hydrodynamization and phenomenological application to HIC

Weak and strong coupling hydrodynamization compared

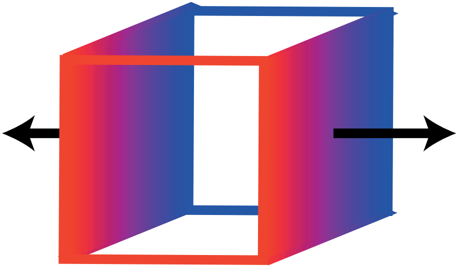
- Question:

  To what extent are the strong coupling and weak coupling hydrodynamizations similar or different?

- Challenge:

  How to setup similar initial condition in theories with different microscopic physics?

Weak and strong coupling hydrodynamization compared

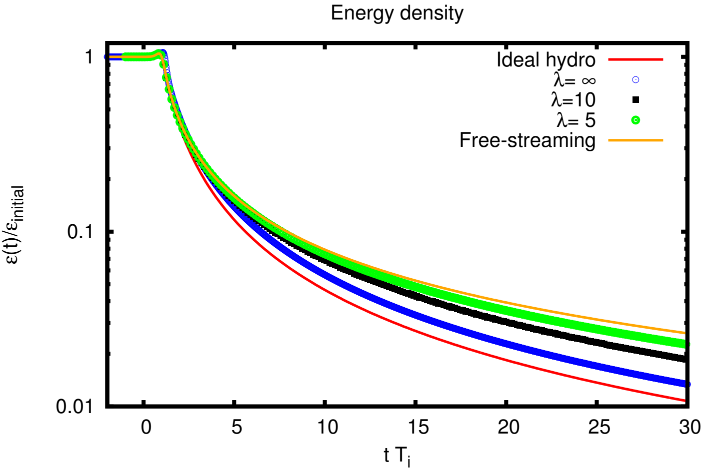


- Start with thermal equilibrium T_i
- perform same *macroscopic* deformation on both

$$ds^2 = -dt^2 + dx^2 + dy^2 + g(t)dL^2$$

  - $g(t \to -\infty) = 1$, Minkowski
  - $g(t \to \infty) = t^2$, Milne

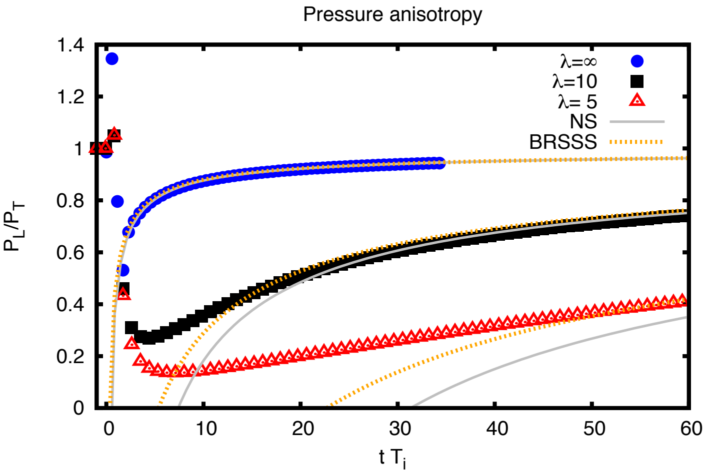
Weak and strong coupling hydrodynamization compared



Keegan et al JHEP 1604 (2016)

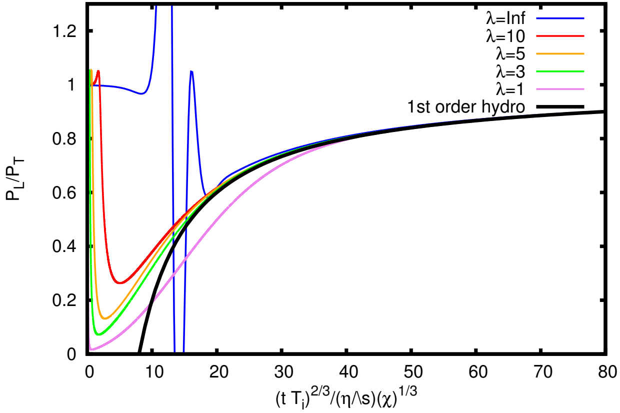
$\lambda = \infty$: N=4 SUSY, $\lambda = 5, 10$: pure gauge

Weak and strong coupling hydrodynamization compared



- Large quantitative difference due to different η/s

Weak and strong coupling hydrodynamization compared

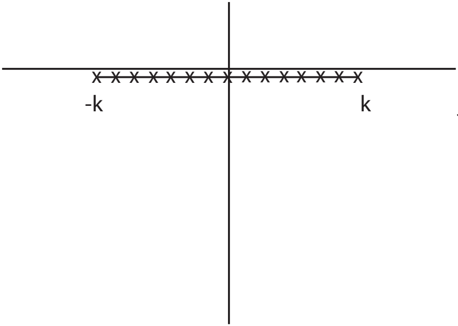


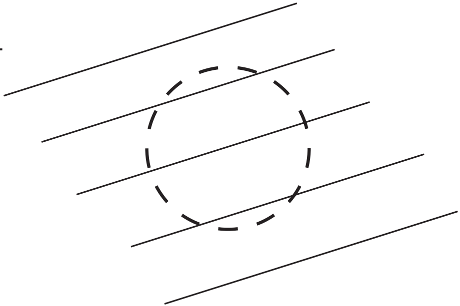
Keegan et al JHEP 1604 (2016)

$$\frac{P_L}{P_T} = 1 - \frac{8}{3}\frac{(\eta/s)}{(tT_i)^{2/3}\chi^{1/3}}, \qquad \chi = \frac{S_{eq}(t \to \infty)}{S_{eq}(t \to -\infty)}$$

- All hydrodynamize at very large anisotropy!

Spectrum of non-hydro modes in weak coupling

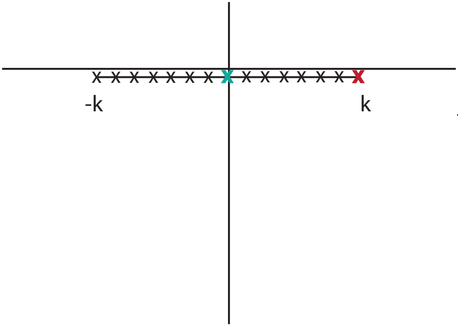


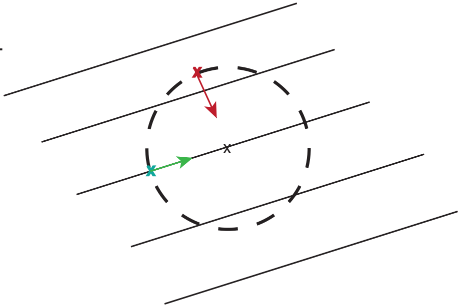


$$G^{00,00}(w,k) = -6(\epsilon + P)\left[1 - \frac{\omega}{4k}\log\left(\frac{\omega - k + i\epsilon}{\omega + k + i\epsilon}\right)\right]$$

$$= -6(\epsilon + P)\left[1 - \frac{\omega}{2}\int \frac{d\Omega}{4\pi}\,\underbrace{\frac{1}{\omega - \mathbf{v}\cdot\mathbf{k} + i\epsilon}}_{\delta(\mathbf{x}-\mathbf{v}t)}\right]$$

Spectrum of non-hydro modes in weak coupling





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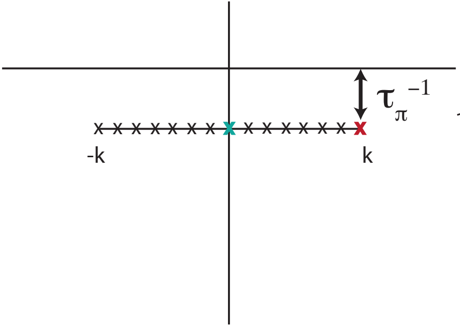
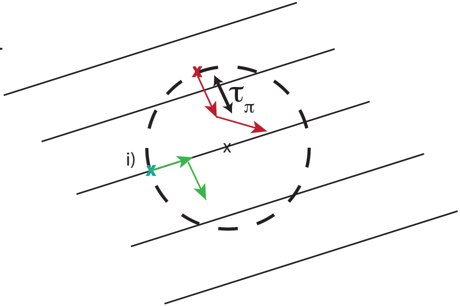
Spectrum of non-hydro modes in weak coupling





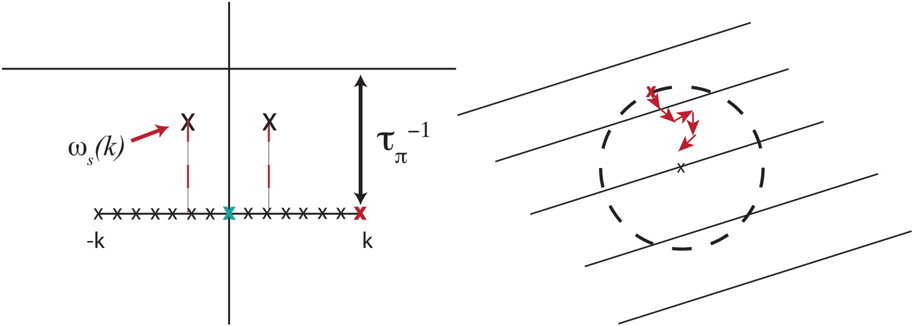
$$p^\nu \partial_\nu f(t, \mathbf{x}, \mathbf{p}) = \frac{p^0}{\tau_\Pi}(f - f_{eq}) \qquad \text{RTA}$$

$$\delta'(\mathbf{x} - \mathbf{v}t) \to \delta'(\mathbf{x} - \mathbf{v}t)\exp(-\tau/\tau_\pi)$$

$$\log\left(\frac{\omega - k}{\omega + k}\right) \to \log\left(\frac{\omega - k + i/\tau_\pi}{\omega + k + i/\tau_\pi}\right)$$

Bit more complicated than that... Romatschke, EPJC76 (2016)

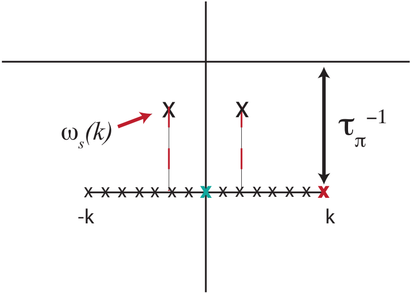
Spectrum of non-hydro modes in weak coupling

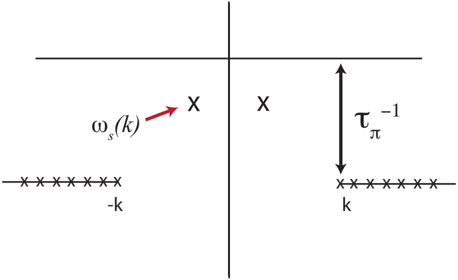


- Enter hydro pole: $k \ll 1/\tau_\pi$

$$\omega_s(k) = \pm c_s k - \frac{i}{2} \underbrace{\frac{\tau_\pi}{5}}_{\eta/(P+\epsilon)} k^2 + \dots$$

Spectrum of non-hydro modes compared





- No singularity at the complex infinity → cut may be deformed

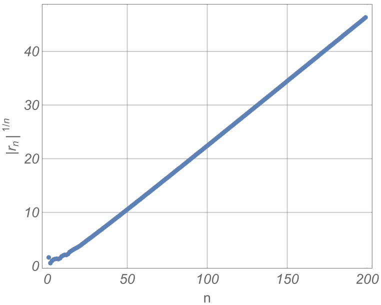
$$\log\left(\frac{\omega - k + i/\tau_\pi}{\omega + k + i/\tau_\pi}\right)$$

Hydrodynamization through decay of non-hydro modes

In both, holography and kinetic theory the hydrodynamical gradient expansion in divergent

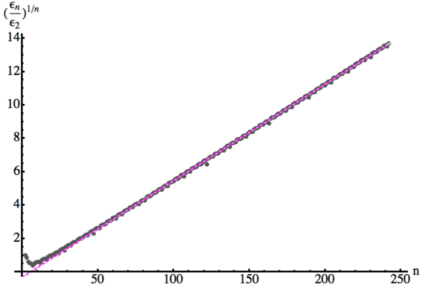
$$R \equiv \frac{P_L - P_T}{P} = \sum_{n=1}^{\infty} r_n (T\tau)^{-n} \approx \frac{8C_\eta}{T\tau} + \frac{16 C_\eta (C_{\tau\Pi} - C_\lambda)}{3(T\tau)^2} + \mathcal{O}(\frac{1}{(T\tau)^3})$$

here $\tau_\Pi = T^{-1}$



RTA

AK, Heller, Spalinski al. 1609.04803



Holography

Heller et al. PRL 110 (2013)

Hydrodynamization through decay of non-hydro modes

- Divergence signals that powerlaw form is not sufficient
- Needs to be supplemented with terms

$$\sim e^{-\xi_0 T\tau} \times (\text{constants of integration})$$

- No surprise? In kinetic theory, need $f(p)$ as an initial condition. In gradient expansion, the only boundary condition T at $t \to \infty$

Hydrodynamization through decay of non-hydro modes

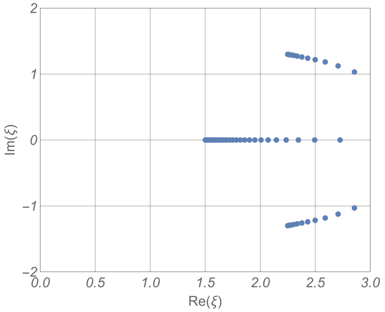
- Find ξ_0 through analytical properties of Borel transform

$$R_B(\xi) = \sum_{n=1}^{\infty} \frac{r_n}{\Gamma(n+b)} \xi^n, \quad R_{I-B}(T\tau) = \frac{1}{T\tau} \int_0^\infty d\xi e^{-\xi/T\tau} \xi^b R_B(\xi)$$

- Exponential decay is governed by the lowest non-hydro mode

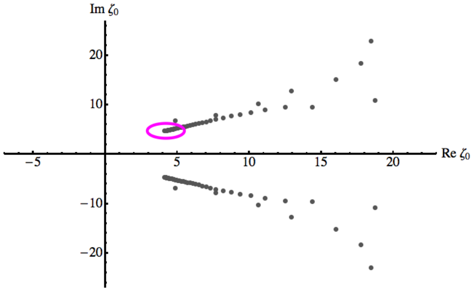
Also in IS hydro

$$e^{-\omega_{nh} \int T(\tau) d\tau} \sim e^{-3/2 \omega_{nh} T\tau} = e^{-\xi_0 T\tau}$$



RTA

AK, Heller, Spalinski al. 1609.04803



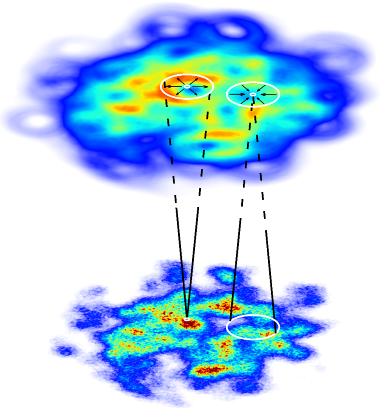
Holography

Heller et al. PRL 110 (2013)

Outline

- Effective kinetic theory
- Hydrodynamization and thermalization at weak coupling in effective kinetic theory
- Apples to apples comparison of weak and strong coupling hydrodynamization
- Green functions of $T^{\mu\nu}$ in during hydrodynamization and phenomenological application to HIC

Transverse dynamics and preflow

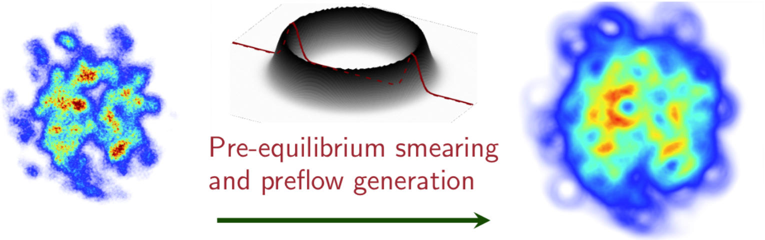


Nuclear radius $R \ll c\tau_i \sim$ Nucleon radius $R_p \ll 1/Q_s$

- Transverse structure small perturbation within the causal horizon
- Linear response theory for the transverse structures

Transverse dynamics and preflow

$$\int d^2\boldsymbol{x}' \underbrace{\frac{\delta e(\tau_0,\boldsymbol{x}')}{e(\tau_0)}} \times \underbrace{E(|\boldsymbol{x}-\boldsymbol{x}'|;\tau,\tau_0)} = \underbrace{\frac{\delta e(\tau,\boldsymbol{x})}{e(\tau)}, \vec{v}(\tau,\boldsymbol{x})}$$



- Green functions on top of non-equilibrium background

Linearized perturbations in EKT

Keegan et al. 1605.04287

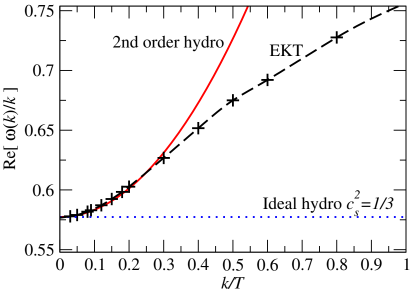
Transverse perturbations characterized by wavenumber $\mathbf{k}$

$$f(\mathbf{x}_\perp, \mathbf{p}) = \bar{f}(\mathbf{p}) + \exp(i\mathbf{x}\cdot\mathbf{k})\delta f(\mathbf{p})$$

$$\left(\partial_\tau - \frac{p_z}{\tau}\partial_{p_z}\right) f = C[f]$$

$$\left(\partial_\tau - \frac{p_z}{\tau}\partial_{p_z} + i\mathbf{k}\cdot\mathbf{p}\right) f = C[\bar{f}, f]$$

- For thermal $\bar{f}$: large wavelenght pert. described by hydro



$$\frac{\omega(k)}{k} = c_s^2 + \frac{4}{3}\frac{\eta}{e+p}\left(c_s\tau_\pi - \frac{2}{3c_s}\frac{\eta}{e+p}\right)k^2$$

- For larger k, $c_s^2 \to 1$, with polynomial decay

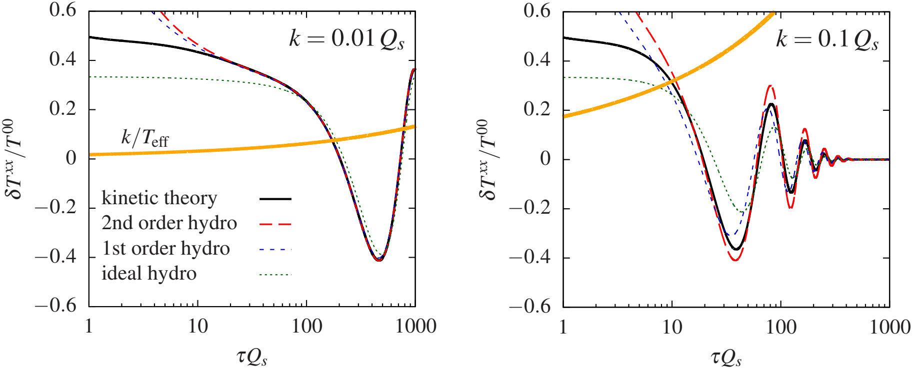
no plot unfortunately...

Hydrodynamization of perturbations

Keegan et al.JHEP 1608 (2016)

$$\delta T^{xx} = \frac{\delta e}{e}\left[\frac{1}{3}e + \frac{1}{3}\eta\tau_\pi k^2 + \frac{\eta}{2\tau} - \frac{2(\lambda_1 - \eta\tau_\pi)}{9\tau^2}\right] - \frac{ik\delta T^{0x}}{e}\left[\eta - \frac{1}{\tau}\left(\frac{\eta^2}{2e} + \frac{\eta\tau_\pi}{2} - \frac{2}{3}\lambda_1\right)\right]$$

$k \sim 1/R_{\text{nucleus}}$ $\qquad$ $k \sim 1/R_{\text{proton}}$



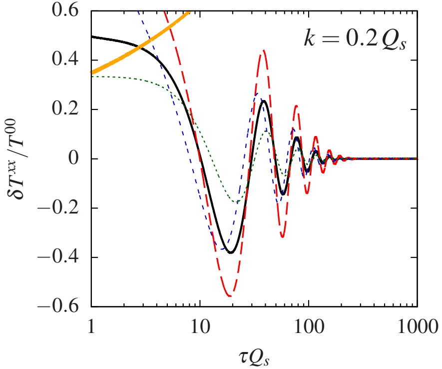
- Perturbations hydrodynamize also at $Q\tau \sim \{10, 20\}$.

Hydrodynamization of perturbations

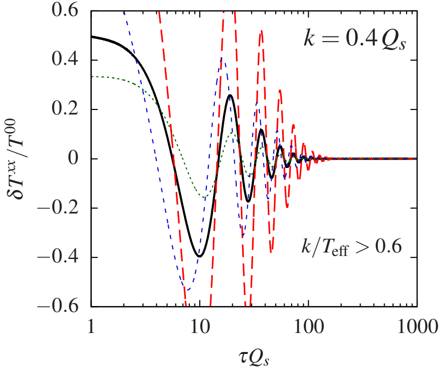
Keegan et al.JHEP 1608 (2016)

$$\delta T^{xx} = \frac{\delta e}{e}\left[\frac{1}{3}e + \frac{1}{3}\eta\tau_\pi k^2 + \frac{\eta}{2\tau} - \frac{2(\lambda_1 - \eta\tau_\pi)}{9\tau^2}\right] - \frac{ik\delta T^{0x}}{e}\left[\eta - \frac{1}{\tau}\left(\frac{\eta^2}{2e} + \frac{\eta\tau_\pi}{2} - \frac{2}{3}\lambda_1\right)\right]$$

$k \sim 1/0.5 R_{\text{nucleus}}$

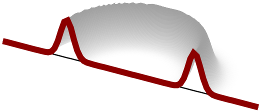


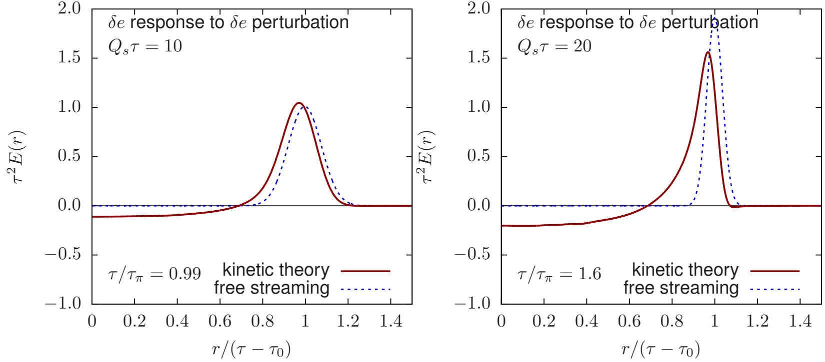
$k \sim 1/0.25 R_{\text{nucleus}}$



- No hydrodynamics for the large-k modes

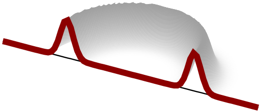
Green function in coordinate space

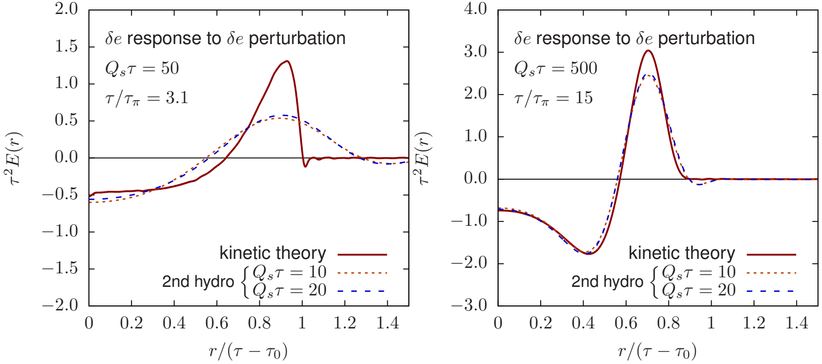




- Nanscent formation of dip in the origin hall mark of hydro

Green function in coordinate space

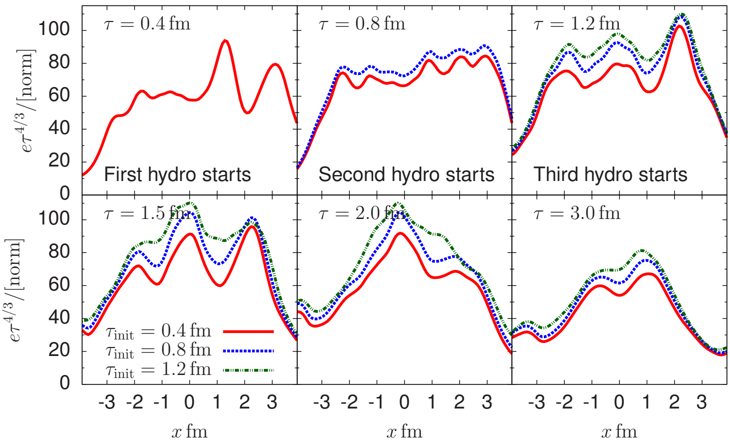




- Evolution after $Q\tau_i > \{10, 20\}$, evolution described by hydro

Transverse dynamics and preflow

- With free streaming pre-equilibrium evolution:

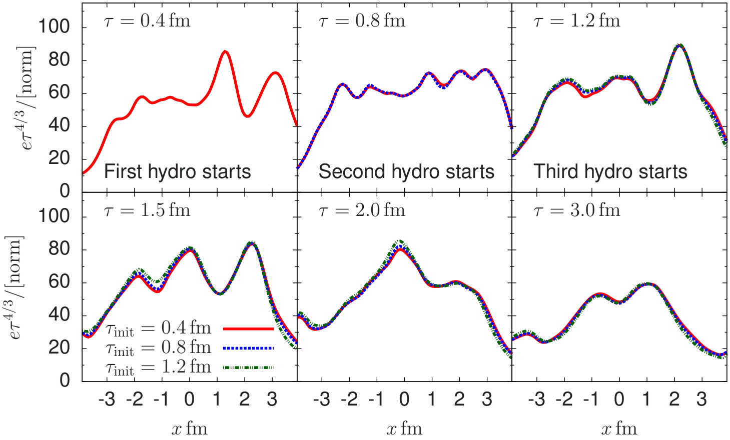


AK, Mazeliauskas, Paquet, Schlichting, Teaney, in progress

- Strong dependence on initialization time!

Transverse dynamics and preflow

- With full EKT pre-equilibrium evolution:



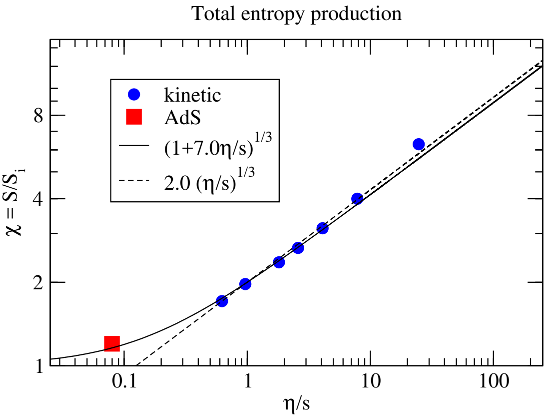
AK, Mazeliauskas, Paquet, Schlichting, Teaney, in progress

- Initialization time removed

Summary

- Weak coupling hydrodynamization quantitatively and qualitatively understood, with some caveats
- Push towards phenomenologically useful pre-equilibrium description
- Some similarities and differences between weak and strong coupling
  - Big quantitative difference in $\eta/s \longrightarrow$ time scales very different
  - Non-hydro modes near equilibrium:
    - Imaginary parts: T in strong, τ_Π in weak coupling
    - Real parts T in strong, and k in weak coupling
  - Similar divergent hydrodynamic series and hydrodynamization through decay of non-hydro modes

Weak and strong coupling hydrodynamization compared



- Strong coupling quite close to where weak coupling goes haywire?

Weak coupling param. estimate for entropy production:

$T_i t_{eq} \sim \frac{T_i}{\lambda^2 T(t_{eq})}$, before free streaming: $T^4(t_{eq}) \sim T_i^4/(T_i t_{eq})$ then $t_{eq} \sim \lambda^{-8/3} \sim (\eta/s)^{4/3}$