SAŠO GROZDANOV

INSTITUUT-LORENTZ FOR THEORETICAL PHYSICS
LEIDEN UNIVERSITY

TOWARDS WEAK COUPLING IN HOLOGRAPHY

WORK IN COLLABORATION WITH J. CASALDERREY-SOLANA, N. KAPLIS, N. POOVUTTIKUL, A. STARINETS AND W. VAN DER SCHEE

OXFORD, 6.3.2017

OUTLINE

- from holography to experiment
- coupling constant dependence and universality in hydrodynamics
- thermalisation and higher-energy spectrum
- heavy ion collisions
- conclusion and future directions

FROM HOLOGRAPHY TO EXPERIMENT

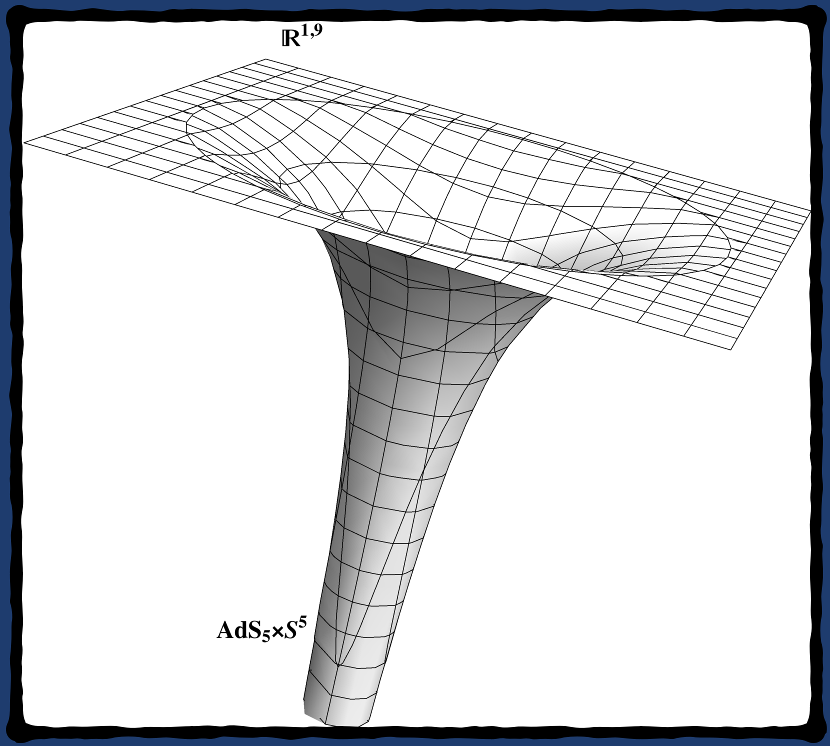
- some strongly coupled field theories have a dual gravitational description (AdS/CFT correspondence)

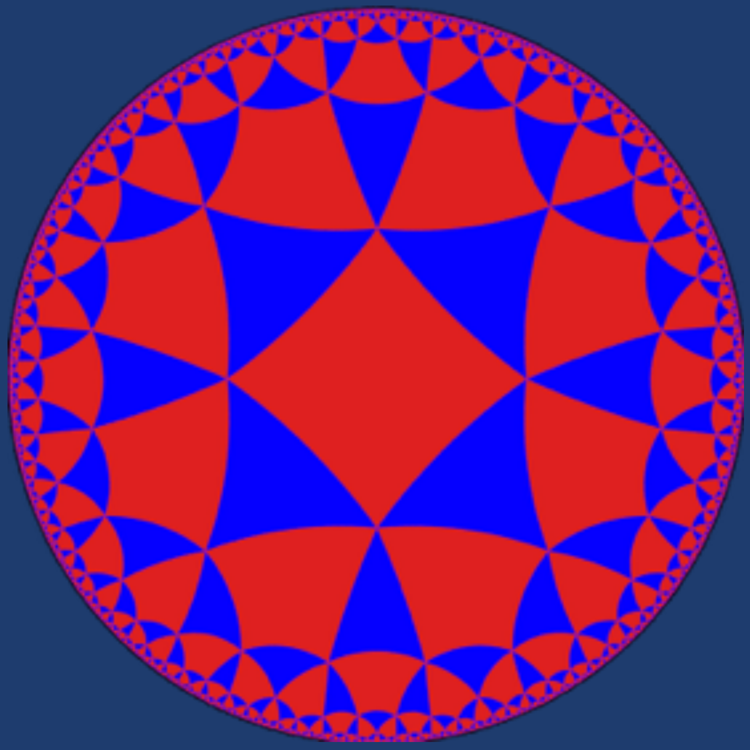
$$Z[\text{field theory}] = Z[\text{string theory}]$$

- originally a duality in type IIB string theory
- so far: universality at strong coupling

- challenges
  - field content (no supersymmetry)
  - away from infinite N
  - $\boxed{\text{away from infinite coupling}}$ $\boxed{\alpha' \propto 1/\lambda^{1/2}}$

$$\lambda \equiv g_{YM}^2 N$$





TWO CLASSES OF THEORIES

- top-down dual of *N=4* theory with 't Hooft coupling corrections from type IIB string theory

$$\boxed{S_{IIB} = \frac{1}{2\kappa_{10}^2}\int d^{10}x\sqrt{-g}\left(R - \frac{1}{2}\left(\partial\phi\right)^2 - \frac{1}{4\cdot 5!}F_5^2 + \gamma e^{-\frac{3}{2}\phi}\mathcal{W} + \dots\right)}$$

$$\gamma = \alpha'^3\zeta(3)/8 \qquad\qquad \alpha'/L^2 = \lambda^{-1/2}$$

$$\mathcal{W} = C^{\alpha\beta\gamma\delta}C_{\mu\beta\gamma\nu}C_{\alpha}{}^{\rho\sigma\mu}C^{\nu}{}_{\rho\sigma\delta} + \frac{1}{2}C^{\alpha\delta\beta\gamma}C_{\mu\nu\beta\gamma}C_{\alpha}{}^{\rho\sigma\mu}C^{\nu}{}_{\rho\sigma\delta}$$

- bottom-up curvature-squared theory with a special, non-perturbative case of Gauss-Bonnet gravity

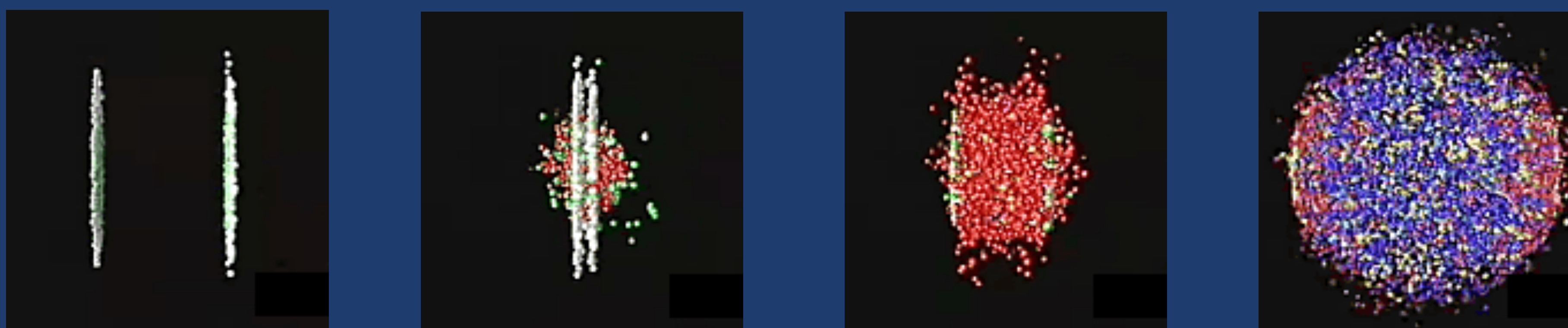
$$S_{R^2} = \frac{1}{2\kappa_5^2}\int d^5x\sqrt{-g}\left[R - 2\Lambda + L^2\left(\alpha_1 R^2 + \alpha_2 R_{\mu\nu}R^{\mu\nu} + \alpha_3 R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}\right)\right]$$

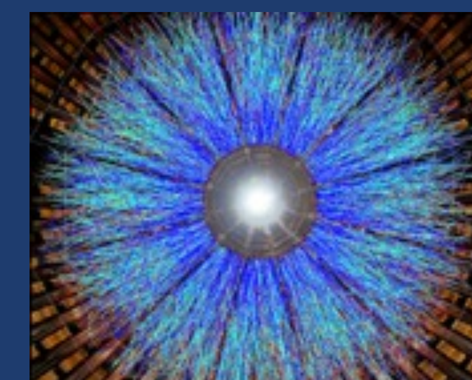
$$S_{GB} = \frac{1}{2\kappa_5^2}\int d^5x\sqrt{-g}\left[R - 2\Lambda + \frac{\lambda_{GB}}{2}L^2\left(R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}\right)\right]$$

HYDRODYNAMICS

HYDRODYNAMICS

- QCD and quark-gluon plasma





- low-energy limit of QFTs (effective field theory)

$$T^{\mu\nu}\left(u^{\lambda}, T, \mu\right) = (\varepsilon + P)\, u^{\mu} u^{\nu} + P g^{\mu\nu} - \eta \sigma^{\mu\nu} - \zeta \nabla \cdot u \Delta^{\mu\nu} + \dots$$

$$\boxed{\nabla_{\mu} T^{\mu\nu} = 0}$$

- tensor structures (phenomenological gradient expansions) with transport coefficients (microscopic)

HYDRODYNAMICS

- conformal (Weyl-covariant) hydrodynamics $T^{\mu}_{\ \mu} = 0$

$$g_{\mu\nu} \to e^{-2\omega(x)} g_{\mu\nu} \qquad\qquad T^{\mu\nu} \to e^{6\omega(x)}\, T^{\mu\nu}$$

- infinite-order asymptotic expansion

$$T^{\mu\nu} = \sum_{n=0}^{\infty} T^{\mu\nu}_{(n)} \longrightarrow \omega = \sum_{n=0}^{\mathcal{O}_H} \alpha_n k^{n+1}$$

- classification of tensors beyond Navier-Stokes

first order: 2 (1 in CFT) - shear and bulk viscosities

second order: 15 (5 in CFT) - relaxation time, … [Israel-Stewart and extensions]

third order: 68 (20 in CFT) - [S. G., Kaplis, PRD 93 (2016) 6, 066012, arXiv:1507.02461]

HYDRODYNAMICS

- diffusion and sound dispersion relations in CFT

$$\text{shear:} \quad \omega = -i\frac{\eta}{\varepsilon+P}k^2 - i\left[\frac{\eta^2\tau_\Pi}{(\varepsilon+P)^2} - \frac{1}{2}\frac{\theta_1}{\varepsilon+P}\right]k^4 + \mathcal{O}\left(k^5\right)$$

$$\text{sound:} \quad \omega = \pm c_s k - i\Gamma_c k^2 \mp \frac{\Gamma_c}{2c_s}\left(\Gamma_c - 2c_s^2\tau_\Pi\right)k^3 - i\left[\frac{8}{9}\frac{\eta^2\tau_\Pi}{(\varepsilon+P)^2} - \frac{1}{3}\frac{\theta_1+\theta_2}{\varepsilon+P}\right]k^4 + \mathcal{O}\left(k^5\right)$$

- loop corrections break analyticity of the gradient expansion (long-time tails), but are *1/N* suppressed [Kovtun, Yaffe (2003)]
- entropy current, constraints on transport and new transport coefficients (anomalies, broken parity)
- non-relativistic hydrodynamics
- hydrodynamics from effective Schwinger-Keldysh field theory with dissipation [Nicolis, et. al.; S. G., Polonyi; Haehl, Loganayagam, Rangamani; de Boer, Heller, Pinzani-Fokeeva; Crossley, Glorioso, Liu]

HYDRODYNAMICS FROM HOLOGRAPHY

- holography can compute microscopic transport coefficients [Policastro, Son, Starinets (2001)]

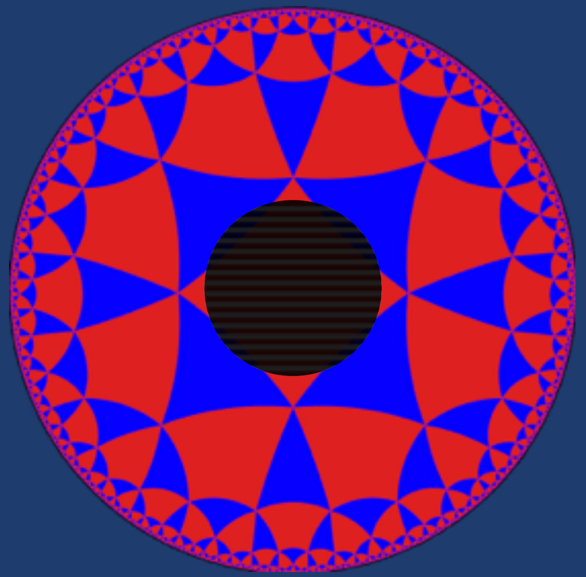
- low-energy limit of QFTs / low-energy gravitational perturbations in backgrounds with black holes

- Green's functions and Kubo formulae

$$\langle T_R^{ab}(0)\rangle = G_R^{ab}(0) - \frac{1}{2}\int d^4x G_{RA}^{ab,cd}(0,x)h_{cd}(x) + \frac{1}{8}\int d^4x d^4y G_{RAA}^{ab,cd,ef}(0,x,y)h_{cd}(x)h_{ef}(y) + \dots$$

- quasi-normal modes [Kovtun, Starinets (2005)]

$$\omega = \sum_{n=0}^{\infty} \alpha_n k^{n+1}$$



- fluid-gravity [Bhattacharyya, Hubeny, Minwalla, Rangamani (2007)]

$N=4$ AT INFINITE COUPLING

- type IIB theory on S^5 dual to $N=4$ supersymmetric Yang-Mills at infinite 't Hooft coupling and infinite N_c

$$\boxed{S = \frac{1}{2\kappa_5^2}\int d^5x\sqrt{-g}\left(R + \frac{12}{L^2}\right)}$$

$$\kappa_5 = 2\pi/N_c$$

- black brane

$$ds^2 = \frac{r_0^2}{u}\left(-f(u)dt^2 + dx^2 + dy^2 + dz^2\right) + \frac{du^2}{4u^2 f(u)} \qquad f(u) = 1 - u^2$$

- use to find field theory stress-energy tensor to third order

$$T^{ab} = \varepsilon u^a u^b + P\Delta^{ab} - \eta\sigma^{ab} + \eta\tau_\Pi\left[{}^{\langle}D\sigma^{ab\rangle} + \frac{1}{3}\sigma^{ab}\left(\nabla\cdot u\right)\right] + \kappa\left[R^{\langle ab\rangle} - 2u_c R^{c\langle ab\rangle d}u_d\right]$$

$$+ \lambda_1\sigma^{\langle a}{}_c\sigma^{b\rangle c} + \lambda_2\sigma^{\langle a}{}_c\Omega^{b\rangle c} + \lambda_3\Omega^{\langle a}{}_c\Omega^{b\rangle c} \boxed{+ \sum_{n=1}^{20}\lambda_n^{(3)}\mathcal{O}_n}$$

$\mathcal{N}=4$ AT INFINITE COUPLING

- transport coefficients [S. G., Kaplis, PRD 93 (2016) 6, 066012, arXiv:1507.02461]

$$\eta = \frac{\pi}{8} N_c^2 T^3$$

$$\tau_\Pi = \frac{(2-\ln 2)}{2\pi T} \qquad \kappa = \frac{N_c^2 T^2}{8} \qquad \lambda_1 = \frac{N_c^2 T^2}{16} \qquad \lambda_2 = -\frac{N_c^2 T^2}{8} \ln 2 \qquad \lambda_3 = 0$$

$$\longrightarrow \boxed{\frac{\eta}{s} = \frac{1}{4\pi}}$$

$$\lambda_1^{(3)} + \lambda_2^{(3)} + \lambda_4^{(3)} \equiv -\theta_1 = -\frac{N_c^2 T}{32\pi}$$

$$\lambda_3^{(3)} + \lambda_5^{(3)} + \lambda_6^{(3)} \equiv -\theta_2 = \frac{N_c^2 T}{384\pi} \left(\frac{\pi^2}{12} + 18 \ln 2 - \ln^2 2 - 22 \right)$$

$$\lambda_1^{(3)} - \lambda_{16}^{(3)} = \frac{N_c^2 T}{16\pi} \left(\frac{\pi^2}{12} + 4 \ln 2 - \ln^2 2 \right)$$

$$\lambda_{17}^{(3)} = \frac{N_c^2 T}{16\pi} \left(\frac{\pi^2}{12} + 2 \ln 2 - \ln^2 2 \right)$$

$$\frac{\lambda_1^{(3)}}{6} + \frac{4\lambda_2^{(3)}}{3} + \frac{4\lambda_3^{(3)}}{3} + \frac{5\lambda_4^{(3)}}{6} + \frac{5\lambda_5^{(3)}}{6} + \frac{4\lambda_6^{(3)}}{3} - \frac{\lambda_7^{(3)}}{2}$$
$$+ \frac{3\lambda_8^{(3)}}{2} + \frac{\lambda_9^{(3)}}{2} - \frac{2\lambda_{10}^{(3)}}{3} - \frac{11\lambda_{11}^{(3)}}{6} - \frac{\lambda_{12}^{(3)}}{3} + \frac{\lambda_{13}^{(3)}}{6} - \lambda_{15}^{(3)} = \frac{N_c^2 T}{648\pi} \left(15 - 2\pi^2 - 45 \ln 2 + 24 \ln^2 2\right)$$

TOP-DOWN CONSTRUCTION

- type IIB action with 't Hooft coupling corrections

$$\boxed{S_{IIB} = \frac{1}{2\kappa_{10}^2}\int d^{10}x\sqrt{-g}\left(R - \frac{1}{2}(\partial\phi)^2 - \frac{1}{4\cdot 5!}F_5^2 + \gamma e^{-\frac{3}{2}\phi}\mathcal{W} + \dots\right)}$$

$$\gamma = \alpha'^3\zeta(3)/8 \qquad\qquad \alpha'/L^2 = \lambda^{-1/2}$$

$$\mathcal{W} = C^{\alpha\beta\gamma\delta}C_{\mu\beta\gamma\nu}C_{\alpha}{}^{\rho\sigma\mu}C^{\nu}{}_{\rho\sigma\delta} + \frac{1}{2}C^{\alpha\delta\beta\gamma}C_{\mu\nu\beta\gamma}C_{\alpha}{}^{\rho\sigma\mu}C^{\nu}{}_{\rho\sigma\delta}$$

- dimensional reduction

$$\boxed{S = \frac{1}{2\kappa_5^2}\int d^5x\sqrt{-g}\left(R + \frac{12}{L^2} + \gamma\mathcal{W}\right)}$$

- black brane

$$ds^2 = \frac{r_0^2}{u}\left(-f(u)Z_t dt^2 + dx^2 + dy^2 + dz^2\right) + Z_u\frac{du^2}{4u^2 f} \qquad\qquad f(u) = 1 - u^2$$

$$Z_t = 1 - 15\gamma\left(5u^2 + 5u^4 - 3u^6\right) \qquad\qquad Z_u = 1 + 15\gamma\left(5u^2 + 5u^4 - 19u^6\right)$$

TOP-DOWN CONSTRUCTION

- *N=4* transport coefficients to second order [S. G., Starinets, JHEP 1503 (2015) 007 arXiv:1412.5685]

$$\eta = \frac{\pi}{8} N_c^2 T^3 \,(1 + 135\gamma + \dots)$$

$$\tau_\Pi = \frac{(2 - \ln 2)}{2\pi T} + \frac{375\gamma}{4\pi T} + \dots$$

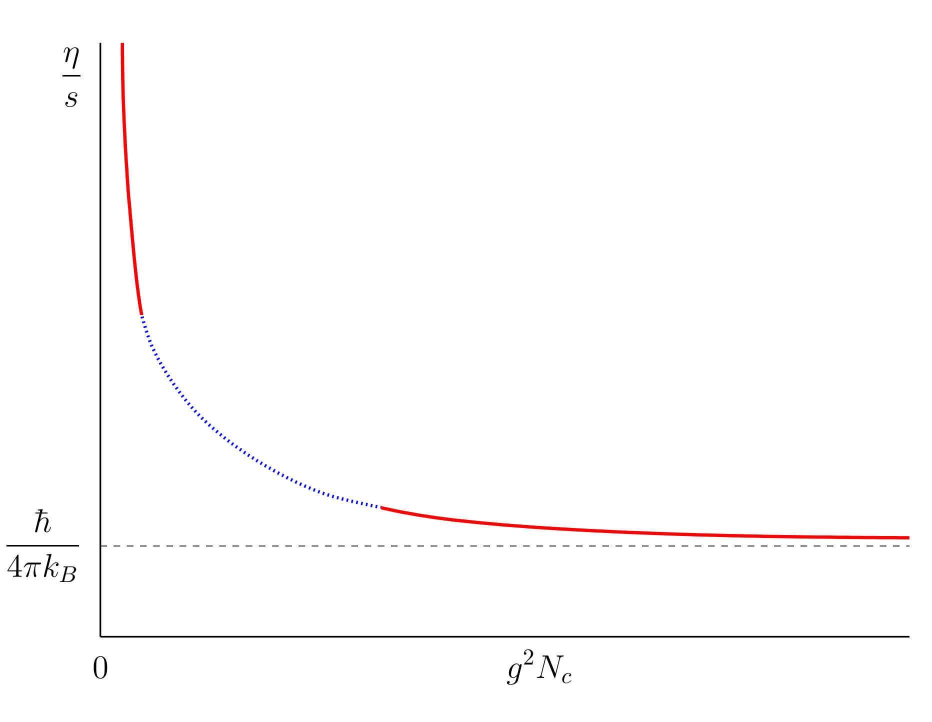
$$\kappa = \frac{N_c^2 T^2}{8} (1 - 10\gamma + \dots)$$

$$\lambda_1 = \frac{N_c^2 T^2}{16} (1 + 350\gamma + \dots)$$

$$\lambda_2 = -\frac{N_c^2 T^2}{16} (2 \ln 2 + 5\,(97 + 54 \ln 2)\,\gamma + \dots)$$

$$\lambda_3 = \frac{25 N_c^2 T^2}{2} \gamma + \dots$$

$$\longrightarrow \quad \boxed{\frac{\eta}{s} = \frac{1}{4\pi} \left(1 + 15\zeta(3)\lambda^{-3/2} + \dots\right)}$$



[Kovtun, Son, Starinets (2005)]

BOTTOM-UP CONSTRUCTION

- curvature-squared theory [S. G., Starinets, JHEP 1503 (2015) 007 arXiv:1412.5685]

$$S_{R^2} = \frac{1}{2\kappa_5^2} \int d^5x \sqrt{-g} \left[R - 2\Lambda + L^2 \left(\alpha_1 R^2 + \alpha_2 R_{\mu\nu} R^{\mu\nu} + \alpha_3 R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} \right) \right]$$

$$\eta = \frac{r_+^3}{2\kappa_5^2} \left(1 - 8\left(5\alpha_1 + \alpha_2\right)\right) + \mathcal{O}(\alpha_i^2)$$

$$\eta\tau_\Pi = \frac{r_+^2 \left(2 - \ln 2\right)}{4\kappa_5^2} \left(1 - \frac{26}{3}\left(5\alpha_1 + \alpha_2\right)\right) - \frac{r_+^2 \left(23 + 5\ln 2\right)}{12\kappa_5^2} \alpha_3 + \mathcal{O}(\alpha_i^2)$$

$$\kappa = \frac{r_+^2}{2\kappa_5^2} \left(1 - \frac{26}{3}\left(5\alpha_1 + \alpha_2\right)\right) - \frac{25 r_+^2}{6\kappa_5^2} \alpha_3 + \mathcal{O}(\alpha_i^2)$$

$$\lambda_1 = \frac{r_+^2}{4\kappa_5^2} \left(1 - \frac{26}{3}\left(5\alpha_1 + \alpha_2\right)\right) - \frac{r_+^2}{12\kappa_5^2} \alpha_3 + \mathcal{O}(\alpha_i^2)$$

$$\lambda_2 = -\frac{r_+^2 \ln 2}{2\kappa_5^2} \left(1 - \frac{26}{3}\left(5\alpha_1 + \alpha_2\right)\right) - \frac{r_+^2 \left(21 + 5\ln 2\right)}{6\kappa_5^2} \alpha_3 + \mathcal{O}(\alpha_i^2)$$

$$\lambda_3 = -\frac{28 r_+^2}{\kappa_5^2} \alpha_3 + \mathcal{O}(\alpha_i^2)$$

BOTTOM-UP CONSTRUCTION

- Gauss-Bonnet theory [S. G., Starinets, Theor. Math. Phys. 182 (2015) 1, 61-73]

$$\boxed{S_{GB} = \frac{1}{2\kappa_5^2}\int d^5x\sqrt{-g}\left[R - 2\Lambda + \frac{\lambda_{GB}}{2}L^2\left(R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}\right)\right]}$$

$$\gamma = \sqrt{1 - 4\lambda_{GB}}$$

$$\eta = s\gamma^2/4\pi$$

$$\tau_\Pi = \frac{1}{2\pi T}\left(\frac{1}{4}(1+\gamma)\left(5+\gamma-\frac{2}{\gamma}\right) - \frac{1}{2}\log\left[\frac{2(1+\gamma)}{\gamma}\right]\right)$$

$$\lambda_1 = \frac{\eta}{2\pi T}\left(\frac{(1+\gamma)\left(3-4\gamma+2\gamma^3\right)}{2\gamma^2}\right)$$

$$\lambda_2 = -\frac{\eta}{\pi T}\left(-\frac{1}{4}(1+\gamma)\left(1+\gamma-\frac{2}{\gamma}\right) + \frac{1}{2}\log\left[\frac{2(1+\gamma)}{\gamma}\right]\right)$$

$$\lambda_3 = -\frac{\eta}{\pi T}\left(\frac{(1+\gamma)\left(3+\gamma-4\gamma^2\right)}{\gamma^2}\right)$$

$$\kappa = \frac{\eta}{\pi T}\left(\frac{(1+\gamma)\left(2\gamma^2-1\right)}{2\gamma^2}\right)$$

$$\theta_1 = \frac{\eta}{8\pi^2 T^2}\gamma\left(2\gamma^2+\gamma-1\right)$$

$$\longrightarrow \quad \boxed{\frac{\eta}{s} = \frac{1}{4\pi}\left(1 - 4\lambda_{GB}\right)}$$

LIMITS OF THE GAUSS-BONNET THEORY

- how can we interpret the extreme limits of the Gauss-Bonnet coupling?
- exact spectrum in the extreme (anomalous) limit of $\lambda_{GB} = 1/4$

$$
\begin{aligned}
&\text{Scalar:} \quad \mathfrak{w} = -i\left(4 + 2n_1 - \sqrt{4 - 3\mathfrak{q}^2}\right), \quad && \mathfrak{w} = -i\left(4 + 2n_2 + \sqrt{4 - 3\mathfrak{q}^2}\right) \\
&\text{Shear:} \quad \mathfrak{w} = -2i\left(1 + n_1\right), && \mathfrak{w} = -2i\left(3 + n_2\right) \\
&\text{Sound:} \quad \mathfrak{w} = -i\left(4 + 2n_1 - \sqrt{4 + \mathfrak{q}^2}\right), && \mathfrak{w} = -i\left(4 + 2n_2 + \sqrt{4 + \mathfrak{q}^2}\right)
\end{aligned}
$$

- in the extreme ``weak" limit $\lambda_{GB} \to -\infty$ there is a curvature singularity, which needs a stringy resolution

$$
\lim_{\lambda_{GB}\to-\infty} S_{GB} = \frac{\lambda_{GB} L^2}{4\kappa_5^2} \int d^5x \sqrt{-g}\left[R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} - \frac{4\Lambda}{\lambda_{GB} L^2}\right]
$$

$$
ds^2 = \sqrt{-\lambda_{GB}}\left[-\frac{\tilde{r}^2}{L^2}\sqrt{1 - \frac{\tilde{r}_+^4}{\tilde{r}^4}}\, dt^2 + \frac{L^2}{\tilde{r}^2\sqrt{1 - \frac{\tilde{r}_+^4}{\tilde{r}^4}}} d\tilde{r}^2 + \frac{\tilde{r}^2}{L^2}\left(dx^2 + dy^2 + dz^2\right)\right]
$$

CHARGE DIFFUSION

- construct the most general four-derivative action [S. G., Starinets (2016) arXiv:1611.07053]

$$S = \frac{1}{2\kappa_5^2}\int d^5x\sqrt{-g}\left[R - 2\Lambda + \mathcal{L}_{GB}\right] + \int d^5x\sqrt{-g}\mathcal{L}_A$$

$$\mathcal{L}_A = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \alpha_4 R F_{\mu\nu}F^{\mu\nu} + \alpha_5 R^{\mu\nu}F_{\mu\rho}F_\nu{}^\rho + \alpha_6 R^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma} + \alpha_7\left(F_{\mu\nu}F^{\mu\nu}\right)^2$$
$$+ \alpha_8\nabla_\mu F_{\rho\sigma}\nabla^\mu F^{\rho\sigma} + \alpha_9\nabla_\mu F_{\rho\sigma}\nabla^\rho F^{\mu\sigma} + \alpha_{10}\nabla_\mu F^{\mu\nu}\nabla^\rho F_{\rho\nu} + \alpha_{11}F^{\mu\nu}F_{\nu\rho}F^{\rho\sigma}F_{\sigma\mu}$$

- make it such that the equations of motion are only second order

$$\mathcal{L}_A = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \beta_1 L^2\left(RF_{\mu\nu}F^{\mu\nu} - 4R^{\mu\nu}F_{\mu\rho}F_\nu{}^\rho + R^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma}\right)$$
$$+ \beta_2 L^2\left(F_{\mu\nu}F^{\mu\nu}\right)^2 + \beta_3 L^2 F^{\mu\nu}F_{\nu\rho}F^{\rho\sigma}F_{\sigma\mu}$$

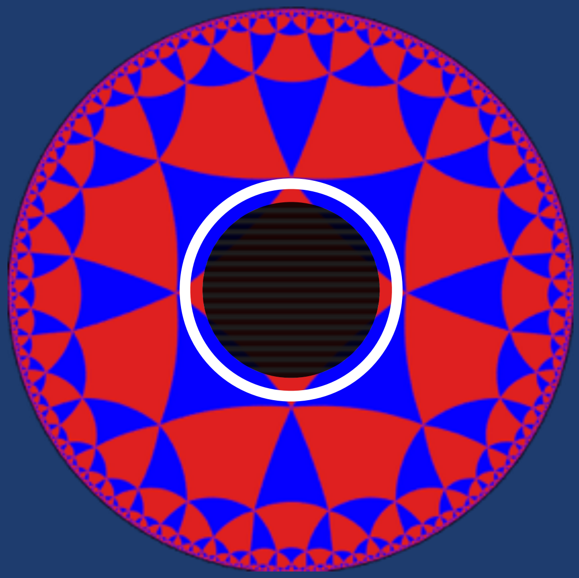
- diffusion

$$\mathcal{D} = \frac{(1+\gamma_{GB})(1+2\beta)\left(\beta+\sqrt{\beta^2-\gamma_{GB}^2}\right)}{6(\beta-1)\left[\beta\left(\beta+\sqrt{\beta^2-\gamma_{GB}^2}\right)-\gamma_{GB}^2\right]}\left\{\sqrt{\left(1-\gamma_{GB}^2\right)\left(\beta^2-\gamma_{GB}^2\right)}\ln\left[\frac{\gamma_{GB}}{1+\sqrt{1-\gamma_{GB}^2}}\right]\right.$$
$$\left. - \left(\beta-\gamma_{GB}^2\right)\ln\left[\frac{\gamma_{GB}}{\beta+\sqrt{\beta^2-\gamma_{GB}^2}}\right]\right\}$$

$$\boxed{\begin{array}{c}\mathcal{D} \neq D = 0 \\ D \neq \mathcal{D} = 0\end{array}}$$

$$\gamma_{GB} = \sqrt{1-4\lambda_{GB}}$$
$$\beta = 1 + 48\beta_1$$

UNIVERSALITY



- membrane paradigm (conserved current)

$$\partial_r \mathcal{J} = 0$$

- first-order hydrodynamics [Kovtun, Policastro, Son, Starinets]

$$\frac{\eta}{s} = \frac{1}{4\pi}$$

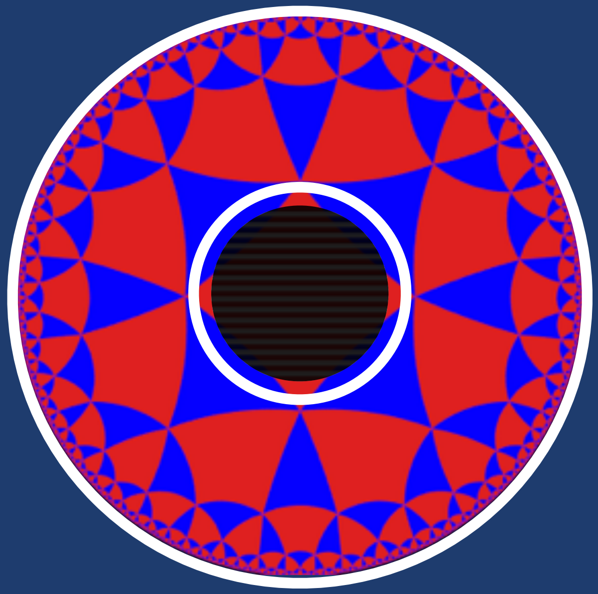
- second-order hydrodynamics [Haack, Yarom (2009); S. G., Starinets, JHEP 1503 (2015) 007 arXiv:1412.5685]

$$2\eta\tau_\Pi - 4\lambda_1 - \lambda_2 = \mathcal{O}\left(\gamma^2\right)$$

$$2\eta\tau_\Pi - 4\lambda_1 - \lambda_2 = \mathcal{O}\left(\alpha_i^2\right)$$

$$2\eta\tau_\Pi - 4\lambda_1 - \lambda_2 = -\frac{\eta}{\pi T}\frac{\left(1-\gamma_{GB}\right)\left(1-\gamma_{GB}^2\right)\left(3+2\gamma_{GB}\right)}{\gamma_{GB}^2} = -\frac{40\lambda_{GB}^2\eta}{\pi T} + \mathcal{O}\left(\lambda_{GB}^3\right)$$

(MORE) UNIVERSALITY



- non-renormalisation of anomalous conductivities
  [Gursoy, Tarrio (2015); S. G., Poovuttikul, JHEP 1609 (2016) 046 arXiv: 1603.08770]

$$\begin{pmatrix} \langle \delta J^\mu \rangle \\ \langle \delta J_5^\mu \rangle \end{pmatrix} = \begin{pmatrix} \sigma_{JB} & \sigma_{J\omega} \\ \sigma_{J_5 B} & \sigma_{J_5\omega} \end{pmatrix} \begin{pmatrix} B^\mu \\ \omega^\mu \end{pmatrix}$$

- universal values

$$\sigma_{J_5 B} = -2\gamma\mu \qquad \sigma_{JB} = -2\gamma\mu_5,$$

$$\sigma_{J_5\omega} = \kappa\mu_5^2 + \gamma\mu^2 + 2\lambda(2\pi T)^2 \qquad \sigma_{J\omega} = 2\gamma\mu_5\mu$$

- bulk with arbitrarily high derivatives (gauge, diffeomorphism)

$$S = \int d^5x\sqrt{-g}\,\{\mathcal{L}\left[A_a, V_a, g_{ab}, \phi_i\right] + \mathcal{L}_{CS}\left[A_a, V_a, g_{ab}\right]\}$$

$$\mathcal{L}_{CS}\left[A_a, V_a, g_{ab}\right] = \epsilon^{abcde} A_a \left(\frac{\kappa}{3} F_{A,bc}F_{A,de} + \gamma F_{V,bc}F_{V,de} + \lambda R^p{}_{qbc} R^q{}_{pde}\right)$$

- proof to all orders in the coupling constant expansion

BEYOND HYDRODYNAMICS

WEAK COUPLING (KINETIC THEORY)

- coupling constant dependence of non-hydrodynamic transport
  [S. G., Kaplis, Starinets, JHEP 1607 (2016) 151 arXiv:1605.02173]
- instead of hydrodynamics, start with (weakly coupled) kinetic theory
- essential concept: quasi-particles
- Boltzmann equation $$\frac{\partial F}{\partial t} + \frac{p_i}{m}\frac{\partial F}{\partial r^i} - \frac{\partial U(r)}{\partial r^i}\frac{\partial F}{\partial p_i} = C[F]$$
- close to equilibrium $$F(t,\mathbf{r},\mathbf{p}) = F_0(\mathbf{r},\mathbf{p})\left[1+\varphi(t,\mathbf{r},\mathbf{p})\right]$$
- resulting equation $$\frac{\partial \varphi}{\partial t} = -\frac{p_i}{m}\frac{\partial \varphi}{\partial r^i} + \frac{\partial U(r)}{\partial r^i}\frac{\partial \varphi}{\partial p_i} + L_0[\varphi]$$
- solution $$\varphi(t,\mathbf{r},\mathbf{p}) = e^{tL}\,\varphi_0(\mathbf{r},\mathbf{p}) = \frac{1}{2\pi i}\int_{\gamma-i\infty}^{\gamma+i\infty} e^{st}\,R_s ds\,\varphi_0(\mathbf{r},\mathbf{p})$$ $$R_s = (sI-L)^{-1}$$

WEAK COUPLING (KINETIC THEORY)

- ansatz for homogeneous eq. distribution $\varphi(t, \mathbf{p}) = e^{-\nu t} h(\mathbf{p})$
- eigenvalue equation for lin. coll. operator $-\nu h = L_0[h]$
- hierarchy of relaxation times $\varphi(t, \mathbf{p}) = \sum_n C_n e^{-\nu_n t} h_n(\mathbf{p})$

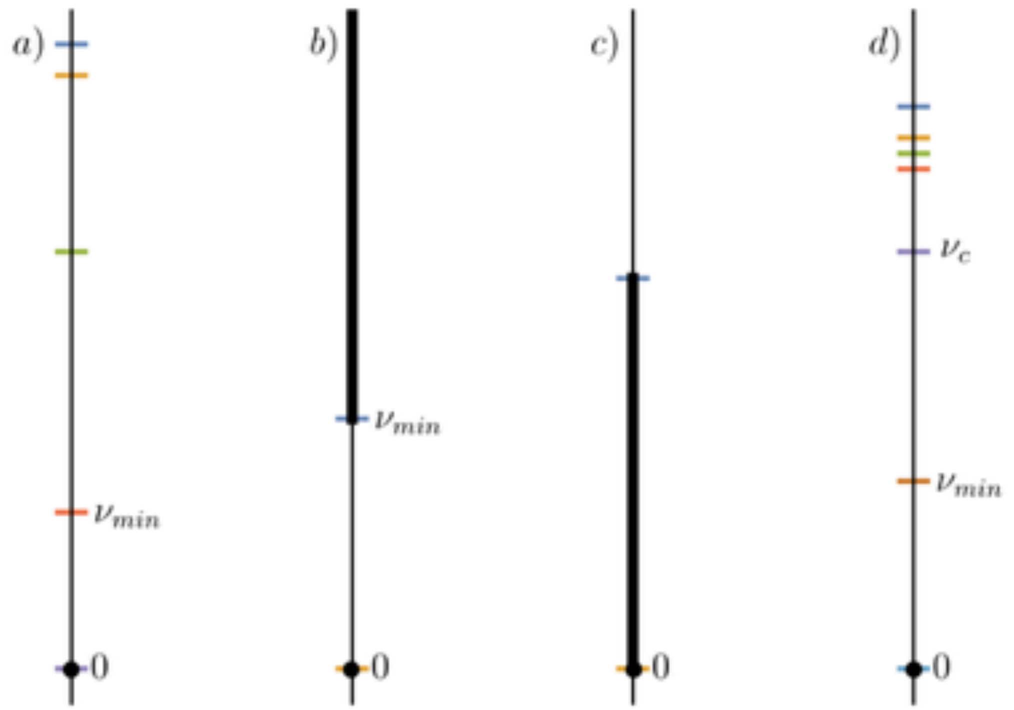


Figure 1: The spectrum of a linear collision operator: a) discrete spectrum, b) continuous spectrum with a gap, realized for the interaction potential $U = \alpha/r^n$, $n > 4$, c) gapless continuous spectrum, realized for the interaction potential $U = \alpha/r^n$, $n < 4$, d) Hod spectrum (see text): $0 \leq \nu_{min} \leq \nu_c$. In all cases, $\nu = 0$ is a degenerate eigenvalue corresponding to hydrodynamic modes (at zero spatial momentum).

dominant:

$$\boxed{\tau_R = 1/\nu_{min}}$$

THERMALISATION (RELAXATION)

- KGB equation

$$\frac{\partial F}{\partial t} + \frac{p_i}{m}\frac{\partial F}{\partial r^i} - \frac{\partial U(r)}{\partial r^i}\frac{\partial F}{\partial p_i} = -\frac{F - F_0}{\tau_R}$$

- kinetic theory predicts

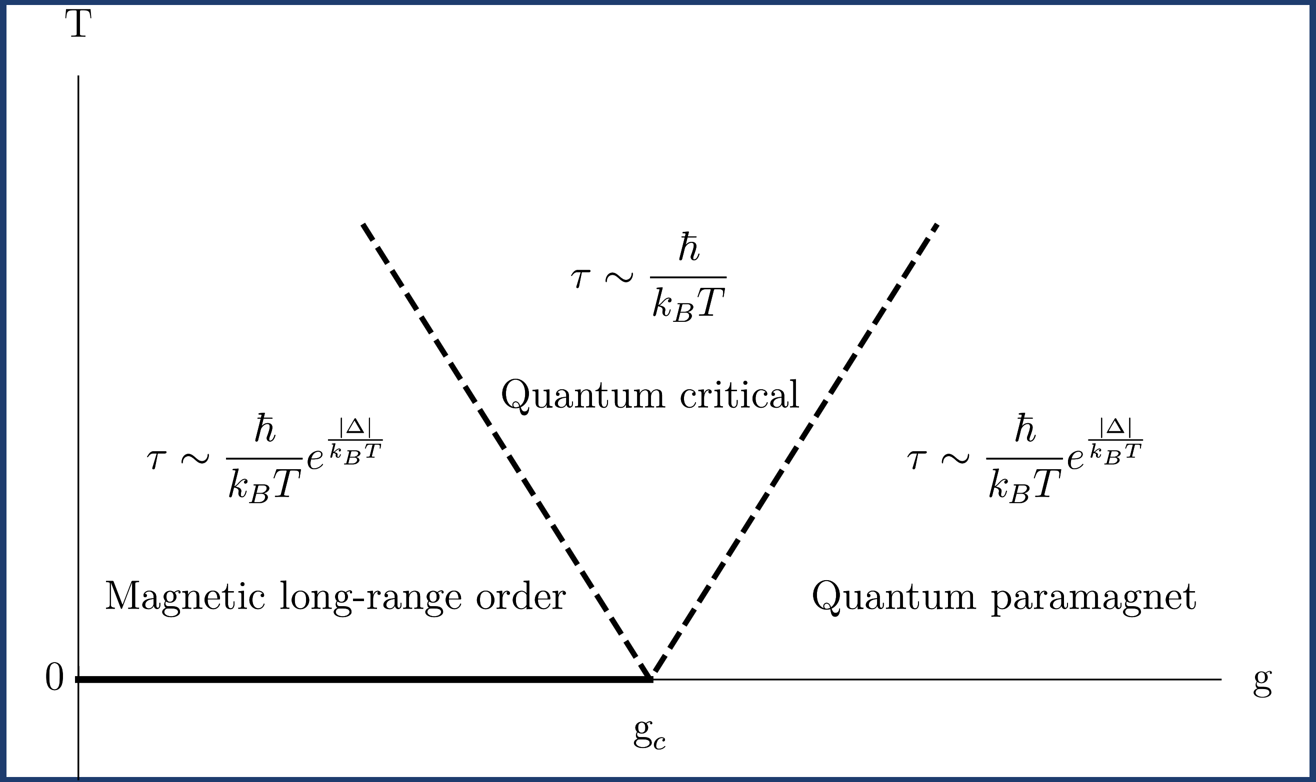
$$\boxed{\eta = \tau_R\, s\, T}$$

- relaxation time bound [Sachdev]

$$\tau_R \geq \mathcal{C}\,\frac{\hbar}{k_B T}$$

- Ising model (BTZ)

$$G^R(\omega, q) = \frac{\mathcal{C}_\Delta}{\pi\,\Gamma^2(\Delta-1)\sin\pi\Delta}\left|\Gamma\left(\frac{\Delta}{2} + \frac{i(\omega-q)}{4\pi T}\right)\Gamma\left(\frac{\Delta}{2} + \frac{i(\omega+q)}{4\pi T}\right)\right|^2$$
$$\times\left[\cosh\frac{q}{2T} - \cos\pi\Delta\cosh\frac{\omega}{2T} + i\sin\pi\Delta\sinh\frac{\omega}{2T}\right]$$



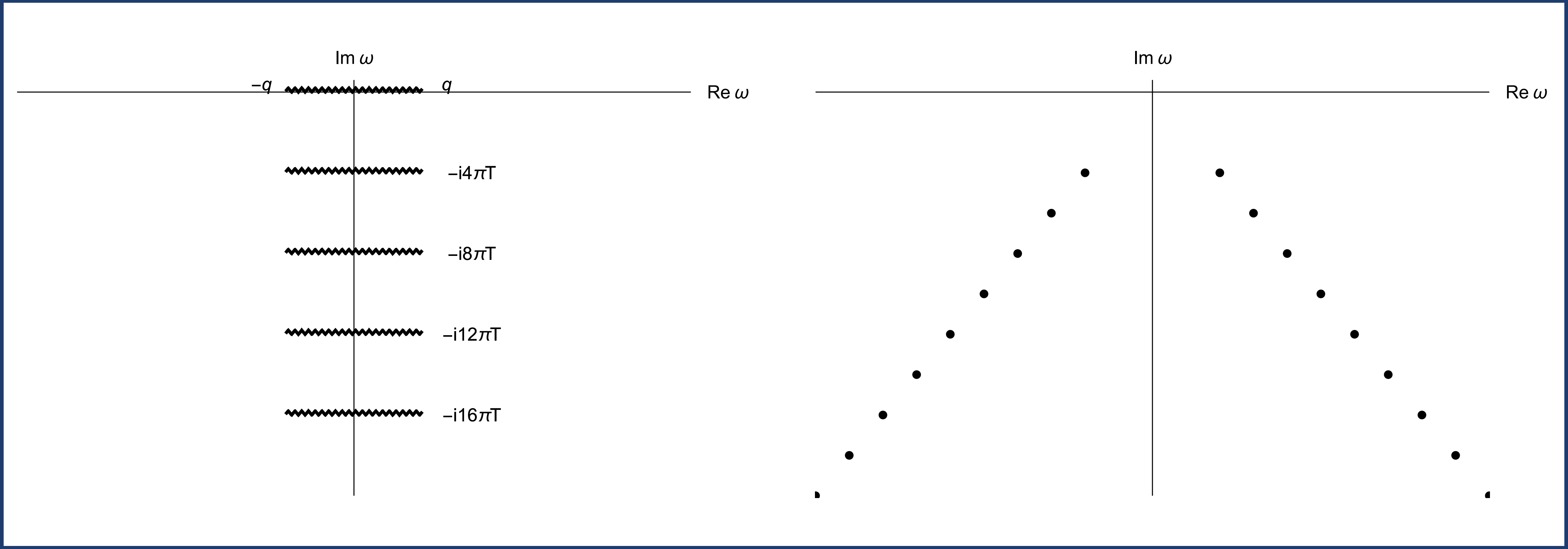
$$\omega = \pm q - i4\pi T\left(n + \frac{\Delta}{2}\right)$$

$$\tau_R = \frac{1}{2\pi\Delta}\,\frac{\hbar}{k_B T}$$

cute: $\Delta = 2 \Rightarrow$ "η/s" $= 1/4\pi$

FULL QUASINORMAL SPECTRUM

- weak/strong coupling (perturbative/holographic quasi-normal mode calculations) [Hartnoll, Kumar (2005)]



$\lambda \to 0$ $\lambda \to \infty$

RESULTS: QNM STRUCTURE

- different trends depending on η/s
- poles become denser (branch cut)
- new poles on imaginary axis

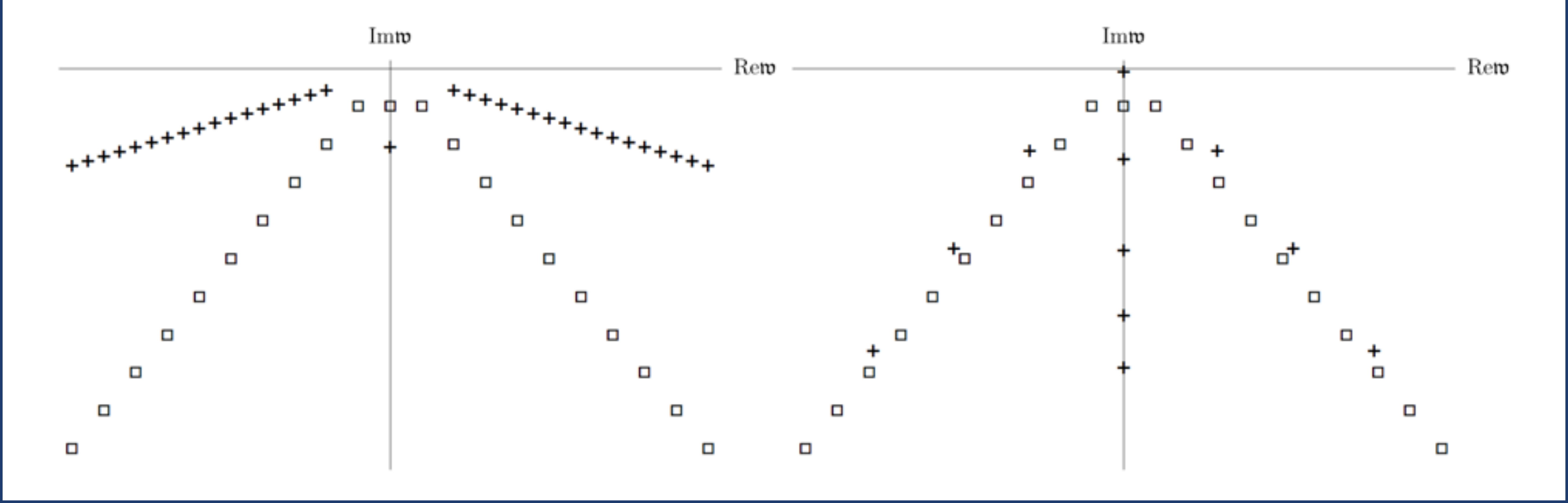
``weak limit''

$\gamma \to \infty$

$\lambda_{GB} \to -\infty$

``anomalous limit''

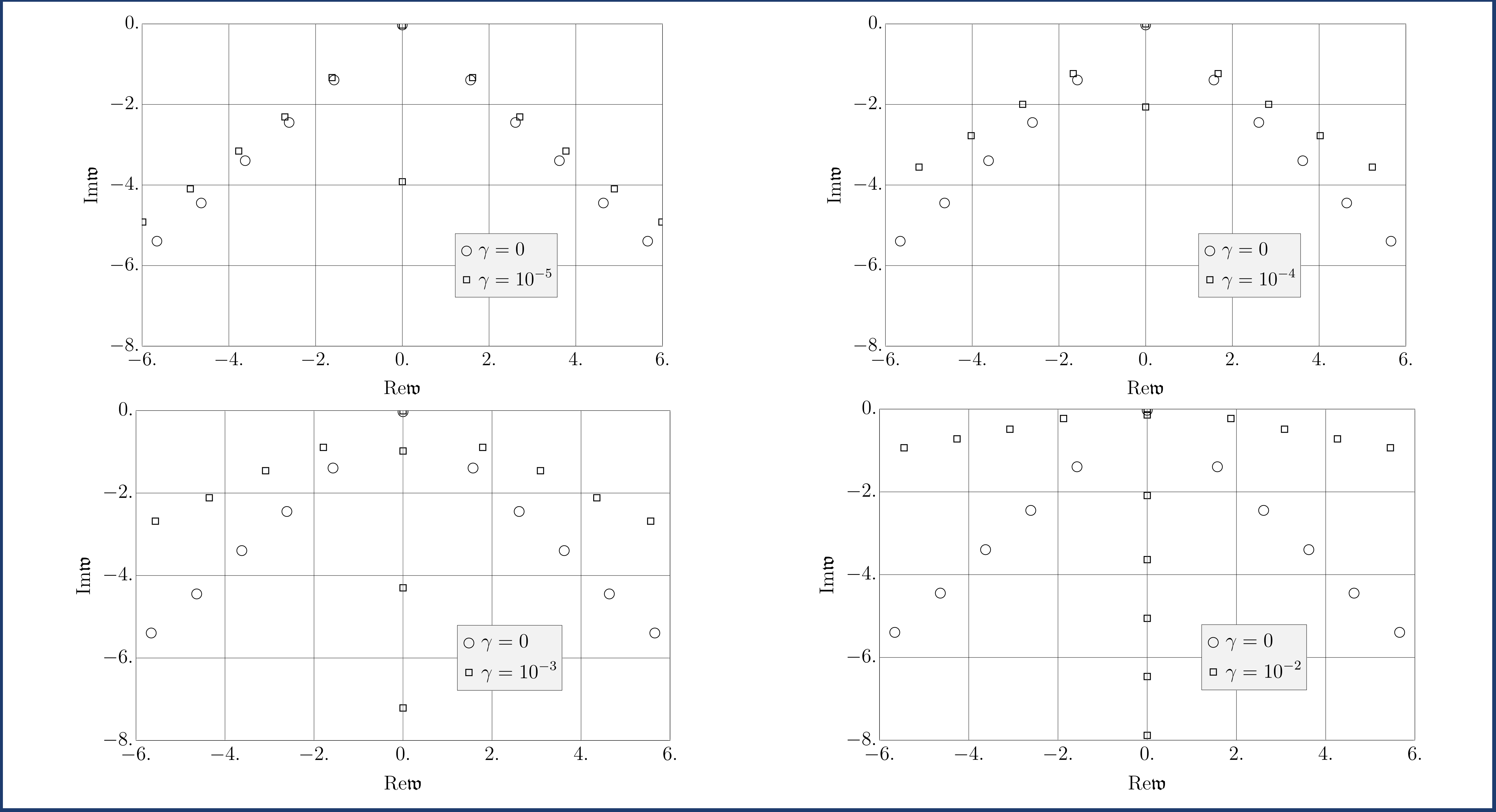
$\lambda_{GB} \to 1/4$



$\eta/s > \hbar/4\pi k_B$ $\qquad$ $\eta/s < \hbar/4\pi k_B$

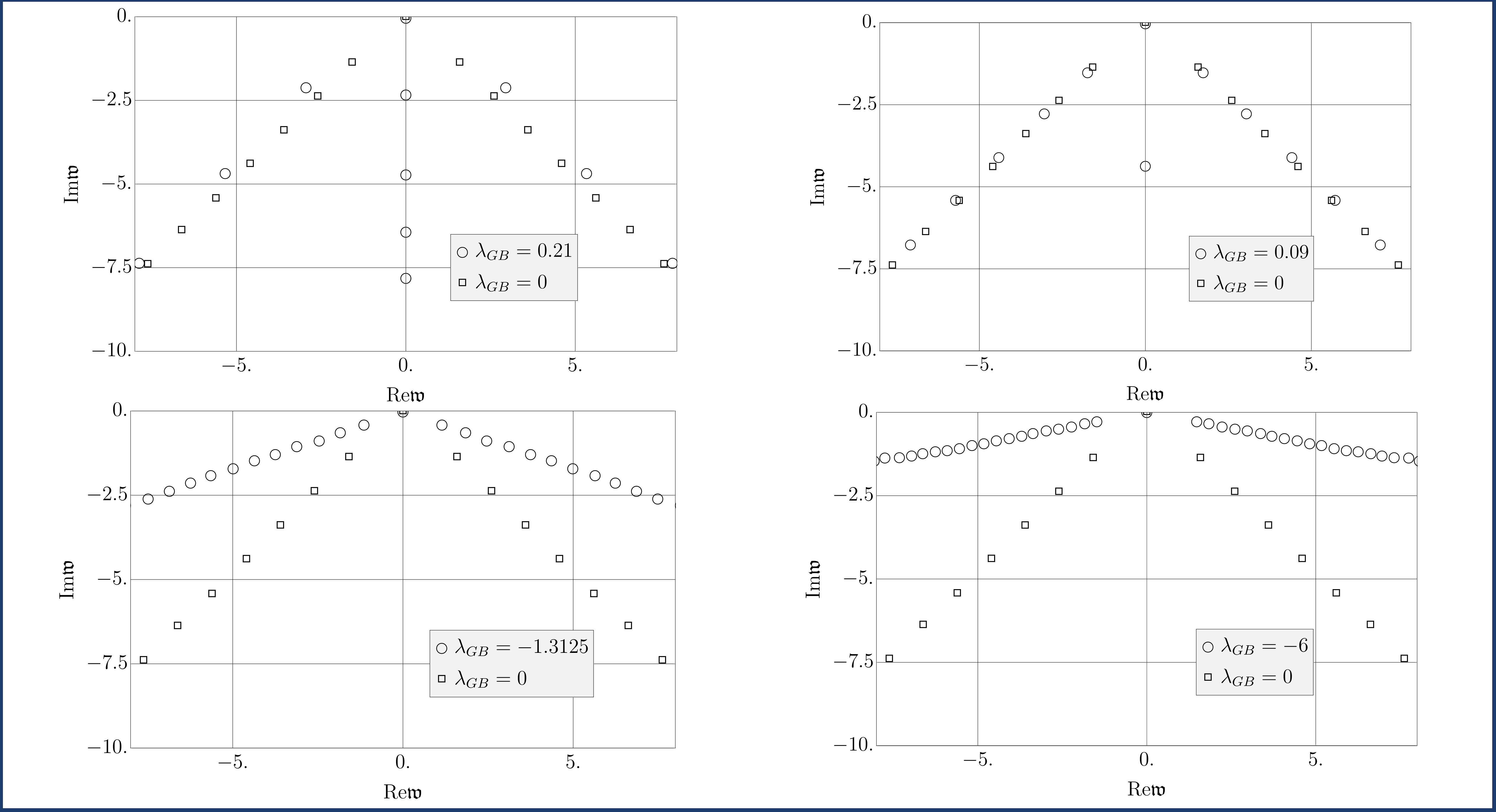
QNM STRUCTURE

- shear channel QNMs in *N=4*



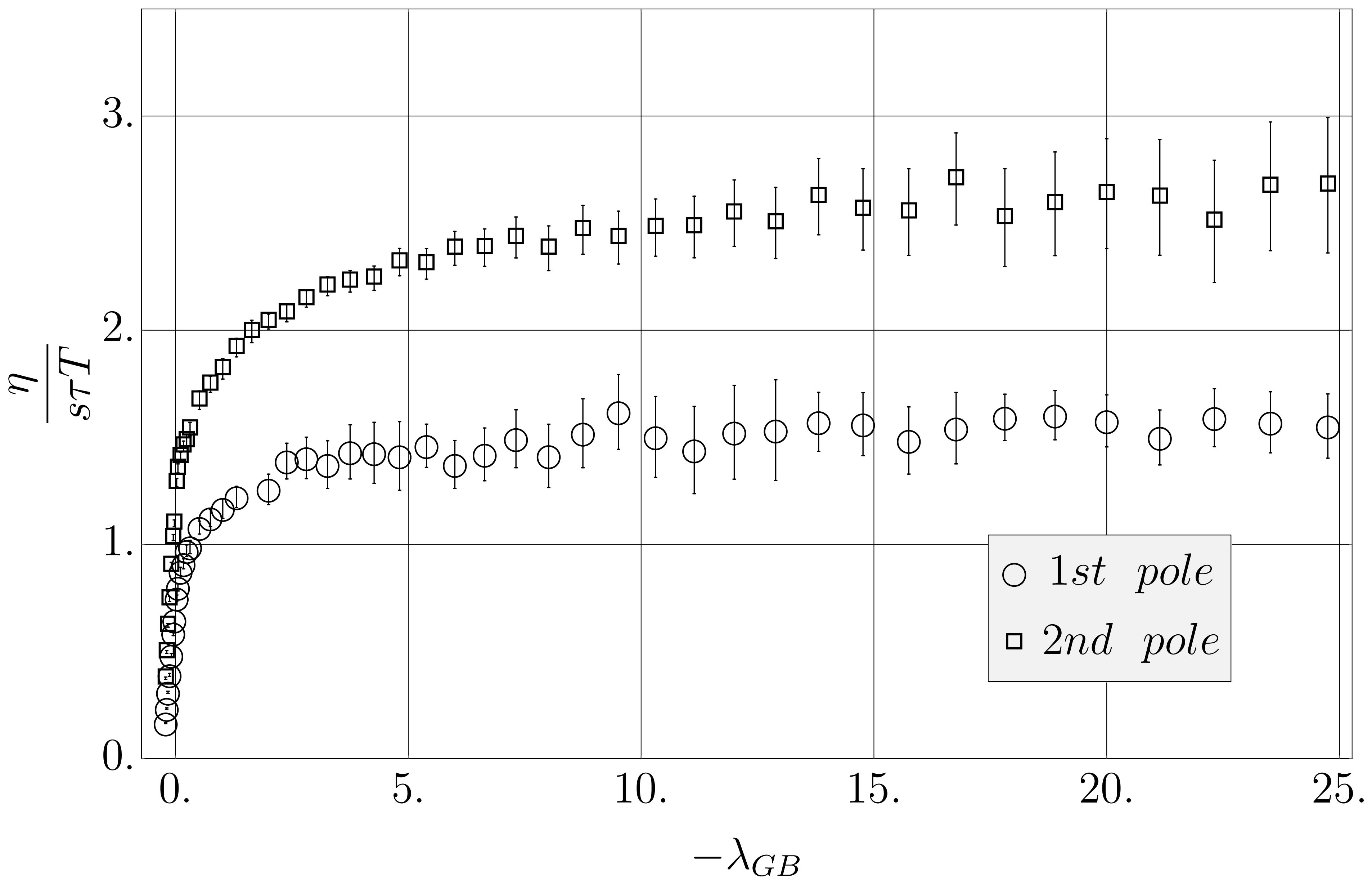
QNM STRUCTURE

- shear channel QNMs in Gauss-Bonnet



KINETIC THEORY RESULT

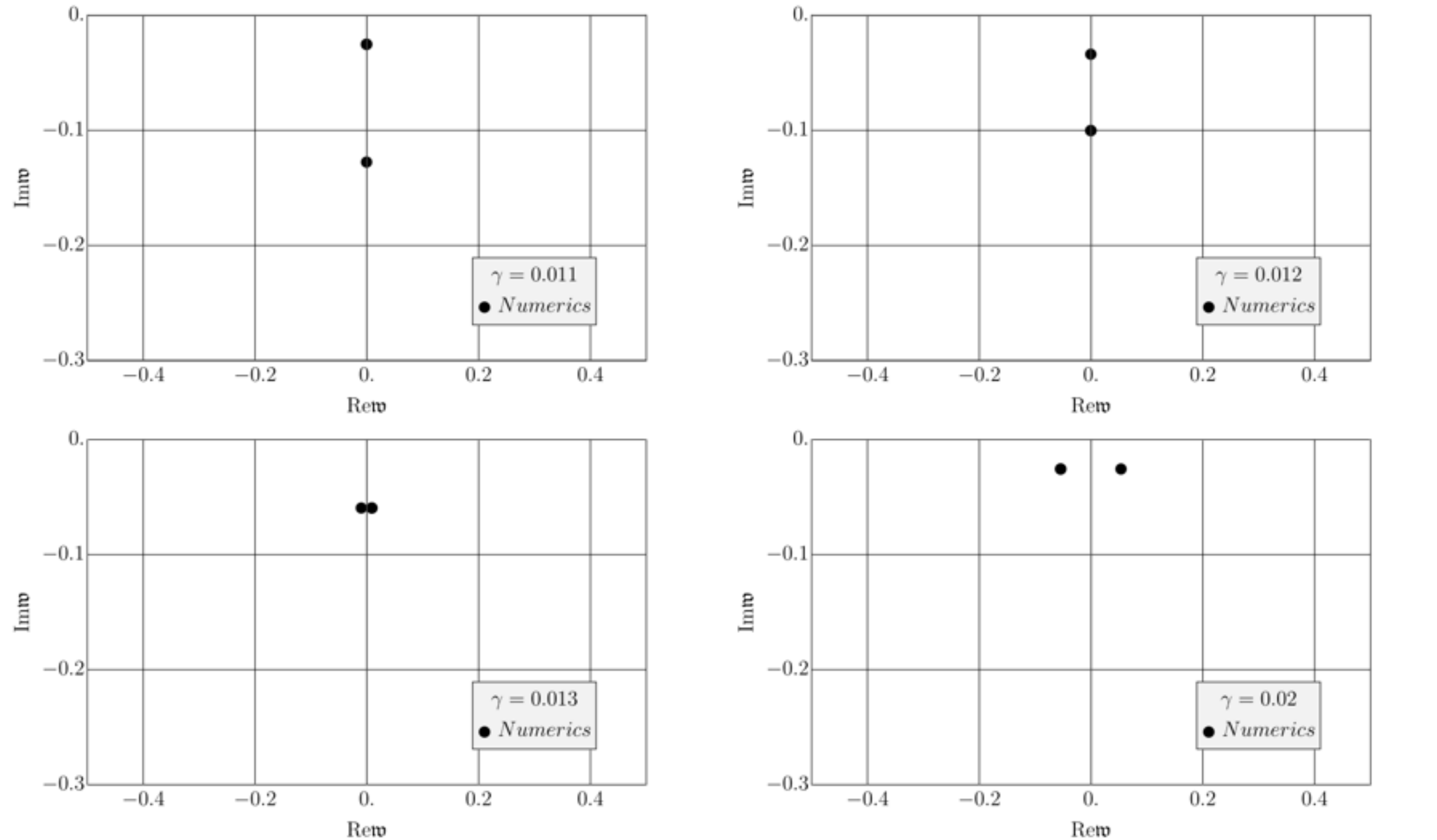
- kinetic theory behaviour quickly approached $\eta/s \sim const\, \tau_R T$



HYDRODYNAMICS

- breakdown of hydrodynamics (diffusion)

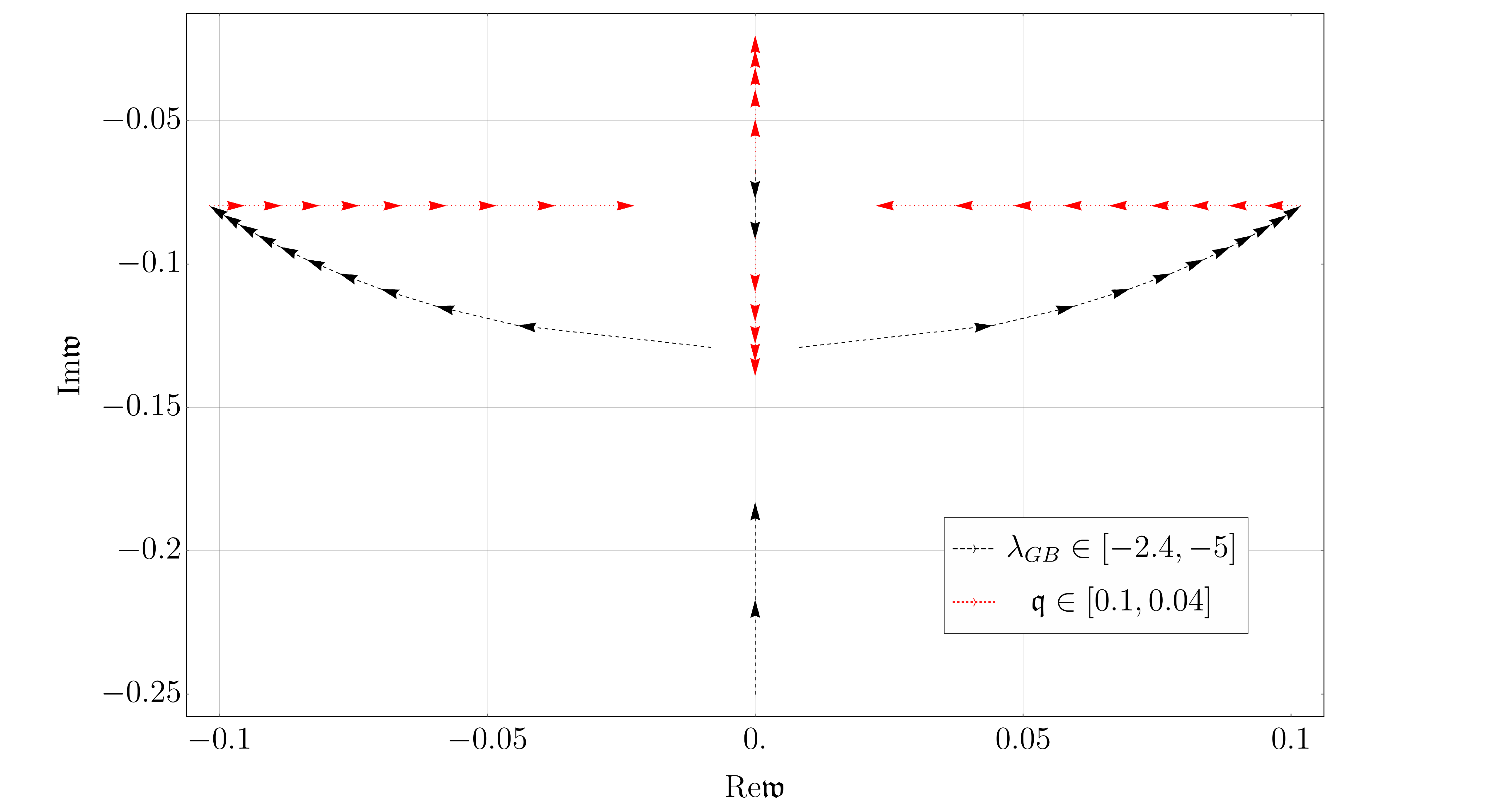
$$\boxed{\mathfrak{q}_c \sim \lambda^{3/4}}$$



HYDRODYNAMICS

- breakdown of hydrodynamics (diffusion)

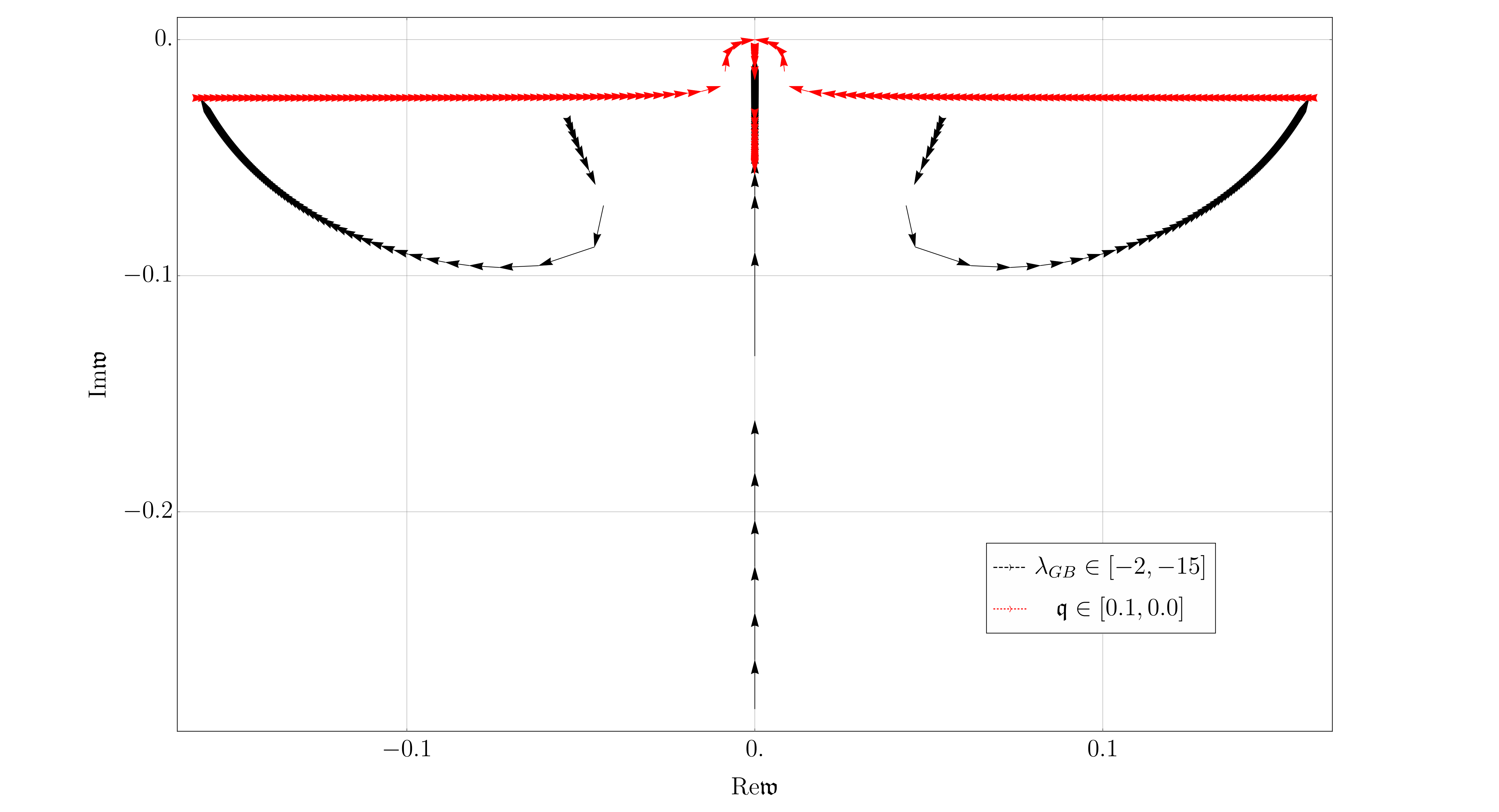
$$\boxed{\mathfrak{q}_c \sim \lambda^{3/4}}$$



HYDRODYNAMICS

- breakdown of hydrodynamics (sound)

$$\boxed{\mathfrak{q}_c \sim \lambda^{3/4}}$$



HYDRODYNAMICS

- breakdown of hydrodynamics due to new poles
- shear and sound correlators in Gauss-Bonnet [S. G., Starinets (2016) arXiv: 1611.07053]

$$G_{xz,xz}(\omega, q) = \frac{\sqrt{2}\pi^3 T^3 \gamma_{GB}^2}{(1+\gamma_{GB})^{3/2}\kappa_5^2}\left(\frac{\omega^2}{i\omega - i\omega^2/\omega_{\mathfrak{g}} - \gamma_{GB}^2 q^2/4\pi T}\right)$$

$$G_{tt,tt}(\omega, q) = \frac{3\sqrt{2}\pi^4 T^4}{(1+\gamma_{GB})^{3/2}\kappa_5^2}\left(\frac{\left(5q^2 - 3\omega^2\right)\left(1 - \omega/\omega_{\mathfrak{g}}\right) - i\gamma_{GB}^2 \omega q^2/\pi T}{\left(3\omega^2 - q^2\right)\left(1 - \omega/\omega_{\mathfrak{g}}\right) + i\gamma_{GB}^2 \omega q^2/\pi T}\right)$$

- dispersion relations

$$\omega_1 = -i\frac{\gamma_{GB}^2}{4\pi T}q^2 \qquad \omega_2 = \omega_{\mathfrak{g}} + i\frac{\gamma_{GB}^2}{4\pi T}q^2$$

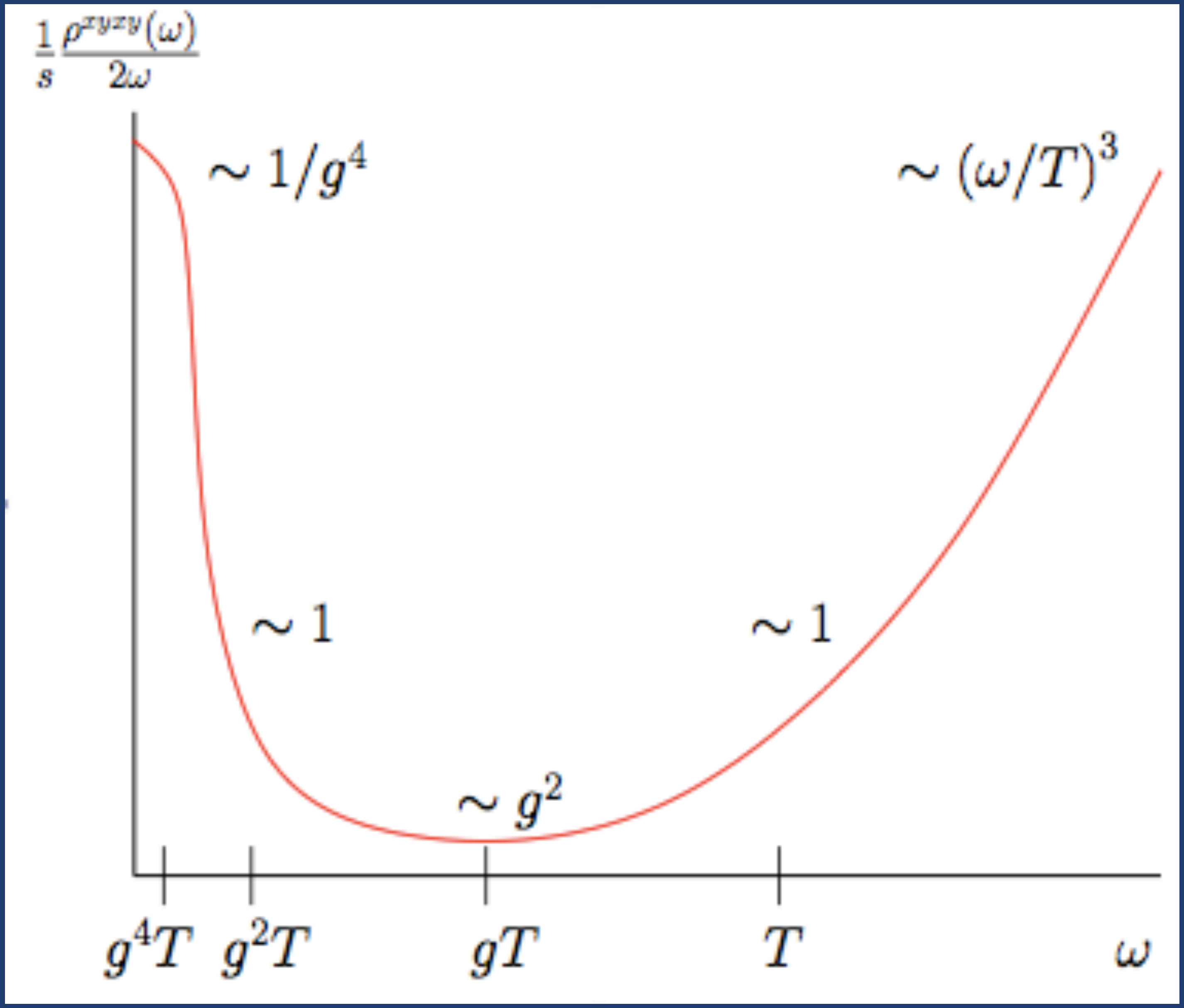
$$\omega_{1,2} = \pm\frac{1}{\sqrt{3}}q - i\frac{\gamma_{GB}^2}{6\pi T}q^2 \qquad \omega_3 = \omega_{\mathfrak{g}} + i\frac{\gamma_{GB}^2}{3\pi T}q^2$$

- gap

$$\omega_{\mathfrak{g}} = -\frac{8\pi T i}{\gamma_{GB}\left(\gamma_{GB}+2\right) - 3 + 2\ln\left(\frac{2}{\gamma_{GB}+1}\right)} \approx -\frac{8\pi T i}{\gamma_{GB}^2}$$

QUASI-PARTICLES (TRANSPORT PEAK)

- quasi-particles appear in the spectrum [Casalderrey-Solana, S. G., Starinets (2017)]
- weak coupling (Boltzmann equation)



QUASI-PARTICLES (TRANSPORT PEAK)

- holographic results in *N=4* theory (scalar channel)

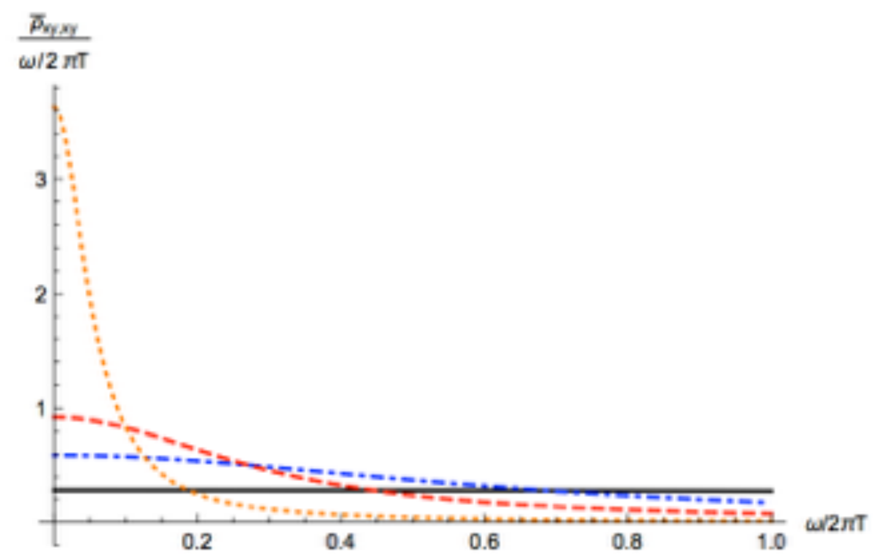


Figure 1: Dimensionless spectral function $\bar{\rho}_{xy,xy}(\omega, q)$ computed in the hydrodynamic approximation for the $\mathcal{N} = 4$ theory with $\gamma = 10^{-3}$ (black, solid), $\gamma = 10^{-2}$ (blue, dot-dashed), $\gamma = 2 \times 10^{-2}$ (red, dashed) and $\gamma = 10^{-1}$ (orange, dotted).

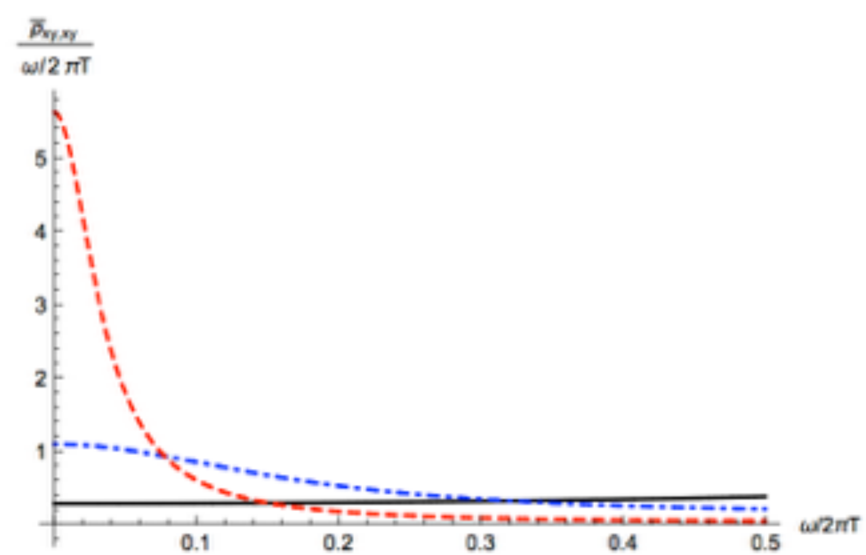


Figure 2: Dimensionless spectral function $\bar{\rho}_{xy,xy}(\omega, q)$ at $q = 0$ computed numerically and plotted for $\gamma = 10^{-3}$ (black, solid), $\gamma = 10^{-2}$ (blue, dot-dashed) and $\gamma = 2 \times 10^{-2}$ (red, dashed). The plot represents the emergence of the transport peak at $\omega = 0$.

QUASI-PARTICLES (TRANSPORT PEAK)

- holographic results in a dual of the Gauss-Bonnet theory (scalar channel)

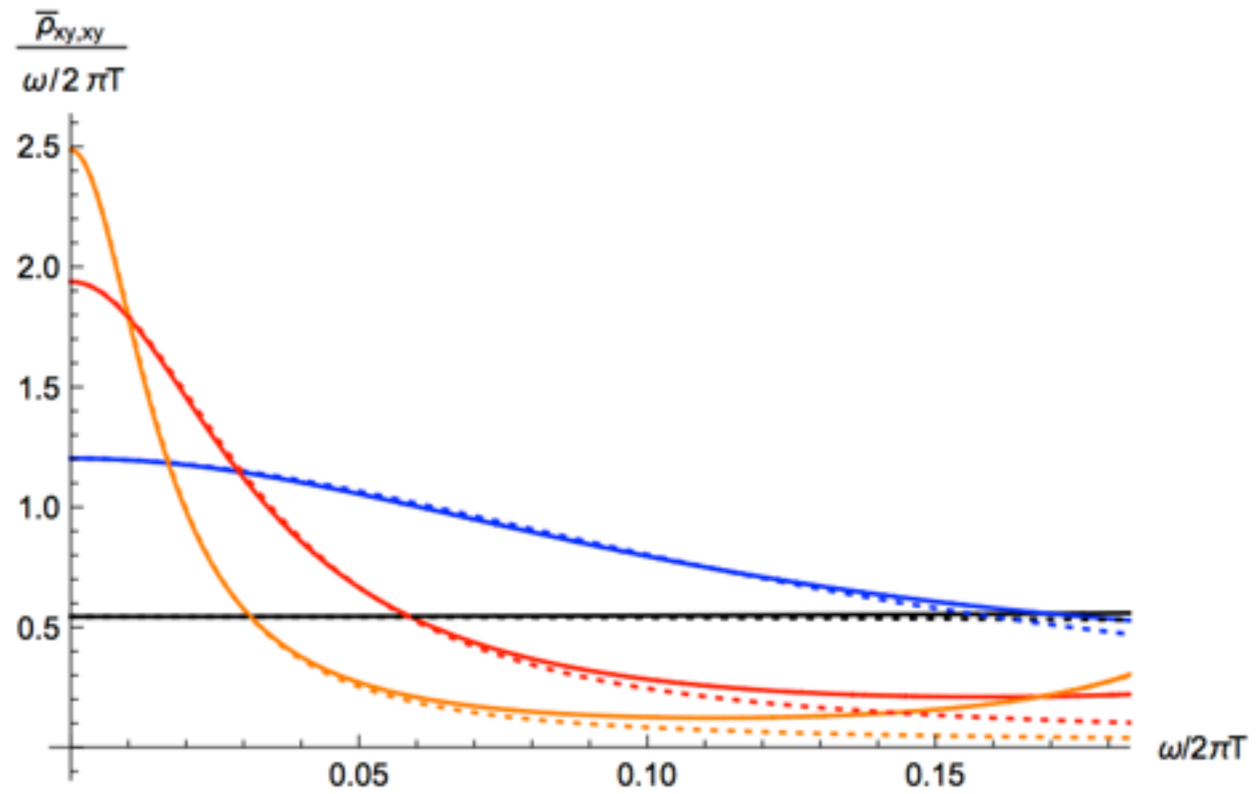


Figure 7: Comparison between analytic (dotted) and numerically computed (solid) spectral function at $q = 0$ for $\gamma_{GB} = 5$ (black), $\gamma_{GB} = 10$ (blue), $\gamma_{GB} = 20$ (red) and $\gamma_{GB} = 30$ (orange).

QUASI-PARTICLES IN $N=4$

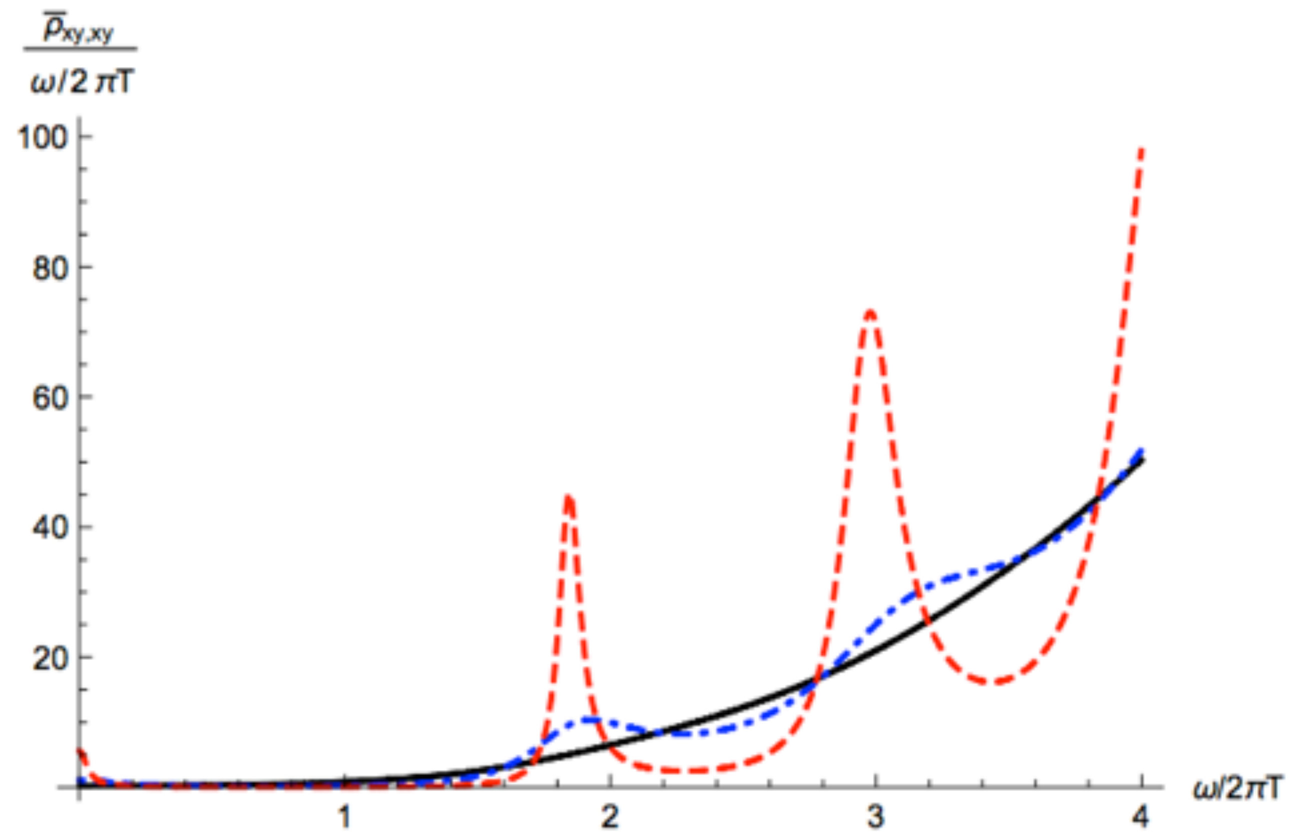


Figure 3: Dimensionless spectral function $\bar{\rho}_{xy,xy}(\omega, q)$ at $q = 0$ computed numerically and plotted for $\gamma = 10^{-3}$ (black, solid), $\gamma = 10^{-2}$ (blue, dot-dashed) and $\gamma = 2 \times 10^{-2}$ (red, dashed). The plot represents the emergence of quasiparticles at intermediate coupling.

QUASI-PARTICLES IN GAUSS-BONNET

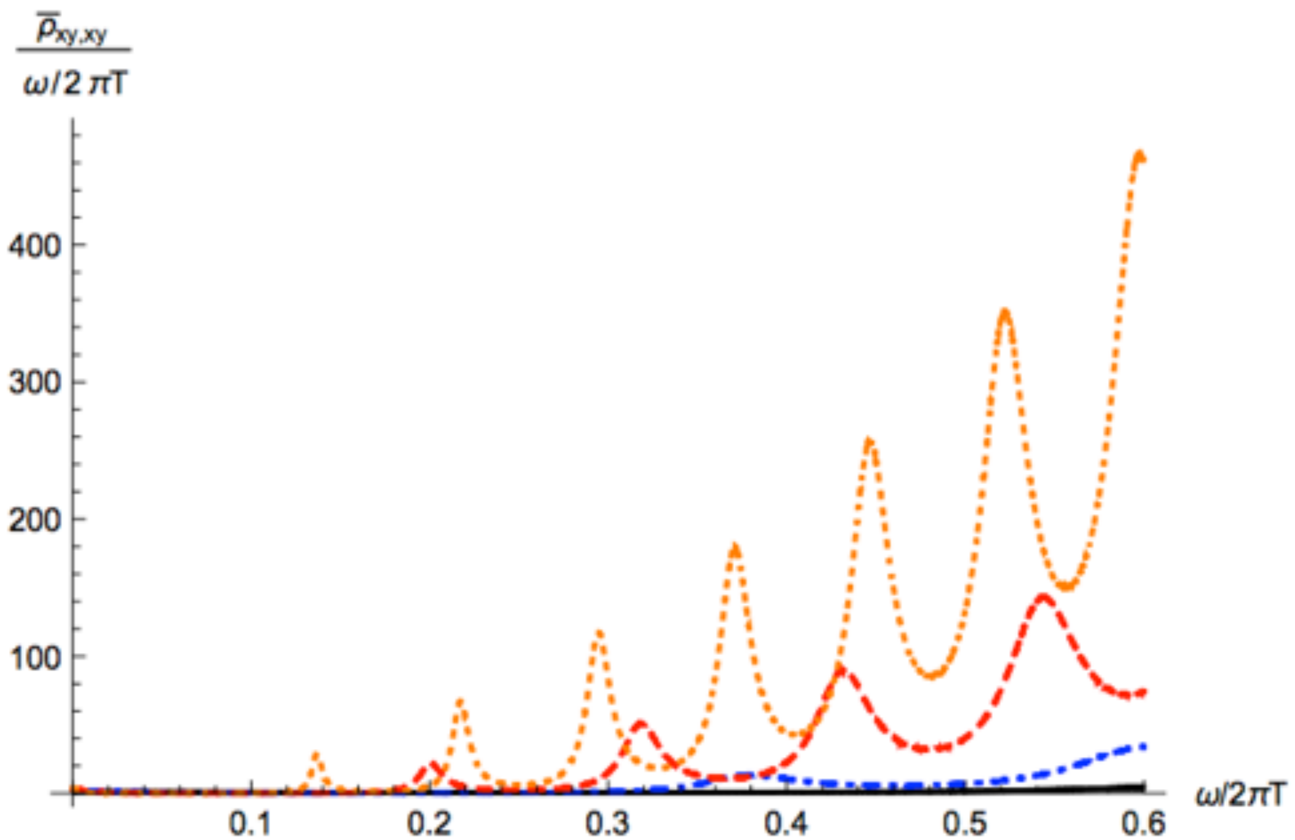


Figure 6: Dimensionless spectral function $\bar{\rho}_{xy,xy}(\omega, q)$ at $q = 0$ computed numerically and plotted for $\gamma_{GB} = 5$ (black, solid), $\gamma_{GB} = 10$ (blue, dot-dashed), $\gamma_{GB} = 20$ (red, dashed) and $\gamma_{GB} = 30$ (orange, dotted). The plot represents the emergence of quasiparticles at intermediate coupling.

QUASI-PARTICLES IN GAUSS-BONNET

- quasi-particles appear also in other channels (shear)

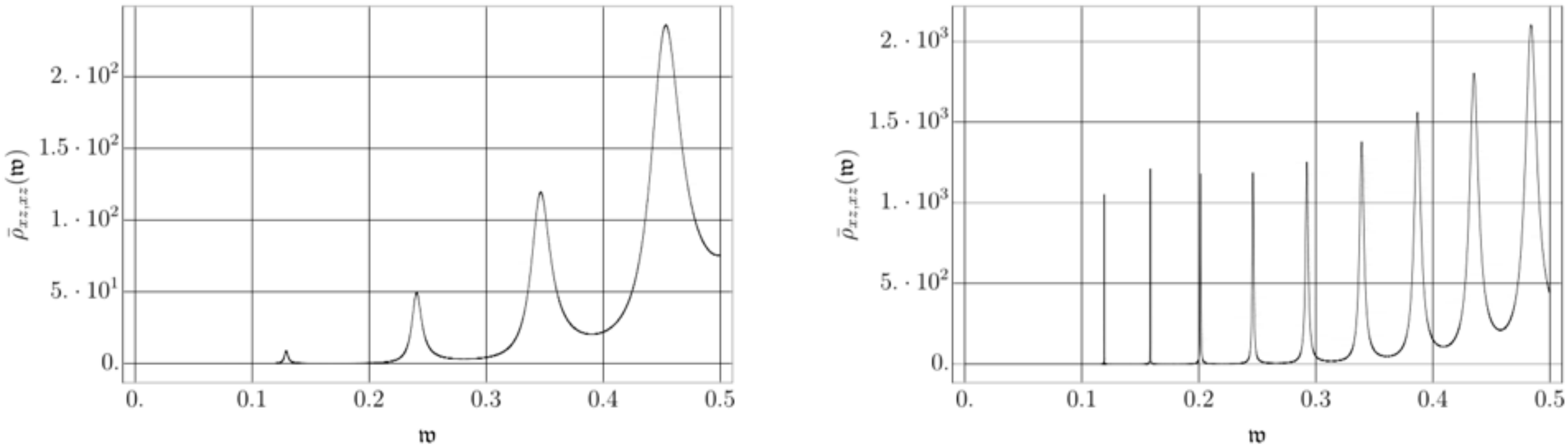


Figure 31: The dimensionless spectral function $\bar{\rho}_{xz,xz}(\mathfrak{w}, \mathfrak{q}, \lambda_{GB})$ in the shear channel of the Gauss-Bonnet theory for $\lambda_{GB} = -100$ (left panel) and $\lambda_{GB} = -500$ (right panel) at $\mathfrak{q} = 0.1$.

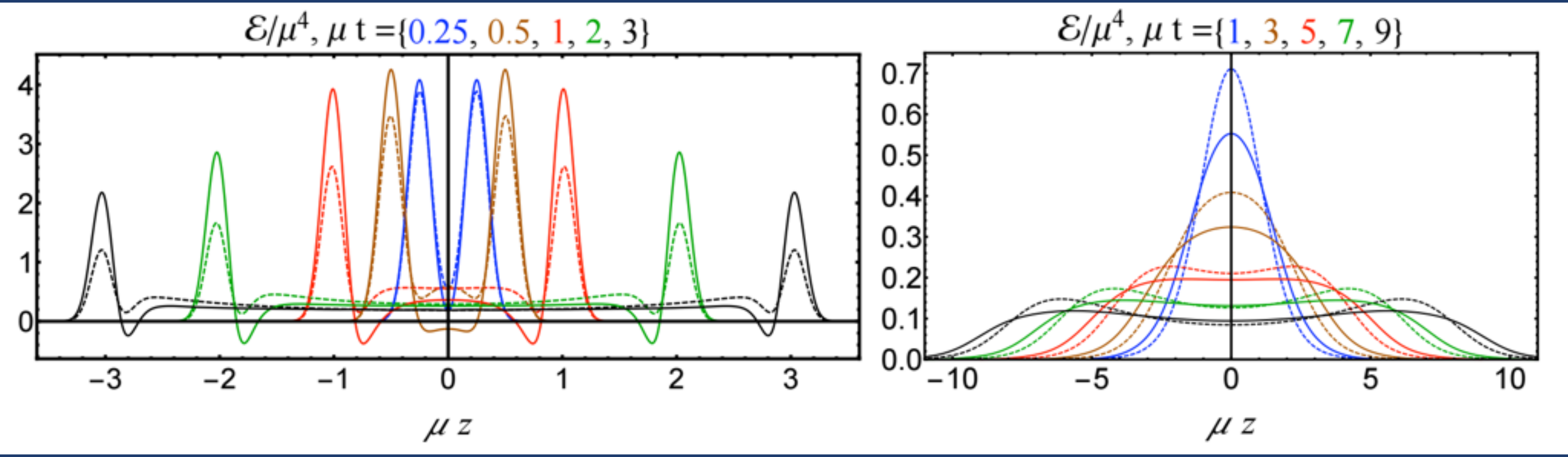
- how do these peaks manifest themselves (if at all) in a weakly coupled field theory like QCD?

HEAVY ION COLLISIONS

HEAVY ION COLLISIONS

- R^2 coupling constant corrections to shockwave collisions
  [S. G., van der Schee (2016) arXiv:1610.08976]

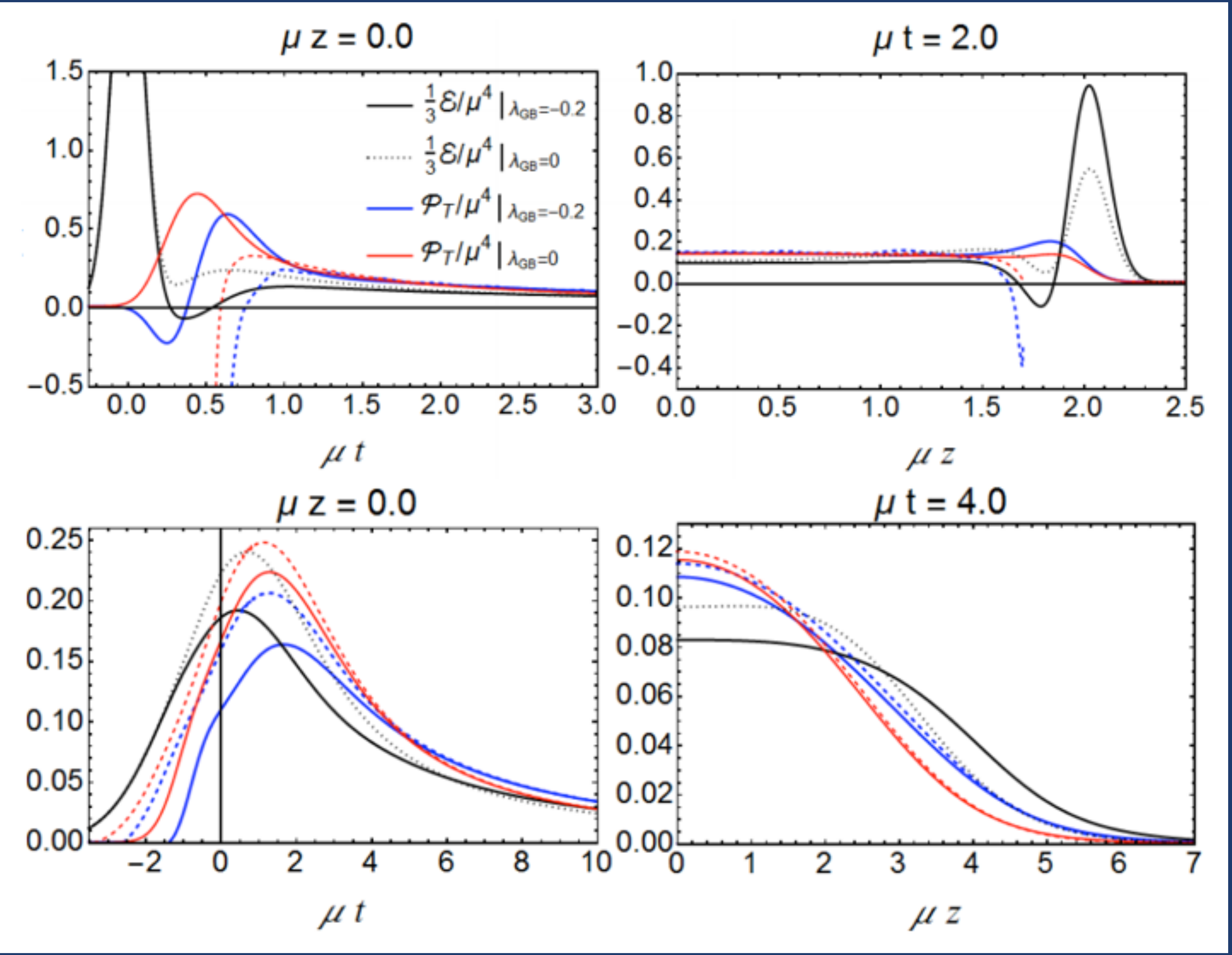
- energy density along the longitudinal direction after collision: less stopping of narrow shocks (*88%* higher energy density on lightcone) and decreased energy density of wide shocks



HEAVY ION COLLISIONS

- delayed hydrodynamisation

$$t_{\rm hyd} T_{\rm hyd} = \{0.41 - 0.52\lambda_{GB},\; 0.43 - 6.3\lambda_{GB}\}$$

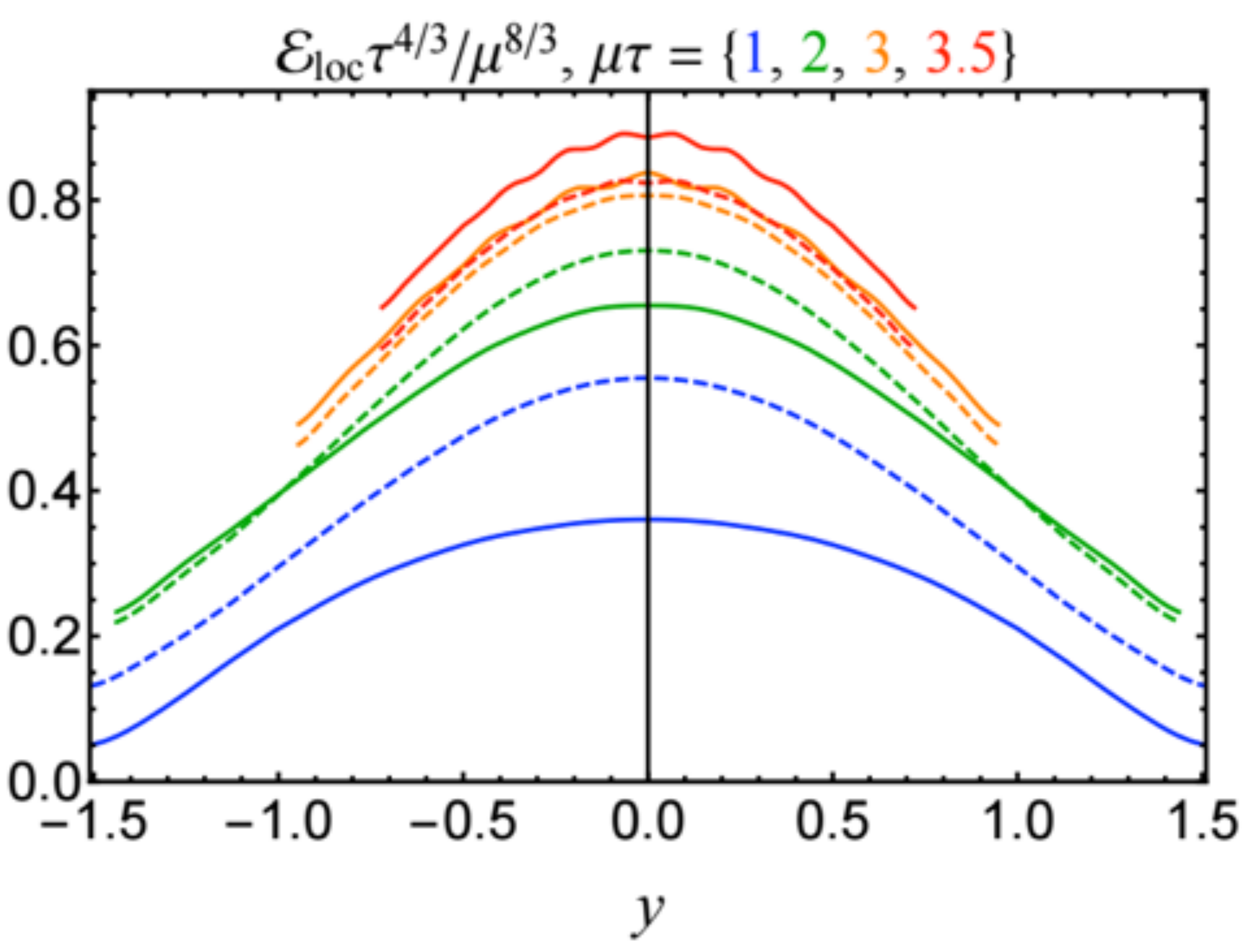


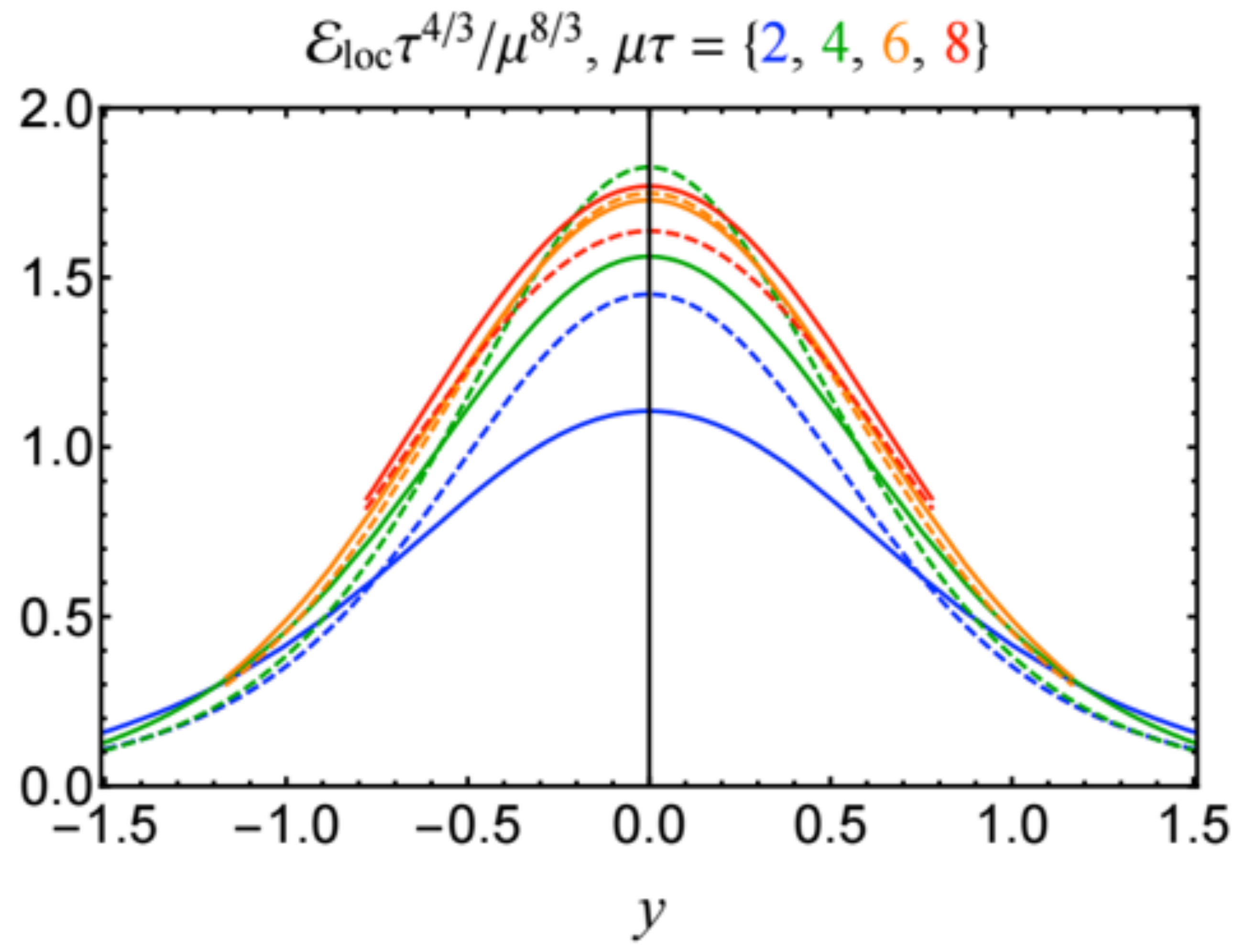
at $\lambda_{GB} = -0.2:$

$\{25\%, 290\%\}$

HEAVY ION COLLISIONS

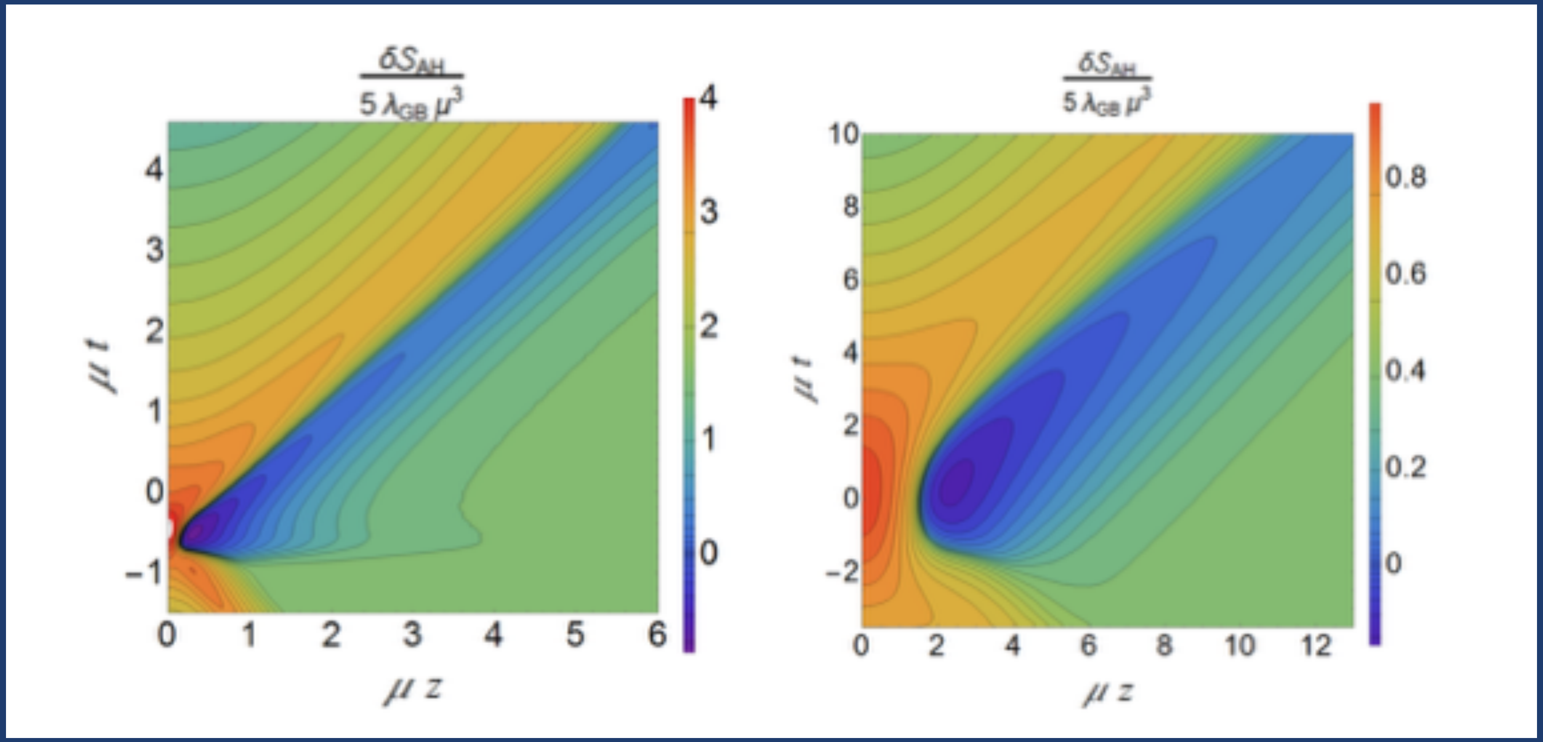
- rapidity profile: start wider and smaller but later become comparable



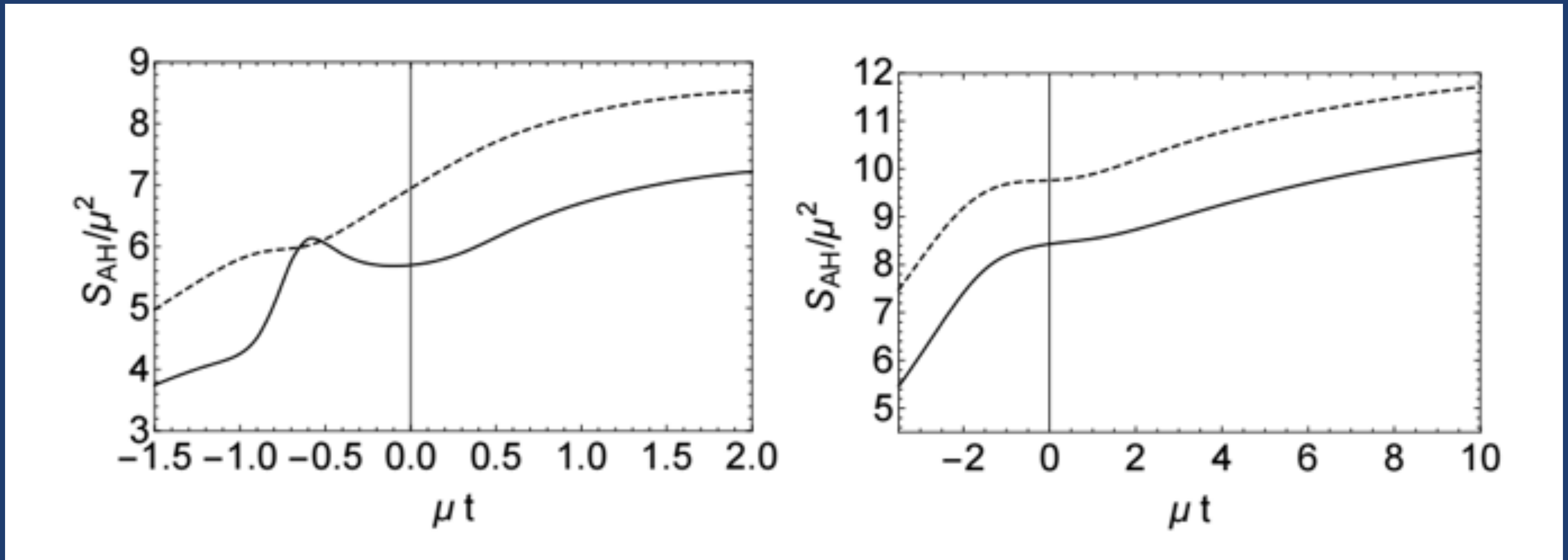


HEAVY ION COLLISIONS

- change in entropy density: enhanced on lightcone, negative in plasma



- total entropy: reduced at intermediate coupling



CONCLUSION AND FUTURE DIRECTIONS

- various weakly coupled properties are recovered remarkably quickly
- the topic of this workshop: how precisely do they match onto those computed from weakly coupled, perturbative physics?
- use interpolations between weakly coupled physics and coupling-dependent holography to understand intermediate coupling
- can we see a signature of higher-QNMs (quasi-particles) in experiments?
- a harder task for the future: understand *1/N* corrections

THANK YOU!