Small black holes and puzzles in thermalization

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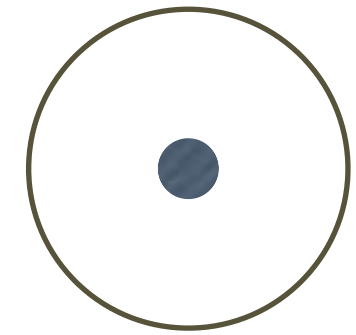
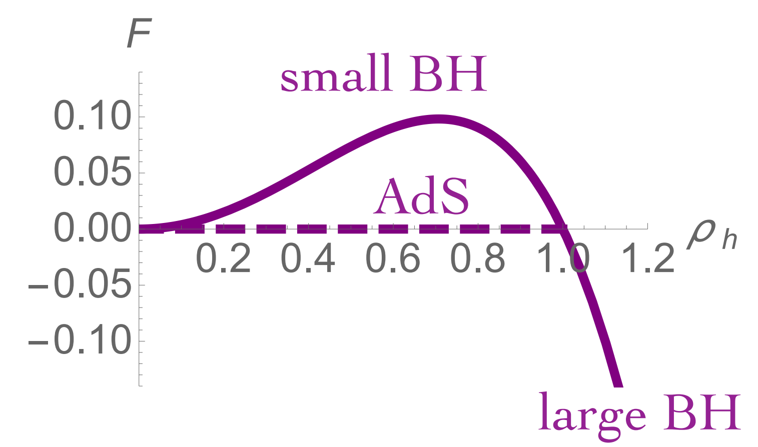
Black holes in global AdS



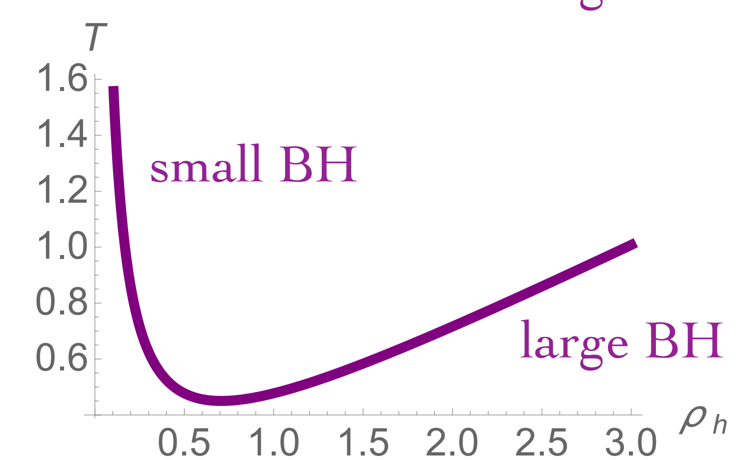
- AdS$_5$-Schwarschild × S^5 metric:

$$ds^2 = -f(\rho)\,dt^2 + \frac{d\rho^2}{f(\rho)} + \rho^2\,d\Omega_3^2 + d\Omega_5^2\,, \qquad f(\rho) = \rho^2 + 1 - (\rho_\mathrm{h}^2 + 1)\,\frac{\rho_\mathrm{h}^2}{\rho^2}$$

- free energy $F = -\frac{\pi}{8}\,\rho_\mathrm{h}^2\,(\rho_\mathrm{h}^2 - 1) + (\mathrm{Casimir})$



- mass $M = \frac{3\pi}{8}\,\rho_\mathrm{h}^2\,(\rho_\mathrm{h}^2 + 1) + (\mathrm{Casimir})$
- temperature $T = \frac{2\rho_\mathrm{h} + \rho_\mathrm{h}^{-1}}{2\pi}$



- "large" BH branch, $\rho_\mathrm{h} > 1$
- "small" BH branch, $\rho_\mathrm{h} < 1$

Large AdS$_5$ black holes:

- Thermodynamically stable
  - Free energy minimum: $F_{\text{BH}} < F_{\text{AdS}}$
  - Positive heat capacity, $\mathrm{d}M/\mathrm{d}T > 0$
- Dynamically stable
  - Im(quasi-normal mode frequencies) < 0
- Dual description of deconfined plasma
  - equilibrium state of $\mathcal{N}=4$ SU(N_c) SYM on $\mathbb{R}\times S^3$
  - $\mathcal{O}(N_c^2)$ free energy, entropy
  - $T \geq T_c = 3/(2\pi)$
- First order Hawking-Page transition at $T = T_c$

SU(N_c) $\mathcal{N}$=4 SYM on $\mathbb{R}\times S^3$

- $N_c \to \infty$ limit = thermodynamic limit
- $T<T_c$: confined phase, $\mathcal{O}(N_c{}^0)$ free energy
    - dual description = "thermal" AdS
- $T>T_c$: deconfined phase, $\mathcal{O}(N_c{}^2)$ free energy
    - dual description = large AdS black hole
- $T=T_c$: first order confinement/deconfinement phase transition
    - dual description = Hawking-Page transition

Small AdS_5 black holes:

- Thermodynamically unstable for $\rho_h < 1$:
  - Non-minimal free energy, $F_{BH} > F_{AdS}$
  - Negative heat capacity for $\rho_h < 1/\sqrt{2}$
  - T diverges as $\rho \to 0$
- Dynamically unstable for $\rho_h < \rho^* = 0.4402$:
  - Unstable modes = deformations of internal S^5 geometry
  - Bifurcations lead to various deformed static solutions
  - Widely expected endpoint: BH localized on S^5
- Holographic interpretation ???
  - Old question …

Hubeny & Rangamani

Gregory & Laflamme; Banks, Douglas, Horowitz & Martinec; Peet & Ross, …

Curtis Asplund & David Berenstein, 0809.0712: *Small AdS black holes from SYM*

"We provide a characterization of the set of configurations in $\mathcal{N} = 4$ SYM theory that are dual to small AdS black holes. Our construction shows that the black hole dual states are approximately thermal on a SU(M) subset of degrees of freedom of a SU(N) gauge theory. M is determined dynamically and the black hole degrees of freedom are dynamically insulated from the rest. These states are localized on the S^5 and have dynamical processes that correspond to matter absorption that make them behave as black objects."

"We will argue… that the configurations dual to a small black hole are given by approximately thermal states in an $M{\times}M$ subsector of the large N gauge field theory. The off-diagonal modes connecting this subsector to the rest of the field theory are heavy, providing some thermos that allows the $M{\times}M$ submatrix degrees of freedom to thermalize without spreading the thermal physics to all the $N{\times}N$ degrees of freedom. We will argue that M and the temperature of the black hole are related. We will also show that these configurations localize in the S^5, that they are black and that they have negative specific heat. This provides strong evidence for their identification as the dual states to small black holes in AdS space."

Niko Jokela, Arttu Pönni & Aleksi Vuorinen, 1508.00859: *Small black holes in global AdS spacetime*

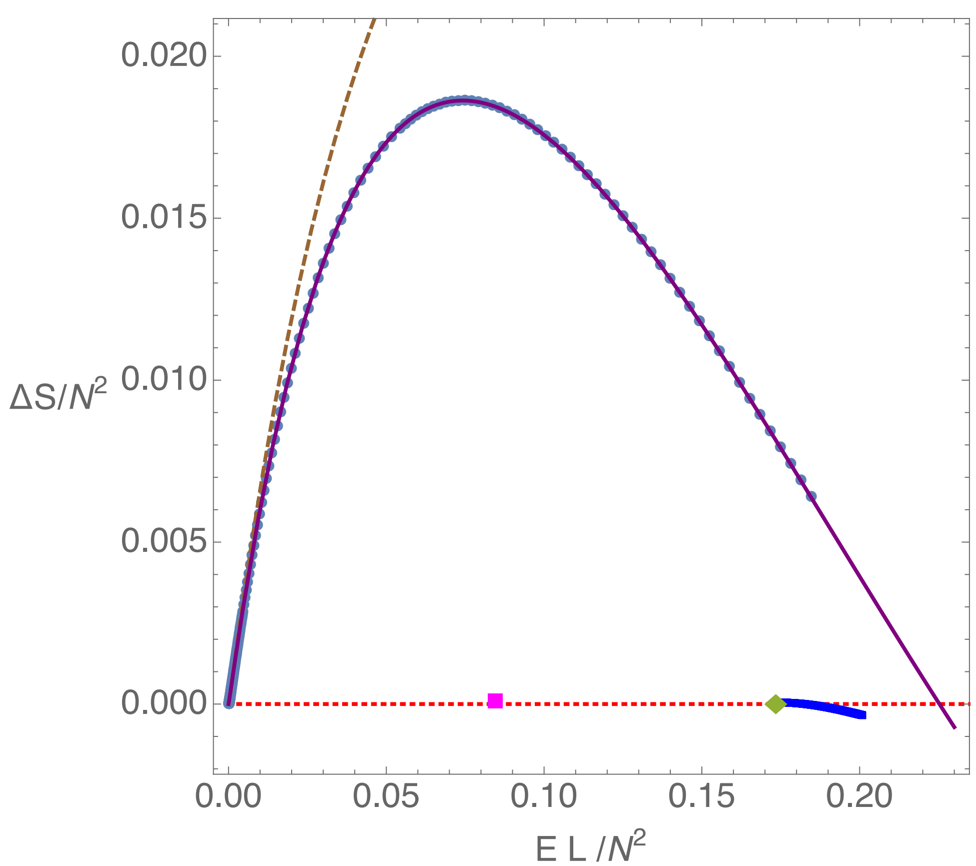
"[Our] results smoothly interpolate between known limits corresponding to large black holes and thermal AdS space. This implies that the quantities are continuous functions of energy density in the microcanonical ensemble, thus smoothly connecting the deconfined and confined phases that are separated by a first order phase transition in the canonical description."

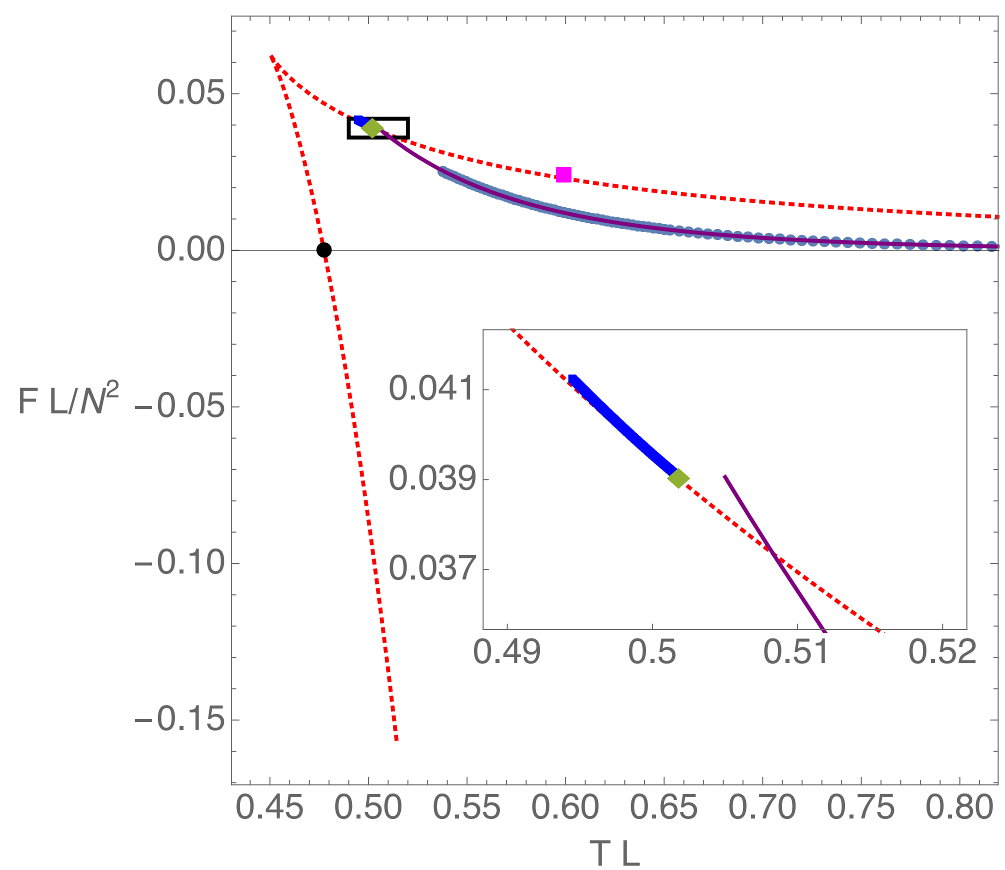
"The space surrounding the small BHs is approximately flat, and as the specific heat of the BH is negative, it evaporates away; as a result, the gravity solution becomes thermal (or pure) AdS with no BH in the bulk."

"[In] the canonical ensemble the dual field theory is known to have a first order deconfinement phase transition at some critical temperature T_c, where several physical quantities exhibit discontinuous behavior. It should be very interesting to investigate what happens to the same observables in the microcanonical ensemble, where the small BH is stable and one can continuously decrease the energy density below the critical one while still residing in the BH phase."

Oscar Dias, Jorge Santos & Benson Way, 1605.04911:
Localised $AdS_5 \times S^5$ black holes

"We numerically construct asymptotically global $AdS_5 \times S^5$ black holes that are localised on the S^5 . These are solutions to type IIB supergravity with S^8 horizon topology that dominate the theory in the microcanonical ensemble at small energies. At higher energies, there is a first-order phase transition to AdS_5-Schwarzschild$\times S^5$. By the AdS/CFT correspondence, this transition is dual to spontaneously breaking the SO(6) R-symmetry of $\mathcal{N} = 4$ super Yang-Mills down to SO(5). We extrapolate the location of this phase transition and compute the expectation value of the resulting scalar operators in the low energy phase."





Masanori Hanada & Jonathan Maltz, 1608.03276:
A proposal of the gauge theory description of the small Schwarzschild black hole in $AdS_5 \times S^5$

"[The] dual gravity description suggests the following in the microcanonical ensemble at strong coupling. The large AdS-BH shrinks as the energy is decreased and the temperature goes down. When the Schwarzschild radius becomes of order R_{AdS}, the BH localizes along the S^5 and can be regarded as a ten-dimensional BH. This transition is of first order, and the BH becomes hotter after the localization. When the Schwarzschild radius becomes much smaller than R_{AdS}, the BH should behave like the 10D Schwarzschild BH in flat spacetime. We will call this localized BH, the small BH. Note that the small BH has a negative specific heat."

- "The large black hole is described by a bound state of all the eigenvalues of scalar fields X_M. All N^2 matrix entries are excited."

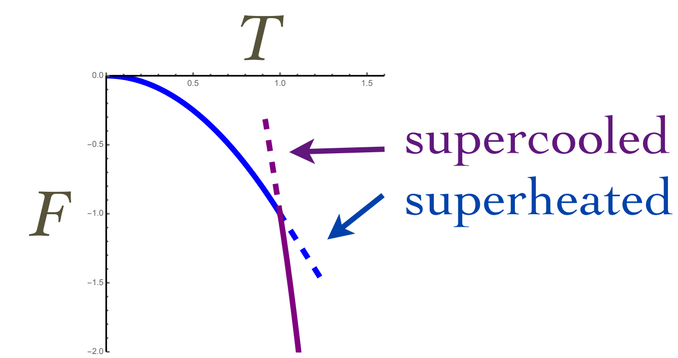
- "Suppose some of the eigenvalues are emitted, after which only $N_{BH} < N$ eigenvalues form a bound state. Such matrices describe the small black hole. The black hole is smaller when N_{BH} is smaller."

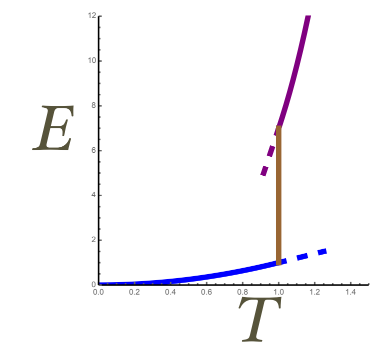
Questions:

- Can microcanonical phase diagram be distinct from canonical?
- How do dynamical instabilities evolve?
  - What is the endpoint?
  - Domain formation?
- What is the complete set of equilibrium states?
  - At $T > T_c$?
  - At $T = T_c$?
  - Are there equilibrium states with spontaneously broken SO(6) R symmetry?
  - Are there equilibrium states with spontaneously broken SO(4) symmetry?
  - Is understanding of the $T>T_c$ canonical ensemble incomplete?

Review: first order transitions

- Thermodynamic limit = volume $V \to \infty$
- 1st order transition = kink in free energy
- Jump in internal energy $E = \dfrac{\partial(\beta F)}{\partial \beta}$
- Latent heat L = discontinuity in internal energy
- Multiple equilibrium states at T=T_c:

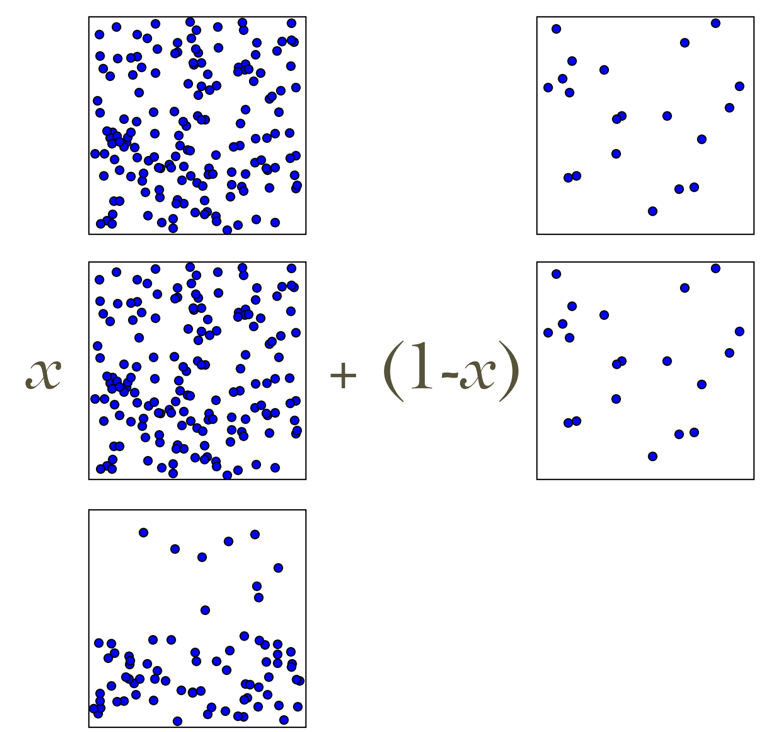




convex polytope

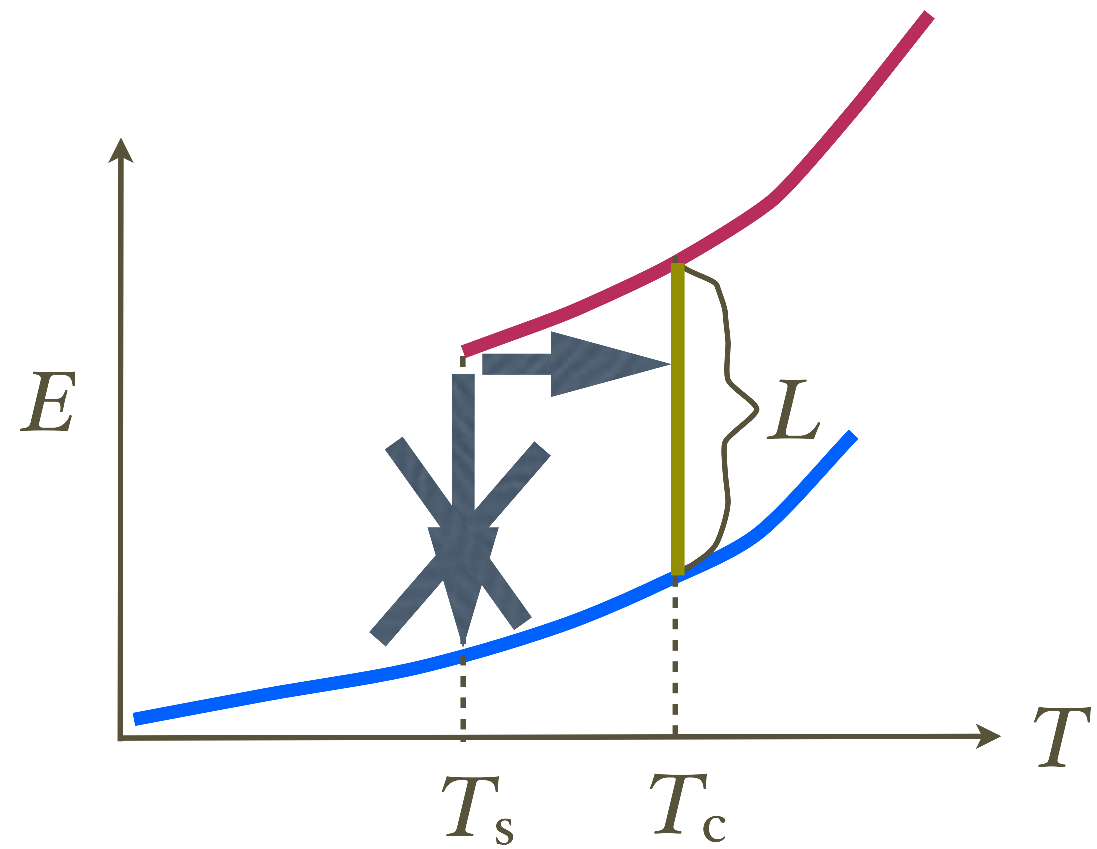
physically realizable, satisfy cluster decomposition

- extremal, homogeneous
- non-extremal, mixed
- extremal, phase separated



Cooling dynamics

- E, T, S all decrease upon cooling
- T=T_c: enter metastable supercooled phase
- T=T_s: spinodal decomposition = limit of metastability
- Re-equilibrate at fixed energy to *phase separated* state at T=T_c provided $E^+(T_s) > E^-(T_c)$



Small black holes & holography

- $\rho^* < \rho_h < 1$: supercooled plasma, stable at $N_c = \infty$
- $\rho_h = \rho^*$: spinodal decomposition threshold
- $\rho_h < \rho^*$: dynamical instability leads to ???

Does system re-equilibrate to new stationary solution with broken $SO(6)_R$ symmetry?

- "Lumpy" $S^3 \times S^5$ horizon topology BH solutions have lower entropy

  O. Dias, J. Santos, B. Way

- Common expectation: BH should undergo Gregory-Laflamme-like instability, develop thin "necks" which break at string scale, settle down to localized S^8 horizon topology BH
- Recent S^8 BH solutions, localized on S^5, have higher entropy but $T > T_c$

  O. Dias, J. Santos, B. Way

Interpretation:

- Can "microcanonical phase diagram" be distinct from canonical?
  - Not in thermodynamic limit:

$$\lim_{N_c \to \infty} \left\langle (\delta T)^2 \right\rangle_{\text{microcanonical}} = \lim_{N_c \to \infty} N_c^{-2} \left\langle (\delta E)^2 \right\rangle_{\text{canonical}} = 0$$

  - Microcanonical & canonical descriptions must be mutually consistent
- Is understanding of $T > T_c$ canonical ensemble completely wrong?
  - Does broken R-symmetry equilibrium phase exist?

no sign, no evidence

- Do extremal phase separated states exist at T_c?
  - No present in weak coupling analysis, $\lambda \ll 1$
  - No "eigenvalue locality", unlike spatial interactions

no sign, no evidence

Re-equilibration possibilities:

- $T_\mathrm{f} > T_\mathrm{c}$?
  - Impossible: insufficient energy, $E(T_\mathrm{s}) < E(T)$ for all $T > T_\mathrm{c}$
- $T_\mathrm{f} < T_\mathrm{c}$?
  - Impossible: excessive energy, $E(T_\mathrm{s}) = \mathcal{O}(N^2) \gg E(T)$ for all $T < T_\mathrm{c}$
- $T_\mathrm{f} = T_\mathrm{c}$?
  - Impossible, unless phase separated extremal equilibrium states exist

Occam's razor:

- Instabilities of small black holes must fail to re-equilibrate
  - Consistent with basic large N_c lore:
    - $N \to \infty$ dynamics of quantum states satisfying large N factorization = classical dynamical system
    - Expect both regular and chaotic regions in phase space
    - Chaotic dynamics need not imply "equilibrating"
  - Finite, closed system, no dissipation
    - Small BH cannot "evaporate away" leaving nothing
- Failure to equilibrate should be testable via GR numerics

Time dependent dynamics?

- Time evolve small BH instability:
  - 10D GR + self dual 5-form, SO(4) × SO(5) invariant geometry

    $$ds^2 = -2A\,dt^2 + \Sigma^2\left(e^{B+H}\,d\alpha^2 + e^{H}\,dS_4^2 + e^{-\frac{5}{3}H-\frac{1}{3}B}\,dS_3^2\right) - 2F\,dt\,d\alpha + 2dtdr$$

    - 10D Einstein equations → 3D PDEs
  - Apply characteristic formulation & numerical methods successfully used to study holographic collisions in 5D GR
    - Hope to follow time evolution, at least for some limited period
  - Project (w. Paul Chesler & Alex Buchel) proceeding slowly
  - No results to show (yet), 😢

Excuses:

- Multiple infinite towers of scalar condensates in addition to $T_{\mu\nu}$
- Complex boundary asymptotics, subtle initial value constraints
  - Dimension 2 condensates ➠ unknown coefficients appear in low-order singular terms of near-boundary expansion
  - Difficult to find subtraction/redefinition scheme which avoids unacceptable precision loss near boundary
- Significantly more challenging than expected/hoped
  - Have not yet succeeded in achieving stable evolution
- Distracted by other projects…

Conclusions:

- Understanding small black holes in global AdS remains a puzzle
- Only plausible scenario is that unstable small black holes fail to thermalize
- Absence of thermalization in states with $\mathcal{O}(N^2)$ energy, despite strong coupling, is reminiscent of recent work on “many body localization”
    - Is there any real connection?
- Interesting case where QFT side information & basic statistical mechanics helps understand classical GR
- Numerics remains challenging, despite symmetry simplifications
    - Is there a better approach?