Neutron star properties from first principles

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Canterbury Tales of Hot QFTs in the LHC Era

Oxford, 14.7.2017

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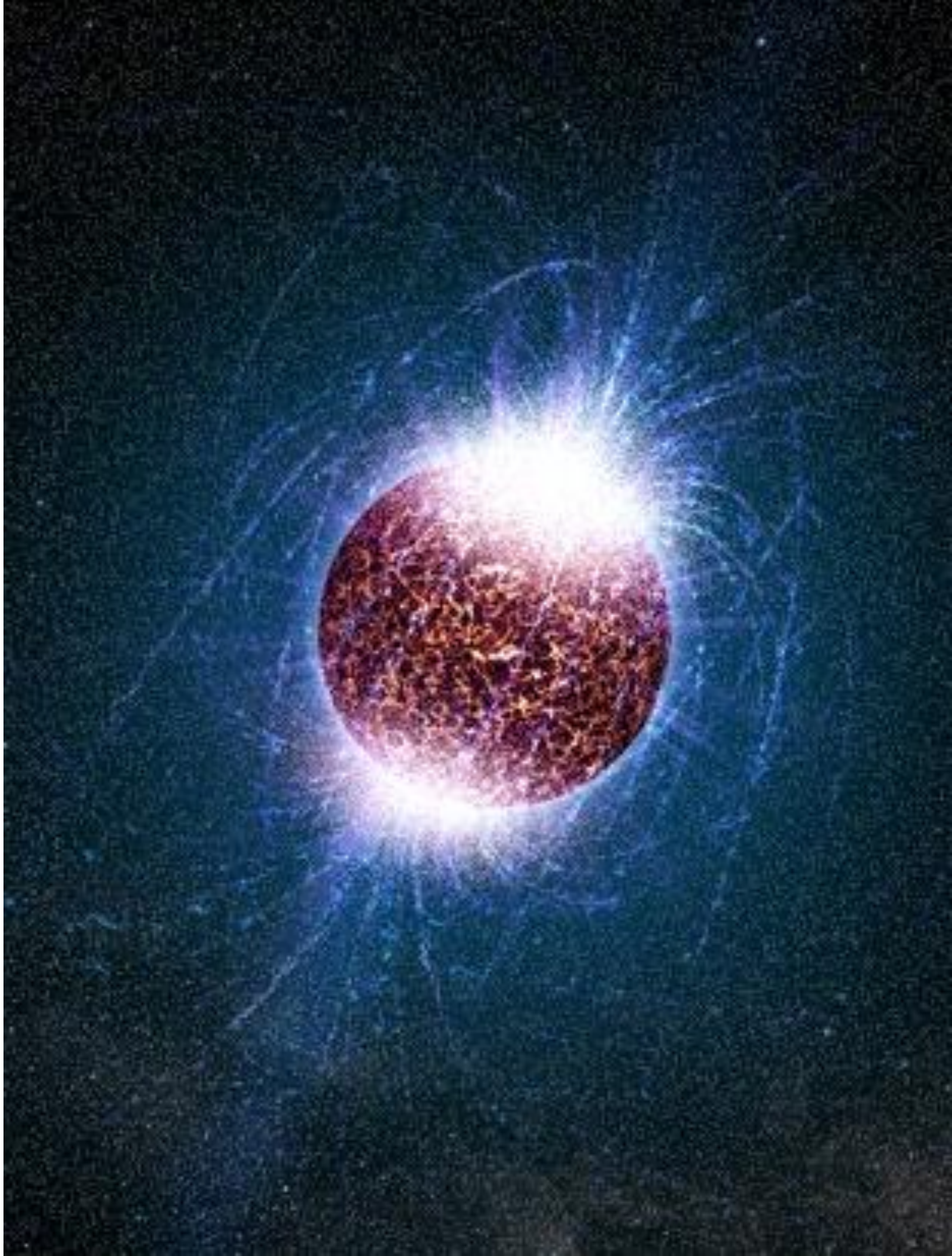


Perturbative QCD at finite density:

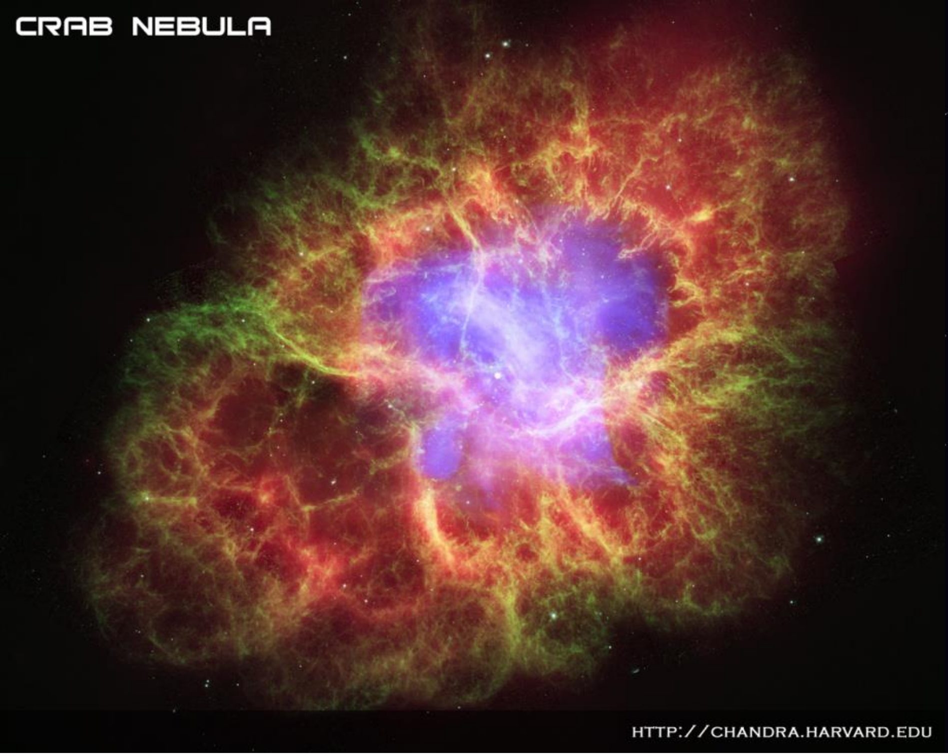
- AV, PRD 68 (2003)
- A. Kurkela, P. Romatschke, AV, PRD 81 (2010)
- A. Kurkela, AV, PRL 117 (2016)
- I. Ghisoiu, T. Gorda, A. Kurkela, P. Romatschke, M. Säppi, AV, Nucl. Phys. B915 (2017)
- I. Ghisoiu, T. Gorda, A. Kurkela, P. Romatschke, M. Säppi, AV, In preparation

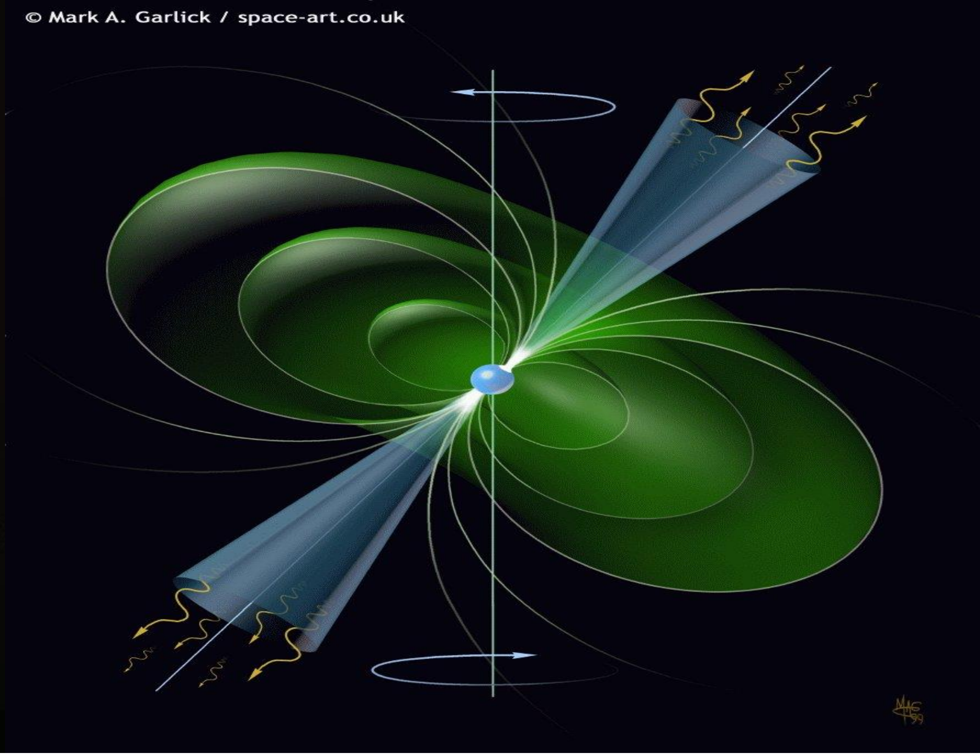
Holographic quark matter:

- C. Hoyos, N. Jokela, D. Rodriguez Fernandez, AV, PRL 117 (2016)
- C. Hoyos, N. Jokela, D. Rodriguez Fernandez, AV, PRD 94 (2016)
- C. Ecker, C. Hoyos, N. Jokela, D. Rodriguez Fernandez, AV, 1707.00521
- E. Annala, C. Ecker, C. Hoyos, N. Jokela, D. Rodriguez Fernandez, AV, In preparation

Application to neutron stars:

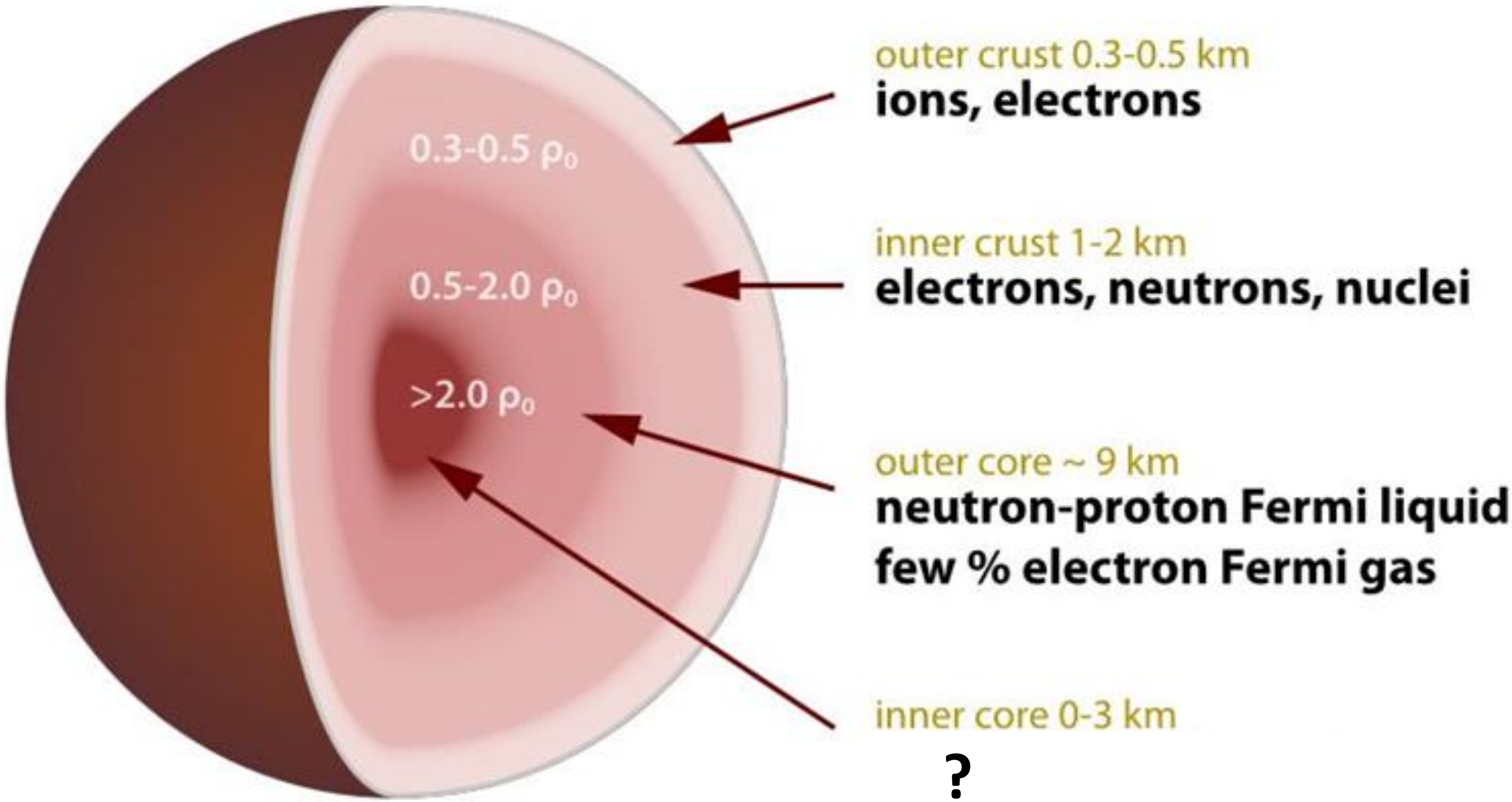
- E. Fraga, A. Kurkela, AV, Astrophys. J. 781 (2014)
- A. Kurkela, E. Fraga, J. Schaffner-Bielich, AV, Astrophys. J. 789 (2014)
- E. Annala, J. Nättilä, A. Kurkela, AV, In preparation



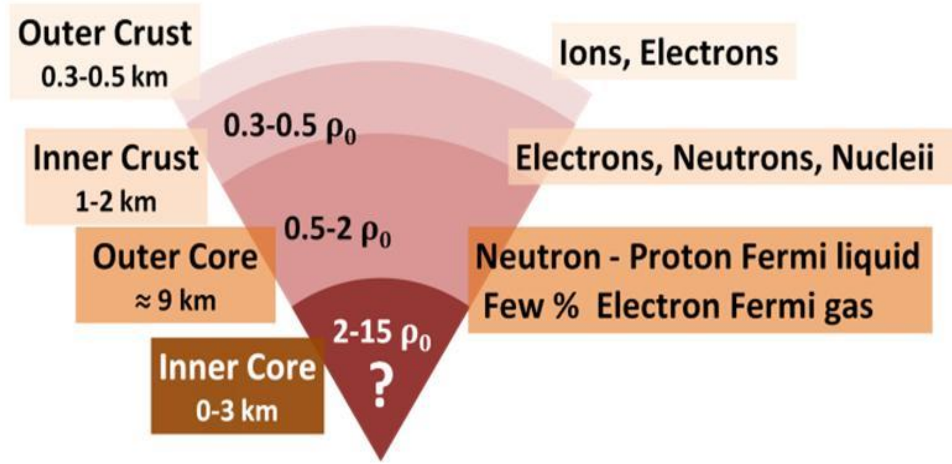


When a hydrogen burning star runs out of fuel:

- $\mathrm{M} \lesssim 9M_{\mathrm{sun}}$ ⇒ White dwarf
- $\mathrm{M} \gtrsim 9M_{\mathrm{sun}}$ ⇒ Supernova explosion
    - $\mathrm{M} \gtrsim 20M_{\mathrm{sun}}$ ⇒ Gravitational collapse into BH
    - $\mathrm{M} \lesssim 20M_{\mathrm{sun}}$ ⇒ Gravitational collapse into…

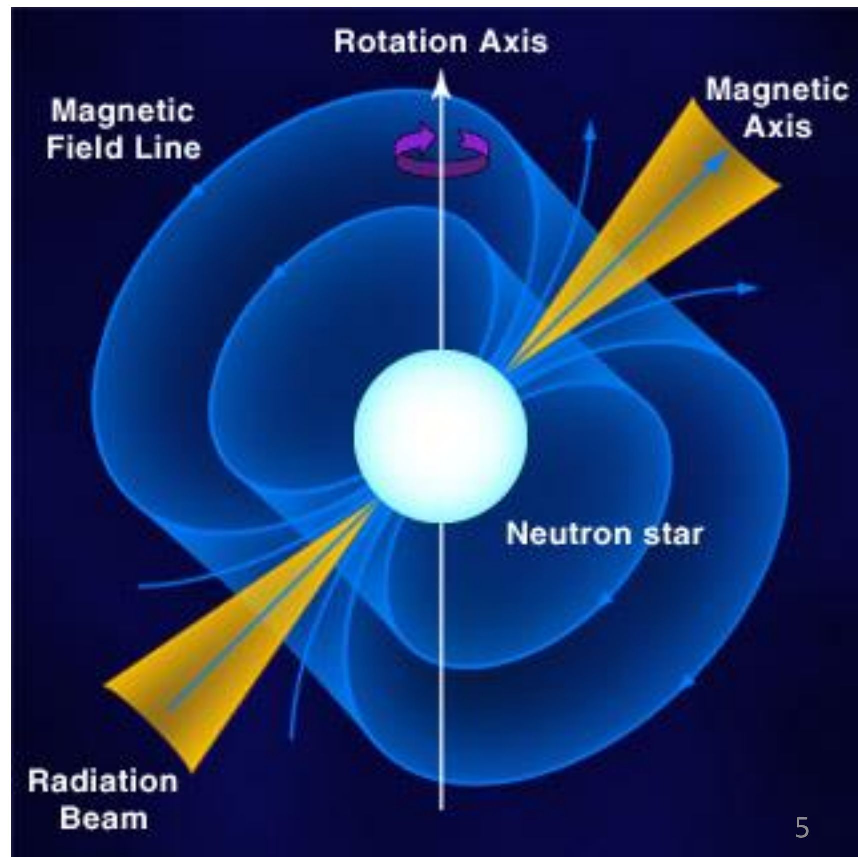

0.3-0.5 ρ_0
0.5-2.0 ρ_0
>2.0 ρ_0
outer crust 0.3-0.5 km
ions, electrons
inner crust 1-2 km
electrons, neutrons, nuclei
outer core ~ 9 km
neutron-proton Fermi liquid
few % electron Fermi gas
inner core 0-3 km
?

NSs unique lab for strong interaction physics: Can we understand their observable properties from first principles, i.e. from QCD?

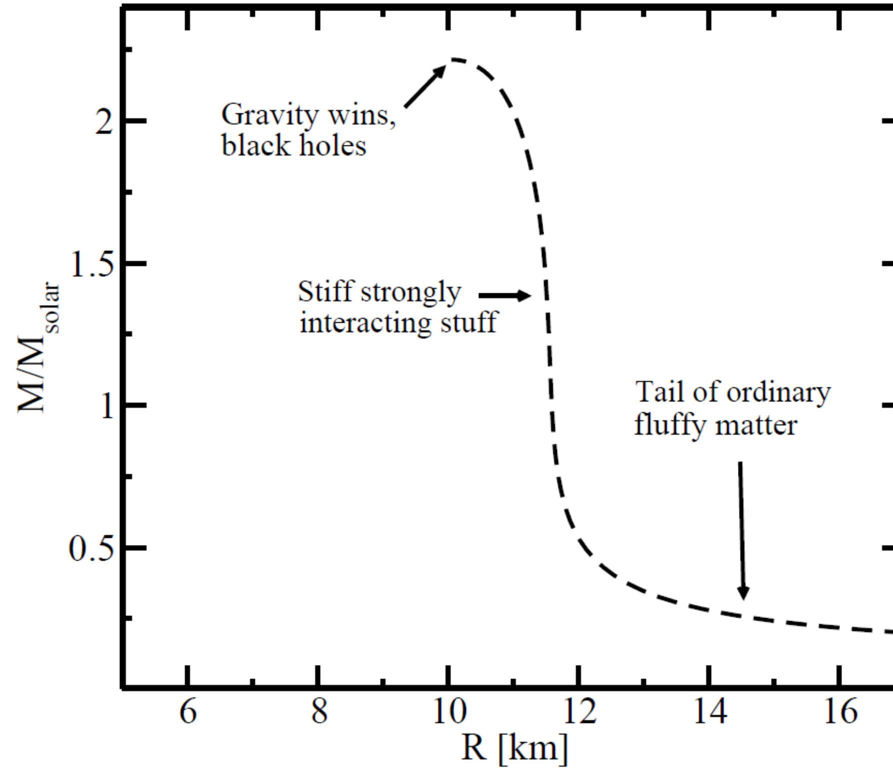


Main characteristics:

- Masses $\lesssim 2M_{\text{sun}}$
- Radii $\approx 12-13$ km
- Spin frequencies $\lesssim$ kHz
- Temperatures $\lesssim$ keV



Physics picture: hydrostatic equilibrium resulting from fierce competition between gravity and the pressure of QCD matter



GR description via Tolman-Oppenheimer-Volkov eqs:

$$\begin{aligned} \frac{dM(r)}{dr} &= 4\pi r^2 \varepsilon(r), \\ \frac{dp(r)}{dr} &= -\frac{G\varepsilon(r)M(r)}{r^2} \frac{(1+p(r)/\varepsilon(r))\left(1+4\pi r^3 p(r)/M(r)\right)}{1-2GM(r)/r} \end{aligned}$$

$$\varepsilon(p) \Rightarrow M(R)$$

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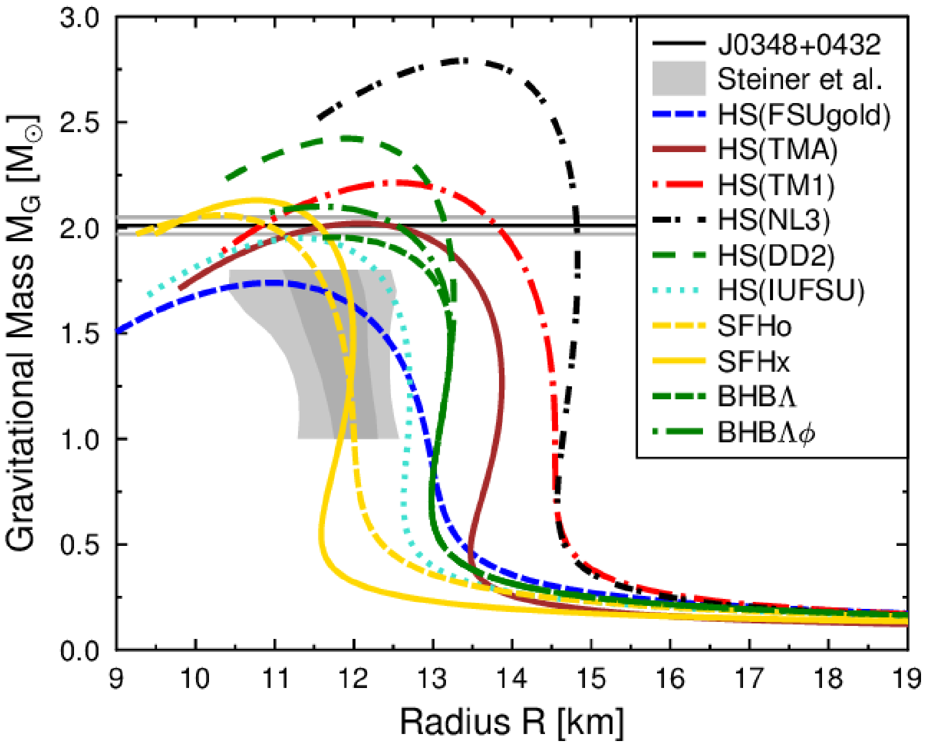

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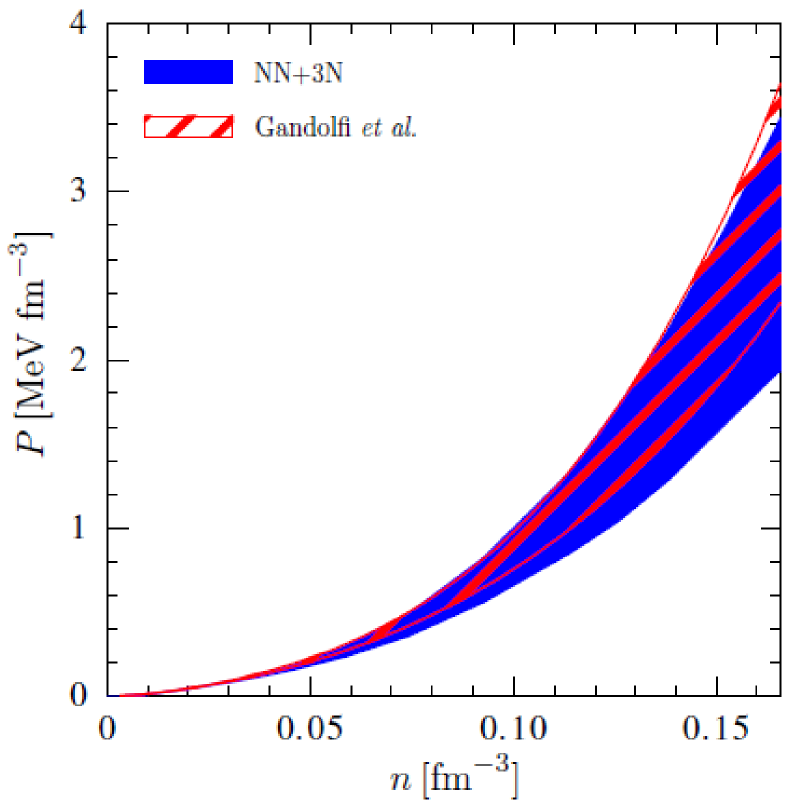
$$\varepsilon(p) \Rightarrow M(R)$$

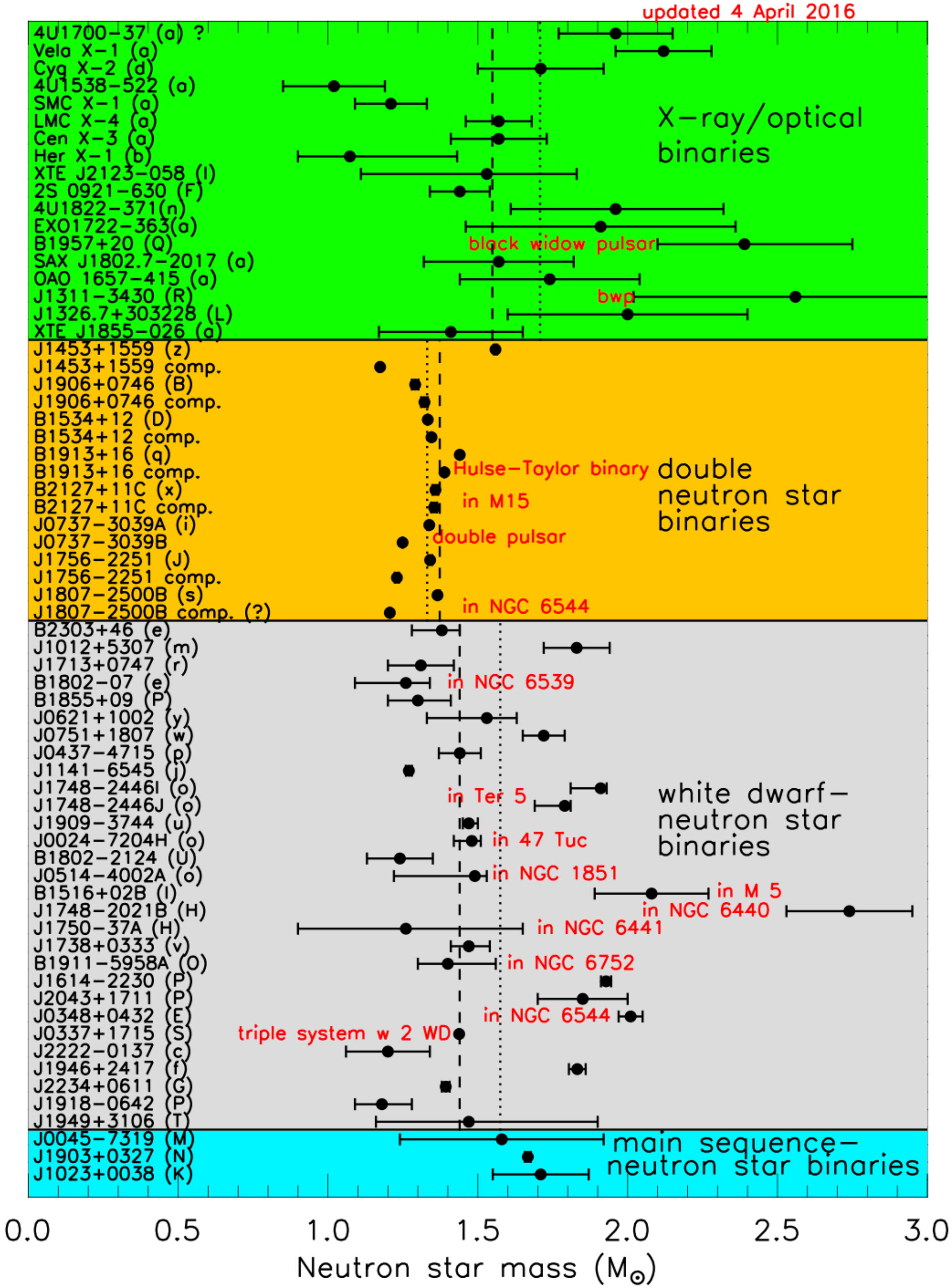
Particle/nuclear theory challenge: find Equation of State of strongly interacting matter that is

- Cold and dense
- Electrically neutral:
  $2/3n_u - n_d/3 - n_s/3 + n_e = 0$
- In beta equilibrium:
  $\mu_B/3 = \mu_d = \mu_s = \mu_u + \mu_e$



Main questions:

- Can we predict future MR measurements?
- Can we infer the QCD matter EoS from observations?
- **Can deconfined matter be found inside the stars?**

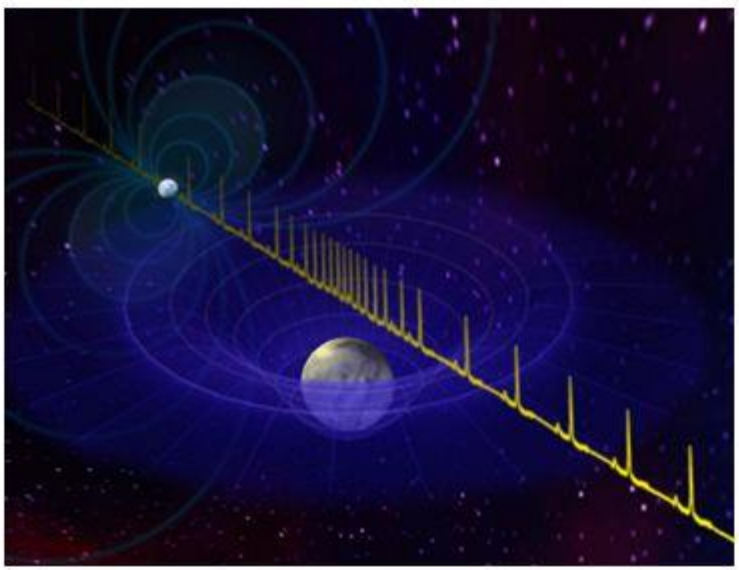

updated 4 April 2016
4U1700−37 (a) ?
Vela X−1 (a)
Cyg X−2 (d)
4U1538−522 (a)
SMC X−1 (a)
LMC X−4 (a)
Cen X−3 (a)
Her X−1 (b)
XTE J2123−058 (I)
2S 0921−630 (F)
4U1822−371(n)
EXO1722−363(a)
B1957+20 (Q)
SAX J1802.7−2017 (a)
OAO 1657−415 (a)
J1311−3430 (R)
J1326.7+303228 (L)
XTE J1855−026 (a)
X−ray/optical binaries
black widow pulsar
bwp
J1453+1559 (z)
J1453+1559 comp.
J1906+0746 (B)
J1906+0746 comp.
B1534+12 (D)
B1534+12 comp.
B1913+16 (q)
B1913+16 comp.
B2127+11C (x)
B2127+11C comp.
J0737−3039A (i)
J0737−3039B
J1756−2251 (J)
J1756−2251 comp.
J1807−2500B (s)
J1807−2500B comp. (?)
Hulse−Taylor binary
in M15
double pulsar
in NGC 6544
double neutron star binaries
B2303+46 (e)
J1012+5307 (m)
J1713+0747 (r)
B1802−07 (e)
B1855+09 (P)
J0621+1002 (y)
J0751+1807 (w)
J0437−4715 (p)
J1141−6545 (j)
J1748−2446I (o)
J1748−2446J (o)
J1909−3744 (u)
J0024−7204H (o)
B1802−2124 (U)
J0514−4002A (o)
B1516+02B (I)
J1748−2021B (H)
J1750−37A (H)
J1738+0333 (v)
B1911−5958A (O)
J1614−2230 (P)
J2043+1711 (P)
J0348+0432 (E)
J0337+1715 (S)
J2222−0137 (c)
J1946+2417 (f)
J2234+0611 (G)
J1918−0642 (P)
J1949+3106 (T)
in NGC 6539
in Ter 5
in 47 Tuc
in NGC 1851
in M 5
in NGC 6440
in NGC 6441
in NGC 6752
in NGC 6544
triple system w 2 WD
white dwarf−neutron star binaries
J0045−7319 (M)
J1903+0327 (N)
J1023+0038 (K)
main sequence−neutron star binaries
0.0
0.5
1.0
1.5
2.0
2.5
3.0
Neutron star mass ($M_\odot$)

LETTER

doi:10.1038/nature09466

A two-solar-mass neutron star measured using Shapiro delay

P. B. Demorest[1], T. Pennucci[2], S. M. Ransom[1], M. S. E. Roberts[3] & J. W. T. Hessels[4,5]



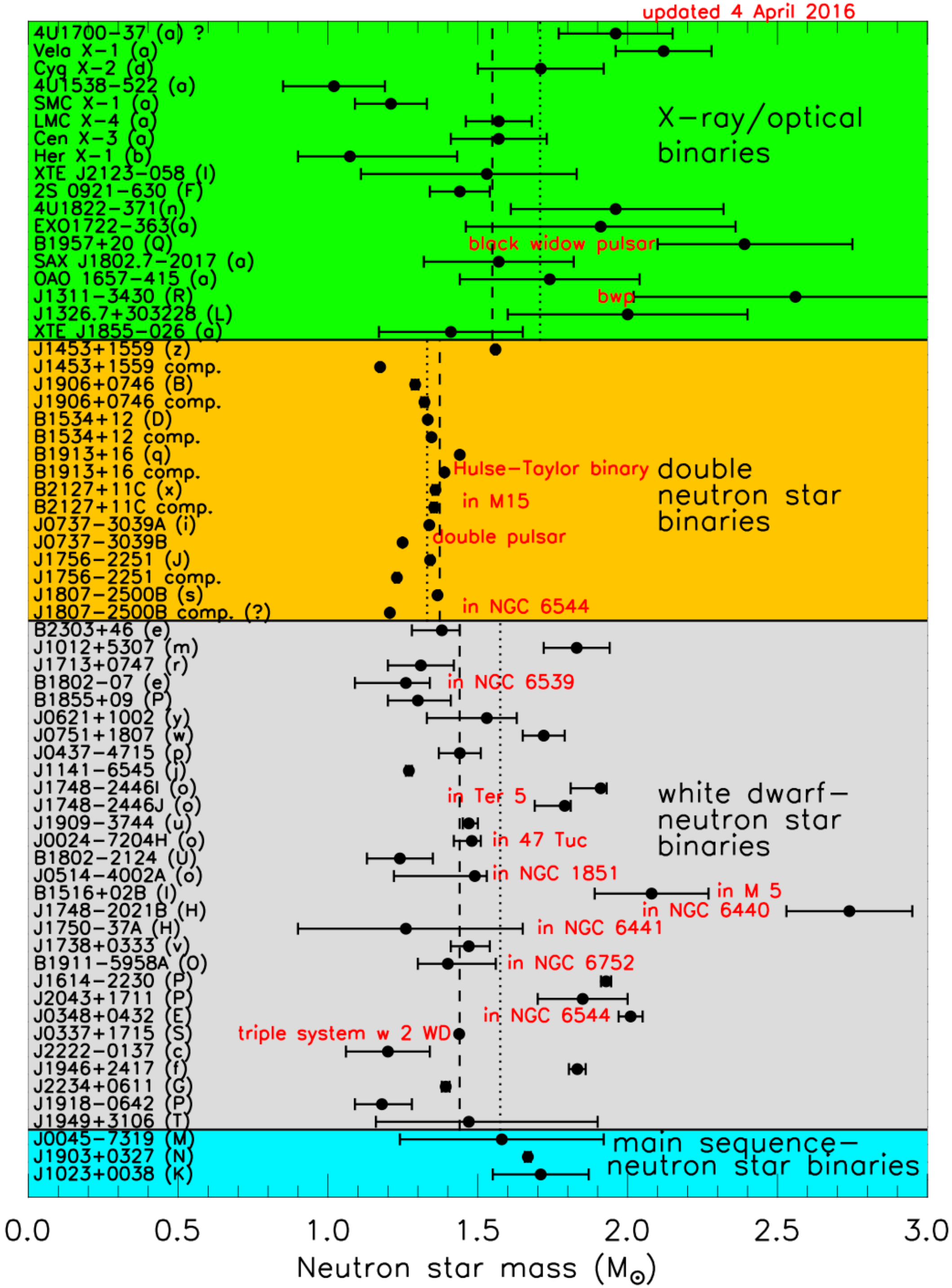
Nature 467, 1081 (Oct. 28, 2010)

PSR J1614-2230

(Millisecond Pulsas & White Dwarf Binary)

1.97 ± 0.04 Msun

(measurement based on Shapiro delay)



By now, two accurate Shapiro delay measurements of two solar mass stars:

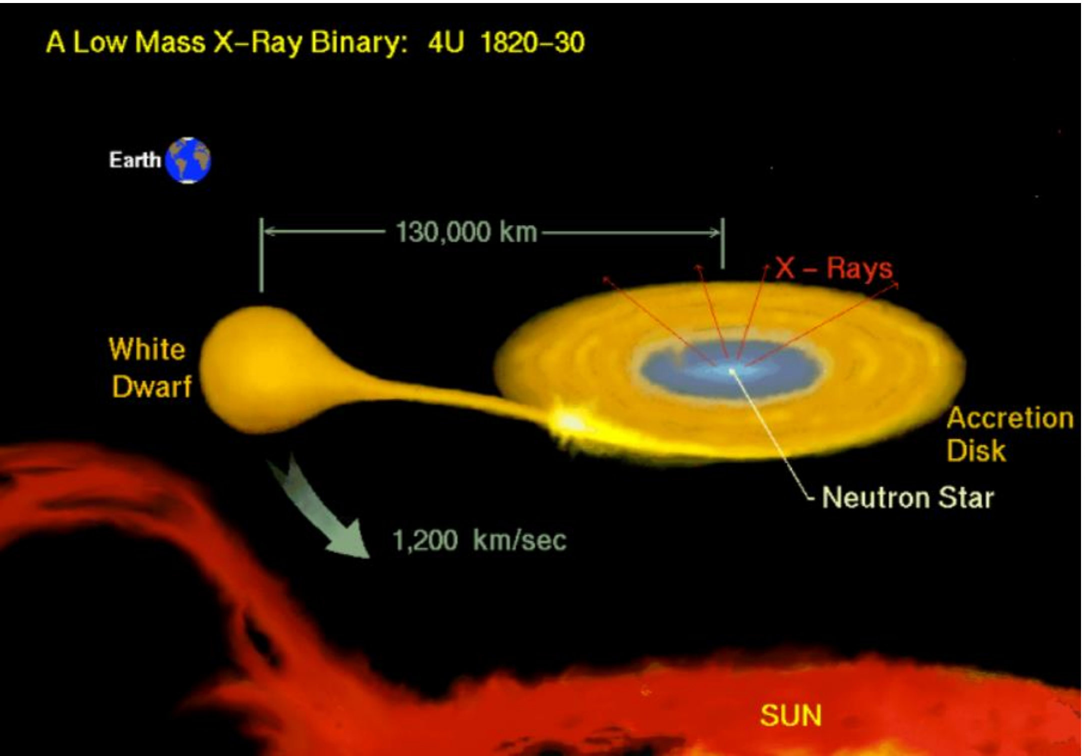
Demorest et al. (2010)

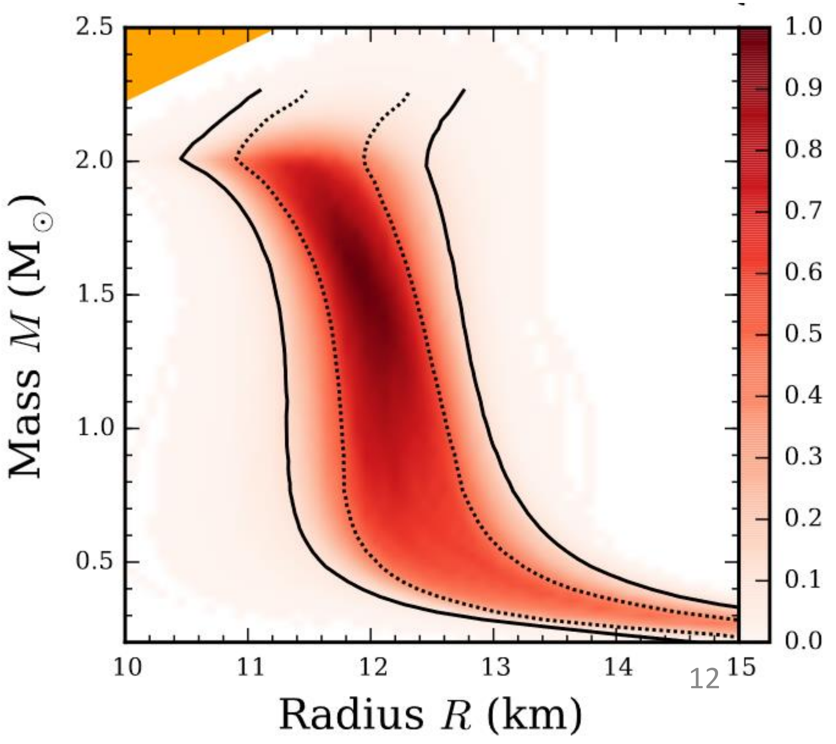
Antoniadis et al. (2013)

$$\therefore \quad M_{\max} > 2M_{\text{sun}}$$

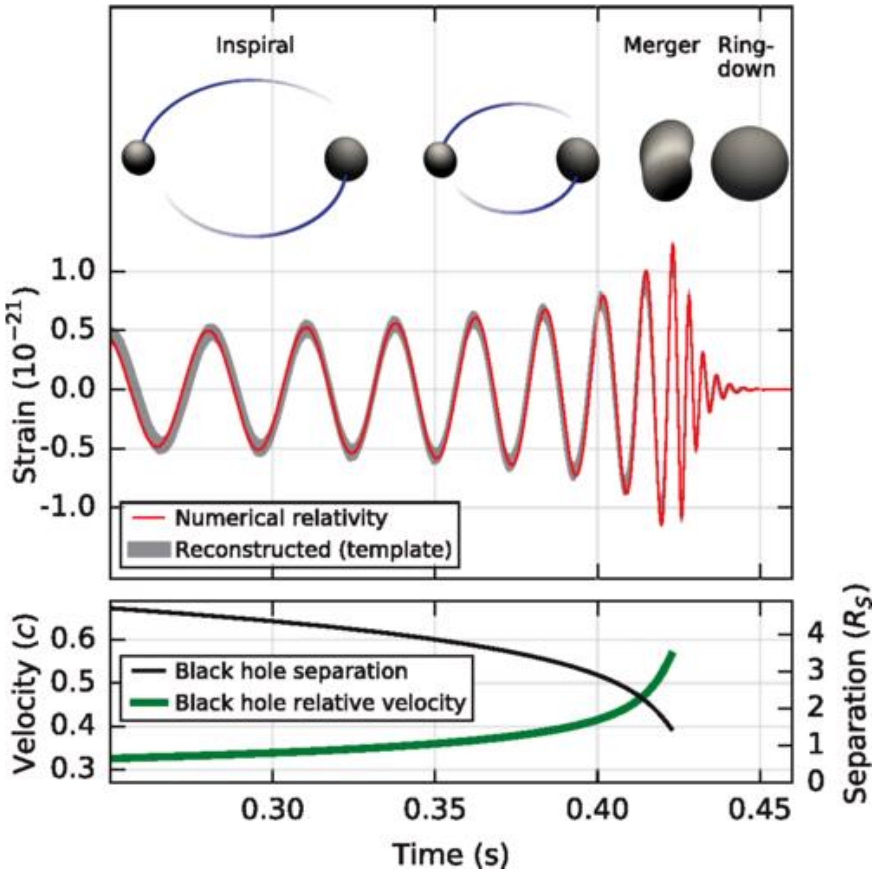
Radius measurements more problematic. Lately increasing precision via cooling of thermonuclear X-ray bursts from NS – white dwarf binaries where NS accretes matter.

E.g. Steiner et al. (2010), Nättilä et al. (2015), ...

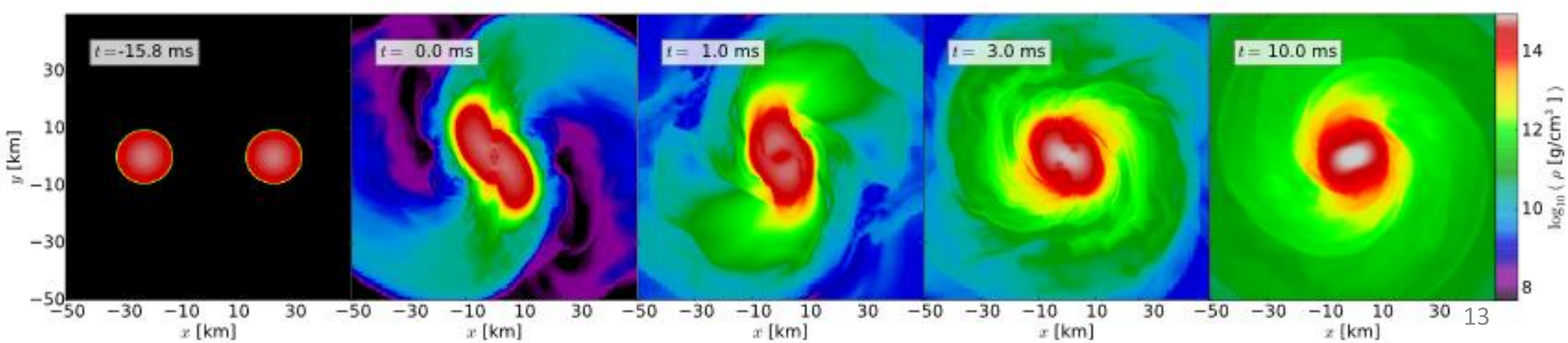




Breakthrough in gravitational wave detection: LIGO observation of BH mergers >1 billion light years away



Future observations of NS mergers potentially ground-breaking: ringdown pattern sensitive to EoS



I. Neutron star Equation of State: Status and quark matter challenge
II. New developments in pQCD: Thermal effects and more loops
III. Strongly coupled quark matter from holography
IV. Final thoughts

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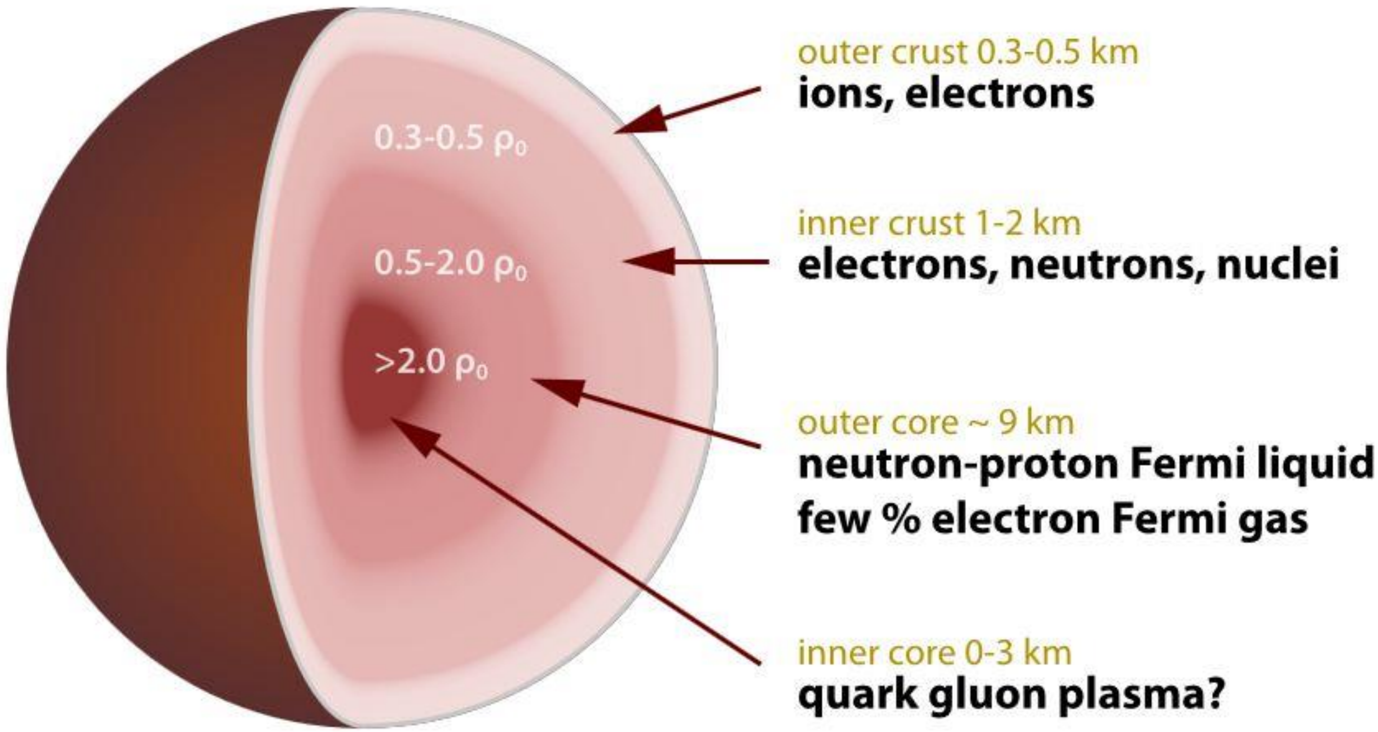
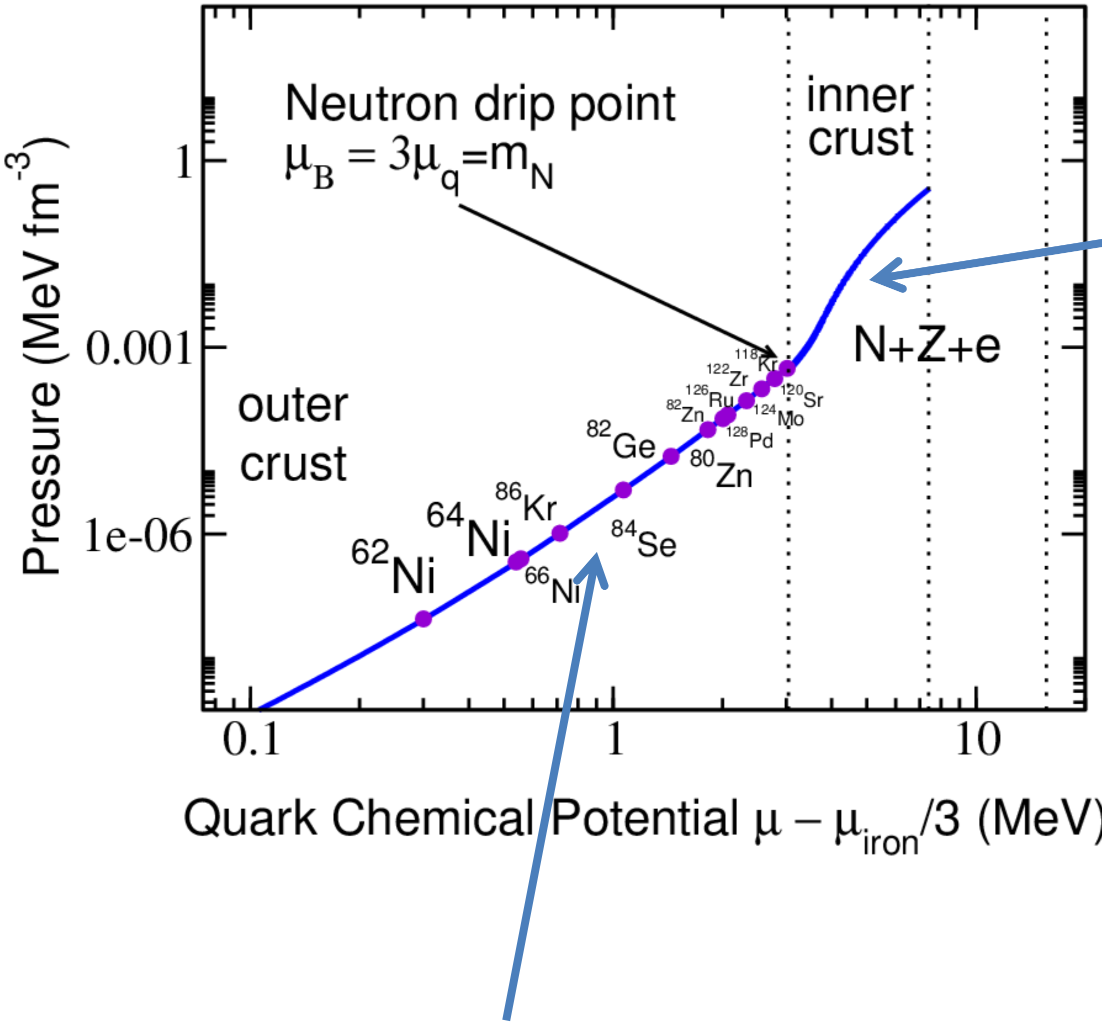


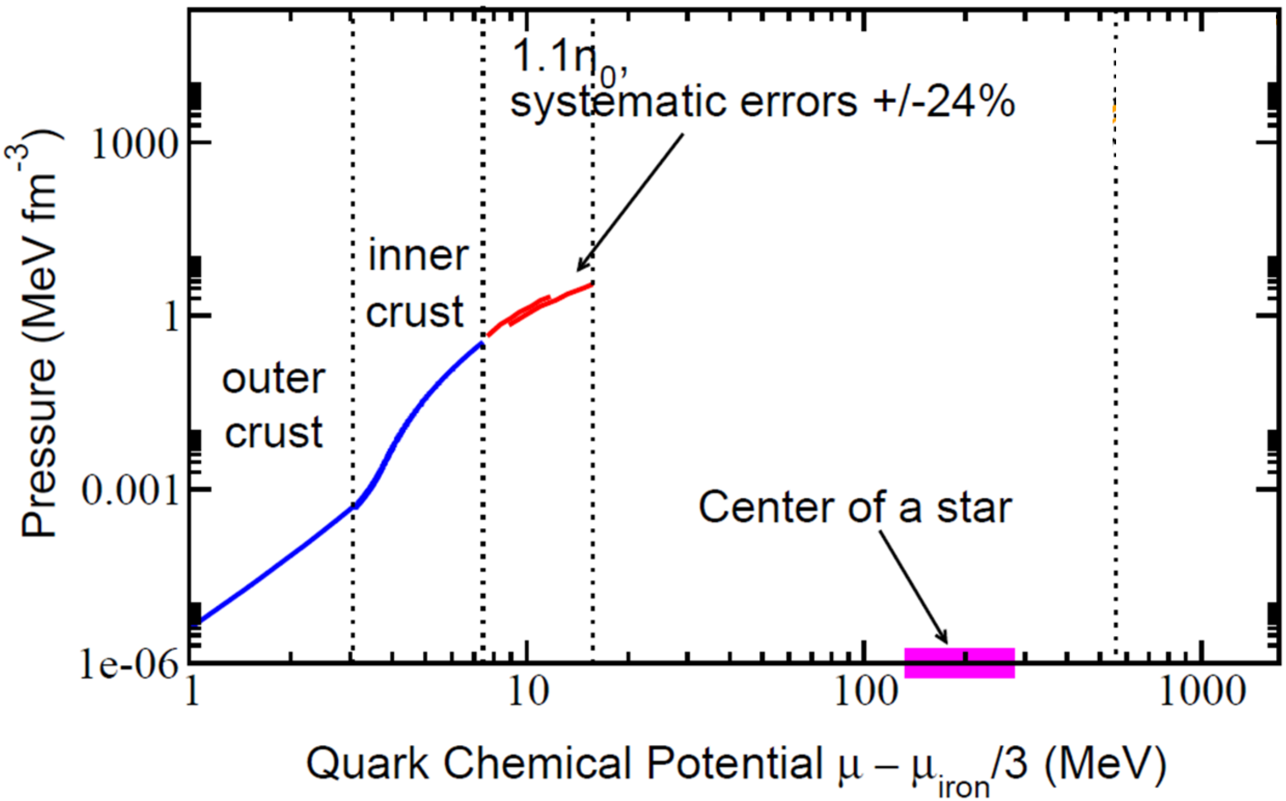
Proceeding inwards from the crust:

- μ_B increases gradually, starting from μ_{Fe}
- Baryon/mass density increases from 0 to beyond $n_s \equiv \rho_0 \approx 0.16/\text{fm}^3 \approx 2 \times 10^{14}\text{g/cm}^3$
- Composition changes from nuclei to neutrons/quarks



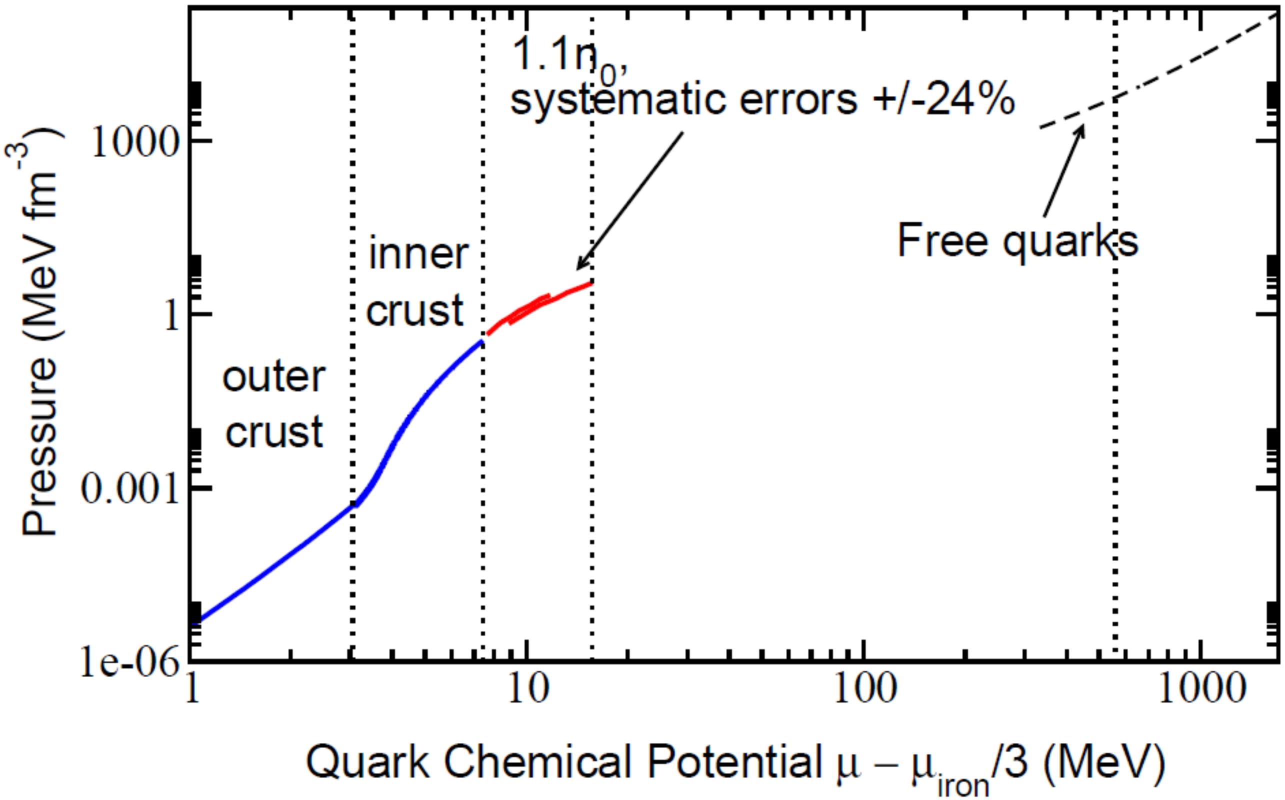
- Neutron gas with nuclei and electrons
- NN interactions important for collective properties; modeled via experimentally highly constrained potential models
- Eventually need 3N interactions, boost corrections,…

- Lattice of increasingly neutron rich nuclei in electron sea; pressure dominated by that of the electron gas
- At zero pressure nuclear ground state $^{56}\mathrm{Fe}$



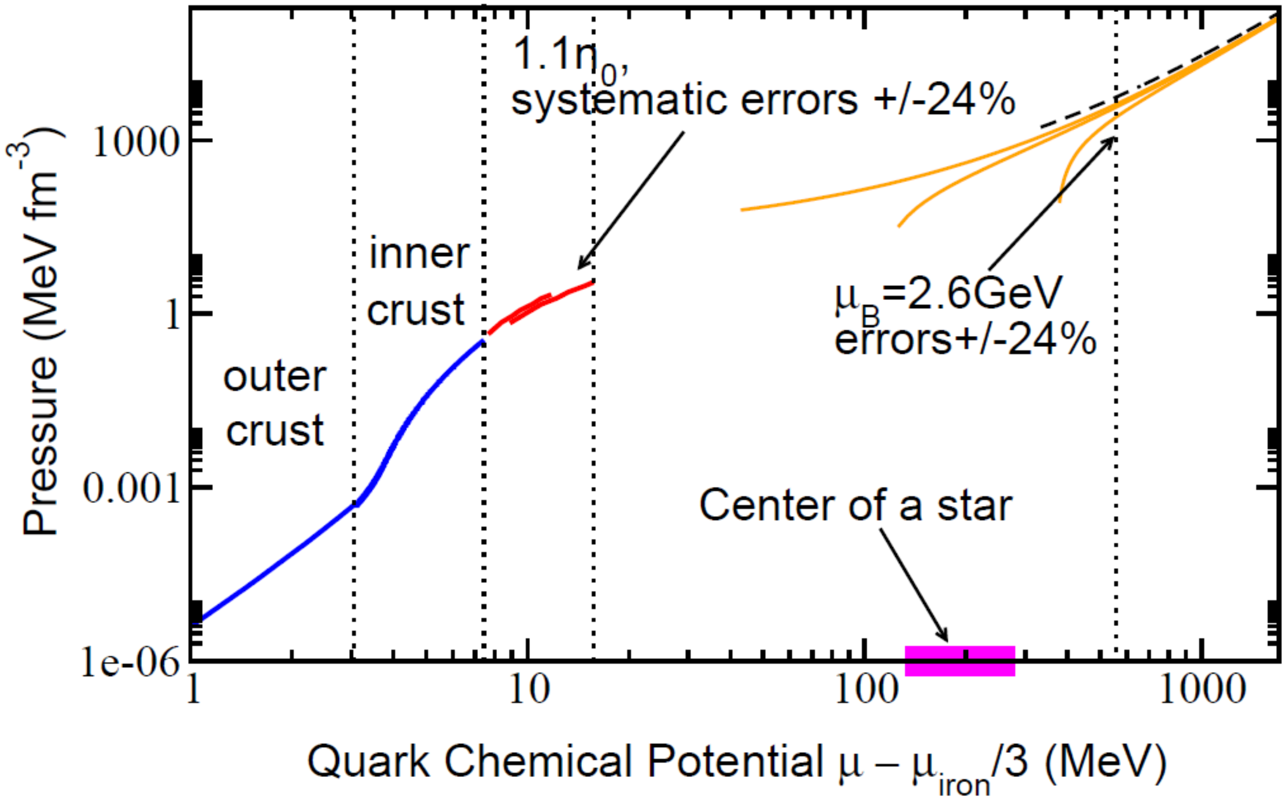
In order to reach (and exceed) nuclear saturation density, need to treat neutron interactions systematically: Chiral Effective Theory

- At $1.1n_s$, current errors $\pm 24\%$ - mostly due to uncertainties in effective theory parameters
- State-of-the-art NNNLO in chiral perturbation theory power counting [Tews et al., PRL 110 (2013), Hebeler et al., APJ 772 (2013)]



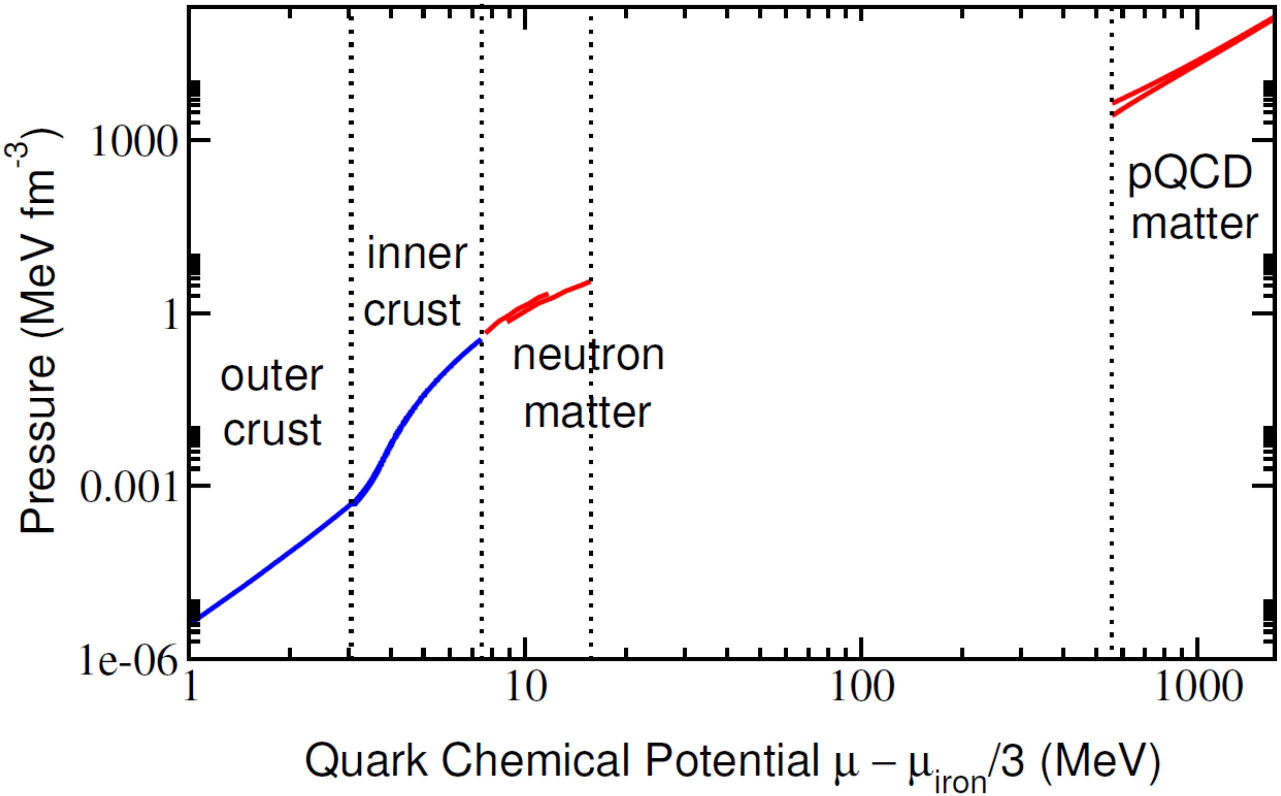
Asymptotic freedom gives asymptotic behavior. However,…

- At interesting densities $(1-15)n_s$ system strongly interacting but no lattice QCD available
- For weak coupling expansions to converge, need to proceed to very high densities



State-of-the-art EoS from perturbative QCD: 3 loops with quark masses [Kurkela, Romatschke, AV, 2009], cf. also [Freedman, McLerran, 1977]

- Uncertainty from renormalization scale dependence
- Result for *unpaired* quark matter; however, pairing contributions subdominant at high densities



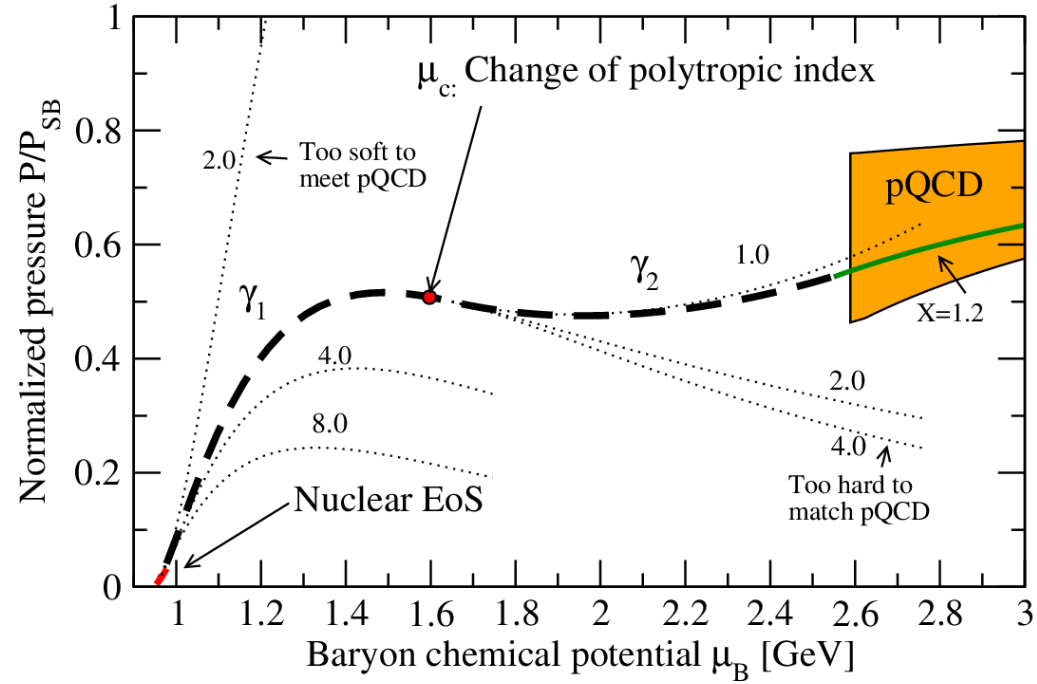
Huge no man's land extending from outer core to densities not realized in physical neutron stars

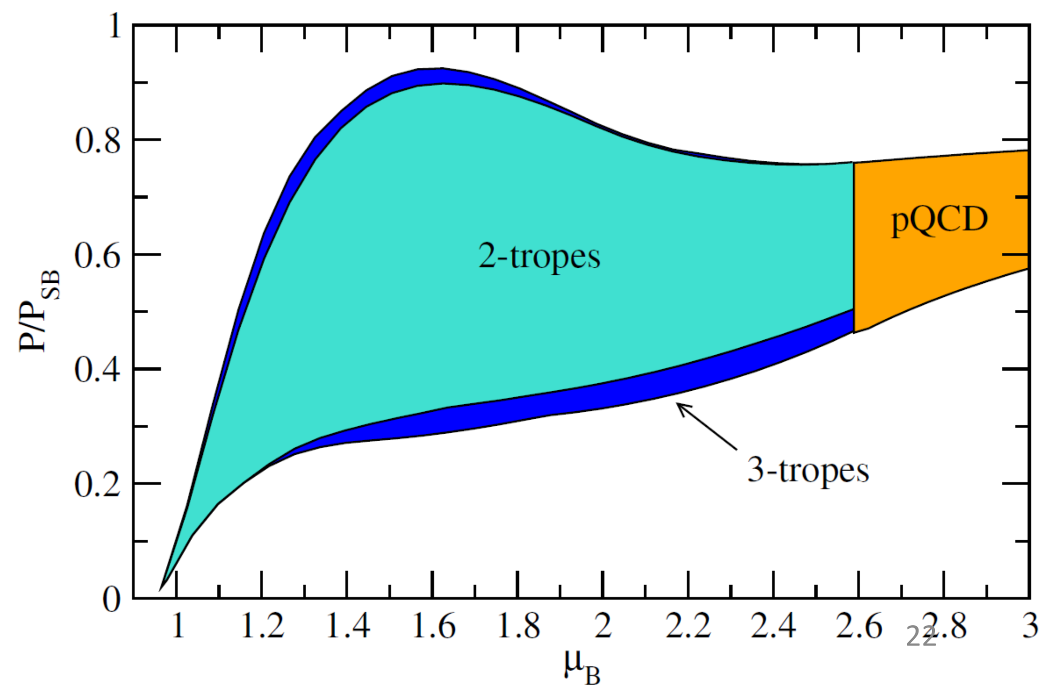
Simplest option: Interpolate EoS between known limits

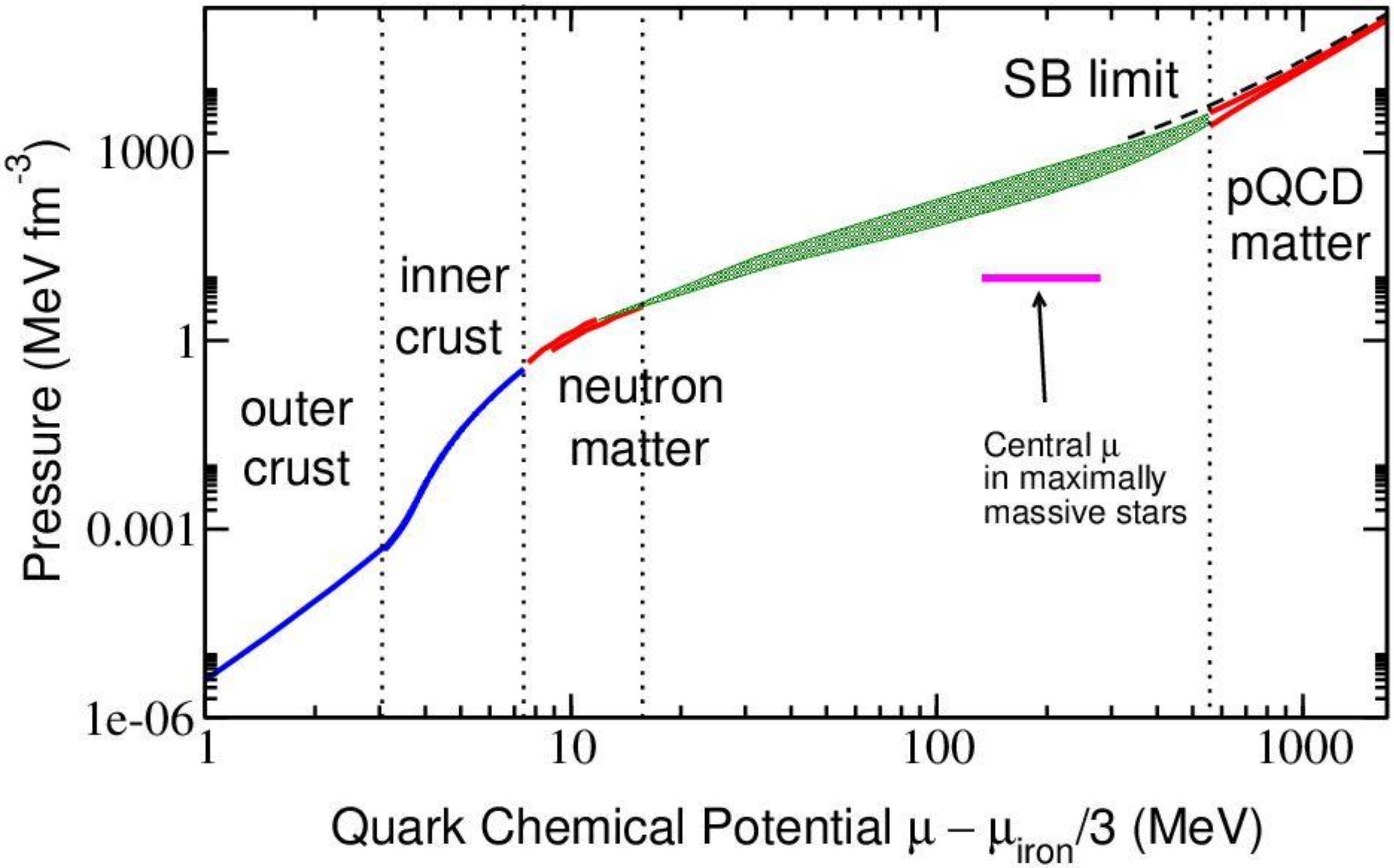
Interpolation using piecewise polytropic EoSs, $p_i(n) = \kappa_i n^{\gamma_i}$, varying all relevant parameters

Require:

1) Smooth matching to nuclear and quark matter EoSs
2) Continuity of p and n when matching monotropes, allowing for one 1st order transition
3) Subluminality
4) Ability to support a two solar mass star





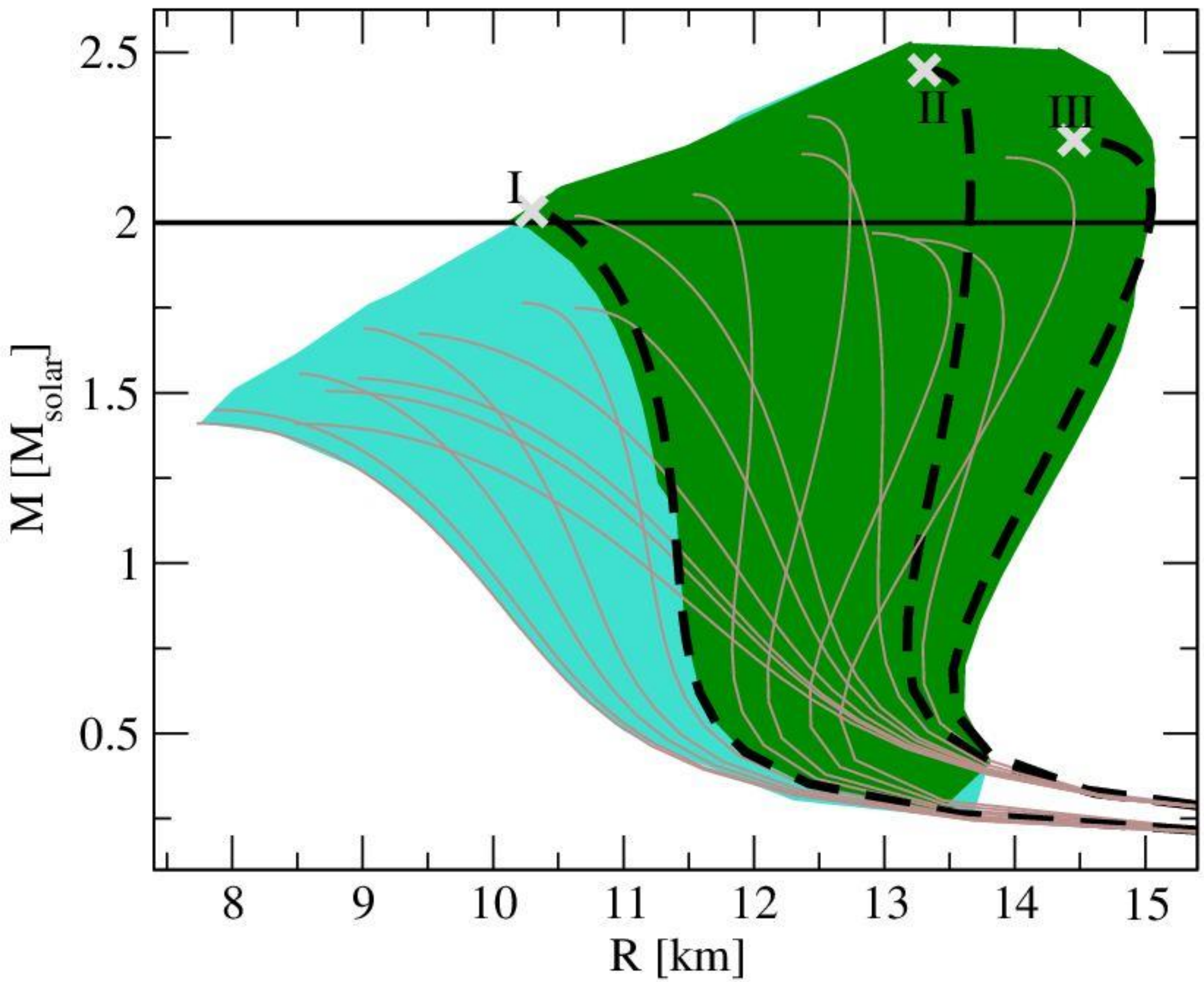


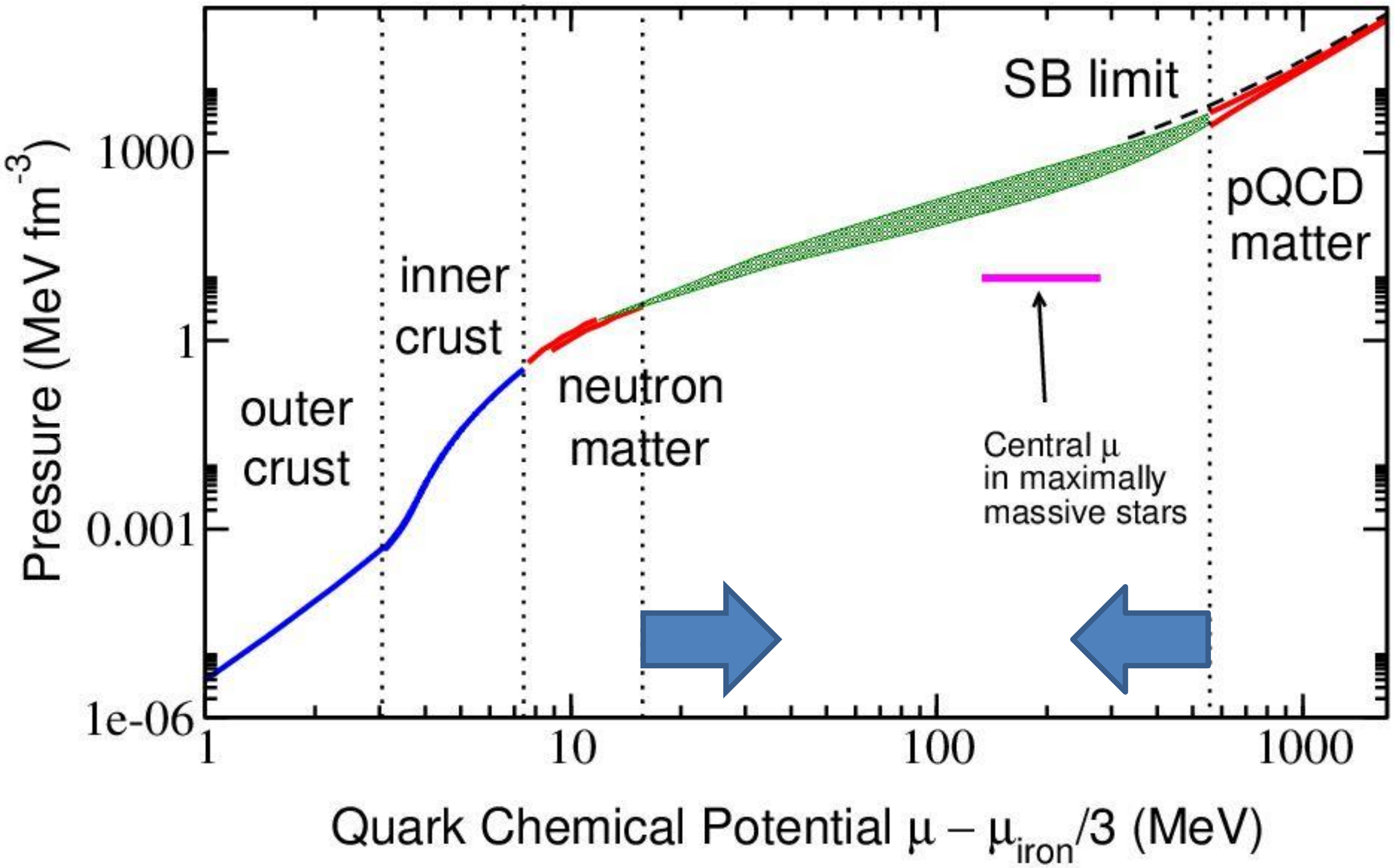
Kurkela, Fraga, Schaffner-Bielich, AV, Astrophys J. 789 (2014)

State-of-the-art EoS at all densities: interpolation between

- CET result for nuclear matter up to saturation density
- pQCD result for quark matter at high densities

Nontrivial insight: *Neutron star EoS constrained by pQCD limit*


2.5
2
1.5
1
0.5
M [M_solar]
8
9
10
11
12
13
14
15
R [km]
I
II
III



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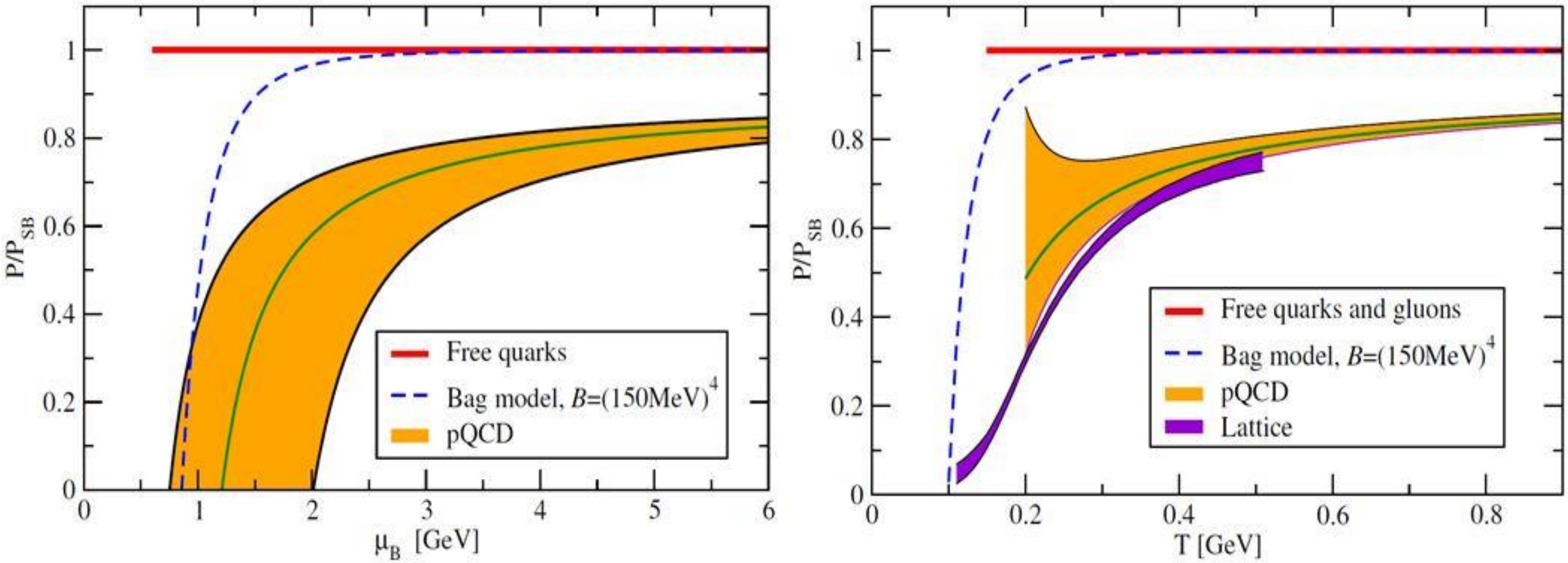

Cold quark matter EoS known to three-loop order, but convergence less than optimal. Therefore need to:

1) Work on extending weak coupling expansion to higher orders [Ghisoiu, Gorda, Kurkela, Romatschke, Säppi, AV]
2) Develop nonperturbative machinery to attack cold quark matter at lower densities [Ecker, Hoyos, Jokela, Rodriguez, AV]

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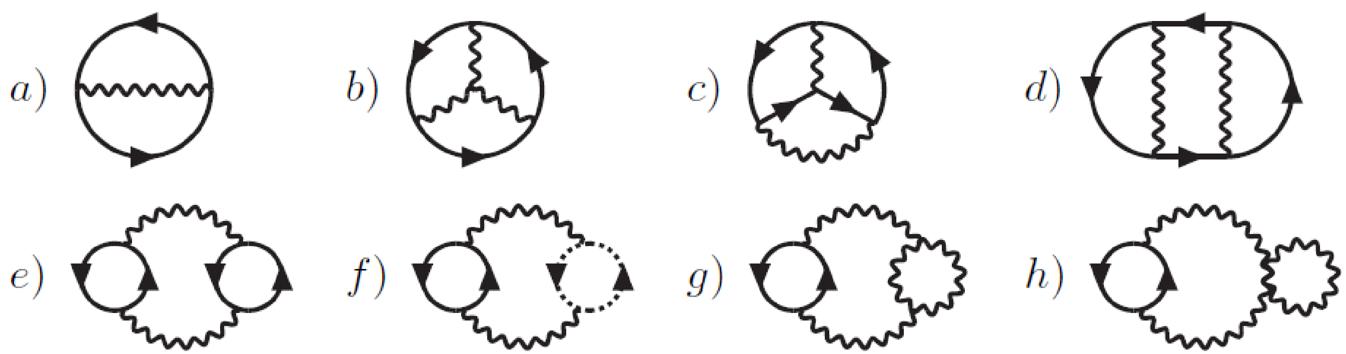
$$\Omega(T,\mu_u,\mu_d,\mu_s,m_s) = -T\log\int \mathcal{D}\bar\psi\mathcal{D}\psi\mathcal{D}A_\mu e^{-\int d^3x\int_0^{1/T}d\tau\mathcal{L}_{\rm QCD}},$$

$$\mathcal{L}_{\rm QCD} = \frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu} + \bar\psi_i(\gamma_\mu D_\mu + m_i - \mu_i\gamma_0)\psi_i$$

Perturbation theory: Expansion of partition function in powers of gauge coupling *g* → Vacuum or bubble diagrams



$$p_n^{\rm bos} = 2\pi nT, \qquad \int\frac{d^{4-2\epsilon}p}{(2\pi)^{4-2\epsilon}} \rightarrow T\sum_{p_n}\int\frac{d^{3-2\epsilon}p}{(2\pi)^{3-2\epsilon}}$$
$$p_n^{\rm ferm} = (2n+1)\pi T - i\mu$$

$$\Omega(T,\mu_u,\mu_d,\mu_s,m_s) = -T\log\int \mathcal{D}\bar{\psi}\mathcal{D}\psi\mathcal{D}A_\mu e^{-\int d^3x\int_0^{1/T} d\tau \mathcal{L}_{\rm QCD}},$$

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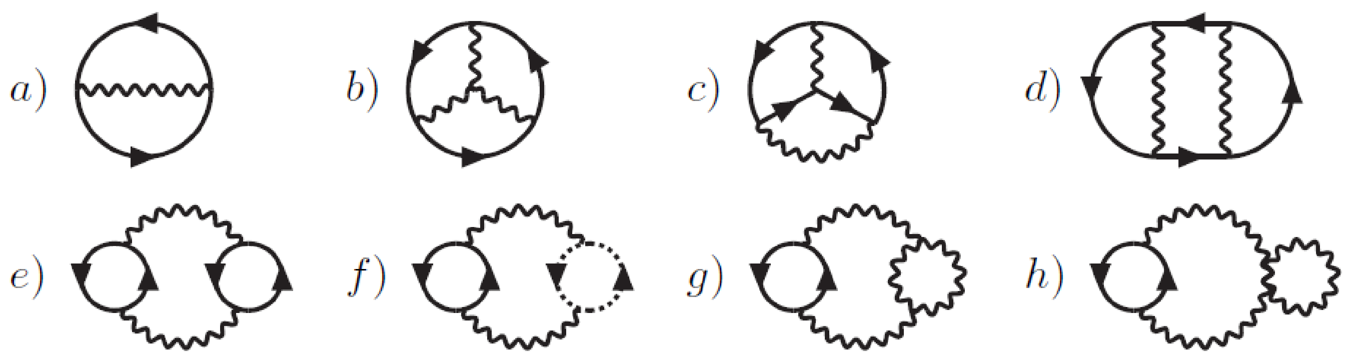
Problem in pQCD: Infrared divergences at three-loop order from long-range (static) gauge fields



$$-\omega^2 + k^2 \to -\omega^2 + k^2 + \Pi(\omega,k)$$

$$\Omega(T,\mu_u,\mu_d,\mu_s,m_s) \;=\; -T\log\int \mathcal{D}\bar{\psi}\mathcal{D}\psi\mathcal{D}A_\mu e^{-\int d^3x\int_0^{1/T} d\tau \mathcal{L}_{\rm QCD}},$$

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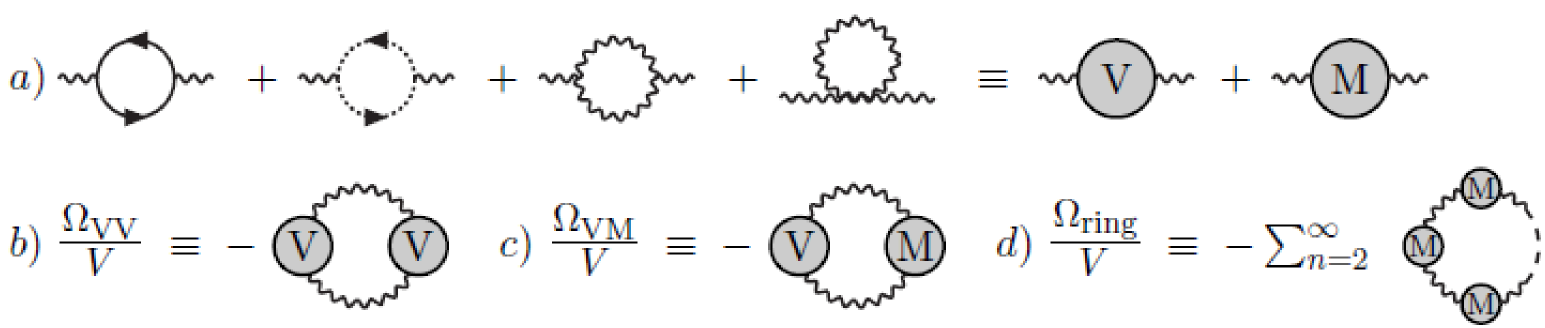
Problem in pQCD: Infrared divergences at three-loop order from long-range (static) gauge fields

Solution: Resummation of IR sensitive contributions to the EoS: Sum certain diagrams to infinite order or use EFT



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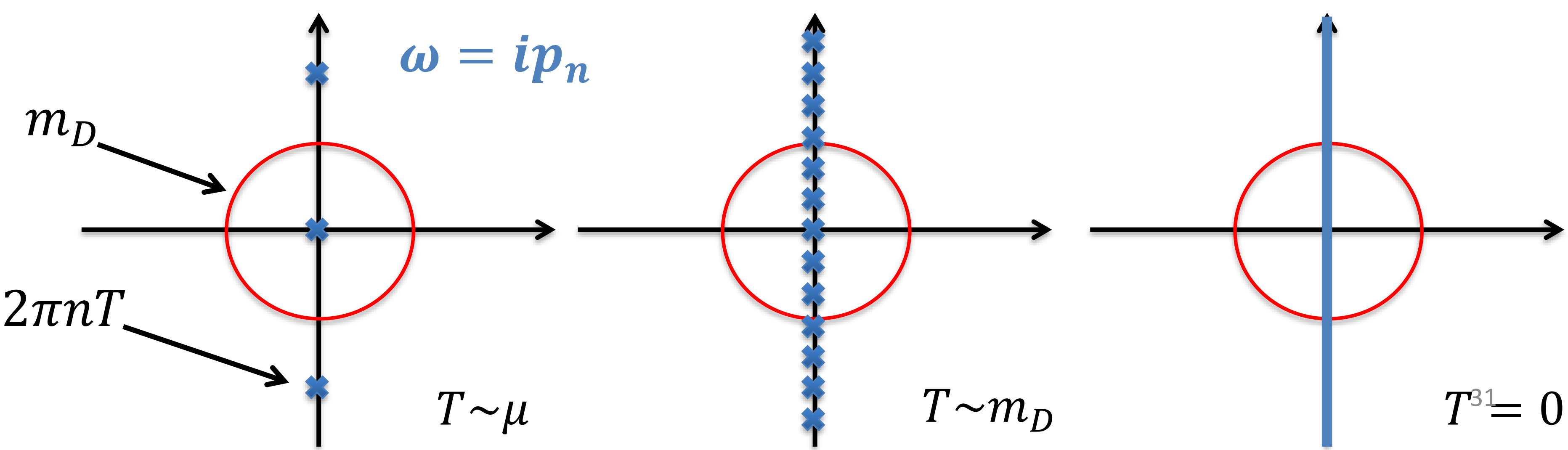

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$$p^{\rm res}_{\rm QCD} = p^{\rm naive}_{\rm QCD} + p^{\rm res}_{\rm DR} - p^{\rm naive}_{\rm DR} + p^{\rm res}_{\rm HTL} - p^{\rm naive}_{\rm HTL}$$

Effective theory for $n=0$ Matsubara mode. Necessary at $T\neq 0$; vanishes when $T\to 0$.

Effective description for $n\neq 0$ Matsubara modes **with $k\leq m_D$**. Dominates in the $T=0$ limit.

$$p_{\mathrm{QCD}}^{\mathrm{res}} = p_{\mathrm{QCD}}^{\mathrm{naive}} + p_{\mathrm{DR}} + p_{\mathrm{HTL}}$$

$$p_{\rm QCD}^{\rm res} = p_{\rm QCD}^{\rm naive} + p_{\rm DR} + p_{\rm HTL}$$

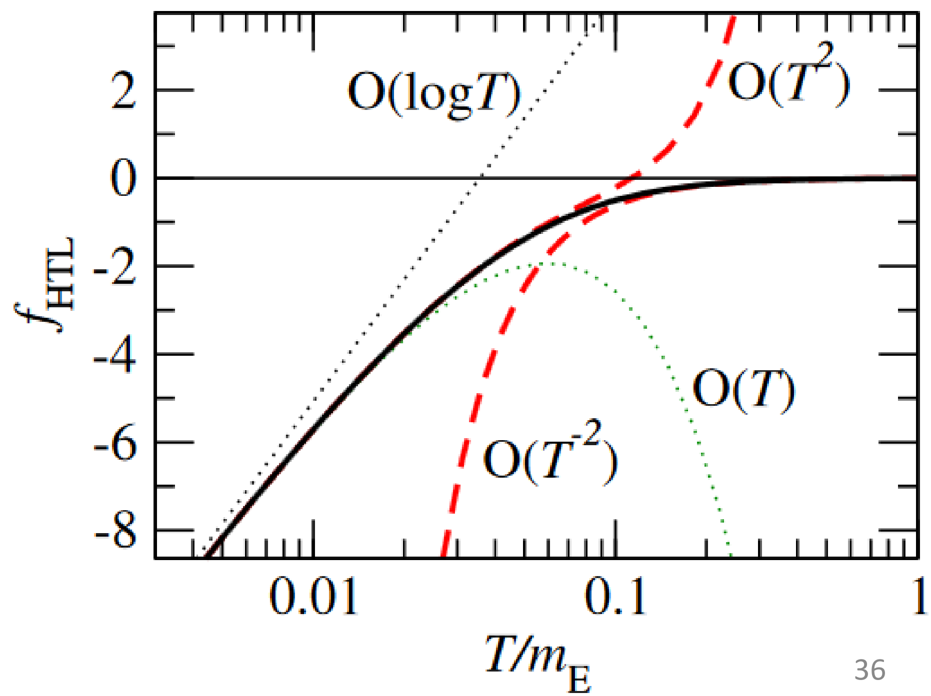
$$
\begin{aligned}
p_1 &= \frac{\pi^2}{45}\frac{T^4}{N_f}\sum_f \left\{ d_A + \left(\frac{7}{4} + 30\bar\mu^2 + 60\bar\mu^4\right) d_F \right\}, \\
p_2 &= -\frac{d_A}{144}\frac{T^4}{N_f}\sum_f \left\{ C_A + \frac{T_F}{2}\left(1+12\bar\mu^2\right)\left(5+12\bar\mu^2\right) \right\}, \\
p_3 &= \frac{d_A T^4}{144(4\pi)^2}\Bigg[\frac{1}{N_f}\sum_f \Bigg\{ C_A^2\left(\frac{12}{\epsilon} + \frac{194}{3}\ln\frac{\bar\Lambda}{4\pi T} + \frac{116}{5} + 4\gamma - \frac{38}{3}\frac{\zeta'(-3)}{\zeta(-3)} + \frac{220}{3}\frac{\zeta'(-1)}{\zeta(-1)}\right) \\
&+ C_A T_F \Bigg(12\left(1+12\bar\mu^2\right)\frac{1}{\epsilon} + \left(\frac{169}{3} + 600\bar\mu^2 - 528\bar\mu^4\right)\ln\frac{\bar\Lambda}{4\pi T} + \frac{1121}{60} + 8\gamma \\
&+ 2\left(127+48\gamma\right)\bar\mu^2 - 644\bar\mu^4 + \frac{268}{15}\frac{\zeta'(-3)}{\zeta(-3)} + \frac{4}{3}\left(11+156\bar\mu^2\right)\frac{\zeta'(-1)}{\zeta(-1)} \\
&+ 24\Big[52\,\aleph(3,z) + 144 i\bar\mu\,\aleph(2,z) + \left(17-92\bar\mu^2\right)\aleph(1,z) + 4i\bar\mu\,\aleph(0,z)\Big]\Bigg) \\
&+ C_F T_F\Bigg(\frac{3}{4}\left(1+4\bar\mu^2\right)\left(35+332\bar\mu^2\right) - 24\left(1-12\bar\mu^2\right)\frac{\zeta'(-1)}{\zeta(-1)} \\
&- 144\Big[12 i\bar\mu\,\aleph(2,z) - 2\left(1+8\bar\mu^2\right)\aleph(1,z) - i\bar\mu\left(1+4\bar\mu^2\right)\aleph(0,z)\Big]\Bigg) \\
&+ T_F^2\Bigg(\frac{4}{3}\left(1+12\bar\mu^2\right)\left(5+12\bar\mu^2\right)\ln\frac{\bar\Lambda}{4\pi T} + \frac{1}{3} + 4\gamma + 8\left(7+12\gamma\right)\bar\mu^2 + 112\bar\mu^4 - \frac{64}{15}\frac{\zeta'(-3)}{\zeta(-3)} \\
&- \frac{32}{3}\left(1+12\bar\mu^2\right)\frac{\zeta'(-1)}{\zeta(-1)} - 96\Big[8\,\aleph(3,z) + 12 i\bar\mu\,\aleph(2,z) - 2\left(1+2\bar\mu^2\right)\aleph(1,z) - i\bar\mu\,\aleph(0,z)\Big]\Bigg)\Bigg\} \\
&+ 288\,T_F^2\frac{1}{N_f^2}\sum_{f\,g}\Bigg\{2\left(1+\gamma\right)\bar\mu_f^2\bar\mu_g^2 - \Big[\aleph(3,z_f+z_g) + \aleph(3,z_f+z_g^*) \\
&+ 4i\bar\mu_f\left(\aleph(2,z_f+z_g) + \aleph(2,z_f+z_g^*)\right) - 4\bar\mu_g^2\,\aleph(1,z_f) - \left(\bar\mu_f+\bar\mu_g\right)^2\aleph(1,z_f+z_g) \\
&- \left(\bar\mu_f-\bar\mu_g\right)^2\aleph(1,z_f+z_g^*) - 4i\bar\mu_f\bar\mu_g^2\,\aleph(0,z_f)\Big]\Bigg\}\Bigg],
\end{aligned}
$$

$$p_{\rm QCD}^{\rm res} \quad = \quad p_{\rm QCD}^{\rm naive} + p_{\rm DR} + p_{\rm HTL}$$

$$\begin{aligned} p_{\rm DR}^{\rm res}/T \quad &= \quad \frac{d_A}{12\pi} m_{\rm E}^3 \\ &+ \quad \frac{d_A C_A}{(4\pi)^2} g_{\rm E}^2 m_{\rm E}^2 \left[-\frac{1}{4\epsilon} - \frac{3}{4} - \ln \frac{\bar{\Lambda}}{2m_{\rm E}} \right] \\ &+ \quad \frac{d_A C_A^2}{(4\pi)^3} g_{\rm E}^4 m_{\rm E} \left[-\frac{89}{24} - \frac{\pi^2}{6} + \frac{11}{6} \ln 2 \right] \\ &+ \quad \mathcal{O}(g^6 \ln g) \end{aligned}$$

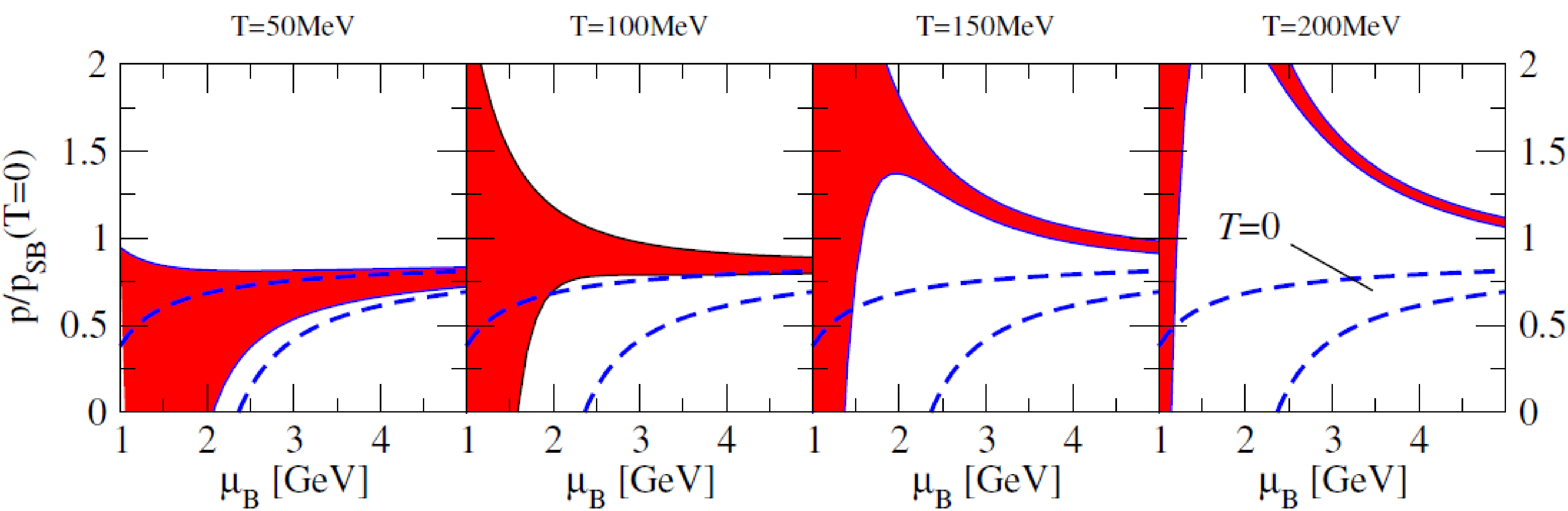
$$p_{\rm QCD}^{\rm res} \quad = \quad p_{\rm QCD}^{\rm naive} + p_{\rm DR} + p_{\rm HTL}$$

$$\begin{aligned} p_{\rm HTL}^{\rm corr} &= -d_A \sum\!\!\!\!\!\!\!\int_K' \left\{ \log\left[1 + \frac{\Pi_{\rm T}(K)}{K^2}\right] - \frac{\Pi_{\rm T}(K)}{K^2} + \frac{\Pi_{\rm T}^2(K)}{2K^4} \right\} \\ &- \frac{d_A}{2} \sum\!\!\!\!\!\!\!\int_K' \left\{ \log\left[1 + \frac{\Pi_{\rm L}(K)}{K^2}\right] - \frac{\Pi_{\rm L}(K)}{K^2} + \frac{\Pi_{\rm L}^2(K)}{2K^4} \right\} \\ &= \frac{d_A m_{\rm E}^4}{256\pi^2} f_{\rm HTL}(T/m_{\rm E}) \end{aligned}$$



$$\Omega(T,\mu_u,\mu_d,\mu_s,m_s) = -T\log\int \mathcal{D}\bar{\psi}\mathcal{D}\psi\mathcal{D}A_\mu e^{-\int d^3x\int_0^{1/T} d\tau\mathcal{L}_{\rm QCD}},$$
$$\mathcal{L}_{\rm QCD} = \frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu} + \bar{\psi}_i(\gamma_\mu D_\mu + m_i - \mu_i\gamma_0)\psi_i$$

New: Analytic result combining DR and HTL resummations → Small temperatures under control [Kurkela, AV, PRL 117, 042501]



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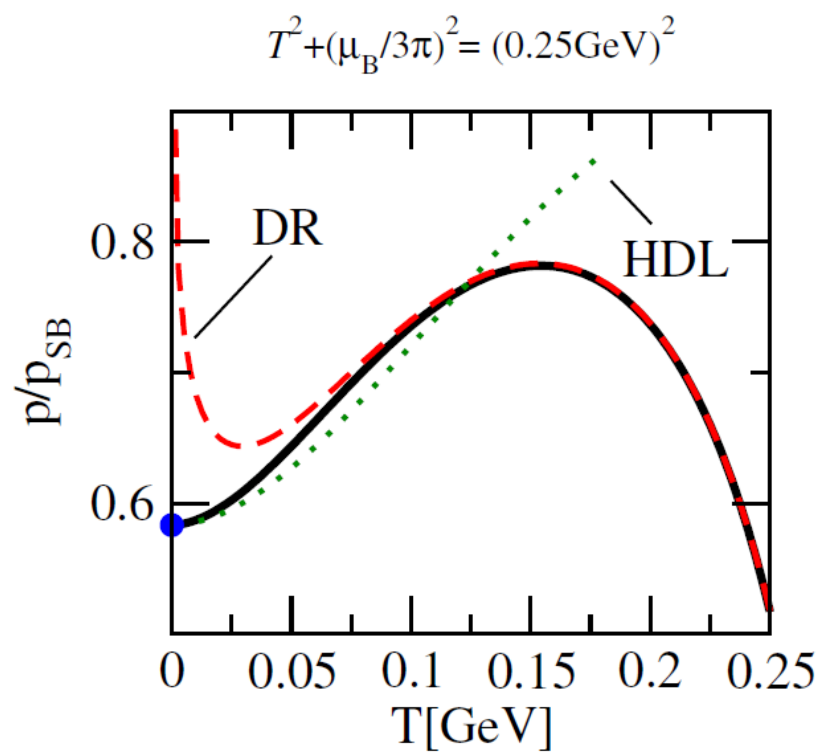

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Main challenge at the moment: Extending $T = 0$ EoS to full four-loop order

$$p^{\rm res}_{\rm QCD} = p^{\rm naive}_{\rm QCD}+p_{\rm soft}$$

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$$p^{\rm res}_{\rm QCD} \;=\; p^{\rm naive}_{\rm QCD}+p_{\rm soft}$$

- Need all four-loop vacuum diagrams in dimensional regularization but with no resummations
- Nice tool for pert. theory at $T = 0, \mu \neq 0$: Cutting rules relating 4d integrals to phase space integrals over on-shell amplitudes [Ghisoiu, Gorda, Kurkela, Romatschke, Säppi, AV, Nucl. Phys. B915 (2017)]
- Result will be a pure $O(g^6)$ contribution

$$\Omega(T,\mu_u,\mu_d,\mu_s,m_s) = -T\log\int \mathcal{D}\bar{\psi}\mathcal{D}\psi\mathcal{D}A_\mu e^{-\int d^3x\int_0^{1/T}d\tau\mathcal{L}_{\rm QCD}},$$

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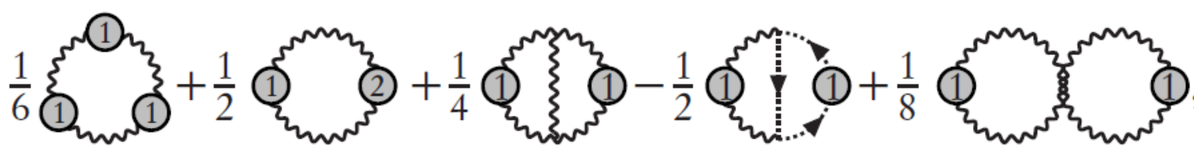
- To consistently determine the soft contributions, need to resum new classes of diagrams, and go up to two-loop level in the gluon self-energy

$-\Omega_{\rm (rings)}\big|_4 =$ 

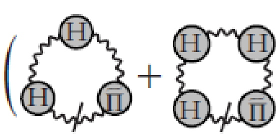
$$\Omega(T,\mu_u,\mu_d,\mu_s,m_s) \;=\; -T\log\int \mathcal{D}\bar{\psi}\mathcal{D}\psi\mathcal{D}A_\mu e^{-\int d^3x \int_0^{1/T} d\tau \mathcal{L}_{\rm QCD}},$$

$$\mathcal{L}_{\rm QCD} \;=\; \frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu} + \bar{\psi}_i(\gamma_\mu D_\mu + m_i - \mu_i\gamma_0)\psi_i$$

Main challenge at the moment: Extending $T=0$ EoS to full four-loop order

$$p_{\rm QCD}^{\rm res} \;=\; p_{\rm QCD}^{\rm naive} + p_{\rm soft}$$

- HTL not enough: need $\Pi_{\rm QCD}$ to higher order in ext. momentum

$$-\Omega_{\rm 1lr}\Big|_{g^6\log g} = \frac{1}{2}\Big($$  $$+\cdots-\Lambda_{\rm 1lr}\Big)$$

$$= \frac{d_A}{2}\sum_{i,j\in{\rm pol.}} e_i e_j \int_P \left[\frac{H_{1i}(\Phi)\,\widetilde{\Pi}_{1j}(\Phi)}{P^2 - H_{1i}(\Phi)} - \frac{H_{1i}(\Phi)\,\widetilde{\Pi}_{1j}(\Phi)}{P^2 + M_{\rm 1lr}^2}\right]$$

$$\Omega(T,\mu_u,\mu_d,\mu_s,m_s) = -T\log\int \mathcal{D}\bar{\psi}\mathcal{D}\psi\mathcal{D}A_\mu e^{-\int d^3x\int_0^{1/T}d\tau\mathcal{L}_{\rm QCD}},$$

$$\mathcal{L}_{\rm QCD} = \frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu} + \bar{\psi}_i(\gamma_\mu D_\mu + m_i - \mu_i\gamma_0)\psi_i$$

Main challenge at the moment: Extending $T = 0$ EoS to full four-loop order

$$p_{\rm QCD}^{\rm res} = p_{\rm QCD}^{\rm naive} + p_{\rm soft}$$

- Result: $O(g^6\ln^2 g)$, $O(g^6\ln g)$, and $O(g^6)$ contributions, of which first two almost there [Ghisoiu, Gorda, Kurkela, Romatschke, Säppi, AV, In preparation]

I. Neutron star Equation of State: Status and quark matter challenge
II. New developments in pQCD: Thermal effects and more loops
III. Strongly coupled quark matter from holography
IV. Final thoughts

Strongly coupled Super Yang-Mills theory a very successful toy model in heavy ion physics. However, in cold and dense QCD:

- Need (finite density of) fundamental flavors, while $N = 4$ SYM only contains adjoint fields
- $N_c = 3$ very important: Baryon structure, color superconductivity, ...
- Need to break SUSY and conformality & impose confinement

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Introduce N_f D7-branes to geometry – corresponds to introduction of N_f fundamental $N = 2$ hypermultiplets to gauge theory

- Theory possesses global $U(N_f) \sim SU(N_f) \times U(1)$ symmetry, with $U(1)$ identifiable with baryon symmetry $U(1)_B$
- Finite density: Turn on gauge field in D-brane worldvolume
- Probe limit $N_f \ll N_c$: Classical SUGRA with no backreaction

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Extrapolate to three colors in quark matter phase:

- In D3-D7 setup always in deconfined phase: Apply only for description of quark matter
- Ignoring quark pairing, large-N_c limit not necessarily a bad approximation for deconfined matter – works nicely at high T, with highly suppressed corrections
- For holographic color superconductivity, cf. talk by Mateos

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At the moment, no holographic dual to QCD exists. However, one can modify the vanilla setup in many ways:

- Break conformal invariance of $N = 4$ SYM in a controlled way → Additional (scalar) field on the gravity side
- Sakai-Sugimoto: Reduces to QCD at low energies
- Bottom-up models: Minimally coupled scalars with hand-picked potential, Veneziano limit IHQCD,...

Simple proof-of-principle: Strongly coupled $N = 2$ SYM matter with $N_c = N_f = 3$, at $T = 0$ [Hoyos, Jokela, Rodriquez, AV, PRL 117, 032501]:

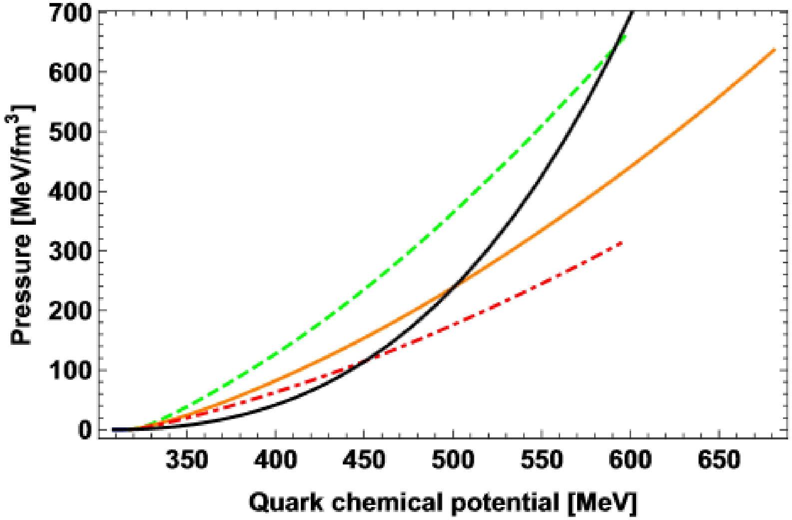
Simple proof-of-principle: Strongly coupled $N = 2$ SYM matter with $N_c = N_f = 3$, at $T = 0$ [Hoyos, Jokela, Rodriquez, AV, PRL 117, 032501]:

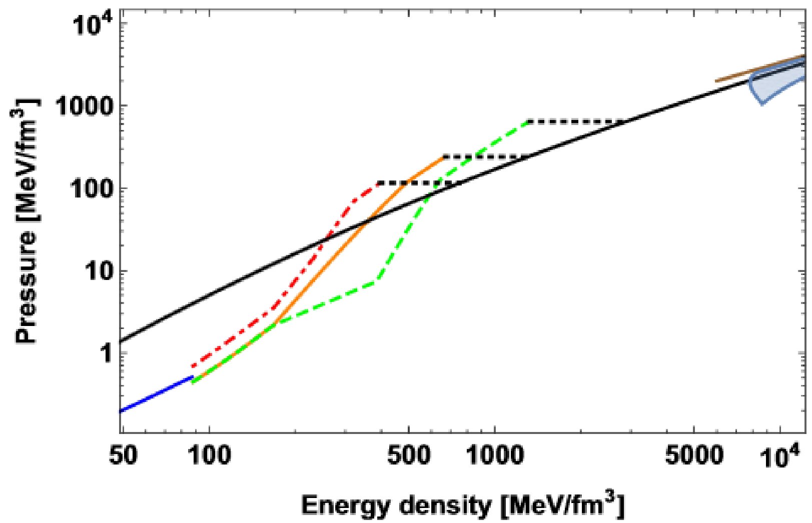
309 MeV from $p(\mu_B) = 0$

$$\varepsilon = 3p + m^2 \sqrt{\frac{N_c N_f}{4\gamma^3 \lambda_{YM}} p} = 3p + \frac{\sqrt{3} m^2}{2\pi} \sqrt{p}$$

$3\pi^2$
from correct UV limit

Matching to state-of-the-art nuclear matter EoSs from CET:





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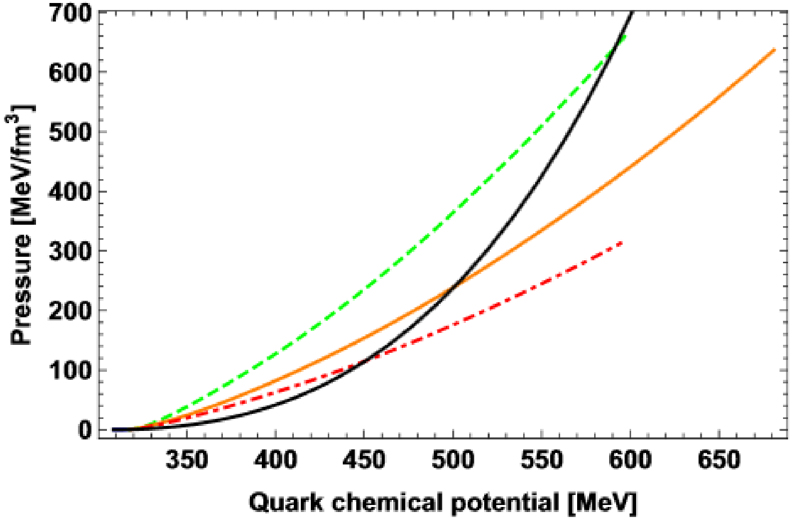
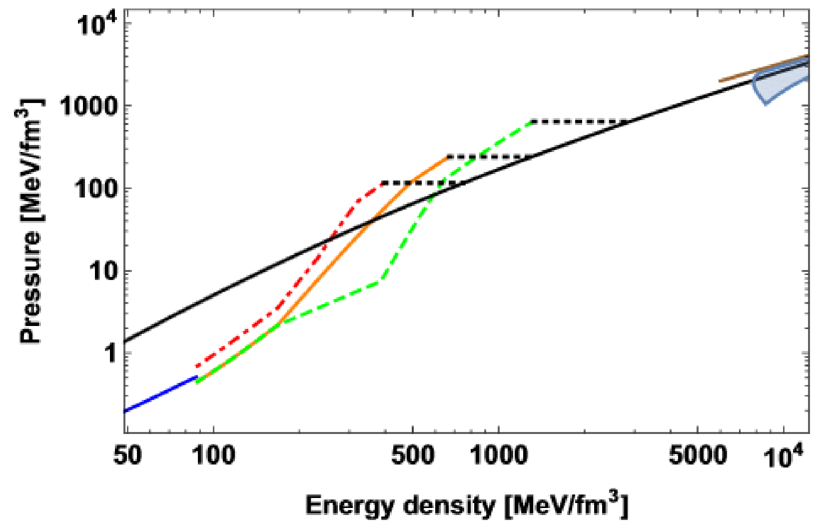
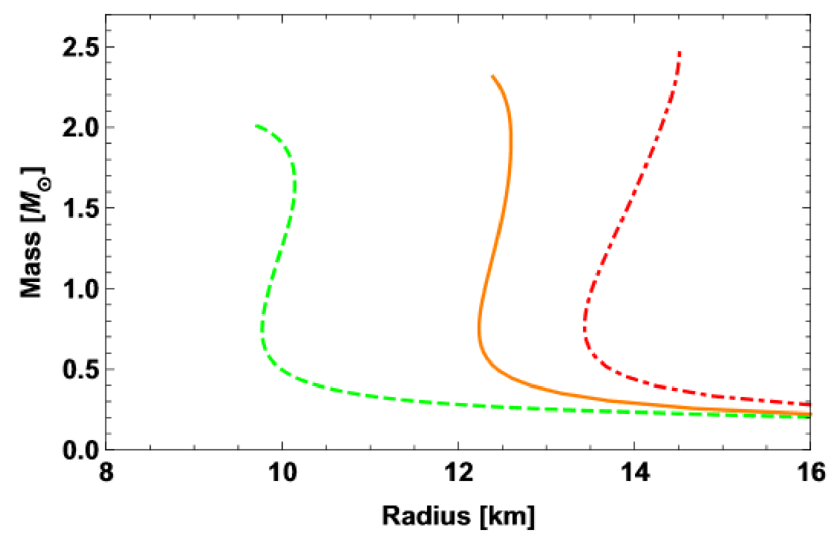




Predictions:

- Strong 1st order transitions at phenomenologically reasonable densities: 2.4-6.9n_s
- No quark matter inside stars (stars become unstable at transition)



Hope: Move beyond proof-of-principle level, and make holography (quantitatively?) relevant for neutron stars

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Lesson from holography in heavy ion physics: Method most powerful when

- Solving physics problems not feasible with traditional methods
- Discovering universal properties of strongly coupled systems

PHYSICAL REVIEW D **80**, 066003 (2009)

Bound on the speed of sound from holography

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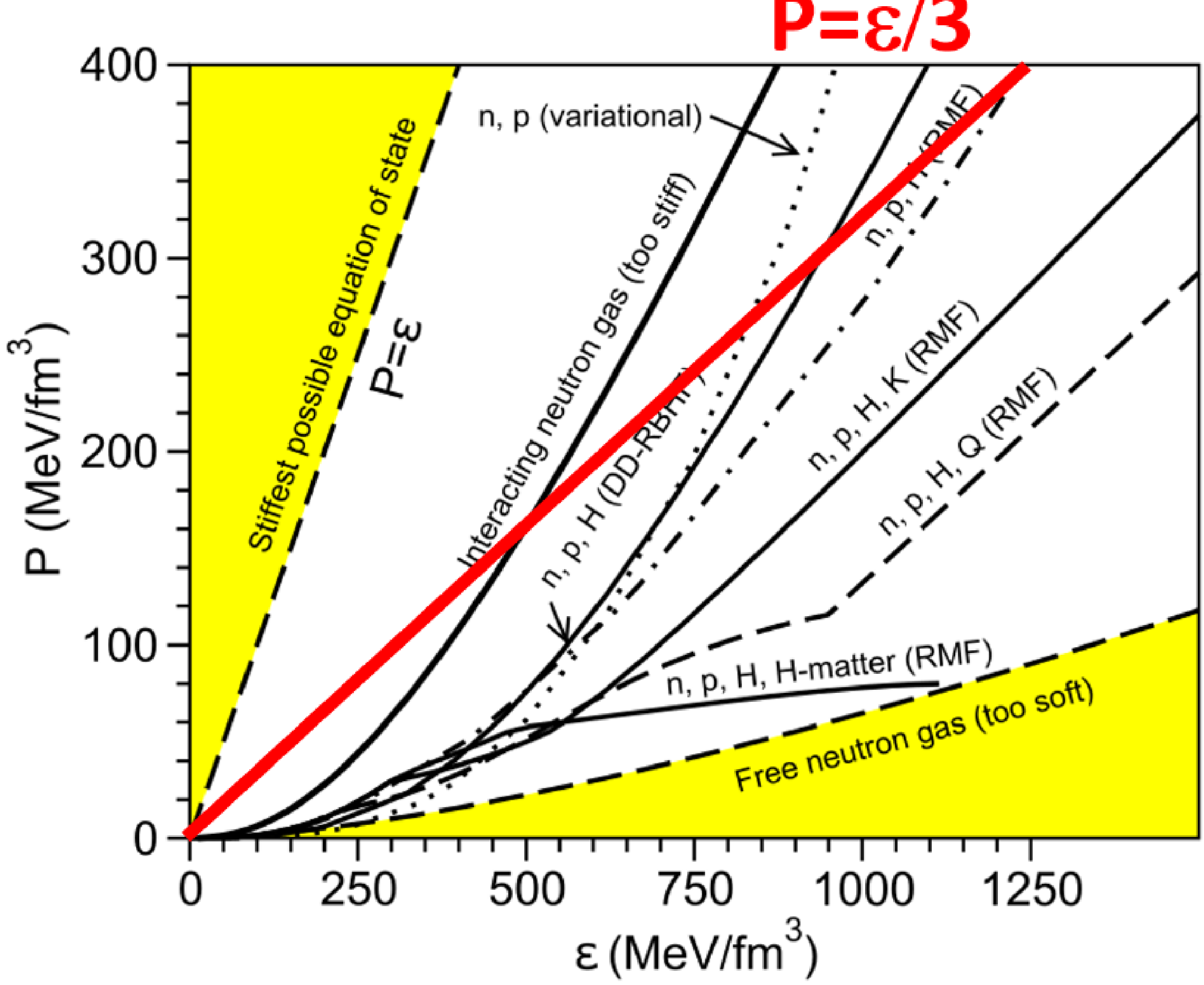
(Received 12 May 2009; published 3 September 2009)

We show that the squared speed of sound v_s^2 is bounded from above at high temperatures by the conformal value of $1/3$ in a class of strongly coupled four-dimensional field theories, given some mild technical assumptions. This class consists of field theories that have gravity duals sourced by a single-scalar field. There are no known examples to date of field theories with gravity duals for which v_s^2 exceeds $1/3$ in energetically favored configurations. We conjecture that $v_s^2 = 1/3$ represents an upper bound for a broad class of four-dimensional theories.

DOI: 10.1103/PhysRevD.80.066003 PACS numbers: 11.25.Tq, 11.15.Pg



P=ε/3
400
300
200
100
0
P (MeV/fm^3)
0
250
500
750
1000
1250
ε (MeV/fm^3)
Stiffest possible equation of state
P=ε
n, p (variational)
Interacting neutron gas (too stiff)
n, p, H (DD-RBHF)
n, p, H (RMF)
n, p, H, K (RMF)
n, p, H, Q (RMF)
n, p, H, H-matter (RMF)
Free neutron gas (too soft)

Unfortunately, bound can be (easily) violated even in asymptotic. AdS models [Ecker, Hoyos, Jokela, Rodriquez, AV, 1707.00521]:

1) Top-down: $N = 4$ SYM with massive gauginos at finite R charge density

$$e^{-1}\mathcal{L} = \frac{1}{4}R - \frac{1}{g^2}F_{\mu\nu}F^{\mu\nu} + \frac{1}{4}(\partial_\mu\phi)^2 + \frac{1}{2}\sinh^2\left(\frac{\phi}{\sqrt{2}}\right)(\partial_\mu\theta - 2A_\mu)^2 - \frac{V(\phi)}{4}$$

$$V(\phi) = -\frac{3g^2}{4}\left(3 + \cosh(\sqrt{2}\phi)\right)$$

2) Bottom-up: Einstein-Maxwell minimally coupled to scalar field with potential

$$\mathcal{V}(\Phi) = -\frac{12}{L^2} + m^2|\Phi|^2 + \frac{V_4}{2L^2}\left(|\Phi|^2\right)^2$$

$$\Phi = \tanh\left(\frac{\phi}{2\sqrt{2}}\right)e^{i\theta}$$

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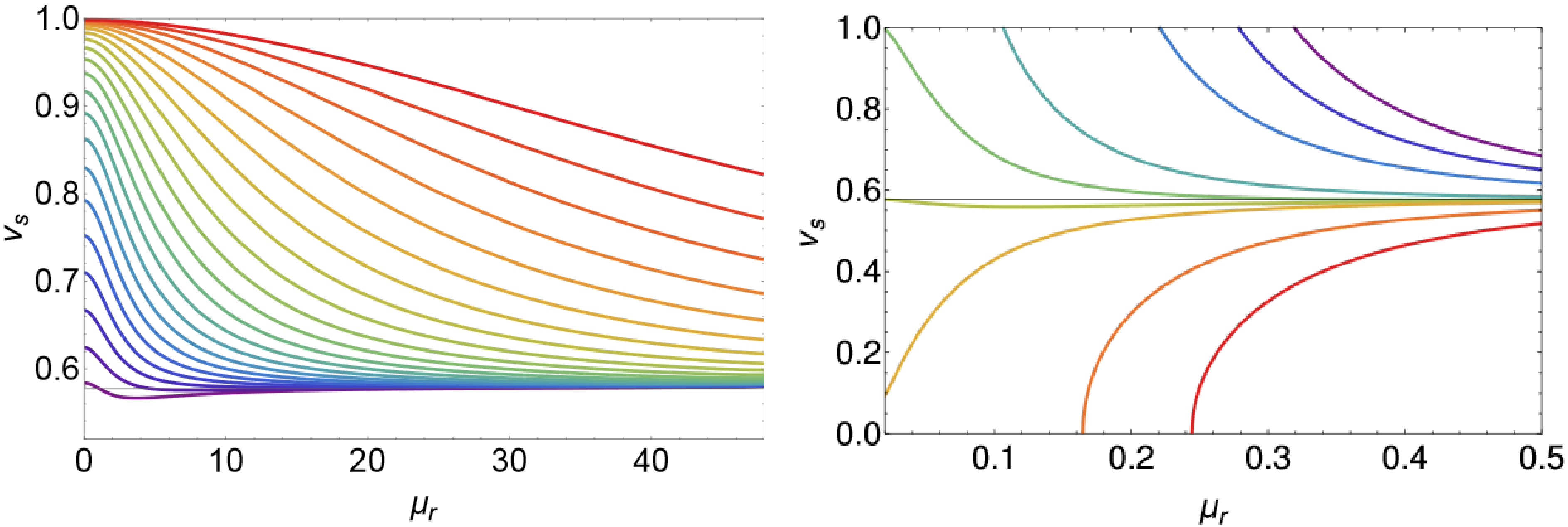

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First attempt unsuccessful, but many alternatives exist: transport properties a particularly promising avenue

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Final thoughts

1. Identifying properties – and identity – of neutron star matter from first principles a hard but perhaps feasible task: Interplay of theory and observations important

2. Particle theorists' main challenge: Fill the gap between known EoSs of nuclear and quark matter – here input from high density limit surprisingly valuable

3. Future: Steady progress with standard tools (CET, pQCD); perhaps surprising leaps with new approaches